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The Burden of Excellence: Endogenous Efficiency Paradoxes under Coopetition*

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Abstract

We analyze a two-stage duopoly where rivals first make non-cooperative demand-expanding investments yielding non-excludable benefits and then compete in product markets. Two efficiency paradoxes emerge endogenously. First, when production technologies are identical, firms with less efficient investment technology earn higher profits. Second, firms disadvantaged in both production and investment can outperform superior rivals. The mechanism is that market-expanding investments benefit all firms while costs fall disproportionately on efficient investors, enabling inefficient firms to free-ride. These paradoxes persist across product differentiation, simultaneous timing, and alternative aggregation technologies. Subsidies intended to remedy market failures paradoxically exacerbate efficiency reversals. While efficiency heterogeneity enhances short-run welfare through complementary effects, it may undermine long-run market selection, potentially causing inefficient monopolization. Our framework applies to brand advertising, platform development, standard-setting, and industry reputation, revealing fundamental tensions between static welfare gains and dynamic efficiency, with implications for competition policy and strategic management in coopetitive markets.

Keywords: Coopetition, efficiency paradox, profit reversals, public goods, market selection, subsidy
JEL classification: D43, H41, L13, L21

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1 Introduction

In many industries, firms simultaneously compete for market share while cooperating on activities that benefit all participants—a phenomenon known as *coopetition* (Brandenburger and Nalebuff, 1996). Representative examples include the California Milk Processor Board's "Got Milk?" campaign by rival dairy processors, Champagne producers' joint investments in quality control through the Comité Champagne, and the formation of the Bluetooth Special Interest Group by leading technology firms to develop common standards.[1] Similar structures appear in retail centers, destination marketing, and industry certification schemes (Bengtsson and Raza-Ullah, 2016). These investments create public-good benefits that raise demand for all participants, yet firms differ in technological capabilities and costs, raising natural questions about who ultimately pays and who ultimately gains under coopetition.

In practice, institutional arrangements often address this asymmetry through differential burden-sharing systems. Examples include tourism marketing districts, urban Business Improvement Districts, and agricultural commodity "checkoff" programs, which typically impose higher contributions on larger or more capable firms.[2] However, the crucial insight is that even without such exogenous institutions, strategic interactions among firms can endogenously generate similar patterns of asymmetric burden-sharing. In public-good demand-expanding investments, benefits accrue non-excludably to all firms while investment decisions are made based on individual efficiency levels, naturally creating a structure where more efficient firms bear larger investment burdens while less efficient firms free-ride on the resulting market expansion.

This endogenous asymmetry poses a fundamental challenge to conventional competition theory. In standard competitive environments, efficiency advantages are expected to translate directly into profit advantages. However, in demand-side coopetition, the investment costs borne by efficient firms may exceed the benefits they capture from expanded market demand. When this occurs, highly efficient firms may systematically earn lower profits than their less efficient rivals—an "efficiency paradox" that emerges purely from strategic interactions. This study identifies the conditions under which such paradoxes arise endogenously in coopetitive markets and provides a theoretical framework for reconsidering the fundamental sources of competitive advantage when firms simultaneously compete and cooperate through market-expanding investments.

We formalize these ideas by constructing a two-stage duopoly model where firms engage in demand-side coopetition. In the first stage, they non-cooperatively choose public-good investments that expand market demand. In the second stage, they engage in Cournot competition in product markets. Our analysis uncovers two counterintuitive results regarding profit rankings and reveals important welfare implications. Contrary to conventional wisdom whereby efficient firms should outperform their ineffi-

[1] Advertising Educational Foundation, case history: "Got Milk?" `https://aef.com/classroom-resources/case-histories/got-milk/`; Comité Champagne overview: `https://www.champagne.fr/en/find-out-more/champagne-industry/comite-champagne`; Bluetooth SIG official resources: "Who we are," `https://www.bluetooth.com/about-us/who-we-are/`.

[2] For instance, in the San Diego Tourism Marketing District, hotels with fewer than 70 rooms are exempt while larger hotels pay a 2% revenue fee that benefits the entire sector, illustrating systematic free-riding by smaller establishments (`https://sdtmd.org/`); New York City's Business Improvement Districts are financed by mandatory assessments that fall disproportionately on large commercial properties (`https://www.nyc.gov/site/sbs/neighborhoods/bids.page`); and agricultural checkoff programs compel major producers to fund generic advertising from which smaller producers also benefit (`https://www.choicesmagazine.org/2006-2/checkoff/2006-2-02.htm`).

cient counterparts, we find that the profit ranking can be reversed under demand-side coopetition—a profit reversal. Our reversals are endogenous: they emerge in equilibrium from non-excludable, demand-raising investments and strategic interaction, rather than from exogenous wedges or policy changes.

The "Single-Dimension Efficiency Paradox" (SDEP) demonstrates that when production technologies are identical, the firm with less efficient investment technology systematically earns greater profits than its more efficient rival. The "Dual-Dimension Efficiency Paradox" (DDEP) reveals that even a firm disadvantaged in both production and investment can outperform a more efficient competitor under certain conditions. The intuition underlying these paradoxes is straightforward: market-expanding investments provide non-excludable benefits to both firms while imposing a disproportionately large investment burden on the more efficient firm. Consequently, the efficient firm shoulders a larger share of total investment while the inefficient firm free-rides on its rival's efforts, potentially reversing conventional profit rankings.

These findings echo concerns observed in real-world contexts where efficient firms bear disproportionate burdens in collective investments. Legal challenges have emerged in cases such as *United Foods v. United States*, 533 U.S. 405 (2001) and *Glickman v. Wileman Brothers & Elliott*, 521 U.S. 457 (1997), where concerns centered on efficient firms bearing excessive burdens while competitors free-ride on collective marketing efforts.[3] Our analysis reveals that similar asymmetric burden-sharing arises endogenously even under purely voluntary investment decisions and in markets with homogeneous products, suggesting that the fundamental tension emerges from the strategic structure of coopetitive markets themselves.

Our welfare analysis reveals a fundamental temporal tension in coopetitive markets. While efficiency heterogeneity unambiguously enhances short-run social welfare through complementary effects on consumer and producer surplus, the long-run consequences may be quite different. The same efficiency differences that maximize static welfare also create conditions that make efficiency paradoxes more likely, potentially undermining market-based selection mechanisms and placing excessive investment burdens on efficient firms.

To establish the robustness of these endogenous efficiency paradoxes, we extend our baseline model along several dimensions. We show that the paradoxes persist even with differentiated products, though their likelihood diminishes as differentiation increases. Analysis of simultaneous coopetition shows that removing strategic commitment amplifies asymmetric burden-sharing and makes the paradoxes more pronounced. Alternative aggregation technologies for the public good—best-shot versus weakest-link—reveal that best-shot technologies make the paradoxes more likely, whereas weakest-link technologies eliminate them entirely. Subsidies intended to remedy market failures can paradoxically exacerbate efficiency reversals.

Moreover, efficiency paradoxes extend beyond profit rankings to distort market structure itself. Our analysis of endogenous entry and exit decisions reveals that these distortions can lead to inefficient monopolization, where less capable firms drive out superior competitors through perverse market dynamics (patterns observed in coopetitive settings such as shopping malls). This creates a critical

[3] Economic research has provided mixed evidence on asymmetric effects in collective advertising (Hamilton et al., 2013; Crespi and Marette, 2002).

policy dilemma: the efficiency differences that benefit society in the short run may simultaneously undermine long-term market efficiency and technological progress.

Our findings have important theoretical and practical implications. On the theoretical side, we extend the cost-paradox literature by demonstrating how endogenous strategic interactions can generate paradoxical profit rankings. We advance the coopetition literature by providing rigorous theoretical foundations for understanding how efficiency asymmetries influence market outcomes when firms simultaneously compete and cooperate. On the practical side, our findings help explain why less efficient firms can survive and thrive in certain industries by free-riding on stronger firms' investments in industry-wide public goods. For policymakers, our framework reveals a crucial trade-off: while efficiency heterogeneity enhances social welfare, it creates conditions where less efficient firms can exploit more efficient rivals, potentially hampering long-term technological progress.

The remainder of this paper proceeds as follows. Section 2 reviews the related literature. Section 3 establishes our theoretical framework and characterizes the subgame-perfect Nash equilibrium. Section 4 demonstrates the efficiency paradoxes and elucidates their economic mechanisms. Section 5 investigates robustness under various alternative specifications. Section 6 analyzes policy interventions and endogenous market structure effects. Section 7 examines the welfare implications of efficiency heterogeneity in coopetitive markets. Section 8 discusses implications and concludes.

2 Related Literature

Situations where firms simultaneously compete and cooperate are known as coopetition, a concept formalized by Brandenburger and Nalebuff (1996) and extensively studied across economics, management, and marketing (Bengtsson and Kock 2000; Ritala and Sainio 2014; Bouncken et al. 2015; Rey and Tirole 2019). In economics, particular attention has been paid to oligopolistic settings with spillover-generating investments. On the supply side, cost-reducing R&D with spillovers and joint research ventures have been analyzed by D'Aspremont and Jacquemin (1988) and Kamien et al. (1992), establishing the foundation for supply-side coopetition. Building on this foundation, Amir and Wooders (2000) analyze a duopoly with one-way spillovers where ex-ante identical firms endogenously adopt asymmetric R&D roles, demonstrating that research joint ventures can improve outcomes under certain conditions. Our study extends these insights to demand-side coopetition with efficiency asymmetries, showing that the efficient firm's disproportionate burden in market-expanding investments can paradoxically lead to systematically higher profits for the inefficient firm—a reversal that persists even under investment cooperation. This finding challenges conventional efficiency-based competitive advantages in coopetitive settings.

In contrast to supply-side coopetition, demand-side coopetition involves investments that expand market demand rather than reduce production costs. Empirical evidence of such arrangements is documented across industries. Teller et al. (2016) study shopping centers where stores jointly invest in marketing and facilities, while Gnyawali and Park (2011) analyze the Samsung–Sony alliance, showing how shared innovation investments can lead to asymmetric benefit appropriation. Among recent theoretical studies, our work extends the framework developed by Hattori and Yoshikawa (2016), who analyze the interaction between free entry and public-good investment under symmetric firm capabilities. Their analysis leaves unexplored how efficiency asymmetries affect profit distributions. Our analysis extends their framework by introducing heterogeneous production and investment efficiencies and examining robustness across various specifications including investment cooperation, product differentiation, and alternative aggregation technologies (best-shot and weakest-link). This reveals that efficiency advantages can become strategic liabilities in coopetitive settings, with the paradox persisting across diverse market configurations. Complementing this line of research, Heywood et al. (2023) analyze the provision of price-excludable public goods by rival duopolists, showing how non-rival consumption characteristics lead to market segmentation. However, their focus on provision of the good itself leaves unexplored how asymmetries in investment efficiency can paradoxically advantage less efficient firms—a central mechanism we identify.

Beyond the coopetition literature, our work also relates to research on firm heterogeneity and market outcomes. The relationship between firm heterogeneity and market outcomes in oligopolistic settings reveals important tensions regarding how efficiency differences affect competitive dynamics and welfare. In Cournot oligopoly with heterogeneous firms, Lahiri and Ono (1988) demonstrate that policies designed to assist less efficient firms can paradoxically reduce total social welfare by shifting production from efficient to inefficient firms. Similarly, Bagwell and Lee (2023) examine the single-sector Melitz-Ottaviano model and find that markets systematically misallocate production toward less efficient firms, revealing how market mechanisms can generate outcomes that undermine efficiency-based selection. However, these studies focus primarily on the direct allocative and welfare effects of efficiency differences. They leave unexplored how such differences interact with strategic investment decisions in coopetitive settings to generate efficiency paradoxes where more capable firms systematically underperform their less efficient rivals through asymmetric investment burden-sharing. We identify this central mechanism in demand-side coopetition and further show that uniform subsidies intended to remedy market failures can counterintuitively exacerbate these efficiency reversals.

Empirical research on generic advertising and collective investment schemes provides additional evidence of similar asymmetric effects. Crespi and Marette (2002) demonstrate theoretically and empirically that generic advertising can harm high-quality producers by eroding product differentiation, using evidence from the California prune industry. Hamilton et al. (2013) show that generic advertising creates demand rotations that disproportionately benefit large-scale producers using U.S. Beef Checkoff Program data.[4] Additional evidence comes from Business Improvement Districts, where empirical studies document redistribution of benefits across firm types (Ellen et al., 2007). Our analysis extends this literature by showing that even with homogeneous products and absent demand rotations, efficiency heterogeneity itself can generate paradoxical profit rankings through asymmetric burden-sharing.

Our work also contributes to the broader efficiency paradox literature in industrial organization theory. Seade (1985) shows that exogenous cost increases can raise profits under certain demand functions, while Amir et al. (2017) demonstrate that efficiency paradoxes can arise endogenously through firms' R&D choices. In the public goods literature initiated by Bergstrom et al. (1986), paradoxical

[4]Zheng et al. (2010) provide comprehensive theoretical analysis of how different types of demand effects from generic advertising—parallel shifts, elastic rotations, and inelastic rotations—create varying profit distributions across firm sizes in asymmetric Cournot markets.

results emerge when higher costs yield higher utilities for voluntary contributors (Buchholz and Konrad 1994; Ihori 1996; Hattori 2005). While these studies focus on efficiency paradoxes in different contexts, we show that in coopetitive markets with efficiency asymmetries, strategic interaction itself generates systematic profit reversals with direct implications for welfare and policy.

3 The Model

We consider a duopoly with two firms ($i \in \{E, I\}$) producing a homogeneous good. The firms engage in demand-side coopetition: they compete in the product market while also making industry-wide public-good investments that raise consumers' willingness to pay for the category (hereafter, public-good investments). We refer to the resulting object as the public good—a shared demand shifter that is non-rival and non-excludable. Throughout, Firm E denotes the firm with more efficient technology in both production and investment, while Firm I denotes the firm with less efficient technology; that is, Firm E is weakly more efficient than Firm I along both dimensions. For a complete summary of notation, see A1 in Appendix A.1.

We analyze the following two-stage game. In the first stage, each firm i non-cooperatively and simultaneously chooses its public-good investment g_i, thereby shifting market demand. In the second stage, each firm i engages in Cournot competition, non-cooperatively and simultaneously choosing its output level, x_i.

The inverse demand is given by $P(X, G)$, where $X \equiv x_E + x_I$ is total output and $G \equiv g_E + g_I$ is the total investment. Firm i's profit function is given by

$$\pi_i \equiv P(X, G)x_i - C_i(x_i) - K_i(g_i).$$

where $C_i(x_i)$ is the production cost and $K_i(g_i)$ is the investment cost function. Here, g_i represents the effectiveness level of investment (a demand-shifting index), while $K_i(g_i)$ denotes the monetary expenditure required to achieve that effectiveness level.

We impose the following standard regularity conditions on the demand and cost functions.

Assumption 1 (Demand and Cost)**.** *The inverse demand function $P(X, G)$ and cost functions $C_i(x_i)$, $K_i(g_i)$ are twice continuously differentiable on their respective domains and satisfy:*

(i) $P_X < 0$, $P_G > 0$.

(ii) *Production costs are ρ_x-th order homogeneous with $\rho_x \geq 1$: $C_i(x_i) = c_i x_i^{\rho_x}$ where $c_I \geq c_E > 0$.*

(iii) *Investment costs are ρ_g-th order homogeneous with $\rho_g > 1$: $K_i(g_i) = k_i g_i^{\rho_g}$ where $k_I > k_E > 0$.*

(iv) $P(0, G) > c_I$ *and* $\lim_{X\to\infty} P(X, G) < c_E$ *for every* $G \geq 0$.

Condition (i) states that inverse demand falls with output and rises with aggregate public-good investment. Conditions (ii) and (iii) normalize both cost functions to start at zero and impose non-decreasing marginal cost; the exponents ρ_x and ρ_g allow their curvature to vary.[5] Condition (iv)

[5]Even under $\rho_x = 1$ (the production cost is linear), an interior Cournot solution still exists because marginal revenue declines in output. For public-good investment we need $\rho_g > 1$; when $\rho_g = 1$ marginal investment cost would be constant, leading to a corner solution.

adds two level bounds: $P(0,G) > c_I$ ensures strictly positive equilibrium output for every $G \geq 0$, while $\lim_{X\to\infty} P(X,G) < c_E$ rules out unbounded output. Together these bounds guarantee that the Cournot equilibrium derived in Section 3.1 is always interior and finite.

3.1 Equilibrium

We solve for the subgame-perfect Nash equilibrium (SPNE) of the two-stage game using backward induction. We begin by characterizing the Cournot equilibrium that arises in the second stage.

Given the first-stage total investment level G, firm i chooses its output x_i to maximize its profit. The first-order condition for profit maximization is

$$P\left(X,G\right) + P_X\left(X,G\right)\, x_i = C_i'(x_i), \qquad i = E, I. \tag{1}$$

Assumption 2 (Cournot Stability)**.** *For every $X, G \geq 0$ the inverse demand function satisfies $P_X(X,G) + X\, P_{XX}(X,G) < 0$.*

Under Assumptions 1 and 2 the marginal-revenue schedule on the left-hand side of (1) is strictly decreasing in x_i because $P_X < 0$ is already imposed in Assumption 1, whereas the marginal cost $C_i'(x_i) = c_i \rho_x x_i^{\rho_x - 1}$ is weakly increasing for all $\rho_x \geq 1$. The additional curvature inequality $P_X + XP_{XX} < 0$ from Assumption 2 is the strict Bulow-Geanakoplos-Klemperer condition; it guarantees that each reaction curve slopes downward and that the joint best-reply mapping is a contraction. Together with the demand-level bounds $P(0,G) > c_I$ and $\lim\limits_{X\to\infty} P(X,G) < c_E$ in Assumption 1, these properties imply that for every aggregate investment level G the Cournot equilibrium $(\tilde{x}_E(G), \tilde{x}_I(G))$ is unique, globally stable, and interior, i.e. $0 < \tilde{x}_i(G) < \infty$. See Vives (1999) for the formal proof.[6]

We denote the equilibrium aggregates by

$$\tilde{X}(G) = \tilde{x}_E(G) + \tilde{x}_I(G), \quad \tilde{P}(G) = P\big(\tilde{X}(G), G\big),$$

and firm-level profits (conditional on own g_i) by

$$\tilde{\pi}_i\big(G, g_i\big) = \tilde{P}(G)\, \tilde{x}_i(G) - C_i\big(\tilde{x}_i(G)\big) - K_i(g_i), \qquad i \in \{E, I\}.$$

In the first stage, each firm chooses its public-good investment g_i—its contribution to the shared demand shifter—while anticipating the second-stage Cournot outcome. Let $\tilde{x}_i(G)$ and $\tilde{X}(G)$ denote the unique second-stage outputs obtained under Assumptions 1 and 2. Applying the envelope theorem, firm i's first-order condition is

$$\left[P_G + P_X \frac{\partial \tilde{x}_j(G)}{\partial G}\right] \tilde{x}_i(G) = K_i'(g_i), \qquad i \in \{E, I\},\ j \neq i. \tag{2}$$

[6] When $\rho_x > 1$ we have $C_i''(x) > 0$, which tightens the contraction bound. Even under linear costs ($\rho_x = 1$), the strict Bulow-Geanakoplos-Klemperer condition keeps the reaction-slope modulus below one, so existence, uniqueness, global stability, and interiority are preserved.

Assumption 3 (Investment Stability). *For every $G \geq 0$ and each firm $i \in \{E, I\}$ define*

$$\mathrm{MB}_i(G) = \left[P_G(\tilde{X}(G), G) + P_X(\tilde{X}(G), G)\, \frac{\partial \tilde{x}_j(G)}{\partial G} \right] \tilde{x}_i(G).$$

The primitives satisfy

$$\sup_{G \geq 0} \left| \frac{d\,\mathrm{MB}_i(G)/dG}{K_i''(g_i)} \right| < 1.$$

Because $K_i(g_i) = k_i g_i^{\rho_g}$ with $\rho_g > 1$ implies $K_i''(g_i) > 0$ for all $g_i > 0$, Assumption 3 ensures that the investment best-reply mapping is a contraction, guaranteeing a unique Nash equilibrium in the investment stage.[7] Hence (2) has a unique interior solution (g_E^*, g_I^*), where starred variables denote equilibrium values.

Let $G^* = g_E^* + g_I^*$. The equilibrium outputs are $x_i^* = \tilde{x}_i(G^*)$ and the equilibrium price is $P^* = P(X^*, G^*)$ with $X^* = x_E^* + x_I^*$. The profile $((x_E^*, x_I^*), (g_E^*, g_I^*))$ is the unique subgame-perfect Nash equilibrium (SPNE) of the two-stage game, and equilibrium profits are

$$\pi_i^* = P^* x_i^* - C_i(x_i^*) - K_i(g_i^*)\,, \qquad i = E, I.$$

With the equilibrium fully characterized, we now turn to how efficiency asymmetries affect the profit ranking of the two firms.

4 Efficiency Paradoxes

This section examines how efficiency asymmetries between firms affect the profit ranking and presents two paradoxical theorems: the Single-Dimension Efficiency Paradox (SDEP) and the Dual-Dimension Efficiency Paradox (DDEP).

The SDEP considers the case where both firms have identical production efficiency but differ only in their public-good investment efficiency. We demonstrate that in this setting, the equilibrium profit of the efficient firm π_E^* is necessarily lower than that of the inefficient firm π_I^*, yielding the counterintuitive result that the firm with lower investment efficiency systematically outperforms its more efficient rival.

The DDEP extends this analysis to situations where the efficient firm enjoys cost advantages in both production and investment dimensions. We show that even when Firm E possesses unambiguous efficiency advantages in both dimensions, its equilibrium profit π_E^* can still fall below that of the inefficient firm π_I^*.

4.1 Single-Dimension Efficiency Paradox

We begin with the benchmark case in which firms differ only in their investment efficiencies while having identical production efficiencies.

[7] Totally differentiating (1) with respect to G and using $P_X + XP_{XX} < 0$ yields $0 < \frac{\partial \tilde{x}_i(G)}{\partial G} \leq \frac{2|P_G|}{|P_X + XP_{XX}|}$. Substituting these bounds into $d\,\mathrm{MB}_i(G)/dG$ shows that this derivative is finite on $[0, \infty)$, so the supremum in Assumption 3 is well defined; see Vives (1999) for details.

Theorem 1 (Single-Dimension Efficiency Paradox). *Under Assumptions 1-3, when $c_I = c_E$ and $k_I > k_E$, we necessarily have $\pi_I^* > \pi_E^*$.*

Proof. See Appendix A.2. □

The SDEP demonstrates that the firm with lower investment efficiency can systematically outperform its more efficient rival. The intuition is straightforward. Public-good investment operates as a shared demand shifter: any firm's outlay raises market demand for both firms. Because Firm E converts resources into the public good at lower cost, it optimally chooses a higher investment level. The resulting demand gain accrues symmetrically, whereas the cost burden does not. Hence, Firm E bears most of the investment expense while Firm I free-rides, and the profit ranking reverses.

4.2 Dual-Dimension Efficiency Paradox

We now extend the analysis to the case where Firm I is disadvantaged in both production and investment efficiencies ($c_I > c_E$ and $k_I > k_E$).

Theorem 2 (Dual-Dimension Efficiency Paradox). *Under Assumptions 1-3, when $c_I > c_E$ and $k_I > k_E$, we have*

$$\pi_I^* > \pi_E^* \iff \frac{1}{\rho_x}[C_I'(x_I^*) - C_E'(x_E^*)]X^* [1 + (\rho_x - 1)\epsilon_D] < K_E(g_E^*) - K_I(g_I^*), \tag{3}$$

and such parameter combinations exist.

Proof. See Appendix A.3. □

Theorem 2 states that the inefficient firm earns strictly more in equilibrium precisely when its investment-burden advantage (right-hand side) outweighs its production-efficiency disadvantage (left-hand side). On the left-hand side, the factor X^*/ρ_x is the Euler scaling implied by $C_i(x) = c_i x^{\rho_x}$: since $x\, C_i'(x) = \rho_x\, C_i(x)$, it converts a per-unit marginal-cost differential into a total-cost differential at the industry scale X^*. The bracketed term $[1 + (\rho_x - 1)\epsilon_D]$ is an amplification factor: stronger cost convexity ρ_x and higher price elasticity of demand $\epsilon_D \equiv -P/(X \cdot P_X)$ (evaluated at X^*) make a given marginal-cost gap translate into a larger output and revenue gap under Cournot competition. The right-hand side measures the investment-burden asymmetry: Public-good investment shifts demand for both firms, but it is cheaper for the efficient firm, so the efficient firm bears a larger share of the investment while Firm I free-rides on the induced market expansion. Hence DDEP arises exactly when this advantage exceeds the production disadvantage.

Paradoxes are more likely when: (i) the investment efficiency gap ($k_I - k_E$) is large, (ii) the production efficiency gap ($c_I - c_E$) is small, and (iii) the market size (X^*) is small. While demand elasticity ϵ_D directly influences the condition, changes in elasticity simultaneously affect all equilibrium values, so whether increased elasticity enhances or diminishes DDEP likelihood depends on the specific functional forms and relative magnitudes of these effects.

While this general condition provides theoretical insights, its practical relevance depends on whether such parameter configurations occur naturally in specific functional forms. Throughout the remainder of this paper, we refer to Theorem 2 as the *efficiency paradox*, with Theorem 1 representing the special case where production efficiencies are identical.

4.3 A Tractable Specification

To illustrate the two efficiency paradoxes, we now specialize the general framework to familiar functional forms. The inverse demand is assumed to be linear, production costs are linear, and investment costs are quadratic[8]:

Assumption 4 (Tractable Specification). *$P(X,G) = a + G - bX$ with $a, b > 0$; $C_i(x_i) = c_i x_i$ with $c_i > 0$; $K_i(g_i) = k_i g_i^2$ with $k_i > 0$.*

Under this assumption, the interiority condition $K_i'(g) > 2P_G/9|P_X|$ reduces to the simple condition $9bk_i > 1$ for $i \in \{E, I\}$.

The second-stage (Cournot) equilibrium is characterized as

$$\tilde{x}_i(G) = \frac{a + G - 2c_i + c_j}{3b}, \quad X = \frac{2(a+G) - c_E - c_I}{3b}.$$

In the first stage, the best-reply function of firm i, $R_i(g_j)$ is obtained by

$$g_i = R_i(g_j) \equiv \frac{a - 2c_i + c_j}{9bk_i - 1} + \frac{1}{9bk_i - 1} g_j,$$

which implies that coopetitive investments are strategic complements. The equilibrium investment g_i^* is obtained by

$$g_i^* = \frac{3bk_j\,(a + c_j - 2c_i) + (c_i - c_j)}{3b\,(9bk_I k_E - k_I - k_E)}.$$

Because $9bk_i > 1$ for all $i \in \{E, I\}$, the denominator is positive and the equilibrium investment is interior.[9]

Substituting these equilibrium values g_I^* and g_E^* into all endogenous variables yields the SPNE profits π_I^* and π_E^*.

Now, setting $c_I = c_E = c$ and $k_I > k_E$ collapses the profit difference to

$$\pi_I^* - \pi_E^* \;=\; \frac{(a-c)^2\, k_I k_E\, (k_I - k_E)}{\left(9bk_I k_E - k_I - k_E\right)^2} > 0,$$

which provides the result of SDEP: the inefficient firm with lower investment efficiency always earns more when production efficiencies are identical.

Next, allowing $\rho_x = 1$, the condition (3) reduces to

$$(c_I - c_E)X^* < K_E^* - K_I^*.$$

The left-hand side is increasing in the gap in marginal production cost $c_I - c_E$, whereas the right-hand

[8]The functional specification employed here builds upon the framework developed by Hattori and Yoshikawa (2016), who examined demand-side coopetition under symmetric cost structures.

[9]The denominator is $3b(9bk_I k_E - k_I - k_E) = 3b[(9bk_E - 1)k_I - k_E)] > 3b(k_I - k_E) > 0$ since $9bk_E > 1$.

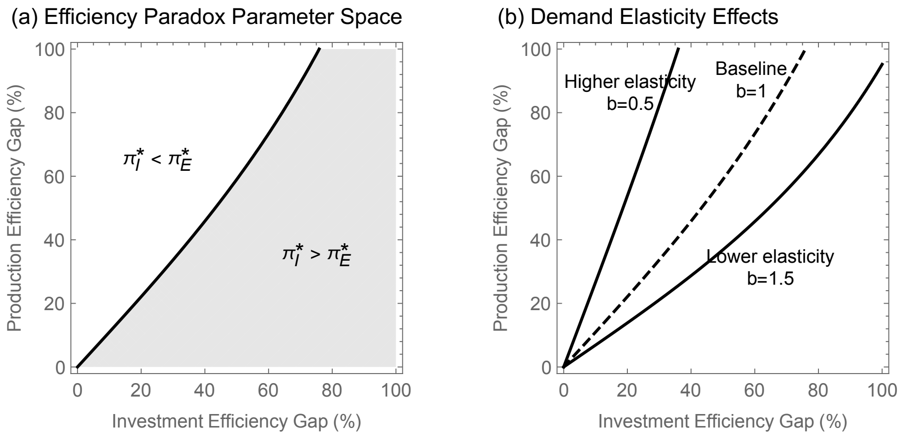

Figure 1: Efficiency Paradoxes under Tractable Specification

Note: Panel (a) shows the baseline parameter region where $\pi_I^* = \pi_E^*$ (solid curve). The shaded area below the curve represents parameter combinations where $\pi_I^* > \pi_E^*$ (efficiency paradoxes occur); above the curve $\pi_I^* < \pi_E^*$ (standard profit ordering). The horizontal axis where production efficiency disadvantage = 0 corresponds to the Single-Dimension Efficiency Paradox (SDEP), while the interior shaded region represents the Dual-Dimension Efficiency Paradox (DDEP). Panel (b) demonstrates robustness across different demand elasticities: higher elasticity ($b = 0.5$, curve above baseline) expands the paradox region, while lower elasticity ($b = 1.5$, curve below baseline) contracts it, compared to the baseline ($b = 1.0$, dashed curve). Parameters: $a = 50$, $c = 0.5$, $k = 2$ for both panels; $b = 1.0$ for panel (a).

side is increasing in the gap in investment cost parameters $k_I - k_E$. Computing this condition explicitly yields:

$$\left\{\frac{3b(a-2c_E+c_I)k_I+c_E-c_I}{3b(a-2c_I+c_E)k_E+c_I-c_E}\right\}^2 < \frac{k_I(9bk_I-1)}{k_E(9bk_E-1)}. \tag{4}$$

To examine how demand elasticity affects the likelihood of efficiency paradoxes, we analyze the comparative statics of condition (4) with respect to b. Denoting the left-hand side and right-hand side as $LHS(b)$ and $RHS(b)$, respectively, we find that in the standard parameter region where the equilibrium remains interior, $RHS(b)$ decreases in b while $LHS(b)$ increases in b. Consequently, higher demand elasticity (smaller b) makes the inequality more easily satisfied (see Figure 1-(b)).

Corollary 1 (Elasticity Effects)**.** *Under Assumption 4, more elastic demand conditions (smaller b) expand the parameter region where efficiency paradoxes emerge.*

The intuition is straightforward. Higher demand elasticity increases equilibrium quantities and investment levels, amplifying the absolute difference in investment costs between firms. This enlarged cost asymmetry makes the paradox more likely to emerge.

To visually examine how efficiency gaps between Firms E and I in production and investment determine the emergence of efficiency paradoxes, we consider the following production and investment cost specification:

$$c_E = c - \delta_c, \quad c_I = c + \delta_c, \quad k_E = k - \delta_k, \quad k_I = k + \delta_k, \tag{5}$$

where $\delta_c \in [0, c)$ and $\delta_k \in [0, k)$ represent cost asymmetry parameters for production and investment, respectively. We focus on parameter configurations that yield interior solutions and stable Nash equilibria. This specification maintains constant average costs across firms while allowing systematic variation in efficiency gaps, thereby isolating the pure effects of asymmetric capabilities on market outcomes.

Figure 1 illustrates the parameter regions where efficiency paradoxes emerge under this specification. The inequality condition traces an upward-sloping boundary in the space defined by relative efficiency disadvantages, with the horizontal axis measuring investment efficiency disadvantage (the percentage by which k_I exceeds k_E) and the vertical axis measuring production efficiency disadvantage (the percentage by which c_I exceeds c_E). The shaded region below this boundary represents parameter combinations where profit rankings are reversed, with the efficiency-disadvantaged firm (Firm I) systematically outperforming its more capable rival. Panel (a) demonstrates that this paradox region is substantial, confirming that the DDEP constitutes a robust phenomenon rather than a knife-edge result, emerging across a broad range of realistic parameter configurations.

Panel (b) demonstrates the effect of demand elasticity on paradox likelihood, with higher elasticity (smaller b) expanding the parameter region where efficiency paradoxes occur. This visual confirmation supports our analytical result and shows that the relationship between demand elasticity and paradox emergence is both systematic and substantial across the parameter space.

This finding resolves the ambiguous condition in Theorem 2 regarding the role of demand elasticity. While the general theoretical condition suggests that elasticity's effect could be unclear due to its influence on both sides of the inequality, the tractable specification reveals that investment differentials unambiguously dominate, making efficiency paradoxes more likely under elastic demand conditions.[10]

5 Extensions: Market Environments and Interaction Modes

This section extends our baseline model in four directions to demonstrate the robustness of efficiency paradoxes. We examine: (i) product differentiation that weakens investment spillovers, (ii) simultaneous coopetition where strategic commitment is removed, (iii) investment cooperation arrangements that alter burden-sharing mechanisms, and (iv) alternative aggregation technologies (best-shot and weakest-link) that reveal the boundaries of our results. These extensions establish the robustness of efficiency paradoxes while identifying conditions under which they are amplified or eliminated.

For tractable exposition, we employ the specification from Section 4.3 (linear demand, linear production costs, and quadratic investment costs) throughout Sections 5-7, unless otherwise noted.

5.1 Product Differentiation and Imperfect Spillovers

While our main analysis considered coopetitive markets with homogeneous Cournot competition and perfect investment spillovers, this subsection examines whether efficiency paradoxes persist when product differentiation reduces the spillover benefits of market-expanding investments. For tractable expo-

[10]For simplicity we focus on the two-firm case. The analysis extends to settings with n_I symmetric inefficient firms and n_E symmetric efficient firms. In such cases, both SDEP and DDEP continue to arise qualitatively: inefficient firms still bear a disproportionate share of coopetitive investment, and profit reversals emerge under analogous thresholds. Increasing the total number of firms generally makes reversals less likely, since each firm's relative investment burden diminishes. With the total number of firms fixed, however, reversals are more likely when inefficient firms are relatively numerous (as the investment burden falls heavily on a few efficient firms), and less likely when efficient firms are relatively numerous (as the burden is shared more broadly).

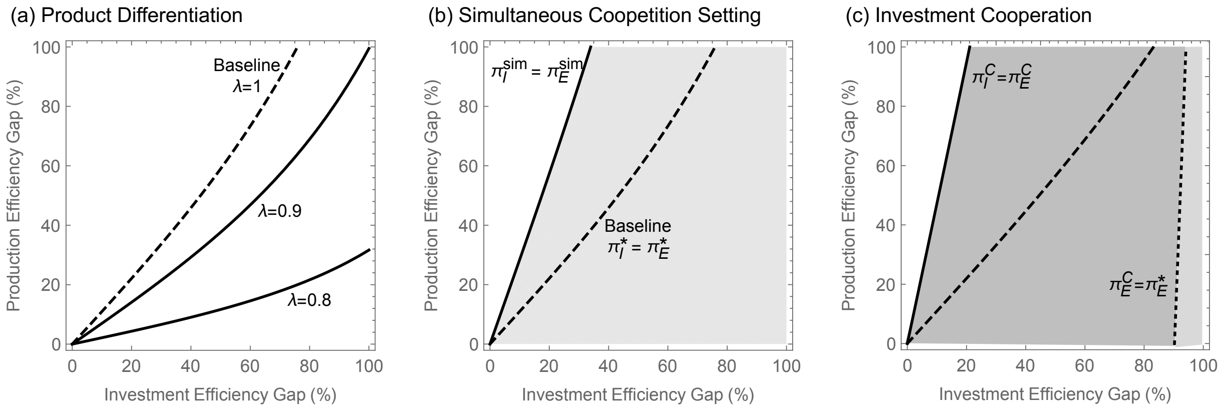

Figure 2: Robustness of Efficiency Paradoxes

Note: The figure demonstrates the robustness of efficiency paradoxes under alternative model specifications. Panel (a): Product differentiation effects with $\lambda = 1$ (homogeneous), $\lambda = 0.9$ (moderate), and $\lambda = 0.8$ (high differentiation). Panel (b): Sequential (dashed) versus simultaneous (solid) coopetition, with shaded area representing efficiency paradox regions under simultaneous coopetition. Panel (c): Investment cooperation with solid curve ($\pi_I^C = \pi_E^C$), dotted curve (Firm E's participation constraint), and dashed curve (baseline paradox boundary). Dark shaded regions indicate efficiency paradoxes under cooperative investment; light shaded regions indicate efficiency paradoxes under non-cooperative investment. Parameters: $a = 50$, $b = 1.0$, $c = 0.5$, $k = 2$.

sition, we employ the functional forms from Section 4.3 (linear demand, linear production costs, and quadratic investment costs).

We introduce a product differentiation parameter $\lambda \in [0, 1]$, where $\lambda = 1$ corresponds to homogeneous products (equivalent to our main analysis) and $\lambda = 0$ represents complete differentiation (akin to separate monopoly markets). Crucially, product differentiation also affects the spillover benefits of market-expanding investments. We capture this simply by modifying the inverse demand function to:

$$P_i = a + (g_i + \lambda g_j) - b(x_i + \lambda x_j). \tag{6}$$

This specification reflects the intuitive notion that greater product differentiation reduces the spillover effects of public-good investments. When products are less substitutable, one firm's market-expanding efforts provide smaller benefits to its rival.

Panel (a) of Figure 2 shows how product differentiation affects the parameter regions where efficiency paradoxes occur. Each curve represents the boundary where $\pi_I^* = \pi_E^*$ for different levels of product differentiation: $\lambda = 1$ (homogeneous products, dashed line), $\lambda = 0.9$ (moderate differentiation, solid line), and $\lambda = 0.8$ (high differentiation, solid line). Below each curve, $\pi_I^* > \pi_E^*$ (efficiency paradoxes occur); above each curve, $\pi_I^* < \pi_E^*$ (standard profit ordering). Greater product differentiation (lower λ) systematically reduces the parameter space where paradoxes emerge, demonstrating that spillover effects are crucial for the emergence of efficiency paradoxes.

This extension confirms the robustness of our main results, demonstrating that efficiency paradoxes are not confined to the special case of homogeneous product competition but can emerge even when products are moderately differentiated. Additionally, we find that replacing quantity competition with price competition under the same product differentiation framework yields qualitatively identical results to those shown in Panel (a) of Figure 2, with efficiency paradoxes persisting under similar parameter configurations for different levels of product differentiation.[11] This consistency across different competitive modes—quantity versus price competition—further demonstrates that our findings represent fundamental features of demand-side coopetition rather than artifacts of specific strategic formulations.

5.2 Simultaneous Coopetition

In the main analysis, we examined a two-stage game where public-good investment choices precede output decisions, reflecting scenarios where investments possess commitment power or represent long-term strategic decisions. This section extends the analysis to examine simultaneous coopetition, where firms choose their output levels and public-good investments simultaneously in a single stage.[12] This extension serves to demonstrate the robustness of our efficiency paradox results—specifically, that these counterintuitive outcomes are not artifacts of a particular game structure but emerge across alternative strategic settings. For tractable exposition, we also employ the functional forms from Section 4.3.

Under simultaneous coopetition, both firms choose their (x_i, g_i) pairs simultaneously, taking the rival's choices as given. The first-order conditions yield $(a+G) - 2bx_i - bx_j = c_i$ and $x_i = 2k_i g_i$. For these conditions to yield a local maximum for each firm we require $4bk_i > 1$ $(i = E, I)$.[13]

The equilibrium investment levels and aggregate investment are characterized by:

$$g_i^{sim} = \frac{2bk_j(a + c_j - 2c_i) + (c_i - c_j)}{2b\,(6bk_ik_j - k_i - k_j)}, \quad G^{sim} = \frac{(a + c_i - 2c_j)k_i + (a + c_j - 2c_i)k_j}{6bk_ik_j - k_i - k_j},$$

where the variables with superscript sim denote equilibrium values under simultaneous coopetition.

A key distinction emerges when comparing simultaneous and sequential coopetition. In the sequential setting, firms internalize the negative strategic effect whereby their first-stage investment increases the rival's second-stage output—captured by the term $P_X \frac{\partial \tilde{x}_j(G)}{\partial G}$ in equation (2). This strategic restraint is absent under simultaneous decision-making, leading firms to invest more aggressively. This effect is formalized by:

$$G^{sim} - G^* = \frac{3bk_ik_j\left[(a + c_i - 2c_j)k_i + (a + c_j - 2c_i)k_j\right]}{(9bk_ik_j - k_i - k_j)(6bk_ik_j - k_i - k_j)} > 0,$$

confirming that aggregate investment is systematically higher under simultaneous coopetition.

The enhanced investment incentive amplifies the asymmetric burden-sharing that drives our effi-

[11] Under price competition, we assume demand functions of the form $x_i = a + g_i + \lambda g_j - p_i + \lambda p_j$ where p_i denotes firm i's price. With appropriate parameter restrictions ensuring interior equilibrium investment levels, the efficiency paradox regions exhibit the same qualitative patterns as those depicted in Panel (a) of Figure 2, confirming the robustness of our results across different competitive modes.

[12] When investment decisions have short-term impacts and are easily reversible—such as generic advertising in daily newspapers or maintenance of shared facilities like shopping mall common areas—they lack strategic commitment value and can be characterized as simultaneous coopetition games.

[13] Given $4bk_i > 1$, the resulting Nash equilibrium is locally stable under best-response dynamics iff $\left|(2bk_E - 1)(2bk_I - 1)/(4bk_E - 1)(4bk_I - 1)\right| < 1$.

ciency paradoxes. Under simultaneous decision-making, the DDEP condition becomes:

$$\left\{\frac{2b(a-2c_E+c_I)k_I+c_E-c_I}{2b(a-2c_I+c_E)k_E+c_I-c_E}\right\}^2 < \frac{k_I(4bk_I-1)}{k_E(4bk_E-1)}. \tag{7}$$

Comparing this condition with the sequential case from equation (4) reveals a systematic shift. For identical cost parameters, the left-hand side is smaller under simultaneous timing, while the right-hand side is larger. Consequently, the inequality is more easily satisfied, making efficiency paradoxes more likely under simultaneous coopetition.

Panel (b) of Figure 2 illustrates this result graphically. The solid curve delineates the parameter region where $\pi_I^{sim} = \pi_E^{sim}$ under simultaneous timing, with the shaded area below representing combinations where efficiency paradoxes occur. The dashed curve shows the corresponding boundary from the sequential analysis. The expansion of the shaded region demonstrates that simultaneous coopetition substantially enlarges the parameter space where the inefficient firm outperforms the more efficient rival.

The intuition underlying this result is clear. Sequential investment timing introduces strategic restraint: firms recognize that their public-good contributions will enhance rivals' subsequent production incentives, creating a moderating force on investment. Under simultaneous timing, this strategic commitment effect disappears, leading to more aggressive investment behavior. Since investment burden asymmetries drive our efficiency paradoxes, the more intensive investment environment under simultaneous timing exacerbates these asymmetries, making profit reversals more likely.

5.3 Investment Cooperation

This subsection examines the impact of inter-firm cooperation, particularly joint determination of public-good investment in the first stage, following the approach of D'Aspremont and Jacquemin (1988).[14] We analyze a simple form of public-good investment cooperation where both firms jointly determine their cooperative investment levels g_I^C and g_E^C to maximize joint profits.[15] We again employ the functional forms from Section 4.3.

Consider the case where both firms can cooperatively choose g_I and g_E in the first stage to maximize joint profits, $\tilde{\pi}_I + \tilde{\pi}_E$, while still competing in the second-stage product market. We assume that investment cooperation is voluntary, with both firms participating only if cooperation yields higher profits than the non-cooperative baseline ($\pi_i^C \geq \pi_i^*$ for $i = E, I$).

The equilibrium investment levels g_i^C and aggregate investment G^C under cooperation are characterized by:

$$g_i^C = \frac{(2a-c_I-c_E)k_j}{9bk_Ik_E-2k_I-2k_E}, \quad G^C = \frac{(2a-c_I-c_E)(k_I+k_E)}{9bk_Ik_E-2k_I-2k_E}.$$

[14] A specific example of cooperative decision-making in demand-expansion investment is the telecommunications network enhancement by French telecommunications carriers. The Fédération Française des Télécoms, which comprises numerous electronic communications operators in France, collaborates with its members to enhance telecommunications networks—spanning both fixed and mobile coverage throughout the country—while focusing on infrastructure and service improvements. See `https://www.fftelecoms.org/nos-champs-daction/`.

[15] Alternative formulations such as Nash bargaining solutions could be considered, but these typically require side payments between firms to achieve the bargained allocation, which we exclude from our analysis for simplicity and realism.

Comparing the investment expenditures of both firms reveals:

$$K_E(g_E^C) - K_I(g_I^C) = (k_I - k_E)\frac{(2a - c_I - c_E)^2 k_I k_E}{(9bk_I k_E - 2k_I - 2k_E)^2} > 0,$$

which confirms that the Single-Dimension Efficiency Paradox persists under cooperative investment.

The explicit condition for efficiency paradoxes under cooperation becomes:

$$27b(c_I - c_E) < \frac{2a + 5c_I - 7c_E}{k_E} - \frac{2a + 5c_E - 7c_I}{k_I}.$$

The individual rationality (IR) conditions ensuring that both Firms E and I satisfy their participation constraints are $\pi_i^C \geq \pi_i^*$ for all $i \in \{E, I\}$.

Panel (c) of Figure 2 illustrates the conditions for efficiency paradoxes under investment cooperation. The solid curve represents the boundary where firms earn equal profits under cooperation ($\pi_I^C = \pi_E^C$). The dashed curve shows Firm E's participation constraint ($\pi_E^C = \pi_E^*$), while the dotted curve depicts the efficiency paradox boundary under non-cooperation ($\pi_I^* = \pi_E^*$).

The dark shaded region shows parameter combinations where cooperation emerges and efficiency paradoxes occur. The light shaded region indicates where Firm E's participation constraint is violated, precluding cooperation, but efficiency paradoxes still arise under the baseline scenario. The analysis demonstrates that investment cooperation expands the parameter space for efficiency paradoxes, allowing inefficient firms to achieve higher profits even within cooperative frameworks.

While we do not provide a detailed analysis here, alternative forms of investment cooperation merit consideration. One possibility involves firms coordinating to maximize joint profits through either uniform investment efforts $\bar{g}^C$ or uniform investment expenditures $\bar{K}^C = k_I g_I = k_E g_E$. Real-world examples include trade associations requiring equal contribution amounts from all members regardless of firm size, and industry consortiums setting flat-rate participation costs for standard-setting activities. Shopping center tenants similarly pay identical common area maintenance fees irrespective of their operational efficiency differences.

Under such uniform coordination schemes, the efficiency paradoxes we have demonstrated cannot arise. Importantly, our analysis focuses on self-enforcing voluntary cooperation where individual rationality constraints ensure participation incentives; we do not require external punishment mechanisms to sustain cooperation. Government-mandated programs with legal enforcement power can sustain cooperation even when individual rationality constraints are violated, potentially expanding the parameter space where efficiency paradoxes emerge, though such mandatory arrangements raise distinct legal and constitutional questions as evidenced by cases such as United Foods v. United States.

5.4 Best-Shot and Weakest-Link

In the main analysis, we assumed that the effective public-good investment follows summation technology, $G = g_E + g_I$, where both firms' investments are perfect substitutes. This section examines two polar alternatives—best-shot and weakest-link technologies—under the general framework without requiring Assumption 4.

Under a best-shot aggregation, the level of the industry public good is determined by the largest individual contribution: $G = \max\{g_E, g_I\}$. Best-shot environments arise when a single leading effort suffices to lift demand for all—e.g., technology standards in which the dominant implementation becomes the industry benchmark, shopping centers where a flagship store generates positive traffic spillovers, and industry reputation where one breakthrough product elevates category demand.

With best-shot technology, the inefficient firm completely free-rides on the efficient firm's investment, since its marginal investment cost is everywhere higher. The efficient firm determines its investment level $g_E^{BS} = G^{BS}$ by equating its marginal cost with the marginal benefit, while the inefficient firm optimally chooses $g_I^{BS} = 0$, where the superscript BS denotes equilibrium values under best-shot technology. This immediately ensures that the Single-Dimension Efficiency Paradox holds with certainty.

Moreover, even when marginal production costs differ, $\pi_I^{BS} > \pi_E^{BS}$ will prevail unless Firm I's production cost disadvantage exceeds its investment burden advantage by a substantial margin. Consequently, Firm E shoulders the entire public-good burden, making the paradoxes even more pronounced than under summation technology.

Under weakest-link technology, the level of the industry public good is determined by the smallest contribution: $G = \min\{g_E, g_I\}$. Weakest-link environments arise when the least-performing firm pins down industry outcomes—e.g., safety regulation where the least compliant firm determines industry-wide risk, supply-chain quality control where the lowest-quality node drives category reputation, and environmental stewardship where the worst offender shapes perceived sectoral harm.

With weakest-link technology, each firm wishes to match—but never exceed—the rival's investment. Consequently, the total amount G is determined by the investment level that maximizes Firm I's profit, where Firm I's marginal benefit equals its marginal cost. Firm E optimally matches this level, yielding a Nash equilibrium where $g_E^{WL} = g_I^{WL}$, where the superscript WL denotes equilibrium values under weakest-link technology.

In equilibrium, both firms invest the same quantity, but the inefficient firm pays strictly more for that quantity while also bearing the same or higher production costs. Consequently, Firm E always attains at least as much profit as Firm I, and the efficiency paradoxes vanish entirely. The likelihood of efficiency paradoxes varies dramatically with the underlying aggregation technology. Moving from summation technology toward perfect substitutability (approaching best-shot conditions), efficiency paradoxes become increasingly likely as the burden of investment falls disproportionately on the efficient firm. Conversely, as technologies exhibit greater complementarity between firms' contributions, ultimately reaching the weakest-link extreme, efficiency paradoxes become less likely and eventually disappear. This spectrum demonstrates that the strategic structure of collective action—not merely cost asymmetries—fundamentally shapes competitive outcomes in coopetitive environments.

6 Policy Interventions and Endogenous Market Structure

This section examines how efficiency paradoxes interact with policy interventions and endogenous market structure. We analyze government subsidies intended to remedy market failures and their paradoxical effects on efficiency reversals, then investigate how these paradoxes distort entry and exit decisions, potentially leading to inefficient monopolization. For tractable exposition, we continue to employ the functional forms from Section 4.3.

6.1 Production and Investment Subsidies

Policymakers may consider subsidy interventions to address underproduction due to oligopolistic market power and underinvestment arising from free-riding in public-good provision. This subsection analyzes how such subsidy policies affect the conditions under which efficiency paradoxes emerge.

We modify the baseline framework to incorporate production subsidies $s_x \in [0, c_i)$ and investment subsidies $s_g \in [0, 1)$, such that production costs become $C_i(x_i) = (c_i - s_x)x_i$ and investment costs become $K_i(g_i) = (1-s_g)k_i g_i^2$. We examine how the introduction of these subsidies shifts the boundary condition for the Dual-Dimension Efficiency Paradox, defined by $\Delta\pi \equiv \pi_I^* - \pi_E^* = 0$.

The DDEP boundary is characterized by the profit differential

$$\Delta\pi \equiv \pi_I^* - \pi_E^* = (1 - s_g)\big(k_E g_E^2 - k_I g_I^2\big) - (c_I - c_E)X$$

equaling zero. From the investment-stage first-order conditions, each firm's investment cost can be expressed as

$$K_i^* = (1 - s_g)k_i(g_i^*)^2 = \frac{(x_i^*)^2}{9(1 - s_g)k_i}.$$

We first evaluate the impact of production subsidies s_x on $\Delta\pi$ at $\{s_x, s_g\} = \{0, 0\}$:

$$\left.\frac{d\Delta\pi}{ds_x}\right|_{(0,0)} = \frac{d}{ds_x}\big(K_E^* - K_I^*\big) - (c_I - c_E)\frac{dX^*}{ds_x}.$$

The first term yields

$$\frac{d}{ds_x}\big(K_E^* - K_I^*\big) = \frac{1}{9}\left(\frac{x_E^*}{k_E} - \frac{x_I^*}{k_I}\right)\frac{dX^*}{ds_x},$$

while the boundary condition $\Delta\pi = 0$ implies the identity

$$(c_I - c_E)X^* = \frac{1}{9}\left(\frac{(x_E^*)^2}{k_E} - \frac{(x_I^*)^2}{k_I}\right).$$

Substituting these relationships and noting that $dX^*/ds_x > 0$, we obtain

$$\left.\frac{d\Delta\pi}{ds_x}\right|_{(0,0)} = \frac{1}{9}\,\frac{x_E^* x_I^*}{X^*}\left(\frac{1}{k_E} - \frac{1}{k_I}\right)\frac{dX^*}{ds_x} > 0,$$

where

$$\frac{dX^*}{ds_x} = \frac{6k_I k_E}{9bk_I k_E - (k_I + k_E)} > 0.$$

Thus, marginal production subsidies expand $\Delta\pi$, making the DDEP more likely to occur. Production subsidies create two opposing effects. They amplify the production cost disadvantage of the inefficient firm (favoring the efficient firm), but they also increase investment levels, which disproportionately burdens the efficient firm (favoring the inefficient firm). Since the latter effect dominates, production subsidies ultimately make the DDEP more likely.

Next, we examine how investment subsidies s_g affect the DDEP threshold. The boundary movement in the vicinity of $(s_x, s_g) = (0, 0)$ is given by

$$\frac{d\Delta\pi}{ds_g} = \frac{d}{ds_g}\big(K_E^* - K_I^*\big) - (c_I - c_E)\frac{dX^*}{ds_g}.$$

From $(1 - s_g)k_i g_i^* = \frac{1}{3}x_i^*$ we have $K_i^* = (x_i^*)^2/\big[9(1 - s_g)k_i\big]$, hence

$$\frac{d}{ds_g}\big(K_E^* - K_I^*\big)\Big|_{(0,0)} = \frac{1}{9}\left(\frac{x_E^{*2}}{k_E} - \frac{x_I^{*2}}{k_I}\right) + \frac{1}{9}\left(\frac{x_E^*}{k_E} - \frac{x_I^*}{k_I}\right)\frac{dX^*}{ds_g}.$$

Using the boundary identity $(c_I - c_E)X^* = \frac{1}{9}\big(\frac{x_E^{*2}}{k_E} - \frac{x_I^{*2}}{k_I}\big)$ gives

$$\frac{d\Delta\pi}{ds_g}\Big|_{(0,0)} = \frac{1}{9}\left(\frac{x_E^{*2}}{k_E} - \frac{x_I^{*2}}{k_I}\right) + \frac{1}{9}\frac{x_E^* x_I^*}{X^*}\left(\frac{1}{k_E} - \frac{1}{k_I}\right)\frac{dX^*}{ds_g}.$$

Finally,

$$\frac{dX^*}{ds_g} = \frac{2(k_I x_E^* + k_E x_I^*)}{9bk_I k_E - (k_I + k_E)} \;>\; 0 \quad \text{so} \quad \frac{d\Delta\pi}{ds_g}\Big|_{(0,0)} > 0,$$

indicating that marginal investment subsidies also promote the DDEP through both direct cost reduction effects and scale effects. These results establish the following proposition:

Proposition 1 (Policy Interventions)**.** *Under Assumption 4 with uniform production subsidies s_x or investment subsidies s_g, efficiency reversals are amplified rather than mitigated.*

Subsidies intended to remedy underproduction and underinvestment amplify asymmetric burden-sharing, compressing the profit advantages that efficient firms would normally enjoy over their inefficient rivals and thereby making the DDEP more likely to emerge. This compression effect has important policy implications. For policymakers with distributional equity motives, such subsidies can serve as effective tools: they simultaneously correct market distortions from imperfect competition and free-riding while reducing profit disparities across firms—provided the compression remains moderate enough that efficient firms still outperform inefficient ones. However, when subsidies push the market into the DDEP region where profit rankings reverse, they undermine efficiency-based competitive advantages and may generate undesirable long-run consequences by allowing competition to select the wrong winners, as we demonstrate in the next subsection.

6.2 Inefficient Market Selection under Efficiency Paradoxes

Our baseline analysis considers a fixed market structure with two active firms. This subsection extends the framework by endogenizing market structure through the introduction of entry and exit decisions. We examine how efficiency paradoxes affect entry and exit decisions, demonstrating that these distortions extend beyond profit rankings to influence market structure itself. The analysis applies under general cost and demand specifications, without requiring Assumption 4.

Consider an extended three-stage game where firms first decide whether to enter the market, then engage in the baseline two-stage coopetition game. Entry requires a common sunk cost $F > 0$. Let

π_i^D denote firm i's equilibrium profit in the baseline duopoly game (net of variable and investment costs, before entry costs) and π_i^M represent its monopoly profit when operating alone.

Entry Stage Analysis. Under certain parameter configurations, the profit reversals that characterize our efficiency paradoxes can lead to perverse entry outcomes. Specifically, when

$$\pi_I^D \geq F > \pi_E^D \quad \text{and} \quad \pi_I^M \geq \pi_I^D,$$

the unique equilibrium in both simultaneous and sequential entry games features Firm I entering while Firm E remains inactive—yielding an inefficient monopoly. The economic logic operates through strategic anticipation: if Firm E were to enter, Firm I would find entry profitable ($\pi_I^D - F \geq 0$), leaving Firm E with negative net profits ($\pi_E^D - F < 0$). Recognizing this outcome, Firm E optimally forgoes entry, while Firm I monopolizes the market. This reasoning applies identically under both simultaneous and sequential timing, establishing $\{E, I\} = \{\text{out}, \text{in}\}$ as the unique Nash equilibrium.

Exit Stage Analysis. Efficiency paradoxes also manifest in exit decisions under adverse market conditions. Consider firms facing a common per-period fixed operating cost $F > 0$ and a negative demand shock $\Delta A < 0$. In both simultaneous and sequential exit games, when the shock reduces firm E's post-shock profit below the fixed cost threshold while Firm I remains viable, the unique equilibrium features Firm E exiting while Firm I continues as a monopolist. This mirrors the entry stage logic: Firm E's higher investment burden makes it cross the viability threshold first, whether entering a profitable market or exiting a declining one.

Proposition 2 (Endogenous Market Structure)**.** *Under Assumption 4 with endogenous entry/exit involving fixed costs $F > 0$, efficiency paradoxes distort entry and exit decisions, potentially leading to inefficient monopolization where less capable firms drive out more efficient competitors.*

Proof. See Appendix A.4. □

Overall, when fixed costs are present and market structure is endogenous, efficiency paradoxes distort selection at both entry and exit margins. Inefficient firms enter markets where efficient rivals cannot operate profitably, while efficient firms exit first during adverse shocks. Both distortions result in inefficient monopolization—the worst outcome for consumer welfare and allocative efficiency.

7 Welfare Analysis

This section examines how production and investment efficiency differentials affect social welfare. For tractable exposition, we employ the functional forms from Section 4.3 (linear demand, linear production costs, and quadratic investment costs).

Classical results by Bergstrom and Varian (1985) demonstrate that under constant marginal production costs, Cournot equilibrium outcomes—including price, total output, and consumer surplus—depend only on the sum of marginal costs, not their distribution across firms. Moreover, production substitution from inefficient to efficient firms enhances welfare through increased producer surplus as cost differentials widen. Our model extends this framework by incorporating investment efficiency differentials alongside production efficiency gaps, allowing us to analyze how both types of efficiency heterogeneity (holding means constant) affect consumer surplus and overall welfare.

Equilibrium consumer surplus (CS^*), producer surplus (PS^*), and welfare (SW^*) are characterized by:

$$\begin{aligned} CS^* &= \int_0^{X^*} P(v, G^*)dv - P^*X^*, \\ PS^* &= \pi_E^* + \pi_I^* \\ SW^* &= CS^* + \pi_E^* + \pi_I^*. \end{aligned}$$

Applying the efficiency differential parameterization from equation (5) to production and investment costs, we first examine the isolated effects of each type of efficiency asymmetry:

$$\left.\frac{\partial CS^*}{\partial \delta_c}\right|_{\delta_k=0} = 0, \tag{8}$$

$$\left.\frac{\partial PS^*}{\partial \delta_c}\right|_{\delta_k=0} = \left.\frac{\partial SW^*}{\partial \delta_c}\right|_{\delta_k=0} = \frac{4(9bk-1)}{9b^2k}\cdot\delta_c > 0, \tag{9}$$

$$\left.\frac{\partial CS^*}{\partial \delta_k}\right|_{\delta_c=0} = \frac{72b(a-c)^2(k_E+k_I)k_Ek_I}{\Lambda^3}\cdot\delta_k > 0, \tag{10}$$

$$\left.\frac{\partial PS^*}{\partial \delta_k}\right|_{\delta_c=0} = \frac{2(a-c)^2(k_E+k_I)\left[18bk_Ek_I+\Lambda\right]}{\Lambda^3}\cdot\delta_k > 0, \tag{11}$$

where $\Lambda \equiv 9bk_Ek_I - (k_E + k_I) > 0$.

These results confirm two distinct welfare channels. Production efficiency differentials alone leave consumer surplus unchanged while enhancing welfare through increased producer surplus via production substitution effects, consistent with Bergstrom and Varian (1985). Investment efficiency differentials enhance both consumer and producer surplus through demand-expansion effects generated by increased total investment.

However, the isolated effects analysis reveals only part of the story. To understand how each type of efficiency asymmetry affects consumer surplus in the presence of the other, we have:

$$\frac{\partial CS^*}{\partial \delta_c} = \frac{8\delta_k\left[2\delta_c\delta_k+3bk_Ek_I(a-c)\right]}{b\Lambda^2} > 0, \tag{12}$$

$$\frac{\partial CS^*}{\partial \delta_k} = \frac{8(3b(a-c)k_Ek_I+2\delta_c\delta_k)\left[\Lambda+6b\delta_k((a-c)k+3\delta_c\delta_k)\right]}{b\Lambda^3} > 0. \tag{13}$$

While production efficiency differentials alone have no impact on consumer surplus (8), they generate positive consumer surplus effects when investment efficiency asymmetries are present. Conversely, the beneficial effect of investment efficiency differentials on consumer surplus (10) is amplified when production efficiency gaps exist. This suggests that the two types of efficiency asymmetries work complementarily rather than additively.

This complementarity is formally confirmed by the interaction effect captured in the cross-partial

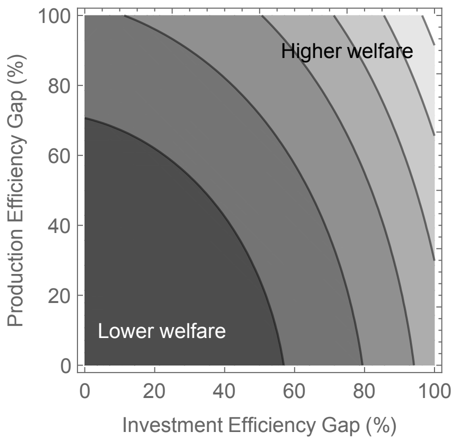

Figure 3: Efficiency Heterogeneity and Social Welfare

Note: The figure shows welfare contours in the space of efficiency disadvantages, where darker regions represent lower social welfare levels. The horizontal axis measures the investment efficiency disadvantage (percentage by which k_I exceeds k_E), while the vertical axis measures the production efficiency disadvantage (percentage by which c_I exceeds c_E). Welfare increases toward the northeast direction, indicating that larger efficiency gaps in both dimensions enhance social welfare. The contour lines represent iso-welfare curves, with each line corresponding to a specific welfare level. Parameters: $a = 50$, $b = 1.0$, $c = 6$, $k = 2$.

derivative of social welfare:

$$\left.\frac{\partial^2 SW^*}{\partial \delta_c \partial \delta_k}\right|_{\delta_k=0} = \frac{36(a-c)k^2}{\Lambda^2} > 0. \tag{14}$$

Proposition 3 (Welfare Implications)**.** *Under Assumption 4, both production and investment efficiency differentials enhance social welfare, with complementary effects such that their interaction yields greater benefits than the sum of individual effects.*

Efficiency differentials in production and investment dimensions exhibit complementary welfare effects, with their interaction yielding greater benefits than the sum of individual effects. While equation (14) evaluates the cross-partial at $\delta_k = 0$, the complementarity result holds more generally when market size $(a - c)$ is sufficiently large relative to cost differentials.

Figure 3 visualizes these complementary effects through welfare contours in the δ_k-δ_c parameter space.[16] The northeast-sloping iso-welfare curves confirm that larger efficiency gaps in both dimensions enhance social welfare, with the steepening contours indicating increasing complementarity between the two types of asymmetries.

The complementarity arises through distinct but interconnected welfare channels. Production efficiency differentials enhance welfare by enabling output reallocation toward more efficient firms, increasing the proportion of total output produced at lower costs. Investment efficiency differentials enhance welfare by increasing aggregate investment levels, which expand market demand and benefit all consumers and firms. Crucially, these two channels interact complementarily: market expansion amplifies the welfare gains from production substitution by creating a larger market where efficient

[16] To enhance the visibility of welfare effects from efficiency differentials, we employ the parameter set $a = 50, b = 1, c = 6, k = 2$. The larger baseline marginal production cost facilitates clearer visualization of how production cost differentials affect welfare.

production advantages become more valuable. Conversely, the presence of more efficient producers magnifies the benefits of demand expansion by ensuring that increased market activity is served more efficiently.

These complementary welfare effects represent a key departure from standard Cournot competition. Beyond the allocative efficiency gains identified by Bergstrom and Varian (1985) and Lahiri and Ono (1988), coopetitive markets generate an additional demand-expansion channel through investment efficiency differentials. Crucially, these two channels interact complementarily—a property absent in standard competition—implying that efficiency heterogeneity in both dimensions maximizes short-run social welfare.

This analysis illuminates a critical temporal dimension in the welfare implications of efficiency heterogeneity. While our static welfare analysis demonstrates unambiguous short-run benefits from efficiency differentials—with production heterogeneity enabling beneficial output reallocation and investment heterogeneity expanding market demand—the dynamic consequences may be quite different. As demonstrated in Section 6.2, efficiency paradoxes distort entry and exit decisions, potentially leading to inefficient monopolization where less capable firms drive out superior competitors. This creates a fundamental policy dilemma: the efficiency differences that maximize short-run welfare may simultaneously undermine the market's long-run ability to select and reward superior performance, potentially hampering technological progress and industry development over time.

8 Concluding Remarks

Excellence can become its own burden when competition meets cooperation. This paper investigates how efficiency asymmetries between firms affect profitability in demand-side coopetition, where rivals compete for market share while investing in industry-wide public-goods that expand overall market demand. Through a two-stage duopoly model with heterogeneous production and investment efficiencies, we demonstrate two counterintuitive results that emerge endogenously from strategic interactions.

We uncover the Single-Dimension Efficiency Paradox, showing that when production efficiencies are identical, the firm with less efficient investment technology systematically earns greater profits than its more efficient rival. We further establish the Dual-Dimension Efficiency Paradox, revealing that even firms disadvantaged in both production and investment can outperform ostensibly superior competitors under certain conditions. These paradoxes arise because market-expanding investments provide equal benefits to all firms while imposing disproportionate burdens on the more efficient investor, echoing Olson's (1965) classic insight regarding 'the exploitation of the great by the small' in collective action settings.

Our robustness analysis demonstrates that these efficiency paradoxes persist across a wide range of alternative specifications. Markets with differentiated products reveal that paradoxes continue under moderate differentiation, though their likelihood diminishes as differentiation increases. The simultaneous coopetition analysis confirms that paradoxes are more pronounced when investments are reversible rather than serving as long-term commitments. Additionally, we show that investment cooperation and aggregation technologies approaching best-shot characteristics make paradoxes more likely, while weakest-link technologies eliminate them entirely.

Building on these findings, our analysis reveals the specific conditions under which efficiency paradoxes are most likely to emerge. We demonstrate that higher demand elasticity amplifies these paradoxical outcomes, suggesting that efficiency paradoxes are more prevalent in luxury goods markets compared to necessity goods, and in short-run versus long-run market conditions. Our theoretical framework helps explain why less efficient firms can survive and even thrive in certain industries, particularly in sectors where firms make industry-wide public-good investments that raise category demand. This perspective helps interpret cases such as tourism, where premium hotels invest in destination marketing that lifts demand for all accommodations; agriculture, where generic advertising by major producers expands entire product categories; and regional food branding, where established producers invest in collective reputation that benefits smaller peers.

These paradoxical outcomes generate important policy and market structure implications that extend far beyond profit rankings. Counterintuitively, subsidies designed to remedy market failures can exacerbate efficiency reversals by amplifying investment asymmetries. Moreover, efficiency paradoxes distort market structure itself, potentially leading to inefficient monopolization through perverse entry and exit dynamics where less capable firms enter markets that efficient rivals cannot operate profitably, and efficient firms exit first during adverse shocks while inferior competitors remain active. These market structure distortions reveal fundamental problems with how markets select firms in coopetitive environments.

Shopping malls provide a particularly compelling illustration of these exit dynamics. Developers coordinate joint promotions and facility investments that generate non-excludable demand spillovers—a quintessential coopetitive setting. Industry observers document that anchor tenants, typically larger and more efficient retailers, increasingly exit malls earlier than smaller inline tenants.[17] Anchor departures are associated with approximately 25% rent reductions for inline tenants (Gatzlaff et al., 1994), while recent evidence confirms substantial anchor-related spillovers (Liu et al., 2024). Our model provides one plausible mechanism for this pattern: when demand-shifting investments are financed through coopetitive burden-sharing, costs fall disproportionately on capable anchor-like participants, potentially inducing earlier exit by efficient firms. While multiple factors—including e-commerce competition and changing consumer preferences—likely contribute to these patterns, our framework identifies an endogenous strategic channel that may complement these other forces.

Beyond exit patterns, geographical indications (GIs) provide complementary evidence from the entry and survival margin. By pooling quality certification and promotion to build category-level reputation, GI systems create demand spillovers that benefit all certified producers. Evidence from French cheese shows that PDO labeling significantly reduces exit risk for smaller firms relative to larger producers (Bontemps et al., 2013), consistent with our prediction that collective demand-expanding investments can systematically advantage less capable firms when more efficient producers bear disproportionate investment burdens (Menapace and Moschini, 2024).

Our welfare analysis further provides crucial insights into the social implications of efficiency heterogeneity in coopetitive markets. We demonstrate that efficiency differentials in production and

[17] Retail Dive, "Anchors, away: How department stores are ditching malls," Nov. 12, 2020, https://www.retaildive.com/news/anchors-away-how-department-stores-are-ditching-malls/588769/; Wall Street Journal, "Most Department Stores Are Leaving Malls. Dillard's Is Buying One," Aug. 25, 2025, https://www.wsj.com/real-estate/commercial/dillards-mall-longview-texas-cd380a0b.

investment dimensions exhibit complementary welfare effects, with production heterogeneity enabling beneficial output reallocation toward more efficient firms while investment heterogeneity expands market demand through increased aggregate investment. This complementarity means that larger efficiency gaps in both dimensions enhance social welfare, with the interaction between the two types of asymmetries yielding greater benefits than the sum of individual effects.

However, integrating these welfare findings with the market structure distortions identified above reveals a fundamental temporal trade-off that lies at the heart of coopetitive markets. While efficiency heterogeneity unambiguously enhances short-run social welfare through complementary effects on consumer and producer surplus, the dynamic consequences through entry and exit decisions suggest that the same heterogeneity that maximizes static welfare may undermine market efficiency over time. This creates a critical policy dilemma: the efficiency differences that benefit society in the short run also create conditions where less efficient firms can exploit more efficient rivals, potentially weakening market-based selection mechanisms and hampering long-term technological progress and industry development.

The empirical implications of our findings await systematic investigation. Future research could examine whether dominant firms with superior efficiency and larger market shares indeed bear disproportionate burdens in coopetitive investments, and whether weaker firms achieve profit reversals by riding on the "shoulders of giants." Additionally, understanding how cost-sharing rules for collective investments are determined in practice represents an important avenue for research, revealing how allocation mechanisms affect competitive outcomes and firms' participation in coopetitive activities.

Beyond empirical investigation, our theoretical framework opens important avenues for policy design research. While our analysis demonstrates that uniform subsidies can paradoxically exacerbate efficiency reversals, the question remains whether carefully designed asymmetric subsidy schemes could mitigate these distortions while preserving welfare benefits. Future research could examine optimal non-uniform subsidy policies that account for firms' heterogeneous efficiencies—for instance, providing higher investment subsidies to efficient firms to offset their disproportionate burden-sharing, or implementing production subsidies that are inversely related to investment efficiency to rebalance competitive incentives.

More broadly, understanding how policy interventions might address the short-run versus long-run welfare trade-offs—whether through investment coordination mechanisms or regulatory frameworks—represents a compelling direction for both theoretical analysis and empirical investigation. The challenge lies in designing interventions that capture the welfare benefits of efficiency heterogeneity while mitigating the competitive distortions that favor less efficient firms. Conversely, investment efficiency differentials may serve as a natural bulwark against excessive market concentration by ensuring that multiple firms remain viable even when efficiency gaps are substantial, thereby preserving competitive pressure and consumer choice.

At the firm level, our analysis provides actionable insights for strategic management practice by demonstrating when efficiency becomes a strategic liability. For efficient firms, we reveal paradoxical strategies such as transferring superior technologies to rivals to reduce disproportionate investment burdens while maintaining market expansion benefits. For less efficient firms, our findings reveal systematic opportunities to enhance competitive position by strategically leveraging industry-wide initiatives without bearing proportional costs.

Appendix

A.1 Notation

Table A1: Notation

Symbol	Name	Definition / Notes
$i \in \{E, I\}$	Firm index	E: relatively more efficient (lower c and/or k); I: relatively less efficient.
x_i	Output of firm i	Decision variable in the production stage.
g_i	Investment of firm i	Non-excludable, demand-raising investment.
$X = \sum_i x_i$	Industry output	Aggregate quantity.
G	Public-good level	Demand shifter generated by $\{g_i\}$. Technology extremes: $G = \max\{g_E, g_I\}$ (best-shot), $G = \min\{g_E, g_I\}$ (weakest-link).
$P(X, G)$	Inverse demand	$P_X < 0$, $P_G > 0$.
$C_i(x_i)$	Production cost	$C_i(x_i) = c_i x_i^{\rho_x}$, with $c_I \geq c_E > 0$, $\rho_x \geq 1$.
$K_i(g_i)$	Investment cost	$K_i(g_i) = k_i g_i^{\rho_g}$, with $k_I \geq k_E > 0$, $\rho_g \geq 1$.
c_i	Cost parameter (prod.)	Shifter in $C_i(\cdot)$.
k_i	Cost parameter (inv.)	Efficiency shifter in $K_i(\cdot)$.
ρ_x	Cost homogeneity (prod.)	Degree of homogeneity of $C_i(\cdot)$.
ρ_g	Cost homogeneity (inv.)	Degree of homogeneity of $K_i(\cdot)$.
ϵ_D	Demand elasticity	Price elasticity of demand at equilibrium: $\epsilon_D \equiv -\frac{\partial X}{\partial P}\frac{P}{X}$.
π_i	Profit of firm i	$\pi_i = P(X, G)\, x_i - C_i(x_i) - K_i(g_i)$.
$\Delta\pi$	Profit difference	$\Delta\pi \equiv \pi_I^* - \pi_E^*$.
δ_c	Symmetric cost gap (prod.)	$c_E = c - \delta_c$, $c_I = c + \delta_c$ with $\delta_c \geq 0$. Equivalently, $c_I - c_E = 2\delta_c$.
δ_k	Symmetric cost gap (inv.)	$k_E = k - \delta_k$, $k_I = k + \delta_k$ with $\delta_k \geq 0$. Equivalently, $k_I - k_E = 2\delta_k$.
s_x	Production subsidy	Per-unit subsidy; local comps at $(s_x, s_g) = (0, 0)$.
s_g	Investment subsidy	Ad-valorem subsidy on $K_i(\cdot)$; local comps at $(s_x, s_g) = (0, 0)$.
$*$	Equilibrium value	SPNE equilibrium object under the specified timing.

A.2 Proof of Theorem 1

Since marginal production costs are identical under $c_I = c_E$, the second-stage first-order condition implies $x_I^* = x_E^*$ in equilibrium, so both firms earn identical revenues.

With ρ_g-th order homogeneous investment costs $K_i(g) = k_i g^{\rho_g}$ where $\rho_g > 1$ and $k_I > k_E > 0$, the first-order conditions give:

$$\rho_g k_I (g_I^*)^{\rho_g - 1} = \rho_g k_E (g_E^*)^{\rho_g - 1} = \text{MB}(G^*).$$

This implies:

$$\frac{g_E^*}{g_I^*} = \left(\frac{k_I}{k_E}\right)^{\frac{1}{\rho_g - 1}} > 1,$$

since $k_I > k_E$ and $\rho_g - 1 > 0$.

The cost difference is:

$$\begin{aligned} K_E(g_E^*) - K_I(g_I^*) &= k_E(g_E^*)^{\rho_g} - k_I(g_I^*)^{\rho_g} \\ &= k_E\left(\frac{k_I}{k_E}\right)^{\frac{\rho_g}{\rho_g-1}}(g_I^*)^{\rho_g} - k_I(g_I^*)^{\rho_g} \\ &= (g_I^*)^{\rho_g}\left[k_E\left(\frac{k_I}{k_E}\right)^{\frac{\rho_g}{\rho_g-1}} - k_I\right] \\ &= (g_I^*)^{\rho_g}k_I\left[\left(\frac{k_I}{k_E}\right)^{\frac{1}{\rho_g-1}} - 1\right] > 0. \end{aligned}$$

The last inequality follows since $k_I > k_E$, $\rho_g - 1 > 0$, and $g_I^* > 0$. Therefore $\pi_I^* > \pi_E^*$. □

A.3 Proof of Theorem 2

The profit difference is:

$$\Delta_\pi \equiv \pi_I^* - \pi_E^* = P^* \cdot (x_I^* - x_E^*) - (C_I - C_E) + K_E - K_I. \tag{A1}$$

Because C_i is an ρ_x-th degree homogeneous function, we have $C_i = \frac{1}{\rho_x}C_i' \cdot x_i$. Substituting this into equation (A1) yields:

$$\Delta_\pi = P^* \cdot (x_I^* - x_E^*) - \frac{1}{\rho_x}(C_I'x_I^* - C_E'x_E^*) + K_E - K_I. \tag{A2}$$

From the second-stage first-order conditions, we have:

$$\begin{aligned} P^* &= C_I' - P_X x_I^*, \\ C_I' - C_E' &= P_X \cdot (x_I^* - x_E^*). \end{aligned} \tag{A3}$$

Using these two conditions, the $P \cdot (x_I^* - x_E^*)$ term in equation (A2) becomes:

$$\begin{aligned} P^* \cdot (x_I^* - x_E^*) &= (C_I' - P_X x_I^*)(x_I^* - x_E^*) = C_I' \cdot (x_I^* - x_E^*) - P_X \cdot (x_I^* - x_E^*)x_I^* \\ &= C_I' \cdot (x_I^* - x_E^*) - (C_I' - C_E')x_I^* = -(C_I'x_E^* - C_E'x_I^*). \end{aligned} \tag{A4}$$

Substituting this into equation (A2) yields:

$$\begin{aligned} \Delta_\pi &= -(C_I'x_E^* - C_E'x_I^*) - \frac{1}{\rho_x}(C_I'x_I^* - C_E'x_E^*) + K_E - K_I \\ &= -\frac{1}{\rho_x}[C_I'x_E^* + (\rho_x - 1)C_I'x_E^* - C_E'x_I^* - (\rho_x - 1)C_E'x_I^* + C_I'x_I^* - C_E'x_E^*] + K_E - K_I \\ &= -\frac{1}{\rho_x}[(C_I' - C_E')X^* + (\rho_x - 1)(C_I'x_E^* - C_E'x_I^*)] + K_E - K_I. \end{aligned}$$

From equation (A4), we have $(C'_I x_E^* - C'_E x_I^*) = -P^* \cdot (x_I^* - x_E^*)$. Substituting this:

$$\Delta_\pi = -\frac{1}{\rho_x}[(C'_I - C'_E)X^* - (\rho_x - 1)P^* \cdot (x_I^* - x_E^*)] + K_E - K_I.$$

Furthermore, from condition (A3), we have $(x_I^* - x_E^*) = (C'_I - C'_E)/P_X$. Substituting this:

$$\begin{aligned}\Delta_\pi &= -\frac{1}{\rho_x}\left[(C'_I - C'_E)X^* - (\rho_x - 1)P^* \cdot \left(\frac{C'_I - C'_E}{P_X}\right)\right] + K_E - K_I \\ &= -\frac{1}{\rho_x}(C'_I - C'_E)X^*\left[1 + (\rho_x - 1)\left(\frac{P}{-P_X \cdot X}\right)\right] + K_E - K_I \\ &= -\frac{1}{\rho_x}(C'_I - C'_E)X^*[1 + (\rho_x - 1)\epsilon_D] + K_E - K_I,\end{aligned}$$

where $\epsilon_D = -\frac{P}{X \cdot P_X}$ is the price elasticity of demand.

Therefore, $\pi_I^* > \pi_E^*$ if and only if:

$$\frac{1}{\rho_x}(C'_I - C'_E)X^*[1 + (\rho_x - 1)\epsilon_D] < K_E - K_I.$$

To establish the existence of parameter combinations satisfying this condition, note that under Assumptions 1 and 2, the equilibrium is unique and continuously differentiable in the parameters (c_I, c_E, k_I, k_E) by the Implicit Function Theorem.

When $c_I = c_E$ and $k_I > k_E$, the left-hand side equals zero while $K_E(g_E^*) - K_I(g_I^*) > 0$, establishing the SDEP. By continuity, there exists some $\bar{\delta} > 0$ such that for all $c_I - c_E \in [0, \bar{\delta}]$, the inequality holds, proving the existence of the DDEP. □

A.4 Proof of Proposition 2

A.4.1 Entry Game

Consider an entry stage where firms simultaneously or sequentially decide whether to enter (requiring sunk cost $F > 0$) before engaging in the baseline two-stage coopetition game. Let π_i^D denote firm i's duopoly profit (before entry costs) and π_i^M its monopoly profit. Assume:

$$\pi_I^D \geq F > \pi_E^D \quad \text{and} \quad \pi_I^M \geq \pi_I^D. \tag{A5}$$

Simultaneous Entry: The payoff matrix is:

	Firm I: IN	Firm I: OUT
Firm E: IN	$\pi_E^D - F, \pi_I^D - F$	$\pi_E^M, 0$
Firm E: OUT	$0, \pi_I^M$	$0, 0$

Since $\pi_E^D - F < 0$ and $\pi_I^D - F \geq 0$, only (OUT, IN) satisfies mutual best-response conditions: Firm E cannot profitably enter given Firm I's entry ($0 > \pi_E^D - F$), while Firm I strictly prefers monopoly entry to staying out ($\pi_I^M > 0$).

Sequential Entry: Consider first the case where Firm E moves first. By backward induction, Firm I chooses IN regardless of Firm E's choice (since $\pi_I^D \geq F$ and $\pi_I^M > 0$). Anticipating this, Firm E optimally chooses OUT (receiving 0) over IN (receiving $\pi_E^D - F < 0$).

If instead Firm I moves first, Firm E's optimal response is OUT when Firm I chooses IN (receiving $0 > \pi_E^D - F$), and IN when Firm I chooses OUT (receiving $\pi_E^M - F$). Anticipating these responses, Firm I optimally chooses IN to secure monopoly profits $\pi_I^M > 0$, inducing Firm E to stay OUT.

In both cases, the unique subgame-perfect equilibrium is (OUT, IN), yielding inefficient monopolization by Firm I.

A.4.2 Exit Game

Consider firms facing fixed operating cost $F > 0$ when a demand shock reduces profits such that:

$$\tilde{\pi}_I^D > F > \tilde{\pi}_E^D \quad \text{and} \quad \tilde{\pi}_I^M > F, \tag{A6}$$

where tildes denote post-shock values.

Simultaneous Exit: The payoff matrix is:

	Firm I: STAY	Firm I: EXIT
Firm E: STAY	$\tilde{\pi}_E^D - F, \tilde{\pi}_I^D - F$	$0, \tilde{\pi}_I^M - F$
Firm E: EXIT	$\tilde{\pi}_E^M - F, 0$	$0, 0$

Since $\tilde{\pi}_E^D - F < 0$ and $\tilde{\pi}_I^D - F > 0$, only (EXIT, STAY) satisfies mutual best responses: Firm E cannot profitably remain ($0 > \tilde{\pi}_E^D - F$), while Firm I strictly prefers monopoly operation to exiting ($\tilde{\pi}_I^M - F > 0$).

Sequential Exit: Consider first the case where Firm E decides first. By backward induction, Firm I chooses STAY regardless of Firm E's choice (since $\tilde{\pi}_I^D > F$ and $\tilde{\pi}_I^M > F$). Anticipating this, Firm E optimally chooses EXIT (receiving 0) over STAY (receiving $\tilde{\pi}_E^D - F < 0$).

If instead Firm I decides first, Firm E's optimal response is EXIT when Firm I chooses STAY (receiving $0 > \tilde{\pi}_E^D - F$), and STAY when Firm I chooses EXIT (receiving $\tilde{\pi}_E^M - F$). Anticipating these responses, Firm I optimally chooses STAY to secure monopoly profits $\tilde{\pi}_I^M - F > 0$, inducing Firm E to EXIT.

In both cases, the unique subgame-perfect equilibrium is (EXIT, STAY), confirming that the efficient firm exits first.

Therefore, efficiency paradoxes distort both entry and exit decisions, leading to inefficient monopolization where less capable firms drive out more efficient competitors. □

A.5 Cross-Partial Derivatives of Welfare by Efficiency Gaps

While equation (14) establishes the complementarity of production and investment efficiency differentials by evaluating the cross-partial derivative at $\delta_k = 0$, the complementarity result holds under more general conditions when market size is sufficiently large relative to cost differentials.

The general cross-partial derivative of social welfare with respect to both types of efficiency gaps is:

$$\frac{\partial^2 SW^*}{\partial\delta_k \partial\delta_c} = \frac{4P(\delta_k)}{b\Lambda^3}$$

where the numerator $P(\delta_k)$ takes the form:

$$P(\delta_k) = (a-c)Q(\delta_k) + \delta_c\delta_k S(\delta_k)$$

with

$$\begin{aligned} Q(\delta_k) &= 9bk^3(9bk-2) + 54bk\delta_k^2 - 81b^2\delta_k^4 \\ S(\delta_k) &= -2k(9bk-8)(9bk-2) + 18b(9bk+4)\delta_k^2 \end{aligned}$$

The sign of the cross-partial derivative depends on $P(\delta_k)$. While $S(\delta_k)$ can be negative, the dominance of the market size term $(a-c)$ and the properties of $Q(\delta_k)$ ensure positive complementarity under reasonable parameter configurations.

Let $D \equiv \delta_k^2 \in [0, k^2)$. Then the function $Q(\delta_k)$ becomes a quadratic in D:

$$Q(D) = -81b^2D^2 + 54bkD + 9bk^3(9bk-2)$$

Since the coefficient of D^2 is negative ($-81b^2 < 0$), $Q(D)$ is a concave quadratic function. Evaluating at the boundary points yields $Q(0) = 9bk^3(9bk-2) > 0$ and $Q(k^2) = 36bk^3 > 0$. Since $Q(D)$ is continuous and positive at both endpoints of a downward-opening parabola, we have $Q(D) > 0$ throughout the interval $D \in [0, k^2)$, which establishes $Q(\delta_k) > 0$ for all feasible values $\delta_k \in [0, k)$.

When market size $(a-c)$ is sufficiently large relative to the efficiency differential parameters δ_c and δ_k, the positive $Q(\delta_k)$ term dominates any negative contribution from $S(\delta_k)$, ensuring $P(\delta_k) > 0$ and thus $\frac{\partial^2 SW^*}{\partial\delta_k \partial\delta_c} > 0$. This confirms that the complementarity of production and investment efficiency differentials extends beyond the special case $\delta_k = 0$ to hold under general parameter configurations with realistic market sizes.

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