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Propagation of Export Shocks: The Great Recession in Japan*

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Abstract

This study analyzes the Japanese economy during the Great Recession period (2007-2009). The Japanese GDP declined considerably during this period, despite little exposure to the US housing market, and exports declined significantly. Motivated by this fact, we construct a multi-sector, multi-region, and small open economy model. Each region has a representative consumer, and regions and sectors are linked through input-output linkages and consumers' final demand. We measure the export shocks in each region using trade statistics. Using our model, we quantitatively evaluate how the decline in export demand propagates throughout the country. We find that export shocks account for a significant portion of the GDP decline in many regions. To inspect the mechanism, we conduct counterfactual exercises in which the change in GDP is decomposed within and across regions, as well as within and across sectors.

Keywords: Great Recession, export demand, regional input-output table, multi-sector model JEL classification: D57, E32, F41, F44, R15

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1 Introduction

The Great Recession, spanning the period of 2007-2009, started with the collapse of the US housing market. It was not only the largest post-war recession in the United States at that time, but it also had a global impact. Japan's real GDP fell by 8.8% from the first quarter of 2008 to the first quarter of 2009. This decline was substantially larger even than the decline of real GDP in the United States during the same period, which was 3.3%. The drop in Japanese exports was even more significant: during the same period, the real value of exports from Japan fell by 36.1%. Given this collapse of exports, a large fraction of which was exports to the United States, it is natural to deduce some of the decline in the Japanese GDP was caused by a reduction in export demand, which arrived in Japan as an exogenous shock.

This event is a rare natural experiment where (i) macroeconomic shocks arrive at a large economy without much anticipation; (ii) shocks ("the impulse") are identifiable at the regional and industry level; and (iii) the regional and sectoral links ("the propagation") can be traced through regional input-output tables. Thus, a detailed examination of this event provides important insight into the macroeconomic propagation mechanism of exogenous shocks. The Japanese government constructs detailed inter-regional input-output tables every five years, which is an essential element in our analysis. It is not common to have such tables—the United States, for example, does not have such comprehensive information on region-industry level linkages. This study also provides a theoretical framework that allows us to conduct counterfactual experiments to examine the propagation process in detail.

We analyze the effect of this large export decline in Japan using a multi-sector, multi-region small open economy model. Shocks to exports from a particular region and industry propagate to other regions through two channels. First, the reduction of demand in one region and industry reduces the intermediate-good demand from another region and industry. Second, the decline in production impoverishes the consumers through the reduction of wage income and profit. The reduction of consumption from another region also acts as a propagation mechanism.

We treat the export demand shock as an exogenous shock from the viewpoint of the Japanese economy. Eaton et al. (2016) estimate a multicountry general equilibrium model and argue that the trade collapse during the Great Recession is mostly caused by the shift in spending away from tradable sectors. Some studies examine the supply-side factors. For example, Amiti and Weinstein (2011) analyze how trade finance affected the decline in exports, and they find that trade finance can explain less than half of the export decline in Japan during this period.

Our study is in the tradition of real business cycle theory as in Prescott (1986). Since Frisch (1933), many macroeconomists have analyzed the business cycle through the lens of shocks and their propagation. Since the 1980s, this approach has given rise to vector autoregressions on one side and the real business cycle approach on the other. Skeptics of real business cycle theory, such as Summers (1986), have criticized the difficulty in interpreting the "shocks," in particular, the technology shocks that were dominant in the early contributions. Cochrane (1994) also emphasizes this difficulty. We consider shocks that are particularly relevant to the Japanese economy during the Great Recession: export demand shocks. The progress we make here is that we can identify the "impulse" in two dimensions (exports from particular regions and industries) and can trace out the propagation process through (i) the inter-regional input-output (IRIO) matrix and (ii) production, consumption, and labor supply decisions in the model.

Many recent studies consider the propagation and amplification of shocks through the IO net-

work. The classic studies are Long and Plosser (1983) and Horvath (2000). This literature mainly focuses on how productivity shock in one industry propagates across different industries and impacts the aggregate economy. This study makes progress over this literature in that we consider the propagation of shocks across regions.

A particularly related study is Caliendo et al. (2018). They analyze the propagation of sectoral productivity shock across regions in the US economy. The difference between this study and Caliendo et al. (2018) is threefold. First, the model characteristics are different. Their model is a closed-economy model with perfect competition based on Eaton and Kortum (2002). Our model is a small open economy model that features monopolistic competition. Second, their analysis focuses on productivity shocks, whereas we consider foreign demand shocks. Third, their regional analysis is based on the Commodity Flow Survey and is limited to the manufacturing sector. Our inter-regional input-output matrix includes all sectors.

The remainder of this paper is organized as follows. Section 2 provides an overview of the main macro facts about the Great Recession in Japan. Section 3 describes the model. Section 4 shows analytical results in a special case to present the main channels of propagation we focus on. Section 5 computes the model and calibrates it to the data. Section 6 simulates the model with the export series. Section 7 conducts counterfactual experiments and decomposes various channels of the change in regional GDP. Section 8 concludes.

2 Overview of the Great Recession in Japan

In this section, we present the general time-series pattern of various statistics from Japan during the Great Recession period. We present statistics from the country overall, across regions, and across industries. Our data mainly comes from public sources: the Ministry of Economy, Trade and Industry (METI), the Research Institute of Economy, Trade and Industry (RIETI), Ministry of Health, Labour and Welfare (MHLW), Ministry of Finance (MoF), and Statistics Bureau of Japan. METI has provided the inter-regional input-output tables (IRIO) until 2005, constructed on 9 regions and x number of industries where $x \in \{12, 29, 53\}$. We reconstruct an IRIO with 26 industries and 9 regions using the tables from 2005 so that the industry classification aligns with the Prefectural Account.¹ We use the JIP database from the RIETI to later compute the technology parameter. The Labour Force Survey (LFS) from the Statistics Bureau and the Monthly Labour Survey (MLS) conducted by the MHLW are used to calibrate parameters related to labor supply for each region.² Finally, to construct regional and sectoral export data, we use the Trade Statistics of Japan (TSJ), constructed by the MoF.

2.1 Time series of GDP and export

Figure 1 plots the time series of the quarterly real GDP (seasonally adjusted, in billions of 2015 dollars), adjusted to annual values. One can see the sharp drop in 2008. Figure 2 draws the time series of the exports of goods and services (seasonally adjusted, in billions of 2015 dollars).³ The decline in 2008 is even more extreme.

¹The Prefectural Account is the prefectural version of the GDP statistics. See, https://www.esri.cao.go.jp/ jp/sna/sonota/kenmin/kenmin_top.html (only the Japanese version is available).

²See, https://www.stat.go.jp/english/index.html.

³Both series are taken from

https://www.esri.cao.go.jp/jp/sna/data/data_list/sokuhou/files/2022/qe221_2/tables/gaku-jk2212.csv.



Figure 1: Time series of Japanese GDP



Figure 2: Time series of Japanese exports



Figure 3: Real sectoral exports: 2008Q3–2010Q4. "TE", "General", "Electric", and "Misc" refer to Transportation Equipment, General Machinery, Electric Machinery, and Miscellaneous (comprising the rest of the 23 industries).

Figure 3 plots the export time series separately across industries. All industries experienced a decline in exports during the Great Recession, and the decline was particularly sharp for the transportation equipment industry.

2.2 Regional heterogeneity

One important element of our analysis is the explicit consideration of regional linkages. Figure 4 describes how we divide Japan into nine regions: Hokkaidō, Tōhoku, Kantō, Chūbu, Kansai,⁴ Chūgoku, Shikoku, Kyūshū, and Okinawa. The division is mainly motivated by the availability of the inter-regional IO matrices. The precise mapping of prefectures into regions is listed in Appendix A. In Figure 4, a thicker color indicates a larger value of regional GDP. Kantō, including the Tōkyō area, is the largest economic region among the nine. Kansai includes the Ōsaka area, which is the second-largest economic region, and Chūbu includes Nagoya, the third-largest economic region. Chūbu also includes the headquarters of Toyota, the largest automaker and auto exporter.

Figure 5 plots the time series of the regional real GDP. All regions except for Okinawa experienced a significant decline in GDP during the Great Recession. We can also observe a large heterogeneity across regions.

Figure 6 draws the export series from 2008Q3 to 2010Q4 for each region as a fraction of 2008Q3 GDP in that region. Drawn from the TSJ, the details of the export data construction can be found in Appendix B. The time series reveals there is a large heterogeneity across regions, both in the composition of industries and the magnitude of the drop in exports during the Great Recession.

⁴Kansai corresponds to "Kinki" in the inter-regional input-output dataset in Japanese. We follow METI's English expression for the region.



Figure 4: Division of Japanese Regions



Figure 5: Regional GDP: 2002Q4-2015Q1



Figure 6: Regional exports: 2008Q3–2010Q4. "TE", "GM", "EM", and "Misc" refer to Transportation Equipment, General Machinery, Electric Machinery, and Miscellaneous (comprising the rest of the 23 industries).

3 Model

Our model is a natural extension of a small open economy real business cycle model to a multisector, multi-region setting. The economy consists of I regions. In each region, S industries operate. An industry is indexed by (s, i), where $s \in [0, S]$ and $i \in [0, I]$. In the model description, we treat s and i as real numbers, although they are integers in the quantitative exercise.

We adopt monopolistic competition as the market structure. Not only is this formulation common in international trade models, but we can also evaluate the effect of price stickiness. Each industry (s, i) is monopolistic, and only one firm produces product (s, i). The production of a good requires capital, labor, and intermediate goods. As we will detail later, we assume that capital and labor inputs are not mobile across regions; that is, industries in region *i* have to use the capital and labor supplied in region *i*. Intermediate goods are mobile; that is, a firm in region *i* can use intermediate inputs from any region.

As in the standard real business cycle model, there is one representative consumer in each region. The representative consumer maximizes the discounted sum of instantaneous utility over the infinite horizon, making consumption-saving decisions and labor supply decisions. Firms make static production decisions, hiring labor, renting capital stock, and purchasing intermediate goods. Each firm's goods are used for consumption, investment, export, and intermediate-good production.

3.1 Representative consumer

The representative consumer in region i maximizes utility

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \left[\frac{(C_{i,t})^{1-\sigma_c} - 1}{1-\sigma_c} - \chi \frac{(N_{i,t})^{1+\zeta}}{1+\zeta} \right]$$

subject to

$$P_{i,t}^{c}C_{i,t} + P_{i,t}^{x}X_{i,t} \le \int_{0}^{S} w_{si,t}n_{si,t}ds + r_{i,t}K_{i,t} + \Pi_{i,t}$$
(1)

and

$$K_{i,t+1} = (1 - \delta)K_{i,t} + X_{i,t},$$

where

$$C_{i,t} = \left[\int_0^S \int_0^I (\xi_{sjc}^i)^{\frac{1}{\sigma}} (c_{sj,t}^i)^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^{Z_t} (\xi_{zf}^i)^{\frac{1}{\sigma}} (c_{zf,t}^i)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}},$$
$$X_{i,t} = \left[\int_0^S \int_0^I (\xi_{sjx}^i)^{\frac{1}{\sigma}} (x_{sj,t}^i)^{\frac{\sigma-1}{\sigma}} dj ds \right]^{\frac{\sigma}{\sigma-1}},$$

and

$$N_{i,t} = \left[\int_0^S (n_{si,t})^{\frac{\tau+1}{\tau}} ds\right]^{\frac{\tau}{\tau+1}}.$$

Here, $C_{i,t}$ is the consumption of composite goods in period t, and $P_{i,t}^c$ is the corresponding price index. The notation c_{sj}^i represents the consumption of good s from region j by agent i and ξ_{sj}^i is a parameter. The consumption goods with c_{zf}^i are imported goods, where f represents "foreign." The imported goods have Z variety, which (is common across regions and) is endogenously determined in equilibrium.⁵ Below, we assume a symmetry for imported goods: $\xi_{zf}^i = \xi_f^i$ for all z. The variable

⁵A similar structure of the small open economy is used by Demidova and Rodríguez-Clare (2009).

 N_i is the composite labor supply. The consumer supplies labor n_{si} for the production of good s. The labor market is perfectly competitive, and the wage rate is w_{si} for industry (s, i). The variable Π_i is the profit from the firms in region i.

The variable $X_{i,t}$ is the investment of composite goods by consumer *i* in period *t*, and $P_{i,t}^x$ is the corresponding price index. The notation x_{sj}^i represents the investment of good *s* from region *j* by region *i* in period *t* and ξ_{sjx}^i is a time-invariant parameter. $K_{i,t}$ is the capital stock in region *i* and period *t*, which is augmented via the region-specific investment good $X_{i,t}$ and depreciates at rate δ every period.

The consumer's optimization implies the Euler equation and the labor supply relationship

$$\left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{-\sigma_c} = \frac{1}{1+\rho} \left(1 + \frac{r_{i,t+1}}{P_{i,t+1}^x} - \delta\right)$$
(2)

and

$$\frac{w_{si,t}}{P_{i,t}^c} = \chi(C_{i,t})^{\sigma_c} (N_{i,t})^{\zeta} \left(\frac{n_{si,t}}{N_{i,t}}\right)^{\frac{1}{\tau}}.$$
(3)

For the consumption of the goods, the consumer allocates consumption across goods by solving the expenditure-minimization problem each period t:

$$\min_{\{c^i_{sj,t}\}_{sj}} \int_0^S \int_0^I p_{sj,t} c^i_{sj,t} dj ds$$

subject to

$$C_{i,t} = \left[\int_0^S \int_0^I (\xi_{sjc}^i)^{\frac{1}{\sigma}} (c_{sj,t}^i)^{\frac{\sigma-1}{\sigma}} dj ds + \int_0^{Z_t} (\xi_{zf}^i)^{\frac{1}{\sigma}} (c_{zf,t}^i)^{\frac{\sigma}{\sigma-1}} dz_t\right]^{\frac{\sigma}{\sigma-1}}$$

Here, $p_{sj,t}$ is the price of good (s, j) in period t, which is common across regions. The prices of the imported goods are assumed to be common at p_f . The solution of the optimization implies the demand for domestic goods

$$c_{sj,t}^{i} = \left(\frac{p_{sj,t}}{P_{i,t}^{c}}\right)^{-\sigma} \xi_{sjc}^{i} C_{i,t},$$

and for imported foreign goods

$$c_{zf,t}^{i} = \left(\frac{p_{zf,t}}{P_{i,t}^{c}}\right)^{-\sigma} \xi_{zf}^{i} C_{i,t},\tag{4}$$

where the price index is written as

$$P_{i,t}^{c} \equiv \left[\int_{0}^{S} \int_{0}^{I} \xi_{sjc}^{i}(p_{si,t})^{1-\sigma} dids + \int_{0}^{Z_{t}} \xi_{zf}^{i}(p_{zf,t})^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}.$$
(5)

Similarly, the household's minimization of investment costs yields

$$x_{sj,t}^{i} = \left(\frac{p_{sj,t}}{P_{i,t}^{x}}\right)^{-\sigma} \xi_{sjx}^{i} X_{i,t}$$

where the price index is given as

$$P_{i,t}^x \equiv \left[\int_0^S \int_0^I \xi_{sjx}^i (p_{sj,t})^{1-\sigma} dj ds\right]^{\frac{1}{1-\sigma}}.$$

3.2 Production

In region i, good h is produced by the production function

$$y_{hi,t} = A_{hi} (M_{hi,t})^{\alpha} (N_{hi,t})^{\beta} (K_{hi,t})^{1-\alpha-\beta},$$

where

$$M_{hi,t} = \left[\int_0^S \int_0^I (\gamma_{sj}^{hi})^{\frac{1}{\sigma}} (m_{sj,t}^{hi})^{\frac{\sigma-1}{\sigma}} dj ds\right]^{\frac{\sigma}{\sigma-1}}.$$

Here, $m_{sj,t}^{hi}$ is intermediate good s from region j used in production of good h in region i and period t, and γ_{sj}^{hi} is a parameter.

The demand function for intermediate goods is

$$m_{sj,t}^{hi} = \left(\frac{p_{sj,t}}{P_{hi,t}^m}\right)^{-\sigma} \gamma_{sj}^{hi} M_{hi,t},$$

where

$$P_{hi,t}^{m} \equiv \left[\int_{0}^{S} \int_{0}^{I} \gamma_{sj}^{hi} (p_{sj,t})^{1-\sigma} dj ds\right]^{\frac{1}{1-\sigma}}.$$
(6)

Thus, the total demand for good (s, j) in period t is, by adding the consumption demand, investment demand, and intermediate good demand,

$$y_{sj,t} = \int_0^I (c_{sj,t}^i + x_{sj,t}^i) di + \int_0^S \int_0^I m_{sj,t}^{hi} didh + y_{sj,t}^f$$

= $(p_{sj,t})^{-\sigma} \left(\int_0^I ((P_{i,t}^c)^{\sigma} \xi_{sjc}^i C_{i,t} + (P_{i,t}^x)^{\sigma} \xi_{sjx}^i X_{i,t}) di + \int_0^S \int_0^I (P_{hi,t}^m)^{\sigma} \gamma_{sj}^{hi} M_{hi,t} didh \right) + y_{sj,t}^f,$

where $y_{sj,t}^{f}$ represents the foreign (export) demand in period t. Assume that foreign demand takes the form

$$y_{sj,t}^f = \omega_{sj,t}^f (p_{sj,t})^{-\sigma} (\bar{P}_t)^{\sigma}, \tag{7}$$

that is, for eign demand has the same price elasticity as domestic demand. \bar{P} is the price level in the foreign country.

Let

$$D_{sj,t} \equiv \left(\int_0^I \left((P_{i,t}^c)^{\sigma} \xi_{sjc}^i C_{i,t} + (P_{i,t}^x)^{\sigma} \xi_{sjx}^i X_{i,t} \right) di + \int_0^S \int_0^I (P_{hi,t}^m)^{\sigma} \gamma_{sj}^{hi} M_{hi,t} di dh \right) + y_{sj,t}^f$$
(8)

so that the demand of good (s, j) can be expressed as $(p_{sj,t})^{-\sigma}D_{sj,t}$. We analyze the firm's problem in two steps. First, the firm chooses the combination of inputs to minimize the unit cost:

$$\min_{M_{sj,t}, N_{sj,t}, K_{sj,t}} P_{sj,t}^m M_{sj,t} + w_{sj,t} N_{sj,t} + (r_{j,t} + \delta) K_{sj,t}$$

subject to

$$1 = A_{sj}(M_{sj,t})^{\alpha}(N_{sj,t})^{\beta}(K_{sj,t})^{1-\alpha-\beta}.$$

The solution yields the unit cost $\lambda_{sj,t}$:

$$\lambda_{sj,t} = \frac{(P_{sj,t}^m)^{\alpha} (w_{sj,t})^{\beta} (r_{j,t} + \delta)^{1-\alpha-\beta}}{A_{sj} \alpha^{\alpha} \beta^{\beta} (1-\alpha-\beta)^{1-\alpha-\beta}}$$
(9)

and the derived factor demand for unit output:

$$M_{sj,t} = \frac{\alpha}{P_{sj,t}^m} \lambda_{sj,t},$$
$$N_{sj,t} = \frac{\beta}{w_{sj,t}} \lambda_{sj,t},$$
$$K_{sj,t} = \frac{1 - \alpha - \beta}{r_{j,t} + \delta} \lambda_{sj,t}.$$

Second, the firm maximizes the profit:

$$\max_{p_{sj,t}} (p_{sj,t} - \lambda_{sj,t}) (p_{sj,t})^{-\sigma} D_{sj,t}$$

The result is the standard constant markup rule:

$$p_{sj,t} = \frac{\sigma}{\sigma - 1} \lambda_{sj,t}.$$
 (10)

Thus the production of good (s, j) is

$$y_{sj,t} = \left(\frac{\sigma}{\sigma - 1}\lambda_{sj,t}\right)^{-\sigma} D_{sj,t}.$$
(11)

The derived factor demand can, therefore, be computed from:

$$M_{sj,t} = \frac{\alpha}{P_{sj,t}^m} \lambda_{sj,t} y_{sj,t},\tag{12}$$

$$N_{sj,t} = \frac{\beta}{w_{sj,t}} \lambda_{sj,t} y_{sj,t},\tag{13}$$

$$K_{sj,t} = \frac{1 - \alpha - \beta}{r_{j,t} + \delta} \lambda_{sj,t} y_{sj,t}.$$
(14)

3.3 Trade balance

We assume the trade balance equation holds at the national level every period:

$$\int_{0}^{Z_{t}} \int_{0}^{I} p_{zf,t} c_{zf,t}^{i} didz = \int_{0}^{S} \int_{0}^{I} p_{si,t} y_{si,t}^{f} dids.$$

The right-hand side is the value of exports. $p_{si,t}$ is the price of good s from region i and $y_{si,t}^{f}$ is the export of good s from region i. The left-hand side is the value of imports. p_{zf} is the price of import good z, which is exogenous from the small open economy assumption. $c_{zf,t}^{i}$ is the consumption of import good z by region i. Using the homogeneity property: $p_{fz} = p_f$ and $c_{zf,t}^{i} = c_{f,t}^{i}$ for all z. Thus

$$Z_t \int_0^I p_{f,t} c_{f,t}^i di = \int_0^S \int_0^I p_{si,t} y_{si,t}^f dids.$$

This equation pins down the variety of imported goods:

$$Z_{t} = \frac{\int_{0}^{S} \int_{0}^{I} p_{si,t} y_{si,t}^{f} dids}{\int_{0}^{I} p_{f,t} c_{f,t}^{i} di}.$$
(15)

3.4 Equilibrium

Market clearing conditions for labor and capital require:

$$N_{si,t} = n_{si,t},$$
$$\int_{0}^{S} K_{si,t} ds = K_{i,t}.$$

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for all (s, i) and i. The total profit income is

$$\Pi_{i,t} = \int_0^S (p_{si,t} - \lambda_{si,t}) y_{si,t} ds.$$
(16)

4 Analysis of a tractable symmetric case

In this subsection, we present an analysis of a special case of the model to obtain an intuition on the propagation mechanism. There are two levels of propagation. First, a foreign demand shock on a sector-regional good propagates to other sectors and regions through the intermediate and final goods transactions. Second, the shock affects even non-tradable goods sectors that mostly serve households in the regions that are not directly affected by the foreign demand shock. The first propagation highlights the amplification through intermediate good sectors, whereas the second propagation shows a multiplier effect through the income of the households.

Sectoral-regional propagation

We use the static version of the model posited in Appendix C. We focus on the case of symmetric firms where goods from all sector-regions enter the CES aggregators symmetrically: $\xi_{sj}^i = \xi_{zf}^i = \gamma_{sj}^{hi} = 1, \forall h, i, j, s$. In addition, regions are assumed symmetric in productivity: $A^{hi} = A, \forall h, i$.

To simplify the expression, let us set S = I = 1 and $A = (\alpha^{\alpha}(1-\alpha)^{1-\alpha})^{-1}$. In this symmetric environment, the consumer's demand for domestic and foreign goods can be written as

$$c_{sj}^i = c = \left(\frac{p}{P}\right)^{-\sigma} C$$

and

$$c_{zf}^i = c_f = \left(\frac{p_f}{P}\right)^{-\sigma} C,$$

where the price index is

$$P = (SIp^{1-\sigma} + Zp_f^{1-\sigma})^{1/(1-\sigma)} = (p^{1-\sigma} + Zp_f^{1-\sigma})^{1/(1-\sigma)}$$

and the aggregate consumption is

$$C = (SIc^{(\sigma-1)/\sigma} + Zc_f^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)} = (c^{(\sigma-1)/\sigma} + Zc_f^{(\sigma-1)/\sigma})^{\sigma/(\sigma-1)}.$$

The intermediate-good demand is

$$m = \left(\frac{p}{P_m}\right)^{-\sigma} M,$$

where

$$P_m = (SI)^{1/(1-\sigma)} p = p.$$
(17)

Thus,

$$M = m$$

in equilibrium. We assume the foreign demand as $y^f = p^{-\sigma} \omega^f$, where ω^f is the foreign demand parameter. The total demand for a good is

$$y = p^{-\sigma}(IP^{\sigma}C + SIP_m^{\sigma}M + \omega^f) = p^{-\sigma}(P^{\sigma}C + P_m^{\sigma}M + \omega^f).$$

The optimal pricing of the monopolist follows the markup rule:

$$p = \frac{\sigma}{\sigma - 1}\lambda,$$

where λ is the unit cost

$$\lambda = P_m^{\alpha} w^{1-\alpha}.$$

Note that (17) implies

$$p = \left(\frac{\sigma}{\sigma - 1}\right)^{\frac{1}{1 - \alpha}} w$$

and thus w/p is constant. Because the (perfectly competitive) final-good sector's production function is $y = AM^{\alpha}N^{1-\alpha}$, the factor demand of the final-good firms are

$$M = \frac{\alpha \lambda y}{P_m}$$

and

$$N = \frac{(1-\alpha)\lambda y}{w}.$$

Hence, $M/N = w/P_m = w/p$, which is constant. Thus, one property of this model is that M and N move together. It also follows that M and N are proportional to y.

The labor supply function is, from the household's optimization,

$$\frac{w}{P} = \chi C^{\sigma_c} N^{\zeta}.$$

Finally, the trade balance is imposed:

$$ZIp_f c_f = SIpy^f,$$

implying

$$Zp_f c_f = py^f.$$

For completeness, the household budget is $PC = E = SwN + \Pi$, where the profit income is $\Pi = S(p - \lambda)y$. Thus, we obtain $PC = S((1 - \alpha)\lambda y + (p - \lambda)y) = S(p - \alpha\lambda)y$. We obtain the following result.

Proposition 1 An equilibrium exists uniquely for each $\omega^f > 0$, and the equilibrium p decreases continuously to 1 when ω^f decreases to 0. Equilibrium consumption C, real wages w, and the price p of domestically produced intermediate goods relative to final goods are increasing functions of ω^f . Hours worked N is decreasing (increasing) in ω^f if $\sigma_c > (<)1$, whereas N is independent of ω^f if $\sigma_c = 1$. *Proof*: See Appendix D.

In Appendix D, we show that equilibrium (p, y) is determined by

$$\omega^{f} = (1 - \alpha + \alpha/\sigma)(p^{\sigma - 1} - 1)py,$$
$$p^{1 - \sigma_{c}} = y^{\zeta + \sigma_{c}}\chi(1 - \alpha)^{\zeta}(1 - \alpha + \alpha/\sigma)^{\sigma_{c}} \left(\frac{\sigma}{\sigma - 1}\right)^{\frac{1 + \alpha\zeta}{1 - \alpha}}$$

The first equation shows that an exogenous increase in ω^f pushes up production revenue py with a multiplier effect $1/(1 - \alpha + \alpha/\sigma)$ which derives from intermediate demand for the good. However, the increase of demand by ω^f also raises intermediate price p and dampens the multiplier py/ω^f in the equation. This secondary effect however does not overturn the primary effect. The first equation can be drawn as a downward-sloping curve on the (y, p) plane. The second equation is downward-sloping if $\sigma_c > 1$ and upward-sloping if $\sigma_c < 1$. Only the first equation shifts with ω^f , and we can see the result on y (therefore N) in Proposition 1 comes from how this second equation is sloped.

The mechanism is reminiscent of the demand externality effect in a monopolistically competitive economy with international trade as in Matsuyama (1992), although our model lacks the increasing returns to scale feature. An increase in foreign demand increases the price of domestically produced intermediate goods and wages. If $\sigma_c = 1$, the equilibrium employment does not change because the income and substitution effects cancel out, but the income is increased, leading to greater consumption. The total value added $(1 - \alpha + \alpha/\sigma)py$ is spent on domestic goods $p^{1-\sigma}C$ and import $Zp_f^{1-\sigma}C$. *C* increases proportionally to revenue py and less than proportionally to ω^f . Hence, the variety of import *Z* expands, and the import share of household consumption increases, reflecting the relative decline of import price p^f to p.

Given this symmetry case as a benchmark, we will explore the heterogeneous responses of sector-regions when the input-output structure varies across regions.

Non-traded goods

A particular dimension of sectoral heterogeneity is the range of the sector's customers. Our model allows some goods to be consumed only within the region where they are produced. A typical non-traded good is a service that caters only to the customers in the region. These non-traded goods may amplify the original export shock. We illustrate this point using a slightly modified model.

We introduce a type of final consumption good C_i^o . The preference weight for C_i^o is zero outside region *i*. Thus, C_i^o is not traded across regions in equilibrium. All the other final goods are bundled into a single consumption good C_i , which is produced by aggregating intermediate inputs purchased from all regions as we specify below. We write the utility function as $U(C_i^o, C_i, N_i) = \frac{((C_i^o)^{1-\alpha}(C_i)^{\alpha})^{1-\sigma_c}-1}{1-\sigma_c} - \chi \frac{N_i^{1+\zeta}}{1+\zeta}$. The production function for the non-traded good is assumed to be linear in labor, $C_i^o = n_i^o$.

The production function for the non-traded good is assumed to be linear in labor, $C_i^o = n_i^o$. The non-traded sector is assumed to be competitive, so the price of C_i^o is equal to the wage in region *i*: $P_o^i = w_i$.

The traded final good C_i is produced competitively using a variety of intermediate goods:

$$C_i = \left(\int_0^I (m_j^i)^{(\sigma-1)/\sigma} dj\right)^{\sigma/(\sigma-1)}$$

The price of C_i is competitively set as $P^i = \left(\int_0^I p_j^{1-\sigma} dj\right)^{1/(1-\sigma)}$ where p_j is the price of intermediate good j.

Here, we assume that intermediate good *i* is produced only by a monopolist residing in region *i*. The production function for the intermediate good is $y_i = A_i n_i$. Final good producer *i*'s intermediate demand for *j* is written as $m_j^i = (p_j/P^i)^{-\sigma}C_i$. Then, the monopolist charges price $p_i = (w_i/A_i)(\sigma/(\sigma-1))$. The labor market clearing condition is $N_i = n_i + n_i^o$.

The representative household in region *i* has a budget constraint $P^iC_i = Y_i$, where the income (including profits) is $Y^i := p_i y_i + w_i n_i^o = w_i ((\sigma/(\sigma-1))n_i + n_i^o)$. The optimal expenditure of households is $P^iC_i = \alpha Y^i$ and $P_o^iC_i^o = (1-\alpha)Y^i$. Since $n_i^o = C_i^o = (1-\alpha)Y^i/w_i = (1-\alpha)((\sigma/(\sigma-1))n_i + n_i^o)$, we have $\alpha n_i^o = (1-\alpha)(\sigma/(\sigma-1))n_i$. Thus, the labor allocated to tradable sector n_i/N_i is constant, which is denoted by $\tilde{\alpha} := n_i/N_i$.

The intermediate market clears as $y_i = \int_0^I m_i^j dj$. This implies $A_i n_i = \int_0^I (p_i/P^j)^{-\sigma} C_j dj$. Let $D := \int_0^I (P^j)^{-\sigma} C_j$ denote the demand shifter for intermediate goods. Then $A_i n_i = p_i^{-\sigma} D$. This is rewritten as $D = A_i \tilde{\alpha} N_i (p_i)^{\sigma} = A_i \tilde{\alpha} N_i ((w_i/A_i)(\sigma/(\sigma-1)))^{\sigma}$.

This indicates that, an exogenous increase in D, caused by the increase in consumption in other regions, increases equilibrium w_i and Y^i in region i. We note that this propagation effect to region i is strengthened if this region has a small $\tilde{\alpha}$, that is, a large weight on non-traded goods. Moreover, this income effect is strengthened if labor supply function N_i is elastic to real wages w_i as we argued in the previous analysis.

5 Computation and calibration

We quantify the model based on the Japanese economy. First, we briefly outline the computational method. Then, we describe how we calibrate the baseline economy.

5.1 Computation

We first compute the model's steady state. We assume the initial steady state is the Japanese economy in 2008Q3 and compute the model steady state with constant parameter values. Then, we compute the perfect-foresight transition dynamics, treating the change in export demand parameter $\omega_{s_{i,t}}^{f}$ as the "MIT shock" (unanticipated shock) targeting the observed export volume.

Steady state: The steady state of the model economy is computed by the following steps.

- 1. Normalize the foreign price level $\bar{P} = 1$. Normalize the import price $p_f = 1$.
- 2. Guess w_{si} for all (s, i), p_{si} for all (s, i), r_i for all i, and Z. Then we can compute price indices P_i^c and P_{si}^m from (5) and (6). The unit cost λ_{si} can be computed from (9) and then p_{si} can be checked by markup formula (10). Thus, for a given (w_{si}, Z, r_t) , we can obtain a p_{si} that is consistent with this (w_{si}, Z, r_i) from this routine.
- 3. Using the computed prices p_{si} , we define the price indices for investment goods P_i^x for each *i*. Given the guessed nominal interest rates r_i , we check if the implied real interest rate r_i/P_i^x is equal to $\rho + \delta$ for each *i* and, if not, update r_i and return to step 2.
- 4. Further guess D_{si} for all (s, i). Then y_{sj} , M_{sj} , N_{sj} , K_i can be computed by (11), (12), (13), (14). Π_i can be computed from (34). Then the expenditure $E_i = \sum_{s=1}^{S} w_{si}n_{si} + \Pi_i$ can be

computed. In the steady state, $X_i = \delta K_i$ has to hold. Then, budget constraint (1) can be used to compute C_i , and the information on M_{sj} and C_i can be used in (8) to check whether the initial guess on D_{si} was correct.

5. Finally, we check (w_{si}, Z) . w_{si} can be checked using (3). To compute Z in (15), note that we already know p_{si} from step 2 and y_{si}^{f} from (7). c_{f}^{i} is given by (4), which is computed using P_{i}^{c} from step 2 and C_{i} from step 4.

Transition dynamics: Consider the time path of new export parameter values from t = 1 (i.e., 2008Q4 in our exercise) onward, and the economy reaches the final steady state at t = T. The final period T is set at 2010Q4.

- 1. Compute the final steady state at period T.
- 2. Guess sequences of nominal interest rates for each region, $\{\tilde{r}_{i,t}\}_{i,t=1,\dots,T-1}$.
- 3. Given $\{\tilde{r}_{i,t}\}_{i,t=1,\dots,T-1}$ and $\{K_{i,T}\}_i$, implement the following subroutine for each t, backward from t = T - 1. The algorithm of this subroutine is the same as that used to solve the steady state except for two points: (1) we do not update the guess on nominal interest rates in this subroutine, and (2) the investment $X_{i,t}$ is given by $K_{i,t+1} - (1 - \delta)K_{i,t}$, not $\delta K_{i,t}$.
 - Normalize the foreign price level $\overline{P} = 1$. Normalize the import price $p_f = 1$.
 - Guess w_{si} for all (s,i), p_{si} for all (s,i), and Z. Then we can compute price indices P_i^c and P_{si}^m from (5) and (6). The unit cost λ_{si} can be computed from (9) and then p_{si} can be checked by markup formula (10). Thus, for a given (w_{si}, Z, r_t) , we can obtain a p_{si} that is consistent with this (w_{si}, Z, r_i) from this routine.
 - Further guess D_{si} for all (s, i). Then y_{sj} , M_{sj} , N_{sj} , K_i can be computed by (11), (12), (13), (14). Π_i can be computed from (16). Then $E_i = \sum_{s=1}^{S} w_{si} n_{si} + \Pi_i$ can be computed. $X_{i,t}$ is given by $K_{i,t+1} (1-\delta)K_{i,t}$. Then, budget constraint (1) can be used to compute C_i , and the information on M_{sj} and C_i can be used in (8) to check whether the initial guess on D_{si} was correct.
 - Finally, we check (w_{si}, Z) . w_{si} can be checked using (3). To compute Z in (15), note that we already know p_{si} from step 2 and y_{si}^{f} from (7). c_{f}^{i} is given by (4), which is computed using P_{i}^{c} from step 2 and C_{i} from step 4.
- 4. Check if the implied allocations satisfy the following conditions for each t. If not, update the guesses on $r_{i,t}$ and return to step 3:
 - t = 2, ..., T 1: Check if the Euler equation (2) are satisfied.
 - t = 1: Check if capital markets clear.
 - The capital supply is fixed to its initial steady state level for each region.
 - The demand is determined by producers' optimal conditions given the nominal interest rates.

5.2 Calibration

We start from the baseline economy in 2008Q3, that is, just before the export shock hits. Our aim here is to set the parameter values so that the equilibrium outcome resembles the data statistics in the Japanese economy at that point. Later, we run experiments to analyze the evolution of the economy afterward.

The consumption share parameters $\{\xi_{sjc}^i\}_{i,sj}$ are calibrated so that the consumption expenditure share of good (s, j), which represents good s produced in region j, by the region i consumer matches the data in the baseline economy. The consumption shares are taken from the inter-regional inputoutput in 2005 (IRIO2005), which is the closest time period before 2008Q3. Similarly, we set the target for $\{\xi_i^f\}_i$ as the GDP share of export goods produced in region i. Parameters governing the demand for the (s, j) production by foreign countries, $\{\omega_{sj}^f\}_{sj}$, are set so that the GDP share of export goods (s, j) matches the data computed in IRIO2005.⁶

In the baseline economy, we assume that the parameter governing the wage elasticity of labor supply choice in each industry, τ , is equal to 1, as in Horvath (2000). The inverse of Frisch elasticity of overall labor supply, ζ , is set to 2.5 based on Kuroda and Yamamoto (2008).⁷ The labor disutility parameter χ_i is calibrated to replicate the regional variation of the employed population in 2008Q3 taken from the LFS, conducted by the MHLW.⁸ Note that the variation in the employed population reflects those in the labor force (or working-age population) and employment rate.⁹ The time discount rate ρ is set to 0.01. As a benchmark, we consider the case of $\sigma_c \rightarrow 1$.

The investment share parameters $\{\xi_{sjx}^i\}_{i,sj}$ are calibrated so that the investment expenditure share of each good sj in region i in the benchmark economy matches the data taken from IRIO2005. The parameter governing the elasticity of substitution, σ , is set to 5.0 in the baseline economy. The parameters governing the cost share of each intermediate good (s, j) for the producer of good hin region i, $\{\gamma_{sj}^{hi}\}_{hi,sj}$, are set so that those in the benchmark matches the data counterparts in IRIO2005. The factor neutral productivity for each industry (s, j), A_{sj} , is given by the product of the industry- and region-specific productivity parameters; that is, $A_{sj} = A_s \times A_j$, where A_s stands for the industry-specific productivity while A_j stands for the region-specific productivity. First, we map the industry classification in the JIP Database to ours and compute the industrial TFP and cost share of intermediate goods for each industry $s(\{\alpha_s\}_s)$. Given all other parameters, the region-specific productivity A_s is pinned down so that the regional variation of the average wage rate in the benchmark replicates the data counterpart computed using the MLS.

Table 1 summarizes the parameter values. The values of A_j and $\{\alpha_s, \beta_s\}_s$ are summarized in Table 6 in Appendix E. The regional parameters A_i , χ_i , and ξ_f^i are summarized in Table 7 in Appendix E. The parameters $\{\xi_{sjc}^i\}_{i,sj}$ $\{\xi_{sjx}^i\}_{i,sj}$ $\{\gamma_{sj}^{hi}\}_{hi,sj}$, and $\{\omega_{sj}^f\}_{sj}$ are too numerous to be summarized in a table and are represented as heatmaps in Figure 28 in Appendix E.

⁶As a result, the export-to-GDP ratio and the share of good (s, j) in the total export match the data.

⁷Kuroda and Yamamoto (2008) estimate the Frisch elasticity in Japan and report that the elasticity on the extensive and intensive margins combined ranges between 0.2 to 0.7 for males. $\zeta = 2.5$ implies the Frisch elasticity of 0.4.

⁸See, https://www.stat.go.jp/english/index.html.

⁹Although it is better to incorporate the variation of working hours per labor force, there are no reliable data disaggregating working hours into each region.

Parameter	Description	Value	Target/Source
Droforon co			
1 Telefence		0.01	A 1
ho	time discount rate	0.01	Assumed
σ_c	curvature	1.0	Assumed
χ_i	disutility of labor supply	Table 7	LFS (2008)
ζ	inverse of Frisch elasticity	2.5	Kuroda and Yamamoto (2008)
au	elasticity of substitution (labor)	1.0	Benchmark in Horvath (2000)
$\{\xi^i_{sjc}\}_{i,sj}$	weight on consumption goods	Figure <mark>28a</mark>	IRIO (2005)
$\{\xi_f^i\}_i$	weight on import goods	Table 7	IRIO (2005)
$\{\omega^f_{sj}\}_{sj}$	weight on export goods	Figure 28d	IRIO (2005)
Technology			
$\{A_{sj}\}_{sj}$	factor neutral productivity	Tables 6 and 7	JIP (2005), MLS (2008)
$\{\alpha_s\}_s$	cost share of intermediate goods	Table 6	JIP (2005)
$\{\beta_s\}_s$	labor share	Table 6	JIP (2005)
σ	elasticity of substitution	5.0	Assumed
$\{\gamma_{sj}^{hi}\}_{hi,sj}$	weight on intermediate goods	Figure 28c	IRIO (2005)
$\{\xi_{six}^i\}_{i,sj}$	weight on investment goods	Figure 28b	IRIO (2005)
δ	capital depreciation rate	0.015	Assumed

Table 1: Summary of the parameter values, their source/reference, and data for setting targets.

6 Simulating the model with the Great Recession export shocks

The primary purpose of building our quantitative model is to analyze the propagation of the export shocks during the Great Recession. Below, we set the time series of $y_{sj,t}^{f}$ so that the time path of the export value replicates the regional export data.

We simulate the export shocks to industry si (i.e., industry s in region i) in period t by changing $y_{si,t}^{f}$ for the following to hold in equilibrium:

$$\frac{y_{si,t}^f}{y_{si,t=0}^f} = \frac{\text{real export of } si \text{ in } t \text{ in data}}{\text{real export of } si \text{ in } t = 0 \text{ in data}},$$

where reference period t = 0 corresponds to the third quarter of 2008 (2008Q3).

A variable of our primary interest is domestic final demand, comprised of consumption and investment. The real consumption is computed excluding the imported goods consumption. Formally, the real consumption for region i in period t, $\bar{C}_{i,t}$, is defined as

$$\bar{C}_{i,t} = \int_0^S \int_0^I p_{sj,t=0} c^i_{sj,t} dj ds,$$

where $p_{sj,t=0}$ is the goods price produced in industry sj in reference period t = 0. Similarly, the real investment for region i in period t, $\bar{X}_{i,t}$, is defined as

$$\bar{X}_{i,t} = \int_0^S \int_0^I p_{sj,t=0} x_{sj,t}^i dj ds,$$



Figure 7: National responses. "Data" plots the fluctuations of HP-filtered variables, normalizing the 2008Q3's value as 1. Consumption data includes only private consumption and excludes government consumption. Similar construction applies to investment and domestic demand.

The national (real) consumption and investment in period t, $\bar{C}_{Japan,t}$ and $\bar{X}_{Japan,t}$, are then defined as

$$\bar{C}_{Japan,t} = \int_0^I \bar{C}_{i,t} di$$

and

$$\bar{X}_{Japan,t} = \int_0^I \bar{X}_{i,t} di.$$

6.1 National level response

Figure 7 draws the time series of domestic final demand, consumption, investment, and export at the national level. Consumption data includes only private consumption and excludes government consumption. Similar construction applies to investment and domestic demand. By construction, the export values from the model exactly match the data. The model accounts for a sizable fraction of the decline in investment and domestic demand each period.

In Panel (a), we draw three different lines: data, the static model, and the dynamic model. The current model is drawn as the dynamic model. The static version of the model, where there



Figure 8: Regional responses

are no investment and capital stock, is described in detail in Appendix C. To the extent that the decline in investment contributes to the decline in domestic demand in the data, the dynamic model better explains the decline in domestic demand than the static model. The model implies a more significant decline in consumption than observed in the data. One reason is that we do not consider the government and thus do not capture an increased government spending observed in the data during the periods.¹⁰ The increased government spending should have increased household disposable income and mitigated the consumption decline to some extent. Thus, abstracting the government spending in the model would lead to a more significant decline in (private) consumption than observed in the data.

6.2 Responses at regional level

Figure 8 compares the regional demand for domestic final goods between the model and data. The model explains the data particularly well for regions such as Chūbu, Kantō, and Kansai, which experienced a large decline in exports.

In contrast, for regions such as Hokkaidō, Shikoku, and Okinawa, where either the export is not a large component of the regional GDP or the export decline was not significant, the model performed poorly in explaining the regional GDP decline.

¹⁰For example, the sum of government consumption and investment increased by 12% in the first quarter of 2009 compared with the third quarter of 2008, corresponding to 3.3% of the domestic demand in the first quarter of 2008.

7 Counterfactual experiments

Using the quantitative model we developed in the previous section, we now explore our research question: how does an export shock propagate? In the following, we run counterfactual experiments to answer this question in different depths. In particular, we run a controlled experiment by feeding the model only the (permanent) shock on $y_{sj,t}^f$ to one region and industry sj for some t (2009Q1) and computing the new steady state, keeping $y_{hi,t}^f$ of the other regions and industries $hi(\neq sj)$ constant. Here, we consider a negative export shock to the transportation equipment industry in the Chūbu region.

7.1 Decomposition

To see how the export demand shock in a region affects other regions, we conduct a decomposition analysis. Our decomposition is based on different demand components. First, note that there are four demand factors in our dynamic model: domestic consumption demand, domestic investment demand, domestic intermediate-good demand, and foreign demand. In the following equation, the first term on the right-hand side is the domestic consumption demand, the second term is the investment demand, the third term is the domestic intermediate-good demand, and the fourth term is the foreign demand for good s produced in region j.

$$y_{sj,t} = \int_0^I (c_{sj,t}^i + x_{sj,t}^i) di + \int_0^S \int_0^I m_{sj,t}^{hi} didh + y_{sj,t}^f.$$

The domestic consumption demand is represented as

$$c_{sj,t}^{i} = \left(\frac{p_{sj,t}}{P_{i,t}^{c}}\right)^{-\sigma} \xi_{sjc}^{i} C_{i,t}.$$
(18)

The domestic investment demand is represented as

$$x_{sj,t}^{i} = \left(\frac{p_{sj,t}}{P_{i,t}^{x}}\right)^{-\sigma} \xi_{sjx}^{i} X_{i,t}.$$
(19)

The domestic intermediate-good demand from industry h in region i is

$$m_{sj,t}^{hi} = \left(\frac{p_{sj,t}}{P_{hi,t}^m}\right)^{-\sigma} \gamma_{sj}^{hi} M_{hi,t}.$$
(20)

Given this background, we compute two economies. The first is the baseline economy (2008Q3) without any shocks. The second is the economy with export shock in 2009Q1, but with only one industry and one region. Then, we keep the shock value as constant at the 2009Q1 level and compute the new steady state. Here, as we mentioned above, we choose the transportation equipment industry in the Chūbu region. The comparison of these two provides the overall changes of the (real) output in each region given this particular shock, where the real output for region i is formulated as follows:

$$\bar{Y}_{i,t} = \int_0^S p_{si,t=0} y_{si,t} ds$$



Figure 9: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

Our decomposition exercise involves two steps. In the first step, we decompose the output change in each region into several factors. More specifically, we consider the following five factors separately in decomposing the output change in region *i*. That is, we change only one of these factors in equations (18), (19), and (20). The first set of factors represents the effect of *prices*. These are $p_{sj,t}$ for all *s* and *j* in (18), (19), and (20) and price indices $P_{i,t}^c$, $P_{i,t}^x$ and $P_{hi,t}^m$ in (18), (19), and (20). Note that foreign price \bar{P} is fixed because of the small open economy assumption. The second to fifth factors are $C_{i,t}$, $X_{i,t}$, $M_{hi,t}$, and $y_{sj,t}^f$. The first step reveals through which factor a region's output is affected but is silent about through which region. The second step then decomposes the contribution of each factor into the regions from whence those effects originate.

Figure 9 plots the decomposition result for the first step. This figure provides insights into how the region-industry-specific export shock propagates. The first takeaway from the figure is that the effect on Chūbu itself is substantially larger than the other regions that are not directly hit by the shock. At the same time, the propagation to the other regions is not negligible. The second takeaway is that even though the shock was on the exports, the other demand components were also affected. For the intermediate goods, this outcome implies that the intra-regional network effect is important. For consumption, the effect on wages and profit does have an effect on consumption demand. Because the demand for goods from Chūbu drops, the price also drops for these goods, mitigating the output effect of shocks.

To focus on the inter-regional propagation, Figure 10 plots the same objects as Figure 9, but excluding the Chūbu region. The triangle dot is the overall effect. Depending on the region, the overall effect can be positive or negative. There are several takeaways. First, geography matters—the closer the region is to Chūbu, the more significant its overall decline tends to be. Second, in terms of components, the negative effects are through consumption and intermediate goods. The effect of investment goods is minimal. Third, the price effects are important. As in the case of Chūbu, the price effect (along the demand curve) mitigates the effect of demand decline. Note that, in addition to the effect of individual good prices, there are effects through price indices. In particular, the trade balance implies that there is less variety of imported goods at the national level. This overall effect implies that the price of imported goods, used only for consumption, relative to



Figure 10: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1), excluding Chūbu.

domestic goods, becomes more expensive. This effect through price indices further stimulates the domestic goods demand. In Okinawa, the combined price effect is, in fact, larger than the demand decline, and the output rises in total.

In the second step of the decomposition, we look at each component—consumption, investment, and intermediate input—separately and decompose the change into their regional contents. Figures 11, 12, and 13 indicate the contributions of each region in accounting for the column region's decline in consumption, investment, and intermediate input demand, respectively.

For investment and intermediate goods, almost all output decline (propagation) is accounted for by the direct decline of demand from Chūbu. Here, again, the IO network is important in propagation. For consumption, there are two important factors. The first is the demand from Chūbu. The second is the own-region demand. The decline of demand from Chūbu, which leads to a decline in production in another region through an inter-regional demand channel, reduces the output in that region, which leads to a decline in wage and profit income. This secondary effect reduces consumption that comes from the own region. This channel turned out to be as important as the inter-regional demand effect.

7.2 The role of price flexibility

The model above is in the tradition of the real business cycle model in that all prices are flexible. In the decomposition, we see that price flexibility does indeed play a role: the effect of price change mitigates the output decline with negative demand shocks. In this section, to further investigate how the prices affect the propagation, we make the opposite assumption: fix all prices at the level of t = 0.

Figure 14 repeats Figure 9 for fixed prices. Comparing these two figures, we can see the main difference for the Chūbu region is the absence of the price effect, which strengthens the negative effect on output. The composition of each component is almost identical.

To see across-region propagation, Figure 15 repeats Figure 10 with fixed prices. Once again, the main difference is the absence of the attenuating effect through price changes. Figures 16, 17,



Figure 11: Changes in consumption with 2009Q1's shock to the TE in Chūbu (2008Q3=1).



Figure 12: Changes in investment with 2009Q1's shock to the TE in Chūbu (2008Q3=1).



Figure 13: Changes in intermediate goods demand with 2009Q1's shock to the TE in Chūbu (2008Q3=1).



Figure 14: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).



Figure 15: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices), excluding Chūbu.



Figure 16: Changes in consumption with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).



Figure 17: Changes in investment with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).



Figure 18: Changes in intermediate goods demand with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).

and 18 plot the results corresponding to Figures 11, 12, and 13 earlier. These figures look almost identical to those for flexible prices.

8 Conclusion

This study constructs a multi-region, multi-sector model to analyze the propagation of export shocks in Japan during the Great Recession period. Our model features monopolistic competition, inter-regional IO linkage, and a representative consumer in each region.

We measure the export shocks in each region through the trade statistics. Calibrating the model to 2008Q3, we examine how the model outcome with export shocks performs compared with the data. We find that the model with flexible price can replicate close to half of the output decline and the entire consumption decline at the macro level. At the regional level, the export shock can particularly be seen to have a large impact on output in regions where export accounts for a large portion of regional GDP.

We run several counterfactual experiments to examine the propagation of shocks across regions and industries. In the first experiment, we feed the model an export shock that hits only a particular industry (transportation equipment) in a particular region (Chūbu). We find that a shock to one region and industry propagates to other regions through the consumption demand and IO linkages. The effect is particularly strong for regions that are geographically closer. The secondary effect of the own consumption decline caused by the income drop is also important. The decline in prices attenuates the negative effects. The second experiment fixes the prices, and we find that both within- and across-region output decline is significantly larger, and there are no mitigating factors.

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Appendix

A Mapping prefectures into regions

Table 2 describes the correspondence between prefectures and the regions we use in the study.

Regions	Prefectures			
Hokkaidō	Hokkaidō			
Tōhoku	Aomori, Iwate, Miyagi, Akita, Yamagata, Fukushima			
$\operatorname{Kant}\bar{\operatorname{o}}$	Ibaraki, Tochigi, Gunma, Saitama, Chiba, Tōkyō,			
	Kanagawa, Niigata, Yamanashi, Nagano, Shizuoka			
Chūbu	Toyama, Ishikawa, Gifu, Aichi, Mie			
Kansai	Fukui, Shiga, Kyōto, Ōsaka, Hyōgo, Nara, Wakayama			
Chūgoku	Tottori, Shimane, Okayama, Hiroshima, Yamaguchi			
Shikoku	Tokushima, Kagawa, Ehime, Kōchi			
Kyūshū	Fukuoka, Saga, Nagasaki, Kumamoto, Ōita, Miyazaki, Kagoshima			
Okinawa	Okinawa			

Table 2: Region classification in our model. This classification is based on the inter-regional inputoutput (2005) provided by the METI.

B Details of the data construction

We use the TSJ constructed by the MoF to construct a quarterly series of the exports for each industry sj.¹¹ The TSJ monthly reports the values of 28 goods exported at ten customs, where the ten customs can be further broken down into 166 offices. We map each office into our region classification and each good into our industry classification and aggregate the raw data to construct the quarterly export series for each region-industry.

We also construct the export series of automobiles for each region using public data.¹² First, the Japan Automobile Manufacturers Association provides data recording monthly production and export of each automobile category (e.g., standard-sized car, bus, truck, etc.) for each carmaker.¹³ Second, for most carmakers, we can check how many (and which category of) cars are produced in which establishment by checking their website or online documents. These two sets of information reveal how many (and which category of) cars are produced and exported from each region. Third, we can compute the prices of each car category using the Current Survey of Production conducted by the METI,¹⁴ which finally enables us to construct the export value series of automobiles for each region.

¹¹See, https://www.customs.go.jp/toukei/info/index_e.htm.

¹²Note that the Auto is included by the Transportation Equipment in our industry classification.

¹³See, https://www.jama.or.jp/english/.

¹⁴See, https://www.meti.go.jp/english/statistics/tyo/seidou/index.html.

C Static model

The baseline model in the main text is a dynamic model with investment. In this section, we consider a static model to contrast with the results in the main text. Even though some portions are straightforward modifications of the baseline model, we allow for some overlap with the main text for the sake of a self-contained exposition of the static model.

C.1 Model setting

The setting is identical to the baseline model, except for the absence of capital stock and investment. Consider a small open economy with I regions. In each region, there are S industries. Thus an industry is indexed by (s, i), where $s \in [0, S]$ and $i \in [0, I]$. Each industry (s, i) is monopolistically competitive; that is, only one firm produces in industry (s, i). The production of a good requires labor and intermediate goods as inputs. Product (s, i) is used for consumption, intermediate goods for production, and export. Each region i has a representative consumer who owns the firm in region i, supplies labor for the firms in region i, and consumes both domestic goods and imported goods.

C.1.1 Representative consumer

The representative consumer in region i maximizes utility

$$U^{i} = \frac{(C^{i})^{1-\sigma_{c}} - 1}{1 - \sigma_{c}} - \chi \frac{(N^{i})^{1+\zeta}}{1 + \zeta}$$

subject to

$$P^i C^i \le \int_0^S w_s^i n_s^i ds + \Pi^i = E^i, \tag{21}$$

where

$$C^{i} = \left[\int_{0}^{S} \int_{0}^{I} (\xi_{sj}^{i})^{\frac{1}{\sigma}} (c_{sj}^{i})^{\frac{\sigma-1}{\sigma}} dj ds + \int_{0}^{Z} (\xi_{zf}^{i})^{\frac{1}{\sigma}} (c_{zf}^{i})^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}$$

and

$$N^{i} = \left[\int_{0}^{S} (n_{s}^{i})^{\frac{\tau+1}{\tau}} ds\right]^{\frac{\tau}{\tau+1}}$$

The notations are the same as our baseline formulation. The variable E^i represents the expenditure of consumer i.

The consumer's optimization implies the labor supply relationship

$$\frac{w_s^i}{P^i} = \chi(C^i)^{\sigma_c} (N^i)^{\zeta} \left(\frac{n_s^i}{N^i}\right)^{\frac{1}{\tau}}.$$
(22)

For the consumption of the goods, the consumer allocates consumption across goods by solving the expenditure-minimization problem

$$\min_{c_{sj}^{i}, c_{zf}^{i}} \quad \int_{0}^{S} \int_{0}^{I} p_{sj} c_{sj}^{i} d_{j} ds + \int_{0}^{Z} p_{f} c_{zf}^{i} dz$$

subject to

$$C^{i} = \left[\int_{0}^{S} \int_{0}^{I} (\xi_{sj}^{i})^{\frac{1}{\sigma}} (c_{sj}^{i})^{\frac{\sigma-1}{\sigma}} dj ds + \int_{0}^{Z} (\xi_{f}^{i})^{\frac{1}{\sigma}} (c_{zf}^{i})^{\frac{\sigma-1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}.$$

Here, p_{sj} is the price of good (s, j), which is common across regions. The prices of the imported goods are assumed to be common at p_f . The solution of the optimization implies the demand for domestic goods

$$c_{sj}^i = \left(\frac{p_{sj}}{P^i}\right)^{-\sigma} \xi_{sj}^i C^i,$$

and for imported foreign goods

$$c_{fz}^{i} = c_{f}^{i} = \left(\frac{p_{f}}{P^{i}}\right)^{-\sigma} \xi_{f}^{i} C^{i}, \qquad (23)$$

where the price index is written as

$$P^{i} \equiv \left[\int_{0}^{S} \int_{0}^{I} \xi^{i}_{sj}(p_{sj})^{1-\sigma} dj ds + Z\xi^{i}_{f}(p_{f})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$
(24)

C.1.2 Production

In region i, good h is produced by the production function

$$y_{hi} = A^{hi} (M^{hi})^{\alpha} (N^{hi})^{1-\alpha},$$

where

$$M^{hi} = \left[\int_0^S \int_0^I (\gamma_{sj}^{hi})^{\frac{1}{\sigma}} (m_{sj}^{hi})^{\frac{\sigma-1}{\sigma}} dj ds\right]^{\frac{\sigma}{\sigma-1}}.$$

Here, m_{sj}^{hi} is intermediate good s from region j used in production of good h in region i and γ_{sj}^{hi} is a parameter.

The demand function for intermediate goods is

$$m_{sj}^{hi} = \left(\frac{p_{sj}}{P^{hi}}\right)^{-\sigma} \gamma_{sj}^{hi} M^{hi},$$

where

$$P^{hi} \equiv \left[\int_0^S \int_0^I \gamma_{sj}^{hi}(p_{sj})^{1-\sigma} dj ds\right]^{\frac{1}{1-\sigma}}.$$
(25)

Thus, the total demand for good (s, j) is, by adding the consumption demand and the intermediate good demand,

$$y_{sj} = \int_0^I c_{sj}^i di + \int_0^S \int_0^I m_{sj}^{hi} didh + y_{sj}^f$$

= $(p_{sj})^{-\sigma} \left(\int_0^I (P^i)^{\sigma} \xi_{sj}^i C^i di + \int_0^S \int_0^I (P^{hi})^{\sigma} \gamma_{sj}^{hi} M^{hi} didh \right) + y_{sj}^f,$

where y_{sj}^{f} represents the foreign (export) demand. Assume that the foreign demand takes the form

$$y_{sj}^f = \omega_{sj}^f (p_{sj})^{-\sigma} (\bar{P})^{\sigma}, \qquad (26)$$

that is, foreign demand has the same price elasticity as domestic demand. \bar{P} is the price level in the foreign country.

Let

$$D_{sj} \equiv \left(\int_0^I (P^i)^\sigma \xi^i_{sj} C^i + \int_0^S \int_0^I (P^{hi})^\sigma \gamma^{hi}_{sj} M^{hi} didh + \omega^f_{sj}(\bar{P})^\sigma\right)$$
(27)

so that the demand of good (s, j) can be expressed as $(p_{sj})^{-\sigma}D_{sj}$ We analyze the firm's problem in two steps. First, the firm chooses the combination of inputs to minimize the unit cost:

$$\min_{M^{sj},N^{sj}} P^{sj}M^{sj} + w_s^j N^{sj}$$

subject to

$$1 = A^{sj} (M^{sj})^{\alpha} (N^{sj})^{1-\alpha}$$

The solution yields the unit cost λ^{sj} :

$$\lambda^{sj} = \frac{(P^{sj})^{\alpha} (w_s^j)^{1-\alpha}}{A^{sj} \alpha^{\alpha} (1-\alpha)^{1-\alpha}}$$
(28)

and the derived factor demand for unit output:

$$M^{sj,1} = \frac{\alpha}{P^{sj}} \lambda^{sj},$$
$$N^{sj,1} = \frac{1-\alpha}{w_s^j} \lambda^{sj}$$

Second, the firm maximizes the profit:

$$\max_{p_{sj}} (p_{sj} - \lambda^{sj})(p_{sj})^{-\sigma} D_{sj}$$

The result is the standard constant markup rule:

$$p_{sj} = \frac{\sigma}{\sigma - 1} \lambda^{sj}.$$
(29)

Thus the production of good (s, j) is

$$y_{sj} = \left(\frac{\sigma}{\sigma - 1}\lambda^{sj}\right)^{-\sigma} D_{sj}.$$
(30)

The derived factor demand can, therefore, be computed from:

$$M^{sj} = \frac{\alpha}{P^{sj}} \lambda^{sj} y_{sj} \tag{31}$$

and

$$N^{sj} = \frac{1-\alpha}{w_s^j} \lambda^{sj} y_{sj}.$$
(32)

C.1.3 Trade balance

As in the baseline model, we assume the trade balance equation holds at the national level:

$$\int_{0}^{Z} \int_{0}^{I} p_{zf} c_{zf}^{i} didz = \int_{0}^{S} \int_{0}^{I} p_{si} y_{si}^{f} dids.$$

The right-hand side is the value of exports. p_{si} is the price of good s from region i and y_{si}^f is the export of good s from region i. The left-hand side is the value of imports. p_{zf} is the price of import good z, which is exogenous from the small open economy assumption. c_{zf}^i is the consumption of import good z by region i. Using the homogeneity property: $p_{zf} = p_f$ and $c_{zf}^i = c_f^i$ for all z. Thus

$$Z\int_0^I p_f c_f^i di = \int_0^S \int_0^I p_{si} y_{si}^f di ds.$$

This equation pins down the variety of imported goods:

$$Z = \frac{\int_0^S \int_0^I p_{si} y_{si}^f dids}{\int_0^I p_f c_f^i di}.$$
(33)

C.1.4 Equilibrium

The labor market equilibrium requires

$$N^{sj} = n_s^i$$

for all (s, i). The total profit income is

$$\Pi^{i} = \int_{0}^{S} (p_{si} - \lambda^{si}) y_{si} ds.$$
(34)

From these two pieces of information and the budget constraint, we can compute the equilibrium value of C^i .

C.2 Computation and calibration

C.2.1 Computation

The equilibrium of the model economy is computed with the following steps. Note that this model is static, and thus, we can compute the model period-by-period.

- 1. Normalize the foreign price level $\bar{P} = 1$. Normalize the import price $p_f = 1$.
- 2. Guess w_s^i for all (s, i) and p_{si} for all (s, i). Guess Z. Then, we can compute price indices P^i and P^{si} from (24) and (25). The unit cost λ^{si} can be computed from (28), and then p_{si} can be checked by markup formula (29). Thus, for a given (w_s^i, Z) , we can obtain p_{si} that is consistent with this (w_s^i, Z) from this routine.
- 3. Further guess D_{si} for all (s, i). Then y_{sj} , M^{sj} , N^{sj} can be computed by (30), (31), (32). Π^i can be computed from (34). Then $E^i = \sum_{s=1}^{S} w_s^i n_s^i + \Pi^i$ can be computed. Budget constraint (21) can be used to compute C^i , and the information on M^{sj} and C^i can be used in (27) to check whether the initial guess on D_{si} was correct.
- 4. Finally, we check (w_s^i, Z) . w_s^i can be checked using (22). To compute Z in (33), note that we already know p_{si} from step 2 and y_{si}^f from (26). c_f^i is given by (23), which can be computed by P^i in step 2 and C^i in step 3.

C.2.2 Calibration

Calibration is similar to the baseline model. We start from the economy in 2008Q3, that is, just before the export shock hits. The consumption share parameters $\{\xi_{sj}^i\}_{i,sj}$ are calibrated so that the consumption expenditure share of good (s, j), which represents good s produced in region j, by the region i consumer matches the data in the baseline economy. The consumption shares are taken from the inter-regional input-output in 2005 (IRIO2005), which is the closest time period before 2008Q3. Similarly, we set the target for $\{\xi_i^f\}_i$ as the GDP share of export goods produced in region i. Parameters governing the demand for the (s, j) production by foreign countries, $\{\omega_{sj}^f\}_{sj}$, are set so that the GDP share of export goods (s, j) matches the data computed in IRIO2005.¹⁵

We assume that the parameter governing the wage elasticity of labor supply choice in each industry, τ , is equal to 1, as in Horvath (2000). The inverse of Frisch elasticity of overall labor

¹⁵As a result, the export-to-GDP ratio and the share of good (s, j) in the total export match the data.

Parameter	Description	Value	Target/Source
Dí			
Preference			
σ_c	curvature	1.0	Assumed
χ_i	disutility of labor supply	Table 5	LFS (2008)
ζ	inverse of Frisch elasticity	2.5	Kuroda and Yamamoto (2008)
au	elasticity of substitution (labor)	1.0	Benchmark in Horvath (2000)
$\{\xi_{sj}^i\}_{i,sj}$	weight on consumption goods	Figure 27a	IRIO (2005)
$\{\omega_{sj}^{\check{f}}\}_{sj}$	weight on export goods	Figure 27c	IRIO (2005)
Technology			
$\{A_{sj}\}_{sj}$	factor neutral productivity	Tables 4 and 5	JIP (2005), MLS (2008)
$\{\alpha_s\}_s$	cost share of intermediate goods	Table 4	JIP (2005)
σ	elasticity of substitution	5.0	Assumed
$\{\gamma_{sj}^{hi}\}_{hi,sj}$	weight on intermediate goods	Figure 27b	IRIO (2005)

Table 3: Summary of the parameter values, their source/reference, and data for setting targets.

supply, ζ , is set to 2.5 based on Kuroda and Yamamoto (2008).¹⁶ The labor disutility parameter χ_i is calibrated to replicate the regional variation of the employed population in 2008Q3 taken from the LFS, conducted by the MHLW.¹⁷ Note that the variation in the employed population reflects those of the labor force (or working-age population) and employment rate.¹⁸ As a benchmark, we consider the case of $\sigma_c \to 1$.

The parameter governing the elasticity of substitution, σ , is assumed to be 5. The parameters governing the cost share of each intermediate good (s, j) for the producer of good h at region i, $\{\gamma_{sj}^{hi}\}_{hi,sj}$, are set so that those in the benchmark matches the data counterparts in IRIO2005. The factor neutral productivity for each industry (s, j), A_{sj} , is given by the product of the industryand region-specific productivity parameters; that is, $A_{sj} = A_s \times A_j$, where A_s stands for the industry-specific productivity while A_j stands for the region-specific productivity. First, we map the industry classification in the JIP Database to ours and compute the industrial TFP and cost share of intermediate goods for each industry s ($\{\alpha_s\}_s$). Given all other parameters, the regionspecific productivity A_s is pinned down so that the regional variation of the average wage rate in the benchmark replicates the data counterpart computed using the MLS.

Table 3 summarizes the parameter values. The values of A_j and $\{\alpha_s\}_s$ are summarized in Table 4 in Appendix E. The regional parameters A_i , χ_i , and ξ_f^i are summarized in Table 5 in Appendix E. The parameters $\{\xi_{sj}^i\}_{i,sj}$ $\{\gamma_{sj}^{hi}\}_{hi,sj}$, and $\{\omega_{sj}^f\}_{sj}$ are too numerous to be summarized in a table and are represented as heatmaps in Figure 27 in Appendix E.

¹⁶Kuroda and Yamamoto (2008) estimate the Frisch elasticity in Japan and report that the elasticity on the extensive and intensive margins combined ranges between 0.2 to 0.7 for males. $\zeta = 2.5$ implies the Frisch elasticity of 0.4.

¹⁷See, https://www.stat.go.jp/english/index.html.

¹⁸Although it is better to incorporate the variation of working hours per labor force, there are no reliable data disaggregating working hours into each region.



Figure 19: National responses. "Data" plots the fluctuations of HP-filtered variables, normalizing 2008Q3's value as 1.

C.3 Simulating the model with the Great Recession export shocks

We repeat the same experiments as in the main text. Below, we set the time series of y_{sj}^{t} so that the time path of the export value replicates the regional export data.

We simulate the export shocks to industry si (i.e., industry s in region i) in period t by changing $y_{si,t}^{f}$ for the following to hold in equilibrium:

$$\frac{y_{s_{i,t}}^{f}}{y_{s_{i,t=0}}^{f}} = \frac{\text{real export of } si \text{ in } t \text{ in data}}{\text{real export of } si \text{ in } t = 0 \text{ in data}},$$

where reference period t = 0 corresponds to the third quarter of 2008 (2008Q3).

A variable of our primary interest is domestic (final) demand, equivalent to consumption in this static model, at the national and regional levels. The real consumption is computed excluding the imported goods consumption. Formally, the real consumption for region i in period t, $\bar{C}_{i,t}$, is defined as

$$\bar{C}_{i,t} = \int_0^S \int_0^I p_{sj,t=0} c_{sj,t}^i dj ds,$$

where $p_{sj,t=0}$ is the goods price produced in industry sj in reference period t = 0. The national (real) consumption in period t, $\bar{C}_{Japan,t}$, is then defined as

$$\bar{C}_{Japan,t} = \int_0^I \bar{C}_{i,t} di.$$

C.4 National level response

Figure 19 draws the domestic final demand and exports at the national level. By construction, the export values from the model exactly match the data. The model accounts for 63.2% of the decline in consumption in 2009Q1 and 19.5% of the decline in average consumption from 2008Q4 to 2009Q4. The demand decline in the static model is more modest than in the dynamic model because the static model does not capture investment. According to the data, investment experienced a substantial decline, which contributed significantly to the decline in domestic demand and GDP.



Figure 20: Regional responses

C.4.1 Responses at regional level

Figure 20 compares the regional demand for domestic final goods between the model and data. The model explains the data particularly well for regions such as Chūbu, Kantō, and Kansai, which experienced a large decline in exports. In contrast, for regions such as Shikoku and Okinawa, where the export shocks were not significant, the model performed poorly in explaining the regional GDP decline.

C.5 Counterfactual experiments

In this section, we run a controlled experiment by feeding the model only the shock on y_{sj}^f to one region and industry, keeping y_{sj}^f of the other regions and industries constant. Here, as in the main text, we consider a negative export shock to the transportation equipment (TE) industry in Chūbu.

C.5.1 Decomposition

To see how the export demand shock in a region affects other regions, we conduct a decomposition analysis. Our decomposition is based on different demand components. In the following equation, the first term on the right-hand side is the domestic consumption demand, the second term is the domestic intermediate-good demand, and the third is the foreign demand for good s produced in region j.

$$y_{sj} = \int_0^I c_{sj}^i di + \int_0^S \int_0^I m_{sj}^{hi} di dh + y_{sj}^f$$



Figure 21: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

The domestic consumption demand is represented as

$$c_{sj}^{i} = \xi_{sj}^{i} \left(\frac{p_{sj}}{P^{i}}\right)^{-\sigma} C^{i}, \tag{35}$$

and the domestic intermediate-good demand from industry h in region i is

$$m_{sj}^{hi} = \gamma_{sj}^{hi} \left(\frac{p_{sj}}{P^{hi}}\right)^{-\sigma} M^{hi}.$$
(36)

Given this background, we compute two economies. The first is the baseline economy (2008Q3) without any shocks. The second is the economy with export shock in 2009Q1, but only one industry and one region. Here, as we mentioned above, we choose the transportation equipment industry in the Chūbu region. The comparison of these two provides the overall changes of the (real) output in each region given this particular shock, where the real output for region i is formulated as follows:

$$\bar{Y}_{i,t} = \int_0^S p_{si,t=0} y_{si,t} ds$$

Our decomposition exercise involves two steps. In the first step, we decompose the output change in each region into several demand factors. More specifically, we consider the following three factors separately in decomposing the output change in region *i*. That is, we change only one of these factors in equations (35) and (36). The first set of factors represents the effect of *prices*. These are $p_{sj,t}$ for all *s* and *j* in (35) and (36), and price indices $P_{i,t}^c$, $P_{i,t}^x$ and $P_{hi,t}^m$ in (35) and (36). Note that foreign price \bar{P} is fixed because of the small open economy assumption. The second and third factors are $C_{i,t}$ and $M_{hi,t}$. The first step reveals through which factor a region's output is affected but is silent about through which region. The second step then decomposes the contribution of each factor into the regions from whence those effects originate.

Figure 21 plots the decomposition result for the first step for regions other than Chūbu. The overall effect, indicated as triangles, can be positive or negative. The closer the region to Chūbu, the more significant its overall decline tends to be. The price changes lead to greater output for each region, reflecting the price decline of domestic goods relative to import goods.



Figure 22: Changes in consumption with 2009Q1's shock to the TE in Chūbu (2008Q3=1).

Figures 22 and 23 indicate the contributions of each region in accounting for the column region's decline in consumption and intermediate goods demand, respectively. The decline in each demand component for each region is largely attributed to a decline in Chūbu's demand for that region. The decline in a region's demand for its region also accounts for the decline, which is particularly important in accounting for consumption decline.

C.6 The role of price flexibility

Once again, we consider a situation where all prices are fixed at the level of t = 0.

Figures 24 to 26 plot the results comparable to those for the flexible price benchmark. Now, all other regions' outputs move negatively. The lack of price effect implies that consumption demand and intermediate-good demand directly affect the output of other regions.



Figure 23: Changes in intermediate goods demand with 2009Q1's shock to the TE in $Ch\bar{u}bu$ (2008Q3=1).



Figure 24: Changes in output with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).



Figure 25: Changes in consumption with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).



Figure 26: Changes in intermediate goods demand with 2009Q1's shock to the TE in Chūbu (2008Q3=1, fixed prices).

D Proof of Proposition 1

Now we set S = I = 1 and $A = (\alpha^{\alpha}(1-\alpha)^{1-\alpha})^{-1}$. Then, $P_m = p$ and $\lambda = p^{\alpha}w^{1-\alpha}$ hold. The equilibrium $(P, p, w, \lambda, y, C, N, M, Z)$ is determined by the following equations.

$$P^{1-\sigma} = p^{1-\sigma} + Z p_f^{1-\sigma} \tag{37}$$

$$Zp_f^{1-\sigma}P^{\sigma}C = p^{1-\sigma}\omega^f \tag{38}$$

$$y = p^{-\sigma} \left(P^{\sigma} C + p^{\sigma} M + \omega^f \right)$$
(39)

$$PC = (1 - \alpha(\sigma - 1)/\sigma)py \tag{40}$$

$$w/P = \chi C^{\sigma_c} N^{\zeta} \tag{41}$$

$$wN = (1 - \alpha)\lambda y$$

$$pM = \alpha\lambda y$$
(42)

These equations indicate that the domestic price system is determined up to relative prices.
Hence we set the numeraire as
$$P = 1$$
. Using (37), (40), and $\lambda/p = (\sigma - 1)/\sigma$, the trade balance (38) is transformed to

 $u = A M^{\alpha} N^{1-\alpha}$

$$\omega^f = (1 - \alpha + \alpha/\sigma)(p^{\sigma-1} - 1)py.$$
(43)

This equation coincides with the market clearing condition for domestic goods (39) combined with the household's budget constraint (40). Since $\omega^f > 0$, (43) implies p > 1.

Equation (42) and $\lambda/w = (\sigma/(\sigma-1))^{\alpha/(1-\alpha)}$ imply that N is a linear function of y only:

$$N = (1 - \alpha)(\sigma/(\sigma - 1))^{\alpha/(1 - \alpha)}y.$$
(44)

Equations (40) and (41) imply $w = \chi \left(py(1 - \alpha + \alpha/\sigma) \right)^{\sigma_c} N^{\zeta}$. Using $w/p = ((\sigma - 1)/\sigma)^{1/(1-\alpha)}$, we obtain

$$p^{1-\sigma_c} = y^{\zeta+\sigma_c} \chi (1-\alpha)^{\zeta} (1-\alpha+\alpha/\sigma)^{\sigma_c} \left(\frac{\sigma}{\sigma-1}\right)^{\frac{1+\alpha_\zeta}{1-\alpha}}.$$
(45)

Equations (43) and (45) determine equilibrium y and p. Combining these, we obtain $\omega^f \propto (p^{\sigma-1} - 1)p^{\frac{\xi+1}{\xi+\sigma_c}}$. This implies that the equilibrium exists uniquely. If $\omega^f = 0$, Z = 0 holds. In this case, the economy is closed and has a unique equilibrium with p = 1. Thus, the equilibrium with $\omega^f > 0$ is continuous with the equilibrium with $\omega^f = 0$ when $\omega^f \searrow 0$.

Moreover, $py \propto p^{\frac{\xi+1}{\xi+\sigma_c}}$. Since C is proportional to py by (40), we obtain that C is increasing in p, and thus increasing in ω^f . Real wage w is increasing in ω^f , since w is proportional to p.

From (45), y is independent of p if $\sigma_c = 1$. Note that y/N is constant as in (44). Hence, if $\sigma_c = 1$, N is constant regardless of ω^f . Using the same equations, we obtain that N is decreasing (increasing) in ω^f if $\sigma_c > (<)1$, respectively. \Box

E Tables and figures for parameter values

E.1 Static model

#	Industry (our classification)	$\operatorname{JIP}(2008)$	A_s	α_s
1	Agriculture, Forestry, Fisheries	1-6	1.000	0.71
2	Mining and Quarrying of Stone and Gravel	7	0.497	0.72
3	Food and Beverage	8-14	2.596	0.79
4	Textile Mill Products	5	2.078	0.66
5	Pulp, Paper, and Paper Products	18	1.631	0.82
6	Chemical Products	23-29	1.975	0.86
7	Petroleum and Coal Products	30,31	3.149	0.98
8	Ceramic, Stone, and Cray Products	32-35	0.813	0.66
9	Iron and Steel	$36,\!37$	3.412	0.90
10	Non-Ferrous Metals	38,39	1.271	0.83
11	Fabricated Metal Products	40,41	2.154	0.63
12	General-Purpose Machinery	42-45	2.879	0.72
13	Electrical Machinery	46-53	3.165	0.75
14	Transportation Equipment	54-56	5.876	0.82
15	Information and Communication Electronics Equipment	57	1.188	0.67
16	Miscellaneous Manufacturing Products	$16,\!17,\!19\text{-}22,\!58,\!59$	2.092	0.69
17	Construction	$60,\!61,\!72$	15.465	0.77
18	Electricity, Gas, Heat Supply and Water	62-66	4.330	0.81
19	Whole Sale and Retail Trade	$67,\!68$	18.296	0.50
20	Finance, Insurance, and Real Estate	69-71	8.289	0.50
21	Transportation	73-77	4.350	0.59
22	Information and Communication	$78,79,\ 90-93$	4.308	0.67
23	Education, Medical, Health Care, and Welfare	80-84,98-107	7.578	0.39
24	Services for Businesses	85-88,	5.754	0.60
25	Services for Consumers	89,94-97	4.802	0.57
26	Others	108	1.389	0.98

Table 4: Industry classification for our model, its correspondence with the JIP Database, their factor neutral productivity (with agriculture=1), and intermediate goods share. Those parameters are computed based on the JIP Database 2008 (https://www.rieti.go.jp/en/database/JIP2008/index.html), provided by the RIETI. We map the industry classification in the JIP Database to ours and compute the industrial TFP and intermediate shares.

E.2 Dynamic model

	Okinawa	Kyushu	Shikoku	Chugoku	Kansai	Chubu	Kanto	Tohoku	Hokkaido
A_i	0.978	1.077	1.172	1.253	1.306	1.348	1.377	1.030	1.000
χ_i	8.8e-7	4.7e-11	1.2e-13	2.5e-14	3.8e-12	3.5e-19	6.5e-9	1.4e-13	1.2e-10
ξ_f^i	1.577	0.173	0.873	0.804	1.243	0.581	1.494	1.557	1.603

Table 5: Parameter values for the regional TFP, disutility of labor, and weight on import goods.



Figure 27: Values for the weight parameters

#	Industry (our classification)	JIP(2008)	A_s	α_s	β_s
1	Agriculture, Forestry, Fisheries	1-6	1.000	0.51	0.21
2	Mining and Quarrying of Stone and Gravel	7	0.497	0.63	0.24
3	Food and Beverage	8-14	2.596	0.72	0.19
4	Textile Mill Products	5	2.078	0.59	0.30
5	Pulp, Paper, and Paper Products	18	1.631	0.74	0.16
6	Chemical Products	23-29	1.975	0.77	0.13
7	Petroleum and Coal Products	30,31	3.149	0.94	0.02
8	Ceramic, Stone, and Cray Products	32-35	0.813	0.58	0.29
9	Iron and Steel	$36,\!37$	3.412	0.82	0.09
10	Non-Ferrous Metals	38,39	1.271	0.76	0.16
11	Fabricated Metal Products	40,41	2.154	0.59	0.35
12	General-Purpose Machinery	42-45	2.879	0.65	0.25
13	Electrical Machinery	46-53	3.165	0.66	0.22
14	Transportation Equipment	54-56	5.876	0.75	0.16
15	Information and Communication Electronics Equipment	57	1.188	0.55	0.28
16	Miscellaneous Manufacturing Products	$16,\!17,\!19\text{-}22,\!58,\!59$	2.092	0.63	0.28
17	Construction	$60,\!61,\!72$	15.465	0.44	0.22
18	Electricity, Gas, Heat Supply and Water	62-66	4.330	0.54	0.14
19	Whole Sale and Retail Trade	$67,\!68$	18.296	0.46	0.45
20	Finance, Insurance, and Real Estate	69-71	8.289	0.42	0.40
21	Transportation	73-77	4.350	0.47	0.34
22	Information and Communication	$78,79,\ 90-93$	4.308	0.56	0.29
23	Education, Medical, Health Care, and Welfare	80-84, 98-107	7.578	0.33	0.52
24	Services for Businesses	85-88,	5.754	0.46	0.34
25	Services for Consumers	89,94-97	4.802	0.50	0.38
26	Others	108	1.389	0.98	0.02

Table 6: Industry classification for our model, its correspondence with the JIP Database, their factor neutral productivity (with agriculture=1), and intermediate goods share. These parameters are computed based on the JIP Database 2008 (https://www.rieti.go.jp/en/database/JIP2008/index.html), provided by the RIETI. We map the industry classification in the JIP Database to ours and compute the industrial TFP and intermediate shares.

	Okinawa	Kyushu	Shikoku	Chugoku	Kansai	Chubu	Kanto	Tohoku	Hokkaido
A_i	0.978	1.077	1.172	1.253	1.306	1.348	1.377	1.030	1.000
χ_i	35212.8	12.8	855.2	89.1	2.2	9.1	8.2	39.6	262.3
ξ_f^i	0.88	0.13	0.24	0.16	0.16	0.11	0.16	0.06	0.10

Table 7: Parameter values for the regional TFP, disutility of labor, and weight on import goods.



Figure 28: Values for the weight parameters.