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INTERNATIONAL LINKS OF INNOVATION PATTERNS

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I. Introduction

The traditional theory of economic growth has concentrated on capital accumulation, exogenous population growth and exogenous technical progress. Indeed, capital accumulation and labor growth proved to be important components in the explanation of growth patterns, but capital accumulation and labor growth alone explain only part of observed growth rates. The rest has been typically attributed to technical progress (see Solow (1957) for the original contribution and Maddison (1977) for a recent discussion of growth accounting).

It has been clear, however, from the very beginning that exogenous technical progress cannot be a satisfactory working hypothesis. Major productivity gains require a deliberate effort of invention an innovation, especially in modern times (see, for example, Freeman (1982)). The latter implies that resources need to be devoted to these activities. In order

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for these resources to be forthcoming, however, the economic system has to properly reward inventors and innovators, thereby providing incentives to engage in these activities. All this is rather self evident, but direct measurement of the contribution of inventions and innovations to economic growth proved to be very difficult (see Griliches (1979)). Part of the difficulty results from the lack of satisfactory data for the problem at hand. But another part of the difficulty results from the lack of a satisfactory theory.

Following the general slowdown of productivity growth in the 1970s the interest in economic growth has been renewed. This has been manifested in the publication of numerous empirical studies that attempt to explain the events of the seventies. More recently, however, the theory of economic growth has also been modified in order to deal with the new concerns. The new approaches emphasize factors that lead to sustained long run growth at rates that are endogenously determined. This constitutes an important departure from traditional theory in which long run growth was exogenously determined. The new studies have proceeded along several lines:

(i) Endogenous population growth (Becker and Murphy (1988).
(ii) The role of public services (Barro (1989)).
(iii) Accumulation of human capital (Ohyama (1989).
(iv) Learning by doing and related spillovers (Romer (1986,1988) and Lucas (1988)).

Although learning by doing has been explored in the earlier literature (see Arrow (1962) for the original contribution as well as Uzawa (1965), Levhari (1966) and Sheshinski (1967)), its recent combination with an explicit treatment of product innovation -- in the form of development of new products or the improvement of existing products -- has yielded important new insights on endogenous technical progress (see, for example, Romer (1988) and Aghion and Howitt (1989)).

This study is concerned with international links of innovation patterns, where innovation drives economic growth. Such links have been explored in a series of papers by Grossman and Helpman (1989a-e) for long run steady states. (Grossman (1989) has also used it to explain Japan's recent performance.) They showed in a variety of models that build on endogenous product innovation how the long run growth rate of a country depends on its own features, features of its trading partners, its own trade policy, trade policy of its trading partners, its own policy towards innovation and imitation and its trading partners' policies towards innovation and imitation. These links proved to be rather involved. For example, a less developed country that encourages imitation of products that have been originally developed in an advanced industrial country may thereby speed up growth around the globe.

In what follows I study the <u>time pattern</u> of endogenous innovation, including out of steady state dynamics. For this purpose I employ a simplified version of a model developed in Grossman and Helpman (1989a). In that model the growth rate is closely associated with the rate of innovation. Two major questions will be addressed:

(i) How do the time patterns of innovation and growth respond to a change in the available resources?

(ii) How do the time patterns of innovation and growth respond to R&D subsidies?

The first question addresses a structural issue while the second addresses a policy problem.

Some interesting results emerge from this analysis. For example, the short run response of the rate of innovation to an R&D subsidy can differ from the long run response, and short run rates of innovation can overshoot or undershoot long run innovation rates. The answers to the policy questions depend on structural parameters as well as on whether the policy active country has a comparative advantage in research and development. A summery of the results is provided in the closing section.

In order to make the paper self contained I present in the next section a simple one country model in which profit seeking entrepreneurs develop new products. An expansion of the menu of available products raises productivity in manufacturing via the refinement of specialization (as in Ethier (1982)) or it raises productivity in consumption via an increase in the available product choice. For this reason a larger product choice is desirable per se. In addition current product development reduces costs of future product innovation. This feature captures the idea that even when targeted at particular products R&D also generates broader knowledge that can be applied to other products. It thereby leads to the accumulation of non-appropriable knowledge capital. Knowledge capital is particularly useful for R&D. Thus, current innovative activities reduce costs of future innovations. This specification leads to endogenous innovation and growth, with their rates depending on available resources and on the government's industrial policy (as in Romer (1988)). In the one-country case the rates of innovation and growth settle down immediately on steady state levels. Therefore in this case there exist no out of steady state daynamics.

In Section III the model is extended to represent a world of two countries. With two countries in place there exist non-trivial out of steady state dynamics. In a stationary environment the rate of innovation may rise or decline over time (depending on initial conditions) until it converges to a steady state. In that section I also analyze the response of its time pattern to changes in available resources.

The model is further extended in Section IV in order to deal with industrial policy. I show how the patterns of innovation responds to R&D subsidies. The resulting changes in these time patterns depend on whether the policy is enacted by a country with comparative advantage or disadvantage in innovation. The paper closes with a summary of the main findings and some concluding comments.

II. The Basic One-Country Model

We consider an isolated economy that is populated by identical individuals with time additive preferences

$$U = \int_0^{\infty} e^{-\rho t} \log D(t)\, dt, \tag{1}$$

where t represents a time index, ρ stands for the subjective discount rate, and $D(t)$ represents a consumption index. There are two alternative interpretations of the consumption index: (a) there exists a single homogeneous final consumer good and $D(t)$ represents its consumption level; and (b) $D(t)$ represents and index of consumed differentiated products. Under both interpretations

$$D(t) = \left[\int_0^{n(t)} x(\omega)^{\alpha}\, d\omega\right]^{1/\alpha}, \quad 0 < \alpha < 1, \tag{2}$$

where ω is an index of differentiated products, $\omega \in [0,\infty)$, and the set of available products at time t equals $[0,n(t)]$; $n(t)$ also represents the measure of differentiated products available at time t; and $x(\omega)$ represents the quantity of variety ω. Under the former interpretation x is a differentiated input and (2) represents a production function. Under the latter interpretation x is a differentiated consumption good and (2) represents a sub-utility function. The following analysis applies to both cases.

The representative consumer maximizes (1) subject to an intertemporal budget constraint

(3) $$\int_0^\infty e^{-R(t)} P(t) D(t)\, dt \leq \Omega(0) ,$$

where $R(t)$ stands for the discount factor from t to 0; $P(t)$ represents the ideal price index associated with $D(t)$ (which equals the price of the consumption good under the first interpretation); $\Omega(0)$ equals the present value of income plus the value of initial asset holdings (to be determined in equilibrium). As is well known the solution to this problem yields

(4) $$\dot{E}/E = r - \rho,$$

where $E=PD$ represents consumer spending and r the interest rate (equal to $\dot{R}$). Hence, the rate of growth of spending equals the difference between the interest rate and the subjective discount rate.

In addition, at every point in time the distribution of spending across different varieties is given by

(5) $$x(\omega) = p(\omega)^{-\epsilon} E / \int_0^n p(\omega')^{1-\epsilon} d\omega' , \quad \omega \in [0,n], \quad \epsilon = 1/(1-\alpha) > 1,$$

where $p(\omega)$ stands for the price of variety ω. Under the first interpretation (5) represents consumer demand functions; under the second it represents the demand functions of profit maximizing atomistic producers of the homogeneous consumption good. The last point becomes evident when one recognizes that under (2) manufacturers of the homogeneous consumption good face constant returns to scale and each one of them takes the measure of products n and total consumer spending E as given.

We are free to choose the time pattern of a nominal variable (the choice of numeraire). It proves convenient to choose $E=1$ for all t. In this case (4) implies

$$(6) \qquad r(t) = \rho,$$

i.e., the nominal interest rate is constant and equal to the subjective discount rate.

The manufacturing know-how of an existing variety ω belongs to an atomistic entrepreneur that developed it in the past. He needs one unit of labor per unit output for every ω. Thus, his marginal manufacturing costs equal the wage rate w. Facing the demand function (5) the supplier of ω maximizes operating profits $p(\omega)x(\omega) - wx(\omega)$ by charging price $p(\omega)=w/\alpha$. Hence, in equilibrium all varieties are equally priced, and we have

$$(7) \qquad p = w/\alpha.$$

This pricing strategy yields operating profits (recall that $E=1$)

$$(8) \qquad \pi = (1-\alpha)/n.$$

Clearly, operating profits of a representative firm vary over time with the measure of available products.

Let $v(t)$ be the time t stock market value of a firm that has the manufacturing know-how of a variety of x. This know-how entitles the firm to a stream of operating profits $\pi(\tau)$, $\tau \geq t$, where π is given in (8). Therefore the value of the firm equals the present value of operating profits. Taking account of (6) this condition reads

$$v(t) = \int_t^{\infty} e^{-\rho\tau}\pi(\tau)\, d\tau,$$

which implies (via differentiation with respect to time) the no arbitrage condition

$$\pi + \dot{v} = \rho v. \tag{9}$$

The left hand side represents the instantaneous reward for owning the firm. It consists of operating profits plus a capital gains term. The right hand side represents instantaneous costs, which equal foregone interest on the value of the firm.

Ownership of a firm can be acquired in two ways: by purchase of an existing company on the stock market at cost v or by the establishment of a new firm. The latter possibility requires to develop a new variety of x. Let a/K be the labor output ratio in product development, where a is a parameter and K represents the stock of knowledge capital in innovation. The interpretation of this coefficient is as follows. Employment of L_n labor units in R&D for a time interval of length dt increases the measure of available products by $dn = (L_n K/a)dt$. Consequently $\dot{n} = L_n K/a$, and the per product cost of product development equals wa/K. In equilibrium the cost of forming a new firm cannot fall short of the price of an existing firm, because if it did entrepreneurs would have had an unbounded demand for labor for R&D purposes. Therefore $wa/K \geq v$. On the other hand, if product development costs exceed the value of an existing firm there can be no innovation. Consequently, as long as there exists active R&D $wa/K = v$.

Knowledge capital grows as a result of experience in R&D. For the purpose of this paper I also assume that this knowledge becomes instantly available to all entrepreneurs. This assumption is rather extreme. Indeed, scientific knowledge spreads very fast through congresses and publications, but experience in product innovation provides an advantage to the innovator for at least a limited period of time. For simplicity, however, I choose the simpler specification and assume $K = n$. Using this equation together with (7) and (8), the no arbitrage condition (9) becomes

$$(1-\alpha)/\alpha pa + \dot{p}/p - \dot{n}/n = \rho. \tag{10}$$

Finally we come to the resource constraint. Our specification of the R&D production function implies that employment in product development equals $a\dot{n}/K$, or $a\dot{n}/n$. Employment in manufacturing equals nx, where output per product equals for all varieties. In fact, $x=1/np$ (see (5) and recall that $E=1$). These considerations imply the following labor market clearing condition

$$a\dot{n}/n + 1/p = L, \tag{11}$$

where L represents the available labor force. The first term on the left hand side represents employment in R&D while the second represents employment in manufacturing. Thus, the equation states that total employment equals the available labor force (it represents a resource constraint).

We are mostly interested in the growth rate. But the question is the growth rate of what? There are two variables in whose growth we might be interested: the consumption index D and the measure of available products; i.e., the rate of innovation. In what follows we concentrate on the rate of innovation, $g=\dot{n}/n$, which is the basic driving force in this economy. It can be shown that the growth rate of consumption, $g_D=\dot{D}/D$, equals

$$g_D = \bar{g}/\alpha - g,$$

where $\bar{g}$ represents the steady state innovation rate. Hence, the short and long run rates of innovation determine the growth rate of consumption.

Equations (10) and (11) can be rewritten as

$$\dot{p}/p = \rho + g - (1-\alpha)/\alpha a p, \tag{12}$$

$$g + 1/ap = H, \tag{13}$$

where $H = L/a$ stands for the effective labor force in terms of R&D. This system represents a differential equation in price plus a side condition that describes the resource constraint. Curve HH in Figure 1 represents the resource constraint (13). It slopes upwards and approaches infinity as g approaches H. The downward slopping curve $\dot{p}=0$ describes stationary points of p. Its properties are derived from (12). The intersection point 1 identifies a steady state equilibrium. Out of steady state the system follows the arrowed path trajectory. Perfect foresight, the consumer's transversality condition, and the lack of profit opportunities in product development imply that the economy has to converge to the steady state. Since point 1 is a source (i.e., an unstable equilibrium point), it implies that a perfect foresight trajectory coincides with the steady state point. Namely, the economy jumps immediately to the steady state.

Now we can derive the effects of a lager labor force and an R&D subsidy on innovation and growth. An economy with a larger labor force has an HH curve further to the right. Consequently, countries with a large resource base feature higher rates of innovation and growth than smaller countries. In the presence of an R&D subsidy a proportion s of gross innovation costs are born by the government, where $0 < s < 1$ equals the subsidy rate. Therefore a typical entrepreneur bears only a proportion (1-s) of his R&D costs. Hence, we need to replace the parameter a on the left hand side of (10) by (1-s)a; an R&D subsidy reduces product development costs and therefore raises the profit <u>rate</u>. This modification implies that on the right hand side of (12) a too has to be multiplied by (1-s). Consequently, an economy with an R&D subsidy has a higher $\dot{p} = 0$ curve, innovates faster, and grows faster. These examples show that the growth rate is indeed endogenous.

Before closing this section it is worth pointing out that (12)-(13)

can be used to derive an autonomous differential equation in g. To do this differentiate (13) with respect to time and substitute (12) and (13) in the resulting equation in order to obtain

$$(14) \qquad \dot{g} = (H - g)[\rho - (1-\alpha)H/\alpha + g/\alpha] \quad \text{for} \quad 0 \leq g \leq H.$$

(The domain restriction guarantees non-negative employment in innovation and manufacturing.) It implies a steady state growth rate

$$(15) \qquad \bar{g} = (1-\alpha)H - \alpha\rho.$$

The steady state is again a source and therefore immediately attained in a perfect foresight equilibrium.

III. A Two Country Model

We have seen in the previous section how the growth rate and the rate of innovation of an isolated economy depend on resources and industrial policy. Those rates did not vary over time. In the two-country extension that follows the world economy does not attain immediately a steady state and growth and innovation rates vary over time. We are interested in the time pattern of innovation. In this section we also investigate how this time pattern responds to changes in the resource base.

Preferences are as before and apply to both countries. I assume free international (financial) capital mobility. Therefore the same interest rate prevails in both countries. In this case (4) applies to each country separately and to the world at large. For current purposes E represents world spending and our numeraire will be $E = 1$ for all t. Consequently (6) remains valid. The demand functions (5) also remain valid.

Now consider country i. At time t its firms posses the know-how to manufacture a measure $n_i(t)$ of differentiated products. The measure of products available in the world equals $n = \Sigma_i n_i$. A typical manufacturer maximizes profits by choosing a price that exceeds marginal costs by a factor of $1/\alpha$. I assume that a unit of output requires one unit of labor in every country. This assumption is inconsequential; it represents a normalization of labor units and saves on notation. Therefore marginal manufacturing costs in country i equal the wage rate in country i, w_i, and we replace (7) by

$$(7') \qquad p_i = w_i/\alpha;$$

i.e., all country-i manufactured varieties are equally priced. On the other hand product prices differ across countries as long as wage rates differ. Output of a variety of country i equals in this case to (see (5))

$$(16) \qquad x_i = p_i^{-\epsilon}/\Sigma_j n_j p_j^{1-\epsilon}.$$

Using this representation profits per product in country i equal

(8')
$$\pi_i = (1-\alpha)p_i^{1-\epsilon}/\Sigma_j n_j p_j^{1-\epsilon}.$$

Now the no arbitrage condition (9) reads

(9')
$$\pi_i + \dot{v}_i = \rho v_i,$$

where v_i stands for the value of a country-i firm. Given (9') the return on equity holdings is the same in both countries.

As before, the value of a firm equals the cost of product development as long as product innovation takes place. Recall that we assumed equal labor input per unit output in manufacturing in both countries. In order to allow for comparative advantage I therefore assume that innovation costs per product equal $w_i a_i/K$ in country i. The first thing to observe about this formulation is that country i has a comparative advantage in R&D relative to manufacturing if and only if $a_i < a_j$, $i \neq j$. The second thing to observe is that the stock of knowledge capital is the same in both countries. Thus, not only does knowledge that has been acquired though product innovation spread to other domestic firms, but it also spreads at an equal pace to foreign firms. This is, of course, an extreme assumption, but it proves to be a convenient working hypothesis. We shall in fact assume that knowledge spreads instantaneously and K=n. (The consequences for steady states of lags in the dissemination of knowledge that differ within and across countries have been studied by Grossman and Helpman (1989a).) Using this specification as well as (7')-(9') the no arbitrage condition can be rewritten in a form similar to (10); i.e.,

$$(10') \qquad n\frac{(1-\alpha)}{\alpha}\frac{p_i^{-\epsilon}/a_i}{\Sigma_j n_j p_j^{1-\epsilon}} + \dot{p}_i/p_i - g = \rho.$$

Finally, the resource constraint becomes

$$(11') \qquad a_i\dot{n}_i/n + n_i p_i^{-\epsilon}/\Sigma_j n_j p_j^{1-\epsilon} = L_i,$$

where L_i stands for the labor force of country i.

This completes the description of the two-country model and we proceed to analyze its dynamics. First note from (10') that the relative price p_i/p_j, $i \neq j$, converges to zero or infinity unless $p_i^{-\epsilon}/a_i = p_j^{-\epsilon}/a_j$ initially, and therefore at each point in time. Consequently, on a perfect foresight equilibrium trajectory relative prices are constant, which implies that there exists a function $q(t)$ such that

$$(17) \qquad p_i = qa_i^{-1/\epsilon} \quad \text{for all } t.$$

Hence, the rate of change of every price equals the rate of change of q, and using (10') and (17)

$$(18) \qquad \dot{q}/q = \rho + g - (1-\alpha)/\alpha q\Sigma_j\sigma_j a_j^{\alpha}.$$

Next, using (17), we calculate from the resource constraint (11') the rate of innovation in country i, $g_i = \dot{n}_i/n_i$, and the world's rate of innovation g:

(19) $$g_i = H_i/\sigma_i - 1/q\Sigma_j\sigma_j a_j^{\alpha},$$

(20) $$g = H - 1/q\Sigma_j\sigma_j a_j^{\alpha},$$

where $\sigma_i = n_i/n$ represents the share of country i in the measure of products; $H_i = L_i/a_i$ stands for effective labor in terms of R&D in country i, and $H = \Sigma_j H_j$. Naturally, g is a weighted average of g_i: $g = \Sigma_j\sigma_j g_j$. By definition, the rate of change of the share σ_i equals g_i minus g. Therefore (19) and (20) imply

(21) $$\dot{\sigma}_i = H_i - \sigma_i H, \quad 0 \leq \sigma_i \leq 1.$$

This is a simple, stable, linear and autonomous differential equation in σ_i. From every initial condition the shares converge monotonically to $\sigma_i = H_i/H$. In steady state the shares are proportional to effective labor. This shows clearly that in a two-country world the steady state will be approached gradually unless the initial ownership of products happens to be proportional to the steady state values.

Equations (18) and (21) form a system of differential equations with side condition (20). This system represents an extension of (12)-(13). It does not lend itself, however, to an easy analysis. For this reason we proceed as follows. Differentiate (20) with respect to time to obtain

$$\dot{g} = (\dot{q}/q + \Sigma_j\dot{\sigma}_j a_j^{\alpha}/\Sigma_j\sigma_j a_j^{\alpha})/q\Sigma_j\sigma_j a_j^{\alpha}.$$

Now use (18), (20) and (21) to obtain

$$(22) \qquad \dot{g} = (H - g)(\alpha\rho - H + \alpha\Sigma_j H_j a_j^\alpha / \Sigma_j \sigma_j a_j^\alpha + g)/\alpha \quad \text{for} \quad 0 \le g \le H.$$

This is the new counterpart of (14). It is straightforward to see that (22) reduces to (14) whenever no country has comparative advantage in R&D. In this special case (22) implies that the equilibrium rate of innovation does not vary over time, independently of whether the country composition of products varies over time. In particular, the innovation rate is as given by (15), which equals the innovation rate that obtains in an integrated world economy with effective labor equal to $H = \Sigma_j H_j$.

Naturally, we are mostly interested in the case in which countries differ in relative efficiency with which they perform product innovation. In this case (21) and (22), which form a system of autonomous differential equations, can be used to analyze equilibrium trajectories of innovation and product shares. Since the sum of product shares equals identically 1, we may use a two equation system consisting of (22) and one equation from (21). For concreteness assume that country $i=2$ has comparative advantage in R&D; i.e., $a_1 > a_2$. Figure 2 depicts the phase diagram of (σ_1, g), where $1-\sigma_1$ has been substituted for σ_2 in (22). Along the vertical line $\dot{\sigma}_1 = 0$ and along the upward slopping curve $\dot{g} = 0$. Their intersection at point 1 identifies the steady state. Evidently, the steady state is saddle path stable. The arrowed path through point 1 describes the perfect foresight equilibrium trajectory.

The time pattern of innovation is apparent from the figure. First consider the case in which the initial composition of products is such that country 1 -- which has comparative disadvantage in R&D -- has a disproportionately small share as compared to its relative effective size. Then country 1 will innovate faster than its trading partner, thereby increasing its product share. The average rate of innovation in the world economy will fall short of its steady state value, but will increase over time. If, on the other hand, country 1 has initially a disproportionately large share of products as compared to its effective relative size, it will innovate at a slower rate than its trading partner. The average rate of innovation will exceed its steady state value and decline over time. The steady state innovation rate is the same as in (15); i.e.,

$$\bar{g} = (1-\alpha)H - \alpha\rho.$$

Therefore cross country differences in relative costs affect out of steady state but not steady state innovation rates (as long as H is the same in both cases). In addition, the larger the world economy in terms of effective labor, the faster steady state innovation. One should, however, bear in mind that more resources are not conductive to innovation in all economic structures (see Grossman and Helpman (1989a,d) on this point).

Next we consider the effects of resources on the time pattern of innovation. Suppose that initially the world is in a steady state, say at point 1 in Figure 2. Trajectory A in Figure 3 describes the time pattern of its innovation rate. Now country 1 -- which has a comparative disadvantage in R&D -- experiences an unexpected permanent increase in labor supply L_1. In this case the $\dot{\sigma}_1 = 0$ line shifts to the right. The $\dot{g} = 0$ curve shifts upwards or downwards at the initial value of σ_1, depending on whether $\alpha < \Sigma_j H_j a_j^{\alpha}/Ha_1^{\alpha}$ or $\alpha > \Sigma_j H_j a_j^{\alpha}/Ha_1^{\alpha}$. In either case, however, the new steady state point is to the North-East of 1 (because the long run rate of innovation increases). The new saddle path is located above the new $\dot{g} = 0$ curve to the left of the new steady state. It should therefore be clear from the dynamics depicted in the figure that in the former case the rate of innovation rises on impact, and increases gradually thereafter together with the share of country 1 in the available products, until they attain new steady state values. Hence, in this case the innovation rate is higher and increasing monotonically over time, as depicted by trajectory B in Figure 3.

In the latter case the rate of innovation may rise or decline on impact. If it rises its time pattern is the same as B in Figure 3. If it declines it remains lower than the original innovation rate for some time. However, since it increases over time until it attains the new steady state value that features faster innovation, it eventually becomes higher than the original rate of innovation and remains higher thereafter. Curve C in Figure 3 describes a trajectory of this nature.

When the labor force of country 2 increases unexpectedly the $\dot{\sigma} = 0$ line shifts to the left and the $\dot{g} = 0$ curve shifts upwards. The new steady state point is North-West of 1, say at point 2. Consequently, since to the right of the new steady state the new saddle path is located above the new $\dot{g} = 0$ curve, the rate of innovation rises on impact, overshoots its long run level, and declines thereafter together with σ_1 until the new steady state is reached, as depicted by the arrowed path leading to point 2. Trajectory D in Figure 3 describes its time pattern.

Our discussion shows clearly that as simple as this two-country model may be it generates rich dynamics. In particular, it shows that short run rates of innovation may overshoot or undershoot long run values, and therefore their time series may exhibit cyclical movements in response to unanticipated permanent shocks. The same also applies to anticipated shocks. To see the latter point consider a case in which initially the world economy is in steady state at point 1. Then it is unexpectedly announced that country 2's labor force L_2 will permanently increase at some future point in time. The response will be an increase in the innovation rate on impact. Afterwards the rate of innovation gradually increases while the composition of products remains constant, as shown by the vertical arrowed path in Figure 4. The initial jump in g ensures that the vertical arrowed path reaches the saddle path through point 2 exactly at the time of change in L_2 (otherwise there would be intertemporal arbitrage opportunities). From that point on the economy follows the arrowed saddle path to the long run equilibrium. Trajectory E in Figure 3 describes the resulting cyclical time pattern of the innovation rate.

IV. R&D Subsidies

Our next task is to investigate the effects of R&D subsidies. We take an R&D subsidy to be a subsidy to innovation. In particular, if gross costs of innovation per product equal $w_i a_i/n$ in country i, in the presence of a subsidy a fraction s_i of these costs are born by the government. Consequently net costs to a product developer equal $(1-s_i)w_ia_i/n$. The general question addressed in this section is: How does an innovation subsidy affect the time pattern of innovation? And more specifically:

(a) How does it affect the long run innovation rates?

(b) The short run innovation rates?

(c) How do the answers to the previous questions depend on whether the subsidizing country has comparative advantage in R&D?

In order to deal with these questions we need to extend the two-country model. We do it by following its development in the previous section. First observe that the presence of subsidies does not change the pricing equation (7'), the profit equation (8') and the no arbitrage condition (9'), because these do not depend directly on the structure of innovation costs. The no arbitrage condition (10') builds, however, on innovation costs. In the presence of subsidies it needs to be modified to

$$n\frac{(1-\alpha)}{\alpha}\,\frac{p_i^{-\epsilon}/(1-s_i)a_i}{\Sigma_j n_i p_j^{1-\epsilon}} + \dot{p}_i/p_i - g = \rho. \tag{23}$$

On the other hand, the resource constraint equation (11') does not depend on innovation costs and subsidies do not affect it. I reproduce it here for convenience:

$$(24) \qquad a_i\dot{n}_i/n + n_i p_i^{-\epsilon}/\Sigma_j n_j p_j^{1-\epsilon} = L_i.$$

Now note from (23) that the relative price p_i/p_j, $i \neq j$, converges to zero or infinity unless $p_i^{-\epsilon}/(1-s_i)a_i = p_j^{-\epsilon}/(1-s_j)a_j$ initially, and therefore also at each point in time. Consequently, on an equilibrium trajectory with perfect foresight there exists a function $q(t)$ such that

$$(25) \qquad p_i = q[(1-s_i)a_i]^{-1/\epsilon} \quad \text{for all } t.$$

Hence, as before, the rate of change of every price equals the rate of change of q, but this time

$$(26) \qquad \dot{q}/q = \rho + g - (1-\alpha)/\alpha q\Sigma_j\sigma_j(1-s_j)^{\alpha}a_j^{\alpha}.$$

Using (24) and (25) we calculate the rates of change of n_i and n:

$$(27) \qquad g_i = H_i/\sigma_i - (1-s_i)/q\Sigma_j\sigma_j(1-s_j)^{\alpha}a_j^{\alpha},$$

$$(28) \qquad g = H - \Sigma_j\sigma_j(1-s_j)/q\Sigma_j\sigma_j(1-s_j)^{\alpha}a_j^{\alpha},$$

which imply:

$$(29) \qquad \dot{\sigma}_i = H_i - \sigma_i\left\{(1-s_i)(H-g)/\Sigma_j\sigma_j(1-s_j) + g\right\} \quad \text{for} \quad 0 \le \sigma_i \le 1,$$

and together with (26) also imply

$$(30) \qquad \dot{g} = (H - g)\Big\{\rho + g + \Sigma_j H_j (1-s_j)^{\alpha} a_j^{\alpha} / \Sigma_j \sigma_j (1-s_j)^{\alpha} a_j^{\alpha}$$
$$- \Sigma_j H_j (1-s_j)/\Sigma_j \sigma_j (1-s_j) - (H - g)\Big[(1-\alpha)/\alpha$$
$$+ \Sigma_j \sigma_j (1-s_j)^{1+\alpha} a_j^{\alpha} / \Sigma_j \sigma_j (1-s_j)^{\alpha} a_j^{\alpha}$$
$$- \Sigma_j \sigma_j (1-s_j)^2 / \Sigma_j \sigma_j (1-s_j)\Big] / \Sigma_j \sigma_j (1-s_j)\Big\} \qquad \text{for} \quad 0 \le g \le H.$$

Using (29) for $i = 1$, $\sigma_2 = 1-\sigma_1$, and (30) we have a system of two differential equations that can be analyzed as before. Naturally, this system reduces to (21) and (22) when the subsidy rates equal zero in both countries. I continue to assume that country 2 has comparative advantage in R&D; i.e., $a_1 > a_2$.

The following analysis deals with the case of small innovation subsidies (large innovation subsidies require a messy analysis). Suppose that initially there are no subsidies at all and the world economy advances on a steady state trajectory with product shares $\sigma_i = H_i/H$ and innovation rate $g = (1-\alpha)H - \alpha\rho$. Then, the government of one of the countries unexpectedly provides a subsidy to its innovators, thereby bearing a proportion s_i of their R&D costs, where $s_i > 0$ is close to zero. A direct calculation from (29) shows that the new $\dot{\sigma}_1 = 0$ curve is downward slopping and located to the right of the old curve when country 1 (with comparative disadvantage in R&D) stimulates innovation. The curve slopes upwards and is located to the left of the old curve when country 2 stimulates innovation. It also shows that the steady state share of country 1 in available products, σ_1, increases in the former case and declines in the latter.

Using (30) we calculate shifts of the $\dot{g} = 0$ curve. At the initial steady state product share the curve shifts up in response to a subsidy in country i if and only if $\alpha < \Sigma_j H_j a_j^\alpha / H a_i^\alpha$. This inequality holds when country 2 (with comparative advantage in R&D) encourages innovation, because in this case the right hand term exceeds one. It may, however, be violated when country 1 encourages innovation. Hence, when country 2 subsidizes R&D the $\dot{g} = 0$ curve shifts upwards, while it may shift upwards or downwards when country 1 subsidizes R&D. Nevertheless, in all cases the post subsidy steady state innovation (and growth) rate exceed the original innovation (and growth) rate. The last point can be seen from the fact that combining the stationarity conditions that result from (29) and (30) one obtains

$$\rho + g = (1-\alpha)(H-g)/\alpha\Sigma_j \sigma_j (1-s_j).$$

This equation implies that the first order effect of s_i on the rate of innovation is positive when evaluated at $s_1 = s_2 = 0$.

The dynamics that result from a subsidy in country 2 are presented in Figure 5. Point 1 represents the original steady state. The broken line curves represent subsidy inclusive $\dot{g} = 0$ and $\dot{\sigma}_1 = 0$ curves. Their intersection at point 2 represents the new steady state. Since the composition of products cannot change at a point in time (but rather through a time consuming innovation processes), σ_1 remains constant on impact. On the other hand the rate of innovation adjusts on impact in order to reach the saddle path that leads to the new steady state. Therefore the rate of innovation rises on impact to the arrowed path -- that represents the new saddle path -- and follows it thereafter. We conclude that an R&D subsidy in country 2 brings about an immediate increase in the rate of innovation up to a level that overshoots the new long run rate of innovation, and the rate of innovation declines thereafter until it converges to the new steady state. At the same time the rate of innovation in country 2 exceeds the rate of innovation in country 1, which feeds the decline of country 1's product share. The cross country difference in innovation rates declines over time, however, until it disappears in the steady state. Naturally, on the new trajectory the average rate of innovation is higher than in the original situation at each point in time. If line A in Figure 3 describes the time pattern of the rate of innovation in the absence of R&D subsidies, then the time pattern of innovation in the presence of an R&D subsidy in country 2 can be described by curve D.

Figure 6 depicts the response to an R&D subsidy in country 1. Point 1 is the original steady state. The broken line curves describe the post subsidy $\dot{g} = 0$ and $\dot{\sigma}_1 = 0$ curves. If $\alpha < \Sigma_j H_j a_j^\alpha / H a_i^\alpha$ the relevant $\dot{g} = 0$ curve passes above point 1, say through point 3. If $\alpha > \Sigma_j H_j a_j^\alpha / H a_i^\alpha$ it passes below point 1, say through point 4. In either case the saddle path trajectory approaching point 2 slopes upwards to the left of 2 and lies above the relevant $\dot{g} = 0$ curve. In the former case the rate of innovation increases on impact and rises thereafter together with σ_1 until they reach point 2. This means that country 1 innovates at a faster pace than country 2, thereby gaining larger shares of products. This case is described by curve B in Figure 3. In the latter case the rate of innovation may increase or decline on impact, but even when it declines it cannot decline below point 4. If the rate of innovation increases on impact the time pattern of innovation is similar to B in Figure 3. If the rate of innovation declines on impact, however, there will be a time interval during which the post subsidy rate of innovation is lower than the original rate of innovation. Afterwards the subsidy induced rate of innovation exceeds the original level. The resulting time pattern is as described by curve E in Figure 3. It is evident from this analysis that small R&D subsidies affect the rate of innovation in a similar way as country size.

V. Summery and Conclusions

Much of economic growth has been driven by inventions and innovations. It is therefore important to study their determinants. In an international environment in which countries trade and learn from each other, these activities affect their R&D performance through various channels, such as through intentional and unintentional technology transfer. They also affect its resource allocation, including the allocation of resources to R&D. For all these reasons the rate of innovation of a country depends on its own features as well as on features of its trading partners.

I have studied in this paper the determinants of innovation rates in a simple international environment. The emphasis has been on the role of resources and R&D subsidies in the determination of short and long run rates of innovation. We have seen that out of steady state the world's average rate of innovation may increase or decline over time, depending on whether the country with comparative advantage in R&D has a disproportionately large or small share of existing products (where the disproportionality is measured relative to the effective resource base). In the former case its innovation rate falls short of the innovation rate of its trading partner, thereby tilting the composition of products in favor of the country with comparative disadvantage in R&D. In the latter case the country with comparative advantage in R&D innovates faster, tilting the composition of products in its own favor.

Starting from a steady state, an increase in the resource base of a country with comparative advantage in R&D brings about an immediate upward jump in the average rate of innovation and to its gradual decline thereafter. Nevertheless the average rate of innovation is larger at each point in time than in the initial steady state. Not only does the larger resource base of a country with comparative advantage in R&D speed up the world's innovation, but its own innovation rate becomes larger than its trading partner's. This leads naturally to a gradual increase in its share of available products.

An increase in the resource base of a country with comparative disadvantage in R&D brings about an increase in the long run average rate of innovation. The short run rate may rise or fall, however. In either case, following the adjustment on impact, the world's innovation rate gradually increases. While this gradual adjustment process takes place the country with comparative disadvantage in R&D innovates at a faster pace and increases its share of available products. If the average innovation rate rises on impact it remains higher forever. Otherwise it remains lower for a limited period of time and becomes higher thereafter.

A small innovation subsidy in the country with comparative advantage in R&D leads to a response of innovation rates that mimic the response of a resource expansion in that country, while an innovation subsidy in the country with comparative disadvantage in R&D leads to a response of the innovation rates that mimics the response of a resource expansion in that country.

These results show that rates of innovation can exhibit complicated patterns. Even in the simple model employed in this paper innovation rates can respond in the short run to structural shifts or economic policies by undershooting or overshooting long run values. They may also rise or decline over time thereafter. A country with comparative advantage in R&D is not guaranteed to innovate faster than its trading partner; the relative speeds of innovation depend on the relationship between the shares of existing products and relative resources. A country whose relative resource base exceeds the relative number of its available products innovates faster than its trading partner independently of whether it has comparative advantage in R&D.

The results reported in this paper are of course not conclusive but rather indicative. They have been derived from a model that is much too simple to have reliable empirical predictions. They show, however, what may happen. We have identified a number of mechanisms that are also relevant in more complicated situations. Further work is needed in order to examine their relative importance.

References

Aghion, Philippe and Howitt, Peter (1989), "A Model Growth Through Creative Destruction," mimeo.

Arrow, Kenneth J. (1962), "The Economic Implications of Learning by Doing," Review of Economic Studies 29, pp.155-173.

Barro, Robert J. (1989), "A Cross-Country Study of Growth, Saving, and Government," Working Paper No. 2855, National Bureau of Economic Research.

Becker, Gary S. and Murphy, Kevin M. (1988), "Economic Growth, Human Capital and Population Growth," presented at the SUNY-Buffalo Conference on "The Problem of Development".

Ethier, Wilfred J. (1982), "National and International Returns to Scale in the Modern Theory of International Trade," American Economic Review 72, pp.389-405.

Freeman, Christopher (1982), The Economics of Industrial Innovation, 2nd Edition (Cambridge, MA: MIT Press).

Griliches, Zvi (1979), "Issues in Assessing the Contribution of Research and Development in Productivity Growth," Bell Journal of Economics 10, pp.92-116.

Grossman, Gene M. (1989), "Explaining Japan's Innovation and Growth: A Model of Quality Competition and Deynamic Comparative Advantage," mimeo.

Grossman, Gene M. and Helpman, Elhanan (1989a),"Comparative Advantage and Long-Run Growth," Working Paper No. 2809, National Bureau of Economic Research. Forthcoming in the American Economic Review.

——— (1989b), "Endogenous Product Cycles," Working Paper No. 2913, National Bureau of Economic Research.

——— (1989c), "Growth and Welfare in a Small Open Economy," Working Paper No. 2970, National Bureau of Economic Research.

——— (1989d), "Quality Ladders in the Theory of Growth," Working Paper No.32-89, The Foerder Institute for Economic Research.

——— (1989e), "Quality Ladders and Product Cycles," in preparation.

Levhari, David (1966), "Further Implications of Learning by Doing," Review of Economic Studies 93, pp.31-38.

Lucas, Robert E. Jr. (1988), "On the Mechanics of Economic Development," Journal of Monetary Economics 22, pp.3-42.

Maddison, Angus (1987), "Growth and Slowdown in Advanced Capitalist Economies: Techniques of Quantitative Assessment," Journal of Economic Literature 25, pp.649-698.

Ohyama, Michihiro (1989), "Human Capital and Endogenous Economic Growth," Discussion Paper No. 8902, Keio Economic Society.

Romer, Paul M. (1986), "Increasing Returns and Long-Run Growth," Journal of Political Economy 94, pp.1002-1037.

——— (1988), "Endogenous Technological Change," presented at the. SUNY-Buffalo Conference on "The Problem of Development".

Sheshinski, Eitan (1967), "Optimal Accumulation with Learning by Doing," in Shell, Karl (ed.), Essays on the Theory of Optimal Economic Growth (Cambridge: The MIT Press).

Solow, Robert M. (1957), "Technical Change and the Aggregate Production Function," Review of Economic and Statistics 39, pp.312-320.

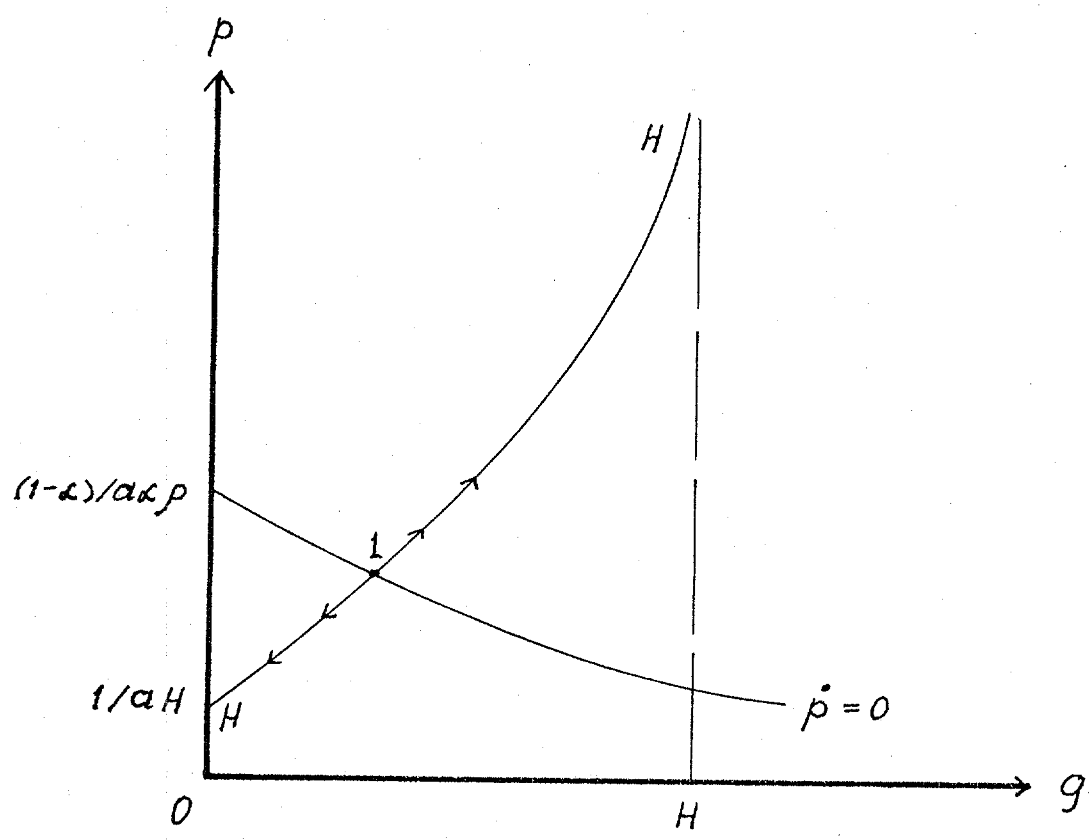


Figure 1

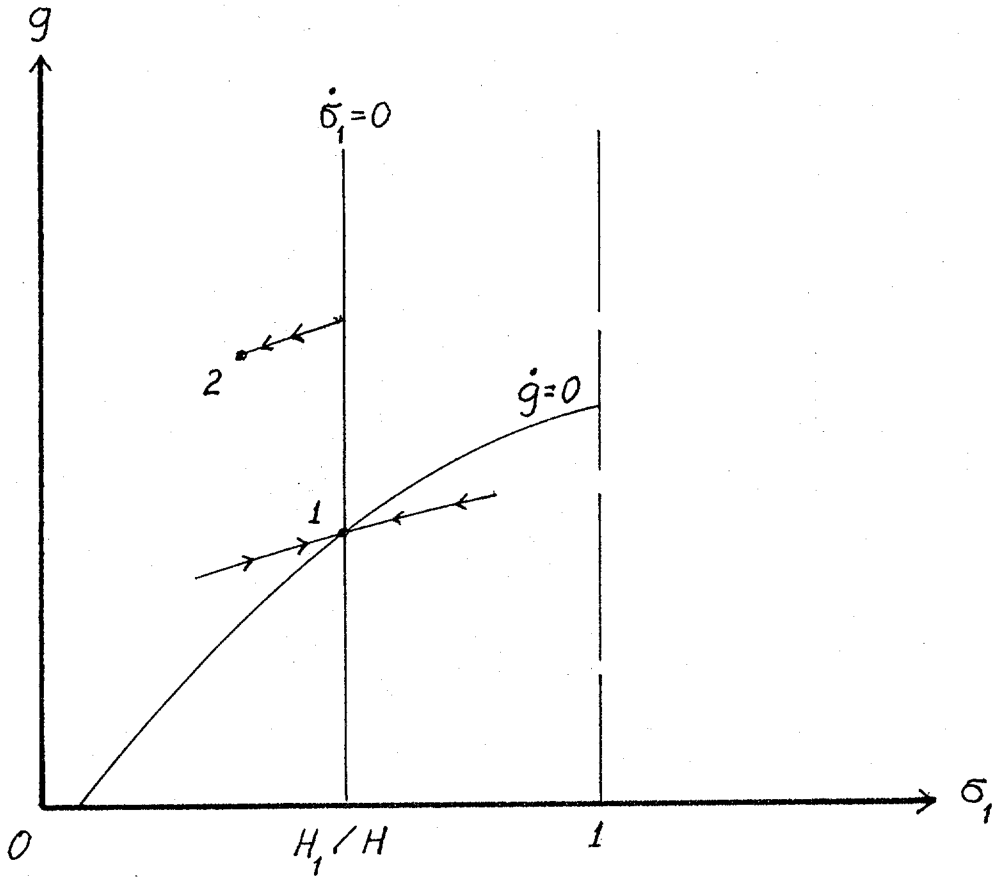


Figure 2

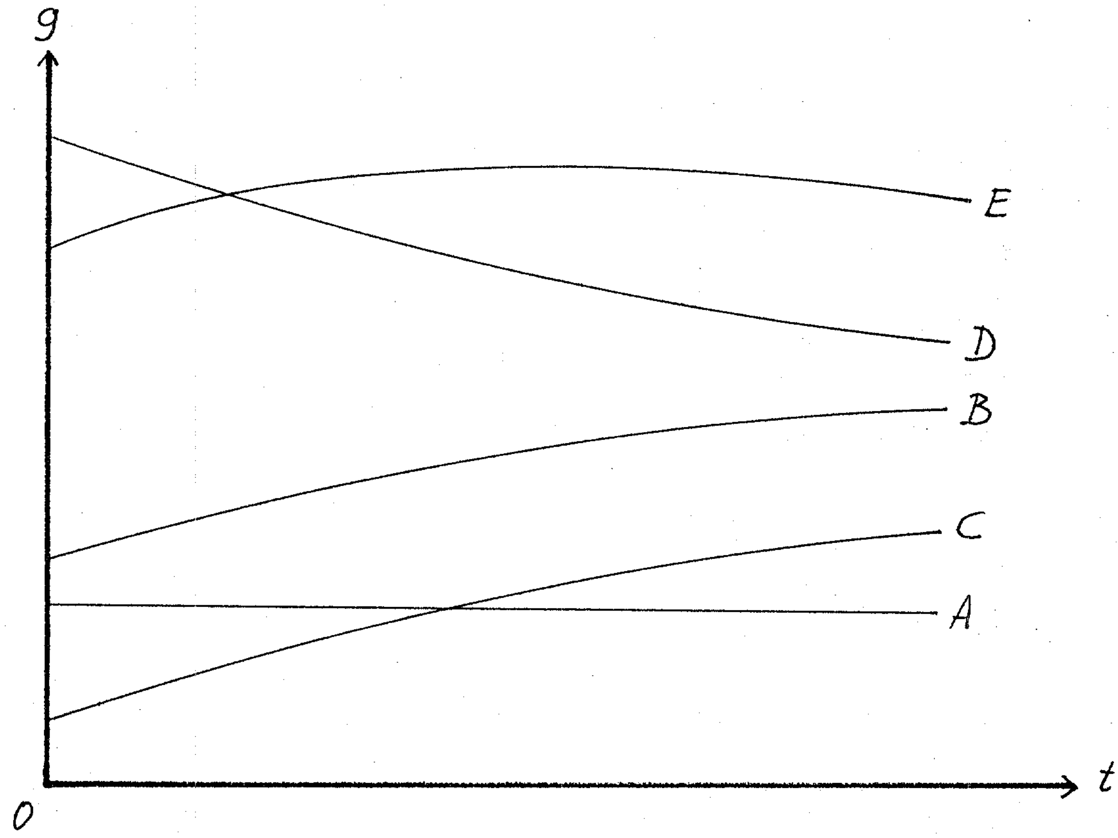
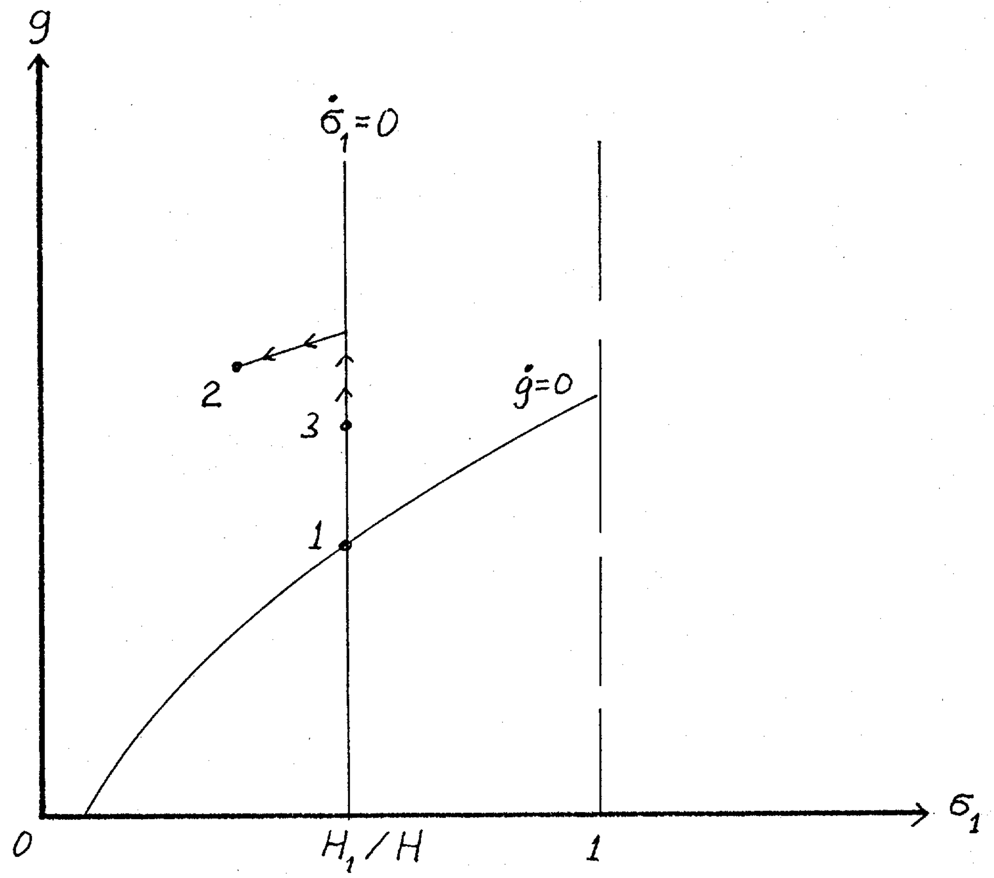


Figure 3



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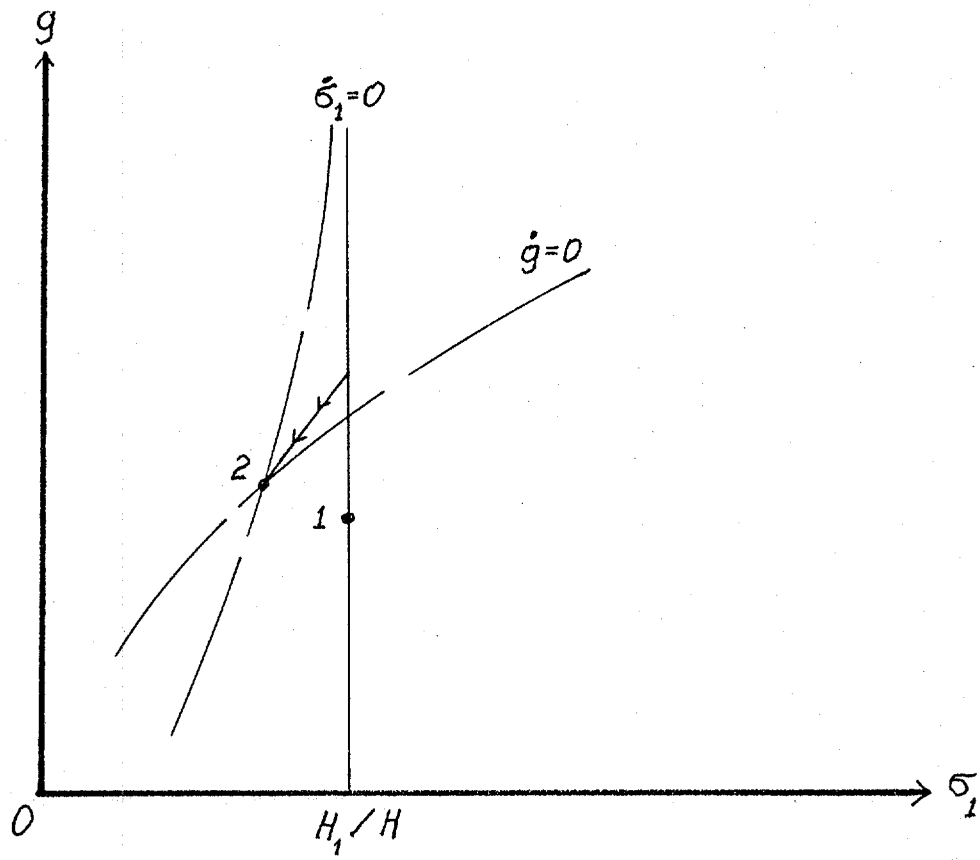

Figure 5

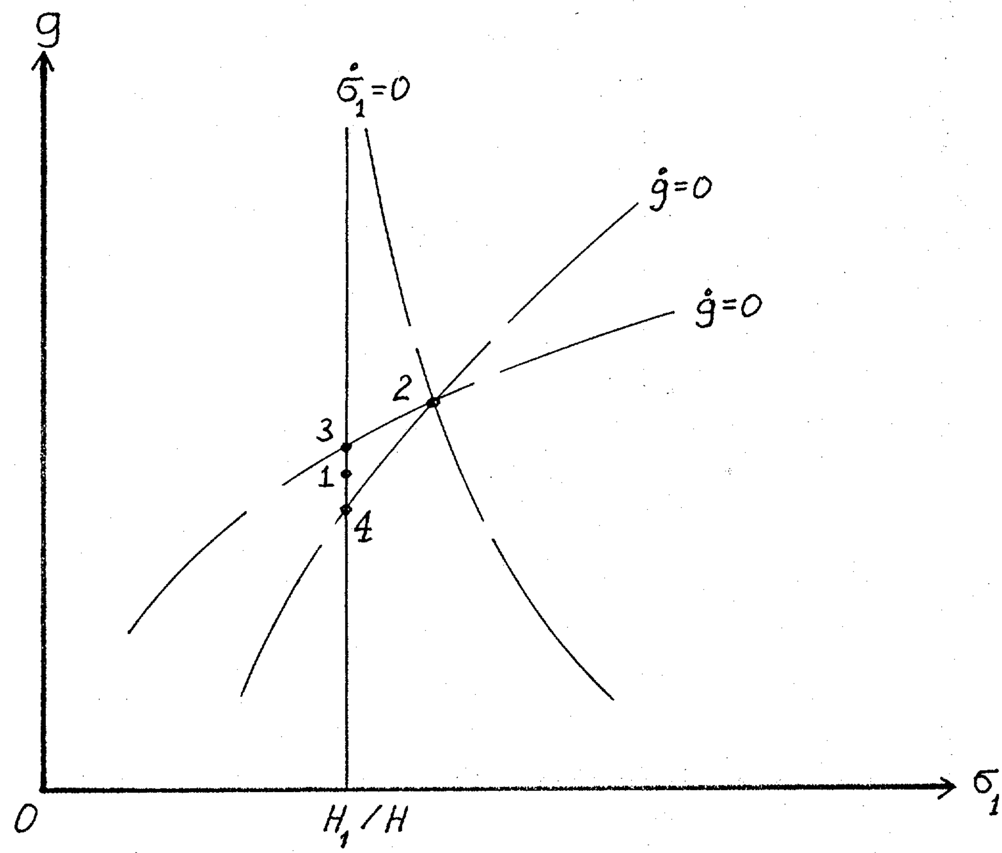

Figure 6