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Debt Overhang as a Delayed Penalty

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This paper proposes a simple general equilibrium model of depression due to a financial shock and subsequent corporate-debt overhang. The “debt overhang problem” in the literature is the inefficiency caused by the penalty to the defaulters, such as the deterrence of new investments or the bankruptcy of debtors. This type of inefficiency is temporary and disappears as the new entrepreneurs enter. In this paper, we examine the inefficiency caused by the delay of the punishment. If the defaults are due to a macroeconomic financial shock, the creditors may hesitate to bankrupt debtors since the defaults are not due to their moral hazard. However, once the creditor rationally decides to keep a debtor unpunished, the trading partners may distrust the debtor’s commitment to the “relation-specific” investments since the creditor may bankrupt the debtor at any time. If the suspicion prevails, chains of production by specialized suppliers are broken down, and the economy is trapped in a persistent stagnation.
1 Introduction

The last decade of the twentieth century is often mentioned as “the Lost Decade” for the Japanese economy. Conventional wisdom is that the delay of the disposal of non-performing loans which mount to more than 10% of GDP has caused the persistent stagnation in Japan. It is regarded as a big puzzle that the Japanese economy did not recover although the short-term rate of interest has been kept nearly 0% for 4 years and there have been successive fiscal expansions resulted in the public debt of 120% of GDP. I present a simple general equilibrium model in which the delay of the disposal of non-performing loans creates multiple equilibria one of which is realized according to people’s expectations.

The focus of the argument is the bargaining problems due to “incomplete contracts” and “highly specific relations” between firms in a chain of production. The importance of “specificity” in macroeconomics is pointed out by Caballero and Hammour (1996) and is applied in a recent macroeconomic study by Blanchard and Kremer (1997). According to Blanchard and Kremer, a relation is called “specific” if there is a joint surplus to the parties from dealing with each other rather than taking their next best alternatives. One example of a specific relation is “keiretsu” between a major car maker and its specialized suppliers.

The story goes as follows. Under the normal circumstances, the long-term relations and the consideration about “good reputations” in the market guarantee that the commitments by firms to the specific relations are fulfilled. The complex chains of production operate fine. Suppose that a large-scaled financial shock occurs and that it makes many firms default. The creditors may hesitate to bankrupt the debtors, since the debtors are not fully responsible for their defaults. If a creditor decides not to bankrupt the defaulter, the firm is kept operating. However, the trading partners of the firm suspect
that the creditor may bankrupt the firm at any time, and that the commit-
ment of the firm to the specific relation would be cancelled by the creditor.
Then the other firms in the production chain would incur losses by com-
mittng to the specific relation, since the debtor may be bankrupted leaving
the production unfinished and their commitment may become worthless. If
this suspicion prevails in the economy, firms lose confidence in committing
to a specific relation with strangers. Thus the chains of production and the
division of labor between firms shrink, and firms undertake less productive
activities that they can carry out in narrow circles, leading the economy to
the persistent stagnation. The decline of observed productivity leads the
decline of asset prices and strengthen the pessimism.

Figure 1 shows the total volume of the bills and checks clearings and the
domestic fund transfer through the inter-bank computer network. This figure
indicates the sharp contraction of business transactions. It may imply that
the networks of production and the division of labor between firms have been
damaged continuously in 1990s.

1.1 Literature

The recent studies in macroeconomics emphasizes the importance of the
credit constraints caused by information asymmetry and the principal-agent
problems. Kiyotaki and Moore (1997) examine the case where the principal-
agent problem limits the amount borrowed by a firm. This limitation ampli-
fies a productivity shock and generates cyclical movements of output. Their
result is that the \textit{ex ante} constraint on the availability of money causes ineff-
ciency. The “financial accelerator” models treat this problem (See, for ex-
ample, Bernanke and Gertler[1989]; Bernanke, Gertler and Gilchrist[1996]).

The consideration about the principal-agent problem also produces the
social norm which works as the \textit{ex post} penalty for the moral hazard. It is the
priority of the existing debt to the new debt. To prevent the debtors from
shirking, there is the practice that a failed debtor cannot get new money unless he proves that he can repay the existing debt. Thus once the debtor failed, he is often forced to stop the business even when its going-concern value is positive. While this penalty to the defaulters guarantees the debtor’s diligence *ex ante*, it causes the *ex post* inefficiency because a valuable business has to be stopped in some case. This inefficiency is called the “debt overhang problem” (Hart[1995]). If a failure of a debtor is idiosyncratic, the debt overhang problem does not make a macroeconomic problem. However, a financial shock such as the asset market collapse makes many debtors distressed simultaneously. Lamont (1995) shows that the simultaneous “debt overhang problems” may generate a stagnant equilibrium by changing macroeconomic expectations.

Note that the financial accelerator models and the Lamont model illustrate the business cycles rather than a persistent stagnation. This is obvious for the financial accelerator models, since the credit constraint does not change the equilibrium but just amplifies the fluctuations from the optimal equilibrium. We can also reason that the inefficiency of the Lamont model, which is a two-period model, cannot continue for a long period. It is because the defaulters are eventually bankrupted by the inefficiency of “debt overhang” such as the halt of operations or the deterrence of new investments. This inefficiency is the “punishment” to the defaulters for moral hazard. The defaulters eventually exit, and then the inefficiency no longer persists. The entry of new entrepreneurs leads to the recovery of the economy. Thus, the recession due to debt overhang in the Lamont model must be temporary.

In the analysis of a persistent stagnation or even standard business cycles, we need to see another inefficiency of debt overhang, i.e., the inefficiency due to the *delay* or “*unfinishedness*” of the penalty, which is treated in the model of this paper.
1.2 Multiple Equilibria due to Debt Overhang

In this section, I will briefly outline the multiplicity of the equilibria due to debt overhang.

The mechanism of the decline of productivity in our model is similar to the model of “disorganization” in the former Soviet Union by Blanchard and Kremer (1997). In their model, the inefficiency due to bargaining problems arises as the coercive power of the central planner is weakened. It is because, in the former Soviet Union, only the coercive power has guaranteed the firms’ commitments to the specific relations. On the other hand, in our model, the control right of the firm is transferred from the management of the firm to the creditor when the default occurs. This shift of the control right makes the debtor’s commitment untrustworthy for its trading partners. Suppose that a good is produced according to either N-Technology or S-Technology. N-Technology is a Leontief type technology in which 2 firms produce different intermediate goods from the labor input and assemble them into the final good. The production function of N-Technology is

\[ y = V(m_i, m_j) = 2 \times \min\{m_i, m_j\}, \]

and

\[ m_i = \Lambda l_i \quad \text{and} \quad m_j = \Lambda l_j, \]

where \( \Lambda (\Lambda > 1) \) is a parameter, and \( l_i \ (l_j) \) is the labor input of firm \( i \) (firm \( j \)). S-Technology is the production by a single firm with the production function:

\[ y = F(l_i) = l_i. \]

We assume that there is the following “specificity” in the relation between firm \( i \) and firm \( j \): the intermediate goods \( m_i \ (m_j) \) creates the joint surplus only with \( m_j \ (m_i) \), and the intermediate goods have no alternative use. We also assume that there is the following “incompleteness” of contract: firm
and firm $j$ cannot fix the contract to divide $y$ before they produce the intermediate goods $m_i$ and $m_j$. Thus, they use Nash bargaining to divide the surplus after they produce the intermediate goods. Therefore, there are three outcomes for one firm. Assuming that each firm is endowed with 1 unit of labor, firm $i$ obtains $\Lambda$ if it chooses N-Technology and firm $j$ fulfills the commitment to produce $m_j$; it obtains 0 if it chooses N-Technology and firm $j$ breaks the commitment; and it obtains 1 if it chooses S-Technology.

Suppose that the manager of a firm has no other choice than to continue production according to the technology chosen. He stops production only when he resigns or is dismissed. We assume that the manager incurs huge private cost by dismissal (or resignation). This private cost might be interpreted as the loss of the “good reputation” and/or “perquisites” from controlling the firm. The private cost for the manager guarantees that the manager fulfills the commitment to produce the intermediate good, and that a firm always obtains $\Lambda$ if it chooses N-Technology. Therefore, in the normal circumstances, all firms choose N-Technology and the economy enjoys high productivity.

Next, suppose that a large-scaled financial shock brought about the defaults of many firms, and that the creditors obtain the discretionary right to dismiss the managers. If the creditors decide not to dismiss the managers right away, the structure of the game changes. The creditor can dismiss the manager in the production process of N-Technology and can cancel the commitment to produce the intermediate good, since the creditor does not have any private cost from breaking the commitment that his debtor made. If there is the prevailing pessimism that the creditor of firm $j$ dismisses the manager of firm $j$ and cancels the supply of $m_j$ with a large probability, then the expected profits of firm $i$ becomes smaller when it chooses N-Technology than when it chooses S-Technology. Therefore, all firms choose S-Technology and the economy suffers from the low productivity, if the pessimism prevails,
while N-Technology is chosen if the optimism prevails.

If the probability of dismissal exceeds a certain threshold, then the adopted technology changes from N-Technology to S-Technology. Then, the probability of dismissal loses the chance to be corrected, and the pessimism is self-reinforced. It seems consistent with the “contagion” of pessimism which has been often observed in the currency and financial crises in 1990s.

This pessimistic equilibrium illustrates the basic idea of this paper: the delay of the disposal of non-performing loans may create the persistent inefficiency by breaking down the coordinations between highly specialized firms, if the macroeconomic expectations deteriorate. We may call this problem as “disorganization due to debt overhang” which leads the economy to an obstinate stagnation. On the other hand, if the defaulters are punished according to the financial contracts, the inefficiency of “debt overhang problem” in the Lamont model may lead the economy into a sharp recession, though it may not last for a long time.

In the following sections, I will examine a general equilibrium model in which the inefficiency due to the delayed disposal of debt overhang generates multiple equilibria. In Section 2, I define the basic ingredients of the model and construct the good equilibrium without debt overhang. In Section 3, I introduce debt overhang in the model and explain how it creates multiple equilibria. In Section 4, the policies for the pessimistic equilibrium are proposed. Section 5 provides concluding remarks.

2 Model

Time is discrete and extends from zero to infinity. There are two kinds of agents: consumers (= workers) and managers of firms. The economy has only one kind of consumer good which is non-storable and is produced from labor and capital. In this simple economy, the only capital is land,
which is non-depletable and is initially owned by consumers. Consumers are
endowed with a unit of labor every period. Consumers are infinitely lived
agents with identical preference. The manager’s interest is to enjoy private
benefit in operating his firm. The manager does not consume the consumer
good. An investor (= a bank = an agent for consumers) gives a manager
the right incentive to exert effort to prevent an accident by financial contract
under which the right to dismiss the manager is given to the investor if the
repayment is less than a minimum amount.

2.1 Consumer

There are continuum\(^1\) of consumers in this economy with their population
measure normalized to one. Each consumer is infinitely lived and has identi-
cal preference. Each consumer is endowed with one unit of land at the initial
period and with one unit of labor every period. Consumers gain utility only
from consuming the consumer good which is not storable. We regard the
consumer good as the numeraire of the economy.

The consumer invests his land in firms through a bank. The banks com-
pete with each other and offer the savings with the market rate of interest
\(r_t\) to consumers. By arbitrage trading, the rate of returns on financial assets
and land become identical. Define \(a_t\) as the total value of asset of a consumer
which is in his bank account. Then the representative consumer solves the
following problem:

\[
\max_{c_t,a_t} \sum_{t=0}^{\infty} \beta^t u(c_t)
\]

subject to

\[
c_t + a_t \leq w_{t-1} + (1 + r_{t-1}) a_{t-1},
\]

where \(u(\cdot)\) is the utility function which is increasing, concave and twice dif-
ferentiable, \(\beta (\leq 1)\) is the discount factor, \(c_t\) is the consumption and \(w_t\) is

\(^1\)A consumer may be indexed by a real number between 0 and 1.
the wage rate for the work in period \( t \). Concerning the budget constraint, we have implicitly assumed that the consumer can freely lend and borrow although the consumer good is non-storable. The consumer solves the problem regarding \( a_0 \) and \( \{r_t, w_t\}_{t=0}^{\infty} \) as given parameters. \( a_0 = Q_0 \) where \( Q_0 \) is the unit price of land in period 0 measured by consumer good.

### 2.2 Production Technology

There are continuum of firm-managers with measure 1. Each manager indexed by a real number between 0 and 1 operates one firm. Suppose that there are constant-returns-to-scale (CRS) technologies for the production of consumer good. In these technologies, labor input \( l_t \) and land input \( k_t \) in the current period are transformed into output of consumer good \( y \) in the next period. Land is not depletable. Thus land input \( k_t \) remains after the production process is over.

For simplicity, we assume that there are only two technologies: S-Technology (Production by a single firm) and N-Technology (Network of production or cooperation between two firms). N-Technology is a simplification of a complex chain of productions. The choice of technology by a firm is observable and the manager of a firm cannot change the choice once he made.

#### 2.2.1 Single production

The production function of S-Technology is

\[
y = A_S k^{1-\alpha} l^\alpha,
\]

where \( 0 < \alpha < 1 \) and \( A_S \) satisfies \( \beta^{-1} < A_S \). When a firm uses S-Technology, it can produce the consumer good by itself, while it needs to cooperate with another firm in order to use N-Technology.
2.2.2 Production by Network of firms

Firms form pairs by random matching when they decide to use N-Technology. Suppose that firm $i$ and firm $j$ form a pair. A firm transforms its labor to intermediate goods. Production process of N-Technology is as follows.

First, firm $i$ transforms its labor ($l_i$) to $m_i$ units of intermediate good which cannot be used in S-Technology, where

$$m_i = \Lambda l_i \ (\Lambda > 1).$$

Firm $i$ and firm $j$ can produce “augmented labor” $z$ by a Leontief technology:

$$z = V(m_i, m_j) = 2 \times \min\{m_i, m_j\}.$$

The intermediate good $m_i$ ($m_j$) is useless without $m_j$ ($m_i$) in production of $z$, and it has no alternative use out of the pair of firm $i$ and firm $j$. This technological constraint on the intermediate goods is the model of “specificity” in this simple economy. We assume the following “incompleteness of contracts”: firm $i$ and firm $j$ can make a contract to divide $z$ only after they produce $m_i$ and $m_j$. Thus $z$ is divided by Nash bargaining. For simplicity, we focus on the symmetric case where all firms employ the same amount of labor: $l$ and produce the same amount of intermediate good: $m$. In this case, firm $i$ takes $z_i$ by Nash bargaining where

$$z_i = \begin{cases} 
\Lambda l_i & \text{if } m_i = m_j > 0, \\
0 & \text{if } m_i > 0 \text{ and } m_j = 0. 
\end{cases}$$

The augmented labor can be used in S-Technology. Thus after the division of augmented labor, firm $i$ produces $y_i$ units of consumer good by S-Technology:

$$y_i = A_S k_i^{1-\alpha} z_i^\alpha = A_N k_i^{1-\alpha} l_i^\alpha,$$

where $A_N \equiv A_S \Lambda^\alpha > A_S$. 

11
2.3 Banks

There are many banks trying to maximize their profits. They compete with each other and offer sure savings to consumers at the market rate of interest. We assume that the number of banks is a very large number $N$. Since there are continuums of consumers and firms, one bank gathers deposit (land) from continuum of consumers whose measure is $\frac{1}{N}$ and invests them into a continuum of firms whose measure is $\frac{1}{N}$, too. By the Law of Large Number applied to the continuum of firms, a bank has a fixed income without any uncertainty while the production of each firm has an idiosyncratic risk of loss by an accident (See Section 2.4.2). Firms employ labor input from consumers (= workers) by “promising” to pay market price of labor in terms of consumer good at the beginning of the next period. Wage payment has the priority over repayment to banks. This priority guarantees that all workers obtain the market wage.

2.4 Model of Corporate Control

There are three stakeholders for one firm: an investor (= a bank), a manager and a worker. A worker works for the firm at market price of labor. The control of the firm is a problem between the investor and the manager.

2.4.1 Dismissal of Manager

In the production process of S-Technology or N-Technology, managers can be dismissed. It is the equivalent of the bankruptcy in the real economy. If a manager is dismissed, he incurs a private cost which can be interpreted as the loss of reputation as a good manager or the loss of perquisites. We assume that they can establish a new firm in the next period. After the dismissal, the investor can carry out the production by itself. Since a bank is less productive than a manager, the output becomes smaller. If a manager is
dismissed in the production process of S-Technology, the remaining outputs are $A_L k^{1-\alpha} l^\alpha$ units of consumer good and $k$ units of land where

$$A_L < A_S.$$ 

Suppose that firm $i$ and firm $j$ form the pair for N-Technology, and the manager of firm $i$ is dismissed before he produces intermediate good, while firm $j$ continues the production of intermediate good. The investor of firm $i$ can continue production of consumer good using labor ($l_i$) and land ($k_i$). He can produce $A_L k_i^{1-\alpha} l_i^\alpha$ units of consumer good. In the meantime, firm $j$ knows the bankruptcy of firm $i$ after it produced intermediate good $m_j$. Since the production of augmented labor is a Leontief, firm $j$ obtains zero unit of augmented labor and firm $j$ cannot use $m_j$ in S-Technology. Therefore, the final output of firm $j$ is zero, when the manager of firm $i$ is dismissed. This condition that firm $j$ loses more than the investor of the bankrupt partner seems plausible since firms are likely to make relation-specific investments in the network of production.

A manager can be dismissed after the production is over and before the production of the next period begins. In this case, the dismissal works as a penalty to the manager.

**Definition 1** The right of corporate control is the discretionary right to dismiss the manager of the firm. The control right is initially owned by the manager.

### 2.4.2 Agency Structure and Manager’s Preference

We introduce a very simple structure of an agency problem in the relation between the investor (= bank) and the manager of a firm. We assume that there is an idiosyncratic risk that the consumer good is lost by an “accident” after the production and before the repayment to the bank.
Assumption 1 An accident occurs with probability $p$ if the manager exerts effort and with probability $P$ if he does not exert effort, where $0 < p < P < 1$. Effort of a manager is not observable for the investor. The firm loses $F$ units of consumer good by an accident. $F$ is a random variable which satisfies that $E[F]$ equals the amount of consumer good produced and there exists $\underline{F} > 0$ such that $F \geq \underline{F}$.

To exert effort makes private cost for the manager. The manager also receives private cost from dismissal. Thus, his preference is as follows.

Assumption 2 (Manager’s Preference)² The manager’s preference is completely independent of his monetary compensation (his salary). The manager’s problem is to maximize the utility function: $U_t^m = -C_L L - C_e e$, where $0 < C_e \ll C_L$, $C_e < (P - p)C_L$, and

\[
L = \begin{cases} 
1 & \text{if the manager is dismissed during period } t, \\
0 & \text{otherwise},
\end{cases}
\]

\[
e = \begin{cases} 
1 & \text{if the manager exerts effort}, \\
0 & \text{otherwise}.
\end{cases}
\]

At the beginning of period $t$, the manager’s problem is to maximize $E[U_t^m]$. A manager cannot change a production technology once chosen. Only if he is dismissed, the firm can change the operation.

2.5 Financial Contract

At the beginning of every period, firms and banks make financial contracts to finance the production of consumer good in the current period. A contract determines (1) allocation of the control right of the firm, (2) the production technology to be chosen by the firm, (3) amount of money (consumer good)

²This type of manager’s preference is a variation of the model in Hart (1995). See the third assumption in page 129 of Hart(1995)
to be borrowed and to be repaid by the firm, and (4) rule for reallocation of the control right. The fourth term is set to induce manager’s effort.

2.5.1 Penalty for Moral Hazard

In this simple economy, the reallocation of control right is the only device to induce the manager to exert effort. It is because the manager’s utility is independent of his salary.

Suppose that the managers’ private costs from effort and dismissal are negligible in social welfare of this economy. The social optimal is attained when managers exert effort. Therefore, the optimal contract determines that the control right is reallocated from the manager to the investor if an accident occurs. The investor is presumed to dismiss the manager immediately when he obtains the control right. With this contract, manager’s expected payoff is

\[ E[U_t^m] = \begin{cases} -pC_L - C_e & \text{if the manager exerts effort}, \\ -PC_L & \text{otherwise}. \end{cases} \]

We have assumed that \( C_L \gg C_e \) and \((P - p)C_L > C_e\). Therefore, managers exert effort and the output is maximized.

In this economy, an accident represents infinitely many events which can damage the business activity of a firm. Therefore, it is plausible to set the following assumption concerning the technology of the contract.

**Assumption 3** An accident is not describable in a contract.

Thus the reallocation of control right must be contingent on the amount of repayment to the bank. We can plausibly assume that this rule is adopted as the social norm that the control right over a defaulter is transferred to the investor.
2.5.2 Optimal Contract

In Definition 1, we have assumed that managers own the control right initially. Suppose that people knows that managers have the control right for all firms. Assumption 2 guarantees that all firms complete the production of intermediate goods if they choose N-Technology. Therefore, all firms choose N-Technology, since managers act as the agents for profit maximizing banks as long as they can avoid the private cost. Given that $Q_t$ is the unit price of land at period $t$, the optimal contract is written as follows.\(^3\)

"Allocate the control right to the manager. The investor lend $Q_t k_t$ units of consumer good to the manager. The manager choose N-Technology. The manager buy $k_t$ units of land and produce the consumer good. The manager repay $Q_{t+1} k_t + A_N k_t^{1-\alpha} l_t^\alpha - w_t l_t$ to the investor. If the repayment is less than $Q_{t+1} k_t + A_N k_t^{1-\alpha} l_t^\alpha - w_t l_t - E_t$, then the control right is given to the investor."

Although the contract is written in terms of consumer good, the actual transaction is that $k_t$ units of land is lent by a bank to the firm and that the firm returns the land to the bank.

Note also that this contract is realized by a combination of debt and equity. Define $\Delta_t \equiv (1 + r_t)^{-1}(Q_{t+1} k_t + A_N k_t^{1-\alpha} l_t^\alpha - w_t l_t - E_t)$ and $E_t \equiv Q_t k_t - \Delta_t$. If the bank gives the firm $\Delta_t$ units as debt and $E_t$ units as equity, this contract is realized. Therefore, the bank in this model represents a complex of equity holders and debt holders in the reality.

The technology and inputs $k_t$ and $l_t$ are determined to maximize the repayment. Since the technology is CRS and the endowment of inputs is $k_t = l_t = 1$, we can focus on a symmetric case where $k_t = l_t = 1$ for all firms when we analyze an equilibrium.

\(^3\)Since consumer good is intermediary of transactions in this economy, payment in a contract is written in terms of consumer good.
2.6 Equilibrium without Debt Overhang

The social optimum is attained in the equilibrium without debt overhang where the optimal contract prevails.

2.6.1 Timing of Events

The timing of events in one period is as follows. After the contract is fixed, the bank credits $Q_t$ units of consumer good to the firm. The firm acquires 1 unit of land from the bank in exchange for $Q_t$ and employs 1 unit of labor. Then firms form pairs for N-Technology and produce output $A_N$. At the beginning of period $t + 1$, the market price of land $Q_{t+1}$ is revealed. After selling the land and paying $w_t$ to the worker, the firm repays $Q_{t+1} + A_N - (\text{the loss by an accident}) - w_t$ to the bank. If the repayment is less than $Q_{t+1} + A_N - w_t - F$, the control right is transferred to the bank. The bank dismisses the manager when it obtains the control right. Then the dismissed managers establish new firms, and banks and firms negotiate the contracts for period $t + 1$.

2.6.2 Equilibrium Allocation

Since there is no technological progress or capital accumulation in this economy, the prices and output in equilibria are constant over time.

Therefore, the market rate of interest $r_t$ is determined by

$$\frac{1}{1 + r_t} = \frac{\beta u'(c_{t+1})}{u'(c_t)} = \beta.$$ 

Since there is a risk of an accident, the competitive wage rate is $w_t = \alpha(1 - p)A_N$. The equilibrium land price $Q_h$ is determined by the following. A bank earns $Q_h + (1 - p)A_N - \alpha(1 - p)A_N$ at the beginning of period $t + 1$ by investing $Q_h$ in period $t$. Since the rate of returns must be consistent with
the market rate of interest \( r_t = \beta^{-1} - 1 \), we have

\[
Q_h = \frac{(1 - \alpha)(1 - p)A_N}{\beta^{-1} - 1}.
\]

In this equilibrium, consumers have asset \( a_t = Q_h \) in their bank accounts. They obtain \( \alpha(1 - p)A_N \) units of consumer good as labor income and \( (1 - \alpha)(1 - p)A_N \) units as interest payment on their assets every period. Thus they consume \( (1 - p)A_N \) units of consumer good every period. Managers who have accidents in period \( t \) are dismissed at the beginning of period \( t + 1 \) and the managers receive private cost of \( C_L \). The dismissed managers establish new firms in period \( t + 1 \) so that the measure of firms remains the same at the beginning of every period. It is easily shown that this equilibrium is stable against small perturbation of prices and interest rate.

3 Equilibrium with Debt Overhang

3.1 Introduction of Debt Overhang

Suppose that the economy was in the equilibrium without debt overhang initially. At period \( t_0 \), the economy receives a exogenous financial shock which hits all firms. This shock reduces firms’ repayment to \( Q_h + A_N - D - w_{t_0} \), where

\[
A_L > D > \max\{E, \frac{P - p}{p} A_S\}.
\]

In this case, the firm’s control right is transferred to the investor according to the equilibrium contract. We assume that the shock is not a real shock and that the total output of the economy in period \( t_0 \) is \( (1 - p)A_N \) which is equal to the amount of previous periods.\(^4\) Assume that this shock is observable for all agents in this economy. Since banks know that the managers are not

\(^{4}\)We do not specify this shock more in detail. The land price decline would be one example if the debtors are to bear the risk of land price fluctuations. Suppose that the financial contract determines that the debtors are to repay \( Q_t \) at period \( t + 1 \) instead of
responsible for their defaults, they have no reason to dismiss the managers. It is because the dismissal is useful only as the penalty to debtor’s moral hazard, while the banks know that there was no moral hazard in this case. Instead of dismissal, they establish $D$ as the manager’s debt overhang that is to be repaid as soon as possible or when they have a positive financial shock.

In summary, banks obtain the control right according to the contract. But they decide not to exert the right. Managers are given temporary and discretionary respite from dismissal by banks.

**Assumption 4** The creditor of debt overhang reserves the right to dismiss the manager. This right is returned to the manager of the debtor firm only if either the debt overhang is fully repaid or the creditor forgives the debt overhang. A manager who has debt overhang with the face value $D$ is obliged to pay a cash flow which has the discounted present value (DPV) of $D$. If the DPV of the repayment becomes smaller than $D$, the creditor dismisses the manager immediately. If the creditor dismisses the manager or forgives the debt overhang, then he loses the right to claim the remaining debt overhang.

Since creditors withhold from exerting their rights voluntarily, they have the discretionary power to decide whether and when they dismiss the managers. Firms need to borrow new money from banks to operate from period $t_0 + 1$ on. The creditor of debt overhang may or may not provide new money for the debtor firm. In the following argument, we assume that the creditor finance the existing debtor.\(^5\)

\(^5\)In the case where the creditor does not provide new money for the debtor and the debtor needs to find a new investor, it is easy to show that the economy is to be trapped in a bad equilibrium by similar argument.
3.2 Market of Loans

We assume that the amount of a firm’s debt overhang is observable. However, since the debt overhang is created by an unexpected financial shock, the market of loans where those debts overhang (or, distressed loans) are traded by banks does not exist at least for a while after the shock.

Assumption 5 The market of debt overhang does not exist.

3.3 Phase Transition of Equilibrium Strategy

One effect of debt overhang is to increase agency cost for banks to prevent the debtor’s moral hazard. Since the banks have to give up $D$ when it dismisses the managers, the dismissal is costly not only for managers but also for banks. The condition (1) for the size of $D$ implies that it is optimal for the creditor not to dismiss the manager even if the manager does not exert effort.

The transfer of the control right of the firm to the creditor creates a choice of actions for the creditor when the firm enters into production by N-Technology. It is the choice whether to allow the debtor to complete the production of intermediate good (action “C”), or to stop the production of intermediate good unfinished by dismissing the manager (action “L”). If the creditor chooses action “L”, he can seize land ($k$) and labor ($l$) before it is transformed to intermediate good ($m$), and can produce $A_L k^{1-\alpha} l^\alpha$ units of consumer good.

Suppose firm $i$ and firm $j$ form a pair and enter into production by N-Technology. The creditors (bank $i$ and bank $j$) simultaneously choose “C” or “L”. If bank $i$ chooses “C” while bank $j$ chooses “L”, firm $i$ cannot produce the consumer goods though it produces $m_i (> 0)$, since $m_j = 0$. Therefore, this game between bank $i$ and bank $j$ is the “battle of the sexes”.\footnote{Since the number of banks is a finite number $N$, bank $i$ happens to be bank $j$ with probability $\frac{1}{N}$. We simply neglect this case because the expected payoff of bank $i$ changes}
in the symmetric case where \( k = l = 1 \) is as follows.\(^7\) Bank \( i \)'s expected gain of consumer goods is \((1 - P)A_N - w_t\) if both banks choose “C”; bank \( i \) obtains \( A_L - w_t - D \) if it chooses “L”; and bank \( i \) obtains \(-w_t\) if bank \( i \) chooses “C” while bank \( j \) chooses “L”. Thus, the assumption that \( A_L > D \) implies that the optimal strategy for bank \( i \) is “C” if bank \( j \) chooses “C”, and is “L” if bank \( j \) chooses “L”.\(^8\) Define \( \pi \) as the subjective probability for people in the economy that the other player of the game chooses “C”. Thus, \( \pi \) is bank \( i \)'s subjective probability that bank \( j \) chooses “C”. Therefore, the expected payoff of bank \( i \) is \( \pi(1 - P)A_N - w_t \) if he chooses “C”, and \( A_L - D - w_t \) if he chooses “L”. Therefore, we have multiple equilibria in this economy. If people have the pessimism that \( \pi < \frac{A_S}{A_N} \), all banks and their debtors choose S-Technology, since \( A_L - D < (1 - P)A_S \). In this case, the pessimism persists since \( \pi \) has no chance to be corrected by banks’ actions.\(^9\) If people have the optimism that \( \pi > \frac{A_S}{A_N} \), all banks and their debtors choose N-Technology and

\(^7\)Note that the residual after the wage payment is the gain for banks because the managers do not demand the consumer good.

\(^8\)If \( D > A_L \), the equilibrium of the game is only one where both banks choose “C”. This result seems counter intuitive. We can modify the manager’s preference to avoid this result as follows. Assume that the manager of firm \( i \) receives the private cost of dismissal \( C_L \) when firm \( j \) does not provide intermediate good \( m_j \) and firm \( i \)'s output becomes zero. This private cost can be interpreted as the loss of the manager’s relation-specific investment. Assume that once the manager receives the private cost \( C_L \), the dismissal does not add any more private cost for him. Thus he resigns as manager of firm \( i \) and defaults on the debt overhang \( D \) when bank \( j \) choose “L”. In this case, bank \( i \)'s payoff is \(-D - w_t\) if it chooses “C” while bank \( j \) chooses “L”. If we make these assumptions on the manager’s preference, the game becomes the battle of sexes no matter how large \( D \) is.

\(^9\)We may assume that \( \pi \) is not known to people just after the financial shock. It is the assumption of Knightian Uncertainty (Epstein and Wang [1994]). Gilboa and Schmeidler (1989) imply that the rational agents under Knightian Uncertainty act assuming the worst case: \( \pi = 0 \). If it is the case, \( \pi \) becomes 0 after the shock and has no chance to be corrected afterwards.
action “C”, and \( \pi \) converges to 1.

The equilibrium contract in the pessimistic case where people expect that \( \pi < \frac{A_S}{A_N} \) is the following:

"The creditor of debt overhang \( D_t \) keeps the control right. The creditor lends \( Q_t \) units of consumer good to debtor. The debtor choose S-Technology. The debtor is to repay \( R_t \geq r_tD_t \) as the repayment to debt overhang and \( Q_{t+1} + A_S - w_t - R_t \) as the repayment for new money \( (Q_t) \), at the beginning of period \( t + 1 \). The debt overhang evolves by \( D_{t+1} = (1 + r_t) D_t - R_t.\)"

In addition to the condition \( R_t \geq r_tD_t \), the equilibrium repayment must satisfy \( 1 + r_t \leq (Q_{t+1} + A_S - w_t - R_t)/Q_t \). It is shown in the following section that \( R_t \) is uniquely determined by \( R_t = r_tD_t \) in the equilibria. Under this contract, even if the total repayment is smaller than or equal to \( Q_{t+1} + A_S - w_t - E \), the creditor does not exert the right to dismiss the manager.

In the optimistic equilibrium where people expect that \( \pi = 1 \), N-Technology is chosen and the repayment for new money changes to \( Q_{t+1} + A_N - w_t - R_t \) in the above equilibrium contract.

But why is it at all that banks do not invest their money in new firms who do not have debt overhang? One simple reason is that there are not enough supply of new entrepreneurs. Another reason is the imperfect financial markets. Suppose that the amount of money that one bank can provide to firms is limited by the imperfection of the market. Then the bank cannot provide new money to its debtor when it invests in a new firm. In this case, the bank must dismiss the manager of the debtor when it invests in a new firm, since the debtor needs to borrow new money in order to continue its operation. Therefore, the investment in a new firm necessitate the creditor to realize the loss of \( D \). If \( D \) is large, the creditor never invest in a new firm.\(^{10}\)

\(^{10}\)If the financial market is competitive and banks can collect additional money from
3.4 Pessimistic Equilibrium

We can show the following claim.

Claim 1 In the equilibrium where the adopted technology does not change, the land price is constant over time. Thus

\[ Q_t = Q_{t_0+1} \text{ for } t \geq t_0 + 1. \]

In this simple economy, the fundamental change is only the choice of production technology. If there is no change of production technology, the price of land is determined by arbitrage trading in the market. It is straightforward that land price does not change over time.

We focus on the prices and allocation of the pessimistic equilibrium in the symmetric case where \( k_t = l_t = 1 \) for all firms. Total return to a bank must have at least the market rate of interest in the equilibrium otherwise banks deposit their money in other banks. Since the market rate of interest is \( \beta^{-1} - 1 \) in an equilibrium where output does not change over time, the following equality must hold:

\[ Q_{t_0+1} + D_{t+1} + (1 - P)A_S - \alpha(1 - P)A_S = \beta^{-1}(Q_{t_0+1} + D_t). \]

The fourth term of the left hand side is the equilibrium wage payment: \( w_t = \alpha(1 - P)A_S \). Define \( Q_t \equiv \frac{(1-\alpha)(1-P)}{\beta^{-1}-1}A_S \). The above equation can be rewritten as \( Q_{t_0+1} + D_{t+1} - Q_t = \beta^{-1}(Q_{t_0+1} + D_t - Q_t) \). Therefore,

\[ Q_{t_0+1} + D_{t+1} - Q_t = \beta^{-t-t_0}(Q_{t_0+1} + D - Q_t) \text{ for } t (> t_0). \]

Consumers by offering a higher interest rate for the deposits, they are forced by market competition to dismiss the managers of the debtor firms and to invest in new firms. Since the total amount of savings in this economy is limited, the competition among banks raise the interest rate until the present value of the repayment from the debtor, discounted by the market rate of interest, becomes less than \( D \). Then all creditors choose to dismiss the debtors and to invest in new firms. In this case, the economy recover from the stagnation quickly if there are sufficient supply of new entrepreneurs.
Since the debt does not diverge in the equilibrium, the equation implies $Q_{t_0+1} = Q_t - D$. Thus Claim 1 implies that $D_t = D$ and $Q_t = Q_t - D$. Therefore, the repayment for the debt overhang $R_t$ is uniquely determined by $R_t = (\beta^{-1} - 1)D$. The consumer’s asset is the sum of the land and the firm’s debt overhang: $Q_t - D + D = Q_t$, while it was $Q_h$ before the shock hit the economy at period $t_0$. Immediately after the debt overhang $D$ is set, the land price declines to $Q_t - D$. Thus the banks and the depositors (consumers) realize the loss of $Q_h - Q_t$ at the beginning of period $t_0 + 1$ after the consumers obtain the income $(1 - p)A_N$ and before they determine their consumption and savings for period $t_0 + 1$. Since N-Technology is adopted in period $t_0$, the aggregate consumption in period $t_0 + 1$ must be $(1 - p)A_N$.

And the aggregate consumption (= the aggregate output) from period $t_0 + 2$ on is $(1 - P)A_S$. Thus the interest rate $r_{t_0+1}$ at period $t_0 + 1$ satisfies

$$r_{t_0+1} = \frac{u'(c_{t_0+1})}{\beta u'(c_{t_0+2})} - 1 = \frac{u'((1 - p)A_N)}{\beta u'((1 - P)A_S)} - 1 < r_t = \beta^{-1} - 1, \text{ for } t \leq t_0, t > t_0 + 1.$$

From period $t_0 + 2$ on, a consumer obtains $\alpha(1 - P)A_S$ units of consumer good as labor income and $(1 - \alpha)(1 - P)A_S$ units as interest payment for his asset. Although the managers do not exert effort, they are dismissed even when they have the accidents. Once the economy is trapped in the pessimistic equilibrium, it cannot escape from this equilibrium unless all people’s confidence is restored simultaneously.

If people become optimistic simultaneously, the economy jumps into the optimistic equilibrium where $Q_t = \frac{(1-\alpha)(1-P)}{\beta-1}A_N - D$, $D_t = D$, $w_t = \alpha(1 - P)A_N$, $r_t = \beta^{-1} - 1$, and the consumption is $(1 - P)A_N$.

4 Policy Implication

Since the pessimistic equilibrium is a competitive equilibrium, market competition cannot recover social optimal unless people’s expectations change si-
multaneously. Thus a public policy is necessary once the economy is trapped in the pessimistic equilibrium. There are three types of possible remedies.

The cause of the pessimistic equilibrium is that the investors obtain the right to dismiss the managers and withhold from exerting it. Thus the first remedy is to make banks exert the right. It is to make banks put their debtors into the bankruptcy procedure. For banks, to let the debtors operate is optimal choice once the economy is trapped in the pessimistic equilibrium. Therefore, the implementation of bankruptcy seems to necessitate a strong compulsion by the regulator. On the other hand, the increase of bankruptcy may cause a sharp recession by credit crunch or by the mechanism of the Lamont model. Thus the aggregate demand management by, for example, injection of public money into the capital account of banks’ balance-sheets is necessary to avoid the deflationary spiral.

The second remedy is to make banks return the control right to the managers. If all managers retrieve the control right, the coordination failure would be solved and N-Technology would be adopted. One way to return the control right to managers is to convert the debt overhang to equity (the debt-equity swap). However, it is a variation of debt forgiveness, which may sow the seeds of moral hazard in the firms’ management if people expect that debt forgiveness will be repeated in the future.

The third remedy is to establish the market of debt overhang (or the market of non-performing loans). The breakdown of confidence in entering into specific relations is caused by the fact that firms have different creditors. Suppose that creditors can trade their claims after the pair of N-Technology is formed. Then one creditor can obtain the claims on both firms’ debt overhang. In this case, the payoff of the creditor is maximized when the output of the pair is maximized. Thus the social optimal is attained. The trade of claims on debts overhang would restore the macroeconomic confidence in business transactions. Since the trade of non-performing loans is beneficial
for banks, it seems likely that they trade their claims voluntarily. However, since the bank loans were not tradable traditionally, it would be very costly for private agents to facilitate the trading of bank loans. The market institutions need to be designed appropriately. For example, to provide a fair accounting rules and an efficient bankruptcy procedure facilitate active trade of non-performing loans.

5 Concluding Remarks

The main result of this paper is that a financial shock on the balance sheet variables can affect the real output by raising people’s suspicion in commitment to highly specific relations in complex chains of productions. The inefficiency comes from the people’s reaction to the shock, which is unexpected by the contracts: to keep the defaulters unpunished.

For this inefficiency, the debt level in the private sector might be a possible target of macroeconomic policy. For example, a publicly coordinated debt reduction program that forces banks to dispose of non performing loans may be effective as a policy to bring back the stagnant economy to the growth path.

There may be another course of argument if we consider the case where an active market of distressed loans and an efficient bankruptcy procedure exist. In this case, the pessimistic equilibrium becomes unstable since people can restore confidence by disposal and trading of debt overhang. We can conjecture that the efficiency in institutions of financial markets is the key factor to maintain and restore the confidence. The reform of market institutions may be, therefore, important to remove future possibilities of obstinate depression.  

\[11\]

\[11\]If a large shock occurs, the quick disposal of non performing loans would create the inefficiency of debt overhang which is treated in the Lamont model, and would lead the
References


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economy to a sharp recession. We may need to adopt the fiscal expansion policy to stop the contraction of the economy. Our point is that the aggregate demand management alone cannot recover the growth path unless the macroeconomic confidence is restored through reliable market institutions.
Figure 1
Checks and Bills Clearing & Fund Transfer

0 1000000 2000000 3000000 4000000 5000000 6000000

100 million Yen

Checks and Bills Clearing (Monthly)
Checks and Bills + Domestic Fund Transfer