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Abstract

Can national governments stop adverse effects of automation on workers from materializing in a world where capital is mobile? We tackle this question by studying tax competition between two governments for internationally mobile capital used in a non-automated sector and an automated sector, in the latter of which labor and capital are perfect substitutes in production. We compare the tax-competition outcome with the outcome in a closed economy without capital mobility and find contrasting results. In the closed-economy case, more efficient automation technology brings a higher wage to workers by allowing governments to choose a higher tax on capital and a lower tax on labor used in production. In the tax-competition case, however, the fear of capital relocation to countries with lower capital tax prevents each government from raising their capital tax and lowering their labor tax. Consequently, the wage paid to workers always declines and the social welfare may also decline if the governments prioritize the earnings of workers.

Keywords: Inequality, Robots, Capital mobility, Technological change
JEL classification: D33, F21, H21, H25, H71, H73, H77

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1 Introduction

The advancement of automation technology in recent years has potentially profound impact on many areas of the economy. Producers adopting automation technology tend to increase their productivity and expand their market (Graetz and Michaels, 2018; Humlum, 2021; Adachi et al., 2023). This efficiency gains may come at a cost in the labor market; there is a growing concern on not only low-skilled jobs being replaced by industrial robots, but also high-skilled jobs by artificial intelligence. Although the empirical findings of automation on labor demand are mixed, some studies show its negative effect on the share of revenues paid to labor in some countries (Humlum, 2021 for Denmark; Acemoglu et al., 2020a and Bonfiglioli et al., 2020 for France; Dauth et al., 2021 for Germany).

There seems a consensus in the public and academics that the wave of increasing automation in the last decade is different from the wave of information and communication technology development started in the 1970s (Baldwin, 2019). One major reason why this time is different is that the unprecedented pace of automation technological progress comes together with the massive speed of globalization, which Baldwin (2019) calls "globotics." A notable aspect of globalization among others is the integration of capital market (Quinn, 1997). Investment liberalization has greatly enhanced mobility of capital, inducing competition between governments for capital, thereby preventing them from taxing it (Devereux et al., 2008). This implies that tax policies of national governments constrained by mobile capital may not be able to stop the adverse effect of automation on workers from materializing.

Against this background, we ask the following questions. How does a progress of automation technology change the optimal tax policies of governments competing with each other for internationally mobile capital? Especially, can governments shield workers from reduced labor demand due to more replacement of workers with capital? Who benefits from the technological progress, workers, capital owners, or the society as a whole? To answer these questions, we build a simple two-country model with an automated and non-automated sector, based on the task-based approach to automation à la Acemoglu and Restrepo (2018, 2020). The automated sector produces an intermediate good by bundling many tasks, which can be done by labor and capital in a perfectly substitutable manner. The non-automated sector uses the intermediate input and capital in a complementary manner to produce a final good. Firms in both sectors as well as task producers are perfectly competitive. There is an aggregate capital which is allocated to the automation and the non-automation sector in a country. We call these two types of capital the automation and the non-automation capital respectively. National governments tax on firms in the two sectors for their use of labor, automation capital, and non-automation capital, to maximize their welfarist objective, i.e., a weighted sum of labor and capital income with a greater weight attached to labor income.

The answers to the questions are sharply different depending on whether capital is internationally mobile or not. When capital is immobile, which we call a closed-economy case, more efficient automation technology raises capital demand and pushes the rental rate of capital upward. The governments fully shift this increased rental rate from capital owners to workers by raising capital tax and reducing labor tax. As a result, labor income and social welfare increase, while capital income does not, despite the fact that the technological progress makes more tasks automated.

When capital is mobile between two symmetric countries, the world capital allocation is determined at the point where the (before-tax) rental rate is equalized between the two countries. The governments facing competition for the mobile tax base cannot fully protect workers from accelerating automation. They end up choosing a lower capital tax and a higher labor tax than in the closed-economy case. Therefore, the effects of the advancement of automation technology are in contrast to those in the closed-economy case. That is, the governments do not change their taxes in response to the technological change and let capital owners take more and workers take less. The social welfare may decrease if the welfare weight on labor income is sufficiently greater. Under the pressure of globalization during the last three decades, governments in developed countries have taxed mobile factors less and less, and immobile mobile factors more and more (Egger et al., 2019; Saez and Zucman, 2019). Our results suggest that the extraordinary rate of automation technological growth in recent years and the near future will pose a further challenge to welfare states.

The contrasting results in the closed-economy and the tax-competition cases hold even when countries are asymmetric in size. In addition, the effect of increasing efficiency in automation capital is distinct from that in non-automation capital, which benefits both workers and capital owners in both cases.

Regardless of capital mobility, the optimal capital taxes are the same for both types of capital. One may wonder why the governments intervene in the firm's choice of automation by setting a higher tax on automation capital than on non-automation capital. However, such discriminatory capital taxes are bad for the economy because in our perfectly competitive framework they distort capital allocation within a country and fails to maximize the final-good output. In line with the production efficiency theorem by Diamond and Mirrlees (1971), the governments should a common tax to the same production factor used in different sectors.

The prior studies on taxation of automation capital, or the so-called robot tax, investigate the conditions under which automation capital should be taxed (Zhang, 2019; Acemoglu et al., 2020b; Koizumi, 2020; Costinot and Werning, 2023; Gasteiger and Prettner, 2022;

Guerreiro et al., 2022; Thuemmel, 2022). Acemoglu et al. (2020b), for example, show the possibility of positive tax on the use of capital in some tasks, due to labor market frictions and restricted tax instruments. Thuemmel (2022) considers three types of capital, one of which is automation capital (or "robots"), and finds a robot tax different from taxes on other capital set by governments who cannot observe the type of heterogeneous workers. These studies focus on the closed-economy and ignore policy interactions between governments arising from mobile factors.

There are extensive studies on capital-tax competition (Zodrow and Mieszkowski, 1986; Wilson, 1986; Sinn, 1997; Keen and Konrad, 2013 for a survey). However, we are not aware of studies examining the effect of automation in this literature. As we show, the implications of technological progress in automation capital are largely different from those in traditional capital. More broadly, our study is related to income-tax competition for mobile workers (Bierbrauer et al., 2013; Lehmann et al., 2014; Janeba and Schulz, 2023). They are also not concerned with automation.

The rest of the paper is organized as follows. The next section describes an automated economy on which governments set tax policies. Section 3 characterizes the optimal tax policy of governments in a closed economy with immobile capital. Section 4 allows for capital mobility and analyzes tax competition between two symmetric countries. Section 5 discusses a few extensions and the final section concludes.

2 The model

We consider an economy with one homogeneous final good, and two primary factors: labor and capital. The final good is produced using capital and an intermediate good, which is a composite of a continuum of tasks done by labor and capital. The intermediate-good sector is automated in that labor and capital are perfect substitutes in task production. We call capital used in the final-good sector (or in the intermediate-good sector) the non-automation capital (or the automation capital).[1]

We here provide a single-country case and introduce later another country in the of tax competition.

[1]Our setting that an aggregate homogeneous capital is used in a non-automated sector and an automated sector can be also found in Gasteiger and Prettner, 2022. Examples of the malleability of capital include coal-fired power plants converted into data centers in the US: see the articles on *The Guardian* by Goldenberg (2015) and on *Energy News Network* by Uhlenhuth (2020).

2.1 Task producer

There are a continuum of tasks indexed by $s \in [0, 1]$. Small indexed tasks can be done using either labor or capital, but large indexed tasks using only labor. The production function of task s producer is given by

$$x(s) = \begin{cases} \gamma^x k(s) + \gamma^l l(s) & \text{if } s \in [0, \overline{\theta}] \\ \gamma^l l(s) & \text{if } s \in (\overline{\theta}, 1] \end{cases},$$

where $x(s)$, $k(s)$, and $l(s)$ are respectively output, capital input, and labor input of task s; γ^x and γ^l are respectively capital and labor productivity; $\overline{\theta}$ is the cutoff index of task at which or below which tasks are automated in the sense that they can be done by either labor and capital.

Cost minimization for task $s \in [0, \overline{\theta}]$ producer is

$$\min_{k(s),l(s)} \ (r + t^x + c)k(s) + (w + t^l)l(s),$$
$$\text{s.t. } x(s) = \gamma^x k(s) + \gamma^l l(s) \geq x,$$

where t^x and t^l are taxes on capital and labor used in task production respectively and x is an exogenous target level of output. In addition to the rental rate and the tax, there is a user cost of capital c, which we broadly interpret as infrastructure or resources and will specify later. A similar specification can be found in Sinn (1997). Cost minimization for task $s \in (\overline{\theta}, 1]$ producer is given analogously.

Letting r and w be the rental rate of capital and the wage respectively, the unit cost or the price of task s is

$$p(s) = \begin{cases} (r + t^x + c)/\gamma^x & \text{if only capital is used} \\ (w + t^l)/\gamma^l & \text{if only labor is used} \end{cases},$$

Since the final-good producer buys the lower-priced task, the production method is chosen as follows:

$$x(s) = \begin{cases} \gamma^x k(s) & \text{if } \dfrac{w + t^l}{r + t^x + c} \geq \dfrac{\gamma^l}{\gamma^x} \\ \gamma^l l(s) & \text{if } \dfrac{w + t^l}{r + t^x + c} < \dfrac{\gamma^l}{\gamma^x} \end{cases}.$$

Intuitively, only capital is used if relative (after-tax) wage is higher than or equal to the relative labor productivity, and only labor is used if the opposite holds. Suppose that the relative wage decreases with θ, which we will see holds true, and let $\widetilde{\theta}$ be the cutoff task at which the relative wage is equal to the relative productivity, $(w + t^l)/(r + t^x + c) = \gamma^l/\gamma^x$.

The production function and the price of task s change at the cutoff task $\theta \equiv \min\{\overline{\theta}, \widetilde{\theta}\}$:

$$x(s) = \begin{cases} \gamma^x k(s) & \text{if} \quad s \in [0, \theta] \\ \gamma^l l(s) & \text{if} \quad s \in (\theta, 1] \end{cases}, \qquad p(s) = \begin{cases} (r + t^x + c)/\gamma^x & \text{if} \quad s \in [0, \theta] \\ (w + t^l)/\gamma^l & \text{if} \quad s \in (\theta, 1] \end{cases}.$$

If $\theta = \widetilde{\theta} \leq \overline{\theta}$, the choice of production method is based on the cost-minimization of the task and the final-good producers. If $\theta = \overline{\theta} < \widetilde{\theta}$, it is purely determined by the available automation technology.

2.2 Final-good producer

As in Acemoglu and Restrepo (2020), the representative final-good producer uses capital K^y and the intermediate good X in production:

$$F(K^y, X) = K^y \left(\alpha - \frac{K^y}{2\beta X} \right),$$
$$\text{where} \quad X \equiv \min_{s \in [0,1]} \{x(s)\},$$

and where α and β are positive constants.[2]

She maximizes the following profit:

$$\pi = K^y \left(\alpha - \frac{K^y}{2\beta X} \right) - (r + t^y + c)K^y - \int_0^1 p(s)x(s)ds,$$

where t^y is taxes on capital used in final-good production and the user cost of capital, c, enters here again.

The profit-maximization problem can be solved in two steps. First, the final-good pro-

[2]We simply call the bundle of tasks the intermediate good. We introduce the intermediate-good sector, despite it not changing any substances, purely for expositionnal purpose.

ducer minimizes the cost of task inputs:

$$\min_{\{x(s)\}_{s\in[0,1]}} \int_0^1 p(s)x(s)ds,$$
$$\text{s.t. } \min_{s\in[0,1]}\{x(s)\} \geq X,$$

The Leontief technology makes the firm choose $x(s) = X$ for all $s \in [0,1]$. The total cost of task inputs is then

$$\int_0^1 p(s)x(s)ds = \left(\int_0^1 p(s)ds\right) X = PX,$$

where

$$\begin{aligned} P &\equiv \int_0^1 p(s)ds \\ &= \int_0^\theta \frac{r+t^x+c}{\gamma^x}ds + \int_\theta^1 \frac{w+t^l}{\gamma^l}ds \\ &= \frac{\theta(r+t^x+c)}{\gamma^x} + \frac{(1-\theta)(w+t^l)}{\gamma^l}. \end{aligned}$$

Then she maximizes the profit by choosing capital K^y and the intermediate input X:

$$\max_{K^y,X} \pi = K^y\left(\alpha - \frac{K^y}{2\beta X}\right) - (r+t^y+c)K^y - PX.$$

From the first-order conditions (FOCs), we have

$$r = \alpha - t^y - c - \frac{K^y}{\beta X},$$
$$P = \frac{1}{2\beta}\left(\frac{K^y}{X}\right)^2.$$

Following Sinn (1997), we specify the unit user-cost of capital as an increasing function of the aggregate capital *operated in the country*, $K = K^x + K^y$, and an decreasing function of the capital endowment, denoted by $\overline{K}$, that is, $c = \delta K/\overline{K}$ with $\delta > 0$ being the intensity. In the closed-economy case where capital does not move, these two types of capital are the same, $K = \overline{K}$, and thus the unit user-cost of capital does not depend on the aggregate capital, $c = \delta$. This specification captures negative externalities of capital usage, for which producers have to pay costs. Examples we have in mind include costs for the congestion of production lines, costs of moving capital equipment to production site, costs for limited computational capacity, and costs for the environmental damage associated with capital usage.

2.3 Market clearing

Letting L be the population of the country, labor-market clearing implies

$$L = \int_{\theta}^{1} l(s)ds = \int_{\theta}^{1} \frac{X}{\gamma^l} ds = \frac{(1-\theta)X}{\gamma^l},$$
$$\text{or } X = \frac{\gamma^l L}{1-\theta}.$$

Letting K^x be the amount of capital allocated to the intermediate-good sector, this must be equal to demand by task producers:

$$K^x = K - K^y = \int_{0}^{\theta} k(s)ds = \int_{0}^{\theta} \frac{X}{\gamma^x} ds = \frac{\theta X}{\gamma^x},$$
$$\text{or } \frac{K^y}{X} = \frac{K}{X} - \frac{\theta}{\gamma^x}.$$

Capital market clearing implies

$$K = K^x + K^y = X\left[\frac{\theta}{\gamma^x} + \beta(\alpha - r - t^y - c)\right].$$

Solving this for the rental rate gives

$$r = \alpha - t^y - c - \frac{1}{\beta}\left(\frac{K}{X} - \frac{\theta}{\gamma^x}\right) = \alpha - t^y - c - \frac{1}{\beta}\left[\frac{K(1-\theta)}{\gamma^l L} - \frac{\theta}{\gamma^x}\right].$$

2.4 Equilibrium

In the following analysis, we focus on the case where automation is not constrained. That is, the cutoff task θ is endogenously pinned down such that $\theta = \widetilde{\theta} \leq \overline{\theta}$. Using the FOCs of the final-good producer, we see

$$\frac{1}{2\beta}\left(\frac{K^y}{X}\right)^2 = P = \frac{\theta(r + t^x + c)}{\gamma^x} + \frac{(1-\theta)(w + t^l)}{\gamma^l}.$$

Factor allocation is determined by the cost-minimization of both producers, at the point where the relative wage is equal to the relative labor productivity, i.e., $(w + t^l)/\gamma^l = (r + t^x + c)/\gamma^x$:

$$\frac{1}{2\beta}\left(\frac{K^y}{X}\right)^2 = \frac{(\theta + 1 - \theta)(r + t^x + c)}{\gamma^x} = \frac{1}{\gamma^x}\left(\alpha + \Delta t - \frac{K^y}{\beta X}\right),$$

which can be solved for K^y/X. Letting $\lambda \equiv K^y/X$ for notational convenience, the solution of the above equation is

$$\lambda \equiv \frac{K^y}{X} = \frac{\sqrt{2\beta\gamma^x(\alpha + \Delta t) + 1} - 1}{\gamma^x}, \tag{1}$$

where $\Delta t = t^x - t^y$ is the difference between the tax on automation capital and the tax on non-automation capital. Due to the constant-returns-to-scale technology in the final-good sector, only the ratio of the non-automation capital to the intermediate good, K^y/X, matters and is pinned down by the tax difference Δt. Since λ is proportional to K^y, it captures the non-automation capital.[3]

The other equilibrium outcomes can be written using λ:

$$\begin{aligned}
&\theta = \frac{\gamma^x(K - \gamma^l\lambda L)}{\gamma^x K + \gamma^l L},\\
&X = \frac{\gamma^x K + \gamma^l L}{1 + \gamma^x\lambda}, \quad K^x = \frac{K - \gamma^l\lambda L}{1 + \gamma^x\lambda}, \quad K^y = K - K^x = \frac{\lambda(\gamma^x K + \gamma^l L)}{1 + \gamma^x\lambda},\\
&r = \alpha - t^y - \delta - \frac{\lambda}{\beta}, \quad w = \frac{\gamma^l\lambda^2}{2\beta} - t^l,
\end{aligned}$$

noting that the unit-user-cost of capital is $c = \delta K/\overline{K} = \delta$ since no capital movement between countries equates capital employed in the country with its capital endowment, $K = \overline{K}$. Given t^y, these variables are affected by the tax difference Δt only through the non-automation-capital-to-intermediates ratio λ.

[3] As we will see in the text, the automation cut-off θ can be written using λ and reduce to

$$\theta = 1 - \frac{\gamma^l L\sqrt{2\beta\gamma^x(\alpha + \Delta t) + 1}}{\gamma^x K + \gamma^l L}.$$

Using this and the factor-market cleaning condition, we can rewrite λ as a function of K^y:

$$\lambda = \frac{K^y}{X} = \frac{(1 - \theta)K^y}{\gamma^l L} = \frac{K^y\sqrt{2\beta\gamma^x(\alpha + \Delta t) + 1}}{\gamma^x K + \gamma^l L}.$$

3 Optimal taxation when capital is internationally immobile

Suppose here that capital is not internationally mobile. The government chooses the combination of taxes, $\{t^x, t^y, t^l\}$, to maximize the social welfare, while earning tax revenues no less than the target level $T \geq 0$ and maintaining non-negative before/after-tax factor prices:

$$\max_{t^x,t^y,t^l} \; G = \phi w L + (1-\phi) r K,$$

$$\text{s.t.} \begin{cases} \sum_{j=x,y} t^j K^j + t^l L \geq T, \\ w \geq 0, \quad w + t^l \geq 0, \\ r \geq 0, \quad r + t^x \geq 0, \quad r + t^y \geq 0, \end{cases}$$

where the social welfare function G is the weighted sum of labor and capital incomes, with ϕ being attached to labor income. The weight ϕ is assumed to be in $(1/2, 1]$, that is, a welfarist government weighing labor income more than capital income. We focus on a range of parameters such that the factor prices under optimal taxation are all non-negative. More specifically, we assume that the parameteruser-cost of capital and

$$\delta \in (0, \overline{\delta}), \qquad \overline{\delta} \equiv \frac{1}{3}\left(\alpha - \frac{\lambda^c}{\beta}\right) > 0, \quad \lambda^c \equiv \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\gamma^x}, \tag{A1}$$

$$T \in [0, \overline{T}), \qquad \overline{T} \equiv \frac{\gamma^l L (\lambda^c)^2}{2\beta}, \tag{A2}$$

$$\frac{K}{L} \in (\gamma^l \lambda^c, \infty), \tag{A3}$$

where we will see shortly that λ^c captures the allocation of capital between the two sectors under optimal taxation.[4] Assumptions (A1) and (A2) state respectively that the user-cost of capital and the revenue target must not be extremely high. Assumption (A3) states that the aggregate capital-labor ratio must not be too small.

Using the equilibrium behavior of producers, we see that the welfare expression can be

[4] We can check that $\alpha > \lambda^c/\beta = (\sqrt{2\alpha\beta\gamma^x + 1} - 1)/(\beta\gamma^x)$ always holds and thus $\overline{\delta} = (\alpha - \lambda^c/\beta)/3$ is always positive.

expressed as

$$\begin{aligned}
G &= \phi wL + (1-\phi)rK \\
&= \phi(wL + rK) - (2\phi - 1)rK \\
&= \phi\left[(w+t^l)L + \sum_{j=x,y}(r+t^j+c)K^j - cK - t^lL - \sum_{j=x,y} t^jK^j\right] - (2\phi-1)rK \\
&\leq \phi\left[\frac{(1-\theta)(w+t^l)}{\gamma^l}\frac{\gamma^l L}{1-\theta} + \frac{\theta(r+t^x+c)}{\gamma^x}\frac{\gamma^x K^x}{\theta} + (r+t^y+c)K^y - cK - T\right] - (2\phi-1)rK \\
&= \phi\left[\frac{(1-\theta)(w+t^l)}{\gamma^l}X + \frac{\theta(r+t^x+c)}{\gamma^x}X + (r+t^y+c)K^y - cK - T\right] - (2\phi-1)rK \\
&= \phi[PX + (r+t^y+c)K^y - cK - T] - (2\phi-1)rK \\
&= \phi[F(K^y, X) - \delta K - T] - (2\phi-1)rK, \qquad (2)
\end{aligned}$$

noting that the unit user-cost of capital is $c = \delta K/\overline{K} = \delta$. The strict equality in the fourth line holds when the labor tax is set such that the government budget is balanced. Clearly, the government always does so to maximize its objective.

As the aggregate capital K and the revenue target T are exogenously given, the government problem is equivalent with maximizing a weighted sum of the final-good output F and capital income rK, up to constant terms:

$$\max_{t^x,t^y} \ \phi F(K^y, X) - (2\phi-1)rK = \phi K^y\left(\alpha - \frac{K^y}{2\beta X}\right) - (2\phi-1)rK.$$

In addition, we know from the closed-economy equilibrium results that $K^y/X \equiv \lambda$ is pinned down by the tax difference $\Delta t = t^x - t^y$ and K^y depends on λ only. The government problem can be reformulated, up to constant terms, as

$$\max_{t^y,\lambda} \ \phi\frac{\lambda(\gamma^x K + \gamma^l L)}{1+\gamma^x\lambda}\left(\alpha - \frac{\lambda}{2\beta}\right) - (2\phi-1)\left(\alpha - t^y - \delta - \frac{\lambda}{\beta}\right)K.$$

The solution of this problem and the associated optimal labor tax are

$$\begin{aligned}
\lambda^c &= \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\gamma^x}, \\
t^{xc} &= t^{yc} = t^c = \alpha - \delta - \frac{\lambda^c}{\beta}, \qquad t^{lc} = \frac{T - t^cK}{L},
\end{aligned}$$

where the superscript c represents the closed economy. Taxes on the two types of capital are equal, at which λ^c is consistent with λ defined in (1), which is determined by the cost-minimization of producers. Since the government emphasizes the worker's interests more, the capital tax is set to the lowest level at which the before-tax rental rate is zero, $r = 0$. Given zero capital income, the government tries to maximize its output and achieves it by setting common capital taxes between the two sectors. To see why this is the case, let us look at the marginal-value product of capital in the non/automation sector respectively:

$$\frac{\partial F}{\partial K^y} = \alpha - \frac{K^y}{\beta X} = r + c + t^y,$$
$$\frac{\partial (PX)}{\partial k(s)} = P\gamma^x = \frac{r + c + t^x}{\gamma^x}\gamma^x = r + c + t^x,$$

noting that the price of the final-good output is normalized to one. If the marginal-value product of capital before taxes in both sectors are equal, the economy maximizes the final-good output. If not, there would be room for increasing the final-good output by reallocating capital between the non-automation and the automation sectors.[5] The government equalizes these two marginal-value products by setting a common tax on both types of capital. The optimal capital allocation represented by λ^c is consistent with the cost-minimization behavior of producers given $\Delta t = t^x - t^y = 0$.

For completeness, we check the range of parameters where all before/after-tax factor prices are non negative. Under assumptions (A1) and (A2), both the after-tax rental rate and the before-tax wage are positive:

$$r^c + t^c = 0 + t^c = \alpha - \delta - \lambda^c/\beta > \alpha - \overline{\delta} - \lambda^c/\beta \geq 0,$$
$$w^c = \frac{\gamma^l(\lambda^c)^2}{2\beta} - t^{lc} = \frac{\gamma^l(\lambda^c)^2}{2\beta} - \frac{T - t^c K}{L} > \frac{\gamma^l(\lambda^c)^2}{2\beta} - \frac{\overline{T}}{L} \geq 0.$$

and obviously the after-tax wage is positive, $w^c + t^{lc} > 0$. We are also interested in whether both the automated and the non-automated tasks coexist, or equivalently, whether the automation cutoff θ being in $(0, 1)$. We can show that this is indeed the case under assumption (A3).[6]

The discussion is summarized as follows.

Proposition 1 (Optimal taxation in a closed economy): *Assume that capital is not internationally mobile, the government puts a higher weight on labor income, $\phi \in (1/2, 1]$,*

[5]The final-good output increases if the non-automation capital, K^y, increases or the intermediate good, X, increases due to an increase in the automation capital, $k(s)$.

[6]See Appendix 1 for details.

and parameters satisfy restrictions (A1) to (A3). The optimal taxes $\{t^{xc}, t^{yc}, t^{lc}\}$ *are such that*

(a) taxes on both automation and non-automation capital are the same and positive, $t^{xc} = t^{yc} = t^c = \alpha - \delta - \lambda^c/\beta > 0$.

(b) capital taxes are set so high that the before-tax rental rate is zero, $r^c = 0$.

(c) the labor tax is set so as to balance the government budget constraint and is negative, $t^{lc} = (T - t^c K)/L < 0$.

Both the automated and the non-automated tasks coexist, $\theta^c \in (0, 1)$. *Capital is allocated between the non-automation and automation sectors in a way that maximizes the final-good output, i.e.,* $\lambda^c = K^{yc}/X^c = \text{argmax}_\lambda\ F(K^y, X)$.

Proof: See Appendix 1.

We then move to the question how the advancement of automation technology, captured by a rise in the productivity of automation capital, γ^x, affects the endogenous variables. First of all, a higher γ^x raises the return to automation capital and thus leads to more automation, $\partial\theta/\partial\gamma^x > 0$, more intermediates, $\partial X^c/\partial\gamma^x > 0$, and a smaller ratio of non-automation capital to intermediates, $\partial\lambda^c/\partial\gamma^x < 0$. The last effect then increases the return to non-automation capital. The rise in capital demand allows the government to set the capital tax higher, $\partial t^c/\partial\gamma^x > 0$, and a lower labor tax, $\partial t^{lc}/\partial\gamma^x < 0$. In terms of the before-tax wage, the effect is always positive despite two opposing forces shown below:

$$\frac{\partial w^c}{\partial \gamma^x} = \underbrace{\frac{\partial}{\partial \gamma^x}\left(\frac{\gamma^l(\lambda^c)^2}{2\beta}\right)}_{<0} \underbrace{-\frac{\partial t^{lc}}{\partial \gamma^x}}_{>0} > 0.$$

While fewer non-automated tasks reduce the labor demand (the first term), the lower labor tax raises it (the second term). Under assumption (A3), the later positive effect always dominates. Since the rental rate is always zero under optimal taxation, this result also implies the positive effect of γ^x on the final-good output, $\partial F^c/\partial\gamma^x > 0$, and the welfare, $\partial G^c/\partial\gamma^x > 0$. So far, it is unambiguous whether the effect of automation technological progress on the variable of interest is positive or negative. However, the effect on capital allocated to the automation sector is mixed. This is because a higher γ^x decreases the automation capital $K^x = \theta X/\gamma^x$ by improving efficiency, while it increases the automation capital by enhancing more automation θ and expanding intermediate production X. The relative magnitude of the negative and positive forces depends on the capital-labor ratio K/L. The resulting effect turns out to be non-negative if $K/L \in (\gamma^l\lambda, \eta]$ and positive otherwise, where η is a threshold value consisting of some parameters other than K/L.[7]

The discussions above are summarized as follows.

Proposition 2 (Effect of automation technology in a closed economy): *Consider the optimal taxation in the closed economy stated in Proposition 1. The effects of an advancement in the automation technology, captured by a higher γ^x, are as follows.*

(a) A higher γ^x increases the range of automated tasks θ^c, the intermediate inputs X^c, the final-good output F^c, the optimal capital tax t^c, the wage w^c, and the social welfare G^c.

(b) It decreases the optimal labor tax t^{lc}.

(c) It increases (or decreases) the automation capital K^{xc}, if the capital-labor ratio is sufficiently low, $K/L \in (\gamma^l\lambda^c, \eta)$ (or sufficiently high, $K/L \in (\eta, \infty)$). The opposite holds for the non-automation capital $K^{yc} = K - K^{xc}$. The before-tax rental rate r^c does not change in γ^x.

Proof: See Appendix 2.

4 Tax competition when capital is internationally mobile

We consider a two-country version of the model where taxes can change the world allocation of internationally mobile capital. We denote the capital endowment in country $i \in \{1, 2\}$ as $\overline{K}_i$, which is distinct from the capital operated there as K_i. Letting κ be the share of capital and labor owned by country 1 and $2K$ (or $2L$) be the world capital (or labor) endowment, we see $\overline{K}_1/(2K) = L_1/(2L) = \kappa$ and $\overline{K}_2/(2K) = L_2/(2L) = 1 - \kappa$. This endowment share captures the relative size of country. Similarly, we denote k as the share of capital employed in country 1, i.e., $K_1 = 2kK$ and $K_2 = 2(1 - k)K$. Due to its mobility, we see $k \neq \kappa$ in general and thus the unit user-cost of capital is no longer constant, $c_1 = \delta K_1/\overline{K}_1 = k/\kappa$. The target tax revenue depends on its country size such that $T_1 = 2\kappa T$ and $T_2 = 2(1 - \kappa)T$. We

[7] The threshold η is given by the greater root of $\mathcal{H}(K/L) \equiv -\alpha\beta(\gamma^x)^2(K/L)^2 - 2\gamma^l(3\alpha\beta\gamma^x + 1)(K/L) + \alpha\beta(\gamma^l)^2(8\alpha\beta\gamma^x + 3) = 0$. See Appendix 2 for details.

assume that both countries have an equal amount of factor endowments, $\kappa = 1/2$, and the same technologies in the main analysis below We also assume that goods and tasks are not internationally traded. This assumption is innocuous in the analysis of symmetric countries where equilibrium prices are the same across the world.[8]

The timing of actions proceeds as follows. First, each government in country $i \in \{1, 2\}$ non-cooperatively chooses the tax on capital in both the automated and non-automated sectors, t_i^x and t_i^y, and the tax on labor, t_i^l, to maximize the national social welfare G_i. Second, capital owners invest their capital in firms and the international allocation of capital is determined. Finally, domestic capital is allocated between the two sectors and production takes place. We solve the problem backward in the case where automation is not constrained and the case where it is constrained.

Third stage: producers behavior and market clearing within country The cost-minimizing behavior of producers in country 1 results in

$$\frac{K_1^y}{X_1} = \frac{\sqrt{2\beta\gamma^x(\alpha + \Delta t_1) + 1} - 1}{\gamma^x} \equiv \lambda_1,$$

which is a function of the capital-tax difference. As we will see in the first stage, the government can lead the economy to any $K_1^y/X_1 = \lambda_1$ by setting Δt_1 accordingly. The FOC of the final-good producer implies

$$K_1 = \frac{\overline{K}_1}{\delta}\left(\alpha - r - t_1^y - \frac{\lambda_1}{\beta}\right).$$

Factor-market clearing conditions determine the automation cutoff and the level of non-automation capital:

$$\theta_1 = \frac{\gamma^x(K_1 - \gamma^l \lambda_1 L_1)}{\gamma^x K_1 + \gamma^l L_1},$$
$$K_1^y = \frac{\lambda_1(\gamma^x K_1 + \gamma^l L_1)}{1 + \gamma^x \lambda_1}.$$

Analogous results hold in country 2.

[8] Assuming the non-tradability of goods and tasks is not totally innocuous in the analysis of asymmetric countries where equilibrium prices in general differ across countries.

Second stage: international capital allocation The world capital-market clearing implies

$$2K = \sum_{i=1}^{2} K_i = \sum_{i=1}^{2} \frac{\overline{K}_i}{\delta}\left(\alpha - r - t_i^y - \frac{\lambda_i}{\beta}\right),$$
$$\text{or } r = \alpha - \delta - \frac{1}{2}\sum_{i=1}^{2}\left(t_i^y + \frac{\lambda_i}{\beta}\right),$$

where we used $\overline{K}_1 = \overline{K}_2 = K$.

First stage: choice of taxes The national social welfare that government 1 tries to maximize is

$$\begin{aligned} G_1 &= \phi w_1 L_1 + (1-\phi) r \overline{K}_1 \\ &\leq \phi[F(K_1^y, X_1) - \delta K_1^2/\overline{K}_1 - T_1 + r(\overline{K}_1 - K_1)] - (2\phi - 1) r \overline{K}_1, \end{aligned} \tag{3}$$

where the strict equality holds if the government budget is balanced through an appropriate choice of labor tax t_1^l. Unlike the social welfare in the closed economy defined in (2), capital mobility allows capital operated in country 1, K_1, to differ from the capital endowment there, $\overline{K}_1$.

From the results of the second and the third stages, we see that maximizing the national welfare by choosing t_1^x and t_1^y is equivalent with maximizing it by choosing λ_1 and t_1^y. The way we formulate the government problem here is similar to the one in the closed-economy case, but there is one big difference. With internationally mobile capital, aggregate capital $K_1 = K - K_2$ is also affected by capital taxes in both countries 1 and 2.

Solving the maximization problem of government 1 gives its best response functions:

$$t_1^y = \frac{\gamma^x(\beta t_2^y + \lambda_2) + 1 + \beta\delta\gamma^x - \sqrt{2\alpha\beta\gamma^x + 1}}{2\beta\gamma^x} + \frac{\delta(2\phi - 1)}{2\phi},$$
$$\lambda_1^* = \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\gamma^x}.$$

It is clear that a heavier weight on labor income ϕ pushes the tax on non-automation capital upward and thus the labor tax downward, leading to a higher before-tax wage.

We obtain the best response functions of government 2 and solve the system of equations for the sub-game perfect Nash equilibrium outcomes:

$$t_1^{y*} = t_2^{y*} = t^* = \frac{\delta(3\phi - 1)}{\phi},$$
$$\lambda_1^* = \lambda_2^* = \lambda^* = \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\gamma^x}.$$

The associated tax on automation capital and the labor tax are respectively $t_1^{x*} = t_2^{x*} = t^*$ and $t_1^{l*} = t_2^{l*} = t^{l*} = (T - t^*K)/L$, noting that the equilibrium $\lambda_i^* = \lambda^*$ is consistent with the cost-minimizing λ defined in (1) only when $\Delta t_i = t_i^x - t_i^y = 0$. The symmetric taxes leads to an equal allocation of aggregate capital between the two countries, $K_1^* = K_2^* = K$. We can show that the symmetric equilibrium above is unique and all the before/after-tax factor prices are all non negative.

The equilibrium sectoral allocation of capital is the same as the one under optimal taxation in the closed economy, $\lambda_i^* = \lambda^c$. This implies that equilibrium outcomes which depend only on λ_i are the same in both the tax-competition and the closed-economy cases. Such outcomes include $\theta_i^* = \theta^c$, $X_i^* = X^c$, $K_i^{x*} = K^{xc}$, and $F_i^* = F^c$. On the other hand, the equilibrium capital tax is not the same in the two cases, $t^* \neq t^c$. As a result, unlike the closed-economy case, the rental rate turns out to be positive and the labor tax may not necessarily be negative.

The results are summarized as follows.

Proposition 3 (Tax competition): *Assume that capital is internationally mobile, the two countries are symmetric, the governments put a higher weight on labor income, $\phi \in (1/2, 1]$, and the parameters satisfy (A1) to (A3). Tax competition leads to unique symmetric equilibrium taxes $\{t_i^{x*}, t_i^{y*}, t_i^{l*}\}$ for country $i \in \{1, 2\}$ such that*

(a) taxes on both automation and non-automation capital are the same and positive, $t_i^{x*} = t_i^{y*} = t_i^* = \delta(3\phi - 1)/\phi > 0$.

(b) capital taxes are set so low that the before-tax rental rate is positive, $r^* > 0$.

(c) the labor tax is set so as to balance the government budget constraint and can be positive, zero, or negative, $t_i^{l*} = (T - t_i^*K)/L \gtreqless 0$.

Aggregate capital is equally allocated between the two countries, $K_1^* = K_2^* = K$. Both the automated and the non-automated tasks coexist, $\theta_i^* \in (0, 1)$. Capital is allocated between the non-automation and automation sectors in a way that maximizes the final-good output, given aggregate capital K_i^*, i.e., $\lambda_i^* = K_i^{y*}/X_i^* = argmax_{\lambda_i}\ F(K_i^y, X_i)$.

Proof: See Appendix 3.

The governments chooses a lower capital tax in tax competition than in the closed economy, $t_i^* < t^c$. This is because capital mobility leads to capital reallocation from high-tax to low-tax countries and thus limits the welfarist governments to put a heavy tax on capital. The lower capital tax implies a higher rental rate $r^* > r^c$, a higher labor tax $t_i^{l*} > t^{lc}$, and a lower before-tax wage $w_i^* < w^c$. Despite the welfarist's motives, the governments become more in favor of capital owners as a result of tax competition. The social welfare indeed declines, $G_i^* < G^c$.

Proposition 4 (Tax competition vs. optimal taxation in a closed economy): *Comparing results in a closed economy summarized in Proposition 1 with those in tax competition in Proposition 3, we obtain the following,*

(a) The labor tax and the before-tax rental rate are higher, $t_i^{l*} > t^c$ and $r^* > r^c$.

(b) The capital tax, the before-tax wage, and the national (and the global) social welfare are lower, $t_i^* < t^c$, $w_i^* < w^c$, and $G_i^* < G^c$.

The other equilibrium outcomes including θ_i^*, X_i^*, F_i^*, and K_i^{x*} are the same in both the closed-economy and tax-competition cases.

Proof: See Appendix 4.

We then look at the effect of the efficiency of automation capital, γ^x. In the closed-economy case, a higher γ^x increases the rental rate due, which is exactly taken out by an increasing capital tax. The higher capital tax allows the national government to lower its labor tax and raise the before-tax wage. In the tax-competition case, by contrast, the capital tax does not respond to γ^x and thus cannot be used to shift capital income to workers. This is good for capital owners, but bad for workers. Despite facing the reduced labor demand due to more efficient automation, each national government cannot compensate for wage reductions. Whether a higher γ^x reduces the social welfare depends on how much emphasis the governments put on loss in labor income relative to gain in capital income. If the welfare weight on labor income ϕ is sufficiently high, more efficient automation leads to a lower social welfare.

The effects on other equilibrium outcomes such as the automation cutoff θ_i^*, the final-good output F_i^*, and the automation capital K_i^{x*} are the same as those in the closed economy, summarized in Proposition 2. This is because they depend only on λ_i and λ_i is the same in both the closed-economy case and the tax-competition cases, $\lambda^* = \lambda^c$.

These results are summarized follows.

Proposition 5 (Effect of automation technology in tax competition): *Consider tax competition summarized in Proposition 3. The effects of an advancement in the automation technology, captured by a higher γ^x, are as follows.*

(a) A higher γ^x increases the before-tax rental rate r^*, while it decreases the before-tax wage w_i^*.

(b) It increases (or decreases) the national social welfare G_i^*, if the welfare weight on labor income is sufficiently low, $\phi \in (1/2, \phi^*)$ (or sufficiently high, $\phi \in (\phi^*, 1]$). The capital tax t_i^* and the labor tax t_i^{l*} do not change in γ^x.

The effects on other outcomes are the same as those in the closed-economy case, stated in Proposition 2: positive effect on θ_i^*, X_i^*, F_i^*; mixed effect on K_i^{x*}.

Proof: See Appendix 5.

5 Discussions and extensions

We here discuss implications and robustness of our main result that an advancement of automation technology is good for the social welfare when capital is immobile, but it is bad when capital is mobile.

5.1 Advancement of the non-automation technology

One may wonder whether a rise in the efficiency of non-automation capital leads to a similar conclusion. However, this is not the case. Specifically, we can think of a higher α, a parameter in the final-good production function F_i, as more efficient non-automation capital, since it raises the marginal product of non-automation capital $\partial F_i/\partial K_i^y$. In contrast to the effect of γ^x, in both the closed-economy and the tax-competition cases, a higher α leads to capital movement from the automation to the non-automation sector and (i)a reduction in the number of tasks automated. More efficient non-automation capital increases final-good production, (ii)boosting demand for the intermediate input. Thanks to (i) and (ii), the before-tax wage and the social welfare rise. In the tax-competition case, in particular, a higher α increases the demand for non-automation capital and, by the no-arbitrage argument, the common before-tax rental rate of both types of capital, which benefits capital owners as well.

We here state only main points and relegate other results and the proof to Appendix 6.

Proposition 6 (Effect of non-automation technology): *Consider optimal taxation in a closed economy and tax competition between two symmetric countries with mobile capital, summarized respectively in Proposition 1 and 3. The effects of an advancement in the non-automation technology, captured by a higher α, are as follows.*

(a) A higher α increases the before-tax wage and the social welfare in both the closed-economy and the tax-competition cases.

(b) In the tax-competition case, a higher α increases the before-tax rental rate, while in the closed-economy case it has no effect on the before-tax rental rate.

5.2 Asymmetric country size

We can easily incorporate asymmetry in country size,: an unequal world distribution of capital and labor endowment, $\overline{K}_1/(2K) = \overline{L}_1/(2L) = (T_1/(2T) =)\kappa \neq 1/2$.[9] To be specific, we assume country 1 is larger than country 2, $\kappa \in (1/2, 1)$. Even with size asymmetry, however, our main conclusion continues to hold that regardless of their size, countries lose from an automation technological progress when capital is internationally mobile.

Literature on tax competition emphasizes the importance of relative size of countries. That is, larger countries with less sensitive capital demand choose a higher capital tax and export capital to smaller countries. Analogous results can be obtained in our extended model where the capital and labor endowments are unevenly distributed between two countries. In our notation, the share of aggregate capital in the larger country, say country 1, denoted by $k = K_1/(2K)$, is smaller than its world share of endowment, $k < \kappa$.

We relegate the proof to Appendix 8.

[9]This asymmetric-country analysis can be viewed as tax competition between developed and developing countries. An alternative way of capturing the size difference in developed and developing countries is to allow for asymmetric capital-labor ratio, say, $\overline{K}_1/(2K) = \overline{L}_1/(2L) = \kappa_1 \neq \kappa_2 = \overline{K}_2/(2K) = \overline{L}_2/(2L)$.

Proposition 7 (Tax competition between unequal-sized countries): *Consider tax competition summarized in Proposition 3 with an exception of country 1 being larger than country 2 in endowment, $\overline{K}_1/(2K) = \overline{L}_1/(2L) = \kappa \in (1/2, 1)$. Then government 1 chooses a higher capital tax and a lower labor tax than government 2, and exports capital to country 2, $K_1^*/(2K) = k < \kappa$. The effects of an advancement of the automation technology, captured by a higher γ^x, are the same as those in the symmetric case stated in Proposition 5. In particular, when the welfare weight on labor income is sufficiently high, a higher γ^x lowers the before-tax wage and the social welfare in both large and small countries.*

6 Conclusion

Using the task-based approach to automation, this study characterizes the optimal capital- and labor-taxes on firms in an economy with both an automated and a non-automated sectors. The optimal tax schedule is very different depending on whether capital used in the two sectors is internationally mobile or not. When capital is mobile, national governments weighing workers more than capital owners set taxes in a way that shifts 100% of capital income to workers. When capital is mobile, by contrast, each national government facing a potential relocation of tax base to low-tax countries can only partially achieve their welfarist objective by setting a capital tax lower than that in the immobile-capital case.

The effect of an advancement of automation technology is even more contrasting in the two cases. In the immobile-capital case, more efficient automation technology raises capital demand and pushes the rental rate upward, which can be fully taken out by national governments. They use the increased tax revenues to cut the labor tax, bringing greater labor income and (national) social welfare. Mobile capital prevents each government from raising their capital tax in response to the increased capital demand due to the advancement of automation technology. Despite their welfarist objective, the governments cannot stop labor income from declining and capital income from rising. Consequently, the social welfare may decline.

According to our results, the governments should not intervene in sectoral allocation of capital by choosing discriminatory tax schedules for automation and non-automation capital. This is certainly at odds with public concerns recent years on job destruction due to automation. We leave to future research the exploration when discriminatory taxes on automation and non-automation capital are justified.

Appendix

Appendix 1: Proof of Proposition 1

The government maximization problem is formulated as

$$\max_{t^x, t^y, t^l} \; G = \phi w L + (1-\phi) r K,$$

$$\text{s.t.} \begin{cases} \sum_{j=x,y} t^j K^j + t^l L \geq T, \\ w \geq 0, \quad w + t^l \geq 0, \\ r \geq 0, \quad r + t^x \geq 0, \quad r + t^y \geq 0. \end{cases}$$

As discussed in the text, the problem can be reformulated as

$$\max_{t^y, \lambda} \; \phi F - (2\phi - 1) r K = \phi \frac{\lambda(\gamma^x K + \gamma^l L)}{1 + \gamma^x \lambda} \left(\alpha - \frac{\lambda}{2\beta} \right) - (2\phi - 1) \left(\alpha - t^y - \delta - \frac{\lambda}{\beta} \right) K,$$

$$\text{s.t.} \begin{cases} w \geq 0, \quad w + t^l \geq 0, \\ r \geq 0, \quad r + t^x \geq 0, \quad r + t^y \geq 0, \end{cases}$$

while ignoring constant terms.

In the following, among other constraints, we consider only $r \geq 0$ and solve the optimal taxes. Then we check if other constraints are satisfied under the optimal taxes.

Set the Lagrangian as

$$\mathcal{L} = \phi \frac{\lambda(\gamma^x K + \gamma^l L)}{1 + \gamma^x \lambda} \left(\alpha - \frac{\lambda}{2\beta} \right) - (2\phi - 1) \left(\alpha - t^y - \delta - \frac{\lambda}{\beta} \right) K + \mu \left(\alpha - t^y - \delta - \frac{\lambda}{\beta} \right),$$

where μ is the Lagrangian multiplier associated with the constraint $r = \alpha - t^y - \delta - \lambda/\beta \geq 0$. The Karush-Kuhn-Tucker conditions are

$$\frac{\partial \mathcal{L}}{\partial t^y} = (2\phi - 1)K - \mu = 0, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0, \tag{A.2}$$

$$\mu \geq 0, \quad \alpha - t^y - \delta - \lambda/\beta \geq 0, \quad \mu(\alpha - t^y - \delta - \lambda/\beta) = 0. \tag{A.3}$$

From (A.1) and (A.3), we see $\mu = K(2\phi - 1) > 0$ and $r = \alpha - t^y - \delta - \lambda/\beta = 0$, implying $t^{yc} = \alpha - \delta - \lambda/\beta$. Substituting these results into (A.2) and solving it for λ gives $\lambda^c = (\sqrt{2\alpha\beta\gamma^x + 1} - 1)/\gamma^x$, noting that the smaller root of (A.2), $-(\sqrt{2\alpha\beta\gamma^x + 1} + 1)/\gamma^x$, is negative and thus irrelevant.

The sufficient condition for the solution to be the global maximizer is that the Lagrangian is concave with respect to (t^y, λ) (Theorem 3.6.1, Sydsæter et al., 2008). This is true from the observations that (a)$\partial^2\mathcal{L}/\partial(t^y)^2 = 0$; (b)$\partial^2\mathcal{L}/\partial\lambda^2 < 0$; (c)$\partial^2\mathcal{L}/(\partial t^y\partial\lambda) = 0$; and (d)$[\partial^2\mathcal{L}/\partial(t^y)^2] \cdot [\partial^2\mathcal{L}/\partial\lambda^2] - [\partial^2\mathcal{L}/(\partial t^y\partial\lambda)]^2 = 0 - 0 = 0$.

For λ^c to be consistent with the sectoral capital allocation determined by the cost-minimizing producers, we must have $\Delta t = t^x - t^y = 0$ or $t^{xc} = t^{yc} = t^c$. On the non-negativity of before/after-tax factor prices, it is immediate to see $r + t^c = t^c = \alpha - \delta - \lambda^c/\beta > 0$ under assumption (A1) and $w^c + t^{lc} = \gamma^l(\lambda^c)^2/(2\beta) > 0$ (see Section 1.5). The before-tax wage is positive because

$$w^c = \frac{\gamma^l(\lambda^c)^2}{2\beta} - t^{lc} = \frac{\gamma^l(\lambda^c)^2}{2\beta} - \frac{T - t^cK}{L} > \frac{\gamma^l(\lambda^c)^2}{2\beta} - \frac{\overline{T}}{L} > 0,$$

where the second last inequality is from assumption (A3) and $t^c > 0$. We can also show that the optimal labor tax t^{lc} is always negative:

$$\begin{aligned}
t^{lc} = \frac{T - t^cK}{L} &< \frac{\overline{T}}{L} - \left(\alpha - \delta - \frac{\lambda^c}{\beta}\right)\frac{K}{L} = \frac{\gamma^l(\lambda^c)^2}{2\beta} - \left(\alpha - \delta - \frac{\lambda^c}{\beta}\right)\frac{K}{L} \\
&\hspace{20em} (\because \text{(A2) } T < \overline{T}) \\
&< \frac{\gamma^l(\lambda^c)^2}{2\beta} - \left(\alpha - \overline{\delta} - \frac{\lambda^c}{\beta}\right)\frac{K}{L} = \frac{\gamma^l(\lambda^c)^2}{2\beta} - \frac{1}{2}\left(\alpha - \frac{\lambda^c}{\beta}\right)\frac{K}{L} \\
&\hspace{20em} (\because \text{(A1) } \delta < \overline{\delta}) \\
&< \frac{\gamma^l(\lambda^c)^2}{2\beta} - \frac{1}{2}\left(\alpha - \frac{\lambda^c}{\beta}\right)\gamma^l\lambda^c = \frac{\gamma^l\lambda^c}{2\beta}(2\lambda^c - \alpha\beta) \quad (\because \text{(A3) } K/L > \gamma^l\lambda^c) \\
&< 0.
\end{aligned}$$

Let us then look at the automation cutoff:

$$\theta^c = \frac{\gamma^x(K - \gamma^l\lambda^cL)}{\gamma^xK + \gamma^lL} = 1 - \frac{\gamma^lL\sqrt{2\alpha\beta\gamma^x + 1}}{\gamma^xK + \gamma^lL},$$

which is obviously less than one. It is positive if

$$\begin{aligned}
&\theta^c > 0, \\
&\text{or } \gamma^x(K/L)^2 + 2\gamma^l(K/L) - 2\alpha\beta\gamma^l \equiv \mathcal{F}(L/L) > 0.
\end{aligned}$$

We can see that this is indeed the case as long as assumption (A3)$K/L \in (\gamma^l\lambda^c, \infty)$ holds from the two observations that $\mathcal{F}(K/L)$ is a quadratic function of K/L with a positive coefficient of $(K/L)^2$; and $\mathcal{F}(K/L)$ has a unique positive root, which is $K/L = \gamma^l\lambda^c$.

Finally we can see that the final-good output F is maximized at λ^c by observing

$$\frac{\partial F}{\partial \lambda} = -\frac{(\gamma^x K + \gamma^l L)(\gamma^x \lambda^2 + 2\lambda - 2\alpha\beta)}{2\beta(1+\gamma^x\lambda)^2},$$

takes zero at $\lambda = \lambda^c > 0$ and the second derivative is negative.

Appendix 2: Proof of Proposition 2

From $\lambda^c = (\sqrt{2\alpha\beta\gamma^x + 1} - 1)/\gamma^x$, we see

$$\frac{\partial \lambda^c}{\partial \gamma^x} = -\frac{\alpha\beta\gamma^x + 1 - \sqrt{2\alpha\beta\gamma^x + 1}}{(\gamma^x)^2\sqrt{2\alpha\beta\gamma^x + 1}} < 0.$$

This result immediately implies

$$\begin{aligned}
&t^c = \alpha - \delta - \lambda^c/\beta : && \frac{\partial t^c}{\partial \gamma^x} = -\frac{1}{\beta}\frac{\partial \lambda^c}{\partial \gamma^x} > 0,\\
&t^{lc} = \frac{T - t^c K}{L} : && \frac{\partial t^{lc}}{\partial \gamma^x} = -\frac{K}{L}\frac{\partial t^c}{\partial \gamma^x} < 0,\\
&\theta^c = \frac{\gamma^x(K - \gamma^l\lambda^c L)}{\gamma^x K + \gamma^l L} : && \frac{\partial \theta^c}{\partial \gamma^x} = -\frac{\gamma^l L[K(2\alpha\beta\gamma^x + 1) - \alpha\beta\gamma^l L]}{(\gamma^x K + \gamma^l L)^2\sqrt{2\alpha\beta\gamma^x + 1}} > 0,\\
&X^c = \frac{\gamma^l L}{1 - \theta^c} : && \frac{\partial X^c}{\partial \gamma^x} = \frac{\gamma^l L}{(1-\theta^c)^2}\frac{\partial \theta^c}{\partial \gamma^x} > 0,\\
&r^c = 0 : && \frac{\partial r^c}{\partial \gamma^x} = 0.
\end{aligned}$$

On the automation cutoff, we see

$$\theta^c = \frac{\gamma^x(K - \gamma^l\lambda^c L)}{\gamma^x K + \gamma^l L} : \qquad \frac{\partial \theta^c}{\partial \gamma^x} = -\frac{\gamma^l L[K(2\alpha\beta\gamma^x + 1) - \alpha\beta\gamma^l L]}{(\gamma^x K + \gamma^l L)^2\sqrt{2\alpha\beta\gamma^x + 1}}.$$

The derivative is positive if $K(2\alpha\beta\gamma^x + 1) - \alpha\beta\gamma^l L > 0$, or $K/L > \alpha\beta\gamma^l/(\alpha\beta\gamma^x + 1)$ holds. This inequality is always satisfied since assumption (A3) implies $K/L > \gamma^l\lambda^c > \alpha\beta\gamma^l/(\alpha\beta\gamma^x + 1)$.

As mentioned in the text, the effect on the before-tax wage is a little complicated:

$$w^c = \frac{\gamma^l(\lambda^c)^2}{2\beta} - t^{lc} : \qquad \frac{\partial w^c}{\partial \gamma^x} = \underbrace{\frac{\partial}{\partial \gamma^x}\left(\frac{\gamma^l(\lambda^c)^2}{2\beta}\right)}_{<0} \underbrace{-\frac{\partial t^{lc}}{\partial \gamma^x}}_{>0},$$

which reduces to

$$\frac{\partial w^c}{\partial \gamma^x} = \frac{\widetilde{\mathcal{G}}}{\beta L(\gamma^x)^3\sqrt{2\alpha\beta\gamma^x+1}},$$
where $\widetilde{\mathcal{G}} \equiv \gamma^x K(\alpha\beta\gamma^x + 1) + \gamma^l L(3\alpha\beta\gamma^x + 2) - [\gamma^x K + \gamma^l L(\alpha\beta\gamma^x + 2)]\sqrt{2\alpha\beta\gamma^x + 1}$.

The derivative is positive if $\widetilde{\mathcal{G}}$ is positive, or an equivalent condition is

$$\gamma^x(K/L)^2 + 2\gamma^l(K/L) - 2\alpha\beta(\gamma^l)^2 \equiv \mathcal{G}(K/L) > 0.$$

This inequality indeed holds under assumption (A3) $K/L \in (\gamma^l\lambda^c, \infty)$ since $\mathcal{G}(K/L) = 0$ at $K/L = \gamma^l\lambda^c$ and $\mathcal{G}'(K/L) > 0$ for $K/L \in (\gamma^l\lambda^c, \infty)$.

This result implies

$$\frac{\partial G^c}{\partial \gamma^x} = \phi L \frac{\partial w^c}{\partial \gamma^x} > 0,$$
$$\frac{\partial F^c}{\partial \gamma^x} = \frac{1}{\phi}\frac{\partial G^c}{\partial \gamma^x} > 0,$$

noting the following relations:

$$\begin{aligned} G^c &= \phi w^c L + (1-\phi) r^c K \\ &= \phi w^c L \\ &= \phi(F^c - \delta K - T), \end{aligned}$$

where $F^c \equiv F(K^{yc}, X^c)$ is the final-good output under optimal taxation.

The effect on the automation capital is

$$K^{xc} = \frac{\theta^c X^c}{\gamma^x} = \frac{\gamma^l\theta^c L}{\gamma^x(1-\theta^c)} : \qquad \frac{\partial K^{xc}}{\partial \gamma^x} = \underbrace{-\frac{\gamma^l\theta^c L}{(\gamma^x)^2(1-\theta^c)}}_{<0} + \underbrace{\frac{\gamma^l L}{\gamma^x(1-\theta^c)^2}\frac{\partial \theta^c}{\partial \gamma^x}}_{>0},$$

which reduces to

$$\frac{\partial K^{xc}}{\partial \gamma^x} = \frac{\widetilde{\mathcal{H}}}{(\gamma^x)^2(2\alpha\beta\gamma^x+1)^{\frac{3}{2}}},$$
where $\widetilde{\mathcal{H}} \equiv \gamma^x L(2\alpha\beta\gamma^x + 1)\sqrt{2\alpha\beta\gamma^x + 1} - [\alpha\beta(\gamma^x)^2 K + \gamma^l L(3\alpha\beta\gamma^x + 1)]$.

The sign of the derivative is determined by that of $\widetilde{\mathcal{H}}$, which reduces to

$$\text{sign}\left(\frac{\partial K^{xc}}{\partial \gamma^x}\right) = \text{sign}\ \widetilde{\mathcal{H}} = \text{sign}\ \mathcal{H},$$
$$\text{where}\ \ \mathcal{H}(K/L) \equiv -\alpha\beta(\gamma^x)^2(K/L)^2 - 2\gamma^l(3\alpha\beta\gamma^x+1)(K/L) + \alpha\beta(\gamma^l)^2(8\alpha\beta\gamma^x+3).$$

Further inspection reveals

$$\mathcal{H}(K/L)\begin{cases}\geq 0 & \text{if } K/L \in (\gamma^l\lambda^c, \eta] \\ < 0 & \text{if } K/L \in (\eta, \infty)\end{cases},$$
$$\text{where}\ \ \mathcal{H}(K/L) = 0 \text{ at } K/L = \eta \equiv \frac{\gamma^l\left[(2\alpha\beta\gamma^x+1)^{\frac{3}{2}} - (3\alpha\beta\gamma^x+1)\right]}{\alpha\beta(\gamma^x)^2}.$$

That is, a rising γ^x increases K^{xc} (or decreases $K^{yc} = K - K^{xc}$) if K/L is sufficiently low, and the reverse is true if K/L is sufficiently high.

Appendix 3: Proof of Proposition 3

The maximization problem of government 1 is formulated as

$$\max_{t_1^x, t_1^y, t_1^l}\ G_1 = \phi w_1 L_1 + (1-\phi)r\overline{K}_1,$$
$$\text{s.t.}\ \begin{cases}\sum_{j=x,y} t_1^j K_1^j + t_1^l L_1 \geq T_1, \\ w_1 \geq 0, \quad w_1 + t_1^l \geq 0, \\ r \geq 0, \quad r + t_1^x \geq 0, \quad r + t_1^y \geq 0.\end{cases}$$

As discussed in the text, with the labor tax chosen so as to balance the budget, the national welfare can be rewritten as

$$G_1 = \phi w_1 L_1 + (1-\phi)r\overline{K}_1 = \phi[F(K_1^y, X_1) - \delta K_1^2/\overline{K}_1 - T_1 + r(\overline{K}_1 - K_1)] - (2\phi-1)r\overline{K}_1,$$
$$\text{where}\ \ F(K_1^y, X_1) = \frac{\lambda_1(\gamma^x K_1 + \gamma^l L_1)}{1+\gamma^x\lambda_1}\left(\alpha - \frac{\lambda_1}{2\beta}\right),$$
$$K_1 = \frac{\overline{K}_1}{\delta}\left(\alpha - r - t_1^y - \frac{\lambda_1}{\beta}\right), \qquad r = \alpha - \delta - \frac{1}{2}\sum_{i=1}^{2}\left(t_i^y + \frac{\lambda_i}{\beta}\right).$$

Since the welfare above is the function of t_1^y and λ_1, government 1 maximizes it by choosing the two variables, given government 2's choice of t_2^y and λ_2.

First we solve the maximization problem while ignoring the non-negativity constraints, and then check whether the equilibrium outcomes satisfy the constraints. As shown in the text, solving the FOCs of government 1 yields the best response functions:

$$t_1^y = \frac{\gamma^x(\beta t_2^y + \lambda_2) + 1 + \beta\delta\gamma^x - \sqrt{2\alpha\beta\gamma^x + 1}}{2\beta\gamma^x} + \frac{\delta(2\phi - 1)}{2\phi},$$
$$\lambda_1^* = \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\gamma^x}.$$

Another solution of FOCs has $\lambda_1 = -(\sqrt{2\alpha\beta\gamma^x + 1} + 1)/\gamma^x < 0$, which is clearly irrelevant. Similarly, we obtain analogous expressions for government 2.

The best response functions of both governments constitute a system of equations of at most first order. Therefore, the solution of it is unique and is given by

$$t_1^{y*} = t_2^{y*} = t^* = \frac{\delta(3\phi - 1)}{\phi},$$
$$\lambda_1^* = \lambda_2^* = \lambda^* = \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\gamma^x} = \lambda^c.$$

The FOCs are also sufficient for maximization from the observations that at (t_i^{y*}, λ_i^*), G_i is concave with respect to (t_i^y, λ_i), that is, (a)$\partial^2 G_i/\partial(t_i^y)^2 < 0$; (b)$\partial^2 G_i/\partial(\lambda_i)^2 < 0$; and (c)$[\partial^2 G_i/\partial(\lambda_i)^2] \cdot [\partial^2 G_i/\partial(t_i^y)^2] - [\partial^2 G_i/(\partial t_i^y \partial\lambda_i)]^2 > 0$. Government i achieves λ_i^* by choosing $\Delta t_i = 0$, or $t_i^{x*} = t_i^{y*} = t^*$ and balances it budget by setting t_i^l to

$$t_1^{l*} = t_2^{l*} = t^{l*} = \frac{T - t^* K}{L},$$

where we used the symmetry of countries, $K_i = K$, $L_i = L$, and $T_i = T$. The optimal labor tax t^{l*} becomes positive if $T > 0$ and $t^* = \delta(3\phi - 1)/\phi > 0$ or δ is sufficiently small and it becomes negative if T is sufficiently small.

Noting $t^* = \delta(3\phi - 1)/\phi \leq 2\delta$ and assumption (A1), we see $t^* < t^c = \alpha - \delta - \lambda^c/\beta$. This implies $r^* = \alpha - \delta - t^* - \lambda^*/\beta > 0$ and $r^* + t^* > 0$. As in the closed-economy case, we can also show $w^* + t^{l*} > 0$ and $w^* > 0$. Since $\lambda^* = \lambda^c$ and $K_i^* = \overline{K}_i = K$ hold, the automation cutoff and the final-good output are also the same as those under optimal taxation in the closed economy, $\theta_i^* = \theta_i^c \in (0, 1)$ and $F(K_i^{y*}, X_i^*) = F(K_i^{yc}, X_i^c)$. As shown in Appendix 1, given K_i, F_i is maximized at $\lambda_i = \lambda_i^*$.

Appendix 4: Proof of Proposition 4

We already know $t_i^* < t^c$ from discussions in the text and Appendix 3. This immediately implies $r^* > r^c$, $t_i^{l*} > t^{lc}$, and $w_i^* < w^c$. A less immediate comparison is about the social welfare:

$$G_i^* - G^c = \frac{K(2\phi-1)\widetilde{\mathcal{I}}}{\phi\beta\gamma^x\sqrt{2\alpha\beta\gamma^x+1}},$$
$$\widetilde{\mathcal{I}} \equiv \phi(2\alpha\beta\gamma^x+1) - [\beta\gamma^x(\phi(\alpha-4\delta)+\delta)+\phi]\sqrt{2\alpha\beta\gamma^x+1},$$

where we can confirm that $\beta\gamma^x(\phi(\alpha-4\delta)+\delta)+\phi \equiv I(\phi)$ is positive by noting (a)$I(1/2)>0$; (b)$I(1)>0$; and (c)$I(\phi)$ is linear in ϕ.

The sign of the welfare difference is determined by that of $\widetilde{\mathcal{I}}$, which reduces to

$$\text{sign}\,(G_i^* - G^c) = \text{sign}\,\widetilde{\mathcal{I}} = \text{sign}\,\mathcal{I},$$
$$\text{where}\;\; \mathcal{I}(\delta) \equiv -\beta\gamma^x(4\phi-1)^2\delta^2 + 2\phi(4\phi-1)(\alpha\beta\gamma^x+1)\delta - (\alpha\phi)^2\beta\gamma^x.$$

Further inspection reveals

$$\mathcal{I}(\delta)\begin{cases} <0 & \text{if } \delta\in(0,\delta^a)\cup(\delta^b,\infty) \\ \geq 0 & \text{if } \delta\in[\delta^a,\delta^b] \end{cases},$$
$$\text{where}\;\; \delta^a \equiv \frac{\phi(\alpha\beta\gamma^x+1-\sqrt{2\alpha\beta\gamma^x+1})}{\beta\gamma^x(4\phi-1)} < \delta^b \equiv \frac{\phi(\alpha\beta\gamma^x+1+\sqrt{2\alpha\beta\gamma^x+1})}{\beta\gamma^x(4\phi-1)}.$$

Since assumption (A1) implies $\delta<\overline{\delta}<\delta^a$, we can conclude $\mathcal{I}(\delta)<0$ for $\delta\in(0,\overline{\delta})$.

Appendix 5: Proof of Proposition 5

We first note that equilibrium $\lambda_i^* = K_i^{y*}/X_i^* = \lambda^*$ under tax competition is the same as the one under optimal taxation in the closed economy, but the equilibrium t_i^* (and t_i^{l*}) is different in the two cases. Some equilibrium outcomes such as θ_i^*, X_i^*, K_i^{y*}, and F_i^* only depend on λ_i^*, so that the effect of a higher γ^x on these outcomes are the same as those stated in Proposition 2.

Other equilibrium outcomes respond to a higher γ^x differently than they do in the closed-economy case, because they depend on t_i^*. We can immediately see

$$t_i^* = \delta(3\phi - 1)/\phi : \qquad \frac{\partial t_i^*}{\partial \gamma^x} = 0,$$

$$t_i^{l*} = \frac{T - t_i^* K}{L} : \qquad \frac{\partial t^{lc}}{\partial \gamma^x} = -\frac{K}{L}\frac{\partial t_i^*}{\partial \gamma^x} = 0,$$

$$r^* = \alpha - \delta - t_i^* - \lambda_i^*/\beta : \qquad \frac{\partial r^*}{\partial \gamma^x} = -\frac{1}{\beta}\frac{\partial \lambda_i^*}{\partial \gamma^x} > 0,$$

$$w_i^* = \frac{\gamma^l (\lambda_i^*)^2}{2\beta} - t_i^{l*} : \qquad \frac{\partial w^*}{\partial \gamma^x} = \frac{\gamma^l \lambda_i^*}{\beta}\frac{\partial \lambda_i^*}{\partial \gamma^x} < 0.$$

The effect on the national welfare is not obvious:

$$\frac{\partial G_i^*}{\partial \gamma^x} = \frac{\widetilde{\mathcal{G}} + (2\phi - 1)\widetilde{\mathcal{J}}}{\beta(\gamma^x)^3\sqrt{2\alpha\beta\gamma^x + 1}},$$

$$\text{where } \widetilde{\mathcal{G}} \equiv \gamma^x K(\alpha\beta\gamma^x + 1) + \gamma^l L(3\alpha\beta\gamma^x + 2) - [\gamma^x K + \gamma^l L(\alpha\beta\gamma^x + 2)]\sqrt{2\alpha\beta\gamma^x + 1},$$

$$\widetilde{\mathcal{J}} \equiv [\gamma^x K - \gamma^l L(\alpha\beta\gamma^x + 2)]\sqrt{2\alpha\beta\gamma^x + 1} - [\gamma^x K(\alpha\beta\gamma^x + 1) - \gamma^l L(3\alpha\beta\gamma^x + 2)].$$

As shown in Appendix 2, we know $\widetilde{\mathcal{G}} > 0$. We can check that an equivalent condition of $\widetilde{\mathcal{J}} < 0$ always holds:

$$\text{sign } \widetilde{\mathcal{J}} = \text{sign } \mathcal{J},$$

$$\text{where } \mathcal{J} \equiv (3\alpha\beta\gamma^x + 2)^2 - (\alpha\beta\gamma^x + 2)^2(2\alpha\beta\gamma^x + 1) = -2(\alpha\beta\gamma^x)^3 < 0.$$

That is, $\partial G_i^*/\partial \gamma^x$ is positive if $\phi = 1/2$ and it is negative if $\phi = 1$. Since $\partial G_i^*/\partial \gamma^x$ is linear in ϕ, there exists $\phi^* \in (1/2, 1)$ such that

$$\frac{\partial G_i^*}{\partial \gamma^x} \begin{cases} > 0 & \text{if } \phi \in (1/2, \phi^*) \\ = 0 & \text{if } \phi = \phi^* \\ < 0 & \text{if } \phi \in (\phi^*, 1] \end{cases}, \quad \text{where } \phi^* \equiv \frac{1}{2}\left(1 + \frac{\widetilde{\mathcal{G}}}{\widetilde{\mathcal{J}}}\right).$$

Appendix 6: Effect of non-automation technology

Closed-economy case The effects of α in an closed economy with internationally immobile capital are as follows:

$$\lambda^c = \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\gamma^x} : \qquad \frac{\partial \lambda^c}{\partial \alpha} = \frac{\beta}{\sqrt{2\alpha\beta\gamma^x + 1}} > 0,$$

$$t^c = \alpha - \delta - \lambda^c/\beta : \qquad \frac{\partial t^c}{\partial \alpha} = \frac{\sqrt{2\alpha\beta\gamma^x + 1} - 1}{\sqrt{2\alpha\beta\gamma^x + 1}} > 0,$$

$$t^{lc} = \frac{T - t^c K}{L} : \qquad \frac{\partial t^{lc}}{\partial \alpha} = -\frac{K}{L}\frac{\partial t^c}{\partial \alpha} < 0,$$

$$w^c = \frac{\gamma^l (\lambda^c)^2}{2\beta} - t^{lc} : \qquad \frac{\partial w^{lc}}{\partial \alpha} = \frac{\gamma^l \lambda^c}{\beta}\frac{\partial \lambda^c}{\partial \alpha} - \frac{\partial t^{lc}}{\partial \alpha} > 0,$$

$$\theta^c = \frac{\gamma^x (K - \gamma^l \lambda^c L)}{\gamma^x K + \gamma^l L} : \qquad \frac{\partial \theta^c}{\partial \alpha} = -\frac{\gamma^x \gamma^l L}{\gamma^x K + \gamma^l L}\frac{\partial \lambda^c}{\partial \alpha} < 0,$$

$$X^c = \frac{\gamma^l L}{1 - \theta^c} : \qquad \frac{\partial X^c}{\partial \alpha} = \frac{\gamma^l L}{(1 - \theta^c)^2}\frac{\partial \theta^c}{\partial \alpha} > 0,$$

$$K^{xc} = \frac{K - \gamma^l \lambda^c L}{1 + \gamma^x \lambda^c} : \qquad \frac{\partial K^x}{\partial \alpha} = -\frac{\beta(\gamma^x K + \gamma^l L)}{(2\alpha\beta\gamma^x + 1)^{\frac{3}{2}}} < 0,$$

$$r^c = 0 : \qquad \frac{\partial r^c}{\partial \alpha} = 0,$$

$$G^c = \phi w^c L : \qquad \frac{\partial G^c}{\partial \alpha} = \phi L \frac{\partial w^c}{\partial \alpha} > 0,$$

$$F^c = w^c L + \delta K + T : \qquad \frac{\partial F^c}{\partial \alpha} = L \frac{\partial w^c}{\partial \alpha} > 0.$$

Tax-competition case The effects of α in an economy with two symmetric countries and international mobile capital are as follows:

$$\lambda_i^* = \frac{\sqrt{2\alpha\beta\gamma^x+1}-1}{\gamma^x}: \qquad \frac{\partial\lambda^*}{\partial\alpha} = \frac{\beta}{\sqrt{2\alpha\beta\gamma^x+1}} > 0,$$

$$t_i^* = \delta(3\phi-1)/\phi: \qquad \frac{\partial t^*}{\partial\alpha} = 0,$$

$$t_i^{l*} = (T_i - t_i^* K_i^*)/L_i: \qquad \frac{\partial t_i^{l*}}{\partial\alpha} = 0,$$

$$w_i^* = \frac{\gamma^l(\lambda_i^*)^2}{2\beta} - t_i^{l*}: \qquad \frac{\partial w_i^*}{\partial\alpha} > 0,$$

$$\theta_i^* = \frac{\gamma^x(K_i^* - \gamma^l\lambda_i^* L_i)}{\gamma^x K_i^* + \gamma^l L_i}: \qquad \frac{\partial\theta_i^*}{\partial\alpha} = -\frac{\gamma^x\gamma^l L}{\gamma^x K + \gamma^l L}\frac{\partial\lambda_i^*}{\partial\alpha} < 0,$$

$$X_i^* = \frac{\gamma^l L_i}{1-\theta_i^*}: \qquad \frac{\partial X_i^*}{\partial\alpha} = \frac{\gamma^l L}{(1-\theta_i^*)^2}\frac{\partial\theta_i^*}{\partial\alpha} > 0,$$

$$K_i^{x*} = \frac{K - \gamma^l\lambda_i^* L_i}{1+\gamma^x\lambda_i^*}: \qquad \frac{\partial K_i^{x*}}{\partial\alpha} = -\frac{\beta(\gamma^x K + \gamma^l L)}{(2\alpha\beta\gamma^x+1)^{\frac{3}{2}}} < 0,$$

$$r^* = \alpha - \delta - t_i^* - \lambda_i^*/\beta: \qquad \frac{\partial r^*}{\partial\alpha} = \frac{\sqrt{2\alpha\beta\gamma^x+1}-1}{\sqrt{2\alpha\beta\gamma^x+1}} > 0,$$

$$F_i^* = \frac{\lambda_i^*(\gamma^x K_i^* + \gamma^l L_i)}{1+\gamma^x\lambda_i^*}\left(\alpha - \frac{\lambda_i^*}{2\beta}\right): \qquad \frac{\partial F_i^*}{\partial\alpha} = \frac{\gamma^x K_i^* + \gamma^l L_i}{\gamma^x}\left(\frac{\sqrt{2\alpha\beta\gamma^x+1}-1}{\sqrt{2\alpha\beta\gamma^x+1}}\right) > 0,$$

$$G_i^* = \phi(F_i^* - \delta K_i^* - T_i) - (2\phi-1)r^*\overline{K}_i: \qquad \frac{\partial G_i^*}{\partial\alpha} = \frac{\left(\sqrt{2\alpha\beta\gamma^x+1}-1\right)\left[(1-\phi)\gamma^x K + \gamma^l L\right]}{\gamma^x\sqrt{2\alpha\beta\gamma^x+1}} > 0,$$

noting that we have $K_i^* = K$ for $i \in \{1,2\}$ in tax-competition equilibrium and symmetric country means $\overline{K}_i/(2K) = \overline{L}_i/(2L) = \overline{T}_i/(2T) = 1/2$.

Appendix 7: Asymmetric country size

We here lay out results of tax competition between unequal-sized countries where country 1 is larger than country 2, i.e., $\overline{K}_1/(2K) = \overline{L}_1/(2L) = \overline{T}_1/(2T) = \kappa \in (1/2, 1)$. The capital taxes in tax-competition equilibrium, common to both types of capital, are

$$t_1^{x*} = t_1^{y*} = t_1^* = \frac{\delta(3\phi-1)}{\phi} + \underbrace{\frac{\delta(2\phi-1)(2\kappa-1)(3\kappa-1)}{3\phi\kappa(1-\kappa)}}_{>0},$$

$$t_2^{x*} = t_2^{y*} = t_2^* = \frac{\delta(3\phi-1)}{\phi} + \underbrace{\frac{\delta(2\phi-1)(2\kappa-1)(3\kappa-2)}{3\phi\kappa(1-\kappa)}}_{\gtreqless 0},$$

where the second term in both t_1^* and t_2^* vanishes when the two countries are of equal size, i.e., $\kappa = 1/2$. Government 1 always sets its capital tax higher than government 2:

$$t_1^* - t_2^* = \frac{\delta(2\phi - 1)(2\kappa - 1)}{3\phi\kappa(1-\kappa)} > 0.$$

The higher capital tax in country 1 leads to capital relocation from country 1 to 2. That is, the share of capital operating in country 1 is smaller than its share of capital endowment:

$$\frac{K_1^*}{2K} = k = \kappa - \frac{(2\phi - 1)(2\kappa - 1)}{3\phi} < \kappa = \frac{\overline{K}_1}{2K}.$$

To see why the capital tax in the larger country is set higher, let us look at the difference in the elasticity of tax base, i.e., capital, with respect to capital tax. In the asymmetric-country setting, given taxes, the rental rate and the capital demand are

$$r = \alpha - \delta - \sum_{i=1}^{2} \kappa_i \left(t_i^y + \frac{\lambda_i}{\beta} \right),$$

$$K_i = \frac{\overline{K}_i}{\delta} \left(\alpha - r - t_i^y - \frac{\lambda_i}{\beta} \right) = \frac{\overline{K}_i}{\delta} \left[\delta + \sum_{i=1}^{2} \kappa_i \left(t_i^y + \frac{\lambda_i}{\beta} \right) - t_i^y - \frac{\lambda_i}{\beta} \right] \quad \text{for } i \in \{1, 2\}.$$

Supposing that the two countries choose taxes as if tax is immobile, the tax-elasticity of capital demand is

$$\varepsilon_1 = -\frac{\partial K_1}{\partial t_1^y} \frac{t_1^y}{K_1} = \frac{(1-\kappa)\overline{K}_1}{\delta} \frac{t^c}{\overline{K}_1/\delta} = (1-\kappa)t^c,$$

$$\varepsilon_2 = -\frac{\partial K_2}{\partial t_2^y} \frac{t_2^y}{K_2} = \frac{\kappa\overline{K}_2}{\delta} \frac{t^c}{\overline{K}_2/\delta} = \kappa t^c,$$

where the variable with superscript c is the outcome of optimal taxation in the closed economy (see Proposition 1). We clearly see $\varepsilon_1 < \varepsilon_2$.

Unlike capital taxes, the ratio of non-automation capital to intermediates, λ_i^*, is not different from that in the closed-economy case, $\lambda_i^* = \lambda^c = (\sqrt{2\alpha\beta\gamma^x + 1} - 1)/\gamma^x$. From this result and the fact that the equilibrium capital taxes do not depend on the automation efficiency γ^x, we can show that a higher γ^x has the same effects on equilibrium outcomes as in the symmetric-country case summarized in Proposition 5.

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