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Fiscal Reform, Government Debt and Female Labor Supply in Japan*

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Abstract

This study extends Hansen and Imrohoroglu (2016) by incorporating female labor supply and a home sector into a growth model to assess Japan's fiscal sustainability and quantify the role of female labor in stabilizing government debt. The model is calibrated to the Japanese economy, which features a sizable gender productivity gap in the market sector, with female labor efficiency below 50% of the male level. Absent policy intervention, model simulations project the debt-to-output ratio to exceed 250% by 2035. Stabilizing debt at 60% of output using the consumption tax alone requires raising the tax rate to 40.9% starting in 2035, followed by a reduction to 24.4% once the target is achieved in 2089. We also find that reforming Japan's current spousal tax treatment is critical. Removing the current spousal tax treatment together with the debt stabilization improves female labor supply and reduces the required consumption tax rate to 33.9% during 2035–2089 and to 18.2% at the target. Additional simulations likewise highlight the importance of gender equality for labor supply and long-run fiscal outcomes.

Keywords: Government Debt, Fiscal Sustainability, Female Labor Supply, Japan.

JEL Classification Numbers: H60, J16, J21, J22, O40.

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1 Introduction

Fiscal sustainability has been a critical policy challenge in Japan. The nation's net debt-to-output ratio has experienced a substantial rise from below 40% in 1995 to 160% in 2020 (Figure 1). This upward trajectory, combined with demographic pressures and rising fiscal expenditures, threatens to push debt levels into unsustainable territory in coming decades. On the other hand, Japan's labor market presents limited conventional options for addressing this challenge. While male prime-age employment rates already reach 90%, suggesting limited room for an increase, significant gender disparities persist in both working hours and working environment. Therefore, the government and researchers have given hope by enhancing the female labor supply, which may mitigate the shrinking workforce problem and serve as a potential solution to the nation's demographic and fiscal challenges.[1]

To assess Japan's fiscal sustainability and investigate the implications of female labor supply, we extend the neoclassical growth model in Hansen and Imrohoroglu (2016) by incorporating male/female time allocation decisions with both market and home production sectors. The model setting of female/male labor allocation is similar to Ngai and Petrongolo (2017), which investigates gender disparities in hours and wages in the US labor market. In the model, male and female household members endogenously allocate their labor between the market and home sectors. The market sector hires labor and rents capital from households, while home production only requires labor input from male/female household members.

We assume households have perfect foresight regarding population growth rates, government policies, and factor prices. Households value consumption of market goods, home services, and government bonds. Following Hansen and Imrohoroglu (2016), we include bonds in the utility function to reflect Japan's strong domestic demand for government bonds. Both the quantity and price of bonds are endogenously determined, while government consumption and transfer payments are exogenous. The government generates revenue by taxing labor income, capital income, and consumption. We assume complete markets. Similar to Hansen and Imrohoroglu (2016), we also assume that if the debt-to-output ratio reaches a threshold (250% in our main analysis), the government cannot continue the bond financing policy and must conduct fiscal reforms, such as tax increases, to bring the ratio down to a sustainable level in the long run (60% in our experiments).

We calibrate the model to the Japanese economy and compute a transition path with initial

[1] See, for example, the discussion in Kitao and Mikoshiba (2020).

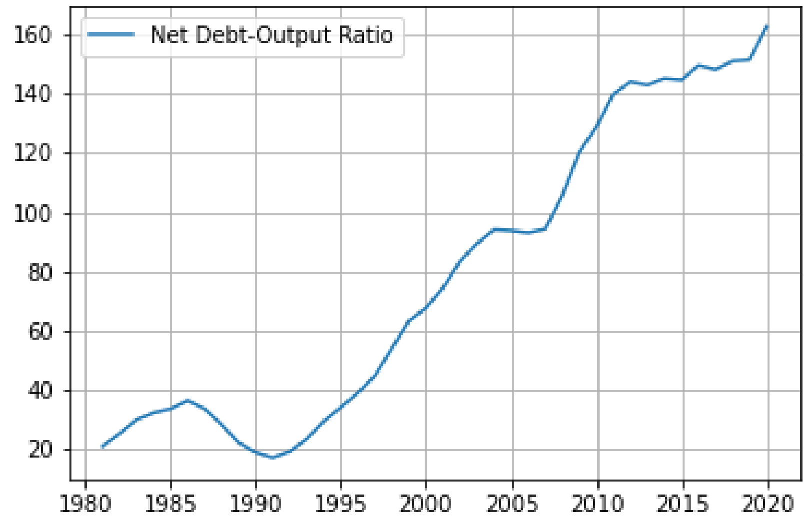

Figure 1: The Japanese net debt-to-output ratio (%)

Source: IMF: World Economic Outlook (2022)

conditions based on the economy in 1981 to a long-run steady state. We use government purchase and transfer payment forecasts from Fukawa and Sato (2009) and projections of future population growth rates produced by the Japanese government to calculate long-run transition dynamics in the model. We also calibrate gender-specific labor market efficiency parameters using Japanese employment and wage data. Our calibration shows that there are significant gender disparities in market labor productivity, with female labor efficiency substantially lower than male counterparts in market sector. In the home sector, however, female workers have relatively higher efficiency. While male and female labor hours exhibit substitutability in market production, they demonstrate complementary relationships in home production.

Building on our calibration results, we assess Japan's fiscal sustainability following a strategy similar to Hansen and Imrohoroglu (2016). In our baseline transition analysis, once the debt-to-output ratio reaches the threshold, we assume the government will solely increase consumption tax to reduce the debt level and to achieve a sustainable debt-to-output level (60% of output) in the long run. Our simulation results show that Japan's net debt-to-output ratio is projected to exceed the threshold (250%) around 2035 given the projected trend of government expenditures without any policy reforms. To achieve fiscal sustainability, the consumption tax rate would need to increase drastically to 40.9% (up from the current 10%) for about 50 years, eventually settling at 24.4% when the target debt level (60% of output) is reached.

Given the calibrated benchmark model, we conduct two key experiments to further examine the role of female labor supply in Japan's fiscal sustainability. Our first experiment examines the elimination of Japan's special tax treatment for low-income or non-working spouses that distorts spousal (typically, female) market labor supply. The current tax system in Japan still promotes the traditional type of family, in which the household head works full time outside and the spouse focuses on domestic family work. If the spouse's labor income is lower than a threshold, the spouse can enjoy all social welfare without paying income tax and the household head can have a tax deduction. This system leads to a distorted labor market outcome that a significant portion of female workers are part-time workers with earnings just below the threshold.

Intuitively, the current spousal tax treatment imposes an implicit opportunity cost to spouses (typically women) from increasing their market labor supply. If the spouse increases the labor supply and the earnings go above the threshold, the spousal income tax rate will jump from 0 to the level as for men, and the household also will lose the original tax deduction. In our growth model with a representative household, we approximate this spousal tax treatment by a high marginal income tax rate faced by the female household member, which discourages the female market labor supply, and a transfer to the household, which captures the tax subsidy as in the current tax system, in the benchmark model. Eliminating the tax treatment in our model is operated by a reduction in the marginal tax rate for the female and a removal of the transfer. This reform encourages female market labor supply and reduces government subsidies, generating a dual fiscal benefit.

Our experiment results demonstrate that removing this tax treatment would encourage female labor supply and yield substantial fiscal savings by broadening the tax base. This reform, if conducted in 2035 together with the debt stabilization, would reduce the required consumption tax to 33.9%, and lower the long-run tax rate to 18.2% when the policy target is achieved.

Our second experiment examines the potential gains from addressing labor market misallocation stemming from undervaluation of women's productivity. The wage and employment data in Japan lead to a large efficiency gap between female and male labor in the market sector where women's efficiency is below 50% of men's in the model. This efficiency gap may be caused by misallocation of labor or discrimination so that women's productivity cannot be well utilized in the market sector. Although we are not discovering the reasons for this gap, we explore a hypothetical scenario that the gender efficiency gap is reduced in the market sector by increasing women's relative productivity from below 50% to 70% of the male level. The results of the counterfactual experiment show a significant expansion in female labor supply and fiscal improvements – a reduction in the required consumption tax for debt stabilization from 2035 to 38.1% and a decrease in the long-run equilibrium tax rate to 22.7%.

Numerous studies have explored Japan's fiscal sustainability challenges, yet relatively few have adopted a dynamic general equilibrium framework with endogenously determined labor supply and government bond holdings. Notable exceptions include the work of Hansen and Imrohoroglu (2016, 2023), which is closely related to our study. Hansen and Imrohoroglu (2016) develop a general equilibrium model incorporating endogenous debt-holding decisions to analyze the additional government revenue or taxes required to reduce Japan's debt to a sustainable long-run level. Their findings indicate that, if the consumption tax is utilized, the rate would need to exceed 60% to effectively reduce the debt. Hansen and Imrohoroglu (2023) extend the model by incorporating the Bank of Japan's government bond purchases and confirm their previous findings, although the timing of required debt stabilization could be delayed. Our paper extends their framework by incorporating endogenous male and female labor supply with time allocation between market and home production sectors that allows us to examine the potential role of female labor supply in government debt stabilization.

Our modeling of the home sector follows the approach of Ngai and Petrongolo's (2017), in which male and female household members endogenously allocate their time between the market and home sectors. Consistent with their U.S.-based findings, our calibration for Japan similarly reveals that male and female labor are substitutes in market production but complementary in home production activities. While existing macroeconomic studies that incorporate home production have primarily focused on structural transformation and business cycle implications, our analysis makes a contribution by specifically investigating how female labor supply may affect fiscal sustainability– a dimension that receives greater attention in aging economies.[2]

[2]For example, Buera and Kobaski (2012) focus on the relatively rapid increase in market services rela-

There are, however, important existing studies regarding the implications of female labor supply on Japan's fiscal condition. For example, Imrohoroglu, Kitao, and Yamada (2016) investigate how an exogenous increase in female labor force participation can help alleviate fiscal imbalances. Their projections indicate that the net debt-to-output ratio would be 477.1% by 2060; however, with higher female labor force participation and an increase in regular employment, this ratio could decrease to 281.2%. Similarly, Kitao and Mikoshiba (2020), using an overlapping generations model with exogenous female/male labor supply, find that greater female labor supply can significantly ease the fiscal pressures associated with population aging in Japan.

Our analysis additionally incorporates endogenous labor supply decisions and the role of gender inequality in the Japanese labor market. The literature documents this phenomenon extensively; Blau and Kahn's (2017) comprehensive review synthesizes theoretical and empirical evidence across economies. Japan-specific research by Hara (2018) identifies a substantial residual gender wage gap unexplained by observable characteristics, suggesting significant discrimination. Our calibration, combining Japanese labor market data with model equilibrium conditions, quantifies these disparities in productivity terms: female workers exhibit less than 50% of male labor efficiency in the market sector. This finding aligns with existing evidence of systemic gender disparities. Our study also has an indication for promoting gender equality in labor markets.

The remainder of the paper is structured as follows. Section 2 presents the model with endogenous labor supply for both females and males, along with the corresponding equilibrium analysis. Section 3 provides details on the calibration of model parameters. Section 4 discusses the quantitative results and policy experiments. Section 5 concludes.

2 Model

Time is discrete and extends to infinity. We consider a neoclassical growth model with two sectors: the market sector and the home sector. The market sector produces consumable investment goods using physical capital and labor, while the home sector provides non-market household services using labor only. The economy is populated by a representative household

tive to home production, while Boerma and Karabarbounis (2021) examine the welfare consequences of the heterogeneity in home production.

consisting of both male and female workers, each of whom can allocate their time between the production of both market goods and household services. We abstract from population growth. All the markets are perfectly competitive. There is no uncertainty in the economy, and households are assumed to have perfect foresight.

2.1 Household

In each period, each household member owns one unit of time, allocating between home and market production: $l_t^g + h_t^g = 1, g = \{f, m\}$. Given initial capital and bond holdings K_0, B_0, in each period t, the representative household chooses its family consumption of market goods C_t, household service C_t^H, market labor supply of the female member l_t^f and of the male member l_t^m, next period government bond holdings B_{t+1}, and next period asset holdings K_{t+1} to maximize lifetime utility:

$$\max_{C_t,\, C_t^H,\, l_t^f,\, l_t^m,\, B_{t+1},\, K_{t+1}} U = \sum_{t=0}^{\infty} \beta^t [\log C_t + \alpha \log C_t^H + \phi log(\mu_t + B_{t+1})], \tag{1}$$

subject to

$$\begin{aligned} K_{t+1} - K_t = (1 - \tau_{k,t})(r_t - \delta)K_t + [1 - (1 - q_{t-1})\tau_{b,t}]B_t + \Lambda_t \\ + (1 - \tau_{l,t}^m)W_t^m l_t^m + (1 - \tau_{l,t}^f)W_t^f l_t^f - q_t B_{t+1} - (1 + \tau_{c,t})C_t, \end{aligned} \tag{2}$$

where q_t is the price of period-t discounted bond B_t, while l_t^f and l_t^m represent female and male's market labor supply in period-t respectively.[3] r_t, q_t, W_t^m, W_t^f are the rate of return to capital, bond price, wage rates of male and female labor in period t respectively. $0 < \beta < 1$ represents the discount factor, while $\alpha > 0$ and $\phi > 0$ represent household's utility weights on home services and bond holdings respectively. $\mu \geq 0$ implies that bond holdings cannot substitute for consumption. $\{\tau_{k,t}, \tau_{l,t}^f, \tau_{l,t}^m, \tau_{b,t}\}$ denote the period-t tax rates on capital income, labor income for females and males, and bond income respectively. $\tau_{c,t}$ denotes the consumption tax rate, the specification of $\tau_{c,t}$ will be introduced later. Λ_t denotes the lump-sum transfer such as public pensions. Each individual owns one unit of labor time

[3] We assume that households hold government bonds due to their safety and liquidity benefits, a perspective well-documented in the literature (e.g., Angeletos, Collard, and Dellas, 2023; Mian, Straub, and Sufi, 2022). Following Hansen and Imrohoroglu (2016), we model the baseline preference for government bonds with the parameter μ.

which can be allocated between market work and home activity. We specify the household service as a CES function of female and male housework time, h_t^f and h_t^m:

$$C_t^H = \left[\varepsilon_H^f (h_t^f)^{\frac{\sigma_H - 1}{\sigma_H}} + \varepsilon_H^m (h_t^m)^{\frac{\sigma_H - 1}{\sigma_H}}\right]^{\frac{\sigma_H}{\sigma_H - 1}}, \tag{3}$$

where parameter $\varepsilon_H^f, \varepsilon_H^m > 0$ represents the productivity in the home sector for female and male labor share respectively, while σ_H is the elasticity of substitution between the housework time for female and male.

The value function of the household's lifetime utility-maximization problem is given below.

$$V(K_t, B_t) = \max_{\{C_t, l_t^f, l_t^m, K_{t+1}, B_{t+1}\}} \left\{\log C_t + \alpha \log C_t^H + \phi \log(\mu_t + B_{t+1}) + \beta V(K_{t+1}, B_{t+1})\right\}$$

$$\begin{aligned}
K_{t+1} - K_t &= (1 - \tau_{k,t})(r_t - \delta)K_t + [1 - (1 - q_{t-1})\tau_{b,t}]B_t + \Lambda_t \\
&\quad + (1 - \tau_{l,t}^m)W_t^m l_t^m + (1 - \tau_{l,t}^f)W_t^f l_t^f \\
&\quad - q_t B_{t+1} - (1 + \tilde{\tau}_{c,t})C_t \\
C_t^H &= \left[\varepsilon_H^f (h_t^f)^{\frac{\sigma_H - 1}{\sigma_H}} + \varepsilon_H^m (h_t^m)^{\frac{\sigma_H - 1}{\sigma_H}}\right]^{\frac{\sigma_H}{\sigma_H - 1}} \\
h_t^f &= 1 - l_t^f; \quad h_t^m = 1 - l_t^m
\end{aligned}$$

For given prices, the first-order conditions for market goods consumption, capital investment, bond purchases, and housework time of female and male are as follows.

$$\frac{1}{\beta(1 + \tau_{c,t})C_t} = \frac{1 + (1 - \tau_{k,t+1})(r_{t+1} - \delta)}{(1 + \tilde{\tau}_{c,t+1})C_{t+1}}, \tag{4}$$

$$\alpha (C_t^H)^{\frac{1-\sigma_H}{\sigma_H}} \varepsilon_H^f (h_t^f)^{-\frac{1}{\sigma_H}} = \frac{(1 - \tau_{l,t}^f)W_t^f}{(1 + \tau_{c,t})C_t}, \tag{5}$$

$$\alpha (C_t^H)^{\frac{1-\sigma_H}{\sigma_H}} \varepsilon_H^m (h_t^m)^{-\frac{1}{\sigma_H}} = \frac{(1 - \tau_{l,t}^m)W_t^m}{(1 + \tau_{c,t})C_t}, \tag{6}$$

$$\frac{\phi}{\mu_t + B_{t+1}} + \frac{\beta[1 - (1 - q_t)\tau_{b,t+1}]}{C_{t+1}(1 + \tilde{\tau}_{c,t+1})} = \frac{q_t}{C_t(1 + \tau_{c,t})}. \tag{7}$$

2.2 Market Production

The representative firm at time t rents capital K_t and employs labor with total working hours L_t to produce investable consumption goods Y_t. The production function follows a constant-returns-to-scale technology as

$$Y_t = A_t K_t^{\theta} L_t^{1-\theta}, \tag{8}$$

where the total working hours follow a CES combination of the market time from the female and male labor force.

$$L_t = \left[\varepsilon^f (l_t^f)^{\frac{\sigma-1}{\sigma}} + \varepsilon^m (l_t^m)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{9}$$

The share of female and male market time is denoted by ε^f and ε^m, representing the working place situations and also the productivity. The market sector elasticity of substitution between female and male labor is σ which differs from that in the home sector. A_t is total factor productivity which grows exogenously subject to

$$A_{t+1} = \gamma_t A_t.$$

Given the rental rate of capital r_t and the wage rates W_t^f and W_t^m for female and male labors, respectively. The firm's profit maximization problem can be written as

$$\max_{K_t, l_t^f, l_t^m} \pi_t = Y_t - r_t K_t - W_t^m l_t^m - W_t^f l_t^f. \tag{10}$$

The optimal conditions of capital, male and female labor inputs are

$$A_t \theta K_t^{\theta-1} L_t^{1-\theta} = r_t, \tag{11}$$

$$A_t (1-\theta) K_t^{\theta} (L_t)^{\frac{1-\theta\sigma}{\sigma}} [\varepsilon^m (l_t^m)^{-\frac{1}{\sigma}}] = W_t^m, \tag{12}$$

$$A_t (1-\theta) K_t^{\theta} (L_t)^{\frac{1-\theta\sigma}{\sigma}} [\varepsilon^f (l_t^f)^{-\frac{1}{\sigma}}] = W_t^f. \tag{13}$$

2.3 Government

The government finances its period-t expenditures through taxation on consumption, capital income, male and female labor income, and bond income, using linear tax rates

$\{\tau_{c,t}, \tau_{k,t}, \tau_{l,t}^m, \tau_{l,t}^f, \tau_{b,t}\}$, respectively. In addition to tax revenues, the government may issue one-period, risk-free discount bonds to finance spending.

Given government consumption G_t and transfer payments Λ_t, the newly issued bonds B_{t+1} are required to satisfy the government ' s budget constraint in period t:

$$\begin{aligned} G_t + \Lambda_t + B_t = q_t B_{t+1} + \tau_{c,t} C_t + \tau_{l,t}^m W_t^m l_t^m + \tau_{l,t}^f W_t^f l_t^f \\ + \tau_{k,t}(r_t - \delta) K_t + \tau_{b,t}(1 - q_{t-1}) B_t. \end{aligned} \tag{14}$$

To ensure long-run fiscal sustainability, the government is assumed to follow a debt stabilization rule as introduced in Hansen and Imrohoroglu (2016, 2023). Specifically, if the debt-to-output ratio B_s/Y_s reaches a threshold b_{max} in period s, the government is required to reduce its debt level to a target ratio b_{target}. This adjustment is implemented through an increase in the consumption tax rate: the government raises the consumption tax rate from $\tau_{c,s}$ to a higher level τ_1 starting from period s and maintains it until the debt-to-output ratio falls below or equals b_{target}. After this target is achieved, the consumption tax rate is reduced to a lower level τ_2.

Thus, if $B_s/Y_s \geq b_{max}$ at some s, the sustainability rule is given by:

$$\tau_{c,t} = \begin{cases} \tau_1 > \tau_{c,s}, & \text{if } B_t/Y_t > b_{\text{target}},\ \forall t \geq s \\ \tau_2, & \text{if } B_t/Y_t \leq b_{\text{target}},\ \forall t \geq s \end{cases}, \tag{15}$$

where τ_1 determines the speed of reducing the debt level. In our benchmark, we choose the minimum tax rate required to guarantee convergence of the debt-to-output ratio to b_{target} as the value for τ_1. Once the target is reached, the tax rate is reduced to $\tau_2 \leq \tau_1$, which is also chosen as the minimum consumption tax rate that ensures the stabilized government debt level satisfies the government ' s budget constraint.

2.4 Dynamic Equilibrium

Given government policies $\{G_t, \Lambda_t, B_t, \tau_{l,t}^f, \tau_{l,t}^m, \tau_{c,t}, \tau_{b,t}, \tau_{k,t}\}_{t=0}^{\infty}$, a debt-sustainability rule $(b_{max}, b_{target}, \tau_1, \tau_2)$, the path of technology $\{A_t\}_{t=0}^{\infty}$, a dynamic competitive equilibrium consists of an allocation $\{C_t, C_t^h, l_t^f, l_t^m, h_t^f, h_t^m, Y_t, K_{t+1}, B_{t+1}\}_{t=0}^{\infty}$, factor prices $\{W_t^f, W_t^m, r_t\}_{t=0}^{\infty}$, and bond prices $\{q_t\}_{t=0}^{\infty}$ such that:

- The allocation solves the household ' s problem.
- The allocation solves the firm ' s profit maximization problem.
- The government budget constraint is satisfied.
- Given a value for b_{max}, the parameters (τ_1, τ_2) in the fiscal sustainability rule are sufficiently large to guarantee convergence of $\frac{B_t}{Y_t} \to b_{target}$.
- The bond market clears.
- The goods market clears:

$$C_t + K_{t+1} - (1-\delta)K_t + G_t = Y_t.$$

2.5 Detrended Equilibrium Conditions and Solution Procedure

To solve the model numerically, the detrended equilibrium conditions are derived in this subsection. For all trending variables, we detrend them by the factor $A_t^{1/(1-\theta)}$ and denote the detrended variables in lowercase letters. For example, the detrended effective per capita capital is given by $k_t = K_t/A_t^{1/(1-\theta)}$, and detrended consumption by $c_t = C_t/A_t^{1/(1-\theta)}$. The same applies to other variables.

$$\begin{aligned}(1+\tau_{c,t})c_t + \gamma_t^{1/(1-\theta)}k_{t+1} + q_t\gamma_t^{1/(1-\theta)}b_{t+1} = {} & [1+(1-\tau_{k,t})(r_t-\delta)]k_t \\ & + [1-(1-q_{t-1})\tau_{b,t}]b_t + \lambda_t \\ & + (1-\tau_{l,t}^m)w_t^m l_t^m + (1-\tau_{l,t}^f)w_t^f l_t^f,\end{aligned} \tag{1'}$$

$$\frac{(1+\tau_{c,t+1})\gamma_t^{1/(1-\theta)}c_{t+1}}{(1+\tau_{c,t})c_t} = \beta[1+(1-\tau_{k,t+1})(r_{t+1}-\delta)], \tag{2'}$$

$$C_t^H = \left[\varepsilon_H^f (h_t^f)^{\frac{\sigma_H - 1}{\sigma_H}} + \varepsilon_H^m (h_t^m)^{\frac{\sigma_H - 1}{\sigma_H}}\right]^{\frac{\sigma_H}{\sigma_H - 1}} \tag{3'}$$

$$1 - l_t^f = h_t^f = \left[\frac{\alpha\varepsilon_H^f(1+\tau_{c,t})c_t(C_t^H)^{\frac{1-\sigma_H}{\sigma_H}}}{(1-\tau_{l,t}^f)w_t^f}\right]^{\sigma_H}, \tag{4'}$$

$$1-l_t^m = h_t^m = \left[\frac{\alpha\varepsilon_H^m(1+\tau_{c,t})c_t(C_t^H)^{\frac{1-\sigma_H}{\sigma_H}}}{(1-\tau_{l,t}^m)w_t^m}\right]^{\sigma_H}, \tag{5'}$$

$$\frac{\phi}{\mu+b_{t+1}} + \frac{\beta[1-(1-q_t)\tau_{b,t+1}]}{(1+\tilde{\tau}_{c,t+1})c_{t+1}} = \frac{q_t\gamma_t^{1/(1-\theta)}}{(1+\tau_{c,t})c_t}. \tag{6'}$$

The profit maximization conditions of firms are given as

$$r_t = \theta k_t^{\theta-1}(L_t)^{1-\theta}, \tag{7'}$$

$$w_t^m = \varepsilon^m(1-\theta)k_t^\theta(L_t)^{\frac{1-\theta\sigma}{\sigma}}(l_t^m)^{-\frac{1}{\sigma}}, \tag{8'}$$

$$w_t^f = \varepsilon^f(1-\theta)k_t^\theta(L_t)^{\frac{1-\theta\sigma}{\sigma}}(l_t^f)^{-\frac{1}{\sigma}}. \tag{9'}$$

The detrended budget constraint of the government with the debt-sustainability rule is

$$\begin{aligned} g_t+\lambda_t+b_t =& q_t\gamma_t^{1/(1-\theta)}b_{t+1}+\tau_{c,t}c_t+\tau_{l,t}^m w_t^m l_t^m+\tau_{l,t}^f w_t^f l_t^f \\ &+\tau_{k,t}(r_t-\delta)k_t+\tau_{b,t}(1-q_{t-1})b_t, \end{aligned}. \tag{10'}$$

$$c_t+k_{t+1}-(1-\delta)k_t+g_t=y_t. \tag{11'}$$

Steady-State Equilibrium (Balanced Growth Path)

Along a balanced growth path (if it exists), all aggregate variables grow at constant rates. Consequently, the detrended economy reaches a steady state. Let $\bar{x}$ denote the steady-state value of variable x. From the household Euler equation, the steady-state rate of return on capital is given by:

$$\bar{r} = \frac{\gamma^{1/(1-\theta)}-\beta}{\beta(1-\tau_k)}+\delta. \tag{16}$$

Using Euler ’ s theorem under constant-returns-to-scale production, where $rk=\theta y$, we derive the steady-state capital-output ratio:

$$\left(\frac{\bar{k}}{\bar{y}}\right) = \frac{\theta(1-\tau_k)}{\gamma^{1/(1-\theta)}/\beta-1+\delta(1-\tau_k)}. \tag{17}$$

From goods market clearing, the steady-state consumption-output ratio satisfies:

$$\left(\frac{\bar{c}}{\bar{y}}\right) = 1 - \left(\gamma^{1/(1-\theta)} - 1 + \delta\right)\left(\frac{\bar{k}}{\bar{y}}\right) - \frac{g}{\bar{y}}. \tag{18}$$

Time allocation decisions for male and female household members satisfy:

$$\bar{l}^m = 1 - \bar{h}^m = 1 - \left[\frac{\alpha(1+\bar{\tau}_c)\varepsilon_H^m(\bar{c}/\bar{y})}{(1-\tau_l^m)\varepsilon^m(1-\theta)}\left(\frac{C^H(\bar{l}^m,\bar{l}^f)^{\frac{1-\sigma_H}{\sigma_H}}}{L(\bar{l}^m,\bar{l}^f)^{\frac{1-\sigma}{\sigma}}}\right)(\bar{l}^m)^{\frac{1}{\sigma}}\right]^{\sigma_H}, \tag{19}$$

$$\bar{l}^f = 1 - \bar{h}^f = 1 - \left[\frac{\alpha(1+\bar{\tau}_c)\varepsilon_H^f(\bar{c}/\bar{y})}{(1-\tau_l^f)\varepsilon^f(1-\theta)}\left(\frac{C^H(\bar{l}^m,\bar{l}^f)^{\frac{1-\sigma_H}{\sigma_H}}}{L(\bar{l}^m,\bar{l}^f)^{\frac{1-\sigma}{\sigma}}}\right)(\bar{l}^f)^{\frac{1}{\sigma}}\right]^{\sigma_H}. \tag{20}$$

The steady-state values of aggregate labor, capital, and output are given by:

$$\bar{L}(\bar{l}^m,\bar{l}^f) = \left[\varepsilon^f(\bar{l}^f)^{\frac{\sigma-1}{\sigma}} + \varepsilon^m(\bar{l}^m)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

$$\bar{k} = \bar{L}\left(\frac{\bar{k}}{\bar{y}}\right)^{\frac{1}{1-\theta}}, \tag{21}$$

$$\bar{y} = \bar{k}^{\theta}\bar{L}^{1-\theta}. \tag{22}$$

The steady-state bond price is derived as:

$$\bar{q} = \frac{\phi(1+\tau_c)(\bar{c}/\bar{y}) + \beta(1-\tau_b)(\mu/\bar{y} + \bar{b}/\bar{y})}{(\mu/\bar{y} + \bar{b}/\bar{y})(\gamma^{1/(1-\theta)} - \beta\tau_b)}. \tag{23}$$

To ensure that the bond-to-output ratio target is satisfied in the long-run steady state (i.e., $\bar{b}/\bar{y} \leq b_{\text{target}}$), the minimum required consumption tax rate is the one that exactly achieves $\bar{b}/\bar{y} = b_{\text{target}}$. We choose this tax rate as τ_2 in the sustainability rule. It can be derived from the government ’ s steady-state budget constraint (Equation 14) as:

$$\tau_2 = \frac{1}{\bar{c}}\left[g + \lambda - \tau_l^m\bar{w}^m\bar{l}^m + \tau_l^f\bar{w}^f\bar{l}^f - \tau_k(\bar{r}-\delta)\bar{k} - (\bar{q}\gamma^{1/(1-\theta)} - 1 + \tau_b(1-\bar{q}))b_{\text{target}}\cdot\bar{y}\right]. \tag{24}$$

Solution Procedure

The initial capital stock and bond holdings in 1980, $(k_0,\ b_0)$, and a sequence $\{\tau^f_{l,t},\ \tau^m_{l,t},\ \tau_{c,t},\ \tau_{k,t},\ \tau_{b,t}, g_t,\ \lambda_t,\ \gamma_t\}_{t=0}^{\infty}$ are taken as given, where each element in the sequence becomes constant beyond some date so that the economy ultimately converges to a steady state as defined above.

We use a shooting algorithm, similar to those in Hayashi and Prescott (2002), Chen et al. (2006), and Hansen and Imrohoroglu (2016), to solve for the sequence of endogenous variables $\{c_t, C^H_t, l^f_t, l^m_t, y_t, k_{t+1}, b_{t+1}, q_t, w^f_t, w^m_t, r_t\}$ at each time t, such that the model's equilibrium conditions are satisfied and the endogenous variables converge to their steady-state values. Note that the fiscal sustainability rule guarantees that the bond-to-output ratio satisfies $b_t/y_t \leq b_{\text{target}}$ in the steady state.

3 Calibration

This section describes the calibration of parameters using Japanese macroeconomic data spanning 1981–2019. The analysis also incorporates projections through 2050, where relevant.

Output and National Accounts: Following Hansen and Imrohoroglu (2023), we measure output Y using an adjusted Gross National Income (GNI) rather than GDP, based on data from the Cabinet Office. Consumption C corresponds to final private consumption expenditures, while investment I includes both net exports and net factor income from abroad. Government expenditures G comprise final government consumption and net land purchases, net of capital depreciation. Consequently, output is defined as:

$$Y = C + I + G.$$

The capital stock includes private fixed assets held both domestically and abroad. For government spending and transfers, we use data from 1981 to 2019 and incorporate projected trends from 2020 to 2050 based on estimates by Fukawa and Sato (2009). By 2050, the government expenditure-to-output ratio is projected to increase by 2.325 percentage points, and the transfer payments-to-output ratio is expected to rise by 3.1 percentage points.

Production Parameters and TFP: We adopt the capital share $\theta = 0.3783$ and depreciation rate $\delta = 0.0842$ from Hansen and Imrohoroglu (2023). Total factor productivity (TFP) is backed out from a Cobb-Douglas production function:

$$A_t = \frac{Y_t}{K_t^{\theta} L_t^{1-\theta}}.$$

The growth rate of TFP, γ_t, is computed as A_{t+1}/A_t for the period 1981-2018. For 2019 and beyond, we follow Hansen and Imrohoroglu (2016) and assume $\gamma_t = 1.015^{1-\theta}$.

Bond Prices: Bond prices in the model lie between 0 and 1. We normalize the bond price using the following equation from Hansen and Imrohoroglu (2023), which incorporates interest payments on public debt (P_t) and the GNI deflator (F_t):

$$q_t = \frac{B_{t+1}/F_t}{(B_{t+1} + P_{t+1})/F_{t+1}}. \tag{25}$$

Labor Supply: Each model period corresponds to one year, and the total time endowment per day is normalized to 24 hours. We assume individuals allocate 14 hours daily to market work and housework, with the remaining 10 hours devoted to primary activities (e.g., sleep, personal care, meals), consistent with the Survey on Time Use and Leisure Activities. Market labor supply is normalized as:

$$l_g = \text{Employment rate}_g \times \frac{\text{Average weekly working hours}_g}{14 \times 7}, \qquad g \in \{m, f\}, \tag{26}$$

where $g = m, f$ denotes males and females, respectively. Housework time is defined residually as $h_g = 1 - l_g$. Employment and working hour data for individuals aged 20-69 are obtained from the Labor Force Survey (1981-2019), published by the Statistical Survey Department, Statistics Bureau, Ministry of Internal Affairs and Communications.

Wage Rate: Hourly wages are calculated by dividing monthly earnings by the total monthly working hours:

$$W_g = \frac{\text{Monthly wages}}{\text{Average weekly working hours} \times 4}, \quad g \in \{m, f\}. \tag{27}$$

The data for the monthly wages are obtained from Monthly Labour Survey, the Ministry of Health, Labour and Welfare (1996-2019). These wage estimates are used exclusively to calibrate parameters through equilibrium conditions. Since our interest lies in the female-to-male wage ratio, we do not normalize wage units to the time endowment scale.

Tax System: Japan introduced a consumption tax in 1989 at a rate of 3%, which was subsequently increased to 5% in 1997, 8% in 2014, and most recently to 10% in 2019. We assume this rate remains constant going forward.

The tax rate on bond interest income, $\tau_{b,t}$, is set at 20% across all periods t, reflecting Japan's standard tax treatment of interest income.[4]

For the capital income tax rate, we follow the measurement approach in Hansen and Imrohoroglu (2023). We utilize the time series data on revenue from corporate income taxes (denoted by REV_{τ_k}) published by the Japanese government. The equilibrium conditions for firms imply $REV_{\tau_k} = \tau_k(r - \delta)K = \tau_k(\theta - \delta\frac{K}{Y})Y$.

Modeling labor income taxation (including the male labor income tax rate, $\tau^m_{l,t}$, and female labor income tax rate, $\tau^f_{l,t}$) requires additional considerations due to Japan ' s spousal tax treatment. We approximate this feature by incorporating two elements, capturing the key mechanisms of the tax treatment, in the model with a representative household framework – 1) a higher average marginal tax rate for women (typically the second earner in a household), reflecting the high opportunity cost for additional spousal labor supply, and 2) a lump-sum transfer to the household, capturing the monetary value of tax subsidies associated with the spousal tax treatment.

First, we account for the opportunity cost of spousal labor supply. In Japan, if one spouse earns less than a threshold (1.03 million yen annually), that income is tax-exempt, and the other spouse can claim a spousal deduction of 380,000 yen. Consequently, many married women work part-time to keep their income below the threshold. According to Yokoyama (2018), over 60% of married women earn less than 1.03 million yen annually, and about 25% earn just below this threshold (more than 40% fall in the 0.8–1.03 million yen range).

When a part-time female worker increases her earnings above the threshold, she faces a discontinuous jump in effective marginal tax burden, not only becoming taxable herself, but also causing her spouse to lose the deduction. Therefore, her effective marginal tax rate includes both the nominal tax on her income and the opportunity cost of forfeiting the spousal

[4]This comprises a 15% national income tax and a 5% local inhabitant tax (National Tax Agency, `https://www.nta.go.jp/taxes/shiraberu/taxanswer/shotoku/1310.htm`).

deduction. In contrast, full-time female workers are subject only to the nominal tax rate.

We assume all male workers are full-time workers and the labor income tax rate for a full-time worker is equal to $\tau_{l,t}^{m}$. The average marginal tax rate for female workers (τ_l^f) in the model is constructed as a weighted average of part-time and full-time female workers:

$$\tau_l^f = f_1(\tau_p^f) + (1 - f_1)\tau_l^m$$

where f_1 is the share of female part-time workers among all female workers, τ_p^f is the effective marginal tax rate faced by part-time workers and τ_l^m is the tax rate faced by full-time female workers, which is the same as that faced by male workers.

To quantify the opportunity cost associated with the spousal tax treatment, denoted by τ_{op}^f, we measure it by an equivalent labor income tax rate:

$$\tau_{op}^f = \frac{\text{Spousal deduction} \times \tau_{l,t}^m}{\tilde{y}_p^f},$$

where $\tilde{y}_p^f$ is the average annual earnings of part-time female workers. The effective marginal tax rate for part-time female workers is then:

$$\tau_p^f = \tau_{op}^f + \tau_{l,t}^m.$$

Based on this construction, we compute $\tau_{l,t}^f$ for each year between 1981 and 2019.

The second element for characterizing the spousal tax treatment is a lump-sum transfer, denoted by S. Since the opportunity cost of the spousal deduction is not collected by the government, it does not constitute actual tax revenue. In contrast, it is a subsidy (tax deduction) to households with spousal income below the threshold. The notional tax revenue from female workers is $\tau_l^f W^f l^f$, but the actual tax revenue received by the government should be from the fraction of women working full-time (above the threshold), $\tau_l^m(1 - f_1)W^f l^f$.

In addition, the opportunity cost is actually the government subsidy to the portion (f_1) of married women working part-time (below the threshold). In the model, it can be expressed as: $\tau_{op}^f(f_1)W^f l^f$.

Therefore, we construct S_t in the model to adjust the difference between the notional tax and the actual tax revenue and also ensure that the representative household do receive a subsidy as provided by the spousal tax treatment.

$$S = \left[\tau_l^f - \tau_l^m(1 - f_1) + \tau_{op}^f(f_1)\right] W^f l^f$$
$$= \left(2\tau_{op}^f + \tau_l^m\right) f_1 W^f l^f.$$

If the government reforms the tax system by eliminating special spousal treatment, this would result in a reduction in $\tau_{l,t}^f$ and the removal of S_t in the model.

Finally, to calibrate $\tau_{l,t}^m$ (applied to all full-time workers), we use time series data on labor income tax revenue, REV_{τ_l}, following Hansen and Imrohoroglu (2023). The equilibrium condition yields:

$$REV_{\tau_l} = \tau_{l,t}^m \left((1 - f_1)w^f l^f + w^m l^m\right) = \tau_{l,t}^m(1 - \theta)Y,$$

where $(1 - f_1)$ is the share of full-time female workers, and θ is the labor income share. Based on this, we construct $\tau_{l,t}^m$ for each year between 1981 and 2019.

3.1 Parameters Calibrated Inside the Model

We determine the remaining parameters using the equilibrium conditions in this model. In the theoretical framework, we define the productivity of men and women in the market sector as ε^m and ε^f respectively, and in the home sector as ε_H^f and ε_H^m. For the calibration, to simplify the calculations without loss of generality, we set male productivity in both home and market sectors to 1, so $\varepsilon^m = 1$ and $\varepsilon_H^m = 1$. This leaves us with four structural parameters in the market and home-production functions, $\sigma_H, \sigma, \varepsilon_H^f$ and ε^f, and four preference parameters in the utility function, β, α, ϕ, and μ. We use point estimation with equilibrium conditions for most of the parameters, and for ϕ and μ we select the values that best align the bond price in our model with the data. These four structural parameters play a crucial role in our model for gender-specific analysis. From the household equilibrium conditions in (3′) and (5′), we derive the housework time ratio of female to male.

$$\frac{h_t^f}{h_t^m} = \left(\frac{\varepsilon_H^f}{\varepsilon_H^m}\right)^{\sigma_H} \left(\frac{w_t^m}{w_t^f}\right)^{\sigma_H}.$$

We assume that ε_H^f is fixed over time. To determine the value of σ_H, we take the logarithmic difference between time 0 and time T, using the equation below. We calculated several values of σ_H with different periods and then averaged them to determine the final value of σ_H.

$$\begin{cases} \ln\left(\frac{h_T^f}{h_T^m}\right) = \ln\left(\frac{\varepsilon_H^f}{\varepsilon_H^m}\right)^{\sigma_H} + \ln\left(\frac{w_T^m}{w_T^f}\right)^{\sigma_H} \\ \ln\left(\frac{h_0^f}{h_0^m}\right) = \ln\left(\frac{\varepsilon_H^f}{\varepsilon_H^m}\right)^{\sigma_H} + \ln\left(\frac{w_0^m}{w_0^f}\right)^{\sigma_H} \end{cases},$$

$$\sigma_H = \frac{\ln\frac{h_T^m}{h_T^f} - \ln\frac{h_0^m}{h_0^f}}{\ln\frac{w_T^f}{w_T^m} - \ln\frac{w_0^f}{w_0^m}}. \tag{28}$$

For given σ_H, we obtain the value of female productivity in home production ε_H^f by

$$\varepsilon_H^f = \left(\frac{h^m}{h^f}\right)^{-\frac{1}{\sigma_H}} \frac{w^f}{w^m}, \tag{29}$$

and we take the sample average to determine the value of ε_H^f. For the female productivity in market production, we use the equilibrium conditions in Eq. (8′) and Eq. (9′) to obtain

$$\frac{l_t^f}{l_t^m} = \sigma \cdot \left(\frac{\varepsilon^f}{\varepsilon^m}\right)^{\sigma} \left(\frac{w_t^m}{w_t^f}\right)^{\sigma}, \tag{30}$$

ε^f is held constant during the first five years of our sample period. Then, using the logarithmic difference with time 0 and T in Eq. (30), we determine the value of σ by the equation below.

$$\sigma = \frac{\ln\frac{l_T^m}{l_T^f} - \ln\frac{l_0^m}{l_0^f}}{\ln\frac{w_T^f}{w_T^m} - \ln\frac{w_0^f}{w_0^m}}, \tag{31}$$

then we take ε^f as a time-varying parameter and calculate the value of ε_t^f in each time with given σ using the equation below.

$$\varepsilon_t^f = \left(\frac{l_t^m}{l_t^f}\right)^{-\frac{1}{\sigma}} \frac{w_t^f}{w_t^m}. \tag{32}$$

The calculation results show that ε_t^f increased from 0.39 in 1996 to 0.48 in 2019, the reason we set ε_t^f as a time-varying parameter is to capture this increasing trend in females' market sector productivity. Our estimation approach captures the aggregate gender wage

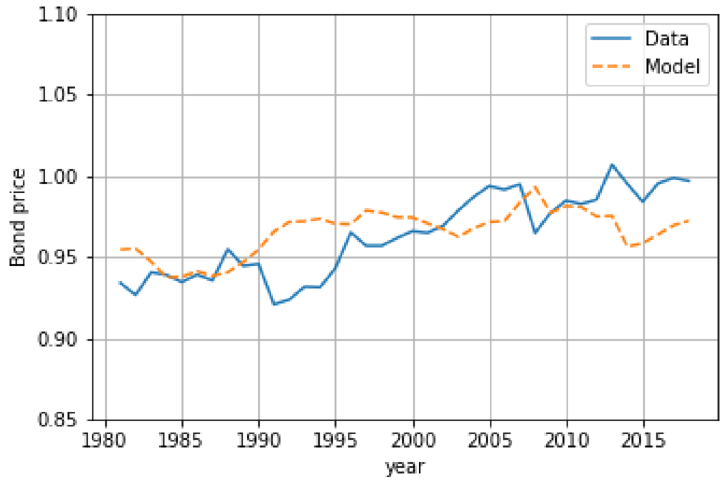

Figure 2: Bond Price

gap without distinguishing between its productivity and discrimination components.[5] In the later quantitative analysis, we will conduct a counterfactual analysis to examine the impact of increased female productivity in the market sector.

Given the parameters and detrended data above, preference parameters β, α, ϕ can be calculated through the following conditions.

$$\beta = \frac{(1+\tilde{\tau}_{c,t+1})\gamma_t^{1/(1-\theta)} c_{t+1}}{(1+\tau_{c,t})c_t[1+(1-\tau_{k,t+1})(\theta\frac{y_{t+1}}{k_{t+1}})-\delta]},$$

$$\alpha = \frac{(1-\theta)\varepsilon^m(1-\tau_{l,t}^m)(h_t^m)^{\frac{1}{\sigma_H}}(L_t)^{\frac{1-\sigma}{\sigma}}}{\varepsilon_H^m(1+\tau_{c,t})(c_t/y_t)(l_t^m)^{\frac{1}{\sigma}}(C_t^H)^{\frac{1-\sigma_H}{\sigma_H}}},$$

$$\phi = (\mu + b_{t+1})\left(\frac{q_t\gamma_t^{1/(1-\theta)}}{\beta(1+\tau_{c,t})c_t} - \frac{\beta[1-(1-q_t)\tau_{b,t+1}]}{(1+\tilde{\tau}_{c,t+1})c_{t+1}}\right).$$

We estimate the preference parameter μ by minimizing the sum of squared differences between model-predicted and observed bond prices:

$$\min_{\mu} \sum_{t=1981}^{2019} \left(q_t^{\text{Data}} - q_t^{\text{Model}}\right)^2,$$

[5]Hara (2016) finds a persistent 18% gender wage gap after controlling for education, experience, and occupation-specific factors. Comparable gaps appear among lawyers (Noonan et al., 2005) and MBA graduates (Bertrand et al., 2010)

following the calibration approach of Hansen and Imrohoroglu (2016). The optimization yields $\mu = 2$ as the value that best matches the empirical bond prices. Figure 2 presents the close correspondence between our model's predictions and the actual data. All parameters determined above are summarized in Table 1.

Table 1: Calibrated Parameters

Calibrated Parameter[a]	Description	Value
β	Time preference	0.9553
ϕ	Relative weight of bond in utility	0.073
μ	Preference parameter	2
α	Relative weight of home goods in utility	1.5
σ_H	Elasticity of substitution in home sector	0.5746
σ	Elasticity of substitution in market sector	1.2820
ε_H^f	Productivity for female in home sector	1.0361
ε^f	Productivity for female in market sector	$0.3947 \sim 0.4802$
Other Parameter[b]	**Description**	**Value**
δ	Depreciation rate	0.0842
θ	Capital share	0.3783
γ	Growth rate of TFP	$1.015^{1-\theta}$

[a] Calibrated parameters are determined by equilibrium conditions using data; μ and ϕ are chosen to match bond price data.
[b] These parameters are borrowed from Hansen and Imrohoroglu (2016).

With the determined parameters, we aim to demonstrate that a steady-state equilibrium exists for both female and male market time. We rewrite the optimal conditions for the household regarding market time as follows:

$$LF(\bar{l}^f, \bar{l}^m) = 1 - \bar{l}^f - \bar{h}^f = 1 - \bar{l}^f - \left[\frac{\alpha(1+\bar{\tau}_c)\varepsilon_H^f(\bar{c}/\bar{y})}{(1-\tau_l^f)\varepsilon^f(1-\theta)} \left(\frac{C^H(\bar{l}^m, \bar{l}^f)^{\frac{1-\sigma_H}{\sigma_H}}}{L(\bar{l}^m, \bar{l}^f)^{\frac{1-\sigma}{\sigma}}} \right) (\bar{l}^f)^{\frac{1}{\sigma}} \right]^{\sigma_H}, \tag{33}$$

$$LM(\bar{l}^f, \bar{l}^m) = 1 - \bar{l}^m - \bar{h}^m = 1 - \bar{l}^m - \left[\frac{\alpha(1+\bar{\tau}_c)\varepsilon_H^m(\bar{c}t/\bar{y})}{(1-\tau_l^m)\varepsilon^m(1-\theta)} \left(\frac{C^H(\bar{l}^m, \bar{l}^f)^{\frac{1-\sigma_H}{\sigma_H}}}{L(\bar{l}^m, \bar{l}^f)^{\frac{1-\sigma}{\sigma}}} \right) (\bar{l}^m)^{\frac{1}{\sigma}} \right]^{\sigma_H}. \tag{34}$$

Eq.(33) and Eq.(34) represent the optimal market working conditions for female and male, respectively. Given the male market time in the interval $(0, 1)$, Eq.(33) shows how the path of female market time changes. Similarly, Eq.(34) illustrates the long-run male market work time for a given female's working time. The calibrated steady-state values for female and male market work time are $(l^f, l^m) = (0.2141, 0.3697)$ as shown in Figure 3.

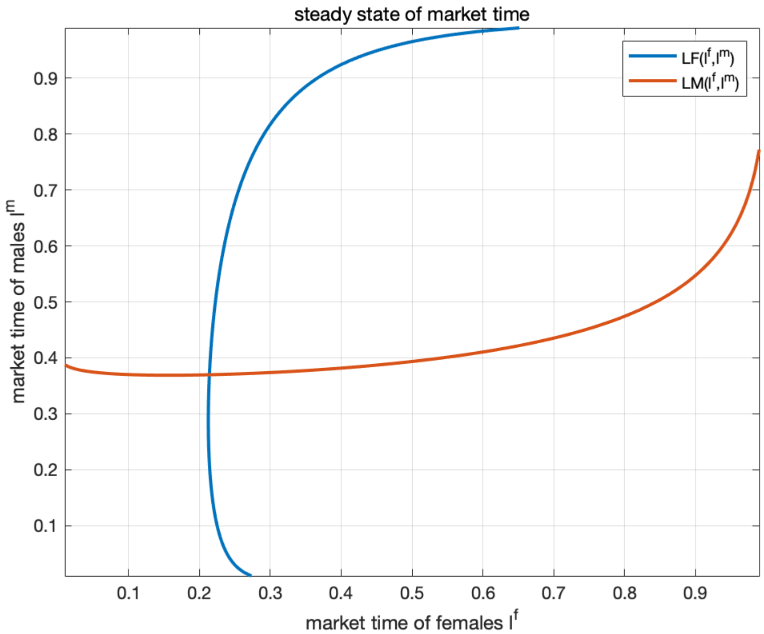

Figure 3: Steady State of (l^f, l^m)

3.2 Model Fit

Prior to conducting out-of-sample projections of debt and other endogenous variables, we assess model performance through historical simulations against Japanese macroeconomic data.

Figure 4 compares market labor time for both females and males between the data and the model. The model predicts a flatter path for market labor hours than is observed in the data. In particular, during the 1990s, labor supply declined significantly due to the reduction in the length of the workweek in Japan, a factor not captured in our benchmark model.

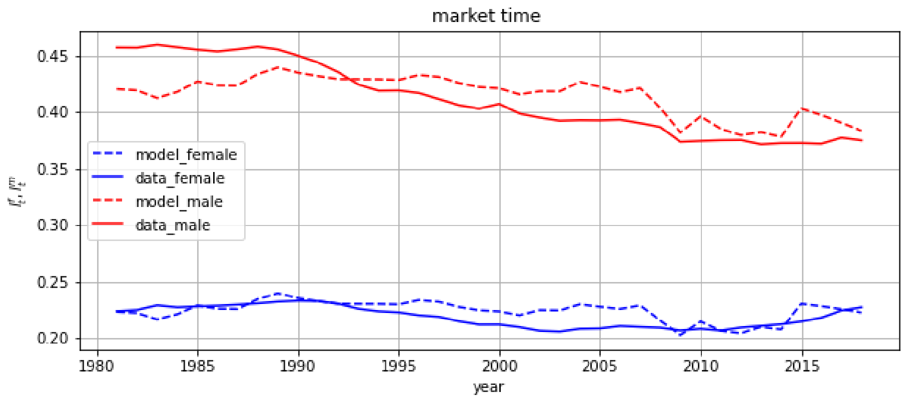

Figure 4: Male and Female Market Labor Hours: Data and Model

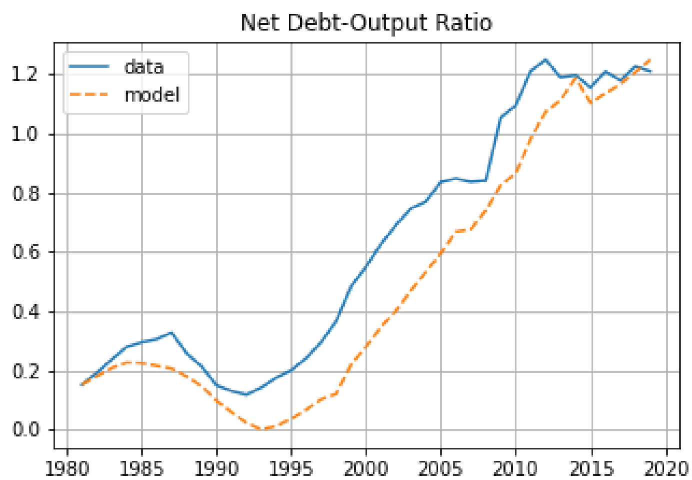

Figure 5: Net Debt-to-Output Ratio: Data and Model

In addition, Figure 5 further displays the debt-to-output ratios from the Japanese data alongside those from our model over the sample period. Overall, while the model abstracts from many details of the Japanese economy, it effectively captures the general patterns of these key endogenous variables.

4 Quantitive Analysis

We conduct a basic transition analysis by simulating the model economy starting from 1981. As previously mentioned, once the government debt-to-output ratio reaches the threshold level b_{max}, the debt sustainability rule is triggered, preventing the government from issuing additional bonds and necessitating an increase in the consumption tax rate to achieve a primary balance.

The details of the debt sustainability rule are described as follows. 1) The target debt-to-output ratio b_{target} along the balanced growth path is set at 60%, a level once considered necessary for membership in the European Monetary Union. 2) The threshold of debt-to-output ratio b_{max}, which serves as the trigger point, is set at 250%. 3) For the subsequent analysis, to facilitate easier comparison of the necessary consumption tax rate across different experiments, τ_1 is chosen to be as small as possible while ensuring that B_t/Y_t can converge to b_{target} by 2089.

4.1 Baseline Experiment

We consider a debt sustainability rule that increases the consumption tax to stabilize the debt and achieve the target debt-to-output ratio of 60% (b_{target}). Suppose that the debt-to-output ratio $B_s/Y_s \geq b_{max}$ at time s, the sustainability rule is given by:

$$\tau_{c,t} = \begin{cases} \tau_1 > \tau_{c,s}, & \text{if } B_t/Y_t > b_{\text{target}},\ \forall t \geq s \\ \tau_2 \leq \tau_1, & \text{if } B_t/Y_t \leq b_{\text{target}},\ \forall t \geq s \end{cases}. \tag{35}$$

The consumption tax rate starts at the actual rate at the beginning of the transition path. Once the debt-to-output ratio reaches the 250% threshold, the sustainability rule is triggered, requiring the consumption tax rate to increase to τ_1. It will be reduced to τ_2 when the debt-to-output ratio satisfies the sustainability target and remains the same to the long run steady state. Government expenditures and transfer payments remain unchanged from the basic transition analysis. The simulation results for the consumption tax transition are shown in Figure 6. The debt-to-output ratio reaches 250% in 2035, necessitating an increase in the consumption tax rate to 40.9% (τ_1). It will then decrease to 24.4% (τ_2) in the long run after the debt reaches the target level in 2089 (see the blue line in Figure 6).

Although the value of τ_1 is chosen to be 40.9% in the baseline experiment, we also conduct experiments with alternative values (42.9% and 44.9%) for comparison, as shown in Figure 6. It can be seen that when the short-term increase in consumption tax is lower, It can be observed that when the short-term increase in the consumption tax is lower, the time

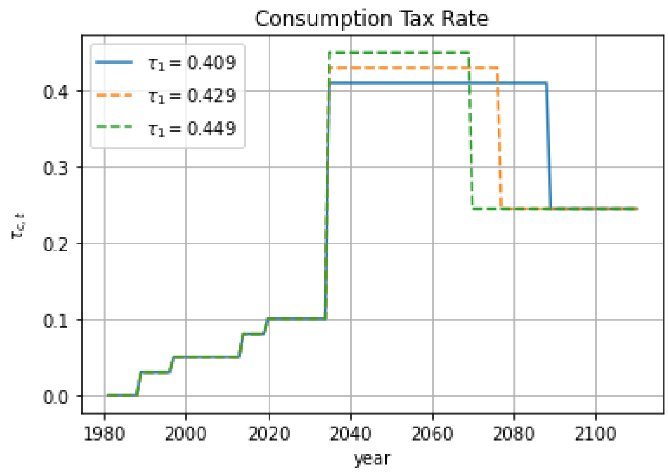

Figure 6: Transition of Consumption Tax (Baseline)

required to achieve the fiscal target is extended (by 12 years and 19 years, respectively).

4.2 Implications of Female Labor Supply

Given the significant gap between male and female market labor supply in Japan, a higher level of female labor supply could potentially improve Japan's fiscal condition. In this subsection, we conduct experiments with various scenarios related to female labor supply to assess its implications for long-term fiscal sustainability. We conduct experiments based on the baseline experiment with a consumption tax-financed debt sustainability rule, considering two counterfactual cases: 1) Abolishing the spousal tax treatment, which lowers the female labor income tax rate ($\tau^f_{l,t}$) and eliminates the subsidy S_t from the total government transfers, and 2) an improvement in the productivity of female market labor (a higher ε^f).

4.2.1 Experiment 1: Abolishing Spousal Tax Treatment

Several amendments to the tax system related to the spousal tax treatment have been made in recent years, and discussions about its abolition are ongoing. In the calibration, we highlighted that due to the inclusion of the spousal deduction, which acts as an opportunity cost for female workers earning more than 1.03 million yen annually, the effective labor income tax rate faced by women is relatively high. We conduct a policy experiment abolishing the spousal tax treatment, which lowers the female labor income tax rate ($\tau^f_{l,t}$) to the male levels ($\tau^m_{l,t}$) and eliminates the subsidy S_t from the total government transfers.

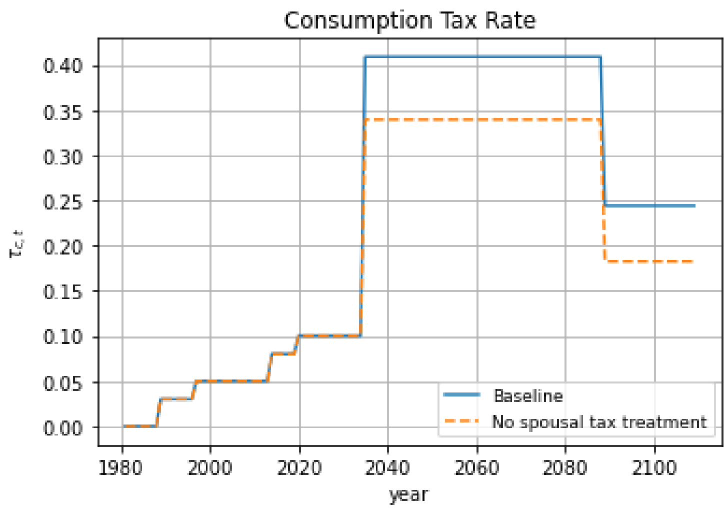

Figure 7: Consumption Tax (Baseline vs. Experiment 1: Tax Reform)

Specifically, we assume that the government conducts the reform at the same time when the sustainability rule is triggered in 2035 (i.e., the debt-to-output ratio exceeds 250%). Figure 7 illustrates the transitions of the required consumption tax rate under the baseline and the counterfactual scenario. If the tax reform is conducted, the consumption tax rate would decrease to 33.9% (dashed line in Figure 7), compared to 40.9% in the baseline case (solid line), with the target debt level being reached in 2089, the same year as in the baseline. The consumption tax τ_2 can also be reduced to 18.2% when the target is reached. In this tax reform case, the required tax rate is lower than the baseline, both in the short and long run.

Figure 8 illustrates the changes in the market time allocation for men and women. The tax reform results in a 3% increase in female market labor supply in both the short and long run. Due to the substitutive relationship between male and female labor, the endogenously determined market time for males decreases by roughly 0.4%.

4.2.2 Experiment 2: Improvement in Women's Relative Productivity

Several studies indicate that there is a significant gender wage gap in Japan, even after controlling for individual characteristics. This gap may be partially attributed to gender discrimination, market distortions, or regulatory factors. In our model, it is captured as a disparity in the productivity of female labor in the market sector relative to the male labor's. Our calibration shows an explicit upward trend in ε^f. This trend might be due to the gradual reduction in gender discrimination, driven by measures such as implementing gender-

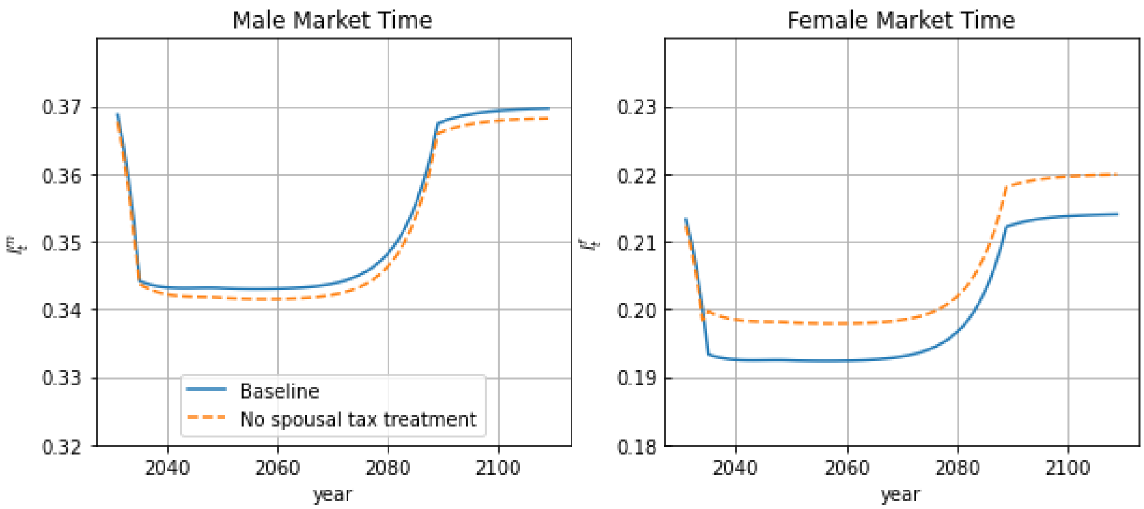

Figure 8: Market Labor (Baseline vs. Experiment 1: Tax Reform)

inclusive policies that promote equal participation of men and women. We hypothesize that a more gender-friendly labor market that reduces the distortions against female workers could substantially boost female labor productivity in the market (ε^f). Based on this hypothesis, we conduct experiments where the relative productivity of women increases once the debt-to-output ratio threshold (250%) is reached, triggering the debt sustainability rule.

We consider two counter-factual experiments: 1) a 25% increase in ε^f that will raise its value from 0.48 to 0.60; 2) a 50% increase in ε^f that will raise it to 0.72. Note that the productivity of men ε^m is normalized to one. So in the first experiment, the female productivity in the market sector is only 60% of men's, and in the second experiment, it is still less than 75% of men's. Figure 9 shows the transitions of consumption tax required in the experiments and the baseline. If ε^f increases 25%, the required consumption tax will be lower at 39.3% when the sustainability rule is triggered (compared to 40.9% in the baseline) and at 23.5% when the target debt level is achieved (see the dashed line in Figure 9). A 50% raise in ε^f leads to a further lower required consumption tax rate at 38.1% when the sustainability rule is triggered and 22.7% when the target debt level is achieved. The changes in men's and women's market time are illustrated in Figure 10. The increase in the relative productivity of women leads to a significant improvement in female labor supply. Specifically, a 25% and 50% increase in ε^f results in a long-term average increase of 13.1% and 23.4% in women's market time, respectively.

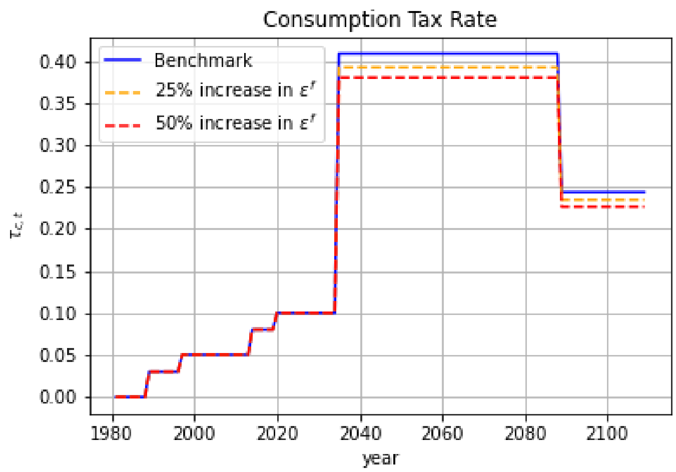

Figure 9: Consumption Tax (Experiment 2: Female Productivity Improvement)

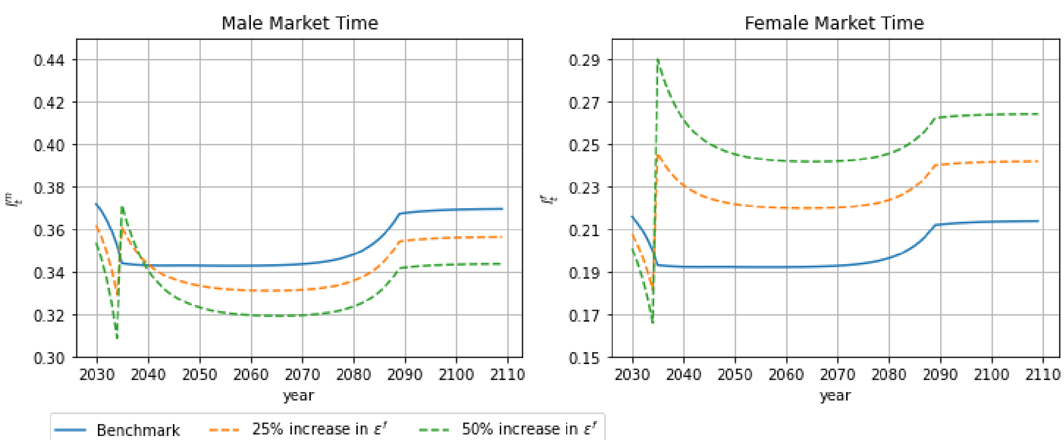

Figure 10: Market Labor (Experiment 2: Female Productivity Improvement)

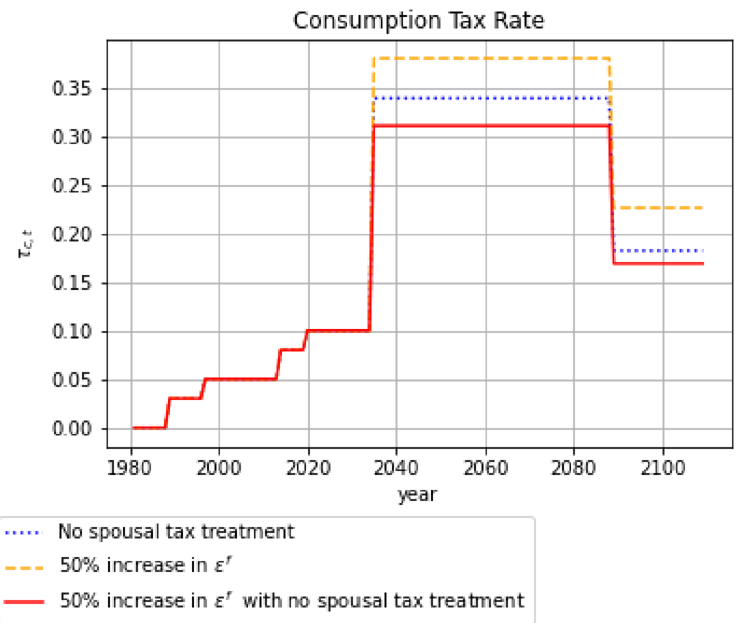

Figure 11: Consumption Tax (Experiment 3: Tax Reform + Productivity Improvement)

4.2.3 Experiment 3: Joint Impact of Tax Reform and Female Productivity Improvement

In this experiment, we analyze the effects of simultaneously increasing women's relative productivity and abolishing the spousal tax treatment (i.e., combining Experiments 1 and 2). The result is shown in Figure 11.

If ε^f is improved by 50% and becomes 72% of men's and the spousal tax treatment is removed (the lower solid line in Figure 11), the required consumption tax rate decreases further to 31.1% when the sustainability rule is triggered, and to 16.9% when the policy target is reached in 2089. We also show the cases of Experiment 1 (the middle dashed line) and Experiment 2 (the upper dotted line) in Figure 11 for comparison.

Figure 12 shows the changes in female (left panel) and male labor supply (right panel) in the market sector. There are corresponding reductions in male market labor supply when female labor supply increases in those scenarios.

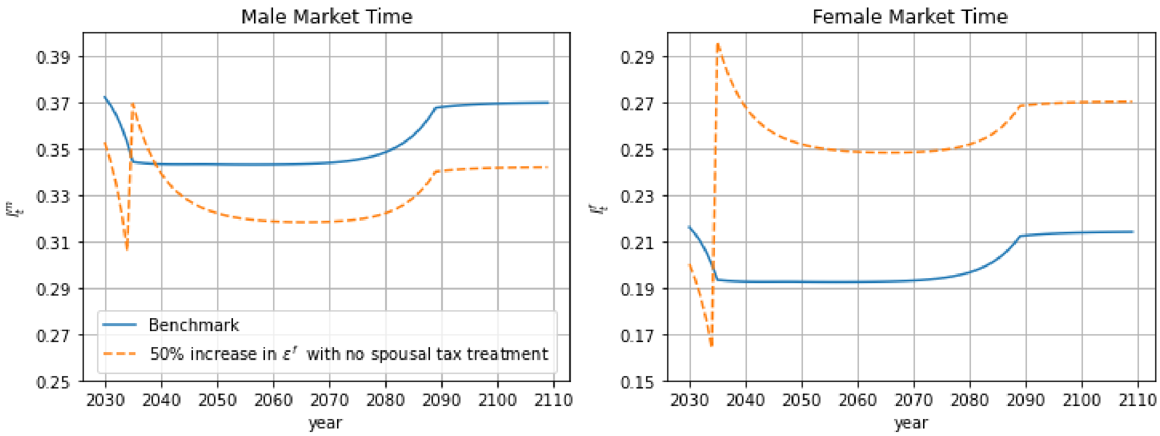

Figure 12: Market Labor (Experiment 3: Tax Reform + Productivity Improvement)

5 Conclusion

The persistent rise in Japan's government debt since the 1990s has resulted in a serious fiscal imbalance. In this paper, we extend the standard growth model by incorporating male and female labor supply decisions across home and market sectors to examine Japan ' s long-run fiscal sustainability and assess the potential role of female labor supply in mitigating the debt burden.

We quantify the magnitude of the fiscal adjustment required to stabilize government debt and the corresponding taxation implications. Our baseline simulation, which maintains government spending and transfer payments while relying solely on consumption tax hikes to stabilize debt, projects that, without reform, the debt-to-output ratio will reach 250% by 2035. If the government implements a fiscal sustainability rule that mandates reducing the debt-to-output ratio to 60%, the consumption tax rate would need to rise sharply to 40.9% until 2089, after which it could be stabilized at 24.4%.

However, policy reforms targeting female labor supply generate significant fiscal benefits. Eliminating the spousal tax treatment reduces the required consumption tax rate to 33.9% upon activation of the sustainability rule, and to 18.2% once the target debt ratio is achieved. In addition, enhancing female market productivity lowers the required tax rate to 38.1% during the debt stabilization and 22.7% in the long run.

Overall, the findings highlight the role of gender-based labor market reforms in achieving fiscal sustainability. Promoting gender equality through tax policy and productivity-enhancing measures not only improves labor allocation efficiency but also strengthens the government's ability to manage long-term debt without excessive reliance on taxation.

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