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Does the supply network shape the firm size distribution? The Japanese case[1]

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Abstract

The paper presents an investigation on how the upward transmission of demand shocks in the Japanese supply network influences the growth rates of firms and, consequently, shapes their size distribution. Through an empirical analysis, analytical decomposition of the growth rates' volatility, and numerical simulations, we obtain several original results. We find that the Japanese supply network has a bow-tie structure in which firms located in the upstream layers display a larger volatility in their growth rates. As a result, the Gibrat's law breaks down for upstream firms, whereas downstream firms are more likely to be located in the power law tail of the size distribution. This pattern is determined by the amplification of demand shocks hitting downstream firms, and the magnitude of this amplification depends on the network structure and on the relative market power of downstream firms. Finally, we observe that in an almost perfectly hierarchical network, the power-law tail in firm size distribution disappears. The paper shows that aggregate demand shocks can affect the economy directly through the reduction in output for downstream firms and indirectly by shaping the firm size distribution.

Keywords: supply network, Hodge potential, demand shock, firm size distribution, Gibrat law
JEL classification: D8, E30, L14

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1 Introduction

Recent literature has challenged the traditional view that firm-level shocks average out, and stressed their impact on macroeconomic volatility due to the presence of input-output links among sectors (Acemoglu et al., 2012) and to the shape of the firm size distribution (Gabaix, 2011). These papers have originated a new approach to the study of business cycles which focuses on micro-level shocks. However, the connection between the two factors determining the macroeconomic effects of firm-level shocks (supply network and firm size distribution) has so far received relatively little attention.

Power law tails emerge as a consequence of the Gibrat law, which states that growth rates are independent from the firm's size. Accordingly, the fact that a power law tail only emerges for very large firms implies that the Gibrat law breaks down for small and medium firms. In order to investigate the breakdown of the Gibrat law, we must consider that firms' growth rates are mutually dependent since firms are connected in the supply and commercial credit network. As a consequence, a causal chain among the firm size distribution, firms' growth rates, and supply network emerges: the structure of the network determines the mechanism of transmission of shocks within the credit network, which in turn affects the growth of firms and the firm size distribution.

The present paper investigates whether the observed breakdown of the Gibrat law in Japan for small and medium firms depends on the structure of the network and on the position of the firms in the supply network. We analyze how demand volatility can be passed by downstream firms to their suppliers, generating higher volatility in the growth rates of the latter and, consequently, limiting their growth. In order to quantify the relative sensitivity of each firm to, respectively, final demand and demand of intermediate inputs, we introduce in economic network modeling the Hodge potential, which can be interpreted as a synthetic indicator of the relative position of a node within the network, given a particular network structure.

We first present an empirical analysis of the Japanese supply network in order to identify its structure and study the connection between the position of firms in the network and the distribution of growth rates. We then use a parsimonious computational model to reproduce the empirical evidence, analyze the shock transmission, and gain insights into the mutual interdependence between the network structure and the firm size distribution.

The empirical analysis reveals that the Japanese supply network has a bow-tie structure and that the Gibrat's law breaks down for upstream firms whereas downstream firms are more likely to be located in the power law tail of the size distribution. The theoretical model explains this pattern as the result of the amplification in the supply chain of demand shocks hitting downstream firms, generating higher volatility in the growth rates of upstream firms. The extent of the amplification depends on two main factors. The first is the network structure, since the amplification of aggregate demand shocks is larger in bow-tie networks than in perfectly hierarchical ones. The second factor is the relative market power of upstream firms, which appears to have an inverse relationship with the variance of their growth rates. The network structure emerges as the main factor affecting the transmission of shocks and, consequently, the shape of the firm size distribution. In the extreme case of an almost perfectly hierarchical network, the power-law tail in firm size distribution disappears.

Our analysis provides three main contributions to the literature. First, the paper offers a novel investigation into the emergence of power law tails in the firm size distribution by discussing the role of the supply network and the interdependence of firms' growth rates. This investigation contributes to and connects the two distinct streams of research that have analyzed the macroeconomic impact of firm-level shocks as dependent on the supply network, on the one hand, and on the firm size distribution, on the other hand. To the best of our knowledge, the only existing study on the interdependence between supply network and firm size distribution is Herskovic et al. (2020) for the US.

The second most relevant contribution is the introduction in economics of the Hodge potential, which can be interpreted as a synthetic indicator of the relative position of a firm within the network. In brief, it quantifies the difference between (direct and indirect) outbound and inbound links as a single number, providing an alternative to the influence vector proposed by Acemoglu et al. (2012). Third, while the transmission of supply-side shocks has been extensively investigated, we add a new perspective by considering demand shocks and focusing on the upstream transmission. In particular, we consider that the transmission of shocks does not only depend on the technological relationship embedded in the production function, but also on the position in the network, the relative market power, and the relative sizes on the node.

The remainder of the paper is structured as follows. Section 2 provides an overview of the existing literature immediately related to this paper. Section 3 presents the characteristics of the Japanese supply network, showing some basic stylized facts. Section 4 introduces the model. Section 5 proposes a simple analytical treatment of the relationship between aggregate shocks and volatility in the different components of the network. Section 6 details the structure of the simulations and presents the results. Finally, section 8 concludes.

2 Related literature

Acemoglu et al. (2012) show that microeconomic shocks, in the presence of input and output linkages, do not average out and their transmission through the supply network can originate aggregate fluctuations. As a consequence, the study of the network structure is fundamental to properly model the volatility of aggregate output. They demonstrate that business cycles can be originated not only by exogenous macro-level factors, but also by idiosyncratic small shocks at the firm level. They identify an influence vector which quantifies, for each sector, the relative weight of its value added, which proxies the centrality of the sector in the transmission of shocks. Accordingly, the productive structure of the economy and, in particular, the input/output links across sectors and firms are crucial in spreading microeconomic shocks to the entire system.

Along the same lines, Carvalho and Gabaix (2013) introduce the concept of fundamental volatility, defined as the volatility associated only to microeconomic shocks, and show that aggregate volatility tracks fundamental volatility. The idea that firm-level fluctuations do not average out was originally put forward by Gabaix (2011). Rather than on input-output linkages, he focuses on the firm size distribution and demonstrates that, when firms are power-law distributed, the micro-level volatility is amplified and can generate aggregate fluctuations.[1] In this paper we mostly abstract from the macroeconomic implications of the firm size distribution and primarily examine the reverse causality, from macro-shocks to the distributions of growth rates and size.

Within the literature stemmed from these contributions, Acemoglu et al. (2017) investigates the connection between the distribution of sectoral shocks and the tail risk in the distribution of macroeconomic fluctuations. Baqaee and Farhi (2019) use the Domar weights derivatives to estimate the second-order nonlinear effects of microeconomic shocks (depending on elasticities of substitution, network linkages, returns to scale, and the extent of factor reallocation) of sectoral shocks. They find that the macroeconomic outcomes of microeconomic shocks are asymmetric, as nonlinearities amplify the impact of negative sectoral shocks and mitigate the impact of positive sectoral shocks. Closer to the present paper is Dhyne et al. (2019), who use firm-level data to quantify the distortions arising from the existence of markup charged by suppliers (assumed to be dependent by their shares of inputs for each buyer).

Our perspective differs from the cited literature, as our primary interest is the firm size distribution, and how it is affected by firm-level shock on the demand side in a supply network. We follow an approach similar to Dhyne et al. (2019) to model market power in a monopolistically competitive network, stripping down the unnecessary details and enriching the analysis by adding the network effect, quantified by the Hodge potential, to the market share as a determinant of firms' markups. This modeling approach allows us to evaluate the role of macro-factors (the network structure) and micro-factors (the position of the single firm within the network) in the transmission of shocks and their final effect on the firm size distribution.

Empirical and theoretical literature have established a connection between the Gibrat law of proportionate effects and power law distribution, showing that the emergence of a power law or Zipf's law in firm size is the consequence of the independence of firms' growth rates from their size.[2] However, as it is known from recent literature (see Aoyama et al., 2010, 2017; Fujiwara, 2020, for example) and as we examine in this paper, Gibrat and Zipf's laws break down for small and medium firms. The Zipf's law is dominated by "a few giants" comprising a considerable fraction of the total size of the economy,

[1] This result is challenged by Dosi et al. (2019), who propose a demand-based granular hypothesis as an alternative to the original supply-side one.

[2] See Bottazzi and Secchi (2003, 2006); Gabaix (1999); Hall (1987); Ijiri and Simon (1977); Luttmer (2007, 2011); Rossi-Hansberg and Wright (2007a,b); Simon (1955, 1960); Steindl (1965) among others, and, more relevant for the target of this paper, Aoyama et al. (2010, 2017); Delli Gatti et al. (2005); Fujiwara (2020); Malevergne et al. (2013).

while equally important are the "many dwarfs", including small and medium firms, the number of which is much larger than that the number of giants. In addition, firms are connected with each other in a giant network with supplier and customer links. Typically, those small and medium firms are suppliers to large firms (see Aoyama et al., 2010, 2017; Fujiwara, Y. and Aoyama, H., 2010; Fujiwara, 2020, and references therein).

As a consequence, the growth rates of firms are not mutually independent, since firms are connected in the supply network and idiosyncratic shocks are transmitted through the networks links. We build on this fact and on the demonstrated connection between the Gibrat law and the power law distribution to investigate how the supply network, by determining the interactions between firms' growth rates, has an effect on the Gibrat law (and consequently on the firm size distribution). More specifically, we analyze whether and why the breakdown of the Gibrat law depends on the relative position of a firm in the network.

Our paper presents a mechanism for the generation of heavy tails in firm size distribution alternative to the shocks in productivity already investigated by the literature (see Aoki and Nirei, 2017; Hopenhayn, 1992; Luttmer, 2007, for example). Carvalho and Grassi (2019) connect this stream of research with the granular hypothesis by modeling the feedback from the macroeconomy to the firm size distribution and deriving the Pareto distribution in steady state as the result of firms' microeconomic optimizing decisions. Our approach relates to this literature by further exploring the dependence between firms' growth and firm size, and innovates, first, by focusing primarily on demand-side shocks, in addition to productivity shocks, and, second, by studying the interdependence of growth rates that arise when firms are mutually connected in the supply network.

Herskovic et al. (2020) share our paper's emphasis on demand shocks in shaping the firm size distribution. Studying the US supply network, they find that firms' growth is negatively correlated with the volatility of sales. They stress the role of demand shocks and their upward transmission, showing that firms with a more concentrated customer base experience higher volatility. Our paper presents several points of departure from Herskovic et al. (2020). First, the availability of network data for Japan dispenses us from assuming a particular network structure. Rather, we replicate the bow-tie structure of the Japanese supply network and let the connection between this structure and the firms size distribution endogenously emerge. Second, studying the actual network structure and link distribution, we verify that, for Japan, the dynamics of firms' size depend on their position in the network and on the structure of the network, rather than on the number of connections. As a consequence, the analysis makes use of the Hodge potential, which represents a further innovation. Finally, given the issues in calculating the moments in distributions with power-law tails, we use a multi-agent computational model to reproduce the full micro-distribution, neatly connecting the emergence of the power law tail as a consequence of the Gibrat law (and its breakdown for small firms), and we are able to endogenously generate firms' growth (which is an exogenously predetermined process in Herskovic et al., 2020).

Finally, Arata and Miyakawa (2022) also investigate the transmission of demand shocks in the Japanese supply network, finding that small and large firms are differently impacted, depending on the nature of the shock. Our paper discusses the transmission demand shocks in the supply network but with the aim to explain the emergence of a particular firms size distribution, which in our paper is endogenous and represents the main object of the analysis. Further, we consider small idiosyncratic firm-level shocks rather than large aggregate disturbances as in Arata and Miyakawa (2022).

3 Firm size, firm growth, and production network of firms

Preliminarily, let us summarize two stylized facts on firm-size and growth.

It is well known that there are empirical laws about the distribution for firm-size and the dynamics for firm's growth. First, denoting by $P(x)$ the probability to observe that a firm's size is greater than x, one observes the so-called Zipf's law, i.e.

$$P(x) \sim x^{-\mu} \tag{1}$$

for several orders of magnitude in the regime of large firms, where μ is numerically close to 1 (see Aoyama et al., 2010; Axtell, 2001, for example).

Second, firm's growth rates are on average independent from its size (see Sutton, 1997, for a readable review). Such a property is known as Gibrat's law of proportionate effect. Denote by x_t the firm's size, being measured typically by sales, profits, number of employees and so forth, at time t. As explained in the following, we will employ annual data, so t denoted a specific year and $t+1$ is the subsequent one. Let us define the growth rate

$$G_t := \frac{x_t}{x_{t-1}}, \tag{2}$$

and its logarithmic variable

$$g_t := \log_{10} G_t = \log_{10} x_t - \log_{10} x_{t-1}. \tag{3}$$

Then Gibrat's law states that g_t is statistically independent of the initial size x_{t-1}. The statistical independence has been verified in the regime of large firms.

These stylized facts concern the stochastic dynamics of firms in an ensemble of firms inside an economy. Models for the dynamics have been studied in a huge literature (see footnote 2)

However, it should be emphasized that these laws are valid for large firms, as we actually observe by employing a large dataset that (1) covers all the regime of firm-size including small and medium firms as well as large ones, and that (2) contains information of production network, namely supplier-customer relationships among the firms.

3.1 Dataset of firms and production network in Japan

For the study on the validity and breakdown of Zipf's law on firm-size distribution and Gibrat's law on firm's growth, it would be ideal to have an exhaustive data set on all the regimes of firm-size, not only large firms such as listed firms but also small and medium firms, at a nationwide scale. We employ such large-scale dataset based on a survey regularly and persistently conducted by one of the leading credit research agencies in Tokyo, Tokyo Shoko Research, Inc., which covers mostly all the active firms in Japan. This is nearly exhaustive in the sense that if a firm A is active as a supplier or as a customer of other firms, such a firm is included in the dataset as far as the other firms need credit information about the firm A, however small it is.

We employ the dataset at two successive years of 2015 and 2016, with a number of firms of more than a million, which covers most of the active firms in Japan. For firm-size x_t, we use annual sales as a proxy and define its growth-rate g_t as in (3), where t refers to the year 2016, and $t-1$ to 2015.

Figure 1 shows the distribution of firm-size for each of the years in all the regimes from 10^5 Yen to 10^{13} Yen, roughly corresponding to 10^3 to 10^{11} in USD/Euro of annual sales. One can see that, on one hand, in the regime of large firms, say larger than 10^9 Yen, the distribution obeys a Zipf's law as in (1). Noting that the distribution in the figure is a complementary cumulative density function (CDF) and that (1) is written in the form of CDF, one can immediately see that the power-law exponent μ is close to 1. On the other hand, for the regime of small and medium firms (sales smaller than 10^9 Yen), there is a deviation from Zipf's law as shown in the CDF.

In addition, the dataset includes the production network or supplier-customer relationships among the firms based on another survery conducted by the credit research agency (see Aoyama et al., 2010, 2017; Fujiwara, Y. and Aoyama, H., 2010, for more details). If firm A sells goods and services to firm B, we define the supplier-customer link from A to B, A $\rightarrow$ B. The number of suppliers f_i of firm A is that of in-coming links $f_i \rightarrow$ A or in-degree, while the number of customers f_i of firm A is that of out-going links A $\rightarrow f_i$ or out-degree.

Figure 2 shows the distribution of in and out degrees, or the number of suppliers and customers, respectively, of each firm. Being similar to the firm-size distribution in the regime of large degrees, the distribution obeys a power-law as in (1). There is a strong correlation between the firm-size and the degrees, as one can naturally expect, for larger firms possess a larger number of suppliers and customers Fujiwara, Y. and Aoyama, H. (2010); Aoyama et al. (2010, 2017).

Now, by employing this dataset, let us turn our attention to Zipf and Gibrat's laws and their validty and breakdown depending on the firm-size in the next section.

3.2 Zipf and Gibrat's laws and firm-size

It is straightforward to empirically study the dynamics of the firms' size. Figure 3 is a scatter plot for the firm size x_{t-1} at time $t-1$ and x_t at time t. As one can see, the growth rate has a peak around $g_t = 0$, because many firms are concentrated along the diagonal 45-degree line. Nevertheless, a fraction of firms can have large positive growth-rate or small negative growth rates.

In order to examine the statistical dependence of the growth-rate g_t on the initial firm-size x_{t-1}, let us divide x_{t-1} into different groups or bins, and calculate the distributions of g_t in the different bins. Figure 4 shows the following results:

- For the regime of large firms, Gibrat's law holds, because the conditional distribution for g_t given the initial size x_{t-1}, $p(g_t|x_{t-1})$ does not depend on x_{t-1} as shown in Figure 4 (a). Here the bins are given by

$$10^9 \times [10^{0.5(n-1)}, 10^{0.5 \cdot n}] \text{ Yen} \qquad \text{for } n = 1, 2, 3, 4 \tag{4}$$

- For the regime of small and medium firms, Gibrat's law breaks down, i.e. $p(g_t|x_{t-1})$ depends on x_{t-1} as shown in Figure 4 (b), where the bins are given by

$$10^6 \times [10^{0.5(n-1)}, 10^{0.5 \cdot n}] \text{ Yen} \qquad \text{for } n = 1, 2, \ldots, 6 \tag{5}$$

- Comparing Figure 4 with the distribution of firms-size in Figure 1, one can see that the boundary between the validity and breakdown of Gibrat's law corresponds to where the distribution deviates from the Zipf's law around 10^9 Yen.

To quantify the statistical dependence of firm's growth on size, we calculate standard deviations from the PDF's in Figures 4 (a) and (b) at each bins given by (4) and (5) to obtain the result in Figure 5. To summarize, one can see that Gibrat's law holds for large firms, but breaks down for small and medium firms.

3.3 Production network of firms

The production network or supplier-customer relationships in our dataset has information about the flow of goods and services from upstream firms to downstream firms. We apply the well-known analysis of "bow-tie" structure (Broder et al., 2000), first by focusing on the connectivity, and then by using the so-called Hodge decomposition.

In general, a network can be represented as a graph $G = (V, E)$, where V and E are the set of vertices and edges, respectively. Let us denote the number of vertices by $|V|$ and edges by $|E|$. In our case, a vertex is a firm, and an edge is a link emanating from supplier to customer. First, one can decompose G into weakly connected components (WCC), i.e. connected components when regarded as an undirected graph. In our case, we found that there exists a giant WCC with $|V| = 1,066,037$ and $|E| = 4,974,802$. In what follows, we focus on the giant WCC, which can be decomposed further into the following components:

GWCC : Giant weakly connected component: the largest connected component when viewed as an undirected graph. An undirected path exists for an arbitrary pair of firms in the component.

GSCC : Giant strongly connected component: the largest connected component when viewed as a directed graph. A directed path exists for an arbitrary pair of firms in the component.

IN : The firms from which the GSCC is reached via a directed path.

OUT : The firms that are reachable from the GSCC via a directed path.

TE : "Tendrils", which include the rest of the GWCC.

It follows from the definitions that

$$\mathrm{GWCC} = \mathrm{GSCC} + \mathrm{IN} + \mathrm{OUT} + \mathrm{TE} \tag{6}$$

Figure 6 depicts the result of bow-tie structure. In (6), GWCC is decomposed into GSCC (49.73%), IN (20.63%), OUT (26.16%), and TE (3.47%) with percentages in parentheses being the fraction in terms of the number of firms contained in each component. GSCC occupies nearly the half of the entire set of firms as a "core", presumably circulating goods and services mutually among them. IN is the portion supplying to the core from the upstream, while OUT is demanding the products in the downstream side.

It should be remarked that the shortest distance from the GSCC to IN or OUT is at most 4, implying that the bow-tie does not have elongated shape like a "bow-tie", but looks like a "walnut". [3]

3.4 Hodge decomposition

The so-called Hodge decomposition of flow on a network is a mathematical method of ranking nodes according to its location in terms of upstream and downstream of the flow (Jiang et al., 2011). The method, also known as Helmholtz-Hodge-Kodaira decomposition, has been used to find such a structure in complex networks (see the applications to economic networks in Iyetomi et al., 2020; Kichikawa et al., 2018).

Let us recapitulate the method briefly only for a binary graph, i.e. without weight. [4]

Consider a directed network $G = (V, E)$ with an adjacency matrix A_{ij}, i.e.

$$A_{ij} = \begin{cases} 1 & \text{if there is a directed link from node } i \text{ to node } j, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

Denote the number of nodes by $N = |V|$. By assumption, $A_{ii} = 0$, i.e. A_{ij} is considered to represent the flow on the network.

[3]See the detailed study on the same Japanese production network's walnut structure in Chakraborty et al. (2018).

[4]See the cited Kichikawa et al. (2018); Iyetomi et al. (2020) for more general cases.

Let us define a "net flow" F_{ij} by

$$F_{ij} = A_{ij} - A_{ji} \ , \tag{8}$$

and a "net weight" w_{ij} by

$$w_{ij} = A_{ij} + A_{ji} \ . \tag{9}$$

It should be remarked that (9) is simply a convention to take into account the effect of mutual links between i and j; one could multiply (9) by a half or an arbitrary positive weight, which actually has little change to the result for a large network.

The Hodge decomposition is given by

$$F_{ij} = w_{ij}(\phi_i - \phi_j) + F_{ij}^{(\mathrm{loop})} \ , \tag{10}$$

where ϕ_i is called a *Hodge potential* of node i, and $F_{ij}^{(\mathrm{loop})}$ is divergence-free by definition, namely

$$\sum_j F_{ij}^{(\mathrm{loop})} = 0 \ , \tag{11}$$

for $i = 1, \ldots, N$. The original flow F_{ij} is decomposed into *gradient flow*, $w_{ij}(\phi_i - \phi_j)$, and *circular flow*, $F_{ij}^{(\mathrm{loop})}$.

From (10) and (11), given F_{ij} and w_{ij}, one has simultaneous linear equations to determine ϕ_i:

$$\sum_j L_{ij}\phi_j = \sum_j F_{ij} \ , \tag{12}$$

for $i = 1, \ldots, N$. Here

$$L_{ij} = \delta_{ij} \sum_k w_{ik} - w_{ij} \ , \tag{13}$$

and δ_{ij} is Kronecker delta, i.e. $\delta_{ii} = 1$ and $\delta_{ij} = 0$ for $i \neq j$.

Note that simultaneous linear equations (12) are not independent of each other. In fact, the summation over i gives zero. This corresponds to the fact that there is a freedom to change the origin of potential arbitrarily. Let us use the convention in the following that the average is zero.

One can prove that if the network is weakly connected, as in our case, the potential can be determined uniquely up to the choice of the origin of the potential.

We apply the method of Hodge decomposition in order to locate an individual firm i in the upstream and downstream layers of the network by using the Hodge potential ϕ_i. The results are summarized in Figure 7. The figure shows the distribution for the Hodge potential of firms in each component of GSCC, IN, OUT and TE (as represented in Figure 6). Recall that the average of all the potentials is 0 by definition. Larger potential implies that the firm is located upstream, while smaller potentials correspond to downstream, as one can see by comparing the histograms for different components. One can see that while the bow-tie analysis can display the relative locations of those components of

GSCC, IN, OUT, and TE, the Hodge decomposition can reveal the individual firm's location in terms of Hodge potential. [5]

4 The model

We proposed a stylized agent-based model, which is stock-flow consistent (albeit with an exogenous foreign sector), to reproduce (in scale) the actual Japanese supply network structure as presented in section 3. It is worth remarking that the initial conditions for all firms, regardless of their position in the network, are drawn from the same distribution. They follow the same behavioral rules, have the same production technology, and are subject to the same productivity shock distribution. The objective is to let the firm size distribution emerges only as a result of network linkages and interactions. For the same reason, the model is simplified to avoid any noise on the final results while at the same time preserving accounting consistency.

Both the intermediate goods market and the final goods market are monopolistically competitive. Each firm i is linked to a randomly predetermined subset of suppliers Ω_i and customers Λ_i. Each firm uses labor, intermediate goods, and raw material as inputs. Mimicking the actual Japanese supply chain, we allocate firms in the three network components IN, GSCC, and OUT, in the same proportions as in the real network. All the components are connected by fixed supply-customer relationships in the intermediate-goods market, but no direct links exist between the IN and OUT components.

Raw materials are all imported and the supply of labor is assumed to be perfectly elastic at the going wage. Firms produce on demand and stock only the excess of intermediate inputs, not their final products.
We abstract from capital accumulation and use. Since we operate on a very short time horizon, production factors are assumed to be strictly complementary.[6] Accordingly, we use the following Leontieff production function:

$$Q_{it} = A_{it} min(\alpha L_{it}, \beta H_{it}, \gamma R_{it}) \tag{14}$$

where A is the total factor productivity which follows a iid stochastic process, L are the units of labor, H is the bundle of intermediate goods, R is the amount of raw material purchased on the international market, and $\alpha, \beta, \gamma > 0$. Since the demand of intermediate products precedes production, firms demand an amount of intermediate goods based on final demand expectations according to

$$H_{it} = \frac{Q^e_{it}}{A_{it}\beta} \tag{15}$$

Expectations are assumed to be adaptive such that $Q^e_{it} = Q_{it-1}$. Accordingly,

[5] For another application of Hodge decomposition for the same Japanese production network, in particular dependence on industrial sector, see Kichikawa et al. (2018).

[6] The modeling choice also finds support in the recent meta-analysis by Gechert et al. (2021) who find a generally low elasticity of substitution between capital and labor.

at the beginning of each period firms' stocks H^s_{it} are updated as follows:

$$H^s_{it} = H^s_{it-1} + H_{it} \tag{16}$$

with H_{it} determined according to (15).
The demand for input j from firm i is

$$H_{ijt} = \frac{P_{jt}^{(1-\eta)}}{(P_t^i)^{(1-\eta)}} H_{it} \ : \ j \in \Omega_i \tag{17}$$

where H_{ij} is the amount of the intermediate goods produced by j used by i and η is the elasticity of substitution across inputs. In order to allocate its demand for inputs, each firm calculates its price index of intermediate goods as

$$P_t^i = \left(\sum_j P_{ij}^{\frac{1-\eta}{\eta}} \right)^{\frac{\eta}{1-\eta}} \ : \ j \in \Omega_i \tag{18}$$

Total final demand is given by the share of profit and labor income that is consumed according to

$$C_t = (1 - \sigma_w - m^w) w \sum_i L_{it-1} + (1 - \sigma_\pi - m^\pi) \sum_i \pi_{it-1} \tag{19}$$

with w is the constant real wage, and $\sigma_w, m^w, \sigma_\pi, m^\pi$ as the constant propensities to save and to import of workers and profit earners, respectively. The amount of consumption going to foreign suppliers is considered as constant (and not as a substitute for domestic products) to simplify the estimation (since in any case the price of foreign firms would be exogenous). Households determine their level of demand for good i as

$$C_{it} = \frac{P_{it}^{(1-\epsilon)}}{P_t^{(1-\epsilon)}} C_t \tag{20}$$

where

$$P_t = \left(\sum_i P_{it}^{\frac{1-\epsilon}{\epsilon}} \right)^{\frac{\epsilon}{1-\epsilon}} \tag{21}$$

is the final goods price index and ϵ is the elasticity of substitution for consumption goods.
For the sake of simplicity, each firm has a unique price for both sales to other firms and to final consumers. Specifically, the price of firm i is optimally set as

$$P_{it} = \frac{\varepsilon_{it}}{\varepsilon_{it} - 1} F_{it} \tag{22}$$

where F_{it} is the average cost and

$$\varepsilon_{it} = \phi_i \eta (1 - s^H_{it}) + (1 - \phi_i) \epsilon (1 - s^c_{it}) \tag{23}$$

where s_{it}^{H} and s_{it}^{c} are the share of the firm in the intermediate goods market and in the final goods market, respectively. The Hodge potential ϕ in (23) is normalized such that $\phi_i \in [0;1]\forall i$. According to equation (23), the demand elasticity for a firm is is a weighted average of the price elasticity of the demand of the final consumption goods and of the intermediate inputs markets, respectively. The weights are given by the market shares, as in Dhyne et al. (2019), and by the Hodge potential. The main reason of correcting the weghts for the Hodge potential is in that it accounts for the network structure and the relative position of the firm in the network, and thus allows for the inclusion of the higher order effects that a change in price or in quantity has on revenues due to possible downstream readjustments. A change in price by an upstream firm (which has a relatively high potential) will generate readjustments in price and quantities in a large number of downstream firms, whereas a change by a more downstream firms (with a relatively low potential) will trigger adjustments on a smaller number of firms. In hierarchical networks, the potential of firms located upstream (downstream) is relatively higher (lower), while in network closer to a circular structure, we can expect the distribution of the potential (before normalization) being more concentrated around 0. In terms of equation (23), upstream firms in circular networks will give relatively higher consideration to the price elasticity of final goods because the readjustment in quantities of final consumers can impact them not only through their downstream customers but also originate higher order effects by indirectly affecting customers at same or higher level in the supply chain. Since the elasticity of input substitution can generally be expected to be lower than the elasticity of substitution among final goods, a more circular network determines a lower market power for upstream firms.

The market shares of each firms in the intermediate goods market and in the final goods market are, respectively,

$$s_{it}^{H} = \sum_{j} \frac{H_{ijt}}{H_j t} \tag{24}$$

$$s_{it}^{C} = \frac{C_{it}}{C_t} = \frac{P_{it}^{(1-\epsilon)}}{P_t^{(1-\epsilon)}} \tag{25}$$

The total demand for firm i is

$$Q_{it} = C_{it} + \sum_{z} H_{zit} + X_{it} \ : \ z \in \Lambda_i \tag{26}$$

where X_{it} is the exports, assumed to be composed only of final goods. Final goods producing firms are assumed to be equally competitive on the domestic and foreign market, so that the shares of consumption calculated according to (20) are also the share of exports.

When the amount of intermediate goods required to satisfy the demand determined in (26) is larger than the firm's stock, the firm increases the demand of inputs to each supplier in the same proportion as in (17). This of course may determine second order adjustments when the suppliers will require quantities of inputs larger than their stocks.

Following Di Guilmi and Carvalho (2017) in assuming a perfectly elastic supply of labor at the going wage w, the quantity of labor, of raw material, and the wage bill are given by, respectively,

$$L_{it} = Q_{it}/\alpha \tag{27}$$

$$R_{it} = Q_{it}/\gamma \tag{28}$$

$$W_{it} = L_{it}w \tag{29}$$

Accordingly, the total average costs are

$$F_{it} = \left(W_{it} + \sum_j P_{jt}H_{jt} + P_{ft}R_{it} \right) /Q_{it} \tag{30}$$

with P_{ft} as the foreign level of price. Profits are calculated as

$$\pi_{it} = Q_{it}\frac{\varepsilon_{it}}{\varepsilon_{it} - 1} \tag{31}$$

Finally, the stock of intermediate inputs is updated according to

$$\Delta H^s_{it} = H^s_{it} - \frac{Q_{it}}{A_{it}\beta} \tag{32}$$

5 Demand volatility transmission

In order to provide a simplified analytical representation of upward shock transmission, let us simplify the network by assuming that the final demand $X + C$ is entirely supplied by OUT firms, which sell only to final consumers. Given the size of the actual sales flows, the simplification does not substantially affect the generality of the analysis.

Abstracting from time indexes to simplify notation, let us a consider a generic GSCC firm whose sales are equal to Q_S and allocated to OUT firms, other GSCC firms, and IN firms according to the following shares: s^O_S, s^S_S, s^I_S, respectively, with $s^O_S + s^S_S + s^I_S = 1$. For the sake of simplicity, we assume that these shares are fixed.

We indicate with Q_O, Q_I the average size of a OUT firm and a IN firm, respectively. Let us consider $[d(X + C) - \beta d(A)]$ as the average percentage change in the demand for intermediate inputs by OUT firms, assuming that they enter symmetrically the final-goods market and considering that the expected change in the TFP A is the same for all firms. Accordingly the expected growth $Q_S g_S$ of a generic firm belonging to the GSCC component can be expressed as

$$\begin{aligned} Q_S\, g_S = s^O_S Q_O[d(X + C) - \beta d(A)] + s^S_S Q_S[dQ_S - \beta d(A)] + \\ + s^I_S Q_I[dQ_I - \beta d(A)] \end{aligned} \tag{33}$$

in which the first term on the r.h.s. is the growth depending on the direct demand by OUT firms, the second term is the demand from other GSCC firms, and the third term is the demand from IN firms. Considering that the demand for inputs from other GSCC firms and from IN firms in turn depends on the change in final demand affecting OUT firms and other GSCC firms, and so on, we can expand (33) as follows

$$
\begin{aligned}
g_S = & \frac{Q_O}{Q_S} s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] + \\
& + s_S^S \left\{ \frac{Q_O}{Q_S} s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] + s_S^S \left\{ s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] \right\} + ... + \right. \\
& \left. + s_S^S s_I^S \{ \frac{Q_O}{Q_S} s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] + s_S^S \left\{ s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] \right\} + ... \} \right\} + \\
& + s_S^S s_I^S \left\{ \frac{Q_O}{Q_S} s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] + s_S^S \left\{ s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] \right\} + ... \right\}
\end{aligned} \tag{34}
$$

which accounts for the feedback effects from IN firms to the other GSCC firms that are interconnected. More precisely, the first line in (34) quantifies the variation in sales due to the change in demand from OUT firms that are directly connected to the GCSS firm. The second and third lines represent the variation due to the other GSCC firms that are directly connected to the GSCC firm, which is given by the variation induced by the OUT firms (second line) and by the IN firms (third line) that are connected to them. Finally, the fourth line is the variation induced by the demand of IN firms that are direct customers of the firm.

Collecting the common term and approximating, we can finally write

$$
\begin{aligned}
g_S = & \frac{Q_O}{Q_S} s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] \{ 1 + s_S^{S2} + s_S^{S3} + ... + s_S^{Sn} + \\
& + s_S^S s_S^I s_I^S (1 + s_S^{S2} + s_S^{S3} + ... + s_S^{Sn}) + s_S^I s_I^S (1 + s_S^{S2} + s_S^{S3} + ... + s_S^{Sn}) \} \\
\approxeq & \frac{Q_O}{Q_S} s_S^O \left[d(X_t + C_t) - \beta d(A_t) \right] \left[1 + s_I^S s_S^I (1 + s_S^S) \right] \frac{1}{1 - s_S^S}
\end{aligned} \tag{35}
$$

The variance of the growth rates of GSCC firms is then given by

$$
V_{gs} = \left[V(X + C) + \beta^2 V(A) \right] \left\{ \left(\frac{Q_O}{Q_S} s_S^O \right) \left[1 + s_I^S s_S^I (1 + s_S^S) \right] \frac{1}{1 - s_S^S} \right\}^2 \tag{36}
$$

$$
= \left[V(X + C) + \beta^2 V(A) \right] \left\{ \left(\frac{Q_O}{Q_S} s_S^O \right) \psi_S \right\}^2 \tag{37}
$$

with $\psi_S \equiv \left[1 + s_I^S s_S^I (1 + s_S^S) \right] \frac{1}{1 - s_S^S}$. Equation (37) shows that the relationship between the variance of the final demand $V(X + C)$ and the volatility of sales for suppliers depends on three factors:

- the relative size of upstream and downstream firms: $\frac{Q_O}{Q_S}$;
- the volatility of productivity $V(A)$;
- the structure of the network, which determines the share of inputs exchanged among the different clusters of the network and within the GSCC: $s_S^O, s_I^S, s_S^I, s_S^S$.

In order to provide an approximate idea of the relevance of the network structure, we focus on the term ψ_S inside the curl brackets since $\left(\frac{Q_O}{Q_S} s_S^O\right)$ is simply the weight of relative size effect. Let us consider the extreme case of a perfectly hierarchical network, in which each component sells its entire production to the component immediately downstream. It is easy to verify that, in this case, $\psi_S = 1$, since $s_S^O = s_I^S = 1, s_S^I = s_S^S = 0$. The variance in this case comes to depend only on the relative size of firms and on the volatility of productivity. In contrast, in case of a network in which circular flows are not null, $s_S^O, s_I^S < 1$ and $s_S^I, s_S^S > 0$. As a consequence, $\psi_S > 1$ and the volatility of the final demand is amplified by horizontal and vertical links and feedback effects in case of bidirectional links.

Finally, assuming away links within the IN portion of the network, it is trivial to derive the relationship between the variance in growth rates for IN firms and the variance for GSCC firms, with the following expression

$$V_{g_I} = \left(\frac{Q_S}{Q_I}\right)^2 [V(g_s) + V(A)] \tag{38}$$

We can use the analysis of section 3 to assess the effects of shocks in final consumption on upstream firms in the case of the Japanese supply network. We know from the already cited Aoyama et al. (2010, 2017); Fujiwara, Y. and Aoyama, H. (2010); Fujiwara (2020) that the size of firms is expected to decline as we move upward in the supply chain, with the largest firms being located downstream. Accordingly, the expected value of the ratio between the size of OUT firms and GSCC firms should be bigger than one: $E\left[\frac{Q_O}{Q_S}\right] > 1$ in (37). The same applies for the ratio $\frac{Q_S}{Q_I}$ in (38).

We also know that, in the case of a perfectly hierarchical network, the Hodge potentials for IN, GSCC, and OUT firms will be equal to, respectively, $\phi_I = 1, \phi_S = 0, \phi_O = -1$. Indeed, these are approximatively the values of the medians as visible in Figure 7. However, the dispersion around the median appears to be quite large in all three components, with a fraction of OUT and GSCC firms having a smaller potential than some IN firms. As a consequence, the Japanese network can be safely described as non-hierarchical and, accordingly, $\psi_S > 1$ in (37). Accordingly, the model predicts that a downstream shock is amplified as its effects are transmitted upward through the different sections of the supply network.

6 Simulations

This section explains how the computer simulations for the model presented in section 4 are implemented and presents the main results.

6.1 Settings

The simulations are based on the stylized network structure of Figure 6. The network is static and links are stochastically created at the beginning of the simulation. The random distribution of connections for each portion of the network is set to mimic the distribution of the potential in the three groups as in Figure 7. The distribution of the potential ϕ (before normalization) over a single run is visualized in figure 8. We further assume that only OUT firms sell in the final goods markets (consumption and export).

The productivity coefficients A for each layer are modeled as an AR(1) process of the type

$$A_{it} = \rho A_{it-1} + \nu_{it} \tag{39}$$

with $\nu_{it} \propto \mathcal{N}(0; \sigma_A)$. For the process in (39), we set $\rho = 0.9, \sigma_A = 0.3$.
The exogenous time series are modeled as stationary in the following way: we detrend the time series of gross export and bilateral exchange rate Yen/USD and calculate the standard deviations of residuals, identifying them as σ_x and σ_f, respectively; finally, the time series of the two variables are obtained as additive shocks on the initial values drawn from the distributions $\mathcal{N}(0; \sigma_x)$ and $\mathcal{N}(0; \sigma_f)$. For both variables, we use St Louis FED data for the period 1991-2008 as reported by the data, to exclude the highly volatile period after the Great Recession, and, converting in indexes, we estimate $\sigma_x = 0.0567, \sigma_f = 0.0099$.

In the simulations, the total number of firms is 2000 and they are allocated to each cluster to mimic the distribution in the real data: 50% in the SCC, 30% in the IN component, and 20% in the OUT component.

Initial conditions are equal for all firms. The initial level A_0 is drawn from a uniform distribution with range [2.5; 97.5]. Production in period 1 is equal for all firms with $Q_{i0} = 0.8$.

The remaining parameters are calibrated as follows: $s_w = 0.1; s_p i = 0.3; m_w = 0.2; m_p i = 0.2; \alpha = 1; \beta = 6; \gamma = .0000001; \eta = 1.10; \epsilon = 1.50; w = .4$.[7] Assuming one simulation period to be equal to a quarter, each simulation is ran for 16 periods, since we aim to replicate the short run change analyzed in the empirical section.

[7]The elasticity of the intermediate inputs is lower than the one for consumption goods to account for the lower substitutability of inputs, although still elastic. According to the empirical literature (Atalay, 2017; Baqaee and Farhi, 2020, among others), the elasticity of substitution is low across industries. Given the scope of the present paper, we abstract from the distinctions between intra and inter industry connections, averaging out the different degrees of input substitutability.

6.2 Results

The simulations qualitatively replicate the evidence reported in section 3, providing some insights on the mechanisms that determine the different growth regimes for the various dimensional classes and position within the network. Let us first present some results generated by single-run simulations before discussing in detail the properties of the model as they emerge from Monte Carlo replications.

The already introduced Figure 8 shows the histogram of the potential in a single simulation. The narrower range is an unavoidable consequence of the lower number of nodes. However, the general structure of the network is comparable, with the distribution of the GSCC centered on 0 and the medians of the IN and OUT equal in absolute value.

The higher degree of dispersion in growth rates for upstream firms is evident in Figure 9. The plot reports the histogram of the average frequencies of growth rates over 100 Monte Carlo replications. The tails of the distributions are considerably fatter for the upper layers of the network, mimicking the pattern observed in Figure 4 for real data. The difference in the levels of volatility in the growth rates is at the root of the size distribution of firms displayed in Figure 10, which shows the size distribution in a single replication, distinguishing firms by their respective network cluster. The results are confirmed by Monte Carlo replications as displayed by Figure 11. Downstream firms are the largest and the power-law tail is composed almost exclusively by firms belonging to the OUT section of the network. GSCC are relatively more frequent in the flatter section of the distribution, out of the power-law tail, whereas IN firms are the smallest. It is worth remarking that the initial conditions for firms in all three clusters are drawn from the same distribution and the size heterogeneity and the power-law tail emerge in the short span of the simulation.

This set of results confirms the hypothesis that downstream demand shocks are amplified in the supply chain, leading to the breakdown of the Gibrat's law for upstream firms and to the power law tail in the firm size distribution for downstream firms. Simulating a a sizable export shock (-30%, roughly the size of the Great Recession shock) in the second last period of the simulation over 100 Monte Carlo replications, the changes in the medians of the distributions of growth rates for OUT, GSCC, and IN firms are, respectively -0.0373, -0.0944, and -0.1113. The shock initially hitting OUT firms is clearly amplified and eventually generates a larger effect for upstream firms. .

We study the role of network structure and market power by means of counter-factual computational experiments. In particular, we examine the size-standard deviation relationship in the baseline scenario, in an almost perfectly hierarchical network, and in a situation in which the elasticity of demand for intermediate goods and final goods are the same, namely $\eta = \epsilon = 1.5$. In the case of an almost perfectly hierarchical network, IN firms sell only to GSCC firms, which in turn sell only to OUT firms. The random links in the adjacency matrix are set up in order to have the (normalized) potentials for all IN, GSCC, and OUT firms of $\phi_I \approxeq 1, \phi_S \approxeq 0.5, \phi_O \approxeq 0$. The results are shown in Figure

12. The top-left panel of Figure 12 reproduces the pattern shown in Figure 5, identifying a scaling regime for small firms, and a Gibrat regime for large firms, in the relationship between firm size and growth volatility. Each dot in the plot represents the standard deviation in growth rates for each bin of the firm size distribution. The results in the plot are the average over 100 Monte Carlo replications. A negative relationship is identifiable for small firms, while no apparent correlation can be detected for the largest ones. Hence, for largest firms Gibrat law applies since their rate of growth is independent from their size, while the same does not hold for the smallest firms, which are upstream in the supply chain, as shown in figure 10. In the top-right panel, elasticity in the intermediate-goods market is higher, implying a relatively smaller market power of suppliers. Besides the general increase in volatility, the most evident effect is the reduction of the Gibrat's range, since only for the top four bins volatility does not change with size. The general reduction in size compared to the blue and black distributions depends on macro-effects, since the tighter market compresses profits and negatively impacts on the final consumption demand. However, a power-law tail in firm size distribution still exists.

Recalling that the competitiveness of firms is stochastic (being dependent on the random process for total factor productivity), higher elasticity of substitution implies expected larger readjustments for upstream firms, as highly price-sensitive customers redistribute their purchases towards the cheaper suppliers to a larger extent. The numerical simulations are likely to underestimate the extent of the redistribution of demand, since the network is static. In reality, major demand shocks can lead to the complete substitution of less efficient suppliers and this effect will be larger the bigger the elasticity of substitution and the customers' market power.

The bottom panels in Figure 12 present the result of a conterfactual experiment, in which the model is simulated in an almost perfect hierarchical network. In this setting, the standard deviation in growth rates appears to be generally increasing with size. When the elasticity in the intermediate goods market is higher the relationship is monotonic. Further, looking also at Figure 13, small firms are no longer necessarily located upstream and, accordingly, the dynamics of upward transmission of shocks changes. As a result, a clear power-law distribution in the right tail is no longer evident. It is also worth remarking that the proportion of GSCC firms among the largest ones in the right tail appears to be larger if compared with Figure 11. The increase in size illustrated in the bottom panels of Figure 12 is a consequence of the lower number of links in the hierarchical network.

To conclude, the network structure appears to affect the firm size distribution to a larger extent than the relative market power of suppliers/customers in the supply chain. The consequences of changes in the market power are a general increase in volatility and a smaller right tail of the distribution, but the Gibrat law still applies for the largest firms. In contrast, a modification in the network structure totally alters the relationship between size and volatility and the Gibrat law appears to no longer be applicable in any range of the firm size distribution.

7 Policy indications

Our results highlight a possible drawback of interconnectedness in complex network structures as the high number of connection exposes suppliers to wider range of idiosyncratic firm-level shocks and can therefore amplify micro-level volatility in case of demand shocks.

In terms of aggregate outcomes, this sort of “upward snowball effect”, as volatility is transmitted to suppliers, can lead to an under-estimation of the overall impact of demand shocks. The first-order or direct effect of demand shocks determine reduction in output for downstream firms and as such it is accounted for in GDP. However, since the particular mechanism of amplification of shocks that we identify shapes the firm size distribution, it also indirectly affects the macroeconomic dynamics because of the *granular hypothesis*. In other words, demand shocks can affect the economy *directly* through the reduction in output for downstream firms (which is recorded in the GDP) and *indirectly* by shaping the firm size distribution and, in particular, altering the size of the power law tail and, as a consequence, the applicability of the granular hypothesis and the way in which micro-level shocks affect the macroeconomy.

8 Conclusions

The paper proposes an investigation of the transmission of demand shocks in the Japanese supply network and, in particular, in their role in determining firms' growth and, as a consequence, their size distribution. Through the analysis of Japanese data, analytical decomposition of the growth rates' volatility, and numerical simulations, we find that: 1) the Japanese supply network has a bow-tie structure in which firms located in the upstream layers display a larger volatility in their growth rates; 2) as a consequence, the Gibrat's law breaks down for upstream firms whereas downstream firms are more likely to be located in the power law tail of the size distribution; 3) this pattern is determined by the amplification of demand shocks hitting downstream firms; 4) the magnitude of this amplification increases the less hierarchical is the network structure and the higher is the market power of downstream firms; 5) in an amost perfectly hierarchical network, the power-law tail in firm size distribution disappears.

The paper provides a primer in the analysis of the implications on the firm size distribution of the upward transmission of demand shocks. The relevance of the network structure (and in particular of horizontal and upward links), of market power, and of relative size of suppliers and customers in the supply network has important consequences in the transmission of demand shocks and, potentially, can affect the impact of supply shocks. The relative market power and the structure of the network are likely to play a role in the modifications of the supply network that are resulting from the Covid-19 shocks.

Future research can extend the analysis to supply shock and further investigate the feedback effects between firm size distribution and macroeconomy (along the lines of Carvalho and Grassi, 2019). The extension could benefit from a more sophisticated model of the macroeconomy, which can be achieved by integrating the current model with the one by Di Guilmi and Fujiwara (2020), allowing for an analysis not only of the real implications of shock transmissions but also of their nominal effects. Further, the introduction of a dynamic network in which firms create or destroy links according to some objective function, can allow for an endogenous network. This extension can introduce another type of feedback and further help the understanding of the generation of power law tails in the firm size distribution.

Acknowledgments

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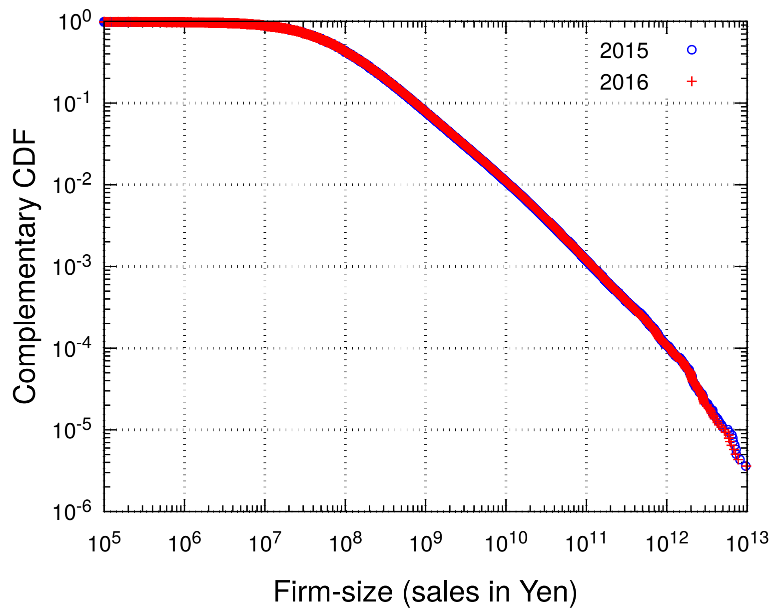


Figure 1: Complementary CDF (cumulative density function) for firm-size of sales (in Yen). The regime of large firms obeys the Zipf's law (1) with μ being close to 1. There is a deviation from the law in the regime of small and medium firms, roughly smaller than 10^9 yen. Two plots correspond to the years of 2015 and 2016.

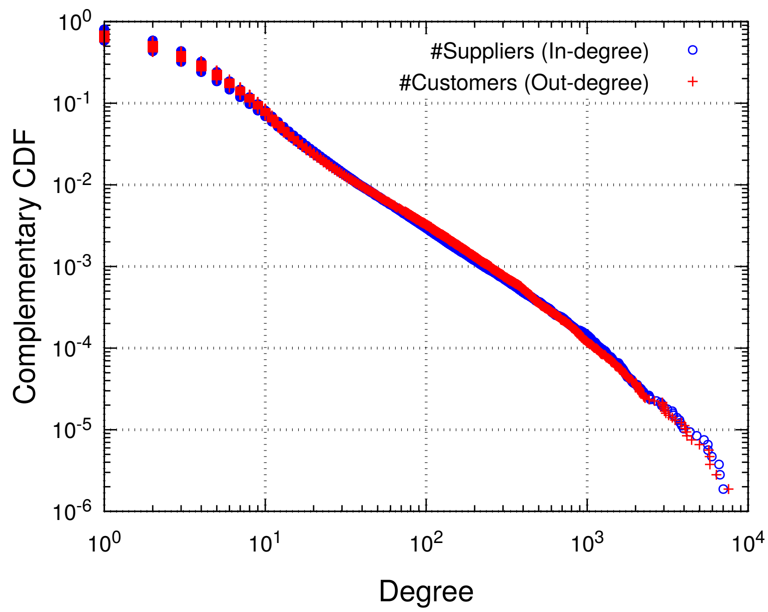


Figure 2: Complementary CDF for the degrees of individual firms. In-degree refers to the number of suppliers of the firm, while out-degree is the number of customers of it. Data for the year 2015. Similarly to the size, both of the in and out degrees obey a power-law in the same functional form of (1). Largeer firms have larger degrees, and vice versa. Similarly for small and medium firms. See Fujiwara, Y. and Aoyama, H. (2010); Aoyama et al. (2010, 2017) for details.

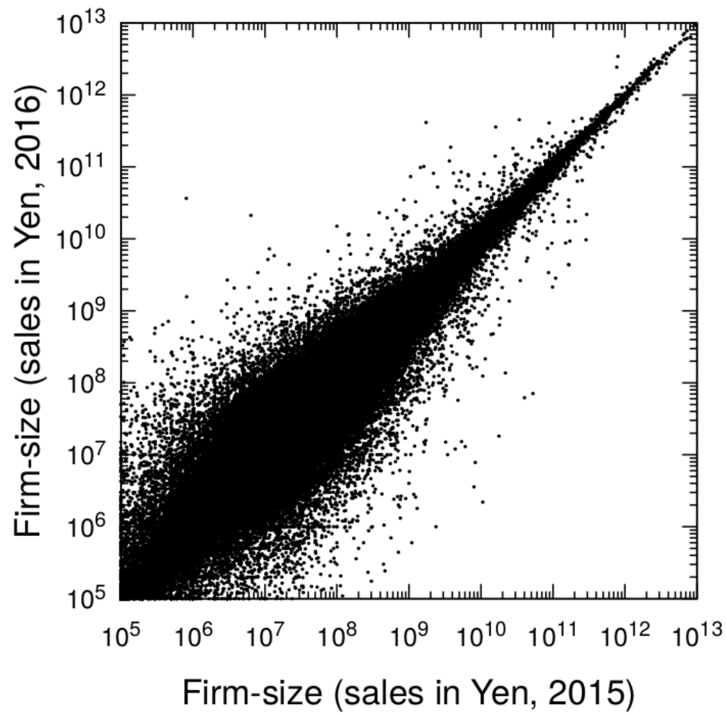
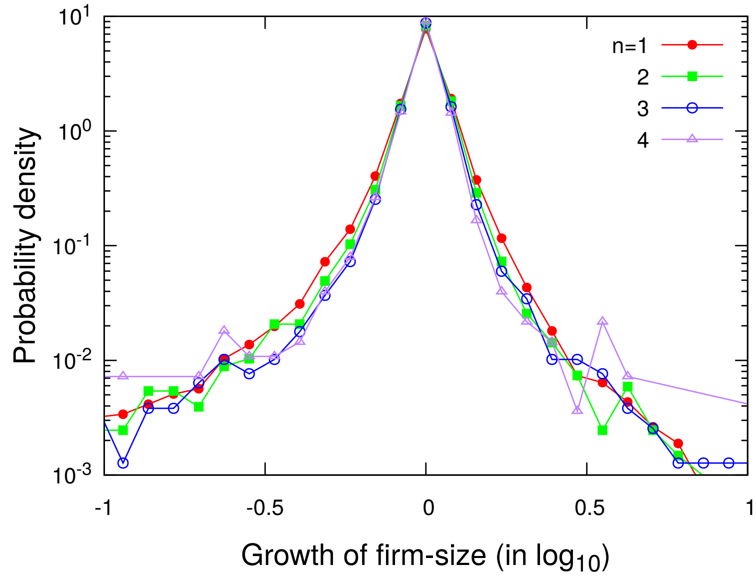
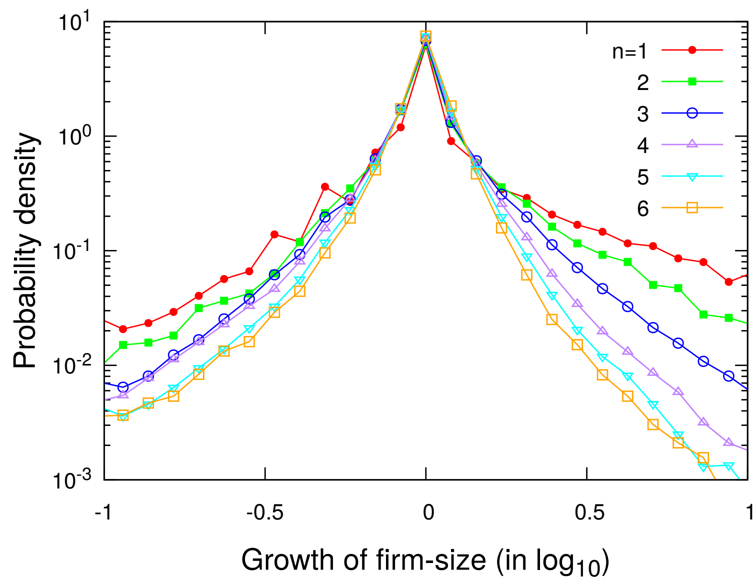


Figure 3: Scatter plot for temporal change of individual firm-size for two successive years of 2015 and 2016. Diagonal 45-degree line correspond to the growth-rate $G_t = 1$ or $g_t = 0$ (see (2), (3)).



(a) For the regime of large-firms



(b) For the regime of small and medium firms

Figure 4: Probability density function (PDF) for growth g_t in the regimes of large firms (a) and small and medium firms (b). (a) $n = 1, 2, 3, 4$ correspond to different bins of initial firm-size x_{t-1} to calculate the growth, which are logarithmically equal-spaced between 10^9 and 10^{11} Yen (see the CDF in Figure 1), i.e. $10^9 \times [10^{0.5(n-1)}, 10^{0.5 \cdot n}]$. Gibrat's law holds, because the PDF for g_t does not depend on x_{t-1}. (b) $n = 1, 2, \ldots, 6$ correspond to bins of $10^6 \times [10^{0.5(n-1)}, 10^{0.5 \cdot n}]$ between 10^6 and 10^9 Yen. Gibrat's law breaks down, because the PDF for g_t depends on x_{t-1}; smaller firms have larger variation in the growth.

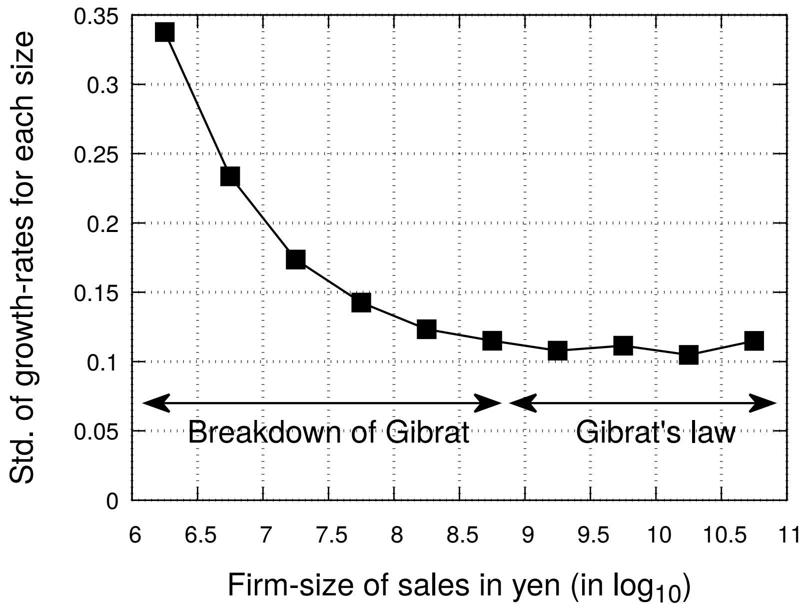


Figure 5: Dependence of standard deviation of growth-rate g_t for the firms binned in different sizes, logarithmically equal-spaced between 10^6 and 10^{11} Yen. Standard deviations are calculated from the PDF's in Figures 4 (a) and (b). One can see that Gibrat's law holds for large firms, but breakdown for small and medium firms.

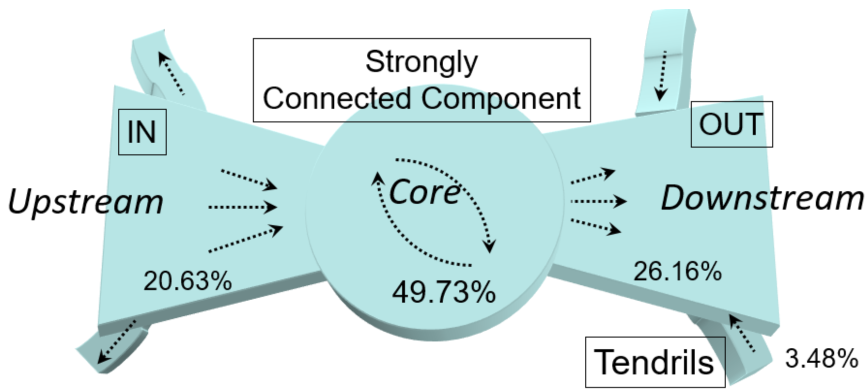


Figure 6: Bow-tie structure for the production network. The "core" is located as a giant strongly connected component (GSCC), which is linked to the IN and OUT components by in-going links into and outgoing links from the GSCC, namely 'upstream" and "downstream". The remaining part is called "tendrills" (TE). Percentage in each component represents the ration of the number of firms included in the component to the total number of firms (1.06 million).

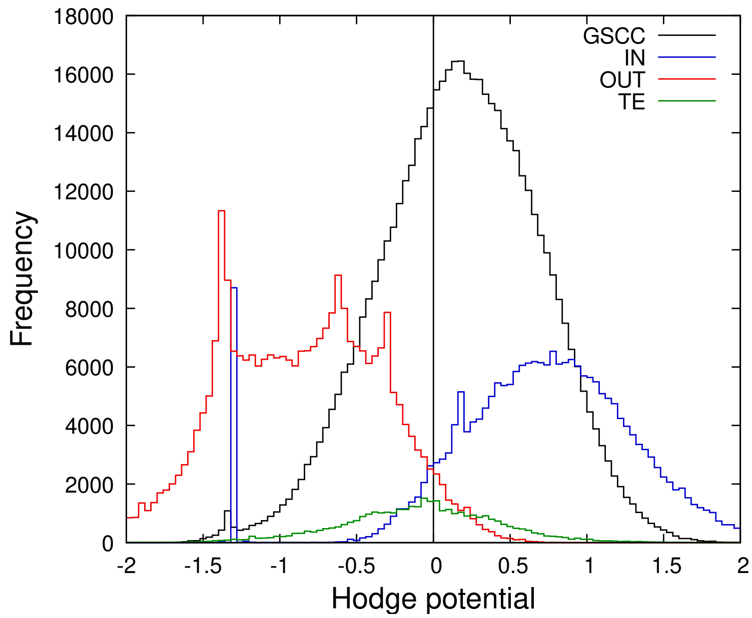


Figure 7: Histogram of the Hodge potential of firms in each component of GSCC, IN, OUT and TE (see Figure 6). The average of all the potentials is 0 by definition (shown as vertical line). Larger potential implies that the firm is located in the upstream, while smaller potentials correspond to downstream.

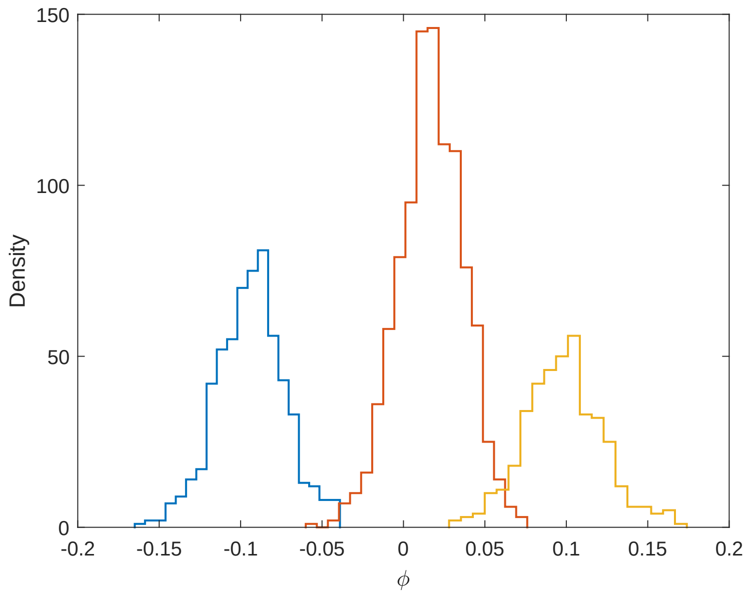


Figure 8: Simulations: histogram of the Hodge potential of firms in each component of GSCC (red), IN (yellow), OUT (blue).

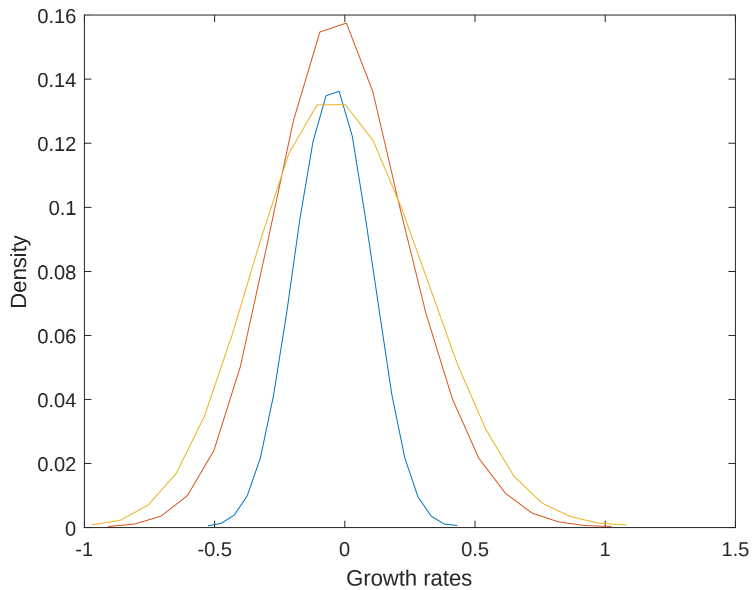


Figure 9: Histogram of growth rates. Blue line: layer 1, red line: layer 2, yellow line: layer 3. Average over 100 Monte Carlo replications.

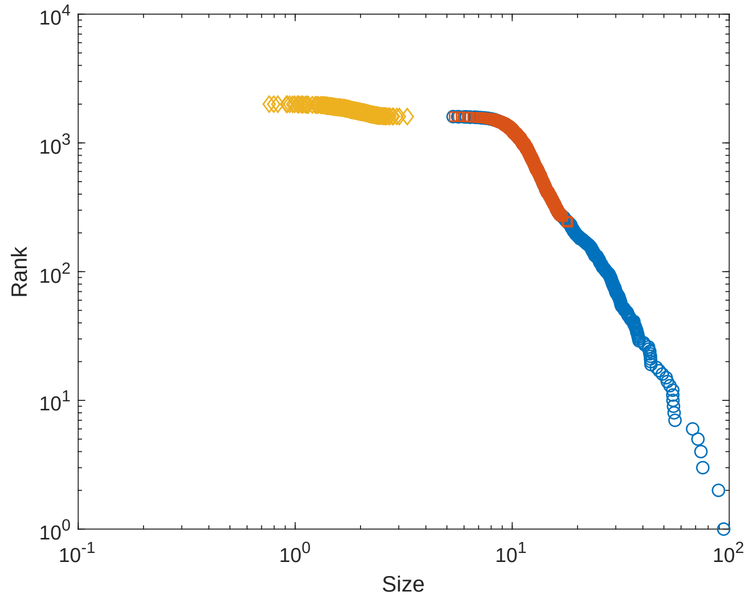


Figure 10: Zipf plot of firm size distribution. Blue circles: OUT, red circles: GSCC, yellow circles: IN.

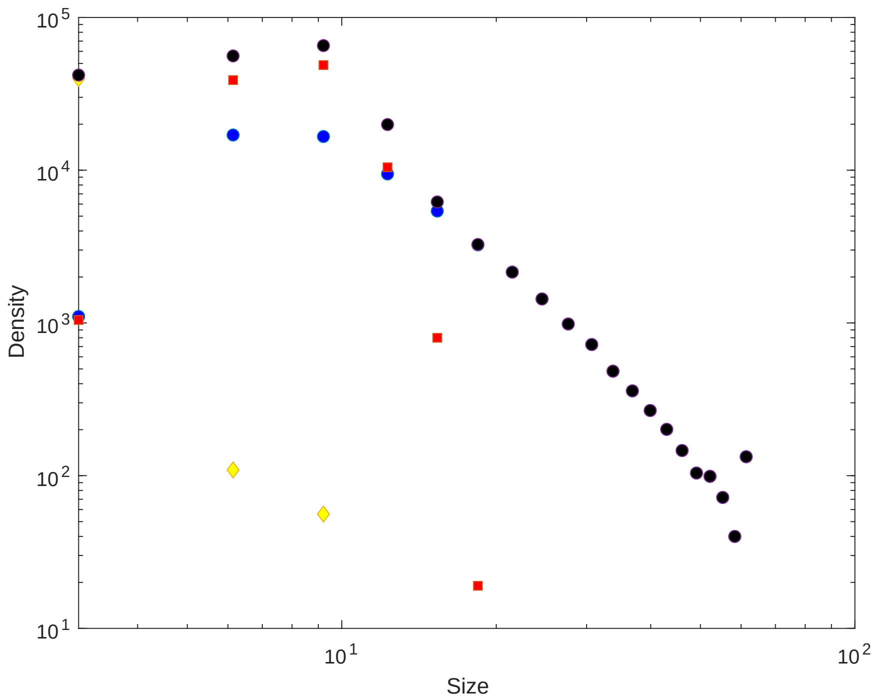


Figure 11: Firm size distribution. Average over 100 Monte Carlo replications. Black circles: total; blue triangles: OUT, red squares: GSCC, yellow diamonds: IN.

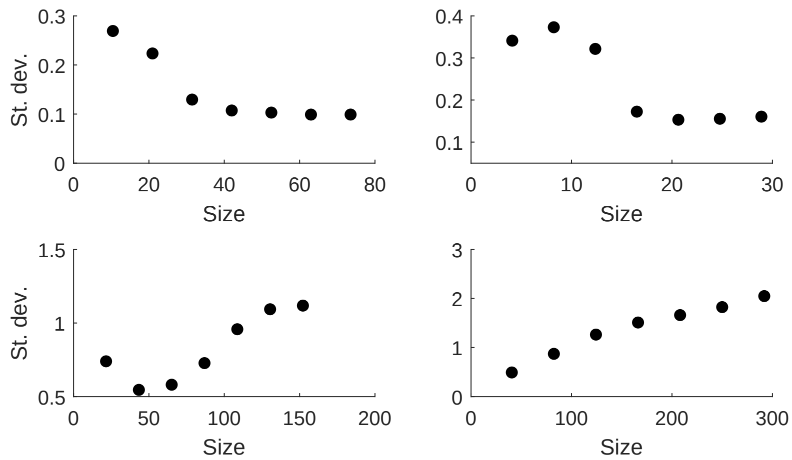


Figure 12: Size versus standard deviation of sales for all firms. Upper panels: baseline bow-tie network structure; lower panels: hierarchical network; left panels: baseline elasticity value: $\eta = 1.1, \epsilon = 1.5$; right panels: $\eta = \epsilon = 1.5$. Average over 100 Monte Carlo replications.

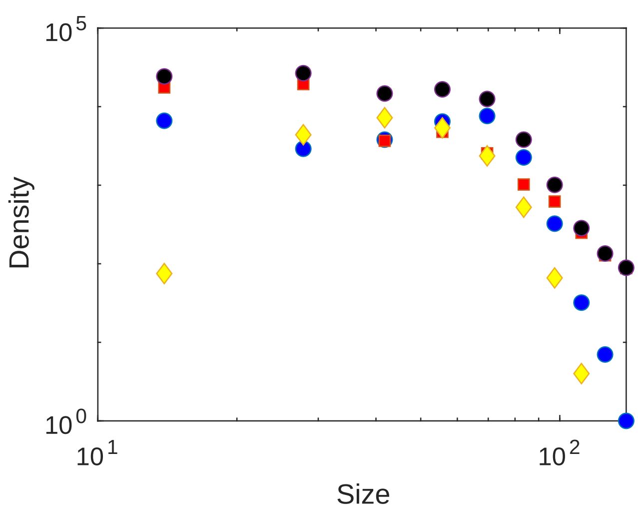


Figure 13: Firm size distribution with hierarchical network. Average over 100 Monte Carlo replications. Black circles: total; blue triangles: OUT, red squares: GSCC, yellow diamonds: IN.

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