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Product Dynamics and Macroeconomic Shocks: Insights from a DSGE model and Japanese Data *

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Abstract

This paper investigates the relationships between aggregate shocks and individual products in the economy, aiming to inform macroeconomic policy and address sectoral imbalances. Using Japanese manufacture census data from 1992 to 2013, we analyze product sales growth (intensive margins) and the number of product-producing plants (extensive margins) to identify patterns and heterogeneity across products and product categories. We employ a structural model to analyze the sources of product business cycles, finding that product-specific demand shocks play a crucial role in explaining product sales dynamics, while both product-specific and plant-product specific shocks are essential for understanding extensive margins. Our findings offer important implications for the design of targeted and effective policies that promote stability, growth, employment, and inclusiveness across diverse sectors of the economy.

Keywords: Product, Plant, DSGE model, Extensive margin, Intensive Margin
JEL classification: D24, E23, E32, L11, L60.

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1 Introduction

Understanding the drivers of business cycles and the heterogeneous responses of different products to aggregate shocks are crucial for informing macroeconomic policy, addressing sectoral imbalances, and improving distribution within an economy. By gaining insights into these dynamics, policymakers can design targeted and effective policies that promote stability, growth, employment, and inclusiveness across diverse sectors of the economy.

To achieve this objective, this paper investigates the relationships between aggregate conditions and individual products in the economy. We examine whether specific patterns exist across products or product categories and explore the similarities and differences between product sales growth (intensive margins) and the number of product-producing plants (extensive margins). Furthermore, we employ a structural model to analyze the sources of product business cycles.

Using Japanese manufacturing census data, which includes over 2,000 products defined at the 6-digit level from 1992 to 2013, we first document some stylized facts about product dynamics, specifically focusing on their similarities and differences. An important advantage of using Japanese data is its annual frequency, which allows for more precise analysis of business cycle dynamics compared to less frequent surveys conducted every five years in the US.

On one hand, we find that total sales exhibit a positive correlation with GDP for the majority of products. Similarly, the growth in the number of product-producing plants is positively correlated with GDP for most products. Additionally, the standard deviations of product sales and plant growth are higher than GDP for all products. On the other hand, we observe significant heterogeneity across products: the cyclicality of sales tends to be higher with a higher share of the product, while the cyclicality of plant growth is independent of the sales share of the product. Furthermore, both the standard deviations of sales and plant growth are lower for products with a higher sales share. We also identify specific patterns depending on product categories.

To gain a deeper understanding of these product dynamics, we develop a theoretical model that captures both intensive and extensive margins. Our approach involves focusing on each product in the sample and analyzing their heterogeneity using the structural model. We explore the driving forces behind different product-specific adjustments in response to aggregate conditions, distinguishing between aggregate shocks that affect both the product and other products and product-specific shocks.

Specifically, we calibrate and estimate parameter values in the theoretical model for over 2,000 products. Our estimation results indicate that individual products are primarily influenced by their unique demand and supply shocks, rather than by generalized market trends or aggregate disruptions. Notably, these product-specific shocks display substantial variation, with some products exhibiting high standard deviations indicative of substantial volatility, while others remain relatively stable. Additionally, the study found a broad spectrum of values for the parameter that manages the influence of overall labor productivity on specific sectors, emphasizing its critical function in aligning the fluctuations in GDP primarily propelled by productivity shocks.

The model successfully replicates the observed product dynamics and heterogeneous patterns observed in the data. We find that product-specific demand shocks play a crucial role in explaining product sales dynamics (intensive margins), while considering not only product-specific shocks but also plant-product specific shocks is essential for understanding extensive margins. Although the overall contribution of shocks in explaining the variance remains similar across product categories, we find that aggregate shocks become more prominent for products with small standard deviations in both intensive and extensive margins.

Our model acknowledges the interaction between specific products and other sectors of the economy, although we simplify the detailed interaction across products by estimating product-specific effects individually. Interestingly, we find that aggregate conditions have limited influence on the majority of products, with only a limited number of products with relatively larger sales shares being affected by aggregate shocks. This result suggests that aggregate stabilization policies, such as monetary and fiscal policy, may be more effective for products with higher sales shares. Furthermore, it implies that for products with lower shares, policies targeting specific products would be more effective in reducing macroeconomic volatility and promoting inclusive macroeconomic fluctuations.

The literature on multi-product producing plants and firms, including works by Feenstra and Ma (2008), Bernard et al. (2010), Eckel and Neary (2010), Nocke and Yeaple (2014), Mayer et al. (2014), and Forslid and Okubo (2023) explores various aspects such as product switching, flexible manufacturing, globalization, market size, firm location, and competition. These studies collectively contribute to our understanding of how these factors interact and shape the behavior, productivity, and international trade of multi-product firms in different economic contexts. Since our focus is on the business cycle aspects of product dynamics, we build a Dynamic stochastic general equilibrium (DSGE) model that shares the features of this class of models, embedding endogenous entry of product-producing plants, as discussed in Bilbiie et al. (2012), Ghironi and Melitz (2005), and Hamano and Zanetti (2017). However, our theoretical model extends this literature by capturing the dynamics of multi-product producing plants and their interactions with specific products and other parts of the economy. In particular, our model is an extended version that simplifies the interaction of a product within the economy, building on the framework proposed by Hamano and Oikawa (2022).

Our paper is also related to the literature that investigates product dynamics, the relationship between product-specific dynamics, aggregate conditions, and the role of various shocks in driving business cycles.

Gabaix (2011) highlights the role of large firms in aggregate fluctuations, while Carvalho and Gabaix (2013) focuses on input-output linkages across firms and sectors. Di Giovanni et al. (2014) emphasize the importance of firm-level heterogeneity and production networks in the transmission of aggregate shocks, while Foerster et al. (2011) disentangle the effects of sector-specific and aggregate shocks on industrial production. These papers provide valuable insights into various aspects of aggregate fluctuations and their relationships with firm and product dynamics. However, our paper takes a different approach by estimating and calibrating product-specific dynamics to characterize their relationships with aggregate conditions.

In addition, Broda and Weinstein (2010) analyze the creation and destruction of products using US microdata, highlighting the significant impact of new products on prices and welfare. Hottman et al. (2016) analyze firm heterogeneity and the sources of variations in firm-level outcomes using the same US microdata. Foster et al. (2016) explore the slow growth of new plants, attributing it to demand rather than supply side differences from older and larger plants. While our findings align with these studies regarding the importance of product-specific demand shocks and the relatively minor role played by plant-specific supply shocks, we employ a structural estimation approach based on maximum likelihood to uncover our results. Furthermore, using the same source of Japanese data, Bernard and Okubo (2016), Borusyak and Okubo (2015), and Dekle et al. (2015) analyze the business cycles of multi-product producing plants. However, our focus is more on the product side and we structurally estimate the driving forces of product business cycles using a DSGE model.

Finally, our paper is also related to research that examines various aspects of product dynamics, including employment growth, innovation, productivity, and the role of product-specific regulations. Criscuolo et al. (2014) and Aghion et al. (2005) investigate the impact of product-specific regulations on employment growth and innovation, respectively, providing insights into the role of regulatory policies in shaping product dynamics. Syverson (2011) reviews the determinants of productivity, including the influence of product-specific regulations, while Bourlès et al. (2013) and Fiori et al. (2012) study the effects of upstream regulations on productivity growth and the interaction between product and labor market deregulation on employment outcomes. Additionally, Hamano and Okubo (2021) investigate period-specific regulation policies and their counterfactual outcomes in the Japanese economy. While these papers share similarities in terms of research questions, our paper takes a different approach in addressing these questions and specifically focuses on the relationship between aggregate conditions, product-specific factors, and their implications for macroeconomic policies.

The structure of our paper is as follows. In Section 2, we describe the data and present stylized facts on product dynamics emerging from Japanese census data. In Section 3, we introduce the theoretical model that captures the business cycle characteristics of product sales and the number of producing plants. We provide details of our calibration and estimation strategy, along with their results, in Section 4. Section 5 compares the implications of the theoretical model, derived from estimated and calibrated parameter values, with the actual data. We discuss the variance decompositions for product sales and product plant growth, highlighting their heterogeneity across products in Section 6. Finally, in Section 7, we provide concluding remarks.

2 Stylized Facts about Product Sales and Plant Dynamics

In this section, we present several stylized facts regarding product sales and plant dynamics. Typically, we investigate the cyclicality of product sales with respect to that of GDP, their standard deviations as well as the number of producing plants and their standard deviations for each product in the sample.

2.1 Data

The Census of Manufacture and Economic Census for Business Activity ('Kogyo Tokei' and 'Keizai sensasu', in Japanese respectively) are provided by the Ministry of Economy, Trade and Industry, and the Ministry of Internal Affairs and Communications. The Census is conducted at the plant level with more than 4 regular employees with annual frequency. The data covers all manufacturing products. Response rates are around 95 percent. The Census of Manufacture includes the number of regular employees and outputs at the 6-digit product level. We specifically use the periods from 1992 to 2013 to use the time-consistent product category as discussed in Pierce and Schott (2012).

2.2 Product Sales Shares

Firstly, we document the sales share of each product in the sample. The sales share for a given product is calculated as the median share of that product in the total annual sales throughout the sample period.[1] As anticipated, the median share of all products is quite small, amounting to 0.000105. Among the 2065 products, only 11 (0.48%) have a share larger than 1%. In Appendix A, Table 7 presents the ranking of products based on their total sales. The data reveals that certain products within the Transportation Equipment category contribute significantly to the Japanese economy, exhibiting high sales shares. In the sample, the sales share of the Transportation Equipment category is the highest, amounting to 16.86%.

The histogram of sales share for each of the 2065 products is given in Figure 15 also in Appendix. In Figure 15, various colors are employed to distinguish the products based on the first two digits of their

[1] In the sample, four products exhibit a zero share throughout the periods. These products are 105191, 105291, 244513, and 314191. Utilizing the mean product share yields nearly identical results, with a correlation coefficient of 0.9986 between the two product shares.

six-digit product code. Table 1 provided below outlines the definitions for each two-digit code.

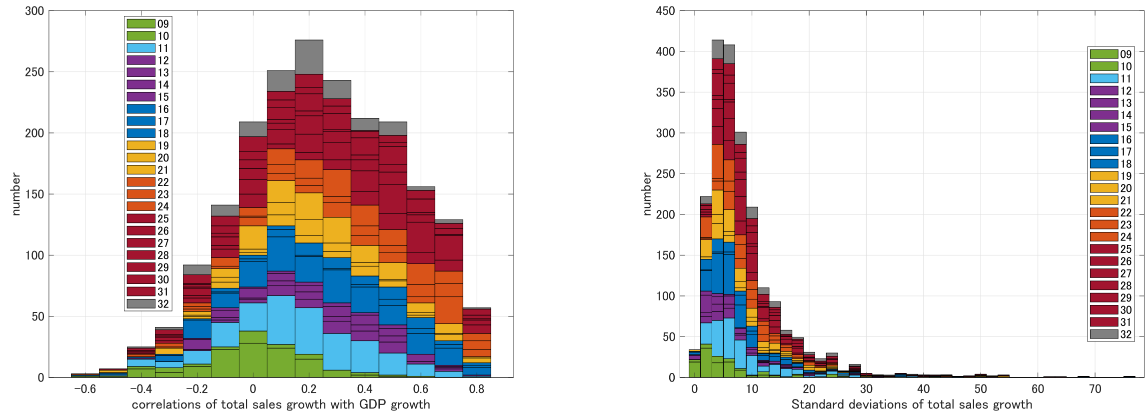
Table 1: Two digit product categories

Code	Description	share
09	FOOD	0.0817
10	BEVERAGES,TOBACCO, AND FEED	0.0337
11	TEXTILE	0.0217
12	Lumber and Wood products	0.0103
13	Furniture and fixtures	0.0089
14	Pulp and paper products	0.0244
15	Printing	0.0264
16	Chemical	0.0787
17	Petroleum and coal products	0.0353
18	Plastic products	0.0371
19	Rubber products	0.0107
20	Leather tanning, leather products and fur skins	0.0021
21	Ceramic, stone and clay products	0.0281
22	Iron and steel	0.0475
23	Non-ferrous metals and products	0.0220
24	Fabricated metal products	0.0526
25	General machinery	0.0394
26	Production machinery	0.0501
27	Business oriented machinery	0.0240
28	Electronicparts, devices and electronic circuits	0.0593
29	Electrical machinery	0.0531
30	Information and communication electronics	0.0454
31	Transportaion equipment	0.1686
32	Other manufacturing	0.0155

2.3 Product Sales Dynamics

In this subsection, we examine the business cycle characteristics of total sales for each product, referred to as *intensive margins.* To quantify, the left panel in Figure 1 depicts the histogram of the correlation between total sales growth and GDP for each of the 2065 products. The correlation coefficients range from -0.572 to 0.895. Despite significant cross-product variation, 84.5% of products exhibit positive correlations with GDP, with a median correlation of 0.2920. The right panel in the figure displays the histogram of standard deviations relative to GDP for each product. The standard deviations of total sales growth for all products exceed the standard deviation of GDP growth, with a median value of 7.638 times that of GDP.

Furthermore, we find that the correlations of the sales growth of products with the growth of GDP tend to be high for the products with higher sales share, while the standard deviations of total sales growth are smaller for the products with higher sales share. Figure 2 shows these patterns with scatter plots. Indeed, the correlation between the correlation of the sales growth with GDP and the product shares is 0.3177 while the correlation between the standard deviations of the sales growth and the product shares is -0.5716 in

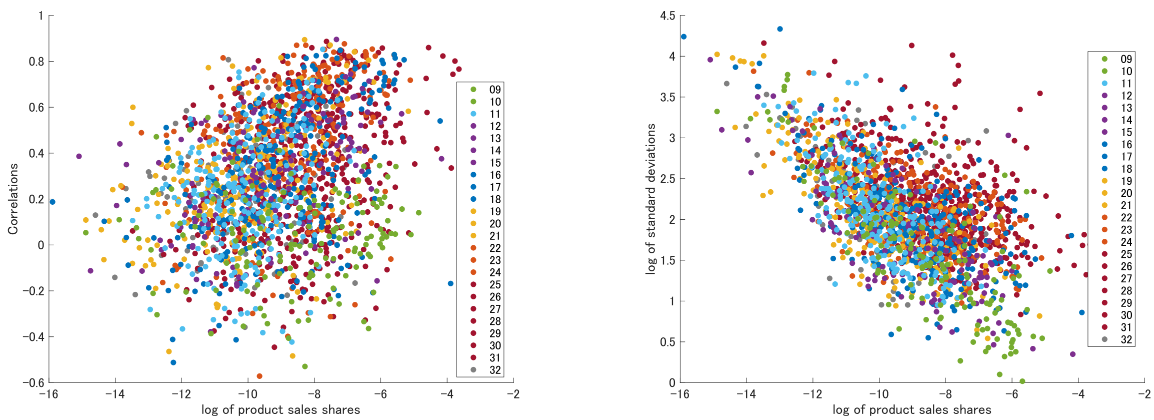


(a) Correlations

(b) Standard deviations

Figure 1: Histogram of Correlations and Standard deviations with respect to GDP: product sales growth

Note: Vertical axes measure the number of products. Horizontal axes represent the correlations between the product sales growth and GDP growth (left panel) or the standard deviations of the product total sales growth (right panel).



(a) Correlations

(b) Standard deviations

Figure 2: Correlations and Standard Deviations with Product sales share

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the product sales growth and GDP growth (left panel) or the standard deviations of the product total sales growth (right panel).

Figure 2.

Are there any systematic patterns according to product categories? We find that all categories of product show a positive relation between the correlation of the sales growth with GDP and the product sales shares, except the products in the categories of Food, and Beverages, Tobacco, and Feed (coded as 09 and 10). As the two panels in Figure 2 reveal, the products in these categories not only show lower correlations with

GDP, but also some products within these categories display lower standard deviations.[2] [3]

In a nutshell, when examining the business cycle characteristics of intensive margins, we find that most products exhibit positive correlations with GDP. Products with higher sales shares generally demonstrate stronger correlations with GDP and lower standard deviations. Meanwhile, consumable goods exhibit lower procyclicality and some of them show low standard deviations of sales.

2.4 Product Plant Dynamics

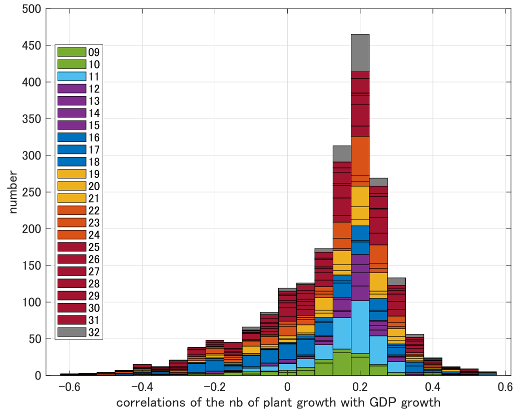
Dynamics of total sales for each product are influenced not only by the expansion or contraction within the existing number of plants, but also by changes in the number of plants producing the product itself. The latter is referred to as *extensive margins* From the data, we observe a similar pattern in the histograms for extensive margins as for intensive margins when it comes to the correlation and standard deviations relative to GDP. As illustrated in the left panel of Figure 3, the correlations between the number of plants producing each product and GDP range from -0.584 to 0.595, with over 83% of products displaying positive correlations between plant growth and GDP. The median correlation is 0.192. The right panel in the figure shows the histogram of standard deviations for the growth in the number of plants producing each product, measured relative to the standard deviation of GDP. For all products, their standard deviations are higher than GDP, with a median value of 8.6718 times that of GDP.

Although the cyclical properties of the number of plants producing each product are similar to those of the sales growth of each product, the pattern of correlations slightly differs with respect to the share of each product as shown in Figure 4. On one hand, the standard deviations of the growth of extensive margins for each product tend to be higher as the share of the product decreases (right panel in Figure 4). The correlation between the standard deviation of the number of product-producing plants and their sales shares is -0.5067. On the other hand, we do not observe any systematic association between the correlations and the sales share of each product (left panel in Figure 4). The correlation between the correlation of the number of product-producing plants and their sales shares is -0.0469. Furthermore, for extensive margins, we don't see any product-category-specific behaviors for both correlations and standard deviations.[4]

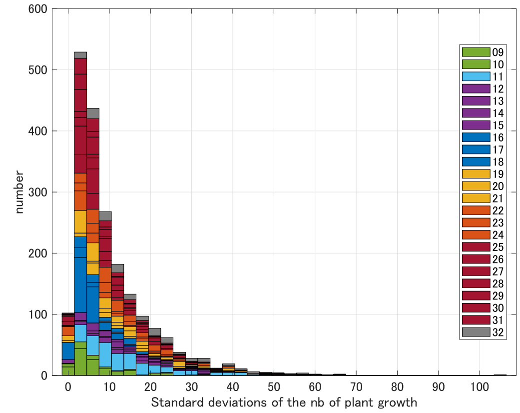
[2] To be precise, the correlations between the correlation of the sales growth with GDP and their sales shares are -0.0399 (09-10), 0.2581 (11), 0.4071 (12-15), 0.4512 (16-18), 0.4234 (19-21), 0.3966 (22-24), 0.2903 (25-31) and 0.1741 (32). The correlations between the standard deviations of the sales growth and their sales shares are -0.8120 (09 -10), -0.6740 (11), -0.7416 (12-15), -0.6206 (16-18), -0.7729 (19-21), -0.5797 (22-24), -0.4355 (25-31) and -0.5560 (32). In the above expressions, the number(s) in parentheses demonstrate the code of the two-digit product categories.

[3] The following 14 products contain no and/or infinite values when calculating the growth rates of total sales and the growth of the number of plants, and are therefore excluded from the analysis presented in this section. These products are 95191, 105191, 105291, 111191, 211111, 221291, 223591, 244323, 244513, 302313, 304191, 313111, 314119, and 314191.

[4] To be precise, the correlations between the correlation of the number of product producing plants with GDP and their sales shares are -0.0159 (09-10), 0.0357 (11), 0.0626 (12-15), 0.1191 (16-18), -0.0035 (19-21), -0.1076 (22-24), -0.0066 (25-31) and -0.1757 (32). The correlations between the standard deviations of the number of product producing plants and their sales shares are -0.3775 (09 -10), -0.3962 (11), -0.4495 (12-15), -0.4700 (16-18), -0.5995 (19-21), -0.4911 (22-24), -0.4861 (25-31) and -0.3954 (32). In the above expressions, the number(s) in parentheses demonstrate the code of the two-digit product categories.



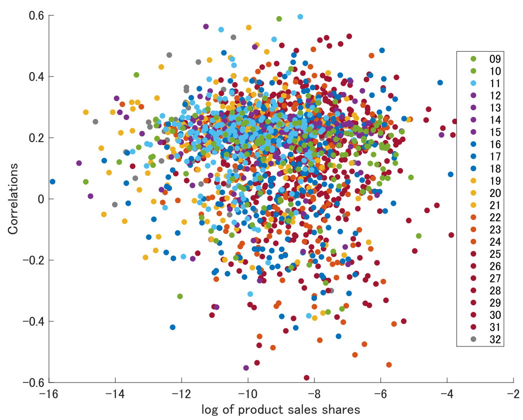
(a) Correlations



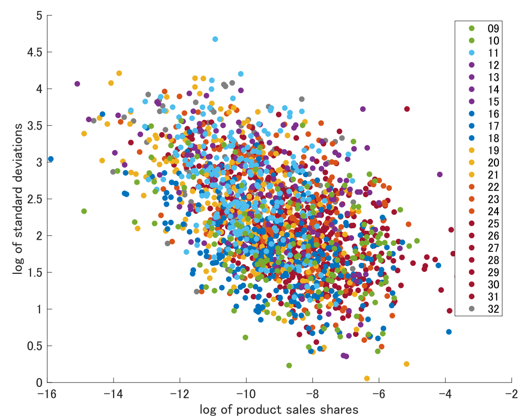
(b) Standard deviations

Figure 3: Histogram of Correlations and Standard deviations with respect to GDP: the number of product-producing plants

Note: Vertical axes measure the number of products. Horizontal axes represent the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).



(a) Correlations



(b) Standard deviations

Figure 4: Correlations and Standard Deviations with Product sales share

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

In summary, when examining the business cycle characteristics of extensive margins, we find a similar pattern to intensive margins regarding correlations and standard deviations relative to GDP. However, unlike intensive margins, there isn't a systematic increase in correlations of product sales growth with higher sales shares, though standard deviations tend to be lower as the sales share increases as a general pattern for all product categories.

3 The Model

To capture the dynamics of each product, we build a parsimonious model based on Hamano and Oikawa (2022), which explores plant dynamics in producing multiple products. In the economy, there are two sectors: a sector where product i is produced, and *other sectors* which are perfectly competitive and indexed with o. Product i is produced by plants that are monopolistically competitive in the market. In each period, there is an H_t mass of entrants. Upon entry, plants draw idiosyncratic productivity φ and a specific taste λ_i for product i. Among the total N_t number of plants, only a subset of $M_{i,t}$ number of plants decide to produce the product, since producing product ırequires operational fixed costs common across all plants. For product i, there are thus $M_{i,t}$ number of product varieties (defined as a combination of firm and product) which are endogenously determined.

3.1 Households

The representative household maximizes expected utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t(j)$, where $0 < \beta < 1$ is the exogenous discount factor. The utility of each individual household j at time t depends on her consumption $C_t(j)$ and supply of labor $L_t(j)$ as follows:

$$U_t(j) = A_t \ln C_t(j) - \chi_t \frac{L_t^{1+\varsigma}(j)}{1+\varsigma},$$

where A_t is an exogenous demand shifter at time t. $\chi_t > 0$ represents the disutility of supplying labor, and $\varsigma > 0$ is the inverse of the Frisch elasticity of labor supply.

Consumption is defined from two sub-baskets as

$$C_t(j) = \left(\frac{C_{i,t}(j)}{\alpha_{i,t}}\right)^{\alpha_{i,t}} \left(\frac{C_{o,t}(j)}{1-\alpha_{i,t}}\right)^{1-\alpha_{i,t}}, \tag{1}$$

where $\alpha_{i,t}$ stands for the stochastic preference weight on consumption of product i, $C_{i,t}(j)$, and $1 - \alpha_{i,t}$ stands for that of others, $C_{o,t}(j)$.

Product i is defined over a continuum of product varieties, Ω_i, and during each period t, only a subset of product varieties, $\Omega_{i,t} \subset \Omega_i$, is available. Each product variety is indexed by $\omega \in \Omega_{i,t}$. Specifically, $C_{i,t}(j)$ is defined as

$$C_{i,t}(j) = \left(\int_{\omega \in \Omega_t} \left(\lambda_i\left(\omega\right) c_{i,t}\left(j,\omega\right)\right)^{1-\frac{1}{\sigma}} d\omega\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $c_{i,t}\left(j,\omega\right)$ is consumption of each product variety ω by households j. $\lambda_i\left(\omega\right)$ is taste or "quality" assigned to each product variety ω. In particular, $\sigma > 1$ is the elasticity of substitution among varieties.

As a result of optimization, demand for each product variety, ω, is given by

$$\lambda_i(\omega) c_{i,t}(j,\omega) = \left(\frac{p_{i,t}(\omega)/\lambda_i(\omega)}{P_{i,t}}\right)^{-\sigma} C_{i,t}(j), \tag{2}$$

where $p_{i,t}(\omega)$ denotes the price of variety ω. $P_{i,t}$ stands for the price index of product i which is defined as

$$P_{i,t} = \left(\int_{\omega\in\Omega_t} \left(\frac{p_{i,t}(\omega)}{\lambda_i(\omega)}\right)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}, \tag{3}$$

Further, given the Cobb-Douglas aggregator as (1), demand for each product basket is found as

$$C_{i,t}(j) = \left(\frac{P_{i,t}}{P_t}\right)^{-1} \alpha_{i,t} C_t(j), \quad C_{o,t}(j) = \left(\frac{P_{o,t}}{P_t}\right)^{-1} \alpha_{i,t} C_t(j) \tag{4}$$

where $P_{o,t}$ denotes the price of other prodcuts. Finally, the price index of aggregate basket $C_t(j)$ is defined as

$$P_t = P_{i,t}^{\alpha_{i,t}} P_{o,t}^{1-\alpha_{i,t}} \tag{5}$$

In what follows, we choose P_t as a numeraire.

3.2 Production, Pricing and Producing Decision in the differentiated Product Sector

In each period, in sector i, a number of new plants, H_t, enter the market. Prior to entry, these plants are identical, but upon entry, each establishment draws a specific productivity level, φ, from a cumulative distribution, $G(\varphi)$, with support on $[\varphi_{\min}, \infty)$ and consumer taste level for product i, λ_i, from a cumulative distribution, $F_i(\lambda_i)$, with support on $[\lambda_{i\min}, \infty)$.

The production of product i requires fixed operational costs of $f_{i,t}/Z_t^{\theta_i}$ in effective labor units in every period where θ_i stands for the spillover from the aggregate labor productivity level Z_t to this specific sector producing product i. To produce $y_{i,t}(\varphi, \lambda_i)$ units, the plant with productivity level φ and taste λ_i demands the following amount of labor:

$$l_t(\varphi) = I_i \left[\frac{y_{i,t}(\varphi,\lambda_i)}{Z_t^{\theta_i}\varphi} + \frac{f_{i,t}}{Z_t^{\theta_i}}\right]. \tag{6}$$

where I_i is an indicator that takes 1 if the plant produces product i and 0 otherwise.

3.2.1 Product Production

The demand for each plant-specific product variety is characterized by equation (2). Profit maximization yields the following optimal price:

$$\rho_{i,t}(\varphi,\lambda_i) = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t^{\theta_i}\varphi}, \tag{7}$$

where $\rho_{i,t}(\varphi,\lambda_i)$ represents the real price of product i produced by a plant with productivity φ and a consumer taste λ_i. w_t is the real wage. Depending on the level of product-specific productivity, φ, and consumer taste λ_i, a product may or may not be produced. Thus, using equation (6), (7) and (4), if production materializes, the following real operational plant-specific profits are generated:

$$d_{i,t}(\varphi,\lambda_i) = \frac{1}{\sigma}\left(\frac{\rho_{i,t}(\varphi,\lambda_i)}{\lambda_i\rho_{i,t}}\right)^{1-\sigma}\alpha_{i,t}C_t(j) - w_t\frac{f_{i,t}}{Z_t^{\theta_i}}.$$

Here $\rho_{i,t} \equiv \frac{P_{i,t}}{P_t}$, which is the real price of the basket of product i.

Since the elasticity of substitution among varieties is assumed to be greater than one ($\sigma > 1$), a lower taste-adjusted real price implies higher profits. Due to the fixed operational costs, among N_t number of potential producers, only a subset number of $M_{i,t}$ plants produce the product i with $d_{i,t}(\varphi,\lambda_i) > 0$. For firm with productivity φ, there exists a zero profit consumer taste cutoff $\lambda_{i,t}^*(\varphi)$ for product i such that

$$d_{i,t}(\varphi,\lambda_i) = \frac{1}{\sigma}\left(\frac{\rho_{i,t}(\varphi,\lambda_i)}{\lambda_{i,t}^*(\varphi)\rho_{i,t}}\right)^{1-\sigma}\alpha_{i,t}C_t(j) - w_t\frac{f_{i,t}}{Z_t^{\theta_i}} > 0, \text{ with } \lambda_i > \lambda_{i,t}^*(\varphi) \tag{8}$$

otherwise, $d_{i,t}(\varphi,\lambda_i) = 0$ with $\lambda_i < \lambda_{i,t}^*(\varphi)$.

Finally, the total profits of a producing plant with productivity φ is thus given by

$$d_t(\varphi) = I_i d_{i,t}(\varphi,\lambda_i).$$

3.2.2 Product Entry and Exit

We assume that plants entered at time t only start producing at time $t+1$. Entrants face sunk entry costs of $f_{E,t}/Z_t^{\theta_i}$ in effective labor units. Plant entry occurs until the expected plant value v_t (which is defined below) is equal to entry costs, leading to the following free entry condition,

$$v_t = w_t\frac{f_{E,t}}{Z_t^{\theta_i}}. \tag{9}$$

The timing of entry and production implies that the number of plants evolves according to the law of motion:

$$N_t = (1-\delta)\left(N_{t-1} + H_{t-1}\right). \tag{10}$$

where δ stands for the rate of plant destruction.

3.2.3 Average Productivity and Profits

A specific average productivity, weighted by consumer tastes of all producers for product i, is defined following Bernard et al. (2010) as

$$\widetilde{\varphi}_{i,t} \equiv \left[\int_{\varphi_{\min}}^{\infty} \widetilde{\lambda}_{i,t}(\varphi) dG(\varphi)\right]^{\frac{1}{\sigma-1}}, \text{where} \quad \widetilde{\lambda}_{i,t}(\varphi) \equiv \int_{\lambda_{i,t}^*(\varphi)}^{\infty} (\lambda_i \varphi)^{\sigma-1} \frac{dF_i(\lambda_i)}{1-F_i(\lambda_{i,t}^*(\varphi))}. \tag{11}$$

In the above expression, $\widetilde{\lambda}_{i,t}(\varphi)$ represents the average productivity-weighted taste of product i for the plant with productivity φ. It summarizes the range of tastes suitable for the production of product i by the plant. The term $\widetilde{\varphi}_{i,t}$ thus contains all the information about the distribution of productivities and consumer tastes. In short, it is interpreted as the taste-weighted-average productivity of product i in the economy. Using this average, the taste-adjusted real price of product i is defined as

$$\widetilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1} \frac{w_t}{Z_t^{\theta_i} \widetilde{\varphi}_{i,t}}.$$

Based on this real price, we also define average profits for each product i as

$$\widetilde{d}_{i,t} = \frac{1}{\sigma} \frac{\rho_{i,t} C_{i,t}(j)}{M_{i,t}} - \frac{w_t f_{i,t}}{Z_t^{\theta_i}} \tag{12}$$

In the above expression, note that the demand and the price index of each product basket i are given by $C_{i,t}(j) = \rho_{i,t}^{-1} \alpha_{i,t} C_t(j)$ and $\rho_{i,t}^{1-\sigma} = M_{i,t} \widetilde{\rho}_{i,t}^{1-\sigma}$, respectively. Similarly, average real profits among all producers are expressed as

$$\widetilde{d}_t = \frac{M_{i,t}}{N_t} \widetilde{d}_{i,t} \tag{13}$$

5

[5] See Appendix A for a detailed derivation.

3.2.4 Parametrization of Productivity and Taste Distribution

To solve the model, we must assume a distribution of productivity levels, φ and λ_i. Specifically, we assume the following Pareto distribution for $G(\varphi)$ and $F_i(\lambda_i)$, respectively:

$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^{\kappa}, \quad F_i(\lambda_i) = 1 - \left(\frac{\lambda_{i\,\min}}{\lambda_i}\right)^{\upsilon}$$

where $\varphi_{\min}$ and $\lambda_{i\,\min}$ are the minimum productivity level, and κ and υ determine the shape of the distribution. The parameter κ and υ index the dispersion of productivity across products. The dispersion decreases as these parameters increase, and the productivity or tastes are concentrated toward the lower bound $\varphi_{\min}$ and $\lambda_{i\,\min}$. In the calibration, we set $\varphi_{\min} = \lambda_{i\,\min} = 1$ without loss of generality. To ensure that variance of the productivity distribution are finite and that the number of products is positive, we assume that $\kappa > \upsilon > \sigma - 1$. With this parametrization, we can express the taste-weighted-average productivity, $\widetilde{\varphi}_{i,t}$, in equation 11 as[6]

$$\widetilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}} \varphi_{\min}\lambda^*_{i,t}(\varphi_{\min}), \tag{14}$$

Noting that we have $M_{i,t} = \int_{\varphi_{\min}}^{\infty} \left[1 - F_i(\lambda^*_{i,t}(\varphi))\right] dG(\varphi)N_t$ and thus the fraction of producing plants can be represented as:

$$\frac{M_{i,t}}{N_t} = \frac{\kappa}{\kappa - \upsilon}\lambda^*_{i,t}(\varphi_{\min})^{-\upsilon}. \tag{15}$$

By combining (14) and (15), we get

$$\widetilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon - (\sigma - 1)}\right]^{\frac{1}{\sigma-1}} \varphi_{\min} \left(\frac{M_{i,t}}{N_t}\frac{\kappa - \upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}}. \tag{16}$$

As mentioned earlier, for the firm with the cutoff level productivity, we can define the zero profit consumer taste cutoff condition as $d_{i,t}\left(\varphi_{\min}, \lambda^*_{i,t}\left(\varphi_{\min}\right)\right) = 0$. This implies:

$$\widetilde{d}_{i,t} = \frac{\sigma - 1}{\upsilon - (\sigma - 1)} w_t \frac{f_{i,t}}{Z_t^{\theta_i}}. \tag{17}$$

[6] Using the zero profits consumer taste cutoff (8) for plant with productivity $\varphi_{\min}$, the consumer taste cutoff of establishment with productivity φ_t, i.e., $\lambda^*_{i,t}(\varphi)$, can be expressed as a function of cutoff productivity level $\varphi_{\min}$ and the consumer taste cutoff of this cutoff firm $\lambda^*_{i,t}(\varphi_{\min})$ as $\lambda^*_{i,t}(\varphi) = \frac{\varphi_{\min}}{\varphi}\lambda^*_{i,t}(\varphi_{\min})$. The expression has a clear interpretation. The cutoff consumer taste of a firm decreases with respect to its own productivity because it allows the firm to produce even with a lower range of taste preferences. It is increasing with respect to $\varphi_{\min}$ and $\lambda^*_{i,t}(\varphi_{\min})$ since a higher value of each intensifies the competition. The above characteristic in turn means that the average taste-weighted productivity $\widetilde{\varphi}_{i,t}$ is expressed in terms of $\varphi_{\min}$ and $\lambda^*_{i,t}(\varphi_{\min})$. Specifically, with the Pareto distribution as in the paper, $\widetilde{\lambda}_{i,t}(\varphi) = \frac{\upsilon}{\upsilon-(\sigma-1)}\left[\varphi_{\min}\lambda^*_{i,t}(\varphi_{\min})\right]^{\sigma-1}$ and thus $\widetilde{\varphi}^{\sigma-1}_{i,t} = \int_{\varphi_{\min}}^{\infty} \widetilde{\lambda}_{i,t}(\varphi)dG(\varphi) = \frac{\upsilon}{\upsilon-(\sigma-1)}\left[\varphi_{\min}\lambda^*_{i,t}(\varphi_{\min})\right]^{\sigma-1}\int_{\varphi_{\min}}^{\infty} dG(\varphi) = \frac{\upsilon}{\upsilon-(\sigma-1)}\left[\varphi_{\min}\lambda^*_{i,t}(\varphi_{\min})\right]^{\sigma-1}$.

3.3 Other Sectors

In the other sectors in which plants are perfectly competitive, real price $\rho_{o,t}$ is equal to the marginal costs as

$$\rho_{o,t} = \frac{w_t}{Z_t}$$

Also, goods market clears as

$$Q_t = C_{o,t}$$

where Q_t stands for the production in the perfectly competitive sector.

3.4 Household Budget Constraints and Intertemporal Problems

The household j receives labor income by supplying labor $L_t(j)$, at wage rate w_t, and gets a share $x_t(j)$ of dividends $\widetilde{d}_t$ and the value v_t of N_t number of existing plants through the mutual fund. The household spends its income on consumption $C_t(j)$, buying $x_{t+1}(j)$ shares of the firm composed of existing products N_t, and new products H_t, at share price v_t. The household budget constraint is thus

$$L_t(j)w_t + x_t(j)N_t\left(v_t + \widetilde{d}_t\right) = C_t(j) + x_{t+1}(j)v_t\left(N_t + H_t\right). \tag{18}$$

During each period t, the household j chooses consumption $C_t(j)$, shareholdings $x_{t+1}(j)$, and the labor supply $L_t(j)$, to maximize the expected utility function subject to the budget constraint (18). The first-order conditions with respect to consumption and labor supply yield the standard labor supply equation

$$\chi_t L_t(j)^{\varsigma} = w_t \Lambda_t(j).$$

where $\Lambda_t(j) = A_t/C_t(j)$ stands for the shadow value of the budget constraint and the marginal utility of consumption.

The first-order condition with respect to shareholdings once combined with the product law of motion (10) and the first-order condition for consumption yields

$$v_t = \beta\left(1-\delta\right)E_t\left[\frac{\Lambda_{t+1}(j)}{\Lambda_t(j)}\left(v_{t+1} + \widetilde{d}_{t+1}\right)\right]. \tag{19}$$

By iterating it forward, v_t is defined as the present discounted value of the stream of current and expected profits $\left\{\widetilde{d}_{s,k}\right\}_{k=t+1}^{\infty}$ as follows

$$v_t = E_t \sum_{k=t+1}^{\infty} \left[\beta\left(1-\delta\right)\right]^{k-t} \left(\frac{\Lambda_t}{\Lambda_k}\right) \widetilde{d}_k. \tag{20}$$

3.5 Model Equilibrium and Solution

In equilibrium, households are symmetric such that $C_t(j) = C_t$, $C_{i,t}(j) = C_{i,t}$, $L_t(j) = L_t$, and $\Lambda_t(j) = \Lambda_t$. Also in equilibrium, by aggregating the budget constraints among households, we get

$$L_t w_t + N_t \widetilde{d}_t = C_t + v_t H_t$$

Further we define average real sales, and total real sales of product i as

$$\widetilde{y}_{i,t} \equiv \sigma\left(\widetilde{d}_{i,t} + w_t \frac{f_{i,t}}{Z_t^{\theta_i}}\right), \quad \mathcal{Y}_{i,t} \equiv M_{i,t}\widetilde{\rho}_{i,t}\widetilde{y}_{i,t}$$

Also real GDP is defined as $Y_t = L_t w_t + N_t \widetilde{d}_t$.[7]

Finally, we assume the following process of aggregate and product specific shocks:

$$\begin{pmatrix} \ln(A_t) \\ \ln(Z_t) \\ \ln(\chi_t/\chi) \\ \ln(f_{E,t}) \\ \ln(\alpha_{i,t}/\alpha_i) \\ \ln(f_{i,t}/f_i) \end{pmatrix} = \begin{pmatrix} \rho_A & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_Z & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\chi & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{f_E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{\alpha_i} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{f_i} \end{pmatrix} \begin{pmatrix} \ln(A_{t-1}) \\ \ln(Z_{t-1}) \\ \ln(\chi_{t-1}/\chi) \\ \ln(f_{E,t-1}) \\ \ln(\alpha_{i,t-1}/\alpha_i) \\ \ln(f_{i,t-1}/f_i) \end{pmatrix} + \begin{pmatrix} \sigma_A\varepsilon_{A,t} \\ \sigma_Z\varepsilon_{Z,t} \\ \sigma_\chi\varepsilon_{\chi,t} \\ \sigma_{f_E}\varepsilon_{f_E,t} \\ \sigma_{\alpha_i}\varepsilon_{\alpha_i,t} \\ \sigma_{f_i}\varepsilon_{f_i,t} \end{pmatrix}$$

where ρ_A, ρ_χ, ρ_Z, ρ_{f_E}, ρ_{α_i}, and ρ_{f_i} refer to the shock persistence and $\varepsilon_{A,t}$, $\varepsilon_{Z,t}$, $\varepsilon_{\chi,t}$, $\varepsilon_{f_E,t}$, $\varepsilon_{\alpha_i,t}$, and $\varepsilon_{f_i,t}$ are normally distributed innovations with zero mean whose variances equal to σ_A^2, σ_χ^2, σ_Z^2, $\sigma_{f_E}^2$, $\sigma_{\alpha_i}^2$, and $\sigma_{f_i}^2$. Among these shocks, $\varepsilon_{A,t}$, $\varepsilon_{Z,t}$, and $\varepsilon_{\chi,t}$ are called "aggregate shocks" in the sense that they influence both specific product and other sectors while $\varepsilon_{f_E,t}$, $\varepsilon_{\alpha_i,t}$, and $\varepsilon_{f_i,t}$ are called *product-specific shocks* since they impact only that specific sector. Table 1 summarizes the benchmark model.

[7] Note that any empirically relevant variable X_t^e is defined as $X_t^e \equiv P_t X_t / P_t^e$ where $\frac{P_t}{P_t^e} = \frac{P_{i,t}^{\alpha_{i,t}} P_{o,t}^{1-\alpha_{i,t}}}{P_{i,t}^{e\alpha_{i,t}} P_{o,t}^{e1-\alpha_{i,t}}} = \left(\frac{P_{i,t}}{P_{i,t}^e}\right)^{\alpha_{i,t}} = \left(\frac{M_{i,t}^{\frac{1}{1-\sigma}}\widetilde{p}_{i,t}}{\widetilde{p}_{i,t}^e}\right)^{\alpha_{i,t}}$ since we have $P_{o,t} = P_{o,t}^e$ without having any measurement errors in other sectors. Given a very tiny weight of $\alpha_{i,t}$ which is close to zero, we assume that there will be a little discrepancy between empirically relevant price index and welfare-consistent index such that $\frac{P_t}{P_t^e} \simeq 1$. This assumption allows us to directly compare actual real time series data and its theoretical counterpart.

Table 2: Summary of the benchmark model

1. Average pricing	$\widetilde{\rho}_{i,t} = \frac{\sigma}{\sigma-1}\frac{w_t}{Z_t^{\theta_i}\widetilde{\varphi}_{i,t}}$
2. Real taste-adjusted price	$\rho_{i,t}^{1-\sigma} = M_{i,t}\widetilde{\rho}_{i,t}^{1-\sigma}$
3. Demand for product	$C_{i,t} = \rho_{i,t}^{-1}\alpha_{i,t}C_t$
4. Demand for other product i	$C_{o,t} = \rho_{o,t}^{-1}\alpha_o C_t$
5. Price index	$1 = \rho_{i,t}^{\alpha_{i,t}}\rho_{o,t}^{1-\alpha_{i,t}}$
6. Average product profits	$\widetilde{d}_{i,t} = \frac{1}{\sigma}\frac{\rho_{i,t}C_{i,t}}{M_{i,t}} - \frac{w_t f_{i,t}}{Z_t^{\theta_i}}$
7. Average profits	$\widetilde{d}_t = \frac{M_{i,t}}{N_t}\widetilde{d}_{i,t}$
8. Consumer taste cutoff	$\widetilde{d}_{i,t} = \frac{\sigma-1}{\upsilon-(\sigma-1)}w_t\frac{f_{i,t}}{Z_t^{\theta_i}}$
9. Taste weighted productivity	$\widetilde{\varphi}_{i,t} = \left[\frac{\upsilon}{\upsilon-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\varphi_{\min}\left(\frac{M_{i,t}}{N_t}\frac{\kappa-\upsilon}{\kappa}\right)^{-\frac{1}{\upsilon}}$
10. Free entry condition	$v_t = w_t\frac{f_{E,t}}{Z_t^{\theta_i}}$
11. Motion of establishments	$N_{t+1} = (1-\delta)(N_t + H_t)$
12. Euler equation	$v_t = \beta(1-\delta)E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(v_{t+1} + \widetilde{d}_{t+1}\right)\right]$
13. Optimal labor supply	$\chi_t L_t^{\varsigma} = w_t\Lambda_t$
14. Definition of discount factor	$\Lambda_t = A_t/C_t$
15. Aggregation	$L_t w_t + N_t\widetilde{d}_t = C_t + v_t H_t$
16. Good market clearing	$Q_t = C_{o,t}$
17. Pricing in other sectors	$\rho_{o,t} = \frac{w_t}{Z_t}$
18. Definition of total sales of product i	$\mathcal{Y}_{i,t} \equiv M_{i,t}\widetilde{\rho}_{i,t}\widetilde{y}_{i,t}$
19. Definition of GDP	$Y_t = L_t w_t + N_t\widetilde{d}_t$

4 Calibration and Estimation

In this section, we present our calibration and estimation strategy. Our aim is to replicate the universe of products and their dynamics as we detailed previously with the help of the theoretical model and identify the source of their variations over the business cycles. To this end, we calibrate and estimate over 2000 products, each defined at a 6-digit level. With the following procedure, we get 2061 sets of calibrated and estimated parameters' value excluding products with zero share, therefore.

4.1 Calibration

We calibrate the parameters of the theoretical model as in Table 3. These parameters' values are either taken from the literature or found from the steady state relations. In calibrating the steady state value of the preference weight of product i in Cobb-Douglas bundles, α_i, we use the sales share of product i in the data as presented in Figure 15. Other parameters' values are assumed to be common for all products. The value of the discount factor β is calibrated so that it gives 4% annual real interest rate as in the literature. The inverse of the Frisch elasticity of labor supply ς is taken from Sugo and Ueda (2008), which estimates the elasticity for the Japanese economy. The elasticity of substitution across product varieties σ is set according to Ghironi and Melitz (2005). The values of parameters υ and κ that shape the taste distribution and

Table 3: Calibration

Common for all products		
β	Discount factor	0.96
ς	Inverse of Frisch elasticity of labor	2.15
σ	Elasticity of substitution of product varieties	3.80
κ	Productivity dispersion	11.5
υ	Taste dispersion	4.18
δ	Exogenous death shock	0.0056
f_e	Entry fixed costs	1
Product Specific		
α_i	Preference weight	from sales data
f_i	Operational Fixed costs	adjusted
χ	Labor supply dis-utility	ajdusted

productivity distribution are set following Hamano and Oikawa (2022). These values satisfy the restriction on the parameter such that $\kappa > \upsilon > \sigma - 1$.

The parameter values related to entry, exit, and selection are calibrated following Hamano and Oikawa (2022). The exogenous establishment destruction rate δ is calibrated so that it replicates the average establishment creation rate in Japan. The operational fixed costs for production of each product, f_i, are set so that it replicates the steady state share of producing plants as $M_i/N = 0.93$. We set entry fixed costs at the steady state as $f_e = 1$ without loss of generality. Finally, the parameter that determines the disutility for the labor supply χ is set such that labor supply at the steady state is unity.[8] We provide a detailed derivation of the steady state in Appendix B.

4.2 Estimation

In the theoretical model with product i and other sectors, the parameters which are related to the propagation of exogenous disturbances are estimated by maximizing the posterior with uninformative priors. In doing so, we use real total sales growth of product i, the growth in the number of producing plant of product i, and real GDP as observables. Because of this estimation strategy, all the estimated parameters become specific to each product, providing 2061 sets of estimated values of the parameter vectors. Table 4 shows our uninformative uniform priors and the resulting median estimates of these parameters. It is noticed that aggregate shocks are less important compared to product-specific demand shocks and product-specific supply shocks. This is the case not only for the median product but also for the universe of products. As Figure 5 and Figure 6 indicate, the standard deviations of these product-specific shocks are more dispersed with the median values which are higher than those of aggregate shocks and other shocks (from Figure 17 to 19 in Appendix). As also the table and the figures show, the persistences of the shocks are estimated to be in

[8]Since we need to adjust this parameter value according to the size of α_i and f_i, it becomes product-specific as well at the steady state.

Table 4: Median Estimation

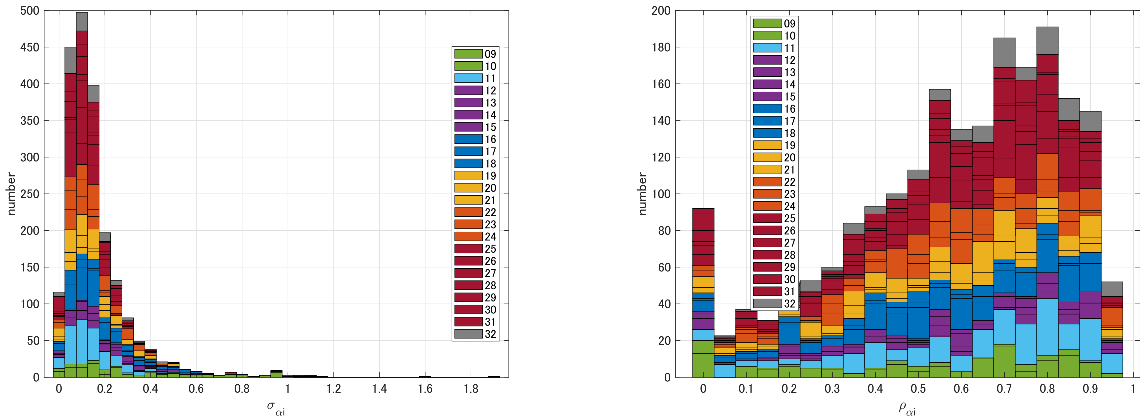
		Low	High	Median estimate
σ_A	std D. of preference shock	0.0001	2.0000	0.0207
σ_Z	Std D. of productivity shock	0.0001	2.0000	0.0144
σ_χ	Std D. of labor disutility shock	0.0001	2.0000	0.0017
σ_{f_E}	Std D. of entry regulation shock	0.0001	2.0000	0.0418
σ_{α_i}	Std D. of product demand shock	0.0001	2.0000	0.1450
σ_{f_i}	Std D. of product supply shock	0.0001	2.0000	0.1866
ρ_A	Persistence of demand shock	0.001	1	0.5213
ρ_Z	Persistence of productivity shock	0.001	1	0.5650
ρ_χ	Persistence of labor disutility shock	0.001	1	0.5505
ρ_{f_E}	Persistence of entry regulation shock	0.001	1	0.8627
$\rho_{f_{\alpha_i}}$	Persistence of establishment regulation shock	0.001	1	0.6510
ρ_{f_i}	Persistence of entry regulation shock	0.001	1	0.3523
θ_i	Adjustment cost for establishment entry	-1	11	-0.1545

the range found in the literature. Product-specific operational fixed cost shock, however, shows relatively lower values of persistence compared to other shocks. The median value of θ_i that governs the spillover of aggregate labor productivity to the sector i is estimated with a slightly negative number. However, it shows a contrasted pattern across products: the values of a large majority of products are centering around the lower bound, and those of other majority of products are found around the upper bound (Figure 16 in Appendix). As we can imagine, this parameter value is of importance in controlling the comovement with GDP driven primarily by the productivity shock.

The primary results of the study suggest that the behavior of individual products is significantly impacted by their own demand and supply shocks, rather than by broader market trends or aggregate shocks. Importantly, there is considerable heterogeneity in these product-specific shocks, with some products experiencing high standard deviation in their shocks, indicating high volatility, while others do not.

5 Theory vs. Data

Equipped with the calibrated and estimated parameter values, we are now prepared to evaluate the performance of the theoretical model. Our investigation begins with an analysis of the median business cycle fit to the observed patterns. Next, we delve into the fit of each product by comparing the data-generated moment distributions with those of the theoretical model, specifically focusing on two observed moments (cyclicality and standard deviations) for two variables (total sales growth and the number of producing plants) for each product. This includes a comparison of the correlations between total product sales growth and GDP, as well as their standard deviations. Additionally, we assess the correlations between the growth of producing plants and GDP, along with their standard deviations. Lastly, we broadly evaluate the capacity of the theoretical

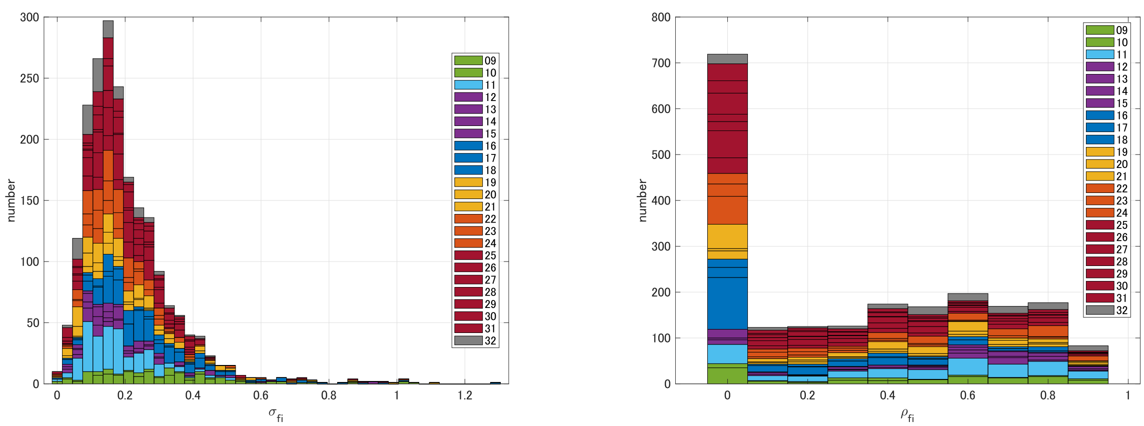


(a) Standard Deviations of the shock

(b) Persistences of the shock

Figure 5: Product specific demand shock

Note: Vertical axes measure the number of products. Horizontal axes represent the standard deviation of the shock (left panel) or its persistence (right panel).



(a) Standard Deviations of the shock

(b) Persistences of the shock

Figure 6: Plant-product specific fixed cost shock

Note: Vertical axes measure the number of products. Horizontal axes represent the standard deviation of the shock (left panel) or its persistence (right panel).

model by comparing its sectoral outcomes with the actual data.

5.1 Business Cycle of the Medium Product

In this section, we present the median business cycle characteristics, focusing on the median correlation of total sales growth with GDP, as well as that of the number of producing plants implied by the theoretical model. Additionally, we explore the standard deviations implied by the theoretical model for these variables.

Our findings reveal that both the median total sales growth and the median number of producing plants

Table 5: Second moments for Median Product

		GDP	Total Sales	Product plant
Corr(Y, X_t)	Data	1.00	0.292	0.192
	Model	1.00	0.111	0.003
Std Relative to Y	Data	1.00	7.638	8.672
	Model	1.00	8.932	15.47

are positively correlated with GDP as shown in Table . However, the theoretical model underpredicts these correlations, providing values of 0.111 and 0.003, respectively, while the observed data indicates correlations of 0.292 and 0.192. Furthermore, we discover that the theoretical model replicates both the median standard deviations of total sales growth and that of the number of producing plants, albeit with slightly higher values (8.932 and 15.47) when compared to the data (7.638 and 8.672).

We conclude that the theoretical model provides a reasonably good fit at the median business cycle. As a next step, we will proceed to explore the capacity of the theoretical model in replicating the business cycles for the entire universe of products, aiming to further understand its strengths and limitations in capturing product dynamics within the context of business cycles.

5.2 Heterogeneity Across Products

5.2.1 Total Sales Growth in the Theoretical Model

We present the model-generated histogram of correlations between the total sales growth of products and GDP growth in the first two panels of Figure 7. The majority of products in the theoretical model exhibit positive correlations. By comparing with the data, we find that the correlation between total sales growth of products and GDP growth increases as the sales share of the product rises, as shown in the first two panels of Figure 8.

To directly assess the model's ability to replicate observed data, we plot data-generated and model-generated correlations for each product. As shown in the left panel of Figure 9, many products are located below the 45-degree line, indicating that the theoretical model tends to underestimate the positive correlations between total sales growth of products and GDP growth. However, for some products, such as consumable goods (coded as 09 and 10), the theoretical model overestimates the correlations, providing positive correlations as opposed to the negative correlations observed in the data.[9]

We also present the model-generated histogram for the standard deviations of total sales growth of

[9] To be precise, the correlations between the correlation of the sales growth with GDP and their sales shares are 0.6313 (09-10), 0.6585 (11), 0.5102 (12-15), 0.4512 (16-18), 0.5516 (19-21), 0.5436 (22-24), 0.4772 (25-31), 0.4067 (32), and 0.5183 for all products. The correlations between the standard deviations of the sales growth and their sales shares are -0.8021 (09 -10), -0.6938 (11), -0.6356 (12-15), -0.6206 (16-18), -0.7338 (19-21), -0.6757 (22-24), -0.5450 (25-31), -0.5630 (32), and -0.6210 for all products. In the above expressions, the number(s) in parentheses demonstrate the code of the two-digit product categories.

products and compare it with the data in the second two panels of Figure 7. By comparing with data distribution, the model-generated one gives a remarkably similar shape of the distirbution. Furthermore, the standard deviations in the theoretical model decrease with higher sales shares of products, just as observed in the data, as shown in the second two panels of Figure 8. The standard deviations exhibit a remarkably good fit with the actual data: as shown in the right panel of Figure 9, where almost all products are located on the 45-degree line, indicating that the theoretical model replicates the size of standard deviations for each product quite precisely.

In conclusion, the theoretical model replicates the distribution of standard deviations among products quite well, while the distribution of product cyclicality is only qualitatively good. We find these results surprising. Remember that we don't target any of these distributions when we estimate and calibrate the parameter values. The estimation and the calibration are independent and product-specific. Nevertheless, our procedure creates a reasonable distribution of business cycle characteristics among products. We see this as an external validity of the theoretical model.

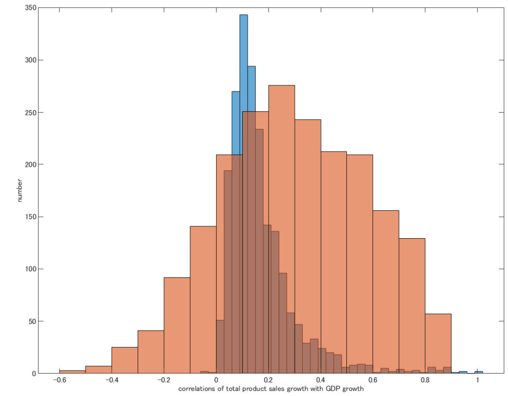
5.2.2 Plant Growth: theory

In this section, we analyze the cyclical behavior of the number of product-producing plants. As depicted in the first two panels of Figure 10, the correlations between the growth in the number of producing plants and GDP growth are positive for the majority of products in the theoretical model. Additionally, the correlation remains stable as the sales share of the product increases, consistent with the actual data (first two panels in Figure 11).

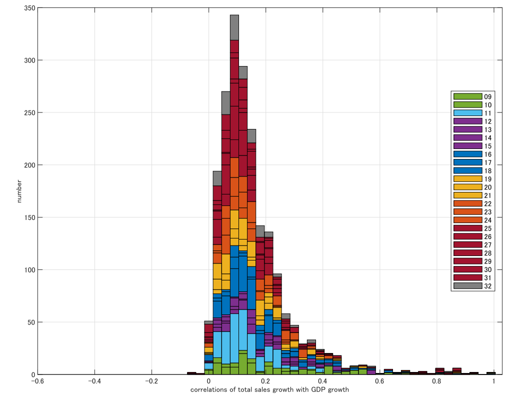
However, as shown in the left panel of Figure 12, more than half of the products are located below the 45-degree line, indicating that the theoretical model largely underestimates the positive correlations of the growth in the number of producing plants of products with GDP growth. We also present the model-generated histogram of the standard deviations of the growth in the number of producing plants of products and compare it with the data in the second two panels of Figure 10.. The standard deviations in the theoretical model decrease with higher sales shares of products, similar to what is observed in the data (second two panels in Figure 11).

The standard deviations of the growth in the number of producing plants of products are close to those obtained with the actual data (12). However, the extent of the match is less compared to that of the total sales growth. To sum up, the theoretical model effectively replicates the distribution of standard deviations among products, but the distribution of cyclicality is only captured to a satisfactory degree qualitatively, similar to the case of total sales growth of products.[10]

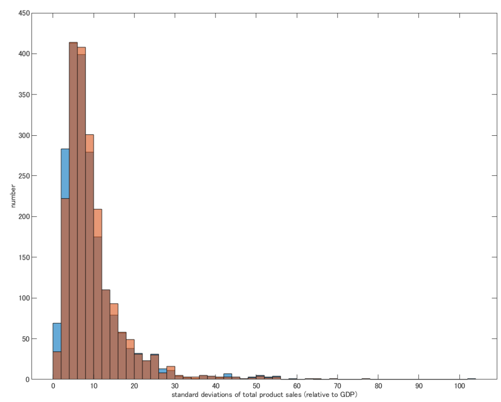
[10] To be precise, the correlations between the correlation of the number of product producing plants with GDP and their



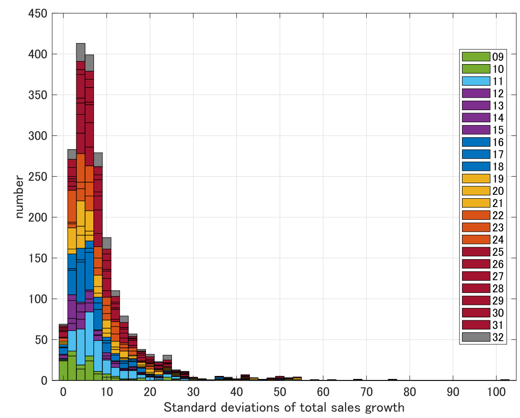
(a) Correlations of total product sales with GDP growth



(b) Correlations of total product sales with GDP growth (theory)



(c) Standard Deviations of total product sales growth



(d) Standard Deviations of total product sales growth (theory)

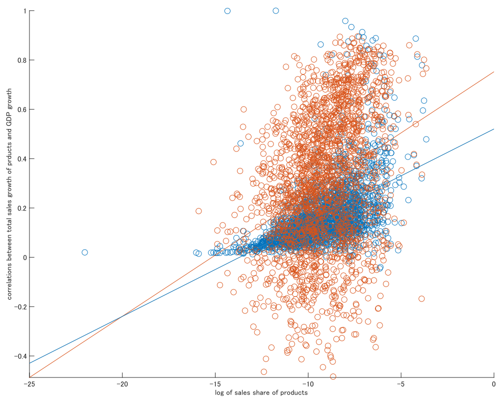
Figure 7: Comparison with data: total sales growth

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

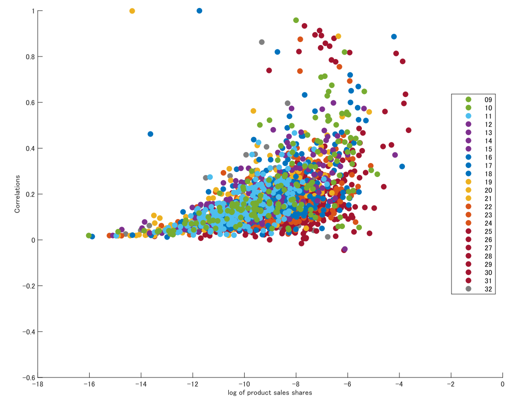
6 Structural Shocks and Variance Decomposition

Having established the validity of the theoretical model, we can now address the central question posed in the introduction: what drives product business cycles? In Table 6, we present the contribution of each of the six shocks implied by the theoretical model in explaining the variance of real GDP, total sales, and the number of producing plants for the average of all products.

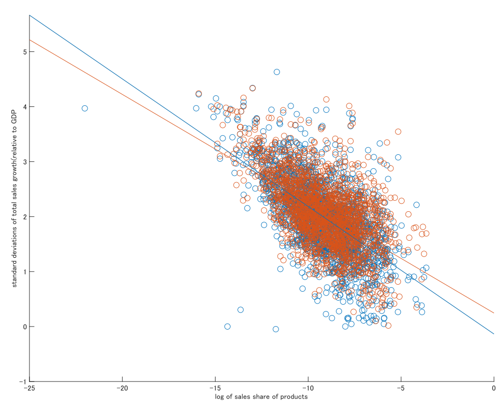
sales shares are 0.2016 (09-10), 0.0432 (11), -0.0585 (12-15), -0.1687 (16-18), -0.0251 (19-21), -0.1688 (22-24), -0.1688 (25–31), 0.0357 (32), and -0.1306 for all products. The correlations between the standard deviations of the number of product producing plants and their sales shares are -0.6627 (09 -10), -0.5708 (11), -0.5825 (12-15), -0.5834 (16-18), -0.6884 (19-21), -0.6675 (22-24), -0.5647 (25-31), -0.4987 (32), and -0.6015 for all products. In the above expressions, the number(s) in parentheses demonstrate the code of the two-digit product categories.



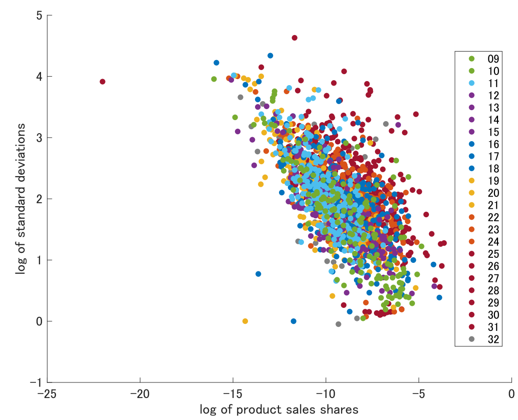
(a) Correlations of total product sales with GDP growth



(b) Correlations of total product sales with GDP growth (theory)



(c) Standard Deviations of total product sales growth



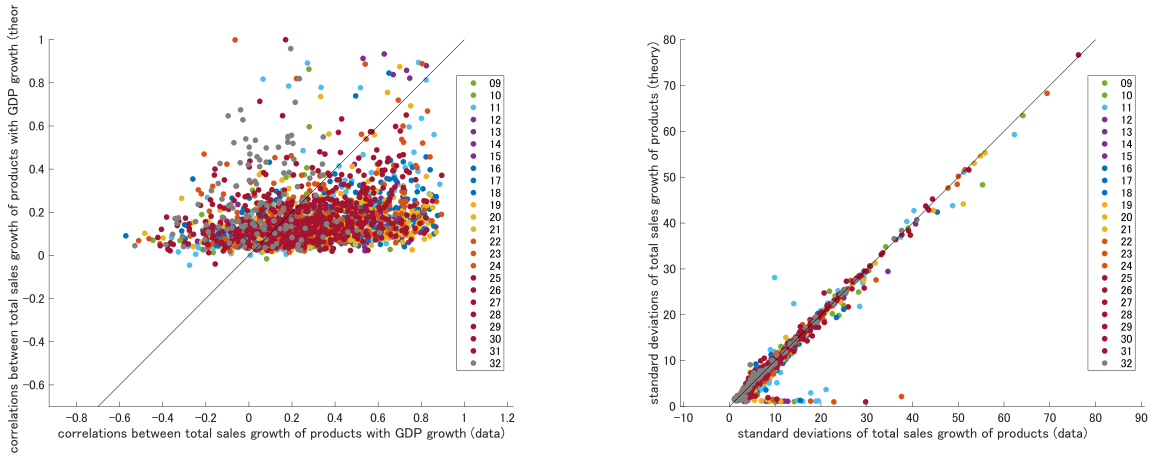
(d) Standard Deviations of total product sales growth (theory)

Figure 8: Product sales shares and product sales growth

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

Our analysis shows that only aggregate shocks ($\varepsilon_{A,t}$, $\varepsilon_{Z,t}$, and $\varepsilon_{\chi,t}$) matter in explaining the variability of real GDP Y, which is consistent with our calibration, where each product has only a tiny weight in the economy. Among these aggregate shocks, the technology shock contributes the most, serving as a driving force of GDP at 54%, followed by the aggregate demand shock at 35%, and the aggregate labor dis-utility shock at 11 %.

For the variability of total sales of the average product $\mathcal{Y}_i$ (i.e., intensive margins), the contribution of the product-specific demand shock ($\varepsilon_{\alpha_i,t}$) is the largest, accounting for more than 94%, while the product-specific shock on operational fixed costs ($\varepsilon_{f_i,t}$) does not play any role in explaining the variance of total sales of the product. Product-specific shock on entry costs ($\varepsilon_{f_E,t}$) accounts for only 0.35% in explaining the variability



(a) Correlations of total product sales with GDP growth

(b) Standard Deviations of total product sales

Figure 9: Data vs. Theory: product sales growth

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

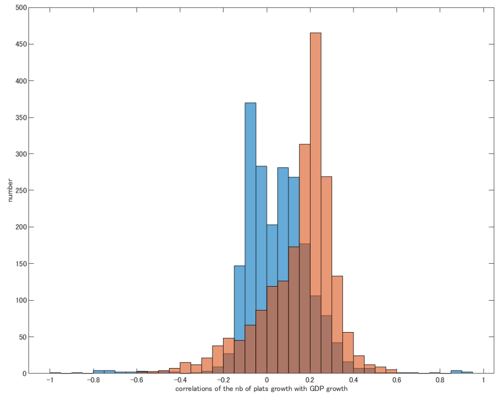
of sales growth. Aggregate shocks have a minor role, with their contributions summing up to around 5%. The result is consistent with the finding in Hottman et al. (2016) who also found the dominance of demand shocks in explaining sales growth at the firm level.

Regarding the number of producing plants for the average product M_i (i.e., extensive margins), both product-specific demand and supply shocks emerge as important drivers of fluctuations. The contribution of product-specific demand shock ($\varepsilon_{\alpha_i,t}$) is 35%, and that of the product-specific shock on operational fixed costs ($\varepsilon_{f_i,t}$) is 60% while the product-specific shock on entry costs ($\varepsilon_{f_E,t}$) plays a very marginal role, amounting to 0.1%. Aggregate shocks play a minor role again, contributing less than 5%.

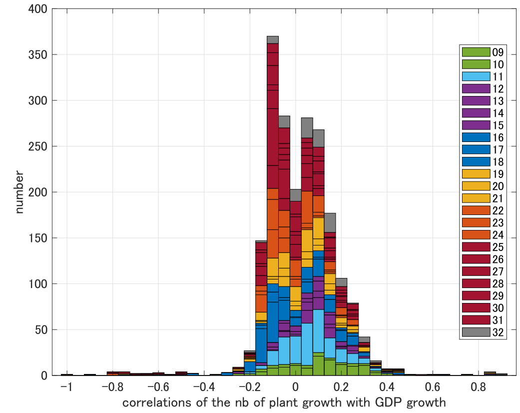
Do these patterns differ among product categories? The answer is not significantly. As Table 8 in the Appendix shows, the averages in each product category reveal a similar pattern of variance decomposition as the average of all products.

Figure 21 shows the histogram of the standard deviations of GDP growth, obtained for each product with the help of our structural model. Consistent with the assumption, the contribution of product-specific shocks to GDP growth is almost zero independent of the product as we have drawn the same conclusion with respect to the average product.

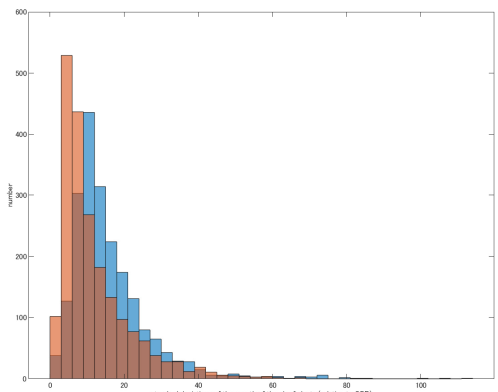
However, a different picture emerges when examining the size of the standard deviations for each product. Figure 13 and Figure 11 display the same histogram of standard deviations for total sales of products and the number of producing plants as Figure 7 and Figure 10. In contrast to these figures, which show product categories with different colors, we represent the contribution of each shock with different colors in the new



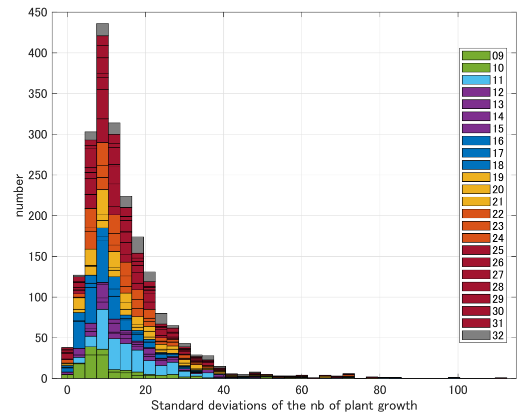
(a) Correlations of plant growth with GDP growth



(b) Correlations of plant growth with GDP growth (theory)



(c) Standard Deviations of plant growth



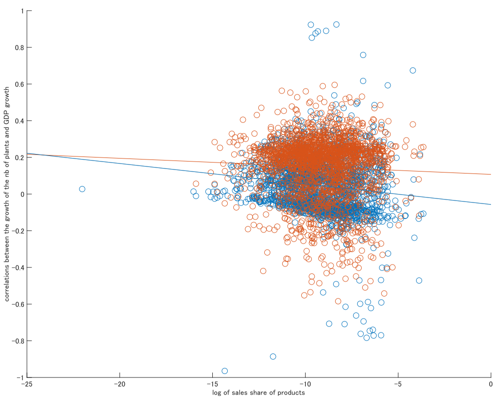
(d) Standard Deviations of plant growth (theory)

Figure 10: Comparison with data: plant growth

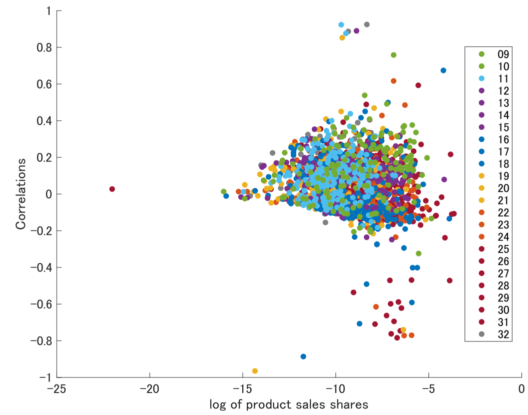
Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

figures.

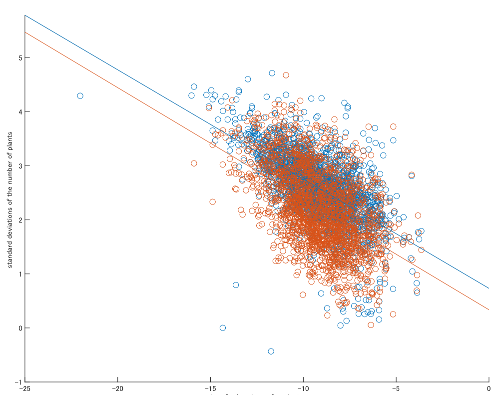
While the overall picture regarding the contribution of shocks in explaining the variance remains broadly similar (significant contribution of product-specific demand shock for product sales and significant contribution of product-specific demand and supply shocks), we observe that for products with small standard deviations, the contribution of aggregate shocks becomes more prominent. The right hand panel of Figure 13 and that of Figure 11 suggest, specifically, among these aggregate shocks, the technology shock plays an important role in explaining the standard deviations of total sales of products, accounting for around 40% when the standard deviations are small and thus close to 1. A similar observation applies for the variability of the number of producing plants: it accounts for around 30% when the standard deviations are small and



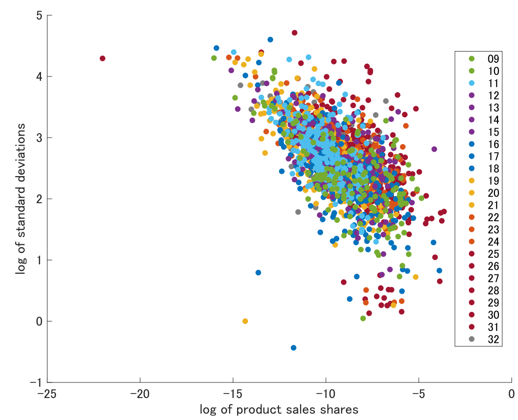
(a) Correlations of plant growth with GDP growth



(b) Correlations of plant growth with GDP growth (theory)



(c) Standard Deviations of plant growth sales growth



(d) Standard Deviations of plant growth growth (theory)

Figure 11: Product sales shares and plant growth

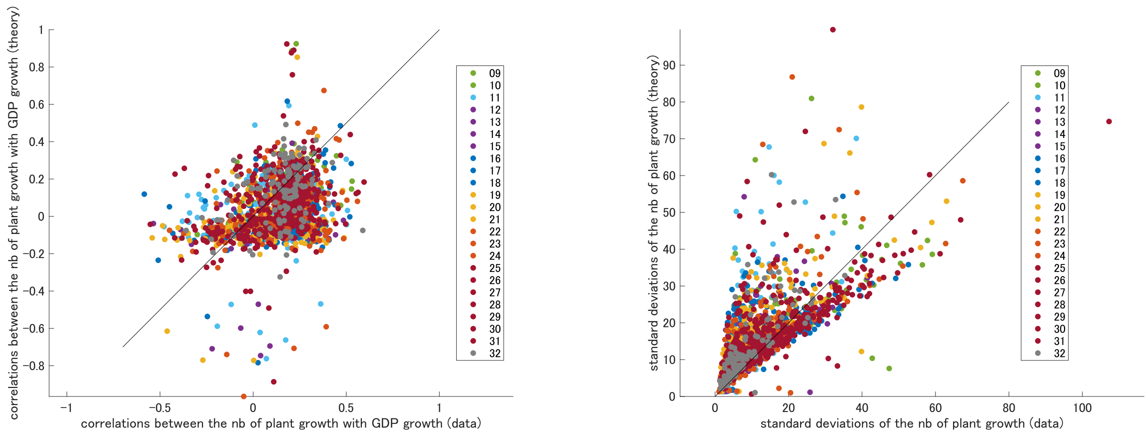
Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

Table 6: Variance Decomposition: Product averages

	σ_A	σ_Z	σ_χ	σ_{f_E}	σ_{α_i}	σ_{f_i}
Y	0.3510	0.5388	0.1073	0.0029	0.0000	0.0000
$\mathcal{Y}_i$	0.0160	0.0288	0.0074	0.0035	0.9442	0.0000
M_i	0.0039	0.0434	0.0019	0.0011	0.3486	0.6011

thus close to 1.

These results imply that for products with high sales shares, the contribution of aggregate shocks becomes more important, as they exhibit low standard deviations in both sales growth and the number of producing plants, as demonstrated in the previous sections.

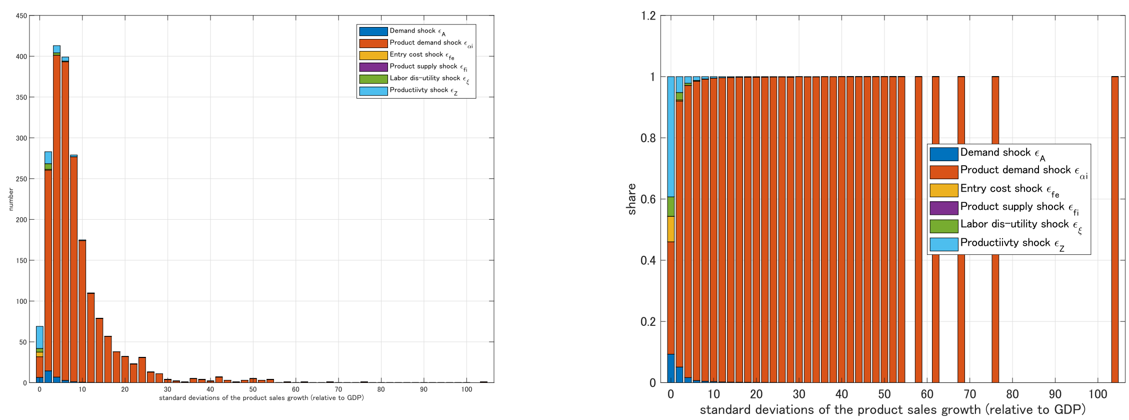


(a) Correlations of plant growth with GDP growth

(b) Standard Deviations of plant growth

Figure 12: Data vs. Theory: plant growth

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).



(a) Variance decomposition of sales growth

(b) Variance decomposition of sales growth (share)

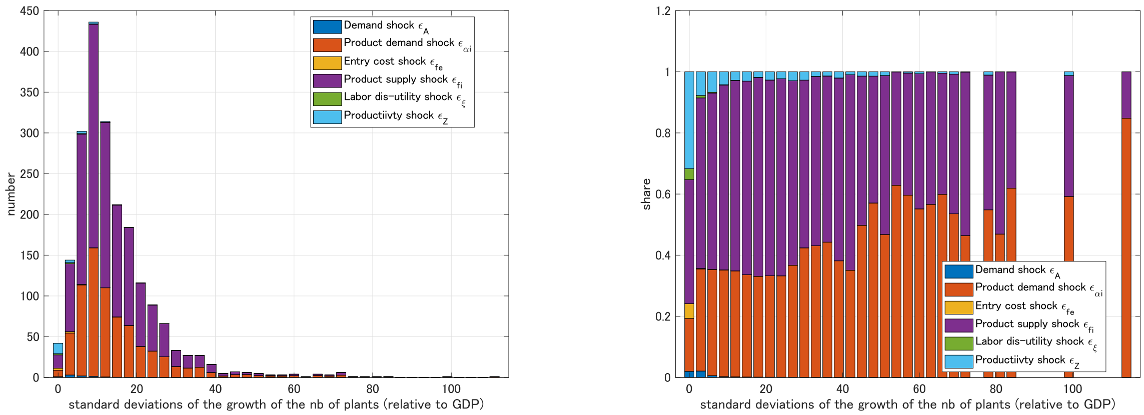
Figure 13: Variance decomposition of the product sales growth

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

7 Conclusion

In conclusion, this study has illuminated the intricate relationships between overall economic conditions and individual product dynamics. By examining Japanese manufacturing census data, we've identified significant heterogeneity in how various products respond to business cycles.

In order to interpret these patterns, we constructed a theoretical model to estimate the individual sources of propagation for each product. The performance of individual products, our results suggest, is primarily



(a) Variance decomposition of plant growth (b) Variance decomposition of plant growth (share)

Figure 14: Variance decomposition of the product plant growth

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

influenced by their unique demand and supply shocks, rather than wider market trends or collective disturbances. We observed a marked variation in these product-specific shocks, with certain products demonstrating significant volatility, while others proved more stable.

Our structural model adeptly replicates these observed product dynamics and heterogeneous patterns, underlining the critical role of product-specific demand shocks in accounting for product sales dynamics (intensive margins), and the necessity of considering both product-specific and plant-product specific shocks for understanding the plant dynamics (extensive margins).

Our findings carry profound implications for macroeconomic policy. Aggregate stabilization policies, like monetary and fiscal initiatives, may be more effective for products with higher sales shares. Conversely, policies targeting specific products could better reduce macroeconomic volatility and promote inclusive macroeconomic fluctuations for products with lower sales shares.

Lastly, the role of global value chains should not be overlooked. As products often exist within intertwined global networks, shocks can reverberate along these chains, affecting product performance in ways that broader analyses might miss. This nuanced understanding can guide policymakers to devise targeted and effective strategies that foster stability, growth, employment, and inclusiveness across diverse sectors. Our study, by enriching the existing literature and deepening our grasp of product dynamics, contributes to the ongoing debate on the drivers of business cycles and the heterogeneous responses of different products to aggregate shocks.

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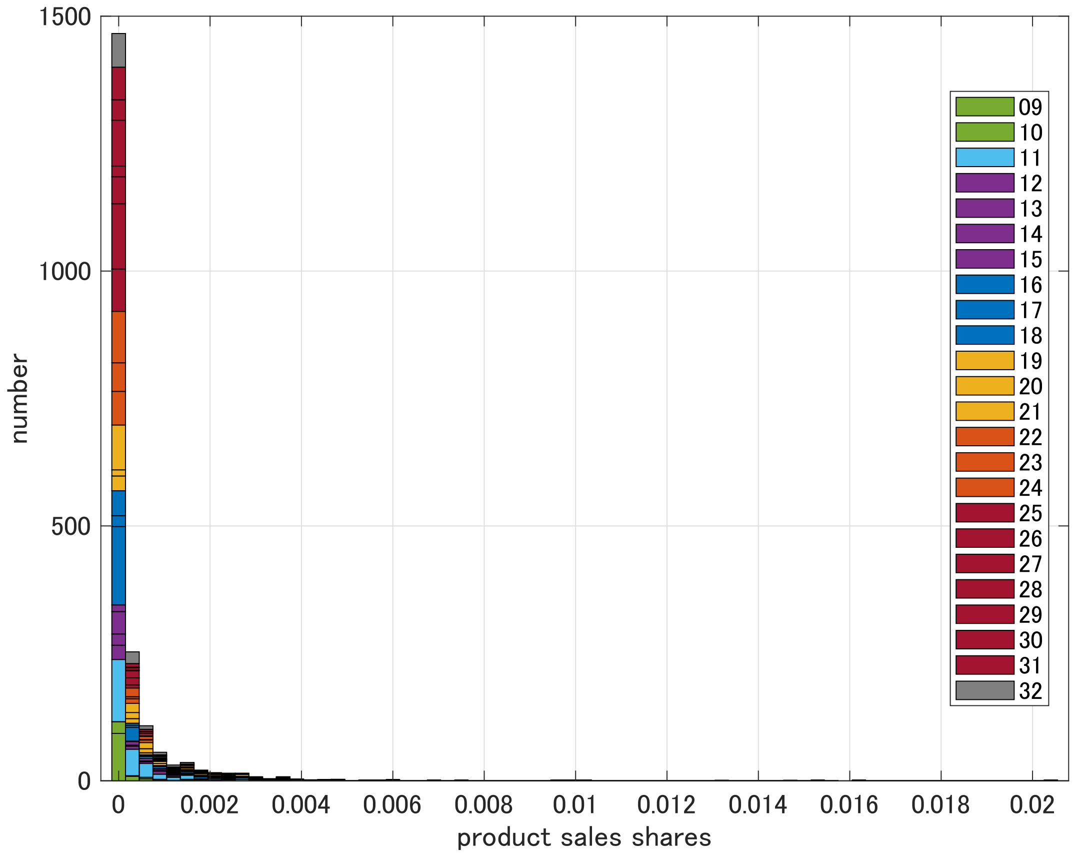

Figure 15: Histogram of total product sales shares

Note: Vertical axes measure the number of products. Horizontal axes represent the median total sales shares of products over the sample periods.

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A Product share ranking

B Estimation

C Variance Decomposition

Table 7: Ranking

Rank	Total Sales	Description	Share
1	311112	Ordinary passenger cars, 2000 ml cylinder capacity or more, including chassis	0.0261
2	311321	Car heaters	0.0232
3	281511	Liquid crystal panel	0.0218
4	311111	Light and small passenger cars, less than 2000 ml cylinder capacity, including chassis	0.0208
5	165111	Medical material preparations	0.0204
6	311315	Parts of driving, transmission and operating units	0.0161
7	151111	Offset printing'	0.0155
8	171111	Gasoline	0.0148
9	281411	Linear circuit	0.0135
10	311314	Parts, attachments and accessories of internal combustion engines for motor vehicles	0.0105
11	311114	Trucks, including tractors	0.0100
12	311317	Parts of chassis and bodies	0.0097
13	105111	Cigarettes, cigars and pipe tobacco	0.0078
14	303111	General lighting bulbs	0.0069
15	102211	Beers	0.0062
16	229111	Steel cuttings	0.0060
17	267111	Processing equipment for wafer process	0.0058
18	212211	Greases made of mineral, animal and vegetable oil purchased	0.0057
19	183211	Automotive plastic products	0.0050
20	311311	Gasoline engines for motor vehicles	0.0050
21	301211	Passenger car bodies	0.0048
22	99934	Cut rice cake and packaged rice cake, except Japanese raw rice cakes	0.0047
23	145311	Cardboard box	0.0046
24	311331	KD sets (passenger cars, buses and trucks)	0.0045
25	311316	Parts of suspension and brake systems	0.0043
26	292221	Parts, attachments and accessories of auxiliary equipment for internal combustion engines	0.0040
27	303113	Parts and accessories for electronic computers	0.0039
28	91111	Chop of meat and frozen meat, except broilers	0.0039
29	101112	Juice	0.0038
30	311213	Truck bodies	0.0038

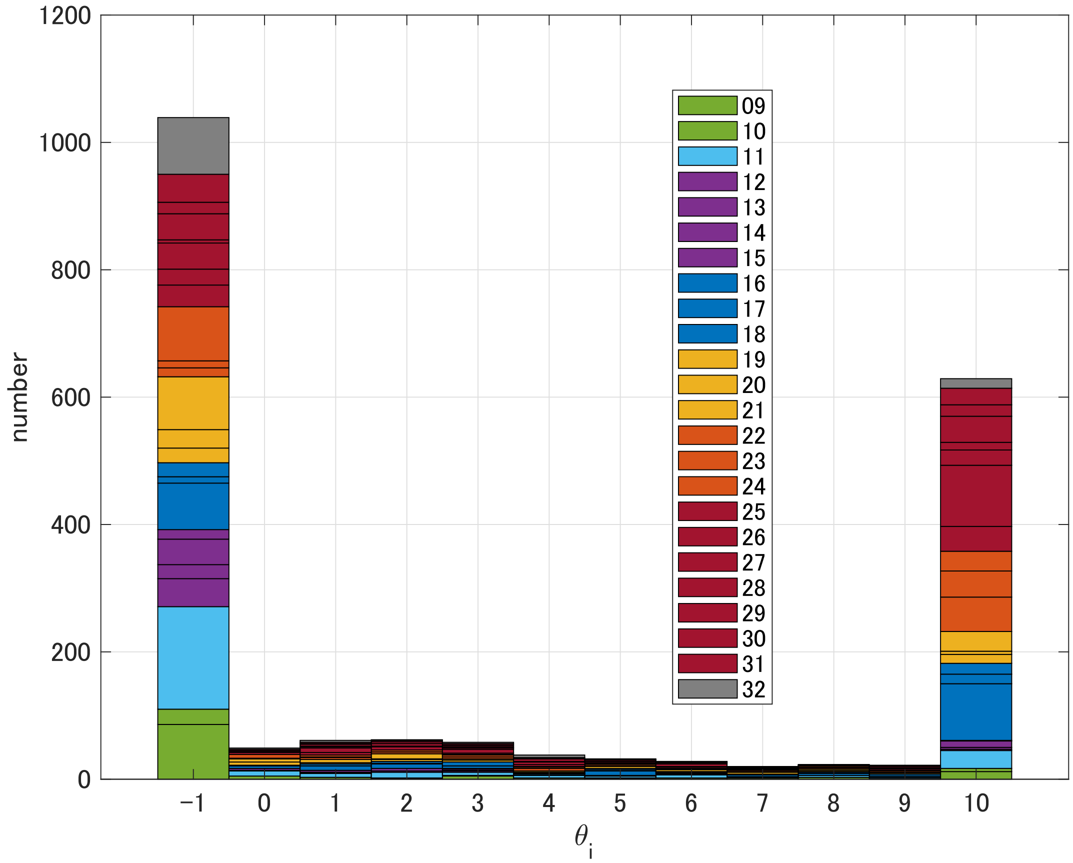
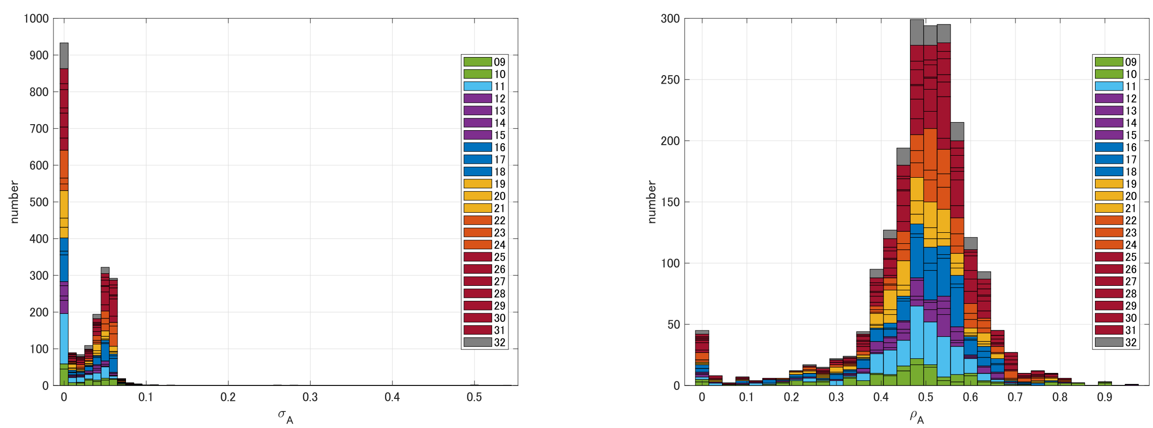


Figure 16: Product-specific productivity spillover: θ_i

Note: Vertical axes measure the number of products. Horizontal axes represent the pproduct-specific productivity spillover: θ_i.

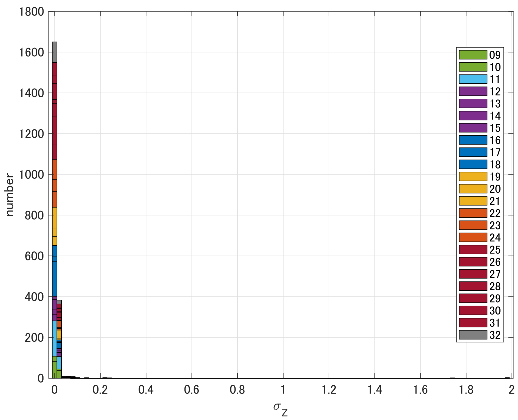


(a) Standard Deviations of the shocks

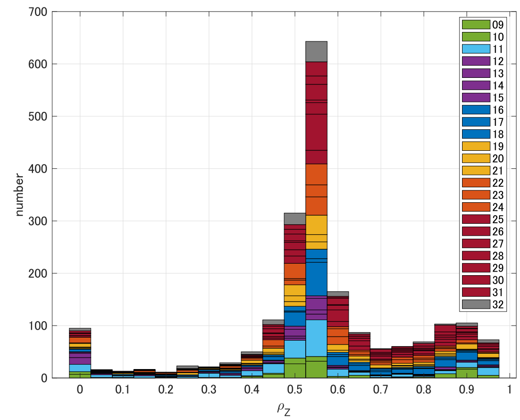
(b) Persistences of the shocks

Figure 17: Aggregate demand shocks

Note: Vertical axes measure the number of products. Horizontal axes represent the standard deviation of the shock (left panel) or its persistence (right panel).



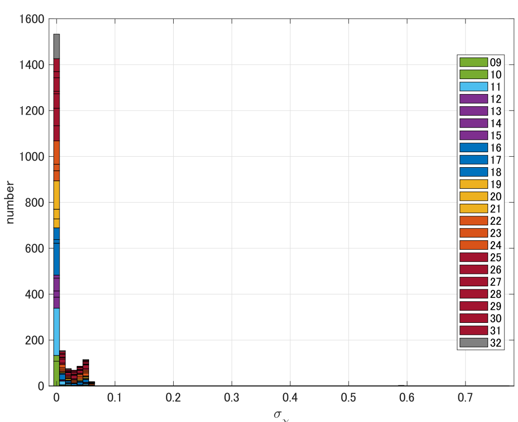
(a) Standard Deviations of the shocks



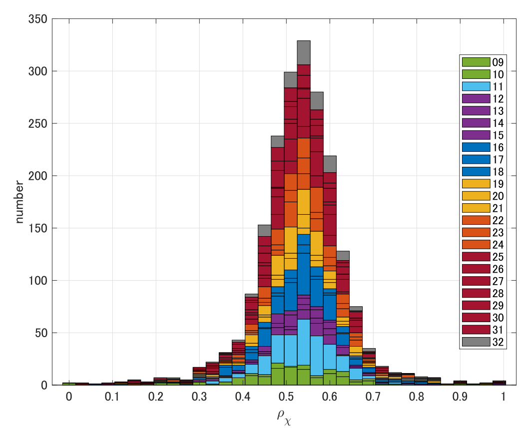
(b) Persistences of the shocks

Figure 18: Aggregate productivity shocks

Note: Vertical axes measure the number of products. Horizontal axes represent the standard deviation of the shock (left panel) or its persistence (right panel).



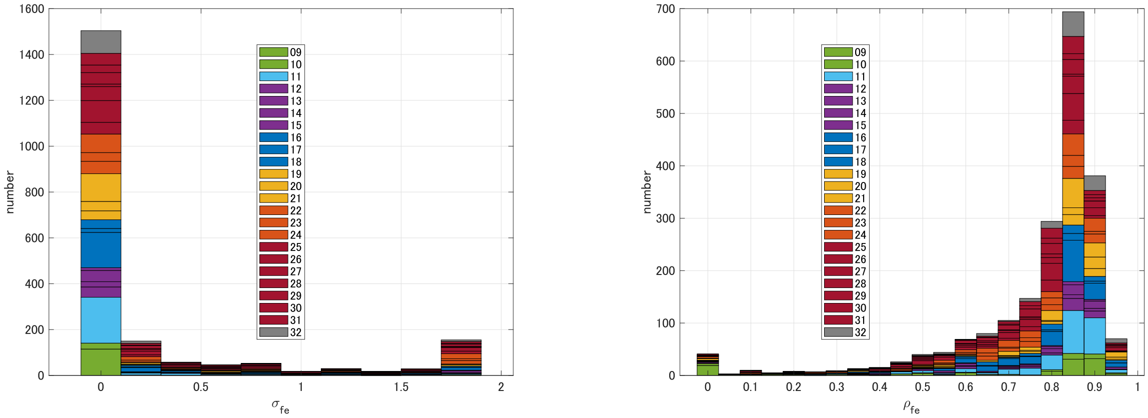
(a) Standard Deviations of the shocks



(b) Persistences of the shocks

Figure 19: Labor dis-utility shocks

Note: Vertical axes measure the number of products. Horizontal axes represent the standard deviation of the shock (left panel) or its persistence (right panel).

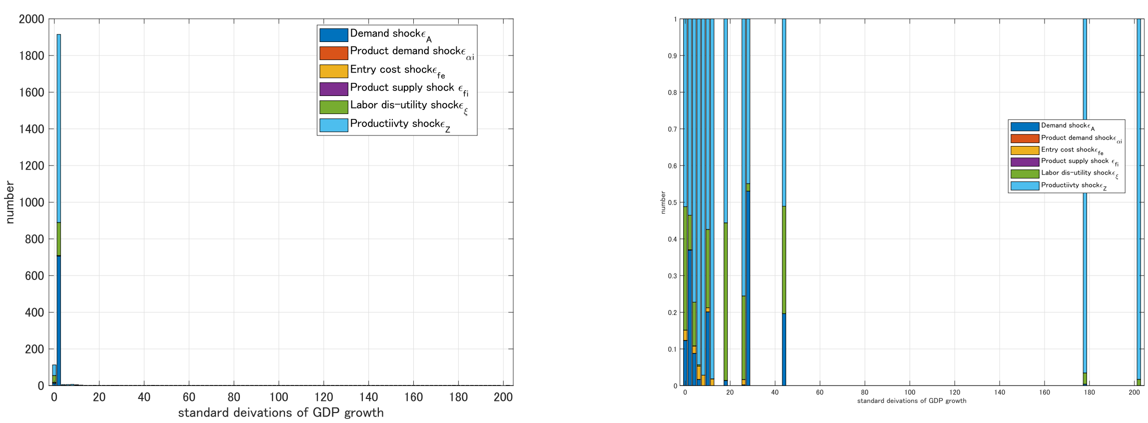


(a) Standard Deviations of the shocks

(b) Persistences of the shocks

Figure 20: Product specific entry cost shocks

Note: Vertical axes measure the number of products. Horizontal axes represent the standard deviation of the shock (left panel) or its persistence (right panel).



(a) Variance decomposition of GDP growth

(b) Variance decomposition of GDP growth (share)

Figure 21: Variance decomposition of GDP growth

Note: Horizontal axes represent product sales share. Vertical axes measure the correlations between the growth in the number of product producing plants and GDP growth (left panel) or the standard deviations of the growth in the number of product producing plants (right panel).

Table 8: Variance Decomposition

Categories		σ_A	σ_Z	σ_χ	σ_{f_E}	σ_{α_i}	σ_{f_i}
09-10	Y	0.2857	0.4530	0.2480	0.0132	0.0000	0.0000
	$\mathcal{Y}_i$	0.0376	0.0408	0.0366	0.0188	0.8662	0.0000
	M_i	0.0043	0.0785	0.0087	0.0024	0.3120	0.5941
11	Y	0.4685	0.4079	0.1222	0.0013	0.0000	0.0000
	$\mathcal{Y}_i$	0.0125	0.0114	0.0052	0.0005	0.9704	0.0000
	M_i	0.0026	0.0484	0.0010	0.0001	0.3082	0.6397
12-15	Y	0.4221	0.4232	0.1531	0.0015	0.0000	0.0000
	$\mathcal{Y}_i$	0.0211	0.0269	0.0113	0.0012	0.9395	0.0000
	M_i	0.0038	0.0415	0.0020	0.0003	0.2458	0.7066
16-18	Y	0.3052	0.6097	0.0824	0.0027	0.0000	0.0000
	$\mathcal{Y}_i$	0.0205	0.0360	0.0067	0.0035	0.9334	0.0000
	M_i	0.0063	0.0421	0.0025	0.0014	0.4073	0.5405
19-21	Y	0.4156	0.4674	0.1158	0.0011	0.0000	0.0000
	$\mathcal{Y}_i$	0.0142	0.0230	0.0057	0.0009	0.9562	0.0000
	M_i	0.0035	0.0482	0.0014	0.0004	0.3425	0.6040
22-24	Y	0.3057	0.6197	0.0728	0.0018	0.0000	0.0000
	$\mathcal{Y}_i$	0.0132	0.0264	0.0041	0.0026	0.9537	0.0000
	M_i	0.0027	0.0311	0.0014	0.0015	0.3343	0.6291
25-31	Y	0.3024	0.6181	0.0767	0.0028	0.0000	0.0000
	$\mathcal{Y}_i$	0.0111	0.0352	0.0028	0.0028	0.9482	0.0000
	M_i	0.0043	0.0339	0.0010	0.0016	0.3987	0.5606
32	Y	0.4302	0.4607	0.1080	0.0010	0.0000	0.0000
	$\mathcal{Y}_i$	0.0116	0.0205	0.0048	0.0033	0.9597	0.0000
	M_i	0.0021	0.0591	0.0007	0.0001	0.2787	0.6593