NOTE: COLLATERAL CONSTRAINTS, EMERGING MARKET BUSINESS CYCLES, AND R&D ACTIVITIES

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[QUESTION] Can the collateral constraint in R&D activities account for the features of the emerging economy?

- Emerging market business cycles exhibit strongly counter-cyclical current accounts, consumption volatility that exceeds income volatility, and " sudden stops " in capital inflows. (Auiar and Gopnath, 2006)

[APPROACH]

- We construct a dynamic general equilibrium model with R&D activities à la Romer (1990).
- In our model, the working capital for the production of effective labor is subject to the collateral constraint because of *the lack of a commitment* problem à la Kiyotaki and Moore (1997).
- Consider the collateral constraints are binding in the emerging economies while they are not binding in the developed ones.
- R&D technology, δ_A, tightness of collateral constraint, ϕ, and Hicks neutral statonaty technology level, ζ are exogenous and AR(1) process.
- Investigate the impulse responses to R&D technology and TFP growth.

[RESULTS]

- The impulse responses of binding and not-binding constraints are almost same. The labor inputs' resonses of binding case are bigger than those of binding case. (Our hypothesis is rejected.)

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1. THE MODEL

HOUSEHOLDS:

$$\max_{c_t,n_t,i_t,k_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{[c_t^{1-\xi}(1-n_t)^{\xi}]^{1-\varepsilon}}{1-\varepsilon}, \tag{1}$$

$$\text{s.t. } c_t + i_t = w_t n_t + r_{k,t} k_{t-1} + \pi_t, \tag{2}$$

$$k_t = (1-\delta_k) k_{t-1} + i_t. \tag{3}$$

EFFECTIVE LABOR PRODUCER:

$$\max_{z_{A,t},z_{y,t},n_{A,t},n_{y,t},a_t} E_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \Big[\hat{w}_t(z_{A,t}+z_{y,t}) - w_t(n_{A,t}+n_{y,t}) + (r_{a,t)} + q_t)a_{t-1} - q_t a_t\Big], \tag{4}$$

$$\text{s.t. } z_{A,t} = n_{A,t}, \tag{5}$$

$$z_{y,t} = n_{y,t}, \tag{6}$$

$$w_t(n_{A,t}+n_{y,t}) = \phi q_t a_{t-1}. \tag{7}$$

FINAL GOODS FIRMS:

$$\max_{y_t,z_{y,t},a_{t-1},x_t} \Big[y_t - \hat{w}_t z_{y,t} - r_{a,t} a_{t-1} - \int_0^{A_{t-1}} p_t(i) x_t(i) di\Big], \tag{8}$$

$$\text{s.t. } y_t = \zeta z_{y,t}^{(1-\gamma)(1-\alpha)} a_{t-1}^{(1-\gamma)\alpha} \Big[\int_0^{A_{t-1}} x_t(i) di\Big]. \tag{9}$$

INTERMEDIATE GOODS FIRMS:

$$P_{A,t}(i) = \frac{1}{\beta^t \lambda_t} E_t \sum_{s=t}^{\infty} \beta^s \lambda_s \Big[p_s(i) x_s(i) - \eta r_{k,s} x_s(i)\Big]. \tag{10}$$

R&D FIRMS:

$$\max_{z_{A,t}} \sum_{t=0}^{\infty} \beta^t \lambda_t \Big[P_{A,t}\delta_A A_t - \hat{w}_t\Big] z_{A,t}. \tag{11}$$

MARKET CLEARING CONDITIONS:

$$c_t + i_t = y_t, \tag{12}$$

$$n_{A,t} + n_{y,t} = n_t, \tag{13}$$

$$a_t = 1, \tag{14}$$

$$A_t = (\delta_A z_{A,t} + 1) A_{t-1}. \tag{15}$$

2. EQUILIBRIUM SYSTEM

The equilibrium system *with binding collateral constraint* is

$$c_t^{(1-\xi)(1-\varepsilon)-1}(1-n_t)^{\xi(1-\varepsilon)} = \lambda_t, \tag{16}$$

$$w_t = \frac{\xi}{1-\xi} \cdot \frac{c_t}{1-n_t}, \tag{17}$$

$$1 = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1 + r_{k,t+1} - \delta_k)\right], \tag{18}$$

$$\hat{w}_t = w_t(1+\theta_t), \tag{19}$$

$$q_t = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left\{r_{a,t+1} + q_{t+1} + \phi\theta_{t+1}q_{t+1}\right\}\right], \tag{20}$$

$$w_t n_t = \phi q_t, \tag{21}$$

$$\hat{w}_t = (1-\gamma)(1-\alpha)\frac{y_t}{n_{y,t}}, \tag{22}$$

$$r_{a,t} = (1-\gamma)\alpha y_t, \tag{23}$$

$$p_t = \gamma n_{y,t}^{(1-\gamma)(1-\alpha)} x_t^{\gamma-1}, \tag{24}$$

$$x_t = \left[\frac{\gamma^2}{\eta r_{k,t}}\right]^{\frac{1}{1-\gamma}} n_{y,t}^{1-\alpha}, \tag{25}$$

$$\hat{w}_t = P_{A,t}\delta_A A_{t-1}, \tag{26}$$

$$n_{y,t} + n_{A,t} = n_t, \tag{27}$$

$$c_t + i_t = y_t, \tag{28}$$

$$A_t = (\delta_{A,t}n_{A,t} + 1)A_{t-1}, \tag{29}$$

$$y_t = \zeta\frac{A_{t-1}^{1-\gamma}}{\eta^\gamma} n_{y,t}^{(1-\gamma)(1-\alpha)} k_{t-1}^\gamma, \tag{30}$$

$$k_{t-1} = \eta x_t A_{t-1}, \tag{31}$$

$$k_t = (1-\delta_k)k_{t-1} + i_t, \tag{32}$$

$$P_{A,t} = E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}P_{A,t+1} + p_t x_t - \eta r_{k,t} x_t\right]. \tag{33}$$

There are 18 equations and 18 variables:

$$c_t,\ n_t,\ n_{A,t},\ n_{y,t},\ \theta_t,\ \lambda_t,\ q_t,\ k_t,\ i_t,$$
$$w_t,\ \hat{w}_t,\ r_{k,t},\ r_{a,t},\ p_t,\ x_t,\ y_t,\ P_{A,t},\ A_t.$$

For the de-trended system, we introduce the following notations:

$$\tilde{\psi}_t := \frac{\psi_t}{A_{t-1}}, \tag{34}$$

for $\psi_t = c_t, q_t, k_t, i_t, w_t, r_{a,t}, y_t$,[1] and

$$\tilde{\lambda}_t := \frac{\lambda_t}{A_{t-1}^{(1-\xi)(1-\varepsilon)-1}}. \tag{35}$$

[1]Note that the variety of goods (or TFP) in the period t is determined at the previous period: A_{t-1}.

The de-trended system is

$$\tilde{c}_t^{(1-\xi)(1-\varepsilon)-1}(1-n_t)^{\xi(1-\varepsilon)} = \tilde{\lambda}_t, \tag{36}$$

$$\tilde{w}_t = \frac{\xi}{1-\xi}\cdot\frac{\tilde{c}_t}{1-n_t}, \tag{37}$$

$$1 = \beta E_t\left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}(1+g_t)^{(1-\xi)(1-\varepsilon)-1}(1+r_{k,t+1}-\delta_k)\right], \tag{38}$$

$$\tilde{q}_t = \beta E_t\left[\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}(1+g_t)^{(1-\xi)(1-\varepsilon)}\left\{\tilde{r}_{a,t+1}+\tilde{q}_{t+1}+\phi\theta_{t+1}\tilde{q}_{t+1}\right\}\right], \tag{39}$$

$$\tilde{w}_t n_t = \phi\tilde{q}_t, \tag{40}$$

$$\tilde{w}_t(1+\theta_t) = (1-\gamma)(1-\alpha)\frac{\tilde{y}_t}{n_{y,t}}, \tag{41}$$

$$\tilde{r}_{a,t} = (1-\gamma)\alpha\tilde{y}_t, \tag{42}$$

$$p_t = \gamma n_{y,t}^{(1-\gamma)(1-\alpha)}x_t^{\gamma-1}, \tag{43}$$

$$x_t = \left[\frac{\gamma^2}{\eta r_{k,t}}\right]^{\frac{1}{1-\gamma}} n_{y,t}^{1-\alpha}, \tag{44}$$

$$\tilde{w}_t(1+\theta_t) = P_{A,t}\delta_A, \tag{45}$$

$$n_{y,t} + n_{A,t} = n_t, \tag{46}$$

$$\tilde{c}_t + \tilde{i}_t = \tilde{y}_t, \tag{47}$$

$$g_t = \delta_{A,t}n_{A,t}, \tag{48}$$

$$\tilde{y}_t = \zeta\frac{1}{\eta^\gamma}n_{y,t}^{(1-\gamma)(1-\alpha)}\left[\frac{\tilde{k}_{t-1}}{1+g_{t-1}}\right]^\gamma, \tag{49}$$

$$\tilde{k}_{t-1} = \eta x_t(1+g_{t-1}), \tag{50}$$

$$\tilde{k}_t = \frac{1-\delta_k}{1+g_{t-1}}\tilde{k}_{t-1} + \tilde{i}_t, \tag{51}$$

$$P_{A,t} = E_t\left[\beta\frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t}(1+g_t)^{(1-\xi)(1-\varepsilon)-1}P_{A,t+1} + p_t x_t - \eta r_{k,t}x_t\right]. \tag{52}$$

where

$$g_{t+1} := \ln\left(\frac{A_{t+1}}{A_t}\right). \tag{53}$$

On the balanced growth path, this system becomes

$$\tilde{c}^{(1-\xi)(1-\varepsilon)-1}(1-n)^{\xi(1-\varepsilon)} = \tilde{\lambda}, \tag{54}$$

$$\tilde{w} = \frac{\xi}{1-\xi}\cdot\frac{\tilde{c}}{1-n}, \tag{55}$$

$$1 = \beta(1+g)^{(1-\xi)(1-\varepsilon)-1}(1+r_k-\delta_k), \tag{56}$$

$$\tilde{q} = \beta(1+g)^{(1-\xi)(1-\varepsilon)}\left\{\tilde{r}_a+\tilde{q}+\phi\theta\tilde{q}\right\}, \tag{57}$$

$$\tilde{w}n = \phi\tilde{q}, \tag{58}$$

$$\tilde{w}(1+\theta) = (1-\gamma)(1-\alpha)\frac{\tilde{y}}{n_y}, \tag{59}$$

$$\tilde{r}_a = (1-\gamma)\alpha\tilde{y}, \tag{60}$$

$$p = \gamma n_y^{(1-\gamma)(1-\alpha)}x^{\gamma-1}, \tag{61}$$

$$x = \left[\frac{\gamma^2}{\eta r_k}\right]^{\frac{1}{1-\gamma}} n_y^{1-\alpha}, \tag{62}$$

$$\tilde{w}(1+\theta) = P_A\delta_A, \tag{63}$$

$$n_y + n_A = n, \tag{64}$$

$$\tilde{c} + \tilde{i} = \tilde{y}, \tag{65}$$

$$g = \delta_A n_A, \tag{66}$$

$$\tilde{y} = \zeta\frac{1}{\eta^\gamma}n_y^{(1-\gamma)(1-\alpha)}\left[\frac{\tilde{k}}{1+g}\right]^\gamma, \tag{67}$$

$$\tilde{k} = \eta x(1+g), \tag{68}$$

$$\left[1-\frac{1-\delta_k}{1+g}\right]\tilde{k} = \tilde{i}, \tag{69}$$

$$\left[1-\beta(1+g)^{(1-\xi)(1-\varepsilon)-1}\right]P_A = (p-\eta r_k)x. \tag{70}$$

There are 17 equations and 17 variables:

$$\tilde{c},\ n,\ n_A,\ n_y,\ \theta,\ \tilde{\lambda},\ \tilde{q},\ \tilde{k},\ \tilde{i},$$
$$\tilde{w},\ r_k,\ \tilde{r}_a,\ p,\ x,\ \tilde{y},\ P_A,\ g.$$

EQUILIBRIUM SYSTEM WITHOUT COLLATERAL CONSTRAINT : If the collateral constraint is not binding, $\theta_t = 0$. The equilibrium system is (16) - (33) except (21) with $\theta_t = 0$. The de-trended system and the balanced growth path system are analogues of those under collateral constraints except (40) and (58) with $\theta_t = 0$ and $\theta = 0$, respectively.

REFERENCES

[1] Auiar, M. and G. Gopnath (2006) "Emerging market business cycles: the cycle is the trend," *Journal of Political Economy* 115, 69-102.

[2] Kiyotaki, N. and J. Moore (1997) "Credit cycles," *Journal of Political Economy* 105, 211-48.

[3] Romer, P. M. (1990) "Endogenous technological change," *Journal of Political Economy* 98, 71-102.

Table 1: Parameter Values

parameter	symbol	value
discount factor	β	.99
curvature of utility function	ε	.6
weight of leasure in utility	ξ	.6
relative share of land to labor in production	α	.01
share of capital in production	γ	.36
steady-state technology growth	$\bar{g}$	.01/4
depreciation rate of capital	δ_k	.025
steady-state tightness of collateral constraint	ϕ	.1
persistence of R&D technology growth	ρ_{δ_A}	.95
persistence of production technology level	ρ_ζ	.95
persistence of tightness of collateral constraint	ρ_ϕ	.95

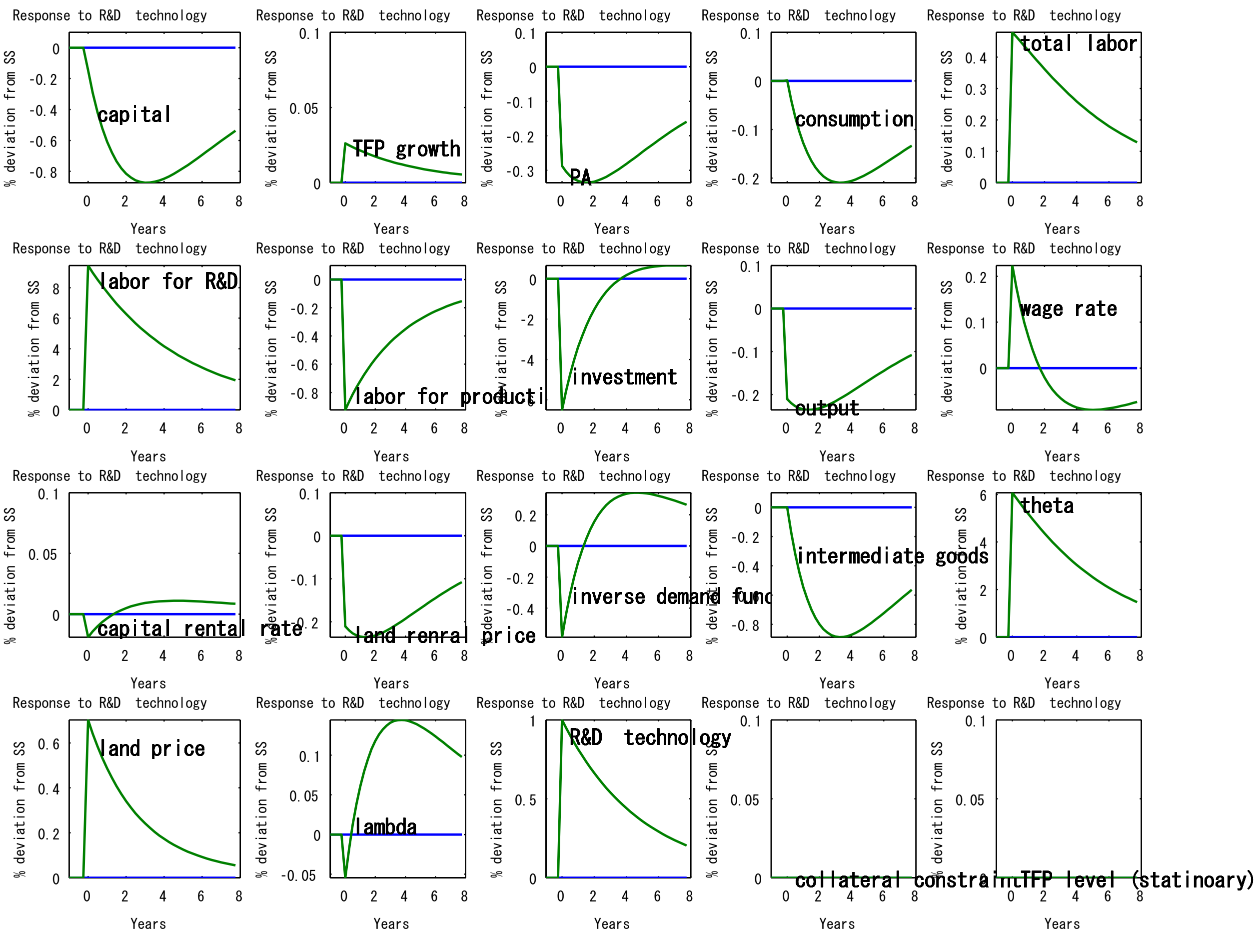

Figure 1: Impulse Response to R&D Technology Level; Binding Collateral Constraint

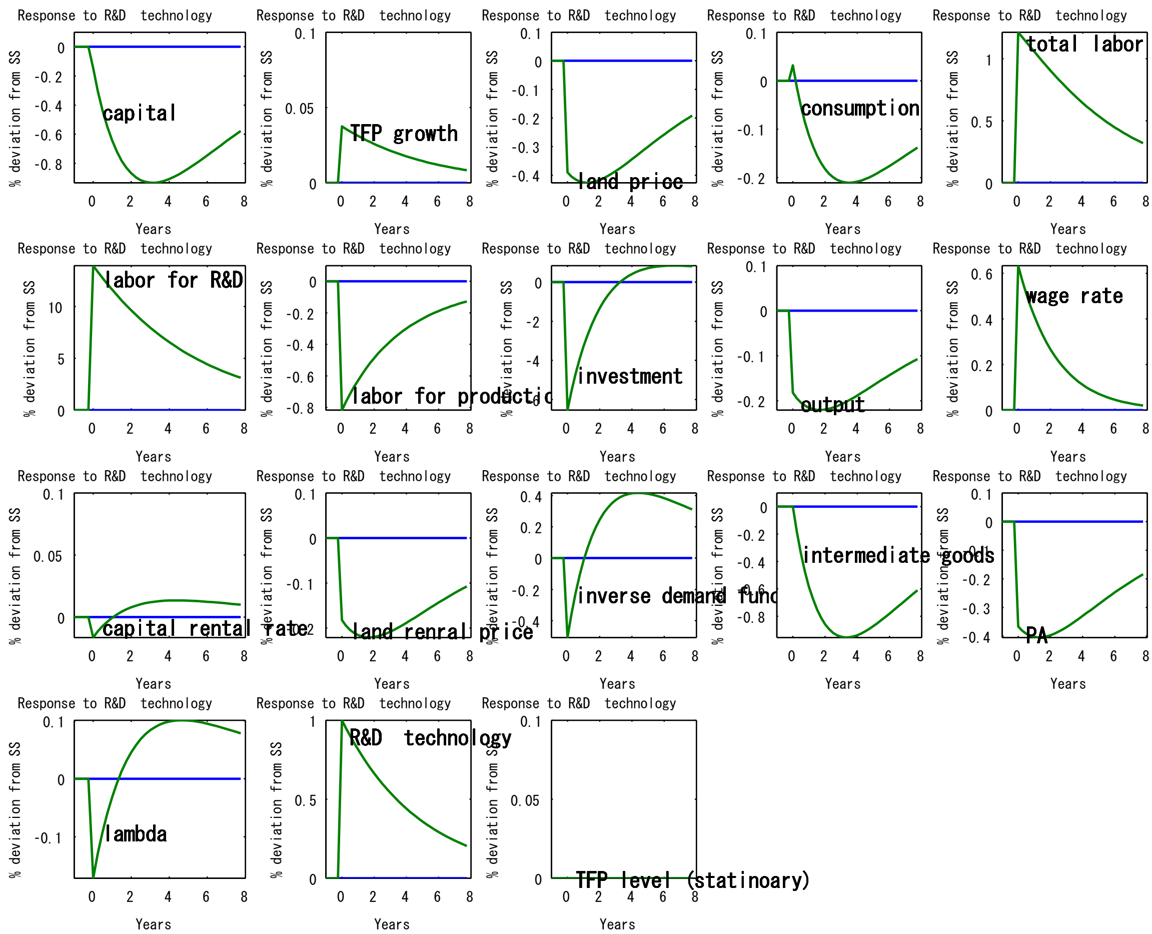

FIGURE 2: IMPULSE RESPONSE TO R&D TECHNOLOGY LEVEL; NO COLLATERAL CONSTRAINT

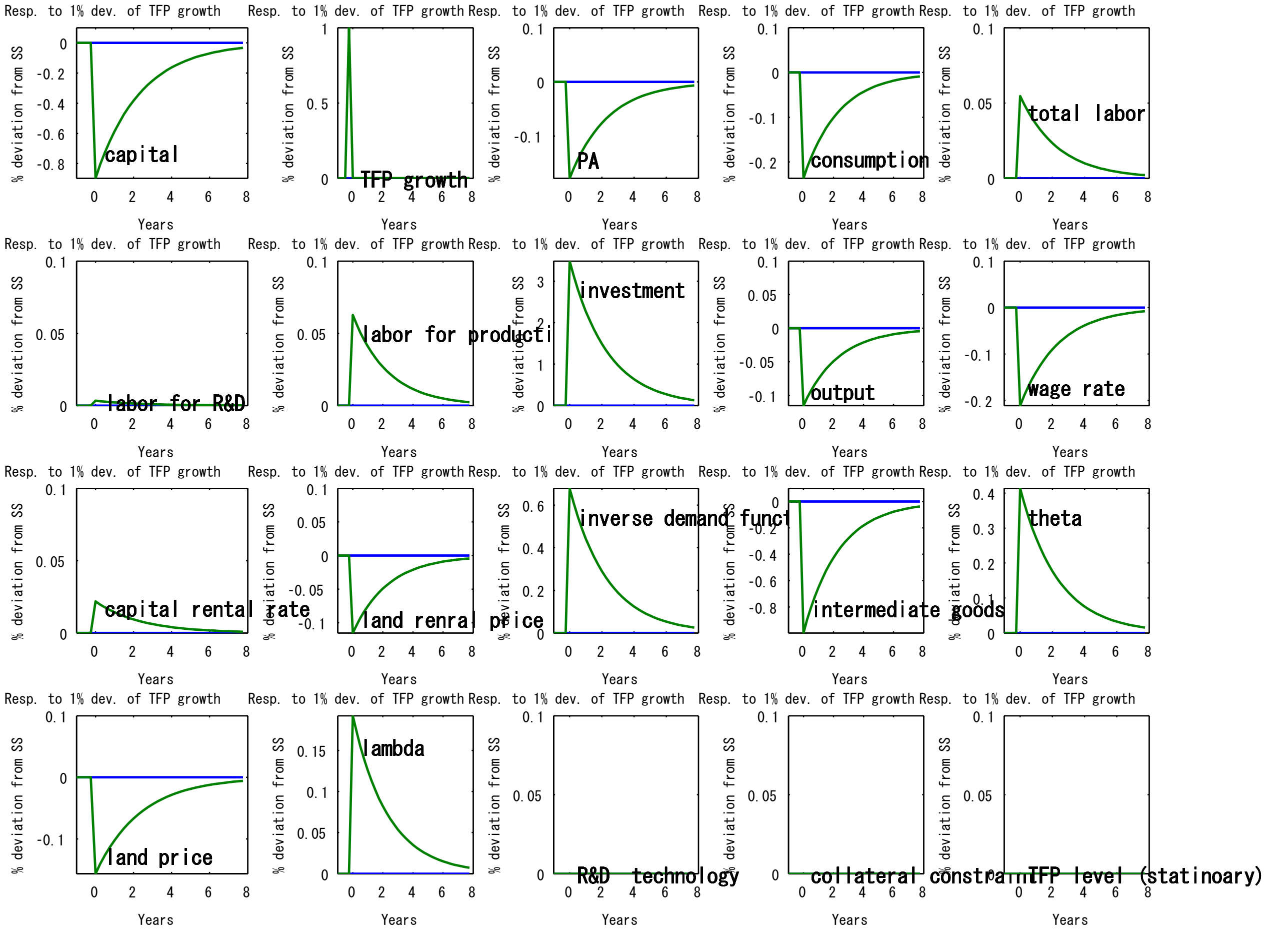

FIGURE 3: IMPULSE RESPONSE TO TFP GROWTH; BINDING COLLATERAL CONSTRAINT

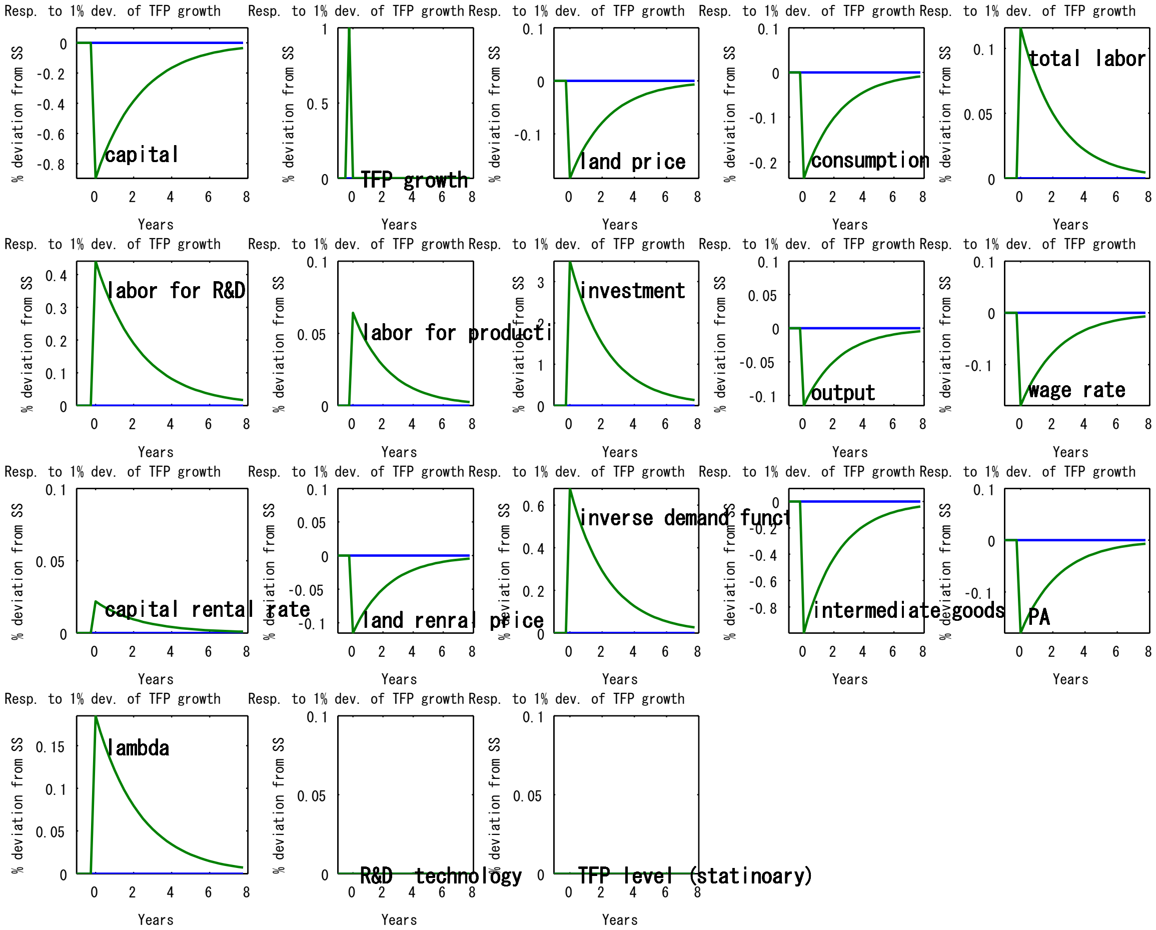

FIGURE 4: IMPULSE RESPONSE TO TFP GROWTH; NO COLLATERAL CONSTRAINT