Technical appendices: Business cycle accounting for the Japanese economy using the parameterized expectations algorithm

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Introduction.

Inaba (2007a) apply the parameterized expectations algorithm (PEA hereafter) to business cycle accounting (BCA hereafter).

The idea of BCA developed by Chari, Kehoe and McGrattan (2002, 2004, 2007) is to assess which wedge is important for the fluctuation of an economy which is assumed to be described as a prototype model with time-varying wedges. These wedges resemble productivity, labor and investment taxes, and government consumption. Since these wedges are measured using the production function and first order conditions to fit the actual macroeconomic data, this method can be interpreted as a generalization of growth accounting.

The PEA introduced by Marcet (1988) is one of the methods to solve the non-linear dynamic stochastic general equilibrium model. Marcet and Lorenzoni (1998) provide applications of PEA to some economic models. The basic idea of the PEA is to approximate the expectation function by a smooth function, in general a polynomial function. The PEA has an advantage[1] that it is simpler and easier to understand and implement than the other non-linear solution methods.[2]

The prototype model

This section describes the prototype model with time-varying wedges: the efficiency wedge A_t, the labor wedge $1 - \tau_{l,t}$, the investment wedge $1/(1 + \tau_{x,t})$,

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[1] There is also a disadvantage that the PEA need a long simulation in order to obtain the fitted coefficients of the approximating function. Therefore the algorithm can be quite computationally demanding.

[2] Chari et al. (2004, 2007) implement BCA using the finite element method for the non-linear solution described by McGrattan(1996).

and the government wedge g_t.

The household maximizes:

$$\max_{c_t, k_{t+1}, l_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t, l_t) N_t \right]$$

subject to

$$c_t + (1 + \tau_{x,t}) \left\{ \frac{N_{t+1}}{N_t} k_{t+1} - k_t \right\} = (1 - \tau_{l,t}) w_t l_t + r_t k_t + T_t, \quad 0 < \beta < 1,$$

where c_t denotes consumption, l_t employment, N_t population, k_t capital stock, w_t the wage rate, r_t the rental rate on capital, T_t the lump-sum taxes per capita. All quantities written in lower case letters denote per-capita quantities except for T_t.

The firm maximizes

$$\max_{k_t, l_t} A_t F(k_t, \gamma^t l_t) - \{r_t + (1 + \tau_{x,t})\delta\} k_t - w_t l_t,$$

where δ denotes the depreciation of capital stock and γ the balanced growth rate of technical progress. The resource constraint is

$$c_t + x_t + g_t = y_t, \tag{1}$$

where x_t is investment, g_t the government consumption and y_t the per-capita output. The law of motion for capital stock is

$$\frac{N_{t+1}}{N_t} k_{t+1} = (1 - \delta) k_t + x_t. \tag{2}$$

The equilibrium is summarized by the resource constraint (1), the law of motion for capital (2), the production function,

$$y_t = A_t F(k_t, \gamma^t l_t), \tag{3}$$

and the first-order condtions,

$$-\frac{U_{l,t}}{U_{c,t}} = (1 - \tau_{l,t}) A_t \gamma^t F_{l,t}, \tag{4}$$

$$U_{c,t}(1 + \tau_{x,t}) = \beta E_t U_{c,t+1} \left[A_{t+1} F_{k,t+1} + (1 - \delta)(1 + \tau_{x,t+1}) \right], \tag{5}$$

where U_{ct}, U_{lt}, F_{lt} and F_{kt} denote the derivatives of the utility function and the production function with respect to their arguments. The functional form of the utility function is given by $U(c, l) = \ln c + \phi \ln(1 - l)$, where $\phi > 0$ is a parameter. Also the functional form of the production function is given by $F(k, l) = k^\alpha l^{1-\alpha}$.

A Accounting procedure

This section provide the accounting procedure to measure actual wedges using PEA.

A.1 Measuring the wedges

We take the government wedge g directly from the data. To obtain the values of the other wedges, we use the data for y_t, l_t, x_t, g_t and N_t, together with a series on k_t constructed from x_t by (2). The efficiency wedge and the labor wedge are directly calculated from (3) and (4). In this paper, to find the actual investment wedge $\tau_{x,t}$, we implement the following algorithm [3].

Algorithm for measuring the wedges

- **Initialization**: *Apply the deterministic method*[4] *of business cycle accounting as described in Kobayashi and Inaba (2006), and regard the derived investment wedge as the initial value of* $\tau_{x,t}^{(0)}$, *and set a stopping parameters* $\epsilon > 0$

- **Step 1**: *Specify a vector AR1 process for the four wedges* $s_t = (\log(A_t), \tau_{l,t}, \tau_{x,t}^{(j)}, \log(g_t))$ *of the form*
$$s_{t+1} = P_0 + P s_t + \eta_{t+1}, \tag{6}$$
where $\eta_t \sim$ *i.i.d.* $N(0, \Omega)$.

- **Step 2**: *Apply the parameterized expectation algorithm to get the non-linear solution of the model. Then we get an approximation function* $\Phi(\cdot)$ *for the expectation function:*
$$E_t U_{c,t+1} \left\{ A_{t+1} F_{k,t+1} + (1-\delta)(1+\tau_{x,t+1}^{(j)}) \right\}.$$
$\Phi(\cdot)$ *is a polynomial function of* k_t, A_t, $\tau_{l,t}$, $\tau_{x,t}^{(j)}$ *and* g_t.

- **Step 3**: *To find the value of* $\hat{\tau}_{x,t}$ *in order to realize the actual data,* c_t *and* l_t, *solve the following equation for* $\hat{\tau}_{x,t}$,
$$U_{c,t}(1+\hat{\tau}_{x,t}) = \Phi(k_t, A_t, \tau_{l,t}, \hat{\tau}_{x,t}, g_t) \tag{7}$$

- **Step 4**: $\tau_{x,t}^{(j+1)} = \nu \hat{\tau}_{x,t} + (1-\nu)\tau_{x,t}^{(j)}, \quad 0 < \nu < 1.$

- **Step 5**: *if* $\| \tau_{x,t}^{(j+1)} - \tau_{x,t}^{(j)} \| < \epsilon$, *STOP; else go to step 1.*

We will explain Step 1 and Step 2 in detail.

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[3] The main difference from the accounting procedure of Chari, Kehoe and McGrattan (2007) is the method to solve the non-linear dynamic stochastic general equilibrium model. While we use the PEA, they use the finite element method described by McGrattan (1996) to solve the model.

[4] For details, see technical appendices Inaba (2007b)

A.2 Estimation for the stochastic process of wedges

In Step 1 above, the OLS estimation of this stochastic process can be non-stationary. We then use the maximum likelihood procedure with a penalty function described in McGrattan (1994) to estimate the parameters P_0, P of the vector AR1 process for the wedges. To ensure stationarity, we add to the likelihood function a penalty term proportional to $\{\max(|\lambda_{\max}| - 0.99, 0)\}^2$, where $\lambda_{\max}$ is the maximal eigenvalue of P. If $\lambda_{\max} < 0.99$, we use the OLS estimation. The detail of this algorithm is following.

Algorithm for Maximum Likelihood with a penalty function.

- **Initialization**: *Specify a vector AR1 process for the four wedges of the form*

$$s_{t+1} = P_0 + P s_t + \eta_{t+1}, \tag{8}$$

where $s_t = (\log(A_t), \tau_{l,t}, \tau_{x,t}, \log(g_t))$ *and* $\eta_t \sim i.i.d.N(0, \Omega)$. *The log likelihood function with a penalty function:*

$$\begin{aligned}\mathscr{L}(P_0, P, \Omega) = &- \frac{Tn}{2}\log(2\pi) + \frac{T}{2}\log\left|\Omega^{-1}\right| \\ &- \frac{1}{2}\sum_{t=1}^{T}\left[(s_{t+1} - P_0 - Ps_t)'\,\Omega^{-1}\,(s_{t+1} - P_0 - Ps_t)\right] \\ &- \gamma * \{\max[|\lambda_{\max} - 0.99|, 0]\}^2.\end{aligned}$$

where $\gamma > 0$ *is a parameter. If* $\lambda_{\max} < 0.99$, *the log-likelihood function is maximized, when* P *is a OLS estimator* $\hat{P}$ *and* Ω *is* $\hat{\Omega} = \frac{1}{T}\sum_1^T (s_{t+1} - \hat{P}_0 - \hat{P}s_t)(s_{t+1} - \hat{P}_0 - \hat{P}s_t)'$, *then STOP; else set the initial value of* Ω *is* $\Omega^{(0)} = \hat{\Omega}$ *go to next step.*

- **Step 2**: *Given* $\Omega^(j)$, *set*

$$P_0^{(j)} = \arg\max_{P_0} \mathscr{L}\left(P_0, P, \Omega^{(j)}\right)$$

$$P^{(j)} = \arg\max_{P} \mathscr{L}\left(P_0, P, \Omega^{(j)}\right)$$

.

- **Step 3**: *Given* $P_0^{(j)}$ *and* $P^{(j)}$, *set*

$$\begin{aligned}\Omega^{(j+1)} &= \arg\max_{\Omega} \mathscr{L}\left(P_0^{(j)}, P^{(j)}, \Omega\right) \\ &= \frac{1}{T}\sum_1^T (s_{t+1} - P_0^{(j)} - P^{(j)}s_t)(s_{t+1} - P_0^{(j)} - P^{(j)}s_t)'\end{aligned}$$

- **Step 3**: *if* $\|\Omega^{(j+1)} - \Omega^{(j+1)}\| < \epsilon$, *where* $\epsilon > 0$ *is a parameter, STOP; else go to step 2.*

A.3 The parameterized expectations algorithm with the moving bound

We use the PEA in step 2 for measuring the wedges. But it is well known that the main drawback of the PEA is that it is not a contraction mapping technique and does not guarantee a solution will be find. Therefore, we modified the PEA following Maliar and Maliar (2003). They discuss a moving bounds method of imposing stability on the PEA to avoid the explosive case due to poor initial parameter values and achieve the enhancement of the convergence property of the PEA. We show the PEA algorithm with the moving bounds following Marcet and Lorenzoni (1998) and Maliar and Maliar (2003).

Consider an economy, which is described by a vector of n variables, z_t, and a vector of w exogenously given shocks, u_t. It is assumed that the process $\{z_t, u_t\}$ is represented by a system

$$g\left(E_t\left[\phi(z_{t+1}, z_t)\right], z_t, z_{t-1}, u_t\right), \quad \text{for all } t, \tag{9}$$

where $g : R^m \times R^n \times R^n \times R^w \to R^q$ and $\phi : R^{2n} \to R^m$; the vector u_t includes all endogenous variables that are inside the expectation, and s_t follows a first-order Markov process. It is assumed that u_t is uniquely determined by (9) if the rest of the arguments is given.

We consider only a recursive solution such that the conditional expectation can be represented by a time-invariant function $\Phi(\mathbf{x}_t) = E_t\left[\phi(z_{t+1}, z_t)\right]$, where $\mathbf{x}_t$ is a finite-dimensional subset of (z_{t-1}, u_t). If the function $\Phi(\cdot)$ cannot derived analytically, we approximate $\Phi(\cdot)$ by a parametric function $\psi(\beta, x)$, $\beta \in R^\nu$. The objective will be to find β^* such that $\phi(\beta^*, \mathbf{x})$ is the best apprication to $\Phi(\mathbf{x})$ given the functional form $\psi(\cdot)$,

$$\beta^* = \arg \min_{\beta \in R^\nu} \|\psi(\beta, x) - \Phi(\mathbf{x})\|.$$

The iterative procedure is as follows.

The parameterized expectation algorithm with the moving bound

- **Initialization**: *Set* $z_t = (c_t, l_t, k_{t+1}, s_t), u_t = s_t$ *and* $\mathbf{x}_t = (k_t, s_t)$. *The function* g *is given by the resouce constraint (1) and the first-order conditions (4), (5). The function* $\phi(z_{t+1}, z_t) \equiv U_{c,t+1}\left\{A_{t+1}F_{k,t+1} + (1-\delta)(1+\tau_{x,t+1})\right\}$. *We set the approximation function of* $\Phi(\cdot)$ *as*

$$\begin{aligned}\psi(\beta, \mathbf{x}) = &\exp(\beta_0 + \beta_1 \ln k_t + \beta_2 \ln A_t + \beta_3 \ln \tau_{l,t} + \beta_4 \ln \tau_{x,t} + \beta_5 \ln g_t \\ &+ \beta_6 (\ln k_t)^2 + \beta_7 (\ln A_t)^2 + \beta_8 (\ln \tau_{l,t})^2 + \beta_9 (\ln \tau_{x,t})^2 + \beta_{10} (\ln g_t)^2 \\ &+ \beta_{11} \ln k_t \ln A_t + \beta_{12} \ln k_t \ln \tau_{l,t} + \beta_{13} \ln k_t \ln \tau_{x,t} + \beta_{14} \ln k_t \ln g_t \\ &+ \beta_{15} \ln A_t \ln \tau_{l,t} + \beta_{16} \ln A_t \ln \tau_{x,t} + \beta_{17} \ln A_t \ln g_t + \beta_{18} \ln \tau_{l,t} \ln \tau_{x,t} \\ &+ \beta_{19} \ln \tau_{l,t} \ln g_t + \beta_{20} \ln \tau_{x,t} \ln g_t).\end{aligned}$$

  For an initial iteration $i = 0$, *fix initial value* $\beta^{(0)} \in R^\nu$. *Fix the upper and lower bounds,* $\underline{k}^{(i)}$ *and* $\bar{k}^{(i)}$, *for the process* $\{k_t(\beta)\}$. *Fix initial conditions* k_0; *draw and fix a random series* $\{s_t\}_{t=1}^T$ *from a given distribution,* *where T is a sufficiently long period so that the series show their stochastic property.*

- **Step 1**: *Replace the conditional expectation in (9) with a function $\phi(\beta^{(i)}, \mathbf{x})$ and compute the inverse of (9) with respect to the second argument to obtain*
$$k_{t+1} = h\left(\phi(\beta^{(i)}, \mathbf{x}_t(\beta^{(i)})), k_t, s_t\right). \quad (10)$$

- **Step 2**: *For a given $\beta^{(i)} \in R^\nu$ and given bounds $\underline{k}$ and $\bar{k}$, recursively calculate $\left\{k_t(\beta^{(i)}), s_t\right\}_{t=1}^{T}$ according to*
$$\begin{aligned} k_{t+1}(\beta^{(i)}) &= \underline{k}^{(i)} && \text{if } k_t(\beta^{(i)}) \geq \underline{k}^{(i)}, \\ k_{t+1}(\beta^{(i)}) &= \bar{k}^{(i)} && \text{if } k_t(\beta^{(i)}) \leq \bar{k}^{(i)}, \\ k_{t+1}(\beta^{(i)}) &= h\left(\phi(\beta^{(i)}, x_t(\beta^{(i)})), k_t, s_t\right) && \text{if } \underline{k}^{(i)} < k_{t+1}(\beta^{(i)}) < \bar{k}^{(i)}. \end{aligned}$$

- **Step 3**: *Find a $G(\beta)$ that satisfies*
$$G(\beta^{(i)}) = \arg\max_{\xi \in R^\nu} \|\phi\left(k_{t+1}(\beta^{(i)})\right) - \psi\left(\xi, \mathbf{x}_t(\beta^{(i)})\right)\|.^5$$

- **Step 4**: *Compute the vector $\beta(i+1)$ for the next iteration,*
$$\beta^{(i+1)} = (1-\mu)\beta^{(i)} + \mu G(\beta^{(i)}), \quad \mu \in (0,1).$$

- **Step 5**: *compute $\underline{k}^{(i+1)}$ and $\bar{k}^{(i+1)}$ for the next iteration,*
$$\begin{aligned} \underline{k}^{(i+1)} &= \underline{k}^{(i)} - \underline{\Delta}^{(i)}, \\ \bar{k}^{(i+1)} &= \bar{k}^{(i)} + \bar{\Delta}^{(i)}, \end{aligned}$$
where $\underline{\Delta}^{(i)}$ and $\bar{\Delta}^{(i)}$ are the corresponding steps.

- **Step 6**: *If $\|\beta^* - G(\beta^*)\| < \epsilon$, where $\epsilon > 0$ is a parameter, and $\underline{k} < k_t(\beta) < \bar{k}$ for all t, STOP; else go to Step 2.*

B Decomposition

In an early version staff paper of Chari, Kehoe and McGrattan (2004), their decomposition method is different from published paper version.[6] Chari, Kehoe and McGrattan (2007b) explain the difference between the CKM (2004) decomposition and CKM (2007) decomposition. [7]

[5] To perform this, one can run a nonlinear least squares regression with the sample $\left\{k_t(\beta^{(i)}), s_t\right\}_{t=1}^{T}$, taking $\phi(k_{t+1}(\beta^{(i)}))$ as a dependent variable, $\phi(\cdot)$ as an explanatory function, and ξ as a parameter vector to be estimated.

[6] Now the staff paper is revised in 2006 and the decomposition method is the same as the published paper.

[7] Our explanation is somewhat different from Chari, Kehoe and McGrattan (2007a). While they assume that the economy experiences one of finitely many events s_t at each period t in the prototype model, we assume that s_t is subject to a VAR(1) process.

B.1 CKM (2004) decomposition

This is the early version of decomposition.

Specify a vector AR1 process for the four wedges of the form

$$s_{t+1} = P_0 + P s_t + \eta_{t+1}, \tag{11}$$

where $s_t = (\log(A_t), \tau_{l,t}, \tau_{x,t}, \log(g_t))$ and $\eta_t \sim$ i.i.d.$N(0, \Omega)$.

B.1.1 The efficiency wedge components in CKM (2004)

Suppose that $y(s_t, k_t)$, $c(s_t, k_t)$, $l(s_t, k_t)$, and $x(s_t, k_t)$ denote the decision rules under (11). Define the efficiency component of the wedges by letting $s_{1t} = (\log A_t, \bar{\tau}_l, \bar{\tau}_x, \log \bar{g})$ be the vector of wedges in which, in period t, the efficiency wedge takes on it period t value while the other wedges take on constant values. We set the constant values to be the average values form 1984 to 1989, while CKM (2004) set the values to be the initial values of each wedges. Then, starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, $y_{1t} = y(s_{1t}, k_t)$, $c_{1t} = c(s_{1t}, k_t)$, $l_{1t} = l(s_{1t}, k_t)$, and $x_{1t} = x(s_{1t}, k_t)$ which we call the *efficiency wedge components* of output, consumption, labor, and investment.

B.1.2 The labor wedge components in CKM (2004)

Use the same decision rules, $y(s_t, k_t)$, $c(s_t, k_t)$, $l(s_t, k_t)$, and $x(s_t, k_t)$. Define the labor component of the wedges by letting $s_{2t} = (\log \bar{A}, \tau_{lt}, \bar{\tau}_x, \log \bar{g})$ be the vector of wedges in which, in period t, the labor wedge takes on it period t value while the other wedges take on constant values. Then, starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, $y_{2t} = y(s_{2t}, k_t)$, $c_{2t} = c(s_{2t}, k_t)$, $l_{2t} = l(s_{2t}, k_t)$, and $x_{2t} = x(s_{2t}, k_t)$ which we call the *labor wedge components* of output, consumption, labor, and investment.

B.1.3 The investment wedge components in CKM (2004)

Use the same decision rules, $y(s_t, k_t)$, $c(s_t, k_t)$, $l(s_t, k_t)$, and $x(s_t, k_t)$. Define the investment component of the wedges by letting $s_{3t} = (\log \bar{A}, \bar{\tau}_l, \tau_{xt}, \log \bar{g})$ be the vector of wedges in which, in period t, the investment wedge takes on it period t value while the other wedges take on constant values. Then, starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, $y_{3t} = y(s_{3t}, k_t)$, $c_{3t} = c(s_{3t}, k_t)$, $l_{3t} = l(s_{3t}, k_t)$, and $x_{3t} = x(s_{3t}, k_t)$ which we call the *investment wedge components* of output, consumption, labor, and investment.

B.1.4 The government wedge components in CKM (2004)

Use the same decision rules, $y(s_t, k_t)$, $c(s_t, k_t)$, $l(s_t, k_t)$, and $x(s_t, k_t)$. Define the government component of the wedges by letting $s_{4t} = (\log \bar{A}, \bar{\tau}_l, \bar{\tau}_x, \log g_t)$ be the vector of wedges in which, in period t, the government wedge takes on it period t value while the other wedges take on constant values. Then, starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, $y_{4t} = y(s_{4t}, k_t)$, $c_{4t} = c(s_{4t}, k_t)$, $l_{4t} = l(s_{4t}, k_t)$, and $x_{4t} = x(s_{4t}, k_t)$ which we call the *government wedge components* of output, consumption, labor, and investment.

B.2 CKM (2007a) decomposition

This is the published version of CKM decomposition which is called this decomposition a theoretically consistent decomposition in CKM (2007b). This decomposition can seem to be theoretically consistent to a deterministic BCA decomposition in Kobayashi and Inaba (2006).

Specify a vector AR(1) process for the four wedges of the form;

$$s_{t+1} = P_0 + P s_t + \eta_{t+1}, \tag{12}$$

where $s_t = (\log A_t, \tau_{lt}, \tau_{xt}, \log g_t)$ and $\eta_t \sim$ i.i.d.$N(0, \Omega)$.

B.2.1 The efficiency wedge components in CKM (2007a)

Assume one to one mapping function;

$$\log A^e(s_t) = \log A_t, \;\; \tau_l^e(s_t) = \bar{\tau}_l, \;\; \tau_x^e(s_t) = \tau_x, \;\text{ and }\; \log g^e(s_t) = \log \bar{g}. \tag{13}$$

To evaluate the effects of the efficiency wedge, we compute the decision rules for the efficiency wedge alone economy, denoted $y^e(s_t, k_t)$, $c^e(s_t, k_t)$, $l^e(s_t, k_t)$, and $x^e(s_t, k_t)$ under an exogenous stochastic process which is a combination with (11) and (13). Starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, y_t^e, c_t^e, l_t^e, and x_t^e which we call the *efficiency wedge components* of output, consumption, labor, and investment.

B.2.2 The labor wedge components in CKM (2007a)

Assume one to one mapping function;

$$\log A^l(s_t) = \log \bar{A}, \;\; \tau_l^l(s_t) = \tau_{lt}, \;\; \tau_x^l(s_t) = \bar{\tau}_x, \;\text{ and }\; \log g^l(s_t) = \log \bar{g}. \tag{14}$$

To evaluate the effects of the labor wedge, we compute the decision rules for the efficiency wedge alone economy, denoted $y^l(s_t, k_t)$, $c^l(s_t, k_t)$, $l^l(s_t, k_t)$, and $x^l(s_t, k_t)$ under an exogenous stochastic process which is a combination with (11) and (14). Starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, y_t^l, c_t^l, l_t^l, and x_t^l which we call the *labor wedge components* of output, consumption, labor, and investment.

B.2.3 The investment wedge components in CKM (2007a)

Assume one to one mapping function;

$$\log A^x(s_t) = \log \bar{A},\ \tau_l^x(s_t) = \bar{\tau}_l,\ \tau_x^x(s_t) = \tau_{xt},\ \text{and}\ \log g^x(s_t) = \log \bar{g}. \quad (15)$$

To evaluate the effects of the investment wedge, we compute the decision rules for the efficiency wedge alone economy, denoted $y^x(s_t, k_t)$, $c^x(s_t, k_t)$, $l^x(s_t, k_t)$, and $x^x(s_t, k_t)$ under an exogenous stochastic process which is a combination with (11) and (15). Starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, y_t^x, c_t^x, l_t^x, and x_t^x which we call the *investment wedge components* of output, consumption, labor, and investment.

B.2.4 The government wedge components in CKM (2007a)

Assume one to one mapping function;

$$\log A^g(s_t) = \log \bar{A},\ \tau_l^g(s_t) = \bar{\tau}_l,\ \tau_x^g(s_t) = \bar{\tau}_x,\ \text{and}\ \log g^g(s_t) = \log g_t. \quad (16)$$

To evaluate the effects of the government wedge, we compute the decision rules for the efficiency wedge alone economy, denoted $y^g(s_t, k_t)$, $c^g(s_t, k_t)$, $l^g(s_t, k_t)$, and $x^g(s_t, k_t)$ under an exogenous stochastic process which is a combination with (11) and (16). Starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, consumption, labor, and investment, y_t^g, c_t^g, l_t^g, and x_t^g which we call the *government wedge components* of output, consumption, labor, and investment.

B.3 Comparing decompositions

CKM (2004) decomposition shows the people's decision for the realized values of random variables, s_{it} at t for $i = 1, \cdots, 4$, where people expect that the exogenous random shocks are subject to (11). CKM (2007b) show that when P is not diagonal, the expected value of target wedge in CKM (2004) does not coincide with the expected value of the wedge in the original stochastic process. Therefore, CKM (2004) decomposition include different forecast effect of the target wedge.

However, CKM (2007b) show that for the most of the experiments the two methodologies yield similar answers in practice. We implement both decompositions. Figure 1 is CKM (2004) decomposition result and Figure 2 is CKM (2007b). We confirm that both results are qualitatively similar.

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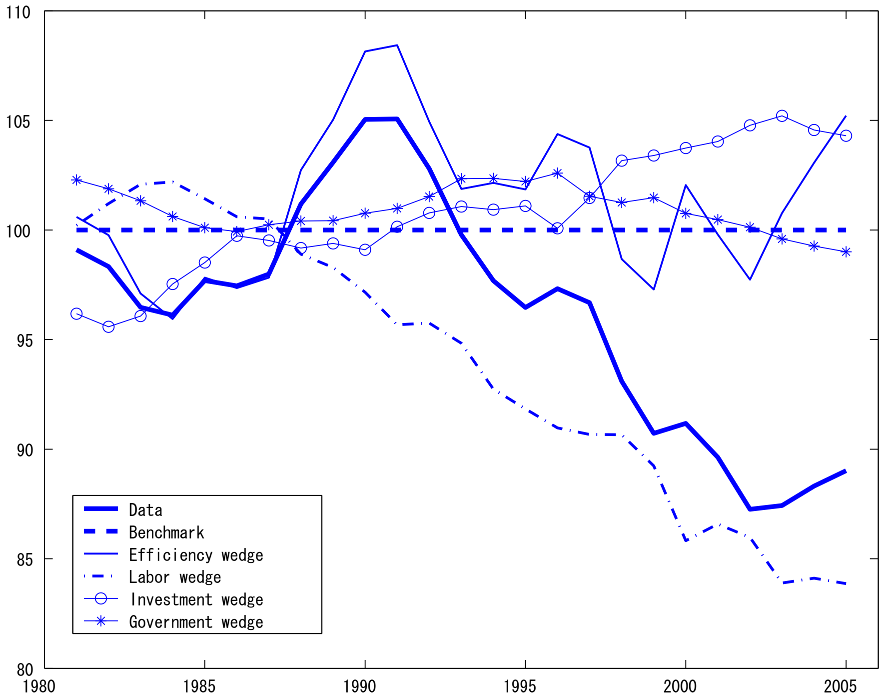

Figure 1: CKM (2004) decomposition of output with just one wedge

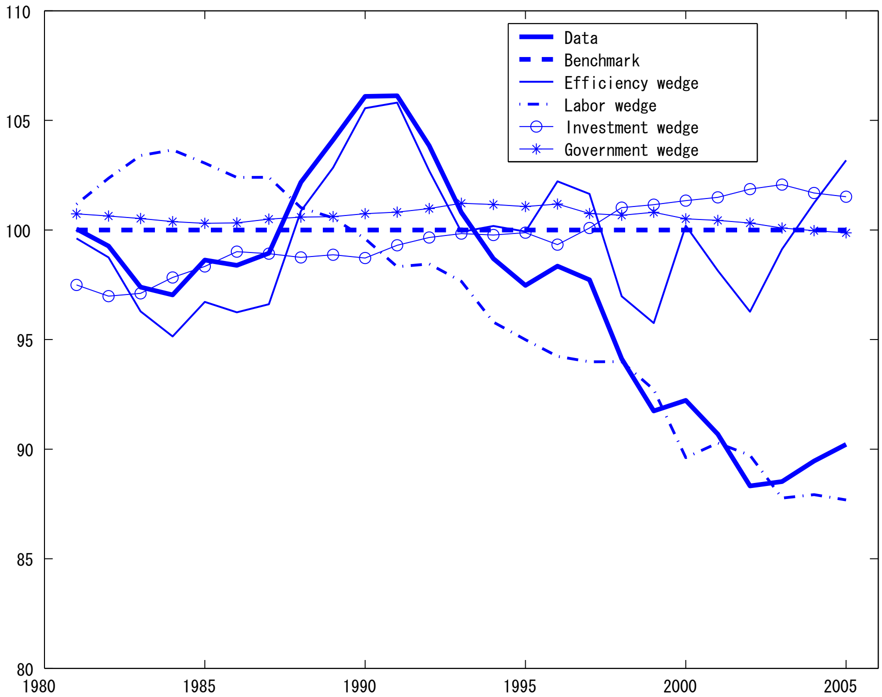

Figure 2: CKM (2007) decomposition of output with just one wedge