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Abstract
This paper explores how firm's internalization strategies - specifically exporting and foreign direct investment (FDI) - relate to their product scope. We develop a model incorporating firm heterogeneity and multi-product firms. The most productive firms engage in FDI and produce the broadest range of products. Firms with intermediate productivity levels tend to export, offering fewer product varieties than FDI firms. In contrast, low-productivity firms typically operate domestically and have the smallest product scope. The model also predicts that the ratio of the product scope of exporters and domestic firms and the ratio of FDI firms and exporters should decline as the difficulty of expanding the product scope increases. The Japanese firm-level data support the theoretical predictions.

Keywords: multi-product firms, heterogeneous firms, export, FDI
JEL classification: F12

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This study utilizes the micro data of the questionnaire information based on "the Basic Survey of Japanese Business Structure and Activities" "Census of Manufacture", and "Survey on Overseas Business Activity", which are conducted by the Ministry of Economy, Trade and Industry (METI), and the Kogyo Toukei converter, which is provided by RIETI.

1 Introduction

It is well-known that firm size and productivity are heterogeneous across firms, and such heterogeneity significantly affects trade behavior (Melitz 2003 and Helpman et al. 2004). Specifically, the most productive firms tend to engage in foreign direct investment (FDI), moderately productive firms are more likely to become exporters, and the least productive firms remain non-exporters. Trade liberalization leads to profit-shifting effects across firm types, with internationalized firms deriving significantly larger benefits and local firms (non-exporters) experiencing profit declines due to intensified competition.

Most firms produce several different products. Multi-product firms, for instance, accounted for 87 percent of total output in the U.S. from 1987 to 1997 (Bernard et al., 2010). Despite this, the dynamics of multi-product firms, which dominate real-world production, remain relatively understudied in the context of international trade and FDI.

This paper incorporates an endogenous product scope into the standard model of heterogeneous firms and trade. It is shown here how product scope depends on the mode of operation: it increases with exporting and it increases even more when a firm conducts FDI. This finding is strongly confirmed in Japanese firm and product level data. Furthermore the model predicts that trade liberalization will decrease the product scope of all types of firms, but less so for exporters. This effect is related to the proximity-concentration trade-off between exporting and FDI, but in our model part of the trade-off manifests through adjustments in product scope. Again data strongly support this result. The model framework we use introduces multiproduct firms *à la* Forslid and Okubo (2023) in the Helpman et al. (2004) framework of trade and FDI with heterogeneous firms.

A number of related papers find that trade liberalization reduces the product scope of firms, as firms concentrate on their core products when trade is liberalized. This effect tends to occur in oligopolistic settings, where the firm has a core product and new products with higher marginal costs compete with those already produced by the firm (the so-called "cannibalization effect").[1]

There is also research showing that trade liberalization affects high and low productivity firms differently. High-productivity firms expand their product scope due to improved access to foreign markets, while low-productivity firms narrow their product scope due to increased competition in the domestic market (Dhingra 2013; Nocke and Yeaple 2014; and Qiu and Zhou

[1] See, for example, Blanchard et al. (2012), Eckel and Neary (2010) and Ju (2003). Similar results are found by Mayer et al. (2014 and 2021), who use a monopolistically competitive model with heterogeneous firms and linear demand. Bernard et al. (2011) have heterogeneous firms that match their "capabilities" to different product attributes (or consumer preferences). Here, trade liberalization can lead to a wider or narrower range of products. Feenstra and Ma (2007) use standard CES preferences but relax the large group assumption. By allowing firms to account for their own effect on the aggregate price index, they obtain a cannibalization effect from new products. Eckel et al. (2015) allow for both vertical (quality) and horizontal (scope) upgrading. Here, trade liberalization (tariff reductions) leads to a narrower product range for all firms.

2013).[2]

We build on the canonical heterogenous firms model of Melitz (2003) and Helpman et al. (2004), where no cannibalization effects are present. Instead we focus on the different effect of trade liberalization on exporters and FDI firms. We introduce multiproduct firms in this framework, and assume that the marginal cost of introducing a new product increases with the distance from a firm's core product, reflecting higher costs for products further from the firm's primary focus. Additionally, we adopt the Melitz (2003) framework to account for firm-level heterogeneity in the marginal cost of their core product. More productive firms, which have lower marginal costs, are more likely to profitably expand their product scope by introducing products further from their core offering. In this model trade liberalization affects both firm selection in to exporting and FDI and the product scope of these firms. Our main results are that the product scope depends on the mode of operation (exporting or FDI), and that trade liberalization, controlling for productivity, reduces the product scope of exporters as well as FDI firms, but less so for exporters. These results are well aligned with our data.

We use Japanese firm- and product-level data, to investigate the predictions of our model. The data reveal that product scope, controlling for productivity, is higher for firms engaging in exporting and is even higher for firms conducting FDI. These findings are consistent with prior studies documenting that multi-product firms are larger and more productive than single-product firms (Bernard et al., 2011; Goldberg et al., 2010) and that they exhibit higher overall productivity (Schoar, 2002). However, contrary to the mentioned studies, we show that exporting and FDI are important determinants of the product scope, even when controlling for productivity. We also show, in the empirical part, stylized evidence in favor of the model's predictions concerning the relative product scope of different types of firms.

The following section sets out the model. Section 3 shows data and stylized facts and Section 4 shows estimation results. Section 5 concludes the paper.

2 Model

2.1 Basics

There are two countries, the Home country and the Foreign country which is denoted by ' $*$ '. There is one primary factor of production, labor L. Labor is perfectly mobile across sectors within each country, but immobile across borders. A homogeneous good is produced under constant returns to scale, whereas differentiated manufactured goods are produced under increasing returns to scale. Firms are heterogeneous with respect to their marginal costs. Each firm produces a range of product varieties. It has one core product, and marginal costs of each new variety increase monotonically as the firm expands its product range.

[2] Bernard et al. (2018) present a very general model where firms chose multiple production locations, multiple export markets, and countries to source from. They show how more productive firms participate more intensively in the world economy along each margin.

All individuals have the utility function

$$U = C_M^{\mu} C_A^{1-\mu}, \quad C_M = \left[\int_{l \in \Psi} c_l^{(\sigma-1)/\sigma} dl \right]^{\sigma/(\sigma-1)}, \tag{1}$$

where $\mu \in (0,1)$, and $\sigma > 1$ are constants. Ψ is the set of consumed varieties. C_M is a consumption index of manufacturing goods, and C_A is consumption of the homogeneous good. c_l is the amount consumed of variety l. Each consumer spends a share μ of his income on manufactures. The total demand for a variety j from firm i in market ν is

$$x_{ij\nu} = \frac{p_{ij\nu}^{-\sigma}}{P_{\nu}^{1-\sigma}} \cdot \mu Y_{\nu}, \tag{2}$$

where $p_{ij\nu}$ is the consumer price of variety j from firm i in market ν, P_{ν} is the CES price index, and Y_{ν} is income (or expenditure) in market v.

On the supply side, the homogeneous-good sector has constant returns and perfect competition. The unit factor requirement of the homogeneous good is one unit of labor. The good is freely traded and chosen as numeraire, implying:

$$p_A = w = 1, \tag{3}$$

where w is the wage of workers in all markets.

The aggregate return in equilibrium equals aggregate operating profit, which is $\mu E^w/\sigma$. Total equilibrium expenditures can be written $E^w = wL^w + \mu E^w/\sigma$. Without loss of generality, we choose units so that $L^w \equiv 1$, which yields $E^w = \frac{\sigma}{\sigma-\mu}$. We assume trade balance, so that income equals expenditure in each market. The income of market ν, is therefore equal to its share of total expenditure:

$$Y_{\nu} = s_{\nu} E^w = s_{\nu} \frac{\sigma}{\sigma - \mu}, \tag{4}$$

with s_v denoting the expenditure share of country v. Y_{ν} is thus constant irrespective of the location of firms; i.e., also out of long-run equilibrium. For ease of notation, we suppress the market subscript where possible in the following.

Manufacturing Firms

We assume a fixed global number (mass) of firms $N^W = n + n^*$, which is normalized to 1, without loss of generality. After having acquired a patent, a firm draws its core marginal cost a_i from a cumulative distribution function $G(a)$, $a \in [0,1]$. The firm thereafter chooses a range of varieties to produce (the product scope), $[0, \overline{m_i}]$, where $\overline{m_i} \geq 0$.[3] Each variety m_{ij} requires an additional fixed cost f in terms of labor (the composite primary factor of production), implying

[3] Firms continue to have zero measure in this set-up. The aggregate CES price index will here be an integral over a surface, where one dimension is the continuum of firms and the other is the product scope. The limiting most productive firm (with a zero marginal cost) will have an infinite product scope, but this firm has a zero measure. Hence the large group assumption of the monopolistically competitive framework remains valid.

a fixed cost $\overline{m_i} f$ for firm i. Furthermore, as the firm moves away from it's core product, the marginal cost of each new variety increases.

The total cost function of a firm i is assumed to be:

$$TC_i = f_E + entry \cos t + \overline{m_i} f + a_i \int_0^{\overline{m_i}} z^{\theta} x_i(z) dz, \tag{5}$$

where f_E is the cost of a patent and the entry cost depends on how and which market the firm enters: f_D is the entry cost of the domestic market, f_X is the entry cost of exporting to the foreign market, and f_I is the entry cost for foreign production (FDI). z is an integration dummy, and the parameter $\theta > 0$ determines how fast the marginal cost increases as a firm expands its product scope. We assume integrated markets: each firm sells all it's varieties in all markets.[4]

Profit maximization by manufacturing firms implies a constant mark-up over the marginal cost of each product variety m_i,

$$p_i = \frac{\sigma}{\sigma - 1} m_i^{\theta} a_i. \tag{6}$$

Geographical distance is represented by trade costs. Shipping the manufactured good involves a frictional trade cost of the "iceberg" form: for one unit of good from market j to arrive in market k, $\tau_{jk} > 1$ units must be shipped. Trade costs are symmetric between markets $\tau_{jk} = \tau \ \forall \ j, k$, and the export price is therefore $p_{ij}\tau$.

The total profit of a firm is given by:

$$\pi_i = \int_0^{\overline{m_i}} \frac{p_i(z) x_i(z)}{\sigma} dz - \overline{m_i} f - f_z, \tag{7}$$

where $f_z \in [f_D, f_X, f_I]$ depending on market entry mode. Exporting involves a higher fixed cost than domestic operation ($f_X > f_D$), while FDI incurs the highest fixed cost ($f_I > f_X$) but avoids variable trade (transportation) costs.

Using (6) and (2), we can write the profit of domestic firms, export firms and FDI firms, respectively:

$$\pi_i^D = \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1 - \theta(\sigma - 1)} a_i^{1-\sigma} B - \overline{m_i} f - f_D, \tag{8}$$

$$\pi_i^X = \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1 - \theta(\sigma - 1)} a_i^{1-\sigma} B + \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1 - \theta(\sigma - 1)} a_i^{1-\sigma} \phi B^* - \overline{m_i} f - f_X, \tag{9}$$

$$\pi_i^I = \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1 - \theta(\sigma - 1)} a_i^{1-\sigma} B + \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1 - \theta(\sigma - 1)} a_i^{1-\sigma} B^* - \overline{m_i} f - f_I, \tag{10}$$

where $B \equiv \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{\mu s E^w}{\sigma \Delta}$, $B^* \equiv \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{\mu(1-s)E^w}{\sigma \Delta^*}$ are measures of market potential that are exogenous from the point of view of an individual firm. $\phi \equiv \tau^{1-\sigma} \in [0,1]$ is the freeness of trade, and

[4] This assumption is also consistent with our data part, since product level sales of foreign affiliates is not available in Japanese data.

$$\Delta \equiv P^{1-\sigma} = s\int_0^{a_D}\left(\int_0^{\overline{m_i}} p_i(z)^{1-\sigma}dz\right)dG(a) + (1-s)\,\phi\int_{a_F^*}^{a_X^*}\left(\int_0^{\overline{m_i}^*} p_i^*(z)^{1-\sigma}dz\right)dG(a)$$

$$+(1-s)\int_0^{a_F^*}\left(\int_0^{\overline{m_i}^*} p_i^*(z)^{1-\sigma}dz\right)dG(a), \tag{11}$$

$$\Delta^* \equiv P^{*(1-\sigma)} = (1-s)\int_0^{a_D^*}\left(\int_0^{\overline{m_i^*}} p_i^*(z)^{1-\sigma}dz\right)dG(a) + s\phi$$

$$\int_{a_F}^{a_X}\left(\int_0^{\overline{m_i}} p_i(z)^{1-\sigma}dz\right)dG(a) \tag{12}$$

$$+s\int_0^{a_F}\left(\int_0^{\overline{m_i}} p_i(z)^{1-\sigma}dz\right)dG(a), \tag{13}$$

where we use s for the domestic endowment (expenditure) share and $1-s$ for the foreign share.

It is seen from (8) that positive profits require the following assumption:

ASSUMPTION 1 $\theta < \frac{1}{\sigma-1}$

The condition relates the decreasing returns to scale in introducing new varieties to the substitutability of varieties, which governs the mark-ups.

We can calculate the profit-maximizing product scope of a firm i for domestic firms, exporters and FDI firms from (8), (9), and (10):

$$\overline{m_i}^D = f^{-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}. \tag{14}$$

$$\overline{m}_i^X = f^{-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}. \tag{15}$$

$$\overline{m}_i^I = f^{-\frac{1}{\theta(\sigma-1)}} \left(B+B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}. \tag{16}$$

The expressions (14) (15) and (16) lead to the following proposition:

Proposition 1 $\overline{m_i}^D \leq \overline{m_i}^X \leq \overline{m_i}^I$

Proof. *Noting that* $\phi \in [0,1]$, *the proposition follows immediately from (14), (15), and (16).* ■

That is, a firm of a given productivity will choose a wider product scope if it engages in exporting, and even wider if it engages in FDI. This hierarchy is confirmed by our empirical findings. Generally the optimal product scope of a firm expands with the market size (B and B^*), and decreases with the firms' core marginal cost, a_i, and with the fixed costs. Firms trade off the increase in fixed cost against the additional operating profit of an extra variety when choosing product scope. More productive firms have lower marginal costs and higher operating profits. Their break-even fixed cost is consequently higher, meaning that such firms will opt for a wider product range. A lower fixed cost will for similar reasons imply a wider product scope. Finally, an increase in market potential enhances the marginal profitability of new varieties, raising the firm's break-even fixed cost and consequently the product range.[5]

It may also be noted from (15) and (16) that trade liberalization will increase the product scope of exporters compared to the export scope of FDI firms. This prediction of the model will be tested below.

The sector cut-off level productivity - where a firm is indifferent between entering and not - is determined separately for each entry mode. For domestic-only firms, the cut-off condition is:

$$\frac{\overline{m_D^D}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_D^{1-\sigma} B = \overline{m_D^D} f + f_D, \tag{17}$$

where $a_D^{1-\sigma}$ is the minimum productivity required for serving the domestic market.

For exporters, the cut-off condition is:

$$\frac{\overline{m_X^X}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_X^{1-\sigma}(B+\phi B^*) - \frac{\overline{m_X^D}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_X^{1-\sigma} B = \overline{m_X^X} f - \overline{m_X^D} f + f_X - f_D. \tag{18}$$

Here $a_X^{1-\sigma}$ denotes the break-even productivity level at which the firm is indifferent between remaining a domestic-only firm and starting to export. The left-hand side measures the additional profits from exporting; the right-hand side reflects the additional fixed and scope-related costs.

For FDI, the cut-off productivity $a_I^{1-\sigma}$ satisfies:

$$\frac{\overline{m_I^I}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{1-\sigma}(B+B^*) - \frac{\overline{m_I^X}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{1-\sigma}(B+\phi B^*) = \overline{m_I^I} f - \overline{m}_I^X f + f_I - f_X \tag{19}$$

At the stage of entry - after paying the entry cost F_E- firms draw $a_i \sim G(a)$. The model is closed with the free-entry condition, which equates the expected profits of entering with the sunk entry cost:

[5] Most multiproduct firm models with cannibalization effects predict a negative association between product scope and market size. However, Qiu and Zhou (2013) finds a positive relationship between product scope and firm level productivity and market size as in the present paper. Dhingra (2013) and Nocke and Yeaple (2014) finds that high productive firms expand their product scope in large markets whereas the opposite is true for low productive firms.

$$F_E = \int_0^{a_D} \left(\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} B - \overline{m_i} f - f_D \right) dG[a] + \tag{20}$$

$$+ \int_{a_I}^{a_X} \left(\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} B + \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} \phi B^* - \overline{m_i} f - f_X \right) dG[a] + \tag{21}$$

$$+ \int_0^{a_I} \left(\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} B + \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} B^* - \overline{m_i} f - f_I \right) dG[a], \tag{22}$$

where the entry cost $F_E = \delta f_E$, and δ is the constant Poisson hazard rate of firm exit. The three cut-off equations together with the free entry condition jointly determine a_D, a_X, a_I and B.

Under symmetry $B = B^*$, the cut-off values can be derived analytically, as shown i Appendix 6.1.[6] The cut-off are given by

$$a_D^k = \frac{F_E}{(\kappa\vartheta - 1) f_D + f_D^{k\theta} (\Phi - 1)^{k\theta} \frac{1}{(f_X - f_D)^{k\theta}} \left(\kappa \frac{\Phi}{(\Phi-1)} (f_X - f_D) - f_X \right) + (f_I - f_X)^{1-k\theta} f_D^{k\theta} \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi \right)^{k\theta} (\kappa - 1)}, \tag{23}$$

$$a_X^k = \frac{\left(\frac{1}{f_X - f_D} \right)^{k\theta} f_D^{k\theta} (\Phi - 1)^{k\theta} F_E}{(\kappa\vartheta - 1) f_D + f_D^{k\theta} (\Phi - 1)^{k\theta} \frac{1}{(f_X - f_D)^{k\theta}} \left(\kappa \frac{\Phi}{(\Phi-1)} (f_X - f_D) - f_X \right) + (f_I - f_X)^{1-k\theta} f_D^{k\theta} \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi \right)^{k\theta} (\kappa - 1)}, \tag{24}$$

and

$$a_I^k = \frac{\left(\frac{1}{f_I - f_X} \right)^{k\theta} f_D^{k\theta} \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi \right)^{k\theta} F_E}{(\kappa\vartheta - 1) f_D + f_D^{k\theta} (\Phi - 1)^{k\theta} \frac{1}{(f_X - f_D)^{k\theta}} \left(\kappa \frac{\Phi}{(\Phi-1)} (f_X - f_D) - f_X \right) + (f_I - f_X)^{1-k\theta} f_D^{k\theta} \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi \right)^{k\theta} (\kappa - 1)}, \tag{25}$$

where $\vartheta \equiv \left(\frac{1}{1-\theta(\sigma-1)} - 1 \right) f^{1-\frac{1}{\theta(\sigma-1)}}$ and $\kappa \equiv \frac{k}{k-\frac{1}{\theta}}$. $\Phi \equiv (1+\phi)^{\frac{1}{\theta(\sigma-1)}}$ is a measure of trade freeness.

The effects of trade liberalization on the cut-offs will depend on parameter values. Consider for example a_D. Trade liberalization (a higher Φ), will increase competition from foreign exporters but it will decrease competition from foreign owned firms (FDI). The effect on a_D will

[6] The assumption that $B = B^*$ implies that the market potential is the same in both markets. This will be guarantied by free entry as long as both countries have the same productivity distribution, the same technology, and symmetric trade costs. Thus, under these conditions the assumption holds even when $L \neq L^*$.

therefore depend on parameter values such as the difference between f_D and f_X. However, the effect of trade liberalization on relative cut-off productivities is easy to establish. The ratio of the cut-offs are given by

$$\frac{a_D}{a_X} = \frac{(f_X - f_D)^\theta}{f_D^\theta (\Phi - 1)^\theta}, \tag{26}$$

$$\frac{a_X}{a_I} = \frac{(f_I - f_X)^\theta (\Phi - 1)^\theta}{(f_X - f_D)^\theta \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi\right)^\theta}, \tag{27}$$

and

$$\frac{a_D}{a_I} = \frac{(f_I - f_X)^\theta}{f_D^\theta \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi\right)^\theta}. \tag{28}$$

As usual, we will assume that parameter values are such that $a_D > a_X > a_I$. More precisely the condition for $\frac{a_D}{a_X} > 1$ is that $\frac{(f_X - f_D)}{f_D(\Phi-1)} > 1$ and the condition for $\frac{a_X}{a_I} > 1$ is $\frac{(f_I - f_X)(\Phi-1)}{(f_X - f_D)\left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi\right)} > 1$. The equations (26), (27), and (28) lead to the following proposition:

Proposition 2 $\frac{d\left(\frac{a_D}{a_X}\right)}{d\Phi} < 0, \quad \frac{d\left(\frac{a_X}{a_I}\right)}{d\Phi} > 0,$ *and* $\frac{d\left(\frac{a_D}{a_I}\right)}{d\Phi} > 0.$

Proof. *The proposition is seen directly from (26), (27), and (28).* ■

The proposition implies that trade liberalization (higher Φ) reduces the productivity gap between domestic-only firms and exporters, but increases the gap between exporters and FDI firms.

Turning to the product scope it is seen from equations (14), (15), and (16), under the assumption that $B = B^*$, that:

$$\frac{\overline{m_i}^X}{\overline{m}_i^D} = (1 + \phi)^{\frac{1}{\theta(\sigma-1)}} > 1, \tag{29}$$

$$\frac{\overline{m_i}^I}{\overline{m}_i^X} = \left(\frac{2}{1 + \phi}\right)^{\frac{1}{\theta(\sigma-1)}} > 1, \tag{30}$$

and

$$\frac{\overline{m_i}^I}{\overline{m}_i^D} = 2^{\frac{1}{\theta(\sigma-1)}} > 1. \tag{31}$$

These ratios imply:

Proposition 3 $\frac{d\left(\frac{\overline{m_i}^X}{\overline{m}_i^D}\right)}{d\phi} > 0$ *and* $\frac{d\left(\frac{\overline{m_i}^I}{\overline{m}_i^X}\right)}{d\phi} < 0.$

Proof. *The proposition follows directly from (29) and (30).* ■

As trade is liberalized (i.e., ϕ increases), the product scope of exporters grows more relative to domestic firms. Conversely, the relative scope advantage of FDI firms over exporters diminishes. We will below present empirical evidence in favor of Proposition 3.

The effects of a higher θ, which governs how hard it is to increase the product scope are given by the following proposition

Proposition 4 $\frac{d\left(\frac{\overline{m}_i^X}{\overline{m}_i^D}\right)}{d\theta} < 0$, $\frac{d\left(\frac{\overline{m}_i^I}{\overline{m}_i^X}\right)}{d\theta} < 0$, *and* $\frac{d\left(\frac{\overline{m}_i^I}{\overline{m}_i^D}\right)}{d\theta} < 0$

Proof. *The proposition follows directly from (29), (30), and (31) noting that all ratios are* < 1. ■

A higher θ, which governs the speed at which marginal cost increases with each new product variety, reduces the product scope for all firm. The negative impact is largest for FDI firms, followed by exporters. Thus, the relative difference in product scope between different types of firms shrinks, as it becomes harder to expand product range.

3 Data and Stylized Facts

3.1 Data

Our empirical analysis employs Japanese firm-level manufacturing data from the Basic Survey of Japanese Business Structure and Activities and the Survey on Overseas Business Activity, which are annual firm-level surveys conducted by METI (Ministry of Economy, Trade, and Industry in Japan). The data set covers the years 1996-2013. The firm-level data is matched with product data from the Census of Manufacture at the 6-digit level by METI, and it contains information on around 11,000 manufacturing firms each year.[7] We use time-consistent product codes à la Pierce and Schott (2012) at the six-digit level, meaning that we have 2060 time-consistent product codes. Our firm-level data includes all manufacturing firms with more than 50 regular employees and at least 30 million Yen (approximately US$275,000) of capital assets. FDI firms are identified by Survey on Overseas Business Activity (METI). The product data refer to domestic and export sales. Due to a lack of product-level data for foreign affiliates, we assume that these patterns proxy reasonably well for the foreign product scope of FDI firms.

3.2 Stylized Facts

In 2012, firms produced on average 2.82 products, with a standard deviation of 3.12. Figure 1 plots kernel densities of product scope for domestic-only firms, exporters, and FDI firms. There is a clear hierarchy: FDI firms > exporters > domestic firms. This ordering mirrors the distribution of firm productivity (TFP). Figure 2 displays the corresponding kernel densities.

[7] The Basic Survey of Japanese Business Structure and Activities provides information on about 12,000-13,000 manufacturing firms per year. The matched sample with the Census of Manufacture contains data on around 11,000 firms.

As expected FDI firms exhibit the highest TFP, followed by exporters then domestic firms. These stylized facts align well with the model's predictions.

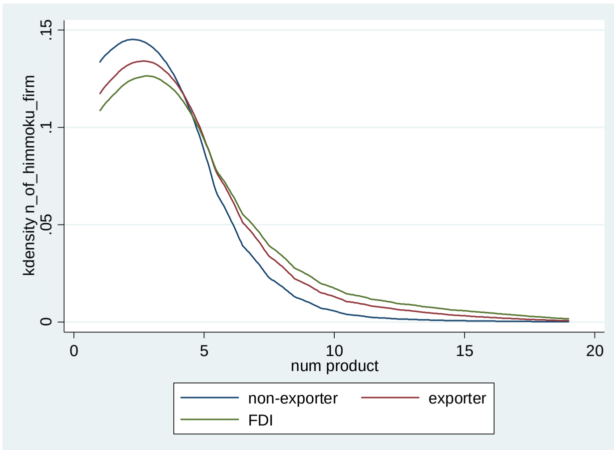

Figure 1: Distribution of product scope

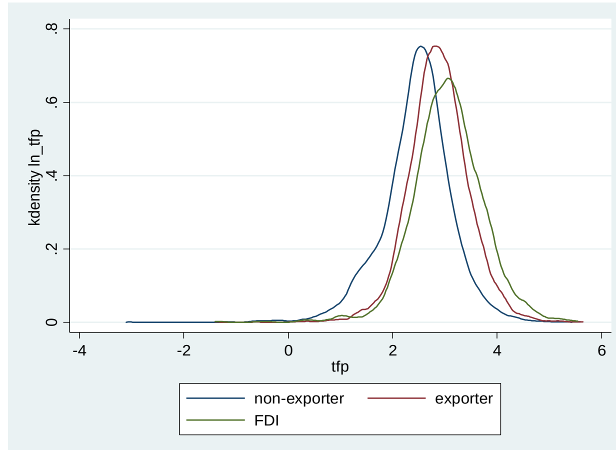

Figure 2: Productivity distributions

A key model parameter is θ, which determines how costly it is to expand product scope. From Proposition 2.1, a higher θ reduces the scope ratios: $\frac{\overline{m_i}^X}{\overline{m}_i^D}$, $\frac{\overline{m_i}^I}{\overline{m}_i^X}$, and $\frac{\overline{m_i}^I}{\overline{m}_i^D}$. To test this, we rank 3-digit manufacturing sectors by inferred θ, based on the assumption that sectors with lower average firm level product scope likely face a higher θ.[8] Table 2 in Appendix 6.2 lists average product scopes by sector. Figures 3a-c plot the corresponding product scope ratios for the ranked sectors. Consistent with the model, all curves exhibit a downward trend. However, factors such as sector-specific fixed costs introduce volatility into the patterns.

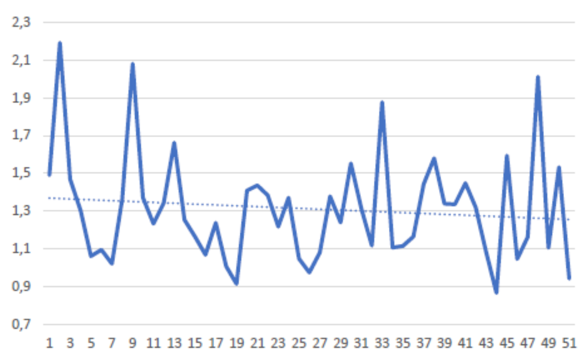

Figure 3a: The ratio of average product scope of exporters and domestic firms in 3-digit sectors

m^X/m^D

[8] Our theoretical framework can easily be extended to a multisector setting by introducing sector specific CES-indices C_{Mi} entering the Cobb-Douglas preferences : $U = C_A^{1-\mu} \prod_i C_{Mi}^{a_i}$.

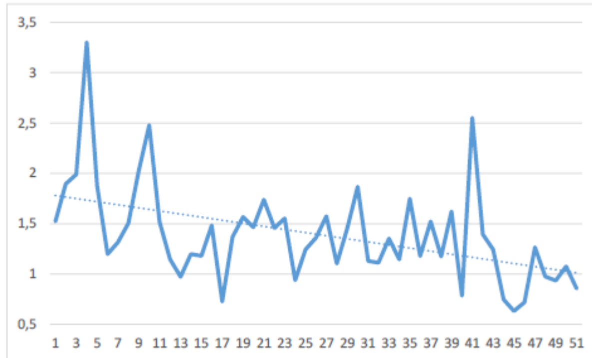

Figure 3b: The ratio of average product scope of FDI firms and exporters in 3-digit sectors m^I/m^X

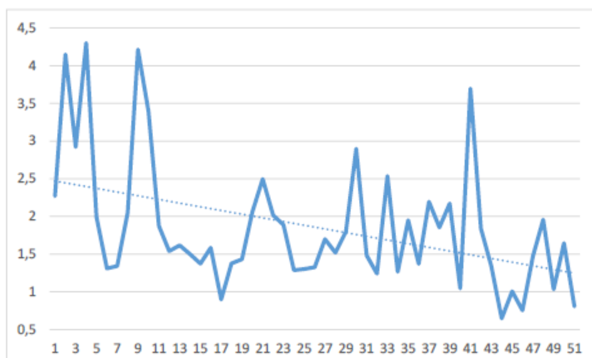

Figure 3c: The ratio of average product scope of FDI firms and domestic firms in 3-digit sectors.m^I/m^D

4 Estimations

4.1 Multi-product and trade/FDI behaviors

Motivated by Proposition 2.1, we examine the relationship between the mode of operation (non-exporting, exporting, and FDI) and the product scope. We estimate the firm-level product scope at the 6-digit product level using a negative binomial regression with panel data from 1998 to 2013:

$$m_{ijt} = \alpha + \beta_1 \log TFP_{i,t} + \beta_2 FDI_{i,t} + \beta_3 Export_{i,t} + \mu_j + \gamma_t + \epsilon_{i,j,t},$$

where $m_{i,j,t}$ is the number of products produced by firm i in sector j at time t. $Export_{i,t}$ is an exporter dummy and $FDI_{i,t}$ is a FDI dummy. μ_j is a 2-digit sector fixed effect, and γ_t is a time fixed effect. If firm i engages exporting and/or FDI in year t, the export and FDI dummies takes the value one, otherwise it is zero, respectively. TFP is calculated using the Levinsohn-Petrin method (Levinsohn and Petrin 2003)[9].

dep var: number prod	IRR	z
TFP	1.27***	42.49
	(0.0072)	
Export	1.17***	26.29
	(0.0070)	
FDI	1.60***	36.54
	(0.0204)	
Year fe	Yes	
2 digit- sector fe	Yes	
Nob	166 170	
Log pseudolikelihood	- 344114.54	
Year: 1998 to 2013		

Table 1: Estimating the product scope using a negative binomial regression. (Robust standard errors in parenthesis.)

Table 1 confirms the theoretical predictions: the coefficient for TFP is significantly larger than one and Export and FDI are both significantly larger than one. Thus, more productive firms tend to produce more products. Furthermore, exporters produce significantly more varieties than domestic-ony firms, and FDI firms significantly more than exporters. These results align with the predicted hierarchy from Proposition 2.1: $\overline{m_i}^D < \overline{m_i}^X < \overline{m_i}^I$.

5 Conclusion

This paper investigates the relationship between firms' internationalization modes - exporting and foreign direct investment (FDI) - and their product scope. We develop a model incorporating firm heterogeneity and endogenous multi-product decisions. The model predicts that the largest and most productive firms engage in FDI and produces the widest product range. Firms with intermediate productivity choose to export but produce fewer varieties relative to FDI firms. Low-productivity firms serve only the domestic market and maintain limited product ranges. Thus, our findings suggest that product scope increase in productivity and the pecking order of trade activities.

[9] To calculate TFP, we use sectorial capital book values from Hosono et al. (2017), whom we thank for providing data.

We validate these predictions using Japanese firm and product level data at the 6-digit level. The empirical evidence supports the theoretical ordering: FDI firms exhibit the greatest product scope, followed by exporters, and then domestic-only firms.

A key innovation in our model is the endogenous product scope, which is determined by how costly it is to expand the product scope (parameter θ). The model predicts that the ratio of the product scope of exporters and domestic firms and the ratio of FDI firms and exporters should decline as θ increases. Stylized facts are consistent with this feature of the model.

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6 Appendix

6.1 Derivation of the cut-off productivities for $B = B^*$

The cut-off condition for FDI firms is from (19):

$$\frac{\overline{m_I^I}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{1-\sigma}(B+B^*) - \frac{\overline{m_I^X}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{1-\sigma}(B+\phi B^*) = \overline{m_I^I} f - \overline{m}_I^X f + f_I - f_X, \tag{32}$$

and the product scope of different firms is from (14), (15) and (16) given by

$$\overline{m_i}^D = f^{-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}. \tag{33}$$

$$\overline{m}_i^X = f^{-\frac{1}{\theta(\sigma-1)}} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}. \tag{34}$$

$$\overline{m}_i^I = f^{-\frac{1}{\theta(\sigma-1)}} (B+B^*)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}. \tag{35}$$

Substituting (35) and (34) into (32) gives

$$\begin{aligned} &\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}} a_I^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{1-\sigma}(B+B^*) - \frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_I^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{1-\sigma}(B+\phi B^*) \\ = \; & f^{-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}} a_I^{-\frac{1}{\theta}} f - f^{-\frac{1}{\theta(\sigma-1)}}(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} f + f_I - f_X. \end{aligned} \tag{36}$$

$$\begin{aligned} &\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{-\frac{1}{\theta}}(B+B^*) - \frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_I^{-\frac{1}{\theta}}(B+\phi B^*) \\ = \; & f^{-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}} a_I^{-\frac{1}{\theta}} f - f^{-\frac{1}{\theta(\sigma-1)}}(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} f + f_I - f_X. \end{aligned} \tag{37}$$

$$\begin{aligned} &\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}\right)^{1-\theta(\sigma-1)} (B+B^*)^{\frac{1}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)} a_I^{-\frac{1}{\theta}} - \frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_I^{-\frac{1}{\theta}} \\ = \; & f^{1-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}} a_I^{-\frac{1}{\theta}} - f^{1-\frac{1}{\theta(\sigma-1)}}(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_I^{-\frac{1}{\theta}} + f_I - f_X. \end{aligned} \tag{38}$$

$$\left(\frac{1}{1-\theta(\sigma-1)} - 1\right)(B+B^*)^{\frac{1}{\theta(\sigma-1)}} - \left(\frac{1}{1-\theta(\sigma-1)} - 1\right)(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} = \frac{f_I - f_X}{f^{1-\frac{1}{\theta(\sigma-1)}}} a_I^{\frac{1}{\theta}} \tag{39}$$

$$(B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} = \frac{f_I - f_X}{\left(\frac{1}{1-\theta(\sigma-1)} - 1\right) f^{1-\frac{1}{\theta(\sigma-1)}}} a_I^{\frac{1}{\theta}} \tag{40}$$

Symmetric Countries

Assuming that $B = B^*$ gives

$$2^{\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} = B^{\frac{1}{\theta(\sigma-1)}} (1+\phi)^{\frac{1}{\theta(\sigma-1)}} + \frac{f_I - f_X}{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)} - 1\right)} a_I^{\frac{1}{\theta}} \tag{41}$$

$$\left(2^{\frac{1}{\theta(\sigma-1)}} - (1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right) B^{\frac{1}{\theta(\sigma-1)}} = \frac{f_I - f_X}{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)} - 1\right)} a_I^{\frac{1}{\theta}} \tag{42}$$

$$a_I^{\frac{1}{\theta}} = \frac{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)} - 1\right)\left(2^{\frac{1}{\theta(\sigma-1)}} - (1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)}{f_I - f_X} B^{\frac{1}{\theta(\sigma-1)}} \tag{43}$$

Next the cut-off equation for exporters is from (18):

$$\frac{\overline{m_X^X}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_X^{1-\sigma}(B+\phi B^*) - \frac{\overline{m_X^D}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_X^{1-\sigma} B = \overline{m_X^X} f - \overline{m_X^D} f + f_X - f_D, \tag{44}$$

Substituting $\overline{m_X^D}$ and $\overline{m_X^X}$ from (33) and (34) gives

$$\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_X^{1-\sigma}(B+\phi B^*) - \frac{\left(f^{-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_X^{1-\sigma} B$$
$$= \overline{m_X^X} f - \overline{m_X^D} f + f_X - f_D, \tag{45}$$

$$\frac{f^{-\frac{1-\theta(\sigma-1)}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)} a_X^{-\frac{1}{\theta}} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - \frac{f^{-\frac{1-\theta(\sigma-1)}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)} a_X^{-\frac{1}{\theta}} B^{\frac{1}{\theta(\sigma-1)}} = \overline{m_X^X} f - \overline{m_X^D} f + f_X - f_D, \tag{46}$$

$$\frac{f^{-\frac{1-\theta(\sigma-1)}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)} a_X^{-\frac{1}{\theta}} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - \frac{f^{-\frac{1-\theta(\sigma-1)}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)} a_X^{-\frac{1}{\theta}} B^{\frac{1}{\theta(\sigma-1)}}$$
$$= f^{-\frac{1}{\theta(\sigma-1)}} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}} f - f^{-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}} f + f_X - f_D, \tag{47}$$

$$\frac{f^{1-\frac{1}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)} a_X^{-\frac{1}{\theta}} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - \frac{f^{1-\frac{1}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)} a_X^{-\frac{1}{\theta}} B^{\frac{1}{\theta(\sigma-1)}}$$
$$= f^{1-\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - f^{1-\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}} B^{\frac{1}{\theta(\sigma-1)}} + f_X - f_D, \tag{48}$$

$$f^{1-\frac{1}{\theta(\sigma-1)}} (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}} \left(\frac{1}{1-\theta(\sigma-1)} - 1\right) + \left(1 - \frac{1}{1-\theta(\sigma-1)}\right) f^{1-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} a_X^{-\frac{1}{\theta}} = f_X - f_D, \tag{49}$$

$$(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - B^{\frac{1}{\theta(\sigma-1)}} = \frac{f_X - f_D}{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)} - 1\right)} a_X^{\frac{1}{\theta}}, \tag{50}$$

$$a_X^{\frac{1}{\theta}} = \frac{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)} - 1\right)}{f_X - f_D} \left((B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - B^{\frac{1}{\theta(\sigma-1)}}\right), \tag{51}$$

Finally the cut-off equation for domestic firms is from (17) given by

$$\frac{\overline{m_D^D}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_D^{1-\sigma} B = \overline{m_D^D} f + f_D, \tag{52}$$

Substituting $\overline{m_D^D}$ from (33) gives

$$\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}a_D^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_D^{1-\sigma}B=f^{-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}a_D^{-\frac{1}{\theta}}f+f_D, \tag{53}$$

$$\frac{1}{1-\theta(\sigma-1)}a_D^{-\frac{1}{\theta}}B^{\frac{1}{\theta(\sigma-1)}}-B^{\frac{1}{\theta(\sigma-1)}}a_D^{-\frac{1}{\theta}}=\frac{f_D}{f^{1-\frac{1}{\theta(\sigma-1)}}}, \tag{54}$$

$$\left(\frac{1}{1-\theta(\sigma-1)}-1\right)B^{\frac{1}{\theta(\sigma-1)}}\frac{f^{1-\frac{1}{\theta(\sigma-1)}}}{f_D}=a_D^{\frac{1}{\theta}}, \tag{55}$$

$$a_D^{\frac{1}{\theta}}=\left(\frac{1}{1-\theta(\sigma-1)}-1\right)B^{\frac{1}{\theta(\sigma-1)}}\frac{f^{1-\frac{1}{\theta(\sigma-1)}}}{f_D}, \tag{56}$$

So cutoff are

$$a_D^{\frac{1}{\theta}}=\left(\frac{1}{1-\theta(\sigma-1)}-1\right)B^{\frac{1}{\theta(\sigma-1)}}\frac{f^{1-\frac{1}{\theta(\sigma-1)}}}{f_D}, \tag{57}$$

$$a_X^{\frac{1}{\theta}}=\frac{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)}-1\right)}{f_X-f_D}\left((B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}-B^{\frac{1}{\theta(\sigma-1)}}\right)=\frac{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)}-1\right)}{f_X-f_D}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)B^{\frac{1}{\theta(\sigma-1)}}, \tag{58}$$

$$a_I^{\frac{1}{\theta}}=\frac{f^{1-\frac{1}{\theta(\sigma-1)}}\left(\frac{1}{1-\theta(\sigma-1)}-1\right)\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)}{f_I-f_X}B^{\frac{1}{\theta(\sigma-1)}} \tag{59}$$

Solving for free entry

The free entry condition is from () given by

$$F_E = \int_0^{a_D}\Big(\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}B-f^{-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}f-f_D)dG[a]+ \tag{60}$$

$$+\int_{a_I}^{a_X}\Big(\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}B+\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}\phi B^*-\overline{m_i}f-f_X)dG[a] \tag{61}$$

$$+\int_0^{a_I}\Big(\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}B+\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}B^*-\overline{m_i}f-f_I)dG[a], \tag{62}$$

To solve this substitute for the $m's$ and solve the integrals.

First integral:

$$\int_0^{a_D}\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}B-f^{-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}f-f_DdG[a]$$

$$=\int_0^{a_D}\frac{f^{1-\frac{1}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)}a_i^{-\frac{1}{\theta}}B^{\frac{1}{\theta(\sigma-1)}}-f^{1-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}-f_DdG[a]$$

$$\int_0^{a_D} \left(\frac{1}{1-\theta(\sigma-1)}-1\right) f^{1-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} - f_D dG[a]$$

$$\int_0^{a_D} \left(\frac{1}{1-\theta(\sigma-1)}-1\right) f^{1-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} ka^{k-1} a_i^{-\frac{1}{\theta}} - f_D ka^{k-1} da$$

$$\left(\frac{1}{1-\theta(\sigma-1)}-1\right) f^{1-\frac{1}{\theta(\sigma-1)}} B^{\frac{1}{\theta(\sigma-1)}} \frac{k}{k-\frac{1}{\theta}} a_D^{k-\frac{1}{\theta}} - n f_D a_D^k$$

Second integral

$$\int_{a_I}^{a_X} (\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} B + \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} \phi B^* - \overline{m_i} f - f_X) dG[a]$$

$$\int_{a_I}^{a_X} (\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} B + \frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} \phi B^* - \overline{m_i} f - f_X) dG[a]$$

$$\overline{m_i}^X = f^{-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}. \tag{63}$$

$$\int_{a_I}^{a_X} (\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} B + \frac{\left(f^{-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)} a_i^{1-\sigma} \phi B^*$$
$$-f^{-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} f - f_X) dG[a]$$

$$\int_{a_I}^{a_X} (\frac{f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}-1}}{1-\theta(\sigma-1)} a_i^{-\frac{1}{\theta}} B + \frac{f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}-1}}{1-\theta(\sigma-1)} a_i^{-\frac{1}{\theta}} \phi B^*$$
$$-f^{-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} f - f_X) dG[a]$$

$$\int_{a_I}^{a_X} (\frac{f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}-1}}{1-\theta(\sigma-1)} a_i^{-\frac{1}{\theta}} \left(B+\phi B^*\right)^* - f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} - f_X) dG[a]$$

$$\int_{a_I}^{a_X} (\left(\frac{1}{1-\theta(\sigma-1)}-1\right) f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} - f_X) dG[a]$$

$$\int_{a_I}^{a_X} (\left(\frac{1}{1-\theta(\sigma-1)}-1\right) f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{-\frac{1}{\theta}} ka^{k-1} - ka^{k-1} f_X) da$$

$$\int_{a_I}^{a_X} (\left(\frac{1}{1-\theta(\sigma-1)}-1\right) f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} a_i^{k-\frac{1}{\theta}-1} k - ka^{k-1} f_X) da$$

$$\left(\frac{1}{1-\theta(\sigma-1)}-1\right) f^{1-\frac{1}{\theta(\sigma-1)}} \left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} \frac{k}{k-\frac{1}{\theta}} \left(a_X^{k-\frac{1}{\theta}} - a_I^{k-\frac{1}{\theta}}\right) - \left(a_X^k - a_I^k\right) f_X$$

Third integral

$$\int_0^{a_I}(\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}B+\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}B^*-\overline{m_i}f-f_I)dG[a],$$

$$\int_0^{a_I}(\frac{\overline{m_i}^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}(B+B^*)-\overline{m_i}f-f_I)dG[a],$$

$$\int_0^{a_I}(\frac{\left(f^{-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}\right)^{1-\theta(\sigma-1)}}{1-\theta(\sigma-1)}a_i^{1-\sigma}(B+B^*)-f^{-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}f-f_I)dG[a],$$

$$\int_0^{a_I}(\frac{f^{1-\frac{1}{\theta(\sigma-1)}}}{1-\theta(\sigma-1)}a_i^{-\frac{1}{\theta}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}-f^{1-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}a_i^{-\frac{1}{\theta}}-f_I)dG[a],$$

$$\int_0^{a_I}(\left(\frac{1}{1-\theta(\sigma-1)}-1\right)f^{1-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}ka^{k-1}a_i^{-\frac{1}{\theta}}-ka^{k-1}f_I)da,$$

$$\left(\frac{1}{1-\theta(\sigma-1)}-1\right)f^{1-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}\frac{k}{k-\frac{1}{\theta}}a_I^{k-\frac{1}{\theta}}-a_I^kf_I,$$

The free entry condition becomes

$$\begin{aligned}&\left(\frac{1}{1-\theta(\sigma-1)}-1\right)f^{1-\frac{1}{\theta(\sigma-1)}}B^{\frac{1}{\theta(\sigma-1)}}\frac{k}{k-\frac{1}{\theta}}a_D^{k-\frac{1}{\theta}}\\&+\left(\frac{1}{1-\theta(\sigma-1)}-1\right)f^{1-\frac{1}{\theta(\sigma-1)}}(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\frac{k}{k-\frac{1}{\theta}}\left(a_X^{k-\frac{1}{\theta}}-a_I^{k-\frac{1}{\theta}}\right)\\&+\left(\frac{1}{1-\theta(\sigma-1)}-1\right)f^{1-\frac{1}{\theta(\sigma-1)}}(B+B^*)^{\frac{1}{\theta(\sigma-1)}}\frac{k}{k-\frac{1}{\theta}}a_I^{k-\frac{1}{\theta}}\\=&\ f_Da_D^k+\left(a_X^k-a_I^k\right)f_X+a_I^kf_I+F_E\end{aligned}$$

$$\begin{aligned}&\vartheta\frac{k}{k-\frac{1}{\theta}}\left(B^{\frac{1}{\theta(\sigma-1)}}a_D^{k-\frac{1}{\theta}}+(B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\left(a_X^{k-\frac{1}{\theta}}-a_I^{k-\frac{1}{\theta}}\right)+(B+B^*)^{\frac{1}{\theta(\sigma-1)}}a_I^{k-\frac{1}{\theta}}\right)\\=&\ f_Da_D^k+\left(a_X^k-a_I^k\right)f_X+a_I^kf_I+F_E\end{aligned} \tag{64}$$

where $\vartheta\equiv\left(\frac{1}{1-\theta(\sigma-1)}-1\right)f^{1-\frac{1}{\theta(\sigma-1)}}$.
Cut-offs are from (57), (58) and (59) given by

$$a_D^{\frac{1}{\theta}}=\vartheta B^{\frac{1}{\theta(\sigma-1)}}\frac{1}{f_D}, \tag{65}$$

$$a_D^k=\vartheta^{k\theta}B^{\frac{k\theta}{\theta(\sigma-1)}}f_D^{-k\theta}, \tag{66}$$

$$a_X^{\frac{1}{\theta}}=\frac{\vartheta}{f_X-f_D}\left((B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}-B^{\frac{1}{\theta(\sigma-1)}}\right), \tag{67}$$

$$a_X^k=\left(\frac{\vartheta}{f_X-f_D}\right)^{k\theta}\left((B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}-B^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta}, \tag{68}$$

$$a_X^{k-\frac{1}{\theta}} = \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta-1} \left((B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - B^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta-1}, \tag{69}$$

$$a_I^{\frac{1}{\theta}} = \frac{\vartheta}{f_I - f_X}\left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right) \tag{70}$$

$$a_I^k = \left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta} \left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} \tag{71}$$

$$a_I^{k-\frac{1}{\theta}} = \left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta-1} \left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta-1} \tag{72}$$

Substitute (65) into (64)

$$\begin{aligned} &\vartheta\frac{k}{k-\frac{1}{\theta}}\left(f_D a_D^k + (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_X^{k-\frac{1}{\theta}} + \left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right) a_I^{k-\frac{1}{\theta}}\right) \\ = \;& f_D a_D^k + \left(a_X^k - a_I^k\right) f_X + a_I^k f_I + F_E \end{aligned}$$

$$\begin{aligned} &\vartheta\frac{k}{k-\frac{1}{\theta}}\left(f_D a_D^k + (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} a_X^{k-\frac{1}{\theta}} + \left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right) a_I^{k-\frac{1}{\theta}}\right) \\ = \;& f_D a_D^k + a_X^k f_X + a_I^k (f_I - f_X) + F_E \end{aligned}$$

Next use solutions for the $a's$ to substitute away all $a's$ except a_D,and use the symmetry assumption that $B = B^*$

$$\begin{aligned} &\vartheta\tfrac{k}{k-\frac{1}{\theta}}\left(f_D a_D^k + (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} \left(\tfrac{\vartheta}{f_X - f_D}\right)^{k\theta-1}\left((B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - B^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta-1}\right) \\ &+\vartheta\tfrac{k}{k-\frac{1}{\theta}}\left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right)\left(\tfrac{\vartheta}{f_I - f_X}\right)^{k\theta-1}\left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta-1} \\ = \;& f_D a_D^k + a_X^k f_X + a_I^k (f_I - f_X) + F_E \end{aligned}$$

$$\begin{aligned} &\vartheta\frac{k}{k-\frac{1}{\theta}}\left(f_D a_D^k + (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta-1}\left((B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}} - B^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta-1}\right) \\ &+\vartheta\frac{k}{k-\frac{1}{\theta}}\left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta-1}\left((B+B^*)^{\frac{1}{\theta(\sigma-1)}} - (B+\phi B^*)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} \\ = \;& f_D a_D^k + a_X^k f_X + a_I^k (f_I - f_X) + F_E \end{aligned}$$

$$\begin{aligned} &\vartheta\frac{k}{k-\frac{1}{\theta}}\left(f_D a_D^k + (1+\phi)^{\frac{1}{\theta(\sigma-1)}} \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta-1} B^{\frac{k}{(\sigma-1)}}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}} - 1\right)^{k\theta-1}\right) \\ &+\vartheta\frac{k}{k-\frac{1}{\theta}}\left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta-1} B^{\frac{k}{\sigma-1}}\left(2^{\frac{1}{\theta(\sigma-1)}} - (1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} \\ = \;& f_D a_D^k + \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta} B^{\frac{k}{\sigma-1}}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}} - 1\right)^{k\theta} f_X \\ &+\left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}} - (1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} B^{\frac{k}{\sigma-1}}(f_I - f_X) + F_E \end{aligned}$$

$$
\begin{aligned}
&\vartheta \frac{k}{k-\frac{1}{\theta}}\left(f_D a_D^k + (1+\phi)^{\frac{1}{\theta(\sigma-1)}}\left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta-1} B^{\frac{k}{(\sigma-1)}}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta-1}\right)\\
&+\vartheta \frac{k}{k-\frac{1}{\theta}}\left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta-1} B^{\frac{k}{\sigma-1}}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta}\\
=\;& f_D a_D^k + \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta} B^{\frac{k}{\sigma-1}}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta} f_X\\
&+\left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} B^{\frac{k}{\sigma-1}}\left(f_I - f_X\right) + F_E
\end{aligned}
$$

To eliminate B use (65)

$$
a_D^{\frac{1}{\theta}} = \vartheta B^{\frac{1}{\theta(\sigma-1)}} \frac{1}{f_D}, \tag{73}
$$

$$
a_D^k f_D^{k\theta} \vartheta^{-k\theta} = B^{\frac{k}{(\sigma-1)}}, \tag{74}
$$

and substitute B into the free entry condition above

$$
\begin{aligned}
&\vartheta \frac{k}{k-\frac{1}{\theta}}\left(f_D a_D^k + (1+\phi)^{\frac{1}{\theta(\sigma-1)}}\left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta-1} a_D^k f_D^{k\theta}\vartheta^{-k\theta}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta-1}\right)\\
&+\vartheta \frac{k}{k-\frac{1}{\theta}}\left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta-1} a_D^k f_D^{k\theta}\vartheta^{-k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta}\\
=\;& f_D a_D^k + \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta} a_D^k f_D^{k\theta}\vartheta^{-k\theta}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta} f_X\\
&+\left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} a_D^k f_D^{k\theta}\vartheta^{-k\theta}\left(f_I - f_X\right) + F_E
\end{aligned}
$$

$$
\begin{aligned}
&\frac{k}{k-\frac{1}{\theta}}\left(\vartheta f_D a_D^k + (1+\phi)^{\frac{1}{\theta(\sigma-1)}}\left(f_X - f_D\right)^{1-k\theta} a_D^k f_D^{k\theta}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta-1}\right)\\
&+\frac{k}{k-\frac{1}{\theta}}\left(f_I - f_X\right)^{1-k\theta} a_D^k f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta}\\
=\;& f_D a_D^k + \left(f_X - f_D\right)^{-k\theta} a_D^k f_D^{k\theta}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta} f_X + \left(f_I - f_X\right)^{1-k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} a_D^k f_D^{k\theta} + F_E
\end{aligned}
$$

$$
\begin{aligned}
&\frac{k}{k-\frac{1}{\theta}}\vartheta f_D a_D^k + \frac{k}{k-\frac{1}{\theta}}(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\left(f_X - f_D\right)^{1-k\theta} a_D^k f_D^{k\theta}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta-1}\\
&+\frac{k}{k-\frac{1}{\theta}}\left(f_I - f_X\right)^{1-k\theta} a_D^k f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta}\\
=\;& f_D a_D^k + \left(f_X - f_D\right)^{-k\theta} a_D^k f_D^{k\theta}\left((1+\phi)^{\frac{1}{\theta(\sigma-1)}}-1\right)^{k\theta} f_X + \left(f_I - f_X\right)^{1-k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-(1+\phi)^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta} a_D^k f_D^{k\theta} + F_E
\end{aligned}
$$

$$
\begin{aligned}
&\frac{k}{k-\frac{1}{\theta}}\vartheta f_D a_D^k + \frac{k}{k-\frac{1}{\theta}}\Phi\left(f_X - f_D\right)^{1-k\theta} a_D^k f_D^{k\theta}\left(\Phi-1\right)^{k\theta-1} \\
&+\frac{k}{k-\frac{1}{\theta}}\left(f_I - f_X\right)^{1-k\theta} a_D^k f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta} \\
= \; & f_D a_D^k + \left(f_X - f_D\right)^{-k\theta} a_D^k f_D^{k\theta}\left(\Phi-1\right)^{k\theta} f_X + \left(f_I - f_X\right)^{1-k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta} a_D^k f_D^{k\theta} + F_E
\end{aligned}
$$

where $\Phi \equiv (1+\phi)^{\frac{1}{\theta(\sigma-1)}}$.

$$
\begin{aligned}
&\kappa\vartheta f_D a_D^k + a_D^k f_D^{k\theta}\left(\Phi-1\right)^{k\theta}\left(f_X - f_D\right)^{-k\theta}\left(\frac{k}{k-\frac{1}{\theta}}\frac{\Phi}{(\Phi-1)}(f_X - f_D) - f_X\right) \\
&+\frac{k}{k-\frac{1}{\theta}}\left(f_I - f_X\right)^{1-k\theta} a_D^k f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta} \\
= \; & f_D a_D^k + \left(f_I - f_X\right)^{1-k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta} a_D^k f_D^{k\theta} + F_E
\end{aligned}
$$

$$
\begin{aligned}
&\left(\kappa\vartheta - 1\right) f_D a_D^k + a_D^k f_D^{k\theta}\left(\Phi-1\right)^{k\theta}\left(f_X - f_D\right)^{-k\theta}\left(\kappa\frac{\Phi}{(\Phi-1)}(f_X - f_D) - f_X\right) \\
&+\left(f_I - f_X\right)^{1-k\theta} a_D^k f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta}\left(\kappa-1\right) \\
= \; & F_E
\end{aligned}
$$

where $\kappa \equiv \frac{k}{k-\frac{1}{\theta}}$. Solving for a_D gives

$$
a_D^k = \frac{F_E}{\left(\kappa\vartheta - 1\right) f_D + f_D^{k\theta}\left(\Phi-1\right)^{k\theta}\left(f_X - f_D\right)^{-k\theta}\left(\kappa\frac{\Phi}{(\Phi-1)}(f_X - f_D) - f_X\right) + \left(f_I - f_X\right)^{1-k\theta} f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta}\left(\kappa-1\right)}
$$

Next solving for a_X. From (58) we have

$$
a_X^k = \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta}\left(\left(B+\phi B^*\right)^{\frac{1}{\theta(\sigma-1)}} - B^{\frac{1}{\theta(\sigma-1)}}\right)^{k\theta}, \tag{75}
$$

rearrange and substitute (74)

$$
a_X^k = \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta} B^{\frac{k}{(\sigma-1)}}\left(\Phi-1\right)^{k\theta} = \left(\frac{\vartheta}{f_X - f_D}\right)^{k\theta} a_D^k f_D^{k\theta}\vartheta^{-k\theta}\left(\Phi-1\right)^{k\theta} = \left(\frac{1}{f_X - f_D}\right)^{k\theta} f_D^{k\theta}\left(\Phi-1\right)^{k\theta} a_D^k, \tag{76}
$$

$$
a_X^k = \frac{\left(\frac{1}{f_X - f_D}\right)^{k\theta} f_D^{k\theta}\left(\Phi-1\right)^{k\theta} F_E}{\left(\kappa\vartheta - 1\right) f_D + f_D^{k\theta}\left(\Phi-1\right)^{k\theta}\left(f_X - f_D\right)^{-k\theta}\left(\kappa\frac{\Phi}{(\Phi-1)}(f_X - f_D) - f_X\right) + \left(f_I - f_X\right)^{1-k\theta} f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta}\left(\kappa-1\right)}, \tag{77}
$$

Finally solving for a_I. Using (59) and (**??**)

$$
a_I^k = \left(\frac{\vartheta}{f_I - f_X}\right)^{k\theta} B^{\frac{k}{\sigma-1}}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta} = \left(\frac{1}{f_I - f_X}\right)^{k\theta} a_D^k f_D^{k\theta}\left(2^{\frac{1}{\theta(\sigma-1)}}-\Phi\right)^{k\theta}
$$

$$a_I^k = \left(\frac{1}{f_I - f_X}\right)^{k\theta} f_D^{k\theta} \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi\right)^{k\theta} a_D^k$$

$$a_I^k = \frac{\left(\frac{1}{f_I - f_X}\right)^{k\theta} f_D^{k\theta} \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi\right)^{k\theta} F_E}{(\kappa\vartheta - 1) f_D + f_D^{k\theta} (\Phi - 1)^{k\theta} (f_X - f_D)^{-k\theta} \left(\kappa \frac{\Phi}{(\Phi-1)} (f_X - f_D) - f_X\right) + (f_I - f_X)^{1-k\theta} f_D^{k\theta} \left(2^{\frac{1}{\theta(\sigma-1)}} - \Phi\right)^{k\theta} (\kappa - 1)} \tag{78}$$

6.2 Ranked sectors

Sector number	Sector name	Average number of products
211	Petroleum refining	8,274306
231	Tires and inner tubes	7,121429
202	Industrial organic chemicals	6,161594
261	Iron and steel	4,96466
271	Smelting and refining of non- ferrous metals	4,440217
201	Chemical fertilizers and industrial inorganic chemicals	4,33642
209	Miscellaneous chemical and allied products	3,873684
301	Industrial electric apparatus	3,866653
142	Oven fabric mills and knit fabrics mills	3,865213
319	Miscellaneous transportation equipment	3,833517
204	Oil and fat products, soaps, synthetic detergents, surface- active agents and paints	3,831665
131	Soft drinks, carbonated water, alcoholic, tea and tobacco	3,825797
293	Office, service industry and household machines	3,600994
302	Household electric appliances	3,595632
291	Metal working machinery	3,588255
322	Optical instruments and lenses	3,490286
123	Flour and grain mill products	3,490234
161	Sawing, planing mills and plywood products	3,470665
292	Special industry machinery and misc.machines and parts	3,337186
272	Non- ferrous metals worked products	3,293872
181	Pulp and paper	3,271513
141	Silk reeling plants and spinning mills, chemical fibers	3,176316
311	Motor vehicles, parts and accessories	3,054638
281	Fabricated constructional and architectural metal products, including fabricated plate work and sheet metal work	3,026196
309	Miscellaneous electrical machinery equipment and supplies	2,901566
219	Miscellaneous petroleum and coal products	2,846386
329	Miscellaneous precision instruments and machinery	2,811362
220	Plastic products, except otherwise classified	2,787515
239	Miscellaneous rubber products	2,781918
121	Livestock products	2,763124
129	Miscellaneous foods and related products	2,756675
321	Medical instruments and apparatus	2,731085
169	Miscellaneous manufacture of wood products, including bamboo and rattan	2,72428
289	Miscellaneous fabricated metal products	2,714337
259	Miscellaneous ceramic, stone and clay products	2,698925
305	Electronic parts and devices	2,669554
262	Miscellaneous iron and steel	2,663235
170	Manufacture of furniture and fixtures	2,641406
182	Paper woked products	2,600499
149	Miscellaneous textile mill products	2,580278
251	Glass and its products	2,551752
252	Cement and its products	2,490204
122	Seafood products	2,36698
192	Publishing industry	2,331646
151	Textile and knitted garments	2,197807
132	Prepared animal foods and organic fertilizers	2,160221
205	Drugs and medicines	2,058957
143	Dyed and finished textiles	2,002167
240	Manufacture of leather tanning, leather products and fur skins	1,816754
193	Printing and allied industries	1,745465
191	Newspaper industries	1,530752

Table 2: Sectors orderd acording to the average product scope