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On the Monetary Endogenous Growth Model with Idiosyncratic Investment Risk

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Abstract

In this paper, we construct a monetary endogenous growth model with idiosyncratic risk in capital formation and study the relationship between monetary policy and economic growth. The model features two types of consumption goods, namely cash goods and credit goods, with money being required to purchase cash goods. We first show that when the inflation rate is low, capital accumulation and economic growth are encouraged. As the inflation rate increases, the economic growth rate begins to decline. Therefore, the nominal interest rate and the economic growth rate have an inverted-U relationship. We next show that the Friedman rule of setting the nominal interest rate to zero is not the optimal monetary policy. Instead, the social welfare-maximizing nominal interest rate is always positive. Inflation stimulates risky investment, which raises the economic growth rate and increases utility. Finally, the introduction of government bonds into our model does not affect resources allocation.

Keywords: Endogenous growth; idiosyncratic risk; Tobin effect; Friedman rule

JEL classification: E5

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1 Introduction

The relationship between monetary policy and economic growth has been one of the important topics in monetary economics. A classic paper of Van der Ploeg and Alogoskoufis (1994) study the AK growth model with overlapping generations, and finds that inflation works as an intergenerational transfer and then enhances capital accumulation and increases the balanced growth rate. The growth-enhancing effect of inflation is also found by Zou (1998) who considers wealth utility in the AK model, Itaya and Mino (2003) who studies the AK model with labor externalities, and Ho et al. (2007) who study a model similar to Itaya and Mino (2003), but assume that seigniorage revenues are redistributed via consumption subsidy. However, in most of the existing papers, capital is accumulated deterministically. As far as we know, how the risk in capital accumulation change the effect of monetary policy on economic growth is not fully understood.

In this paper, we construct the monetary endogenous growth model with idiosyncratic risk in capital formation, and study the relationship between economic growth and monetary policy. In our model, a continuum of the entrepreneurs lives forever, consumes the two final goods, the cash good and the credit goods, holds money and capital, and engages in the risky capital production. The utility function of the entrepreneur is logarithmic. Money is used for cash good consumption.

The final good firm produces final good by using physical capital. The production function is AK type and is linear in capital. If the production of the capital good is deterministic, then our model coincides with the textbook style AK growth model, and the economic growth rate is equal to the marginal product of capital (A) minus discount factor (ρ). Production of capital needs the final good. Producing one unit of capital needs one unit of the final good on average, but the productivity is subject to idiosyncratic shocks. The newly produced capital is traded in the competitive capital market, and the price of capital is endogenously determined.

We first show that there exists a balanced growth path in which the price of capital is constant and the balanced growth rate is lower than the one in the deterministic AK model ($A - \rho$). We next show that for some parameter values, positive nominal interest rate

enhances raises economic growth rate as long as the nominal rate is low, but the growth rate begins to fall when the rate becomes higher. It is well-known that a positive nominal interest rate can be interpreted as a consumption tax on the cash good consumption. When the nominal rate is low, the increase in the rate reduces the total consumption (i.e., the sum of the cash good and credit good consumption) of the entrepreneur by the income effect, and the entrepreneur spends more resources on the risky investment. However, when the nominal interest rate exceeds some threshold value, the entrepreneur begins to substitute credit good for cash good. The substitution increases total consumption and reduces investment. Therefore the nominal interest rate has an inverted-U relationship with economic growth rate.

We next show that the Friedman rule is not optimal. When there exists an idiosyncratic risk, the economic growth rate is lower than the optimal growth rate in the deterministic economy. A positive nominal interest rate is welfare improving because it enhances capital accumulation and makes the growth rate closer to the first best rate. It is true that a deviation from the Friedman rule creates a distortion in consumption of the two final goods, but the first order effect is negligible around the rule. Therefore the Friedman rule is not optimal in our model.

Many authors study the optimality of the rule of setting the nominal interest rate to zero. The deflationary policy rule is optimal in most of the basic monetary models including the money-in-the-utility function model, the cash-credit model and the monetary search model. The nominal rate works as a distortionary tax on the money holdings, and basically it should be removed and the government should instead rely on other taxes such as labor income tax. However, the zero interest rate rule is not optimal in our model with agent heterogeneity.

As an extension, we consider the economy where the government can issue bonds in addition to money. We find that the government bonds does not change the equilibrium allocation. Therefore our results are unaffected by the introduction of the government bonds. We also provide the numerical example. In the example, the growth maximizing nominal interest rate is around 10% while the welfare maximizing rate is around 5%.

1.1 Literature review

Our paper is based on Hori and Im (2023, henceforth HI) who studies the asset price bubbles in the endogenous growth model with the idiosyncratic risk in investment. They show that the balanced growth rate in the bubbly economy may be greater than the growth rate in the bubble-less economy when the degree of the shock is sufficiently large. The asset price bubbles raise the entrepreneur's wealth, increase the demand for the final good and reduce the price of capital, and then raises the rate of return on capital. HI argue that this "wealth effect" is the main reason why the asset bubble increases economic growth. Although their results are influential and interesting, their model is non-monetary and it is not clear how monetary policy affects the economic growth in their economy.

Here we incorporate money which provides the liquidity service into HI, and study how nominal growth rate of money supply affects the economic growth and welfare. Although money and bubbles are usually seen as the similar financial assets, the bubbly economy differs crucially from the monetary economy. In HI, the rate of return on bubbles are the same as the real interest rate (r), and it also equals to the balanced growth rate (g) as long as the bubble does not burst. In our model, on the other hand, the rate of return on money (i.e, minus inflation rate) is lower than the real interest rate (r) because money provides a liquidity, and the real rate is greater than the economic growth rate so as to ensure that the present discounted value of the lump-sum monetary transfer (helicopter money) is well-defined. Therefore the sign of $r - g$ is also different between our model and HI.

Our model is closely related to Di tella (2020) who also studies the monetary AK-type endogenous growth model with idiosyncratic risks. One important difference between the two models is in the properties of the risks. In Di tella (2020), the individual receives the idiosyncratic income shock which is proportional to the individual capital level, while in our model, the individual receives risk on the productivity of capital goods production. Another difference is that Di tella (2020) studies the money-in-the utility function model, while we study the cash-credit model. Otherwise the two models are similar. However, the results are quite different. First, the non-monetary equilibrium in Di tella (2020)

achieves the first best level of investment and then the economic growth rate equals to the difference between the productivity A and the discount factor ρ (i.e., $A - \rho$) just like the deterministic AK model with logarithmic utility function. In our model, the economic growth rate in the non-monetary economy is lower than the one in Di tella (2020) and depends on the size of the idiosyncratic risks. Second, in Di tella (2020), the monetary policy is super-neutral, while it is not in our model¹. Third, the welfare maximizing nominal interest rate is zero in Di tella (2020) while it is strictly positive in our model. The optimal nominal interest rate is zero in Di tella (2020) mainly because in his model, the non-monetary equilibrium allocation achieves the first best.

Our paper is also close to Brunnermeier and Sannikov (2016) who study the welfare effect of monetary policy in the economy where the individuals are subject to the idiosyncratic income shock which is proportional to the individual capital holdings. They find that the economic growth rate monotonistically increases with money growth rate and that the welfare maximizing money growth rate increases in the size of the idiosyncratic risk. In Brunnermeier and Sannikov (2016), money is like asset bubbles and provides no liquidity service. Money is the only safe assets in their model and the effect of the positive nominal interest rate is uncertain. In this paper we focus on the welfare and growth effects of the positive nominal interest rate.

Recent papers theoretically study the non-monotonic relationship between monetary policy and economic growth in the R&D based endogenous growth (see Chu et al. (2019), Hu et al., (2021), Zheng et al. (2021), and Huang et al. (2023)). In their models, R&D activities are subject to the cash-in-advance constraints. Our paper finds out that agent heterogeneity also generates the nonmonotone relationship between monetary policy and growth. There are also some papers who empirically find the inverted-U shaped relationship between inflation and growth (see Bick (2010), Lopez-Villavicencio and Mignon (2011) and He (2023) for example). Our paper is consistent with their findings.

The rest of this paper is organized as follows. Section 2 describes the basic structure of

¹Di tella (2020) argues that for some utility function, inflation rate affects the economic growth rate. However, the Friedman rule is always optimal in his model.

the model. Section 3 investigates wealth inequality. Section 4 considers several extensions. Section 5 concludes the paper.

2 The model

In this section, we provide an overview of our model.

2.1 Production

Time is continuous. The economy consists of the final good and capital. Our set-up is similar to Cooley and Hansen (1989) and Jones and Manuelli (1995) who study the cash-credit model with capital accumulation. All markets are competitive. The final good firm produces the final good by using capital as a sole input. The production function is linear in capital:

$$Y_t = AK_t, \tag{1}$$

where Y_t is the output at time t , $A > 0$ is the productivity parameter, and K_t is the capital at time t . There is no capital depreciation.

Producing one unit of cash good needs one unit of final good. Similarly, one unit of the final good is needed for producing one unit of credit good. Thus the nominal prices of the cash good, credit good, and the final good are the same. Let P_t be a nominal price of the final good. Also, let $\pi_t = \dot{P}_t/P_t$ denote the inflation rate. The cash good is used only for consumption. The credit good is used either for consumption or for risky investment. There is a continuum of entrepreneurs with measure one lives forever. Each entrepreneur is indexed by $j \in [0, 1]$. When the entrepreneur j spends $X_{j,t}$ units of the final goods for dt periods of time, the amount of the capital produced is given by

$$X_{j,t}dt + \sigma X_{j,t}dW_{j,t}, \tag{2}$$

where $W_{j,t}$ is a standard Brownian motion, and σ is the standard deviations. The parameter σ is a positive constant. The term $dW_{j,t}$ represents the idiosyncratic investment

shocks which is identically and independently distributed. We have $(dt)^2 = dt dW_{j,t} = 0$ and $(dW_{j,t})^2 = dt$.

2.2 Entrepreneurs

The entrepreneurs are homogeneous ex ante and is born with capital K_0 and money M_0 at the beginning of period 0. Each entrepreneur receives capital income and monetary transfer, consumes the cash good and the credit good, engages in risky investment, and holds capital and money as an asset. While the purchases of cash good needs cash in advance, the purchases of credit goods does not need money.

Entrepreneur j maximizes the following expected intertemporal utility:

$$\mathcal{U}_t^j = E_t \left[\int_t^\infty e^{-\rho(s-t)} U(C_{j,s}^1, C_{j,s}^2) ds \right], \quad (3)$$

where $C_{j,t}^1$ is the consumption of the cash good, and $C_{j,t}^2$ is the consumption of the credit good, and $\rho > 0$ is the discount factor. We follow Chari and Kehoe (1998) and Siu (2004), and assume that the instantaneous utility function U is homothetic and is given by

$$U(C_t^1, C_t^2) = \ln\{(1 - \alpha)(C_t^1)^\nu + \alpha(C_t^2)^\nu\}^{1/\nu}$$

where $\alpha \in (0, 1)$ shows the relative importance of consuming credit good, and ν shows the degree of substitution. We assume that the constant ν satisfies $\nu \in (0, 1)$.

The entrepreneur j is subject to the following budget constraint expressed in real terms:

$$v_t dK_{j,t} + \frac{dM_{j,t}}{P_t} = (r_t v_t K_{j,t} - C_{j,t}^1 - C_{j,t}^2 - X_{j,t} + T_t) dt + v_t X_{j,t} (dt + \sigma dW_{j,t}), \quad (4)$$

where v_t is the price of capital in terms of the final good, r_t is the real interest rate, $K_{j,t}$ is the capital holdings, $M_{j,t}$ is the nominal balances, $X_{j,t}$ is the risky investment, T_t is the lump-sum transfer from the government and P_t is the nominal price of the final good. The term $v_t X_{j,t} (dt + \sigma dW_{j,t})$ is the revenue from selling the newly produced capital good. We assume that the lump-sum transfer T_t is the same across all the entrepreneurs. Let $\pi_t = \dot{P}_t / P_t$ denote the inflation rate. The initial capital $K_{j,0} = K_0 > 0$ and the initial

nominal balances $M_{j,0} = M_0 > 0$ is exogenous, and the same across all the entrepreneurs. Following Di tella (2020), we also impose the solvency constraint (natural debt limit) on the entrepreneur.

In addition to the budget constraint, the entrepreneur j is subject to the following liquidity constraint on cash good consumption:

$$P_t C_{j,t}^1 \leq M_{j,t}. \quad (5)$$

We restrict our attention to the case in which the liquidity constraint holds with equality and then the cash good consumption is equal to the real money balance: $C_{j,t}^1 = M_{j,t}/P_t$. In that case, the instantaneous utility is re-expressed as the function of the total consumption $C_{j,t} \equiv C_{j,t}^1 + C_{j,t}^2$, and the real money balance $M_{j,t}/P_t$:

$$U(C_{j,t}^1, C_{j,t}^2) = u(C_{j,t}, M_{j,t}/P_t),$$

where the function u is defined as $u(c, m) \equiv U(m, c - m) = \ln[\{(1 - \alpha)m^\nu + \alpha(c - m)^\nu\}^{1/\nu}]$. Our model can therefore be interpreted as the money-in-the utility function model with the instantaneous utility function u . The function u is written as $u(c, m) = \ln c + u(1, m/c)$, and the second term $u(1, m/c) = \ln[\{(1 - \alpha)(m/c)^\nu + \alpha(1 - m/c)^\nu\}^{1/\nu}]$ is maximized when the money-consumption ratio m/c is equal to $[1 + (1/\alpha - 1)^{1/(1-\nu)}]^{-1}$. In this case, money is satiated. Monetary satiation is achieved when the nominal interest rate is zero and the opportunity cost of holding money is zero.

We put the following restriction on the parameter values.

Assumption 1: The parameters satisfy either one of the followings constraints

$$\text{Case 1 : } \rho < \min \left\{ \frac{A^2 \sigma^2}{4}, \frac{1}{\sigma^2} \right\}, \quad (6)$$

$$\text{Case 2 : } \sigma < \min \left\{ \frac{\sqrt{\rho}}{2A}, \frac{1}{2\sqrt{\rho}} \right\} \text{ and } \frac{A}{2} < \rho < A. \quad (7)$$

This assumption requires that either the individuals are sufficiently patient or the size of the shock is sufficiently small.

One can easily check that as far as Assumption 1 holds, we have

$$0 < A - \rho, \quad (8)$$

$$\sigma \sqrt{\rho} < 1. \quad (9)$$

Later we use the inequalities for showing the existence of the equilibrium path.²

The entrepreneurs rent capital to the final good firm. The profit maximization problem of the firm is denoted as $\max_{K_t}\{(A - r_t v_t)K_t\}$. In the competitive equilibrium, the real interest rate is given by $r_t = r(v_t) \equiv A/v_t$.

2.3 Central bank

The central bank controls the growth rate of money supply M_t^s . Let μ_t denote the growth rate of money at time t . Money supply evolves according to

$$\frac{dM_t^s}{dt} = \mu_t M_t^s. \quad (10)$$

Newly printed money is redistributed to the entrepreneur by a lump-sum. The government transfer in real terms is given by $T_t = \mu_t \frac{M_t^s}{P_t}$. The present discounted value of the government transfer in real terms is

$$H_t = \int_t^\infty T_s \exp(-\int_t^s r_j dj) ds. \quad (11)$$

In this paper, we assume that the central bank controls the nominal interest rate i by choosing money growth rate μ . We characterize the equilibrium path as a function of the nominal rate i . We also assume that the government sets i to be the time-independent constant. By the Fisher equation, $i = r_t + \pi_t$. Later we study the relationship between the growth rate of money μ_t and the nominal rate i . In the following, we focus on the case where the nominal interest rate is strictly positive, and consider the Friedman rule as a limit.

2.4 HJB equation

We follow Di tella (2020), and let $\omega_{j,t} = v_t K_{j,t} + M_{j,t}/P_t + H_t$ denote the total wealth of the entrepreneur j . Since the function $u(c, m)$ is homothetic and logarithmic, we can

²If Case 1 of the Assumption 1 holds, $\rho < \sqrt{\frac{A^2 \sigma^2}{4}} \cdot \frac{1}{\sigma^2} = \frac{A}{2} < A$. If Case 2 holds, then immediately we obtain $A > \rho$.

re-express the utility function as

$$u(C_{j,t}, M_{j,t}/P_t) = u(c_{j,t}, m_{j,t}) + \ln \omega_{j,t}.$$

where $c_{j,t} \equiv \frac{C_{j,t}}{\omega_{j,t}}$ is the consumption share and $m_{j,t} \equiv (M_{j,t}/P_t)/\omega_{j,t}$ is the liquidity share. Let $V_{j,t}$ denote the value function of the entrepreneur j at time t . Also let $x_{j,t} = \frac{X_{j,t}}{\omega_{j,t}}$ denote the share of the risky investment. The HJB equation of the entrepreneur j at time t is written as

$$\rho V_{j,t} = \max_{c,m,x} \left\{ u(c_{j,t}, m_{j,t}) + E \left[\frac{dV_{j,t}}{dt} \right] \right\} + \ln \omega_{j,t} \quad (12)$$

subject to the budget constraint

$$d\omega_{j,t} = [r_t - i_t m_{j,t} - c_{j,t} + x_{j,t}(v_t - 1)]\omega_{j,t}dt + \sigma v_t x_{j,t} \omega_{j,t} dW_{j,t}, \quad (13)$$

with solvency constraint $\omega_{j,t} \geq 0$. As Di Tella (2020) show, the solvency constraint is equivalent to the natural debt limit.

In the following, we look for an equilibrium allocation in which the price of capital v and then the real interest rate $r(v)$ are time-independent constant. We guess that v is constant, and the value function has a form $V_{j,t} = b_1 \log \omega_{j,t} + b_2$ where $b_1 > 0$ and b_2 are unknown constant. From Ito lemma ($dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial \omega} d\omega + \frac{1}{2} \frac{\partial^2 V}{\partial \omega^2} (d\omega)^2$), we can simplify the HJB equation (12) as

$$\rho b_1 \log \omega_{j,t} + \rho b_2 = \max_{c,m,x} \left\{ u(c, m) + b_1[r - im - c] + b_1 x(v - 1) - b_1 \frac{(\sigma vx)^2}{2} \right\} + \log \omega_{j,t}. \quad (14)$$

Equation (14) holds for all $\omega_{j,t}$ and then our guess on the value function holds true if the constant b_1 and b_2 are given by

$$\begin{aligned} b_1 &= 1/\rho, \\ b_2 &= \frac{1}{\rho} \max_{c,m,x} \left\{ u(c, m) + \frac{r - c - im + (v - 1)x}{\rho} - \frac{(\sigma vx)^2}{2\rho} \right\}. \end{aligned}$$

In the following section we show that there exists the equilibrium growth path along which the capital price is constant.

2.5 Equilibrium allocation

In this subsection, we characterize solution to the HJB equation (14). In Eq. (14), the optimization problem on the investment share x which must be strictly positive in equilibrium is well-defined only if

$$v > 1. \quad (15)$$

If Eq. (15) holds, the first order conditions on c, m, x are respectively given by

$$\rho u_c(c, m) = 1, \quad (16)$$

$$\rho u_m(c, m) = i. \quad (17)$$

$$x = x(v) \equiv \frac{v-1}{\sigma^2 v^2}. \quad (18)$$

Later we show that the equilibrium path satisfies Eq. (15). The following lemma characterizes the consumption share and the liquidity share as a function of the nominal interest rate.

Lemma 1 *The consumption share c and the liquidity share m are the functions of the nominal rate i ;*

$$c = c(i) \equiv \rho \frac{\left\{ \frac{\alpha}{1-\alpha} (1+i) \right\}^{1/(1-\nu)} + 1}{\left\{ \frac{\alpha}{1-\alpha} (1+i) \right\}^{1/(1-\nu)} + 1 + i}. \quad (19)$$

$$m = m(i) \equiv \rho \frac{1}{\left\{ \frac{\alpha}{1-\alpha} (1+i) \right\}^{1/(1-\nu)} + 1 + i} \quad (20)$$

The total expenditure share on consumption and liquidity is constant:

$$c + im = \rho. \quad (21)$$

The function $c(i)$ is U-shaped and satisfies $c(0) = c(\infty) = \rho$, $c'(0) < 0$ and $c(i) < \rho$ for all $i > 0$. There exists $i^ > 0$ that minimizes $c(i)$.*

Proof. See Appendix A. ■

As the nominal rate i goes up, consumption first decreases because money becomes costly (i.e. income effect). But as the nominal rate goes up further, consumption begins to increase because the entrepreneur substitutes credit good for cash good (i.e., substitution

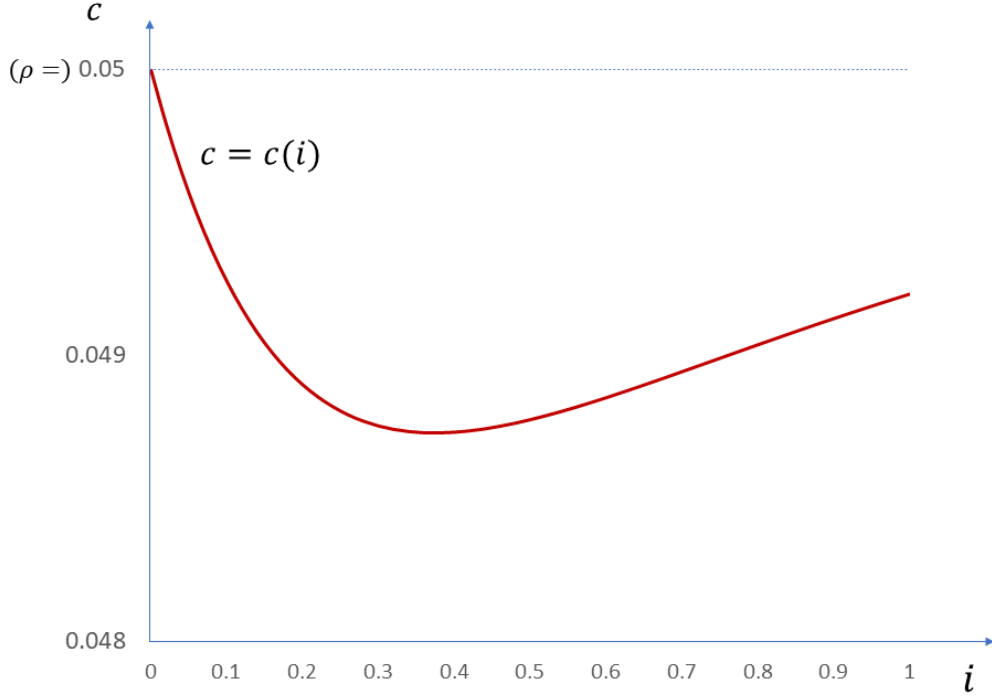


Figure 1: Balanced growth equilibrium

effect). Therefore the function $c(i)$ is not monotone. This property crucially affects the relationship between monetary policy and economic growth. Figure 1 shows the function $c(i)$ when $\rho = 0.05$, $\alpha = 0.57$ and $\nu = 0.83$. The values of α and ν come from Chari and Kehoe (1998) (see page. 100). It shows that i^* is around 0.23.

The next lemma characterizes the equilibrium allocation of each individual.

Lemma 2 *Given the capital price v and the nominal rate i , the equilibrium allocation of the entrepreneur j , $\{C_{j,t}^1, C_{j,t}^2, X_{j,t}, \omega_{j,t}\}$ satisfies*

$$C_{j,t}^1 = m(i)\omega_{j,t}, \quad (22)$$

$$C_{j,t}^2 = \{c(i) - m(i)\}\omega_{j,t} \quad (23)$$

$$X_{j,t} = x(v)\omega_{j,t}, \quad (24)$$

$$d\omega_{j,t} = \{r(v) - \rho + (v - 1)x(v)\}\omega_{j,t}dt + x\omega_{j,t}\sigma v dW_{j,t}. \quad (25)$$

Proof. See Appendix B. ■

3 Balanced growth path

In this section, we characterize the balanced growth path.

3.1 Evolution of aggregate wealth

The resource constraint is

$$AK_t = C_t^1 + C_t^2 + X_t, \quad (26)$$

where $C_t^1 \equiv \int_0^1 C_{j,t}^1 dj$ is the aggregate consumption of the cash good and $C_t^2 \equiv \int_0^1 C_{j,t}^2 dj$ is the aggregate consumption of the credit good. We let $C_t = C_t^1 + C_t^2$ denote the total consumption. The aggregate capital $K_t \equiv \int_0^1 K_{j,t} dj$ is accumulated according to

$$\dot{K}_t = X_t, \quad (27)$$

where $X_t \equiv \int_0^1 X_{j,t} dj$ is the aggregate investment. The total wealth $\omega_t \equiv \int_0^1 \omega_{j,t} dj$ is equal to $vK_t + M_t/P_t + H_t$, where $M_t \equiv \int_0^1 M_{j,t} dj$ is the aggregate nominal balance.

In the following, we characterize a *balanced growth path (BGP)* which the equilibrium allocation $\{C_t^1, C_t^2, X_t, K_t, \omega_t, v\}$ such that the price of capital v is constant, and the growth rate of the variables C_t^1 , C_t^2 , M_t/P_t and K_t are the same constant. Let g denote the balanced growth rate.

Along the BGP, we have $H_t = \frac{\mu}{r-g} \frac{M_t}{P_t}$. The term is well-defined only if the real rate of return r is greater than the economic growth rate g :

$$r - g > 0. \quad (28)$$

Moreover, from the Fisher identity $i - r = \pi$, the growth rate of money supply is $\mu = \pi + g = i - r + g$. Thus the total wealth ω_t is given by

$$\omega_t = vK_t + \frac{1}{r-g} i \frac{M_t}{P_t}. \quad (29)$$

Just as Di Tella (2020) and HI note, the idiosyncratic risk $dW_{j,t}$ disappears in the aggregate. Eq. (25) then implies that the total wealth evolves according to

$$\frac{d\omega_t}{dt} = \{r(v) - \rho + (v-1)x(v)\}\omega_t. \quad (30)$$

The consumption share c and the liquidity share m are constant and are independent of each entrepreneur's wealth. Therefore the variables C_t^1 , C_t^2 , X_t are all proportional to ω_t :

$$C_t^1 = \frac{M_t}{P_t} = m(i)\omega_t, \quad (31)$$

$$C_t^2 = \{c(i) - m(i)\}\omega_t, \quad (32)$$

$$X_t = x(v)\omega_t. \quad (33)$$

The BGP is determined by Eqs. (27), (29), (30), (31), (32), and (33). One can easily show that these equations together imply the feasibility constraint (26). Along the BGP, the aggregate money demand $M_t = \int_0^1 M_{j,t} dj$ is equal to the money supply M_t^s .

3.2 Euler equation

The balanced growth rate g coincides with the growth rate of the wealth ω_t and then Eq. (30) implies that $g = r(v) - \rho + (v - 1)x(v)$. Let us introduce a new variable $z \equiv 1 - 1/v$ which increases in v , and satisfies $z \in (0, 1)$ if $v > 1$. We have $r = A(1 - z)$, and $(v - 1)x = \sigma^{-2}z^2$. We derive an equation which expresses the balanced growth rate g as a function of z .

$$g = g_1(z) \equiv A(1 - z) - \rho + \sigma^{-2}z^2. \quad (34)$$

Eq. (34) comes from the utility maximization problem of the entrepreneur, and then we refer to Eq. (34) as the Euler equation (EU), which contains two endogenous variables $\{g, z\}$.

If there is no uncertainty in investment technology (i.e, $\sigma = 0$), the price of the capital v is equal to its marginal cost, 1 and then $z = 0$. Eq. (34) is simplified as $g = A - \rho$. This is simply the Euler equation in the deterministic AK growth model with log utility. The term $\sigma^{-2}z^2$ in Eq. (34) indicates that the risk in investment raises the expected return on the investment.

3.3 Capital accumulation equation

In the following, we derive another equation that relates the balanced growth rate g with z . Dividing both sides of Eq. (29) by ω_t and using Eq. (21), we obtain

$$1 = vk + \frac{\rho - c}{r - g},$$

where $c = c(i)$. Eq. (34) implies that $r - g = \rho - \sigma^{-2}z^2$. Moreover, the balanced growth rate is equal to capital growth rate, which is x/k according to Eq. (27). Therefore we have

$$g = g_2(z; c) \equiv \frac{z}{\sigma^2} \frac{\rho\sigma^2 - z^2}{c\sigma^2 - z^2}. \quad (35)$$

In the following, we refer to Eq. (35) as a capital accumulation equation (CA). Note the consumption share $c = c(i)$ is strictly less than ρ as long as $i > 0$.

From Eq. (34), the constraint (28) holds if and only if $z < \sigma\sqrt{\rho}$. On the other hand, Eq. (15) which ensures the positive investment is re-expressed as $0 < z < 1$. From Eq. (8), the two constraints on z are consolidated as

$$0 < z < \sigma\sqrt{\rho}. \quad (36)$$

3.4 Balanced growth equilibrium

Equilibrium values of $\{g, z\}$ are determined by EU equation (34) and CA equation (35). The EU curve $g = g_1(z)$ is U-shaped and decreases in z if $z < \bar{z}_1 \equiv \frac{A\sigma^2}{2}$. It is decreasing in z (or equivalently decreasing in v) because the increase in the capital price reduces the rate of return on capital. Moreover, $g_1(0) = A - \rho > 0$.

On the other hand, the CA curve $g = g_2(z; c)$ is an increasing function in z , satisfies $g_2(0; c) = 0 < g_1(0)$ and it diverges to $+\infty$ if z goes to $\bar{z}_2 \equiv \sigma\sqrt{c} (< \sigma\sqrt{\rho})$. The function is well-defined only when $z \in [0, \bar{z}_2)$. The function g_2 is increasing in z (or equivalently increasing in v) because the entrepreneurs try to spend more resources on risky investment when the capital price v goes up.

From Assumption 1, $\bar{z}_2 (< \sigma\sqrt{\rho}) < \bar{z}_1$. Therefore, as Figure 2 shows, the downward sloping EU curve, and upward sloping CA curve cross only once within the interval $[0, \bar{z}_2)$.

The intersection determines the equilibrium values of $\{g, z\}$. The equilibrium value of z is determined by

$$g_1(z) = g_2(z; c). \quad (37)$$

We let $z(c)$ denote a unique solution to Eq. (37). It always satisfies Eq. (36). We can express the equilibrium value of the growth rate g as a function of c : $g = g(c) \equiv g_1(z(c))$. The next proposition characterizes the BGP.

Proposition 1 *When the nominal interest rate is $i > 0$, the balanced growth equilibrium allocation $\{C_t^1, C_t^2, K_t, X_t\}$ is determined by the following equations.*

$$K_t = \exp(gt)K_0, \quad (38)$$

$$X_t = gK_t, \quad (39)$$

$$C_t^1 = \frac{m(i)}{c(i)}(A - g)K_t \quad (40)$$

$$C_t^2 = \frac{c(i) - m(i)}{c(i)}(A - g)K_t. \quad (41)$$

Here the balanced growth rate g is equal to $g = g(c(i))$.

Proof. See Appendix C. ■

The equilibrium price of capital is $v = 1/(1 - z(c(i)))$ and the equilibrium real interest rate is $r = A(1 - z(c(i)))$. The equilibrium price of the good is $P_t = \frac{M_t^s}{C_t^1}$. The growth rate of the money supply which sets the nominal rate to i is determined by $\mu = g(c(i)) + i - A\{1 - z(c(i))\}$. As is clear from Figure 1, the balanced growth rate is always lower than the balanced growth rate without uncertainty, $g_1(0) = A - \rho$.

3.5 Relationship between nominal rate and growth

The function $g_2(z; c)$ decreases in the consumption share c . In Fig 1, we see that when c goes up, the upward-sloping CA curve (i.e, $g = g_2(z; c)$) shifts down while the downward-sloping EU curve is unchanged. Thus the equilibrium value of z increases while the

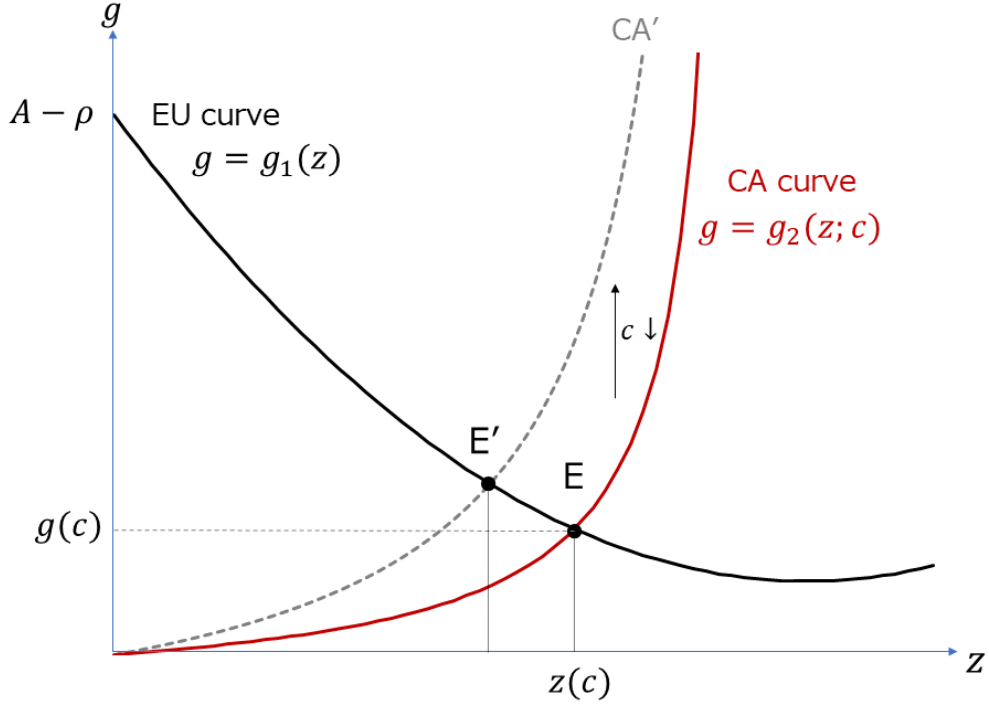


Figure 2: Balanced growth equilibrium

growth rate g decreases. Therefore the function $z(c)$ is an increasing function of c while the function $g(c)$ is a decreasing function of c .³

As we show in Lemma 1, the function $c(i)$ is a U-shaped function of i and is minimized when $i = i^*$. We also have $g'(c) < 0$. Therefore the balanced growth rate $g = g(c(i))$ is maximized when $i = i^*$. We have our main proposition on the relationship between the balanced growth rate $g = g(c(i))$ and the nominal rate i .

Proposition 2 *The equilibrium growth rate g is inverted U-shaped as a function of the nominal rate i . It is maximized when the nominal rate is equal to i^* .*

In our model, a positive nominal interest rate can be interpreted as a consumption tax on the cash good. When the nominal rate is low, the increase in the rate reduces the

³Differencing the equation $g_1(z(c)) = g_2(z(c); c)$ by c , we get

$$z'(c) = \frac{g_{2,c}}{g_{1,z} - g_{2,z}}.$$

Since $g_{2,c} < 0$, $g_{1,z} > 0$ and $g_{2,z} < 0$, we get $z'(c) < 0$. This inequality always holds.

total consumption of the entrepreneur by the income effect, and instead of consumption, the entrepreneur tries to use more resources on the risky investment. This is growth-enhancing. However, when the nominal interest rate exceeds some threshold value, the entrepreneur begins to substitute cash good consumption for credit good consumption. The substitution increases total consumption and then reduces the risky investment. This is growth-reducing. Therefore the relationship between nominal rate and the growth rate is not monotone.

The non-monotone relationship between nominal interest rate and growth crucially depends on the existence of the idiosyncratic risk. As the size of the shock σ converges to zero, the price of capital converges to one, and the equilibrium growth rate converges to $A - \rho$, which is independent of the nominal interest rate i . As long as $\sigma > 0$, the balanced growth rate is less than $A - \rho$.

Under the Friedman rule of setting nominal interest rate i to zero, the expenditure share of liquidity, $im(i)$ disappears, and the consumption share $c(i)$ equals to ρ . In that case, the function $g_2(z; c) = g_2(z; \rho)$ is equal to $\frac{z}{\sigma^2}$, and we can explicitly derive the equilibrium growth rate, say g_{FR} :

$$g_{FR} = \frac{2(A - \rho)}{1 + \sigma^2 A + \sqrt{(1 + \sigma^2 A)^2 - 4\sigma^2(A - \rho)}}. \quad (42)$$

When $r - g > 0$ and $i = 0$, the total wealth ω coincides with the market value of capital vk , and is independent of money holdings. The growth rate therefore coincides the bubbleless economic growth rate in HI.⁴ As σ^2 converges to 0, the growth rate g_{FR} converges to $A - \rho$, which is the balanced growth rate in the deterministic AK model with logarithmic utility.

Comparison with Di tella (2020)

The non-monetary growth rate (42) is different from that in Di tella (2020). In Di tella (2020) the non-monetary growth rate is independent of the risk and is always equal to $A - \rho$. The difference comes from the characteristics of the idiosyncratic risks. In Di tella

⁴In Proposition 2 and 3, Hori and Im (2023) show that the economic growth rate is equal to $g = (1 - V_L)/\sigma^2$, where V_L solves $A = \rho/V_L + (1 - V_L)/\sigma^2$. With some algebra, we get $\sigma^2 g^2 - (1 + A\sigma^2)g + (A - \rho) = 0$. One can easily show that the solution to the quadratic equation is equal to the above value.

(2020), the individual receives the endowment shock (in terms of the final good), and the size of the shock is proportional to the individual capital holdings. In the non-monetary setting, the budget constraint of the individual in Di Tella (2020) is written as

$$dK_{j,t} = (AK_{j,t} - C_{j,t})dt + K_{j,t}\sigma dW_{j,t}.$$

One can easily show that the value function of the individual is equal to $\frac{1}{\rho} \ln K_{j,t}$ plus constant, and the optimal consumption rule is given by $C_{j,t} = \rho K_{j,t}$. Thus the individual capital stock evolves according to $dK_{j,t} = (A - \rho)K_{j,t}dt + K_{j,t}\sigma dW_{j,t}$. Since the shock $dW_{j,t}$ does not affect the aggregate variables, the aggregate capital K_t evolves according to $dK_t = (A - \rho)K_tdt$. Therefore the capital growth rate is $A - \rho$.

In our model, the idiosyncratic risk is on the productivity of the capital good, not the endowment, and it affects the price of the capital good. The endogeneity of the capital price crucially affects the relationship between risk and growth.

Comparison with HI

Although our model is based on HI, the equilibrium path is significantly different from that in HI mainly because our model satisfies $r > g$. In our model, equilibrium allocation is characterized by Eq. (34) and Eq. (35). It is true that the Euler equation (34) holds in HI. However, capital accumulation equation (35) does not hold. In HI, if the bubble bursting probability is zero, we have $r = g$. Therefore from Eq. (34), the price of capital is $v = 1/\{1 - \rho\sqrt{\sigma}\}$, and the bubbly growth rate is $A(1 - \rho\sqrt{\sigma})$.⁵

3.6 Introduction of the government bonds

In this subsection, we consider a case in which the government can issue the government bond in addition to money. Let B_t denote the amount of the government bonds. Also, let $\tilde{T}_t$ denote the lump-sum transfer from the government. The budget constraint of the government is

$$\dot{B}_t = rB_t + \tilde{T}_t - \mu \frac{M_t}{P_t}.$$

⁵This coincides with equations (18c) and (18e) in HI when the capital depreciation rate δ and the bubble bursting probability μ are both equal to zero.

Here we focus on the BGP where the growth rate of the government bonds are equal to the economic growth rate g . The above equation implies

$$\tilde{T}_t = \mu \frac{M_t}{P_t} - (r - g)B_t.$$

Let $\tilde{\omega}_t = v_t K_t + B_{j,t} + M_{j,t}/P_t + \tilde{H}_t$ denote the total wealth of the entrepreneurs, where $\tilde{H}_t = \frac{\tilde{T}_t}{r-g} = \frac{\mu M_t}{r-g} - B_t$ is the present discounted value of the government transfer. The value of wealth which is related to the government is equal to

$$\tilde{\omega}_t = v_t K_t + M_t/P_t + \frac{\mu M_t}{r-g}.$$

Note that the term B_t disappears and $\tilde{\omega}$ coincides with the case without the bond, ω . Therefore the introduction of the government bonds does not change the BGP at all. Ricardian equivalence theorem holds in our model.

4 Welfare

Here we assume the initial individual wealth is the same across all the entrepreneurs who are homogeneous ex ante. Total consumption at time 0 satisfies

$$AK_0 - X_0 = K_0(A - g) = C_0 = c\omega_0$$

Thus the total initial wealth is equal to $\omega_0 = c^{-1}(A - g)K_0$.

In the following, we study the optimality of the Friedman rule which set the nominal interest rate to zero. Recall that $z'(c) > 0$, $g'(c) < 0$ and $c'(0) < 0$. Therefore in the neighborhood of the Friedman rule, positive nominal interest rate reduces $z = z(c(i))$ while it raises $g = g(c(i))$.

We measure the social welfare W as the utility of the ex-ante homogeneous entrepreneur at time zero, $V_{j,0} = b_1 \ln \omega_0 + \frac{b_2}{\rho}$. It is expressed as

$$W = \frac{1}{\rho} u(1, m/c) + \frac{\ln(A - g)}{\rho} + \frac{\ln K_0}{\rho} + \frac{r}{\rho^2} + \frac{z^2}{2\rho^2 \sigma^2} - 1 \quad (43)$$

Substituting Eq. (34) into the equation above, we have

$$W = F(g) + \frac{1}{\rho} u(1, m/c) - \frac{z^2}{2\rho^2 \sigma^2}, \quad (44)$$

Here the first term $F(g) = \frac{1}{\rho} \ln(A - g) + \frac{g}{\rho^2} + \frac{\ln K_0}{\rho}$ is the intertemporal utility in the deterministic AK growth model when the growth rate is g . The second term $u(1, m/c)/\rho$ represents the utility from holding money. Finally, the third term $\frac{z^2}{2\rho\sigma^2}$ represents the utility loss from idiosyncratic risk.

The optimal growth rate in the deterministic AK model with logarithmic utility function is $A - \rho$, and one can easily check that the function $F(g)$ is maximized when $g = A - \rho$. This implies that $F'(g) > 0$ if $g < A - \rho$. We have already shown that the balanced growth rate in our model with idiosyncratic risk is lower than $A - \rho$, then $F'(g_{FR}) > 0$ where g_{FR} is the balanced growth rate under the Friedman rule. As the balanced growth rate increases with i in the neighborhood of the Friedman rule, the term $F(g)$ increases with i as long as i is small.

The second term $u(1, m/c)$ is maximized under the Friedman rule. This is because the opportunity cost of holding money is minimized under the rule. This also implies that the first order effect of the change in i on the term $u(1, m/c)$ is zero. In other words, when $i = 0$, we have

$$\frac{d}{di} u \left(1, \frac{m(i)}{c(i)} \right) = 0$$

Finally, the third term of the welfare $\frac{z^2}{2\rho\sigma^2}$ decreases with i around the Friedman rule, because the nominal interest rate reduces z (see Fig 2). Therefore $\frac{dW}{di} > 0$ when $i = 0$. We now have the following proposition.

Proposition 3 *The Friedman rule is not the optimal monetary policy. There exists positive nominal rate $i^{**} > 0$ which maximizes welfare.*

Proof. See Appendix D. ■

5 Numerical illustration

In this section, we show the numerical example. We choose a discount factor $\rho = 0.03$. We set the marginal product of capital A to 0.1. With respect to the utility parameters α and ν , Chari and Kehoe (1998) (see page. 100) assume that $\alpha = 0.57$ and $\nu = 0.83$.

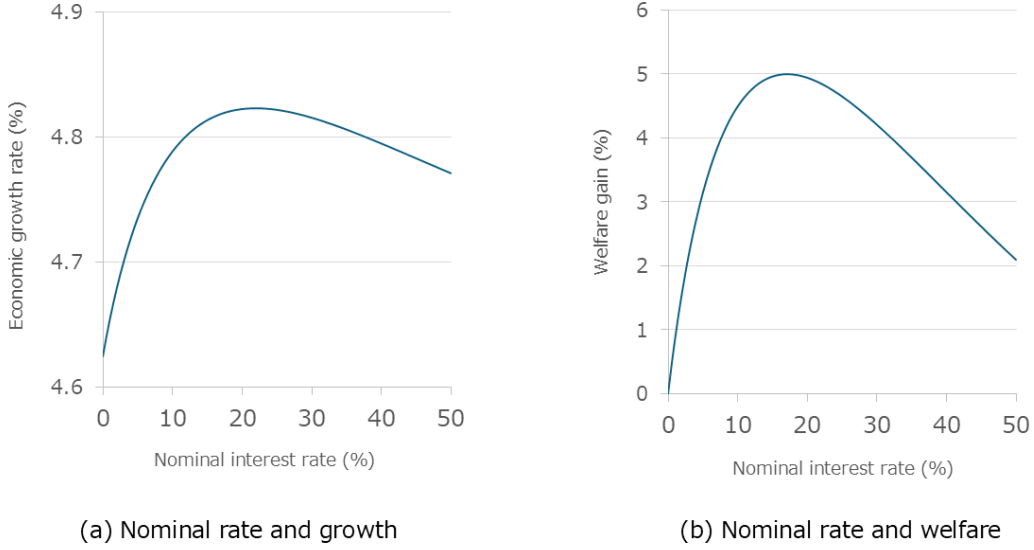


Figure 3: Growth and welfare

On the other hand, Siu (2004) estimate that $\alpha = 0.62$ and $\nu = 0.79$.⁶ Finally, we set the idiosyncratic risk parameter σ to 4.0.

We first show the relationship between the nominal rate and growth. Figure 3 (a) shows the balanced growth rate as a function of the nominal rate i . The growth maximizing nominal rate is around 20%, and the gain in the economic growth rate by deviating from the Friedman rule is about 0.04%.

We next study the relationship between the nominal rate and social welfare. We define the welfare difference between the economy under the positive nominal rate and the economy under the Friedman rule as a consumption-equivalent Δ that satisfies

$$E_t \left[\int_0^\infty e^{-\rho s} U((1 + \Delta)C_{j,s}^1(0), (1 + \Delta)C_{j,s}^2(0)) ds \right] = E_t \left[\int_0^\infty e^{-\rho s} U(C_{j,s}^1(i), C_{j,s}^2(i)) ds \right],$$

where $C_{j,s}^1(i)$ ($C_{j,s}^2(i)$) is the equilibrium consumption level of the cash good (credit good) when the nominal rate is i . Figure 3 (b) shows the consumption equivalent Δ as a function of the nominal rate i . The welfare maximizing nominal rate is around 15%, and the welfare gain from deviating from the Friedman rule is around 5%.

⁶Siu (2004) estimate that $\alpha = 0.62$ and $\nu = 0.79$.

6 Conclusion

In this paper, we construct the monetary endogenous growth model with idiosyncratic risk in capital formation, and study the relationship between economic growth and monetary policy. We adopt the cash-credit good economy. We show that inflation enhances risky investment and raises economic growth as long as the inflation rate is low. However, the inflation begins to reduce economic growth rate when the inflation rate becomes high. We also show that the Friedman rule of setting the nominal interest rate to zero is never optimal and welfare-maximizing nominal interest rate is strictly positive. We finally show that Ricardian equivalence theorem holds and the introduction of the government bond does not change the equilibrium allocation.

Appendix

The Appendix provides proofs for propositions.

A Proof of Lemma 1

We first show Eq. (21). The utility function u satisfies $u(\lambda c, \lambda m) = \ln \lambda + u(c, m)$ for all $\lambda > 0$. Differentiating both sides of the equation by λ and set λ to one, we get $cu_c + mu_m = 1$. Substituting the first order conditions (16) and (17) into the equation yields Eq. (21).

We next show Eqs. (19) and (20). The marginal utilities are written as

$$\begin{aligned} \ln[\{(1 - \alpha)m^\nu + \alpha(c - m)^\nu\}^{1/\nu}] \\ u_c &= \alpha(c - m)^{\nu-1} (\alpha(c - m)^\nu + (1 - \alpha)m^\nu)^{1/\nu-1} \\ u_m &= \{(1 - \alpha)m^{\nu-1} - \alpha(c - m)^{\nu-1}\} (\alpha(c - m)^\nu + (1 - \alpha)m^\nu)^{1/\nu-1} \end{aligned}$$

From the first order conditions (16) and (17), we have

$$\frac{u_m}{u_c} = \frac{1 - \alpha}{\alpha} \left(\frac{c}{m} - 1 \right)^{1-\nu} - 1 = i$$

Thus we get

$$c = \left\{ \left(\frac{\alpha(1 + i)}{1 - \alpha} \right)^{1/(1-\nu)} + 1 \right\} m. \quad (45)$$

From Eq. (21), we get Eqs. (19) and (20). Clearly $c(0) = \rho$. Moreover, since $0 < \nu < 1$, $1/(1 - \nu) > 1$. Thus $c(+\infty) = \rho$.

We finally show the non-monotonicity of $c(i)$. With some algebra, we have

$$c'(i) = \rho \frac{(\frac{\nu}{1-\nu}i - 1)\tilde{\alpha}(1 + i)^{\nu/(1-\nu)} - 1}{[\tilde{\alpha}(1 + i)^{1/(1-\nu)} + 1 + i]^2}$$

where $\tilde{\alpha} = (\frac{\alpha}{1-\alpha})^{1/(1-\nu)}$. The function $(\frac{\nu}{1-\nu}i - 1)\tilde{\alpha}(1 + i)^{\nu/(1-\nu)}$ is negative if $i < \frac{1-\nu}{\nu}$, and is positive and is increasing in i when $i > \frac{1-\nu}{\nu}$. It goes to infinity as i goes to infinity. Therefore there exists unique $i^* (> \frac{1-\nu}{\nu} > 0)$ such as $c'(i^*) = 0$. For such i^* , $c'(i) < 0$ if $i < i^*$, and $c'(i) > 0$ if $i > i^*$.

B Proof of Lemma 2

From Eq. (20), $M_{j,t}/P_t = m(i)\omega_{j,t}$. Since the liquidity constraint on the cash good binds, $C_{j,t}^1 = M_{j,t}/P_t$. Thus we obtain Eq. (22). From Eq. (19), $C_{j,t}^1 + C_{j,t}^2 = c(i)\omega_{j,t}$. Thus we get Eq. (23). Eq. (24) is obvious from Eq. (18). Finally, substituting Eqs. (21) and (22) into Eq. (13) yields Eq. (25).

C Proof of Proposition 1

Eq. (38) is obvious. Eq. (39) comes from Eq. (27). Substitution of Eq. (39) into the resource constraint (39) yields $C_t^1 + C_t^2 = (A - g)K_t$. The ratio of cash good consumption over the credit good consumption is $C_t^1/\{C_t^1 + C_t^2\} = \frac{m(i)}{c(i)}$. Thus we obtain Eqs. (40) and (41). When the nominal interest rate is i , the consumption share is $c(i)$, and the balanced growth rate is equal to $g = g(c(i))$.

D Proof of Proposition 3

As the nominal rate i goes to infinity, the term $c(i)$ goes to $\rho = c(0)$. Thus the balanced growth rate g and variable z returns to the levels under the Friedman rule. Thus

$$\lim_{i \rightarrow \infty} \left\{ F(g) - \frac{z^2}{2\rho\sigma^2} \right\} = F(g_{FR}) - \frac{z_{FR}^2}{2\rho\sigma^2}$$

On the other hand, the utility from money balance $U(1, \phi)$ is a strictly decreasing function of i and then

$$\lim_{i \rightarrow \infty} u(1, m(i)/c(i)) < u(1, m(0)/c(0))$$

Therefore the equilibrium welfare falls short of the one under the Friedman rule when the nominal rate is sufficiently high:

$$\lim_{i \rightarrow \infty} W(i) < V_{j,0}(0).$$

This implies that for some strictly positive i , $W(i)$ is maximized.

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