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Firm-level Study on the Global Connection through Stock Ownership Relations*

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Abstract
The progress of globalization has made economic relations around the world ever more complex, including stock ownership. Here we analyze the Global Equity Ownership database compiled by Thomson Reuters Corporation for the period from 1997 to 2020, currently known as the Refinitiv Ownership and Profiles database. The comprehensive database enables us to construct a stock ownership network for every year that classifies firms and shareholders as nodes and stock ownership relationships as directed links. By adopting network-theoretic methods, we elucidate how firms and investors are connected globally through their stock ownership relations. We pay special attention to the role of Japan in the network. We find that the Japanese firms are discriminated from those in the other countries by strong correlation between their in-degree and out-degree. Such peculiarity in the node properties leads to a loosely coupled cluster constituted dominantly by most Japanese firms with multilateral cross shareholdings. This fact is confirmed by the theory for random directed networks and by randomization simulations. We also address the current issue of publicly listed parent/subsidiary pairs of firms by simulating what would happen to the cooperative ownership structure in Japan if those pairs of firms are combined into one.

Keywords: stock ownership network, bow-tie decomposition, strongly connected component, multilateral cross shareholding, publicly listed parent/subsidiary pair
JEL classification: C55, F21, F62

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I. Introduction

Recent globalization processes have made economic relations around the world ever more complex. Stock ownership is one example of this. There are approximately 41,000 listed firms in the world, with a combined market value of more than 80 trillion dollars in 2017; the total market value of the firms is comparable to the global GDP.

A number of studies have been conducted on stock ownership relations. Many previous studies were subject to a narrow scope, selecting large companies and looking only at their local web of interconnections (Brioschi et al., 1989; Baldone et al., 1998; Porta et al., 1998; La Porta et al., 1999; Claessens et al., 2000; Faccio and Lang, 2002; Chapelle, 2004; Chapelle and Szafarz, 2005; Gutiérrez and Pombo, 2009). They have usually identified the ultimate owners of those important firms within the small networks. From the viewpoint of complex networks, the ownership relationship was explored at a national level, e.g., in Germany (Kogut and Walker, 2001), between European countries and the United States Windolf (2002), Italy (Corrado and Zollo, 2006; Rotundo and D'Arcangelis, 2010), and the Gulf Cooperation Council countries (Santos, 2015). Recently, Battiston et al. (Glattfelder and Battiston, 2009; Vitali et al., 2011; Glattfelder, 2012; Vitali and Battiston, 2014) carried out a series of studies on the ownership network constructed on a global scale, which was distinguished from previous works with respect to the exhaustiveness of the data they used, together with their own methodology.

If some shares of firm A are held by firm B, the shareholders of firm B thereby indirectly own part of the value of firm A. And, if the shares of firm B are held by firm C, the shareholders of the latter are in effect part owners of firm A. Such a sequence of stock holding relationships has been progressively established, even across national borders, resulting in a highly complex international network, and so the recent accelerated globalization processes make it increasingly uncertain as to who the genuine owners of a given firm are.

Furthermore, in the above example, what would happen if firm C were to possess shares of firm A? Such a formation of a triangular investment loop is a typical example of multilateral cross-holdings, giving rise to a kind of relationship that takes the form of an *ouroboros*. In fact, circular ownership loops are often observed in Japan and Korea, and are known as *keiretsu* (Futatsugi, 1973; Lincoln et al., 1992; Flath, 1992; Lincoln et al., 1996; McGuire and Dow, 2009; Waldenberger, 2016) and *chaebol* (Bae et al., 2002; Almeida et al., 2011; Park et al., 2020), respectively.

The concentration of wealth is increasing in various economic sectors around the world. the stock holdings of the top three passive index funds located in the United States, for instance, which referred to as the Big Three (Fichtner et al., 2017), are even bigger than the country's federal budget. We have already shed (Kichikawa et al., 2021) new light on global stock ownership by making full use of a comprehensive dataset available to us, together with the state-of-the-art methods of network science (Barabási, 2016; Aoyama et al., 2017). We constructed ownership networks annually on the basis of the database that includes virtually all of the world's listed firms and their market capitalizations. Such an approach enabled us to address the question of who possesses whom through a web of stock ownership relations on a global scale. We thus discussed the issue from a wide variety of viewpoints, including the hierarchical positions of countries, the distribution of corporate equity holdings within them, cross-ownership between them, and the progressive dominance of institutional investors based in the United States.

In this paper we turn our attention to the role of the Japanese firms in the global ownership network. The Japanese firms have invested in R&D from a long-term perspective because they tend to pursue long-term growth as a management strategy with stable shareholders and investment. Long-term business practices among firms, one of the characteristics of such management, have advantages in manufacturing integral-type products such as automobiles. It attracts us to see how the Japanese-style management is reflected in the global stock ownership structure. In fact, we find that the Japanese firms are discriminated from those in the other countries by strong correlation between their in-degree and out-degree. The notable peculiarity in the node properties leads to a loosely coupled cluster constituted dominantly by most of the Japanese firms with multilateral cross shareholdings. This fact is confirmed by the theory for directed random network and by randomization simulations. We also address the current issue of publicly listed parent/subsidiary pairs of firms in the Japanese stock markets (Kobayashi and Yamada, 2000; Kobayashi et al., 2000). Firms listed on stock exchanges along with their subsidiaries have come under closer scrutiny by investors due to continuing concerns over corporate governance. We simulate what would happen to the cooperative ownership structure in Japan if those pairs of firms are combined into one.

II. Stock Ownership Data

In this study, we analyze the Global Equity Ownership database[1] complied by Thomson Reuters Corp. which is a proprietary database of daily financial transactions of firms over the world, for the period from 1997 to 2020. From this database, we construct a stock ownership network for every year that classifies firms and shareholders as nodes and stock ownership relationships as directed links. The direction of a link between an owner and a stock issuer is defined so as to coincide with that of the ownership (from the owner to stock issuer), and its weight is to be the current price of the shares of the issuer held by the owner. Shares issued by mutual funds and real estate investment trusts (REITs) are not included in this study.

Once the links appear as transaction relationships between owners and issuers, we essentially maintain them. However, we must track whether links are alive or dead. Unfortunately, information on the current status of nodes is not available. In an alternative manner, we removed links that did not appear throughout one year from the database if both of their end nodes had never traded with others in the year; these nodes were regarded as being dead. On the other hand, we maintained such an inactive link if either of its end nodes featured any transactions during the year.

We emphasize that this database is not ready for the construction of ownership networks. This is because stock owners and issuers are separately treated in the database. Since listed firms can play both roles, we have to identify firms which are stock owners as well as issuers. The task is not so easy as described in the Appendix A, where more detailed information is available on how we constructed networks from the original database.

Table 1 shows basic information on the global ownership network constructed for every year from 1997 through 2020; the total numbers of nodes and links are given, together with the number of listed firms.

Many of real-world networks are characterized by the scale-free nature; a scale-free network is a network whose degree distribution follows a power law, at least asymptotically. However, the networks thus constructed have no such a characteristic property as often observed in complex networks. Figures 1 and 2 indicate that the degree distributions are close

[1]Currently known as the Refinitive Ownership and Profiles (ROP) database. The RIETI enabled us to access this database.

to the log-normal distribution. The functional modeling of the degree distributions for listed firms within countries is discussed below, which provides us with fundamental knowledge to delve into the ownership network structure.

Table 1. Temporal change of the total numbers of nodes, links and listed firms in the global ownership networks based on the Thomson Reuters Global Equity Ownership data.

year	#nodes	#links	#listed firms
1997	48971	658837	20847
1998	67312	780851	22622
1999	85252	957440	24238
2000	104811	1094045	25661
2001	119200	1182893	25853
2002	131264	1204631	25978
2003	147471	1327455	26794
2004	188032	1433706	27926
2005	224474	1592234	30182
2006	264293	1796924	31720
2007	317710	2104944	35352
2008	338317	2066089	36381
2009	357826	2117965	37523
2010	370393	2255207	38398
2011	384267	2291905	38955
2012	378434	2357983	39012
2013	380460	2594554	39237
2014	384865	2810195	39668
2015	401560	3040563	40368
2016	413565	3225174	40528
2017	428296	3423172	40999
2018	433620	3473816	41424
2019	414799	3587785	41497
2020	385612	3641237	40711

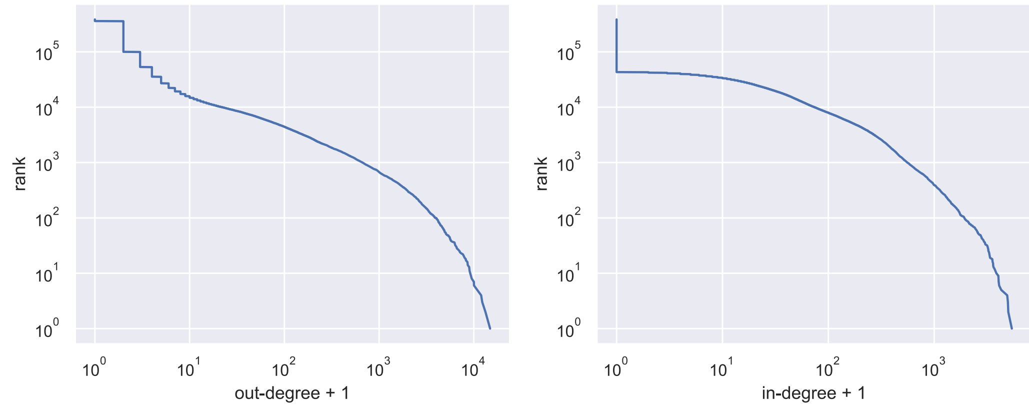

Figure 1. Distributions of the out-degrees (left panel) and the in-degrees (right panel) of nodes of the stock ownership network in 2020. Note that both axes are in a logarithmic scale.

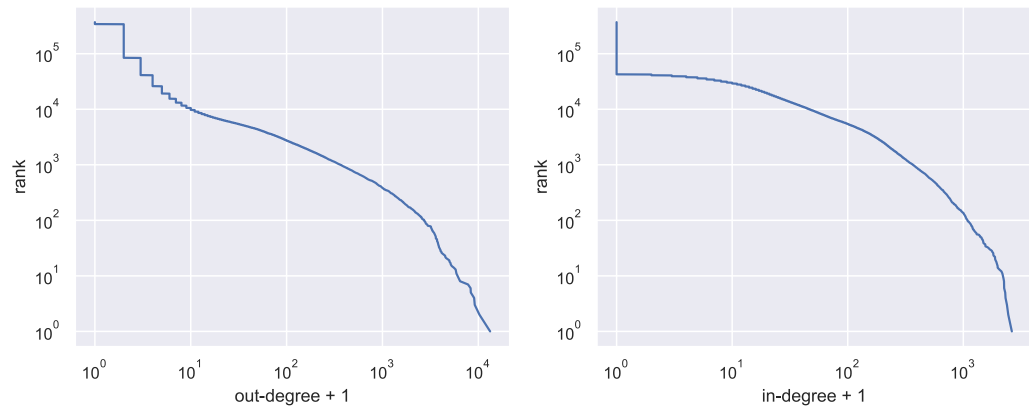

Figure 2. Same as Fig. 1, but in 2010.

The probability density function for the log-normal distribution is given by

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}x} \exp\left[-\frac{(\ln x-\mu)^2}{2\sigma^2}\right], \tag{1}$$

where the two parameters μ and σ are to be optimized to reproduce the empirical distribution of the in-degree or out-degree of listed firms in each country. The mean $\overline{x}$ and median $\tilde{x}$ of the log-normal distribution are given by $\exp(\mu+\sigma^2/2)$ and $\exp(\mu)$, respectively. The fitted value of μ thus gives information on the typical value of the distribution. Also we can learn to what extent the distribution is right skewed from the fitted value of $\sigma^2 = 2\log(\overline{x}/\tilde{x})$.

The fitting results for U.S. and Japan are shown in Tables 2 and 3, respectively. The accuracy of the log-normal function fittings is displayed in Figs. 3 and 4. The median of the in-degree distribution in U.S. is about three times as large as the corresponding median in Japan. And the distribution in U.S. is more right skewed than that in Japan. These comparisons between the two countries are also true for the out-degree distribution. The Appendix B shows the results for China, Korea, and Taiwan. We find no much difference in the in-degree distribution between those countries and Japan. As regards the out-degree distribution, however, the East Asian countries are well distinguished from Japan; they have the median about one third smaller than Japan has.

Table 2. Log-normal function fitting of the in-degree and out-degree distributions for the listed firms in U.S., where the parameters μ and σ were determined by the maximum likelihood method.

	k_{in}			k_{out}		
year	μ	σ	log-likelihood	μ	σ	log-likelihood
2007	4.2	1.4	−3.04e+04	2.9	2.0	−2.28e+03
2008	4.2	1.4	−2.89e+04	2.8	2.0	−2.23e+03
2009	4.3	1.5	−2.80e+04	2.9	2.0	−2.19e+03
2010	4.4	1.5	−2.80e+04	2.9	2.0	−2.18e+03
2011	4.4	1.4	−2.72e+04	3.0	2.0	−2.22e+03
2012	4.5	1.4	−2.69e+04	3.0	2.0	−2.23e+03
2013	4.6	1.4	−2.73e+04	3.0	2.0	−2.29e+03
2014	4.7	1.4	−2.80e+04	3.1	2.0	−2.33e+03
2015	4.7	1.4	−2.81e+04	3.1	2.0	−2.41e+03
2016	4.8	1.4	−2.78e+04	3.1	2.0	−2.45e+03
2017	4.8	1.4	−2.76e+04	3.2	2.0	−2.42e+03
2018	4.9	1.4	−2.74e+04	3.1	1.9	−2.44e+03
2019	4.9	1.4	−2.73e+04	3.2	1.9	−2.55e+03
2020	4.9	1.3	−2.86e+04	3.3	1.9	−2.54e+03

Table 3. Same as Table 2, but in Japan.

	k_{in}			k_{out}		
year	μ	σ	log-likelihood	μ	σ	log-likelihood
2007	3.3	1.0	−1.97e+04	2.0	0.8	−9.10e+03
2008	3.4	1.0	−1.96e+04	2.0	0.8	−9.05e+03
2009	3.4	1.0	−1.93e+04	2.0	0.8	−9.14e+03
2010	3.5	1.0	−1.91e+04	2.0	0.8	−9.14e+03
2011	3.5	1.0	−1.87e+04	2.1	0.8	−9.09e+03
2012	3.5	1.0	−1.87e+04	2.1	0.9	−9.14e+03
2013	3.6	1.0	−1.90e+04	2.1	0.9	−9.27e+03
2014	3.7	1.0	−1.93e+04	2.3	1.0	−1.01e+04
2015	3.7	1.0	−2.00e+04	2.3	1.0	−1.03e+04
2016	3.8	1.0	−2.05e+04	2.3	1.0	−1.03e+04
2017	3.8	1.0	−2.08e+04	2.3	1.0	−1.03e+04
2018	3.8	1.0	−2.11e+04	2.3	1.0	−1.03e+04
2019	3.9	1.0	−2.14e+04	2.4	1.0	−1.05e+04
2020	3.9	1.0	−2.16e+04	2.4	1.1	−1.05e+04

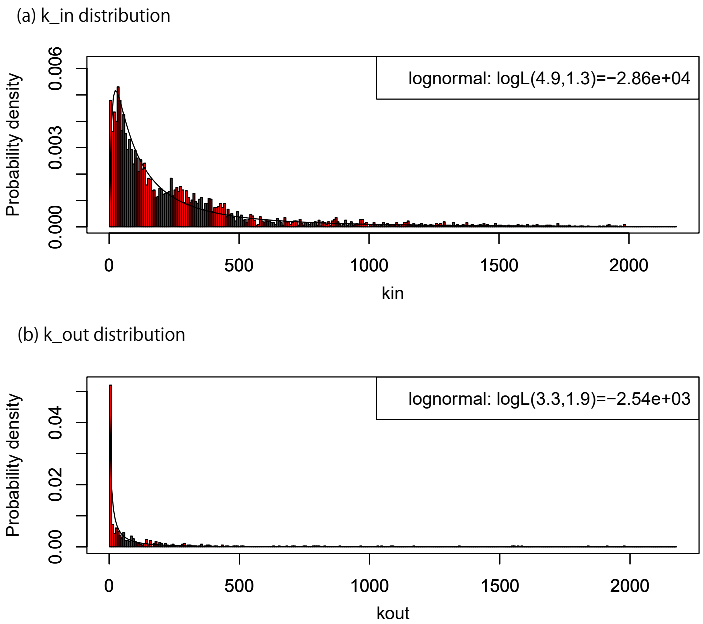

Figure 3. Distributions of the in-degrees (upper panel) and out-degrees (lower panel) of listed firms in U.S. in 2020 with the log-normal function fitting depicted by solid line.

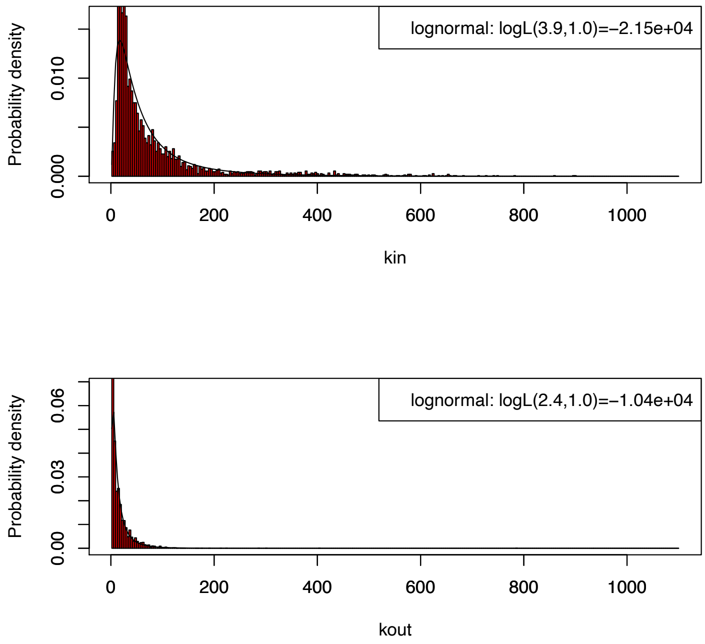

Figure 4. Same as Fig. 3, but in Japan.

III. Comprehensiveness of the Ownership Networks

A series of the results as will be presented in this section allows us refer to our networks constructed from the Thomson Reuters database as *global* ownership networks.

A. *Numbers of domestic listed firms*

Figures 5 and 6 compare the numbers of listed firms for the G20 countries in the global ownership network in 2020 and 2010, respectively, with the corresponding data drawn from the World Bank[2]. We see that most of the listed firms are included in the database for the major countries.

[2]`https://data.worldbank.org`

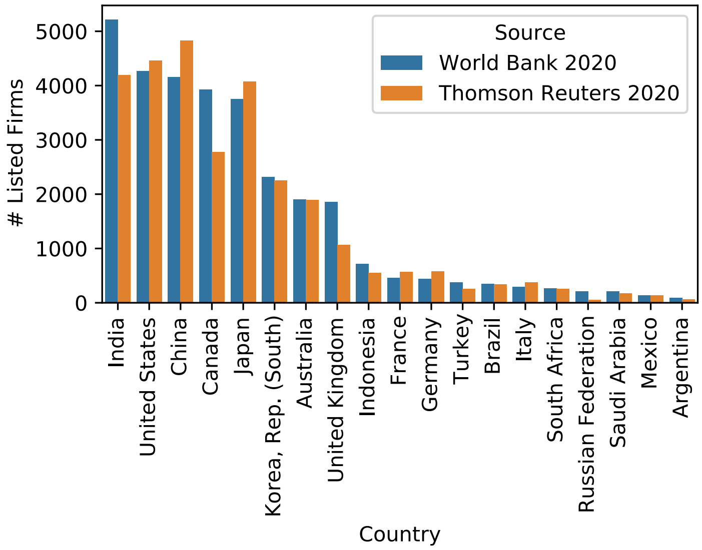

Figure 5. Coverage of the Thomson Reuters Global Equity Ownership data used in this study with respect to the total number of domestic listed firms in 2020. The orange bar represents the total number of those firms included in the data set for each country. The blue bar represents the corresponding data compiled by the World Bank.

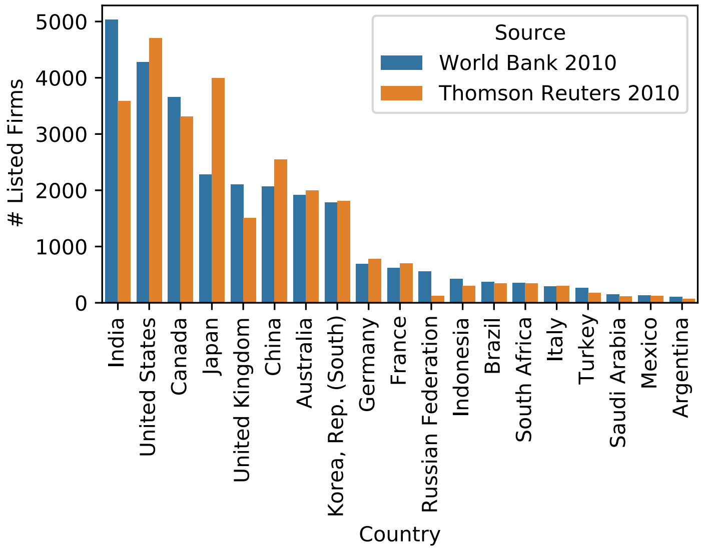

Figure 6. The same as Fig. 5, but for 2010. Note that there exists a notable discrepancy between the World Bank data and the Thomson Reuters database with respect to the number of Japanese listed firms. According to our own counting, however, the correct number is 3,664 in that time, which agrees quite well with the corresponding result based on the Thomson Reuters database. We discovered that the World Bank data omitted firms belonging to minor markets, such as the JASDAQ. The World Bank data first began to incorporate the firms in the JASDAQ in 2013.

B. *Domestic market capitalizations*

Figures 7 and 8 compare the total market capitalization in the global ownership network with the corresponding World Bank data for the same years as in Figs. 1 and 2. Here, the market capitalization of each country in the global ownership network is calculated by summing up the weights of the incoming links of all nodes belonging to the country. The coverage of our network data is almost exhaustive in each country with respect to the market capitalization, as well as the number of listed firms in both years.

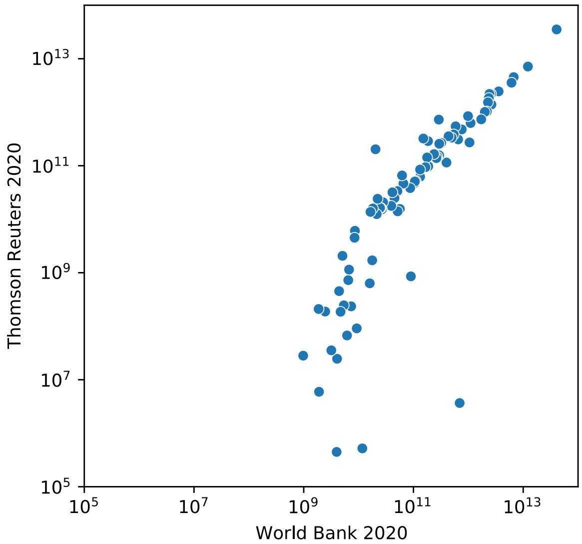

Figure 7. Coverage of the Thomson Reuters Global Equity Ownership data used in this study with respect to market capitalization in 2020. The market capitalization (in units of USD) of each country, based on the dataset, is plotted against the calculations of the World Bank.

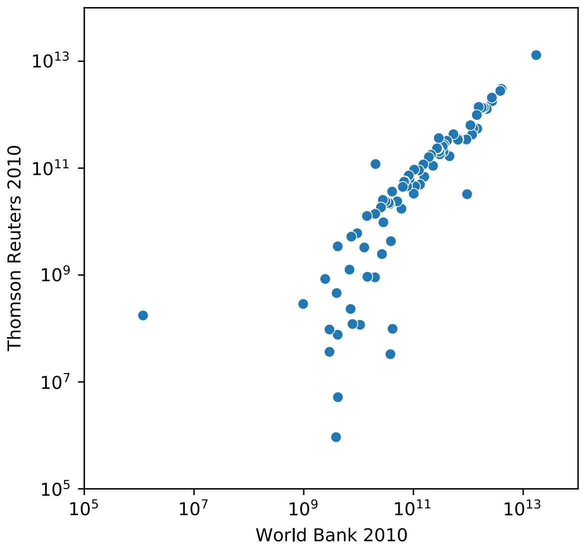

Figure 8. Same as Fig. 7, but in 2010.

C. *FDI and FPI*

Foreign Direct Investment (FDI) and Foreign Portfolio Investment (FPI) provide us with alternative points of view for understanding complex international relations. The FDI focuses attention on how attractive is a country for another country to economically control by investment, while the FPI stresses how attractive is a country for another country to obtain profits by investment. The International Monetary Fund (IMF) makes us accessible to very detailed bilateral information on both FDI[3] and FPI[4] at a country level.

From the Thomson Reuters database, we can estimate the stock value of FDI from country A to country B by collecting shareholding relations with share of 10% or more from investors in country A to issuers in country B. For estimation of the stock value of FPI between two countries, we can take the same procedure as the FDI estimation but with shareholding relations less than 10% instead. Figure 9 compares the Thomson Reuters values with the corresponding IMF values for the FDI and FPI relationships established mutually between the G20 countries in 2020. The two results agree reasonably well with each other. It should be noted that the present estimation on the FDI gives a lower bound for the actual FDI value, because the FDI is also made in the form of inter-firm debt. This fact is clearly reflected in the figure.

We thus see that the ownership database prepared here has potential applications for firm-level investigations of the FDI and FPI issues. Furthermore, we can study *indirect* investment from one country into another country through firms in the third country.

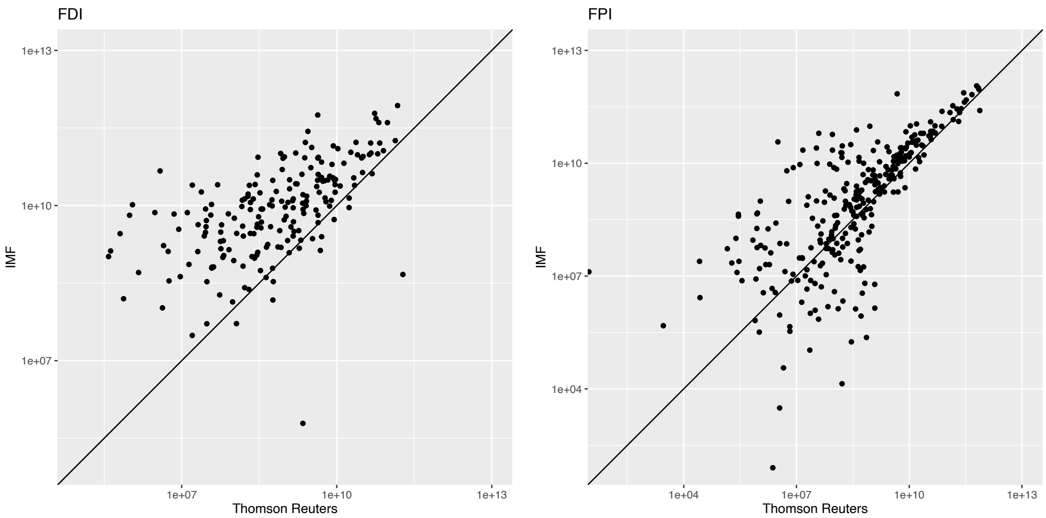

Figure 9. Comparison regarding the bilateral FDI (left panel) and FPI (right panel) positions for the G20 countries in 2020 between the Thomson Reuters data plotted on the horizontal axis and the IMF data on the vertical axis, where the FDI and FPI positions are measured by their stock values in units of USD. Note that both axes have a logarithmic scale and the points should be aligned on the diagonal line if the results based on the two datasets completely agree with each other.

[3]`https://data.imf.org/?sk=40313609-F037-48C1-84B1-E1F1CE54D6D5`

[4]`https://data.imf.org/?sk=B981B4E3-4E58-467E-9B90-9DE0C3367363`

IV. Bow-tie Structure

The bow-tie decomposition (Broder et al., 2000; Dorogovtsev et al., 2001a; Ma and Zeng, 2003; Yang et al., 2011) has been applied to a wide variety of directed networks such as the World Wide Web and metabolic network models to elucidate flow structure in these complex systems. The decomposition of a directed network begins with identifying the largest strongly connected component, which is referred to as the *giant* strongly connected component (GSCC) herein, even if it is in fact not especially large compared to the entire network. In the strongly connected components, any pairs of nodes are connected bidirectionally by two directed paths. Then, the nodes are classified according to how they are connected to the GSCC: IN, In-tendrils, OUT, OUT-tendrils, TUBE, and the others. The IN component is a collection of nodes that have a path to the GSCC, but no reverse path to come back from it. The OUT component is defined the other way around; that is, a collection of nodes that are only reachable from the GSCC. The IN component may have leaf nodes from which the GSCC cannot be reached; those nodes are referred to as IN-tendrils. On the other hand, the tendrils of the OUT component are its leaf nodes which are directly connected to the OUT component. The TUBE is a collection of nodes that bypass from the IN to the OUT and not through GSCC. The remaining nodes are referred to as the others. On the basis of these definitions, the classification scheme of nodes provides an overview of the hierarchical structure of a directed network. In Fig. 10, the bow-tie structure of a directed network is illustrated in a schematic way.

Figure 11 shows the temporal change in the number of nodes belonging to each of the bow-tie components. As demonstrated in Fig. 12, where only the listed firms are counted in Fig. 11, the relative size of the GSCC dramatically increases around 2004 and is fairly stable beyond 2005. More than half of the listed firms in the world have been incorporated into the principal bow-tie components, GSCC, IN, and OUT, since 2010; furthermore, the fraction of those firms is gradually increasing. The Japanese firms are dominant constituents of the GSCC in the ownership networks. For instance, Japan occupies 2,923 nodes out of 4,043 nodes in the GSCC of the network in 2020. The second and third important countries fo the GSCC are China with 279 nodes and U.S. with 269 nodes, followed by Australia with 107 nodes, Hong Kong with 56 nodes, Taiwan with 49 nodes, Korea with 47 nodes, Thailand with 43 nodes, Canada with 33 nodes, and Norway with 25 nodes.

Figure 13 shows 3D visualization of the network in 2020, accompanied by the illumination of the principal bow-tie components, that is, IN, GSCC, and OUT, with different colors. The GSCC is located at the center of the network. It is fairly prolonged and sandwiched by the IN component on the top and the OUT component on the bottom. Tendrils and the other nodes surround the network's core components. Figure 14 projects the GSCC of the network as shown in Fig. 13 onto the x-y plane and also highlights the firms in the top 7 countries constituting the GSCC. Basically, the firms belonging to the individual countries except U.S. are distributed radially forming a folding fan shape, while the American firms are positioned around the center of the folding fan. The Japanese firms are Chinese firms are distributed almost oppositely. The Austrian and Korean firms are located between the two countries. The Taiwanese firms are well overlapped with the Japanese cluster. On the other hand, the firms in Hong Kong form one cluster with the Chinese firms as expected.

The optimized layout of the ownership network in 3D space was obtained by incorporating information on the Helmholtz–Hodge potential into individual nodes. The Helmholtz–Hodge decomposition (Jiang et al., 2011; Johnson and Goldring, 2013; Bhatia et al., 2013; Kichikawa et al., 2019a) allows us to uniquely decompose the flow structure in a directed network into gradient and circular flow components; the gradient flow always runs from a node with higher potential to one with lower potential like a stream of water. The nodes are aligned in the z direction in accordance with their potential values. According to the definition of the Helmholtz–Hodge potential, essentially, the ownership flows from the top to the bottom; that is, nodes on the downstream side are owned by those on the upstream side. The remaining x and y coordinates of nodes were determined by minimizing the potential energy in a spring-electric model, in which the nodes with direct ownership relationships are connected to one another by a spring and all of the nodes have an identical electric charge to prevent the disconnected nodes from being closely coordinated in an accidental manner.

Recently, we have developed (Kichikawa et al., 2019b,a) a software for the visualization of directed networks called the Flow Analysis tools for Large-scale Complex Networks (FALCON)[5]. As is appreciated in Fig. 13, the FALCON is highly useful for gaining an overview on the hierarchical flow layout of a directed network including its bow-tie structure.

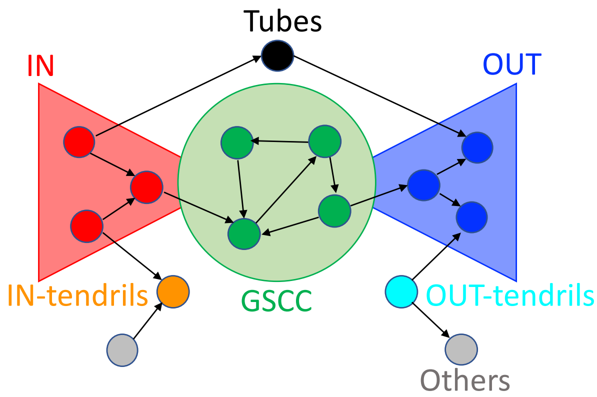

Figure 10. Schematic diagram of the bow-tie structure of a directed network. The color coding of the nodes is as follows: red for the IN nodes, green for the GSCC nodes, blue for the OUT nodes, orange for the IN-tendril nodes, cyan for the OUT-tendril nodes, black for the TUBE nodes, and gray for the other nodes.

[5] https://github.com/ykichikawa/FALCON

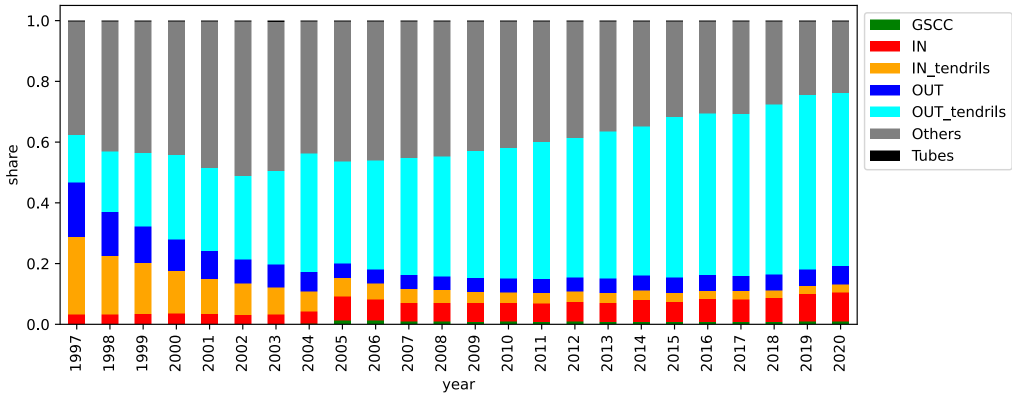

Figure 11. Temporal change of the bow-tie structure in the stock ownership networks. The vertical axis shows the share of each bow-tie component in the form of a stacked bar chart.

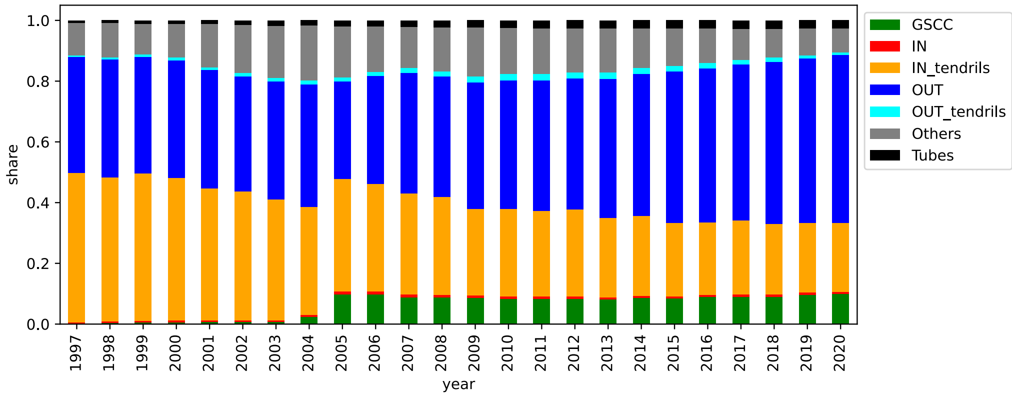

Figure 12. Same as Fig. 11, but only listed firms are counted in evaluating the relative size of each bow-tie component.

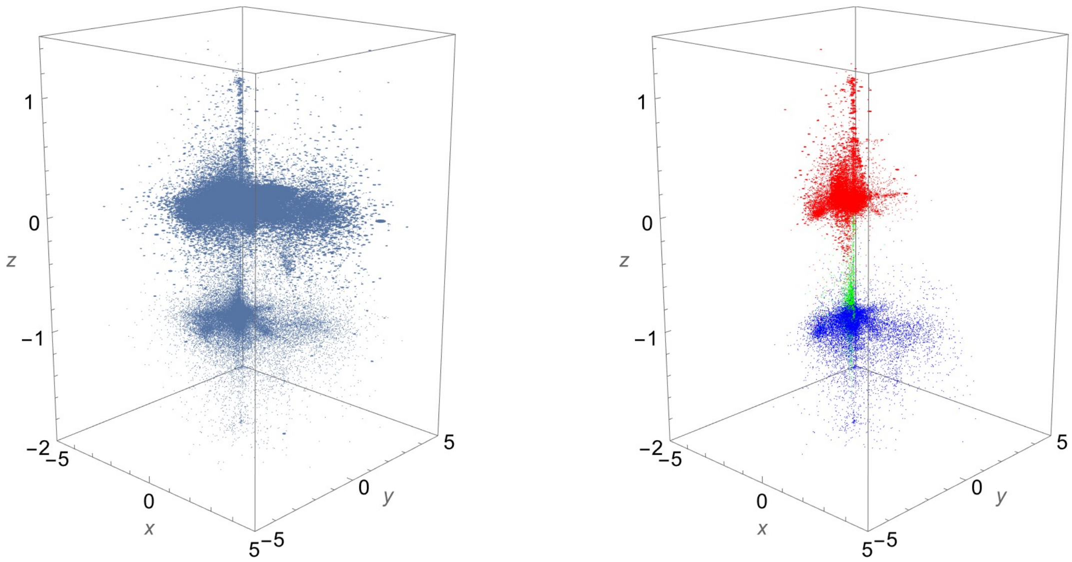

Figure 13. 3D visualization of the stock ownership network in 2020 using FALCON. The left figure show the whole network. The right figure illuminates only the IN component (red), the GSCC (green), and the OUT component (blue) of the network as shown in the left figure.

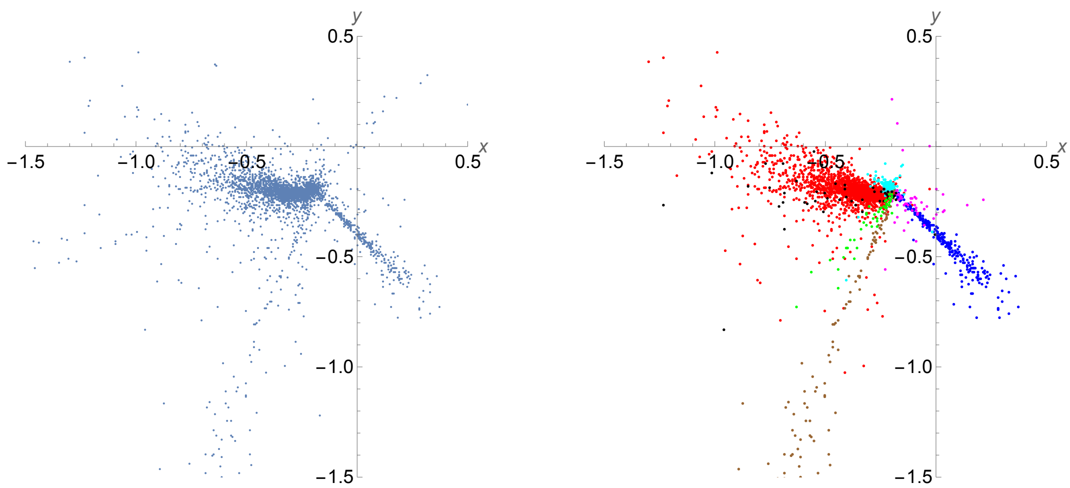

Figure 14. The GSCC of the network in Fig. 13 projected onto the x-y plane. The left figure shows all the firms belonging to the GSCC and the right figure illuminates only firms of the top 7 countries, including Japan (red), China (blue), U.S. (cyan), Australia (brown), Hong Kong (Magenta), Taiwan (black), and Korea (green).

V. Multilateral Cross Shareholdings

As has been already pointed out, the Japanese firms play a major role in forming the GSCC in the global ownership networks. Since any pairs of nodes in a strongly-connected component have a path in each direction between them, many of the Japanese firms form a group of multilateral cross shareholdings. Here we find that such circular ownership structure is not intentionally created, but instead is ascribed to the strong correlation between the in-degree and out-degree of the Japanese firms, which distinguishes them from firms in other countries.

A. *Origin of the GSCC*

To measure how tightly firms are connected with each other in the GSCC, we reconstructed the ownership networks by dismissing links which have shareholding ratio below a cutoff η and then applied the bow-tie decomposition to the reconstructed networks with varied η. The results for the strongly connected components of the network in 2020 are summarized in Table 4. The GSCC breaks into pieces rapidly beyond $\eta = 0.01$, indicating that the firms in the GSCC form a loosely coupled cluster with no strong ownership strategies. This fact is highly opposed to the idea of *keiretsu* and *chaebol* in Japan and Korea, respectively. We thus speculate that the formation of the GSCC have an intrinsic origin different from those business conglomerate systems.

To confirm our speculation, we carried out the two kinds of randomization experiments on the global ownership networks that we have constructed. In the first experiment, referred to as full randomization, the links were rewired randomly even across countries. In fact, however, there is a strong tendency that investors and firms have ownership with firms in the same country as they belong to. To take account of such bias in the formation of ownership linkage, we prepared the second randomization experiment, referred to as partial randomization, by imposing the restriction that the links are rewired within countries. We remark that these randomization processes preserve the in-degree and out-degree of each node.

As an illustrative example, Table 5 gives the experimental results for the network in 2020. One can see to what extent the randomization of linkage affects the GSCC of the network by monitoring the number of nodes in the major countries comprising the network. Japan is a dominant country for the GSCC modified by the partial randomization and its dominance is carried over to the full randomization. We have thus confirmed the speculation is correct. In passing we note that the numbers of the GSCC nodes in Korea and Taiwan dramatically increases by the partial randomization. This indicates that there is a factor(s) to depress the formation of circular ownership reflected in the GSCC for those countries.

Table 4. Dependence of the strongly connected components of the ownership network in 2020 on the cutoff η for share holdings, where links with shareholding ratio below η are dismissed. The notations, #nodes, #links, and #sccs, represent the number of nodes, the number of links, and the number of strongly connected components, respectively, in the largest weakly connected component of the network; scc1 and scc2, the size of the largest and the second largest strongly connected components, respectively, measured by the number of nodes in each of them.

η	#nodes	#links	#sccs	scc1	scc2
0.00	385612	3641237	367	4043	38
0.01	199427	491396	396	1206	26
0.02	156918	322605	416	22	19
0.03	129399	237898	322	12	6
0.04	108777	183867	259	6	6
0.05	92833	146171	218	6	6
0.06	79948	118424	181	6	6
0.07	69218	97177	162	5	5
0.08	59973	80645	141	5	4
0.09	52415	67648	122	4	4
0.10	45656	56987	111	4	4

Table 5. Effects of randomization of the ownership network in 2020 on the major countries constituting the GSCC of the network, measured by the number of nodes in each country. The randomization of the network refers to random rewiring of links with the in-degree and out-degree of each node preserved. The randomization processes denoted by Original, Partial and Full stand for no randomization, random rewiring of links within countries, and full randomization across countries, respectively. The average and the associated standard deviation for the number of nodes in the GSCC classified by countries was computed with 1000 samples.

	Original	Partial	Full
Japan	2923	3170.2 ± 17.1	2253.7 ± 28.9
China	279	406.4 ± 42.0	547.5 ± 19.6
U.S.	269	434.0 ± 17.2	477.8 ± 15.6
Australia	107	211.1 ± 16.3	254.5 ± 12.5
Hong Kong	56	95.4 ± 8.1	100.5 ± 8.0
Taiwan	49	361.5 ± 20.8	281.7 ± 12.8
Korea	47	397.7 ± 27.4	329.6 ± 17.5
Thailand	43	75.3 ± 15.5	94.7 ± 6.8
Canada	33	113.8 ± 10.0	122.4 ± 7.9
Norway	25	32.1 ± 3.7	29.1 ± 3.5
All countries	4043	6243.3 ± 102.3	5867.0 ± 106.2

B. *In-degree and out-degree correlation*

The theoretical study on the bow-tie structure of random directed networks (Newman et al., 2001; Dorogovtsev et al., 2001b) provides an idea to understand the origin of formation of the GSCC dominated by the Japanese firms. It is ascribed to strong correlation between the in-degree and the out-degree of them as shown in the Appendix D.

To ascertain the idea empirically, we invoke the joint cumulative distribution function $F(k_{\rm in}, k_{\rm out})$ for the in-degree $k_{\rm in}$ and the out-degree $k_{\rm out}$ of nodes represented in terms of the copula $C(u_{\rm in}, u_{\rm out})$:

$$F(k_{\rm in}, k_{\rm out}) = C(u_{\rm in}, u_{\rm out}) \ , \tag{2}$$

where $u_{\rm in}$ and $u_{\rm out}$ are the cumulative probabilities for $k_{\rm in}$ and $k_{\rm out}$,

$$u_{\rm in} = F_{\rm in}(k_{\rm in}) \ , \tag{3}$$

$$u_{\rm out} = F_{\rm out}(k_{\rm out}) \ . \tag{4}$$

The joint probability density function $f(k_{\rm in}, k_{\rm out})$ for $k_{\rm in}$ and $k_{\rm out}$ is then derived as

$$f(k_{\rm in}, k_{\rm out}) = f_{\rm in}(k_{\rm in}) f_{\rm out}(k_{\rm out}) c(u_{\rm in}, u_{\rm out}) \ , \tag{5}$$

where $c(u_{\rm in}, u_{\rm out})$ is the copula density defined by

$$c(u_{\rm in}, u_{\rm out}) = \frac{\partial^2 C(u_{\rm in}, u_{\rm out})}{\partial u_{\rm in} \partial u_{\rm out}} \ . \tag{6}$$

Here the two representative copulas, Gumbel and Clayton, were tested to quantify the degree of the correlation between $u_{\rm in}$ and $u_{\rm out}$ of nodes in the major countries constituting the GSCC, that is, Japan, U.S., China, Korea, and Taiwan. The Gumbel and Clayton copulas are members of the Archimedean family, characterized by a single parameter θ as

$$C_{\rm G}\left(u_1, u_2; \theta\right) = \exp\left[-\left\{\left(-\log u_1\right)^\theta + \left(-\log u_2\right)^\theta\right\}^{1/\theta}\right] \ \ (1 \le \theta < \infty) \tag{7}$$

$$C_{\rm C}\left(u_1, u_2; \theta\right) = \left[\max\left(u_1^{-\theta} + u_2^{-\theta} - 1, 0\right)\right]^{-1/\theta} \ \ (-1 \le \theta < \infty) \tag{8}$$

The Gumbel copula exhibits greater dependence in the upper tail than in the lower tail. On the other hand, the Clayton copula has correlation characteristics opposite to the Gumbel copula and furthermore it can accommodate negative correlations with a negative value of θ. The Gumbel and Clayton copulas reduce to the independence copula $C(u, v) = uv$ at $\theta = 1$ and $\theta = 0$, respectively. In addition, we note that the Kendall rank correlation coefficient τ is expressed for these copulas in a compact form as

$$\tau_{\rm G} = 1 - \frac{1}{\theta}, \tag{9}$$

$$\tau_{\rm C} = \frac{\theta}{\theta + 2}. \tag{10}$$

The empirical copulas for U.S. and Japan are demonstrated in Figs. 15 and 16, respectively. The results for the copula modeling are given in Tables 6 and 7, for the two countries. The results for the remaining three countries are deferred to the Appendix C. The Gumbel copula works better than the Clayton copula for the countries excluding U.S., for which the Clayton copula is selected instead. Certainly, the firms in Japan have much stronger correlation between $k_{\rm in}$ and $k_{\rm out}$ than those in the other countries; the firms in U.S. show even slight negative correlation.

It immediately comes into question why the Japanese firms are so exceptional as regards the in-degree and out-degree correlation. A simple hypothesis is that both $k_{\rm in}$ and $k_{\rm out}$ of a firm is proportional to its size as measured by sales or number of employees, resulting in strong correlation between the two degrees through the hidden variable. We examined this idea empirically by adopting the market capitalization of each firm as a proxy for its size. The results are summarized in Table 8. The hypothesis works very well for the in-degree of the firms over the world, indicating diversification of the direct ownership in a global scale. In contrast, the hypothesis is mainly adaptable for the out-degree of the Japanese firms[6]. We may ascribe this peculiarity of Japan to the Japanese-style management. According to the business strategy, firms in Japan like to have long-standing relationship with their affiliated and partners. Eventually, larger firms tend to have a larger number of the out-degree.

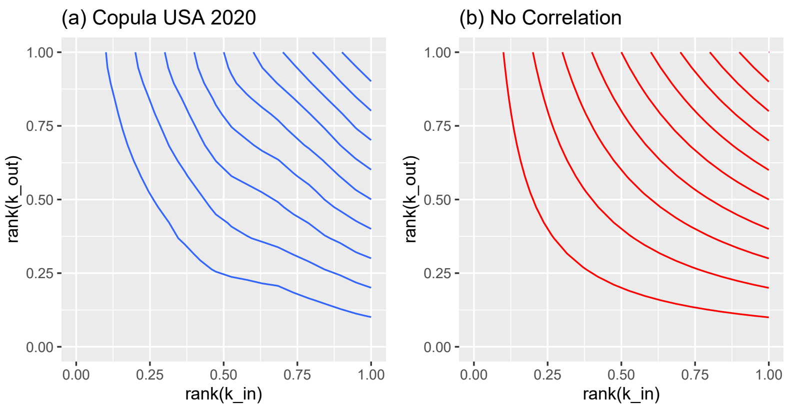

Figure 15. Copula representation of the in-degree and out-degree correlations for firms in U.S. in 2020, where the empirical copula function (left panel) and the no correlation copula (right panel) are compared in the form of a contour plot with equally spaced levels ranging from 0.1 to 0.9.

[6]Korea and Taiwan also show rather strong correlation between $k_{\rm in}$ and $k_{\rm out}$ of their firms. As has already been remarked, however, the out-degree distribution itself in those countries is not so widely spread toward large values as compared with that in Japan.

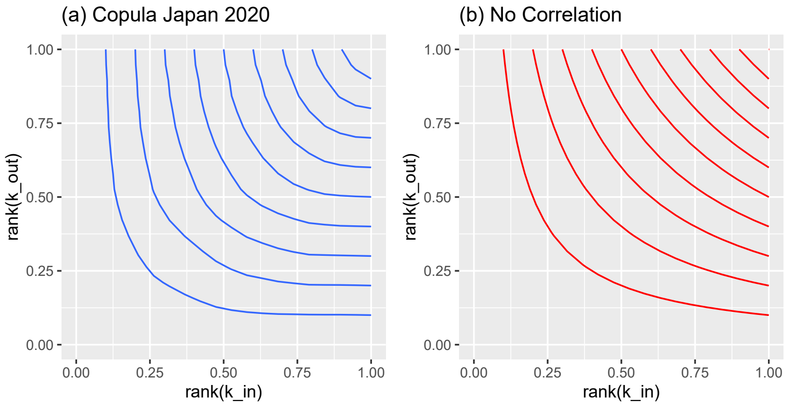

Figure 16. Same as Fig. 15, but in Japan.

Table 6. Selection of the Gumbel or Clayton copula model for the in-degree and out-degree correlation of listed firms in U.S. based on the least square error (LSE) method.

	Gumbel Copula		Clayton Copula	
year	θ	LSE	θ	LSE
2007	1.018188	0.077784	0.023900	0.079114
2008	1.000046	0.068452	−0.030946	0.066518
2009	1.000046	0.073181	−0.021269	0.072273
2010	1.000046	0.063636	−0.026115	0.062262
2011	1.000046	0.062020	−0.009231	0.061850
2012	1.000046	0.056784	−0.053061	0.050943
2013	1.000046	0.070442	−0.061583	0.062494
2014	1.000046	0.065926	−0.069375	0.055735
2015	1.000046	0.073278	−0.086556	0.057179
2016	1.000046	0.085942	−0.130618	0.047538
2017	1.000046	0.114128	−0.169252	0.046922
2018	1.000046	0.111730	−0.173368	0.040959
2019	1.000046	0.116059	−0.184490	0.035138
2020	1.000046	0.132112	−0.199540	0.036312

Table 7. Same as Table 6, but in Japan.

	Gumbel Copula		Clayton Copula	
year	θ	LSE	θ	LSE
2007	1.572868	0.006824	1.059167	0.123189
2008	1.608425	0.006201	1.131977	0.126342
2009	1.594041	0.012961	1.091230	0.156188
2010	1.587152	0.012179	1.079315	0.148647
2011	1.586469	0.011265	1.078499	0.146528
2012	1.571721	0.013574	1.044910	0.154517
2013	1.550083	0.017586	0.994723	0.165452
2014	1.634143	0.021864	1.170229	0.177621
2015	1.617185	0.023000	1.132735	0.181055
2016	1.607418	0.024239	1.110278	0.186008
2017	1.598101	0.024206	1.090881	0.184004
2018	1.589198	0.024262	1.071183	0.185692
2019	1.589122	0.023089	1.073317	0.177241
2020	1.583976	0.024341	1.060815	0.180558

Table 8. Correlations between the in-/out-degree and the market capitalization (MC) of the listed firms for all the countries, Japan, U.S., China, Korea, and Taiwan in 2020, 2015, and 2010. The degree of the correlations is measured by Spearman's rho along with Kendall's tau given in the parentheses.

		k_{in}-MC	k_{out}-MC
2020	All countries	0.785 (0.604)	0.181 (0.133)
	Japan	0.864 (0.694)	0.407 (0.286)
	U.S.	0.955 (0.837)	−0.086 (−0.057)
	China	0.798 (0.624)	0.239 (0.185)
	Korea	0.822 (0.649)	0.289 (0.228)
	Taiwan	0.864 (0.695)	0.318 (0.239)
2015	All countries	0.758 (0.579)	0.169 (0.125)
	Japan	0.866 (0.699)	0.442 (0.312)
	U.S.	0.957 (0.833)	−0.018 (−0.008)
	China	0.646 (0.478)	0.199 (0.154)
	Korea	0.859 (0.690)	0.364 (0.289)
	Taiwan	0.855 (0.689)	0.345 (0.263)
2010	All countries	0.737 (0.560)	0.178 (0.131)
	Japan	0.848 (0.679)	0.461 (0.331)
	U.S.	0.953 (0.832)	0.050 (0.045)
	China	0.469 (0.335)	0.186 (0.144)
	Korea	0.812 (0.645)	0.420 (0.332)
	Taiwan	0.846 (0.672)	0.340 (0.261)

VI. Publicly Listed Parent/Subsidiary Pairs

Finally, we approach the publicly listed parent/subsidiary pairs issue in Japan as an application of the global ownership networks that we have constructed in this study.

As shown in Figs. 17 and 19 dealing with the network in 2020 and 2010 respectively, in fact, we find that a considerable number of firms are involved in such an important issue. Those figures show the number of nodes reduced by merging of Japanese firms as a function of the threshold η for share holdings, where the firms connected with shareholding ratio over η are merged. When the threshold is set to be $\eta = 0.5$, about 500 firms are deleted in the whole network and more than half of them belongs to the GSCC in both years. When η is further reduced to 0.1, about 2500 firms are deleted in total with 2000 firms in the GSCC. We thus see that the GSCC, which we speculate manifests the Japanese-type management, is highly vulnerable to the merging procedure.

How can we reduce the number of the publicly listed parent/subsidiary pairs not changing the essential ownership structure in the current economy, i.e., preserving the GSCC? A possible solution to this question is to add the restriction on merging of the parent-child pairs that at least either of the pairs does not belong to the GSCC. If we adopt the modified merging procedure, we obtain the results in Figs. 18 and 20 corresponding to Figs. 17 and 19, respectively. As expected, the total number of deleted firms is significantly decreased, for instance, to about 500 from 2500 at $\eta = 0.1$. Remarkably, however, the GSCC is stable at least up to $\eta = 0.1$.

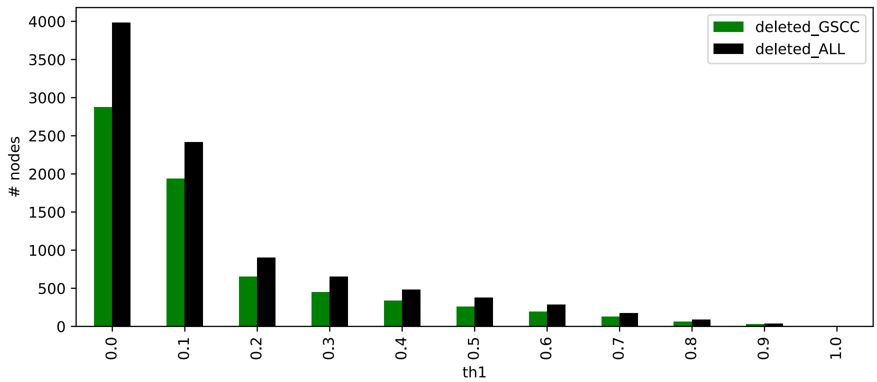

Figure 17. A possible scenario for dissolution of the publicly listed parent/subsidiary pairs of Japanese firms in the ownership network in 2020, showing dependence of the number of nodes reduced by merging of such pairs of Japanese firms in the whole network (black bars) and that in the giant strongly-connected component (green bars) on the threshold η for share holdings, where the firms connected with shareholding ratio over η are merged.

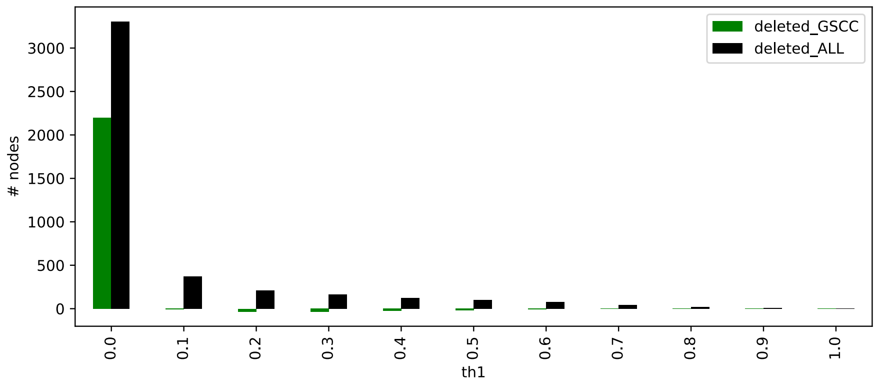

Figure 18. Same as Fig. 17, but with the restriction on merging of parent-child pairs that at least either of the pairs does not belong to the GSCC.

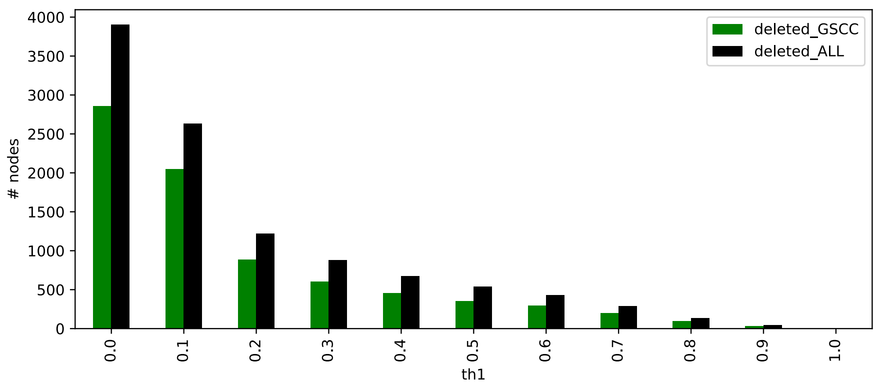

Figure 19. Same as Fig. 17, but in 2010.

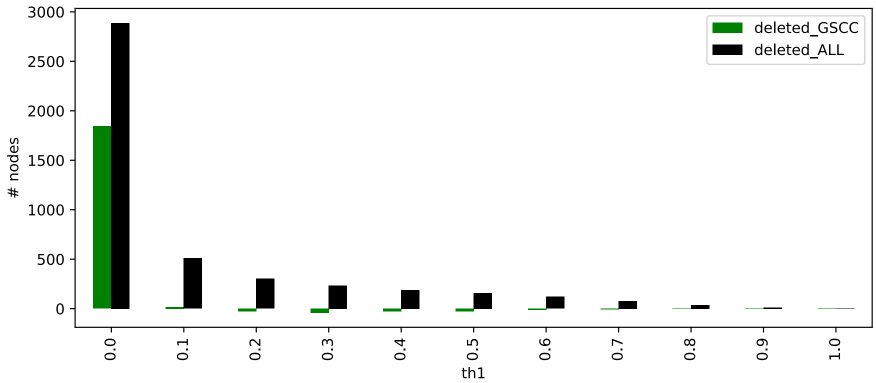

Figure 20. Same as Fig. 19, but with the restriction on merging of parent-child pairs that at least either of the pairs does not belong to the GSCC.

VII. Summary

In this paper we analyzed the Global Equity Ownership database complied by Thomson Reuters Corporation for the period from 1997 to 2020, currently known as the Refinitive Ownership and Profiles database. The comprehensive database, containing information on virtually all the world's listed firms and their market capitalizations, enables us to construct a stock ownership network for every year that classifies firms and shareholders as nodes and stock ownership relationships as directed links. The direction of a link between an owner and a stock issuer is defined so as to coincide with that of the ownership (from the owner to stock issuer), and its weight is to be the current price (absolute or relative value) of the shares of the issuer held by the owner. Also we demonstrated that the ownership database prepared here has potential applications for firm-level investigations of the FDI and FPI issues.

By adopting network-theoretic methods, we were successful in elucidating how firms and investors are connected globally through the stock ownership relations. We developed our previous work and focused here on the position occupied by the Japanese listed firms in the global ownership networks. The bow-tie decomposition of the networks shows that the Japanese firms make a dominant contribution to the GSCC. Since any pairs of nodes in a strongly-connected component have a path in each direction between them, many of the Japanese firms form a group of multilateral cross shareholdings. The analysis with varied cutoff for share holdings to identify minor links shows that the firms are loosely coupled to each other in the GSCC. The randomization simulations together with the theory for directed random network indicates that such circular ownership structure is not intentionally created but instead is ascribed to the strong correlation between the in-degree and out-degree of the Japanese firms, which distinguishes them from firms in other countries. As an application of the global ownership networks that we have constructed here, we addressed the current issue of publicly listed parent/subsidiary pairs of firms in Japan and worked out a procedure of merging parent-child pairs of firms for making a soft landing of the economy.

Appendix A. Details on Construction of the Ownership Networks

Here, we describe the detailed steps that we took to construct the global ownership networks utilized in the present study from the Global Equity Ownership data from Thomson Reuters. In fact, the construction process was not an easy task.

The raw data consists of the following data files:

- Historical Holdings File
- Security File (History)
- Owner File (History)
- Owner Issue Map File

These data files contain a wide range of ownership information for the period 1997-2017. The Holdings File matches security IDs with owner IDs and can thereby enable an owner's portfolio to be constructed or the owners of a given security described. The Security File is a list that links a security ID with its issuer name, issuer ID, and issuing country ID. Moreover, the Owner File is a list that links the owner ID with the owner Name and country ID.

We created network data for each year using the security owner and issuer as nodes and their ownership relations as links. Data for each year was generated using the latest reporting date for each security up to the given year. However, as different IDs are assigned to the same company for the owner ID and issuer ID, the company (such as a listed company), which can be both the owner and issuer, becomes multiple nodes, and in order to unify these, it is necessary to establish one-to-one correspondence between the owner ID and issuer ID for those who are not only owners but also issuers. However, the mapping list provided by Thomson Reuters that links the owner ID to the issuer ID is far from complete. Therefore, we used the owner name, issuer name, country ID, and issuing country ID to complement the mapping list with name identification. The procedure for creating the new mapping list was as follows:

1. Anything that existed in the mapping list provided to Thomson Reuters would be used with priority.
2. Words with notational fluctuations were unified into one word (for example, "corp" and "corporation" were unified into "co")
3. Names were compared word by word for issuers and owners coming from the same country. Those without country information were compared with companies in all countries.
4. If a word matched in the owner name and issuer Name, a score that was inversely proportional to the frequency of the word was given (for example, the words "Industry" and "Insurance" have a low score due to their high frequency, and a high score if a unique name such as "Toyota" matches).
5. The value obtained by dividing the sum of the scores of the matching words by the sum of the scores of all of the unique words of both was defined as the similarity, and a pair with a similarity of 0.9 or more was obtained as a matched pair. When being matched with multiples, only the highest one was taken.

In this way, we obtained a new matching list in which 12,478 firms have been pinned down by our own name identification scheme in addition to 12,227 firms by Thomson Reuters. As was demonstrated in Section II, the global ownership database constructed in this way encompassed virtually all of the world's listed firms.

Appendix B. Log-normal Modeling of the Degree Distributions

Here is detailed information on the log-normal function fitting of the in-degree and out-degree distributions for listed firms in China, Korea, and Taiwan.

Table 9. Log-normal function fitting of the in-degree and out-degree distributions for the listed firms in China, where the parameters μ and σ were determined by the maximum likelihood method.

	k_{in}			k_{out}		
year	μ	σ	log-likelihood	μ	σ	log-likelihood
2007	3.3	0.7	−7.84e+03	1.2	0.9	−5.12e+02
2008	3.4	0.7	−8.43e+03	1.2	0.9	−5.80e+02
2009	3.5	0.7	−9.41e+03	1.2	0.9	−6.43e+02
2010	3.5	0.8	−1.15e+04	1.3	1.0	−7.08e+02
2011	3.6	0.9	−1.32e+04	1.3	1.0	−8.20e+02
2012	3.6	0.9	−1.42e+04	1.3	1.0	−8.83e+02
2013	3.8	0.9	−1.47e+04	1.3	1.0	−9.31e+02
2014	3.8	1.0	−1.62e+04	1.3	1.1	−1.00e+03
2015	3.9	1.0	−1.81e+04	1.3	1.1	−1.13e+03
2016	3.9	1.0	−1.94e+04	1.3	1.1	−1.25e+03
2017	3.8	1.0	−2.14e+04	1.3	1.1	−1.32e+03
2018	3.7	1.1	−2.26e+04	1.3	1.1	−1.39e+03
2019	3.7	1.1	−2.39e+04	1.4	1.1	−1.49e+03
2020	3.7	1.1	−2.50e+04	1.4	1.1	−1.53e+03

Table 10. Same as Table 9, but in Korea.

	k_{in}			k_{out}		
year	μ	σ	log-likelihood	μ	σ	log-likelihood
2007	2.5	1.1	$-3.61e+03$	1.1	0.6	$-2.97e+02$
2008	2.5	1.1	$-3.64e+03$	1.2	0.6	$-3.38e+02$
2009	2.4	1.0	$-6.12e+03$	1.1	0.6	$-4.63e+02$
2010	2.6	1.0	$-6.96e+03$	1.1	0.7	$-5.46e+02$
2011	2.6	1.1	$-7.20e+03$	1.1	0.7	$-5.71e+02$
2012	2.6	1.1	$-7.19e+03$	1.2	0.7	$-5.67e+02$
2013	2.7	1.1	$-7.39e+03$	1.1	0.7	$-5.95e+02$
2014	2.8	1.1	$-7.70e+03$	1.1	0.7	$-6.05e+02$
2015	2.8	1.1	$-8.70e+03$	1.1	0.7	$-6.80e+02$
2016	2.8	1.1	$-9.13e+03$	1.2	0.7	$-7.28e+02$
2017	2.9	1.1	$-9.54e+03$	1.2	0.7	$-7.59e+02$
2018	2.9	1.1	$-9.83e+03$	1.2	0.7	$-7.69e+02$
2019	2.9	1.1	$-1.02e+04$	1.2	0.7	$-8.11e+02$
2020	2.9	1.1	$-9.78e+03$	1.1	0.7	$-8.05e+02$

Table 11. Same as Table 9, but in Taiwan.

	k_{in}			k_{out}		
year	μ	σ	log-likelihood	μ	σ	log-likelihood
2007	3.1	0.7	$-5.35e+03$	1.3	0.8	$-5.34e+02$
2008	3.1	0.7	$-5.43e+03$	1.3	0.9	$-5.89e+02$
2009	3.1	0.7	$-5.49e+03$	1.3	0.9	$-6.26e+02$
2010	3.2	0.7	$-5.88e+03$	1.3	0.9	$-6.89e+02$
2011	3.3	0.7	$-6.15e+03$	1.3	0.9	$-7.46e+02$
2012	3.2	0.7	$-6.22e+03$	1.3	0.9	$-7.82e+02$
2013	3.2	0.7	$-6.51e+03$	1.3	0.9	$-8.12e+02$
2014	3.2	0.8	$-6.80e+03$	1.4	0.9	$-8.41e+02$
2015	3.2	0.8	$-7.02e+03$	1.4	0.9	$-8.68e+02$
2016	3.3	0.8	$-7.18e+03$	1.4	0.9	$-9.01e+02$
2017	3.3	0.8	$-7.35e+03$	1.4	0.9	$-9.34e+02$
2018	3.3	0.8	$-7.49e+03$	1.4	0.9	$-9.65e+02$
2019	3.3	0.8	$-7.56e+03$	1.4	0.9	$-9.98e+02$
2020	3.2	0.8	$-7.54e+03$	1.4	0.9	$-1.01e+03$

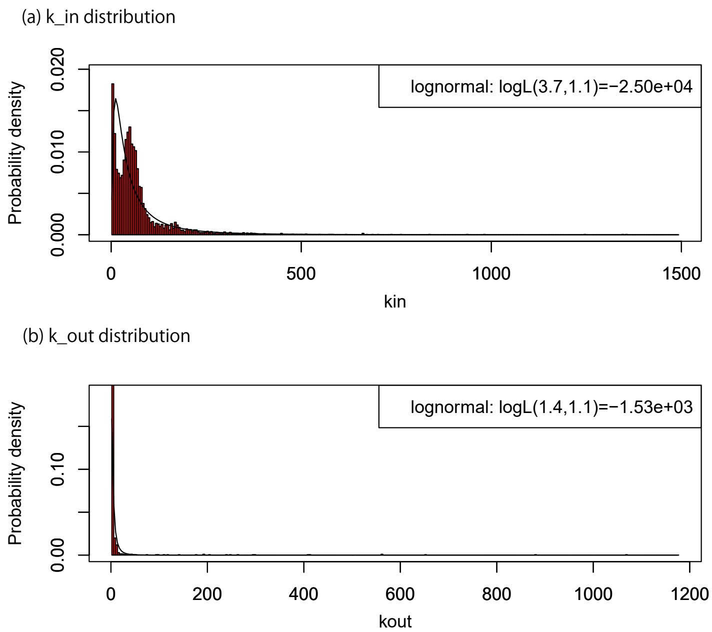

Figure 21. Distributions of the in-degrees (upper panel) and out-degrees (lower panel) of listed firms in China in 2020 with the log-normal function fitting depicted by solid line.

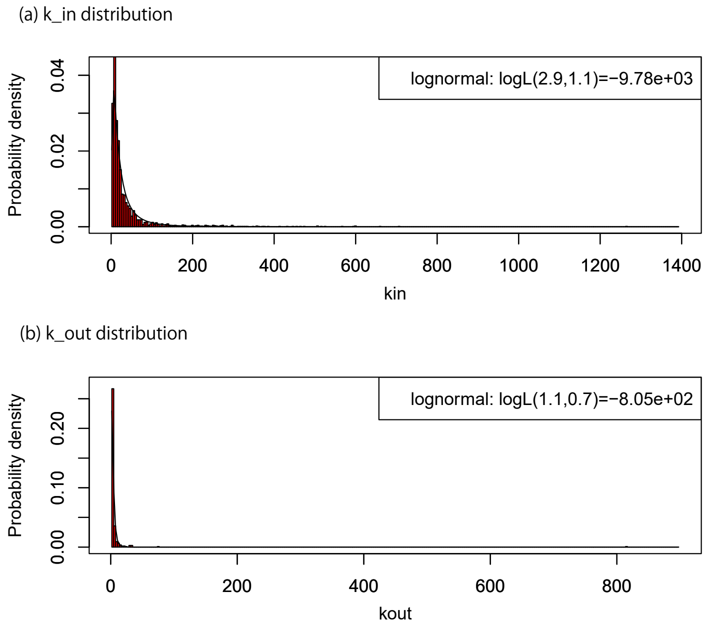

Figure 22. Same as Fig. 21, but in Korea.

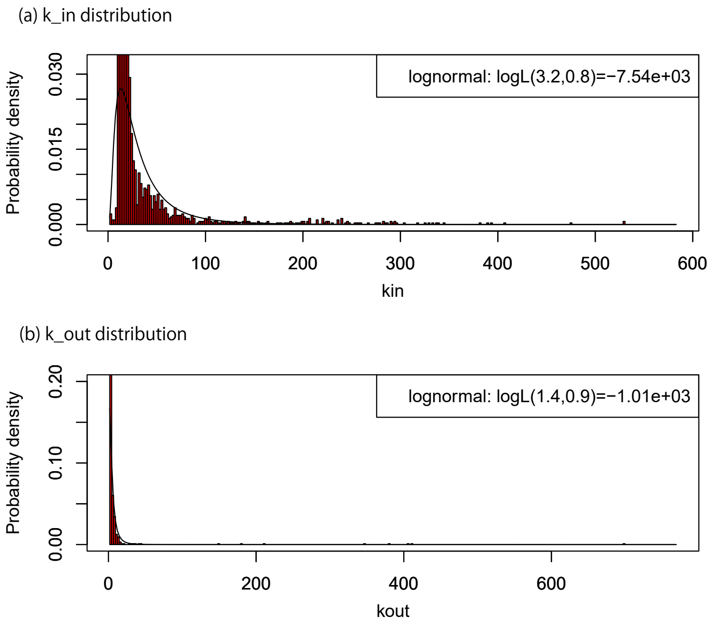

Figure 23. Same as Fig. 21, but in Taiwan.

Appendix C. Copula Modeling of the In-degree and Out-degree Correlations

Here is detailed information on the copula modeling of the in-degree and out-degree correlations for listed firms in China, Korea, and Taiwan.

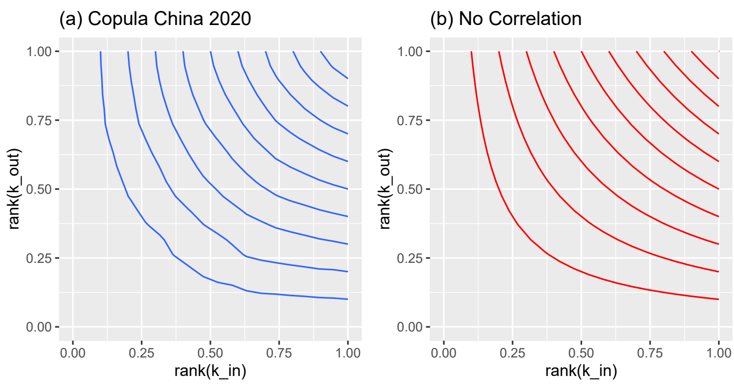

Figure 24. Copula representation of the in-degree and out-degree correlations for firms in China in 2020, where the empirical copula function (left panel) and the no correlation copula (right panel) are compared in the form of a contour plot with equally spaced levels ranging from 0.1 to 0.9.

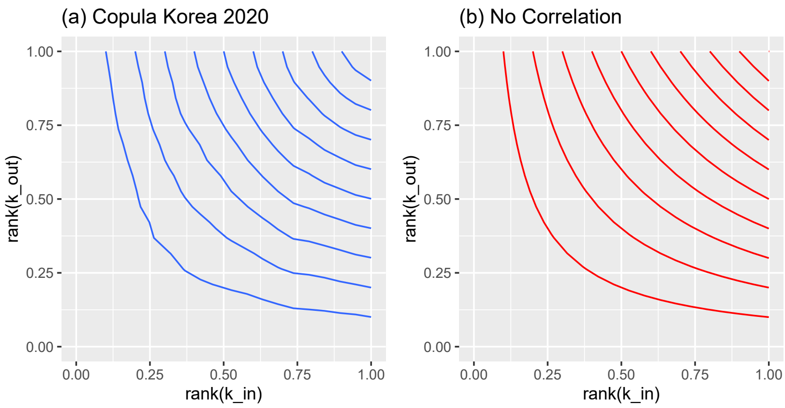

Figure 25. Same as Fig. 15, but in Korea.

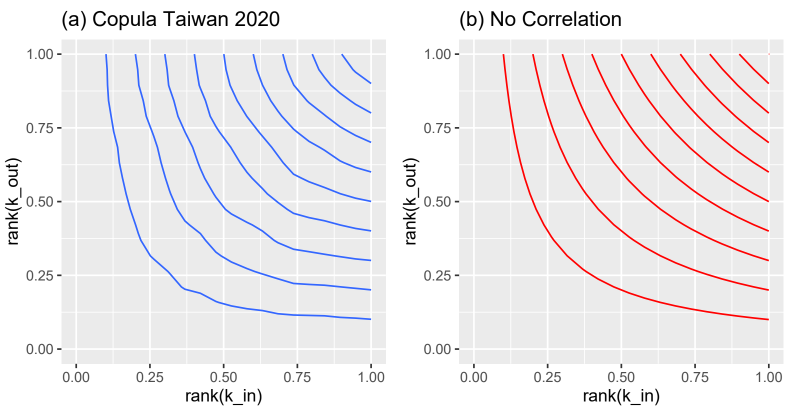

Figure 26. Same as Fig. 15, but in Taiwan.

Table 12. Selection of the Gumbel or Clayton copula model for the in-degree and out-degree correlation of listed firms in China based on the least square error (LSE) method.

	Gumbel Copula		Clayton Copula	
year	θ	LSE	θ	LSE
2007	1.033451	0.027051	0.024111	0.034064
2008	1.039498	0.022259	0.037434	0.030791
2009	1.026442	0.020845	0.013510	0.025645
2010	1.030888	0.032098	0.022730	0.038090
2011	1.065867	0.036835	0.073288	0.056706
2012	1.089054	0.023858	0.118607	0.051057
2013	1.097140	0.029919	0.128131	0.062263
2014	1.114879	0.023871	0.162525	0.061638
2015	1.120117	0.021490	0.170571	0.061931
2016	1.127622	0.021686	0.185331	0.064231
2017	1.120671	0.038394	0.157721	0.086787
2018	1.120003	0.040644	0.154751	0.089569
2019	1.162231	0.037255	0.229511	0.106136
2020	1.171814	0.033261	0.251950	0.102517

Table 13. Same as Table 12, but in Korea.

	Gumbel Copula		Clayton Copula	
year	θ	LSE	θ	LSE
2007	1.217648	0.061672	0.324149	0.159811
2008	1.201634	0.055561	0.290308	0.151428
2009	1.136626	0.088007	0.157655	0.159928
2010	1.160746	0.087989	0.198872	0.174758
2011	1.193418	0.075321	0.262667	0.176235
2012	1.214720	0.066701	0.304373	0.175472
2013	1.165872	0.066184	0.218593	0.148369
2014	1.184028	0.061701	0.255082	0.150408
2015	1.210680	0.048847	0.313794	0.140888
2016	1.191220	0.038800	0.286772	0.116141
2017	1.177774	0.042906	0.254159	0.121227
2018	1.131704	0.046291	0.169313	0.104174
2019	1.096600	0.056286	0.099774	0.099213
2020	1.108629	0.051225	0.123327	0.099666

Table 14. Same as Table 12, but in Taiwan.

year	Gumbel Copula		Clayton Copula	
	θ	LSE	θ	LSE
2007	1.280085	0.022154	0.477271	0.102096
2008	1.286535	0.017629	0.499590	0.085855
2009	1.272333	0.010100	0.487711	0.057606
2010	1.279593	0.009802	0.487864	0.075104
2011	1.276977	0.011045	0.483674	0.075007
2012	1.306402	0.012481	0.555903	0.060978
2013	1.277994	0.014694	0.484951	0.079966
2014	1.270334	0.012320	0.467911	0.078828
2015	1.275896	0.019827	0.471470	0.095376
2016	1.252800	0.018276	0.432001	0.084012
2017	1.229448	0.028277	0.379103	0.098432
2018	1.247942	0.023417	0.412611	0.100037
2019	1.248335	0.021709	0.411834	0.099800
2020	1.243611	0.020622	0.408489	0.091998

Appendix D. The GSCC for random directed networks

Dorogovtsev et al. (2001b) worked out two demonstrative models to discuss the bow-tie structure of random directed networks theoretically.

One of the models is specified by the following joint probability function of in-degree k_{in} and out-degree k_{out}:

$$f(k_{\text{in}}, k_{\text{out}}) = f(k_{\text{in}}) f(k_{\text{out}}) \ , \tag{D.1}$$

with

$$f(k) = p(\delta_{k,0} + \delta_{k,1}) + (1 - 2p)\delta_{k,3} \ . \tag{D.2}$$

In this model k_{in} and k_{out} are completely independent of each other. In contrast, the other model is characterized by the strong in- and out-degree correlation with

$$f(k_{\text{in}}, k_{\text{out}}) = p(\delta_{k_{\text{in}},0}\delta_{k_{\text{out}},1} + \delta_{k_{\text{in}},1}\delta_{k_{\text{out}},0}) + (1 - 2p)\delta_{k_{\text{in}},3}\delta_{k_{\text{out}},3} \ , \tag{D.3}$$

where the correlation coefficient ρ between k_{in} and k_{out} varies with p as $\rho = (12 - 25p)/(13 - 25p)$; $\rho = 2/3, 9/11, 7/8$ at $p = 0.4, 0.3, 0.2$, repectively. Note that these two models share the same marginal distributions for k_{in} and k_{out}, given by Eq. (D.2).

The theory (Dorogovtsev et al., 2001b) takes advantage of the generating function $\Phi(x, y)$ for $f(k_{\text{in}}, k_{\text{out}})$ defined as

$$\Phi(x, y) = \sum_{k_{\text{in}}, k_{\text{out}}} f(k_{\text{in}}, k_{\text{out}}) x^{k_{\text{in}}} y^{k_{\text{out}}}. \tag{D.4}$$

The relative sizes I and O of GIN (GSCC+IN) and GOUT (GSCC+OUT) are given by

$$I = 1 - \Phi(x_c, 1), \quad O = 1 - \Phi(1, y_c), \tag{D.5}$$

where x_c and y_c are the nontrivial solutions of the equations:

$$x_c = \frac{1}{z^{(d)}} \left. \frac{\partial \Phi(x_c, y)}{\partial y} \right|_{y=1}, \quad y_c = \frac{1}{z^{(d)}} \left. \frac{\partial \Phi(x, y_c)}{\partial x} \right|_{x=1}, \tag{D.6}$$

with the average $z^{(d)}$ of the in-degree (out-degree); $z^{(d)} = 3 - 5p$ in the two demonstrative models. The generating function formalism then provides us with the formula for the relative size S of the GSCC:

$$\begin{aligned} S &= \sum_{k_{\text{in}}, k_{\text{out}}} f(k_{\text{in}}, k_{\text{out}}) \left(1 - x_c^{k_{\text{in}}}\right) \left(1 - y_c^{k_{\text{out}}}\right) \\ &= 1 - \Phi(x_c, 1) - \Phi(1, y_c) + \Phi(x_c, y_c). \end{aligned} \tag{D.7}$$

Figure 27 displays the relative sizes of the bow-tie components, S, I, and O as a function of p in the two random directed network models. We see that the GSCC grows up almost linearly with increment of $z^{(d)}$ for both models. Also we learn that the strong correlation between k_{in} and k_{out} significantly enhances formation of the GSCC by comparing the two results for S obtained at the same p. We empirically confirmed these theoretical results in Fig. 27 by generating random directed networks of 1,000 nodes in line with the two models.

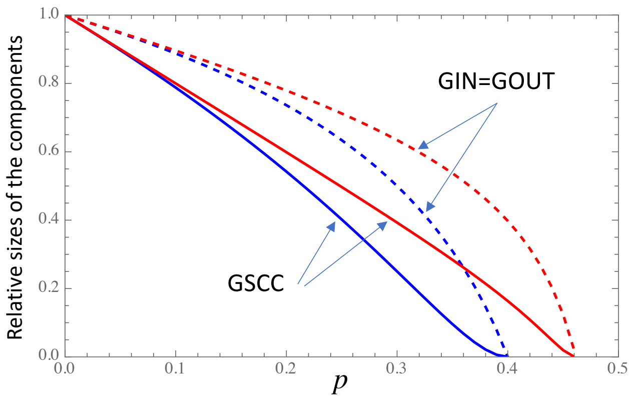

Figure 27. Evolution of the bow-tie components, GSCC, GIN (GSCC+IN), and GOUT (GSCC+OUT) with change of p in the two random directed network models due to Dorogovtsev et al. (2001b). The parameter p controls the joint probability function of in-degree and out-degree in the models with the common marginal distributions; the in-degree and out-degree distributions are identical because of the symmetric nature of the models. The model specified by Eq. (D.1) with Eq. (D.2), the results for which are shown by blue curves, has no in-degree and out-degree correlations at all. In contrast, the other model specified by Eq. (D.3), the results for which are shown by red curves, has strong in-degree and out-degree correlations implemented. Note that the critical point p_c at which the GSCC emerges is 0.4 and 0.462 in the former and latter models, respectively.

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