RIETI Discussion Paper Series 22-E-039

Is Empirical Granularity High Enough to Cause Aggregate Fluctuations? The closeness to Gaussian

Yoshiyuki ARATA
RIETI

Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
https://www.rieti.go.jp/en/

RIETI Discussion Paper Series 22-E-039
April 2022

Is empirical granularity high enough to cause aggregate fluctuations? The closeness to Gaussian

Yoshiyuki ARATA*†
Research Institute of Economy, Trade and Industry

Abstract

Recent studies (e.g., Gabaix (2011)) argue that because of the high heterogeneity of firm size, microeconomic shocks generate aggregate fluctuations (i.e., the granular hypothesis). This paper tests whether empirical granularity is high enough to explain fluctuations in the GDP growth rate using firm-level data in G7 countries. I find that even when the central limit theorem does not hold, microeconomic shocks cancel each other out, and thus, the distribution of aggregate output induced by microeconomic shocks is very close to a Gaussian. In other words, the observed heterogeneity of firm size in all G7 countries is not high enough to prevent the averaging effect of microeconomic shocks. Furthermore, because of the closeness to a Gaussian, microeconomic shocks with export/import relations would result in no tail dependence of aggregate fluctuations. Since the empirical GDP growth rates deviate from a Gaussian in the tail region and show positive tail dependence across countries, the granular hypothesis cannot explain these tail features of the GDP growth rates.

Keywords: Micro-origin of aggregate fluctuations; Granular hypothesis; Tail probability; Tail dependence
JEL codes: E32, E23, D57

*E-mail: y.arata0325@gmail.com

†This study was conducted as a part of the Project "Heterogeneity across Agents and Sustainability of the Japanese Economy" undertaken at RIETI. This study was supported by JSPS (KAKENHI Grant Numbers 18K12788 and 21K13265). Conflicts of interest: none.

1 Introduction

What is the cause of aggregate fluctuations (see **Figure 1**)? For this fundamental question in macroeconomics, recent studies (e.g., Gabaix (2011)) argues that microeconomic shocks to firms substantial generate aggregate fluctuations. This is due to the high heterogeneity of firm sizes (i.e., granularity), that is, microeconomic shocks to giant firms cannot be averaged out even at the aggregate level and contribute to aggregate fluctuations. Since the high heterogeneity of firm sizes is one of the stylized facts, this granular view has been widely accepted in the recent literature.[1]

However, the *distribution shape* of aggregate output induced by microeconomic shocks, given an empirical granularity of firm sizes, is not fully analyzed in previous studies. In particular, can the granular view explain the important features of the distribution of the GDP growth rates? For example, the distribution of the GDP growth rates deviate from Gaussian in the tail region, that is, the probability that an economy experiences a large shock is higher than that predicted by Gaussian.[2] Furthermore, **Figure 1** shows that when an economy experiences a rare event, other economies are likely to experience a rare event at the same time (i.e., the tail dependence). In order to assess the importance of the granular view, it is necessary to examine whether the micro-originated aggregate fluctuations is consistent with these features of the GDP growth rates.

This paper analyzes whether an empirical granularity is large enough to explain the distribution properties of aggregate fluctuations using firm-level data in G7 countries. The main idea of my analysis is to quantify how close to Gaussian the distribution of aggregate output induced by microeconomic shocks is. Given the empirical granularity for G7 countries, I find that the resultant distribution of aggregate output is very close to Gaussian. This means that although the central limit theorem (CLT) does not hold due to the high granularity, the averaging effect is still dominant, that is, most of the variation of microeconomic shocks are canceled each other out. Since the empirical GDP growth rates deviate from a Gaussian distribution in the tail regions, I conclude that the granular view cannot explain the tail properties of the GDP growth rates.

My analysis focuses on the empirical facts of three distributions: the distribution of firm sizes, the distribution of firm growth rates, and the distribution of the GDP growth rates. First, it is well known in the literature that the firm size measured by annual sales revenues are highly heterogeneous and follows Zipf's law, that is, the firm size distribution has Pareto tail with an exponent close to 1.[3] In my analysis, using firm-level data taken from Orbis, which is compiled by Bereau van Dijk, I confirm that Zipf's law holds for G7 countries. This high heterogeneity of firm sizes is the basis of the granular view. Second, I focus on the distribution properties of the sales growth rates. In an empirical literature, it is empirically shown

[1] In the following, I use the two terms "heterogeneity" and "granularity" interchangeably.

[2] See Section 2.1.

[3] See, for example, Axtell (2001) and Gabaix (2009).

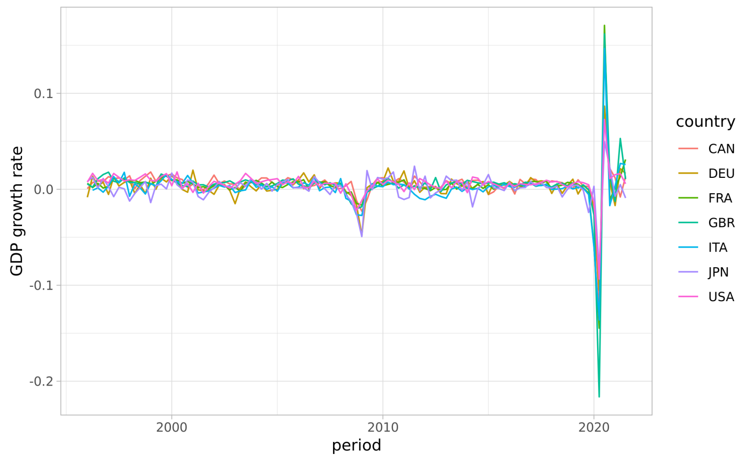


Figure 1: Time series of the GDP growth rates across G7 countries from 1994Q1 to 2021Q3. See the explanation in **Table 1**.

that the distribution of sales growth rates deviates from Gaussian but is close to a Laplace distribution.[4] The Laplace distribution is characterizes by the sharp peak around the center and a fatter tail than Gaussian. In particular, the fatter tail of the Laplace distribution suggests that the probability of a rapid growth (or shrinkage) is larger than that predicted by Gaussian. For example, if such a shock hits giant firms in an economy, whose existence is suggested by Zipf's law, the impact of microeconomic shocks may be relevant even at t he aggregate level. In my analysis, I use the sales growth rates and these distribution properties as proxies for productivity (TFP) shocks and their distribution properties. Third, I focus on the distribution tail of the GDP growth rates. Its deviation from Gaussian has been documented by recent empirical studies, that is, the probability of rare events (such as severe economic downturns) is larger than that predicted by Gaussian.[5] In addition to that, I consider the bivariate distribution of the GDP growth rates for two countries and analyze its dependence structure by using the copula method. I find that there exists significant tail dependence across countries; that is, rare events (i.e., large negative/positive shocks) for two countries are likely to occur simultaneously. The research question in this paper is to test whether these three distribution properties (firm size, sales growth rates, and GDP growth rates) are consistent with each other, given the granular idea.

To analyze how these distribution properties are related with each other, I use probabilistic methods given in Section 3. I focus on the distinction between asymptotic and non-asymptotic results: The asymptotic

[4] See, for example, Coad (2009), Dosi et al. (2017), Bottazzi and Secchi (2006), and Arata (2019). Since the firm-level TFP is estimated from firms' sales revenue (i.e., TFPR), I assume that the distribution of TFP growth rates inherits the Laplace shape of the distribution of the sales growth rates. The Laplace distribution for TFP shocks is also used in Acemoglu et al. (2017), though their empirical analysis is at sector-level.

[5] See, for example, Fagiolo et al. (2008), Cúrdia et al. (2014), and Clark and Ravazzolo (2015). In particular, Adrian et al. (2019) focus on the evolution of the GDP growth rate distribution over time and use a skewed student's t distribution for approximation.

results mean the properties of aggregate output as the number of firm tends to infinity. The slow decay rates of the variance (Gabaix (2011)) and tail probability (Acemoglu et al. (2017)) and the condition of the convergence to Gaussian (Acemoglu et al. (2012)) are examples of the asymptotic results. In contrast, the non-asymptotic results mean the properties of aggregate output with a fixed (and finite) number of firms, that is, firm sizes are empirically given. In particular, in my analysis, I use saddlepoint method to approximate the distribution of aggregate output with given firm sizes. Although most of previous studies rely on the asymptotic results (this is why the granular view is widely accepted), the asymptotic and non-asymptotic results do not necessarily give the same implication about micro-originated aggregate fluctuations. For example, there is possibility that while the distribution of aggregate output does not converge to Gaussian at the limit, the distribution of aggregate output with given firm sizes are very close to Gaussian. My analysis shows that this possibility is crucial in the analysis of micro-originated aggregate fluctuations.

The main finding of this paper is given in Section 4. First, I find that because of the granularity of firm sizes, the CLT does not hold for all countries, as expected by the granular hypothesis. That is, the distribution of aggregate output induced by microeconomic shocks does not converge to Gaussian. On the other hand, given the empirical granularity (and Laplace assumption about microeconomic shocks), I find that the distribution of aggregate output turns out to be very close to a Gaussian. This result is not contradiction: the non-convergence of to Gaussian does not imply that the distribution is far from Gaussian. My analysis reveals that even when the CLT fails, the averaging effect does not cease to work and makes the resultant distribution close to a Gaussian. Furthermore, the closeness to Gaussian has another implication for the two-dimensional case. I find that because of the closeness to Gaussian, there would be no tail dependence between two countries; that is, it is highly unlikely that two countries experience rare events simultaneously, which is inconsistent with the empirical counterparts. Thus, I conclude that microeconomic shocks cannot explain the observed tail features of the GDP growth rates. Put differently, the empirical granularity is not high enough to generate substantial aggregate fluctuations.

Related literature

This paper belongs to the recent literature on the micro origin of aggregate fluctuations (see Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for a survey). In particular, this paper is closely related to studies that analyze the relation between the granularity and distribution properties of aggregate output: Gabaix (2011), which proposes the granular hypothesis, shows that if the firm size distribution has Pareto's tail with exponent close to 1, the micro-originated aggregate variance decays slowly as $n \to \infty$. Closely related to Gabaix (2011), Acemoglu et al. (2012) focus on the heterogeneity of an input-output network and provide the condition that the distribution of aggregate output converges to Gaussian. Acemoglu et al. (2017) focus on the tail probability of aggregate output and show that it decays slowly as $n \to \infty$ when the granularity is high. In contrast, this paper argues that these slow decay rates and non-convergence to

Gaussian found in literature do not necessarily implies the empirical relevance of the granular view. This is because these results are asymptotic results as $n \to \infty$ and non-asymptotic results are more important to assess its empirical relevance. In particular, by extending the methods developed in Arata (2021) and Arata and Miyakawa (2022), my analysis shows that the resultant distribution of aggregate output with empirical granularity is very close to Gaussian, which contradicts the empirical properties of the GDP growth rates.[6]

This paper contributes to the empirical literature on the relevance of the granular hypothesis. Carvalho and Gabaix (2013), Di Giovanni et al. (2014), and Stella (2015) analyze firm-level data and argue that the variance of the GDP growth rates is associated with the granularity in an economy. Magerman et al. (2017) and Miranda-Pinto (2021) use an empirical (firm-level/sector-level) input-output data and show that because of the heterogeneity of the network, microeconomic shocks are an important source of aggregate fluctuations. In contrast to these studies, my analysis focuses not only the variance but the distribution shape of aggregate output including the tail probability of aggregate output. In particular, by focusing on the closeness to the Gaussian distribution, I find that microeconomic shocks would generate only small fluctuations (measured by the variance), and not a large deviation (measured by the tail probability), in aggregate output.

Furthermore, my analysis about the bivariate distribution of aggregate output for two countries is related to international business cycle synchronization. For example, Di Giovanni and Levchenko (2012) and Di Giovanni et al. (2018) argue that because of the granularity and import/export relations, aggregate output for two countries comove. In contrast, my analysis shows that although import/export relationships generate economically significant positive correlation, the closeness to Gaussian (copula) implies that there is no tail dependence for the two countries. That is, it is unlikely that two economies experience a rare event simultaneously.

Outline of this paper

This paper is organized as follows. Section 2 overviews firm-level data and the GDP time series. Section 3 provides probabilistic methods to characterize the distribution of aggregate output. Section 4 provides empirical results. Section 5 concludes this paper. The Appendix provides the proofs of propositions and robustness check of my empirical results.

2 Overview of data

This section overviews data used in my analysis. In our analysis, I focus on the G7 countries (i.e., Canada, France, Germany, Italy, Japan, the UK, the US). Section 2.1 examines the GDP growth rates and

[6] In Arata and Miyakawa (2022), I use the Edgeworth expansion to approximate the distribution of aggregate output. Although the Edgeworth expansion is one of the widely used methods, it is known that the approximation is not reliable when the true distribution is far from Gaussian. See, for example, Kolassa (2006). In this paper, I use the saddlepoint approximation, which provides more accurate approximation especially for the tail regions.

country	count	mean	sd	mad	max	min	since when
CAN	242	0.0075	0.0128	0.0073	0.0860	-0.1170	1961Q2
DEU	122	0.0030	0.0152	0.0069	0.0866	-0.1053	1991Q2
FRA	166	0.0042	0.0188	0.0044	0.1708	-0.1447	1980Q2
GBR	266	0.0057	0.0195	0.0060	0.1619	-0.2163	1955Q2
ITA	102	0.0011	0.0222	0.0052	0.1466	-0.1362	1996Q2
JPN	110	0.0016	0.0136	0.0079	0.0498	-0.0839	1994Q2
USA	298	0.0076	0.0116	0.0069	0.0728	-0.0936	1947Q2

Table 1: Summary statistics of the GDP growth rates for G7 countries. The GDP growth rate is defined as $g_t := \log(\text{GDP}_t) - \log(\text{GDP}_{t-1})$. The time series are based on real GDP (i.e., adjusted for price changes) and also adjusted for seasonal influences.

their cross-sectional dependence. Section 2.2 describes the firm size distribution. Section 2.3 describes the distribution of sales growth rates.

2.1 GDP growth rate

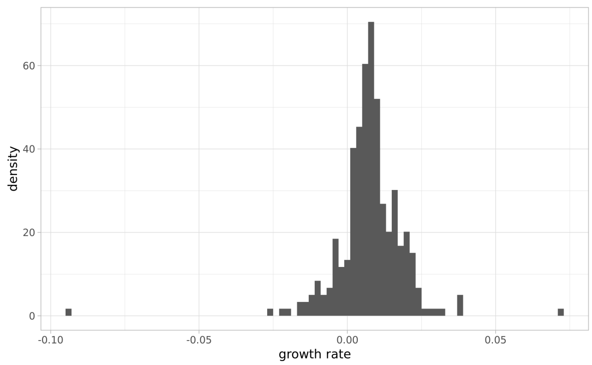
I analyze the quarterly growth rates of seasonally-adjusted real GDP for G7 countries. The data are taken from OECD database. Let g_t be the growth rate of GDP defined by the log difference of GDP over two successive periods, that is, $g_t := \log \text{GDP}_t - \log \text{GDP}_{t-1}$. The time series of g_t for G7 countries are depicted in **Figure 1**. Their summary statistics are given in **Table 1**.

Let us consider the distribution properties of the GDP growth rates. As an example, **Figure 2** shows that the histogram and QQ plot of the GDP growth rates for the US.[7] As indicated by both of these figures, one of the important features of the distribution of the GDP growth rates is the deviation from Gaussian especially in the tail region. That is, the probability that an economy experiences a large deviation is higher than predicted by a Gaussian distribution. For later purpose, we compare the counter cumulative distribution function (CCDF) of negative GDP growth rates with its Gaussian counterpart, in which the standard deviation is estimated by the robust consistent estimator under the Gaussian assumption. **Figure 3** shows that even when the consistent estimator is given, the Gaussian assumption underestimates the tail probability of the GDP growth rates. This suggests that to explain the important features of the GDP growth rates, it is necessary to discard the Gaussian assumption, and without the Gaussian assumption, the variance is not a good measure for the tail probability of a random variable.[8]

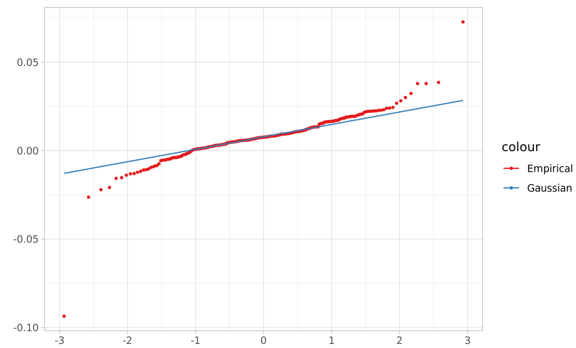
The above discussion shows that the Gaussian assumption is not adequate for the univariate case, that is, the distribution of the GDP growth rates for a single country. The same idea can be applied to the

[7] I confirm that similar deviation from Gaussian can be observed for other countries.

[8] For this point, see Acemoglu et al. (2017).



(a) Histogram



(b) QQ plot

Figure 2: Distribution of the GDP growth rates in the US. In Panel (b), the straight line represents a Gaussian distribution, that is, if the GDP growth rates follow a Gaussian distribution, sample points would lie on the straight line.

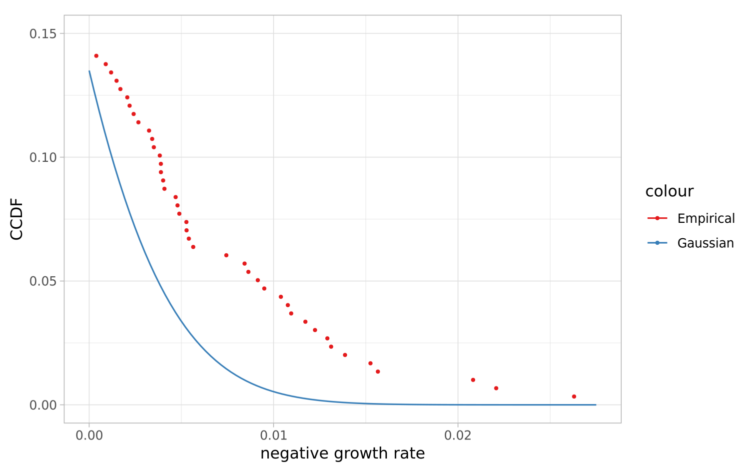
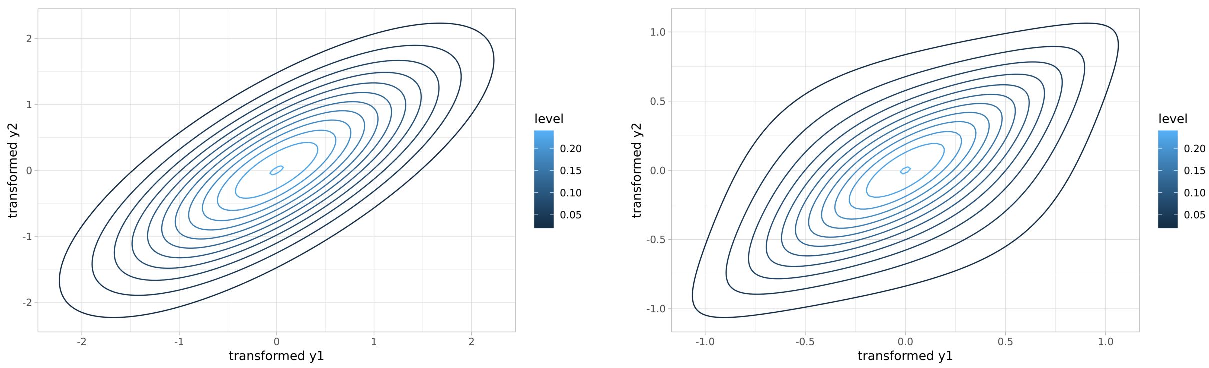


Figure 3: Tail probability. The counter cumulative distribution function of the negative GDP growth rates and its Gaussian counterpart are plotted. For the parameters of the Gaussian distribution, we use the median and median absolute deviation, respectively.



(a) Gaussian copula **(b)** t copula

Figure 4: Example of copulas with normal scores.

bivariate case, that is, the dependence structure of the GDP growth rates for two countries. To make this point rigorous, let us consider the copula C defined by the following identity:

$$F(y) = C(F_1(y_1), F_2(y_2)),\ y \in \mathbb{R}^2$$

where F is the bivariate distribution function of the GDP growth rates for two countries, and F_1, F_2 are its marginal (univariate) distributions. This equation decompose F into two parts: the copula function C representing the dependence structure and marginal distributions F_1 and F_2. Note that the copula function C assumes the quantiles as arguments, which are independent of the marginal distributions. It can be shown that C is uniquely determined given F, and recover the original bivariate distribution.

An important example of the copula is the Gaussian copula derived from the two-dimensional Gaussian distribution. The left panel of **Figure 4** depicts the Gaussian copula with normal scores, showing the elliptic shape.[9] Another important example of the copula is the t copula derived from the two-dimensional t distribution, which is depicted in the right panel of **Figure 4**. Compared to the Gaussian copula, one of the features of t copula is the diamond shape, that is, it is more likely that extremes of two variables occur simultaneously than predicted by the Gaussian copula. This example shows that even when the correlation coefficient is same as each other, the magnitude of the tail dependence can be different depending on the underlying copula.

[9]Normal scores mean the transformation of both axes by applying the quantile function of the standard Gaussian distribution to the marginal distributions of y_1 and y_2. More precisely, instead of y_1 and y_2, we consider the transformed samples defined as follows:

$$\left(\Phi^{-1}(F_1(y_1)), \Phi^{-1}(F_2(y_2))\right)$$

where Φ is the standard Gaussian distribution. In practice, the true marginal distributions are unknown, we use their empirical distributions. The rationale behind normal scores is that this transformation erases the differences coming from those of the marginal distributions and that the resultant scatter plot can highlight the differences of the dependence structure, that is, its copula function. In particular, since the scatter plot of the transformed samples would be a Gaussian distribution for the bivariate Gaussian case, the normal scores highlight departures from the Gaussian copula.

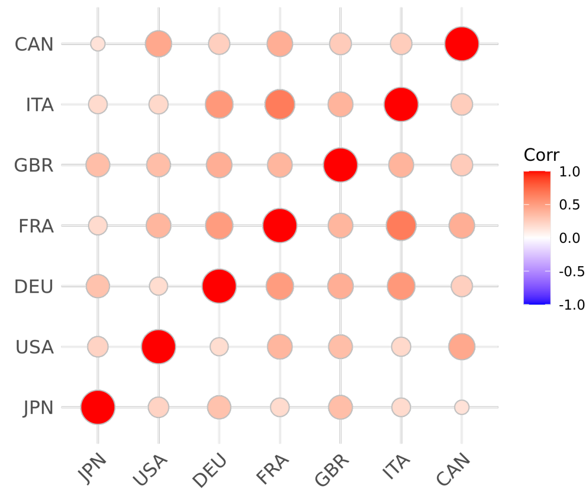


Figure 5: Correlation matrix of the GDP growth rates for G7 countries. For the correlation coefficient, Spearman's ρ is used.

To measure the tail dependence more concretely, I use the following coefficient of the tail dependence: for two random variables X_1 and X_2,

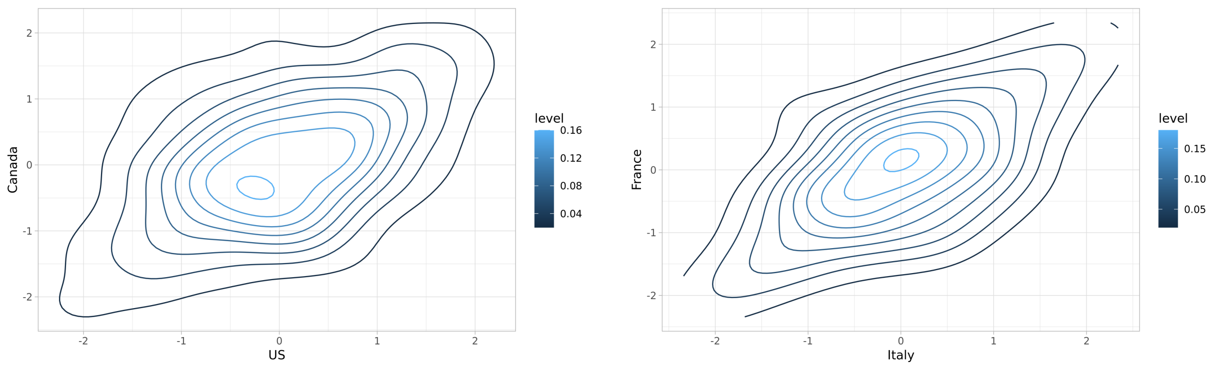
$$\lambda_l := \lim_{u \downarrow 0} \mathbb{P}(X_2 \leq F_2^{\leftarrow}(u) \mid X_1 \leq F_1^{\leftarrow}(u)) = \lim_{u \downarrow 0} C(u,u)/u$$

$$\lambda_u := \lim_{u \uparrow 1} \mathbb{P}(X_2 > F_2^{\leftarrow}(u) \mid X_1 > F_1^{\leftarrow}(u)) = \lim_{u \uparrow 1} (1 - C(u,u))/(1-u)$$

This measures the likelihood of the rare event of X_2 conditional on the rare event of X_1. One can show that for the Gaussian copula, λ_l and λ_u are equal to 0, regardless of the parameter ρ. That is, when the dependence structure of two random variables is described by the Gaussian copula, it is unlikely that both variables experience a large deviation simultaneously. This tail independence is an important feature of the Gaussian copula. In contrast, one can shows that when the dependence structure follows the t copula, $\lambda_l = \lambda_u > 0$. There is a positive probability that the rare events of both two variables occur simultaneously.

What does the empirical copular of the GDP growth rates for two countries look like ? As an example, let us consider the copula of the GDP growth rates for Italy and France, the US and Canada, which are the combination having higher Spearman's ρ (see **Figure 5**). **Figure 6** gives the density estimates with normal scores of the GDP growth rates. Both figures suggest that the empirical copula look more similar to the diamond shape of t copula and put more weights on the lower-left and up-right corners. To quantify the tail dependence, I parametrically estimate the student's t copula by the maximum likelihood method. I find that the estimate of $\lambda_l = \lambda_u$ is 0.344 for the US and Canada and 0.554 for Italy and France. This result suggests that the empirical dependence structure does not follow Gaussian copula, and extremes of both variables are more likely to occur simultaneously than expected from Gaussian copula.

To summarize, there are two deviations from Gaussian: the tail probability (for univariate case) is significantly larger than that of Gaussian, and the tail dependence is substantial, rather than 0 predicted



(a) the US and Canada

(b) Italy and France

Figure 6: Density estimate with normal scores.

country	count	mean	sd	q1	median	q3	max	min
CA	10000	194850.1	1303227.5	35000.00	35000.0	75000.0	45760100	21300
DE	10000	440924.3	2434467.0	79441.50	124351.0	254662.0	95415683	55480
FR	10000	302433.3	1563325.2	63500.75	93831.5	176983.0	70548393	47694
GB	10000	571784.2	3746157.1	89065.00	140267.5	300321.2	180816000	64500
IT	10000	190507.6	677621.9	50250.00	73535.0	134790.8	25202300	37913
JP	10000	702073.6	2316519.8	160650.00	243902.0	496106.2	106245756	118102
US	10000	2314779.1	11462766.3	262979.25	468784.0	1273778.8	559151000	174000

Table 2: Summary statistics of variables for individual firms. The unit of sales is a thousand USD.

by Gaussian copula. Thus, the empirical question to answer in this paper is to test whether the empirical granularity is large enough to explain these deviation from Gaussian. In Section 2.2, I review the empirical granularity for G7 countries.

2.2 Firm size distribution

To measure the empirical granularity for each G7 country, I analyze annual firm sales revenues in 2020 taken from Orbis, which is provided by Bureau van Dijk. I use unconsolidated firms as the definition of firm. I exclude firms in the sector of banking, insurance, and government. I focus on the 10,000 largest firms for each country. Their summary statistics are given in **Table 2**.

One of the stylized facts regarding firm size is its high heterogeneity across firms. In particular, it is known that the distribution of firm sizes has Pareto's tail with an exponent close to 1: there exists some large $x^* > 0$ such that

$$P(\text{sale} > x) \sim Cx^{-\alpha} \quad \text{for } x > x^*.$$

This empirical regularity is called Zipf's law. I confirm that Zipf's law holds in my samples. **Figure 7**

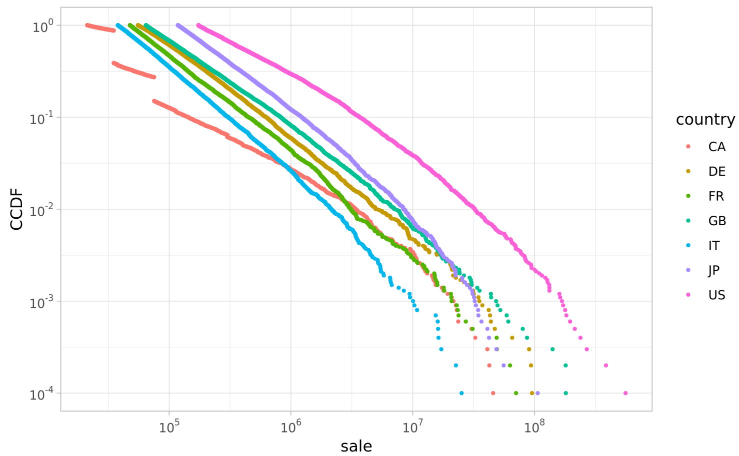


Figure 7: CCDF of sales. The CCDF is plotted in the log-log scale.

plots the counter cumulative distribution function (CCDF) of firm sizes for G7 countries in the log-log scale. Although the firm size itself tends to be large in larger economies, the decay rate of the probability is very similar to each other and the CCDF is close to a straight line. Thus, consistent with previous studies, the tail of the distribution of firm sizes can be approximated by Pareto's tail.

2.3 Firm growth rates

The third empirical stylized fact that I use in my analysis is the distribution of sales growth rates. As in the GDP growth rates, the growth rate for each firm is defined by the log difference of sales, that is, $g_t := \log(\text{sale}_t) - \log(\text{sale}_{t-1})$. The summary statistics of the sales growth rates are given in **Figure 3**.

Let us assume that for each country, firm i's sales growth rate is iid random variables drawn from a common distribution function. Under this assumption, the left panel of **Figure 8** shows the density estimate of the standardized growth rates. Interestingly, as long as location and scale parameters are adjusted, the density of growth rates for G7 countries lie on the same curve. In particular, as described in the right figure, this curve deviates from Gaussian but is well approximated by a Laplace distribution.

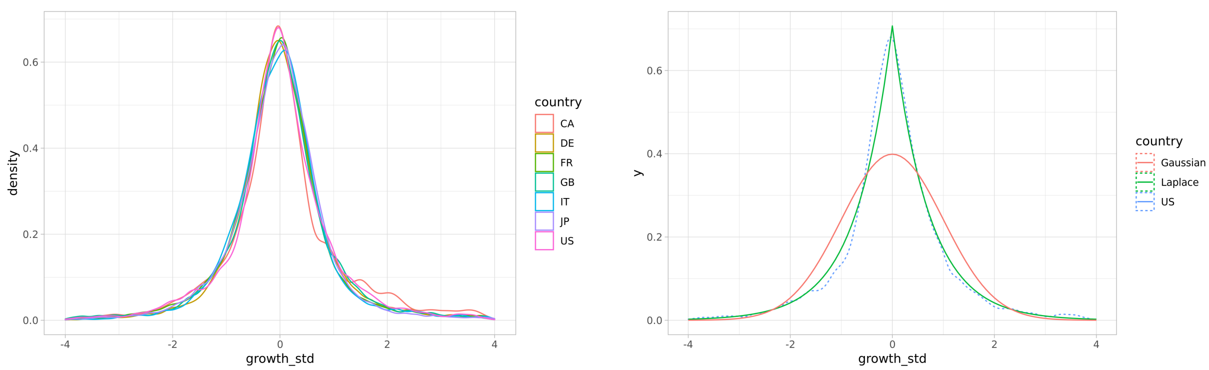
$$f(x; \mu, b) = \frac{1}{2b} \exp\left(-\frac{|x-\mu|}{b}\right)$$

where μ and b is the location and scale parameters. The variance of the Laplace distribution is given by $2b^2$.

Intuitively, since a Laplace distribution has a fatter tail than Gaussian, the probability that a firm experiences a rapid expansion is higher than that predicted by Gaussian. But, since a Laplace distribution has finite moment for all orders, the tail probability is not so heavy compared to power-law tails. Another important feature of the Laplace distribution is that the density sharply peaks around 0, meaning that the most of firms do not increase or decrease its size. In particular, the excess kurtosis, which is defined by the kurtosis minus 3, is equal to 3. Since the excess kurtosis is equal to 0 for Gaussian distribution, the excess

country	count	mean	sd	q1	median	q3	μ	σ
CA	1421	0.165	0.703	-0.090	0.068	0.265	0.053	0.201
DE	9634	0.089	0.573	-0.050	0.035	0.129	0.031	0.122
FR	9785	0.071	0.435	-0.053	0.041	0.131	0.038	0.127
GB	9199	0.069	0.649	-0.128	0.016	0.163	0.011	0.176
IT	9906	0.083	0.462	-0.070	0.048	0.160	0.042	0.146
JP	9945	-0.030	0.284	-0.119	-0.036	0.043	-0.036	0.111
US	4531	0.049	0.357	-0.068	0.032	0.145	0.030	0.146

Table 3: Summary statistics of the sales growth rates. The maximum likelihood estimates of the parameters of a Laplace distribution are also given.



(a) G7 countries **(b)** Comparison with Gaussian and Laplace distributions

Figure 8: The density estimates of the standardized growth rates. In panel (b), the density estimate for the US is plotted for comparison purpose.

kurtosis is one of the measures to distinguish the two distributions.

In this section, I provided three distribution properties: the deviation from Gaussian for the GDP growth rate (for uni- and bivariate cases), Zipf's law, and the Laplace distribution of the sales growth rates. The next section shows how these three properties are linked with each other.

3 Methods

I provide methods to analyze the distribution of aggregate output induced by microeconomic shocks. In Section 3.1, I provide the asymptotic behavior of the aggregate variance and its relation to the CLT condition. In Section 3.2, I review the saddlepoint method to approximate the distribution of aggregate output.

3.1 Variance and the CLT

Suppose that aggregate output Z can be represented as the weighted sum of microeconomic shocks $\epsilon_1, ..., \epsilon_n$ with weights $w_1, ..., w_n$:

$$Z := \sum_i w_i \epsilon_i \tag{1}$$

where n is the number of firms and w_i represents the impact of firm i's microeconomic shock on aggregate output. I assume that microeconomic shocks are independent and identically distributed with mean 0 and variance σ_ϵ^2. Under this assumption, the variance of Z is given by

$$\sigma_Z^2 = \sigma_\epsilon^2 \|w\|_2^2$$

where the ℓ^2-norm is defined as $\|w\|_2 := \sqrt{\sum_i w_i^2}$. For example, if an economy is homogeneous (i.e., $w_i = 1/n$ for all i), the variance of Z decays at the rate of n^{-1}.

Next, let us consider the CLT condition for Z. When the heterogeneity of the weights is low, the classical Lindeberg-Feller CLT characterizes the condition of the convergence to Gaussian:

Theorem 3.1 (Lindeberg-Feller CLT; see, e.g., Theorem 4.7 in Petrov (1995))**.** *Let $X_1, ..., X_n$ be independent random variables with mean 0 and finite variance σ_i^2. Let $s_n := \sum_i \sigma_i^2$ and $Z_n := s_n^{-1/2} \sum_i X_i$. If Lindeberg's condition, that is,*

$$s_n^{-1} \sum_i \mathbb{E}\left[X_i^2 \cdot 1_{\{|X_i| \geq \varepsilon s_n^{1/2}\}}\right] \to 0$$

holds, then Z_n converges in distribution to Gaussian.

Lindeberg's condition is related to the heterogeneity of the weights. In particular, Lindeberg's condition means that the maximum of the variance of components needs to be sufficiently small compared to the variance of their sum:

$$s_n^{-1} \max_{1 \leq i \leq n} \sigma_i^2 \to 0 \tag{2}$$

Furthermore, under the condition Eq.(2), one can show that the Lindeberg's condition is equivalent to the convergence of Z_n to Gaussian. For this reason, the classical Lindeberg-Feller CLT is for the case where no component dominates the whole.

Let us return to my case where $X_i = w_i \epsilon_i$ for all i. Thus, Eq.(2) is reduced to

$$\frac{\max_i w_i^2}{\sum_i w_i^2} = \frac{\|w\|_\infty^2}{\|w\|_2^2} (=: r_{\max}) \to 0 \tag{3}$$

where $\|w\|_\infty$ and $\|w\|_2$ are the ℓ^∞- and ℓ^2-norms of the weights, that is, $\|w\|_2^2 := \sum_i w_i^2$ and $\|w\|_\infty^2 := \max_i(w_i^2)$. Let denote the ratio in Eq.(3) by $r_{\max}$. One might ask whether the condition Eq.(3) holds when the heterogeneity of the weights is high. For this question, Proposition 3.5 in Arata (2021) characterizes the asymptotic behavior of $r_{\max}$ when the distribution of the weights has a Pareto tail.

Proposition 3.2 (Proposition 3.5 in Arata (2021))**.** *Suppose that the weights are independently drawn from a common distribution with Pareto's tail with an exponent $0 < \alpha < 2$. Then, $\lim_{n\to\infty} r_{\max}$ is a non-degenerate* *random variable and*

$$\lim_{n\to\infty} Er_{\max} > 0$$

This propositions means that under the high heterogeneity of firm sizes, the variance contribution from the largest firm is still dominant even at the limit of n. For this reason, one cannot use the classical CLT to analyze the distribution of aggregate output. In other words, it is necessary to consider non-classical setting where $r_{\max} \not\to 0$.[10]

There is another important perspective about how the condition Eq.(3) is related to the Gaussian distribution. Let X_1 and X_2 be independent random variables, and consider the distribution of the sum $X_1 + X_2$. It is well known that if X_1 and X_2 are Gaussian, the sum also follows Gaussian. Cramer's decomposition theorem states that its converse also holds true; that is, if a Gaussian random variable can be represented as the sum of two independent random variables, then the two random variables are Gaussian. In other words, for the sum of independent random variables to be Gaussian, each component comprising the sum must follow a Gaussian distribution. Thus, since the condition Eq.(3) means that a significant component of the sum Z does not follow Gaussian, the distribution of Z does not converge to Gaussian. This point is formalized by Theorem 1(c) in Acemoglu et al. (2012).

Proposition 3.3 (Theorem 1(c) in Acemoglu et al. (2012))**.** *Suppose that* $\epsilon_1, ..., \epsilon_N$ *are not Gaussian random variables and that* $\lim_{n\to\infty} r_{\max} \not\to 0$. *Then,* $\frac{1}{\sigma_\epsilon \|w\|_2} Z$ *does not converge to a Gaussian distribution.*

It should be noted, however, that the other condition that microeconomic shocks are not Gaussian random variables is necessary for the non-convergence to Gaussian. Otherwise, since the sum of independent Gaussian random variables are Gaussian, Z is Gaussian for all n. In light of this fact, one might think that if the distribution of microeconomic shocks is sufficiently close to Gaussian, the resultant distribution of Z is also close to Gaussian. For example, consider the two independent random variables X_1 and X_2, and suppose that X_1 is large and Gaussian and X_2 is tiny and not Gaussian. By Cramer's decomposition theorem, the sum $X_1 + X_2$ does not follow a Gaussian distribution. But, since X_1 dominates the sum and is Gaussian, one would expect that the distribution of the sum is mainly determined by X_1 and thus close to Gaussian. In general, the fact that the distribution does not converge to Gaussian does not necessarily mean that the distribution is far from Gaussian. As shown in the following results by Zolotarev (1997), the closeness to Gaussian depends on two factors: the granularity of the components and the closeness of the distribution of each component to Gaussian.

Theorem 3.4 (Theorem 2.3.2 in Zolotarev (1997))**.**

$$\rho(F_n, \Phi) \leq 4\mu_n^{1/4},\ n \geq 1$$

[10] In the filed of probability theory, the CLT under this condition is called non-classical CLT. See the monograph by Zolotarev (1997).

where

$$\mu_n^2 := \min_{\varepsilon>0} \max \left\{ \varepsilon^2, 2\sum_j \int_{|x|>\epsilon} |x||F_{nj}(x) - \Phi_{nj}(x)|dx \right\}$$

In Theorem 3.4, μ_n^2 can be small in two ways: When the granularity is low, that is, each component is arbitrarily small as $n \to \infty$, μ_n^2 becomes arbitrarily small. Even though the granularity is high, if the distribution of the component is close to Gaussian, μ_n^2 becomes arbitrarily small. Thus, the closeness of the distribution of Z is determined by these two factors.

Given these theoretical results above, the research question in this paper can be summarized as follows: given the empirical granularity and Laplace assumption about microeconomic shocks, how close to Gaussian is the distribution of Z? This is a non-asymptotic result with a fixed and finite n, in contrast to the asymptotic result given in Proposition 3.3. Although the calculating the values of μ_n is an option to get the upper bound of the deviation from Gaussian, the value of Levy's metric has no simple interpretation. In the following, I review the method called saddlepoint approximation, by which I approximate the distribution of Z with given weights.

3.2 Approximation of the distribution

The previous section considers the asymptotic properties of the distribution of Z as $n \to \infty$. Here, I review the saddlepoint approximation method to approximate the distribution of Z non-asymptotically, i.e. with weights fixed. This method is based on the one-to-one correspondence between the distribution and its corresponding cumulant generating function.

Let $K_X(t)$ denote the cumulant generating function of random variable X: for some $t \in \mathbb{R}$,

$$K_X(t) := \log(E \exp(tX))$$

The jth-order cumulant κ_j is defined as its jth-order derivative at 0. For example, if X follows a Gaussian distribution with 0 and σ^2, the corresponding cumulant generating function is $K_X(t) = \frac{1}{2}\sigma^2 t^2$, and $\kappa_1 = 0$, $\kappa_2 = \sigma^2$, $\kappa_j = 0$ for $j \geq 3$.

The cumulant generating function has the following useful property: If X_1 and X_2 are independent random variables, the cumulant generating function of the sum $X_1 + X_2$ satisfies

$$K_{X+Y}(t) = K_X(t) + K_Y(t)$$

This property is useful in our analysis because Z is the weighted sum of iid random variables.

To approximate the distribution of Z with given weights, it is necessary to specify the distribution of microeconomic shocks. In our analysis, I assume that microeconomic shocks follow a Laplace distribution:[11]

Assumption 3.1. Microeconomic shocks follows a Laplace distribution.

[11] In empirical literature on the firm growth dynamics, it is well known that the growth rate of firm size follows a Laplace distribution.

The cumulant generating function of the Laplace distribution is given by $K_X(t) = -\log(1 - b^2t^2)$ for t with $|t| < 1/b$. Thus, under Assumption 3.1, the cumulant generating function of Z is given by

$$K_Z(t) = -\sum_i \log(1 - w_i^2 b^2 t^2) \tag{4}$$

Since a distribution is uniquely determined by its cumulant generating function, the relation between the distribution of Z and weights can be clarified by analyzing Eq.(4). For example, if the heterogeneity of the weights is low (e.g., $w_i = 1/n$), one has $\log(1 - w_i^2 b^2 t^2) \approx w_i^2 b^2 t^2$. Thus, Eq.(4) is reduced to

$$K_Z(t) \approx \sum_i w_i^2 b^2 t^2 = \|w\|_2^2 b^2 t^2$$

Since the right-hand side is the cumulant generating function of Gaussian with variance $2b^2\|w\|_2^2$, this means that the distribution of Z is very close to the Gaussian, as expected by the CLT. In other words, by measuring the difference between the cumulant generating functions of Z and Gaussian, one can analyze how close (or far) to the Gaussian the distribution of Z is. This is the main idea of the saddlepoint approximation.

Formally, the saddlepoint approximation of the probability density is given as follows:[12]

Proposition 3.5 (see, e.g., Chapter 2 in Butler (2007))**.** *The approximation of the density is given by*

$$\widehat{f}(x) := \frac{1}{\sqrt{2\pi K''(\widehat{s})}} \exp\{K(\widehat{s}) - \widehat{s}x\}$$

where $\widehat{s} := \widehat{s}(x)$ is the unique solution to the following saddlepoint equation.

$$K'(\widehat{s}) = x$$

For example, if the distribution is Gaussian, the saddlepoint equation reduced to $\widehat{s} = x$. Thus, the saddlepoint approximation reproduces the exact probability density of the Gaussian distribution. It is known that as the distribution is closer to Gaussian, the saddlepoint approximation becomes more accurate.[13]

It is worth mentioning that the saddlepoint approximation is a non-asymptotic method with a fixed n; that is, weights are given empirically and fixed. This is in sharp contrast with the asymptotic method given in Section 3.1, in which the number of firms tends to ∞. The implications drawn from these two methods do not necessarily coincide with each other. That is, there is the possibility that the distribution of Z does not converge to Gaussian as $n \to \infty$ (i.e., asymptotic result), while the distribution of Z with given weights

[12] More precisely, the saddlepoint approximation is the consequence of Laplace's approximation.

[13] Similarly, the saddlepoint approximation of a distribution function is given as follows: The estimate is given by

$$\widehat{F}(x) := \begin{cases} \Phi(\widehat{w}) + \phi(\widehat{w})(1/\widehat{w} - 1/\widehat{u}) & \text{if } x \neq \mu \\ \frac{1}{2} + \frac{K'''(0)}{6\sqrt{2\pi}K''(0)^{3/2}} & \text{if } x = \mu \end{cases}$$

$$\widehat{w} := \text{sgn}(\widehat{s})\sqrt{2\{\widehat{s}x - K(\widehat{s})\}},\ \widehat{u} := \widehat{s}\sqrt{K''(\widehat{s})}$$

where $\widehat{s}$ is the unique solution to saddlepoint eq $K'(\widehat{s}) = x$, and ϕ and Φ are the standard normal density and distribution function, respectively. Theoretically, the saddlepoint approximation of a distribution function is just the integration of the saddlepoint approximation of the probability density function. The only difficulty is that through carrying out the integration, Temme's approximation is used.

is very close to Gaussian (i.e., non-asymptotic result). This case actually happens, as discussed in the next section.

4 Empirical Results

I approximate the distribution of Z by saddlepoint approximation. In Section 4.1, I show that the univariate distribution of Z is very close to the Gaussian under Assumption 3.1. In Section 4.2, I consider the bivariate case of the GDP growth rates, that is, the two-dimensional distribution of the GDP growth rates for two countries, and analyze its dependence structure.

4.1 One-dimensional distribution

Let us analyze the distribution of aggregate output for the univariate case, given the empirical granularity in an economy. First, consider the aggregate variance induced by microeconomic shocks and the CLT condition discussed in Section 3.1. **Table 4** provides the norms of the weights for G7 countries. For the US case, if the standard deviation of the growth rates for firms 14.6% at the annual basis, σ_Z at quarterly basis, which corresponds to **Table 1**, is given by

$$\sigma_Z = (1/2) \times 14.6\% \times 0.056 = 0.41\%$$

where $1/2$ is introduced due to the assumption that microeconomic shocks are iid. Comparison with **Table 1** suggests that microeconomic shocks are important source of the aggregate variance. That is, because of the granularity, microeconomic shocks do not die out at the aggregate level, consistent with the granular hypothesis.

Note that the non-negligible aggregate variance is mainly driven by shocks to the largest firm. **Table 4** shows the ratio of the two norms of the weights for G7 countries, e.g., $r_{\max} = 0.23$ for the US. That is, 23% of the micro-originated aggregate variance come from the contribution of the largest firm only. Similar to the US case, the high presence of the largest firm (i.e., $r_{\max}$ is significantly larger than 0) is observed for other G7 countries. As discussed in Section 3, owing to this high presence of the largest firm, the CLT does not hold, and the distribution does not converge to Gaussian. That is, in the sense of asymptotics, the distribution of aggregate output is different from Gaussian.

However, as emphasized in Section 3, the fact that the CLT does not hold does not necessarily imply that the distribution of aggregate output is far from a Gaussian. In the following, I show that the distribution of aggregate output with weights empirically given turns out to be very close to Gaussian. Before applying the saddlepoint approximation, consider how the cumulant generating function of Z changes through the summation. For example, if the single firm dominates an economy completely (i.e., $w_1 = 1$ and $w_i = 0$ for all $i \neq 1$), the cumulant generating function of Z is equivalent to that of the Laplace distribution. As the largest firm become less dominant in an economy, the averaging effect works more, and the distribution of Z

country	count	ℓ^2-norm	max	ratio
CA	10000	0.076	0.026	0.121
DE	10000	0.061	0.024	0.149
FR	10000	0.056	0.025	0.196
GB	10000	0.131	0.063	0.228
IT	10000	0.033	0.012	0.128
JP	10000	0.047	0.021	0.193
US	10000	0.056	0.027	0.229

Table 4: Norms of the weights for G7 countries.

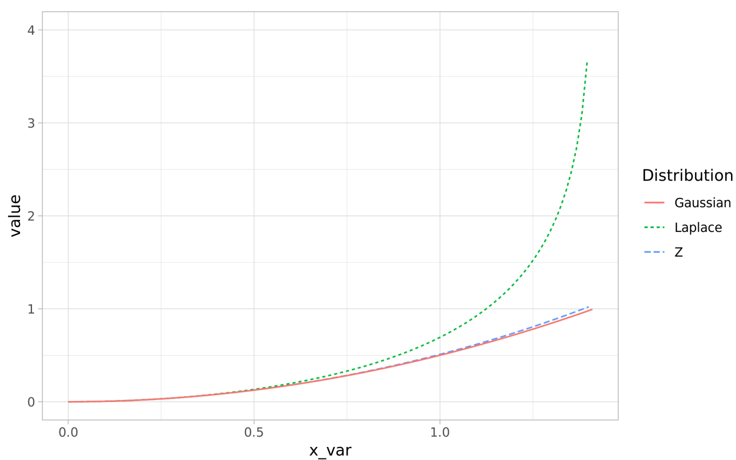


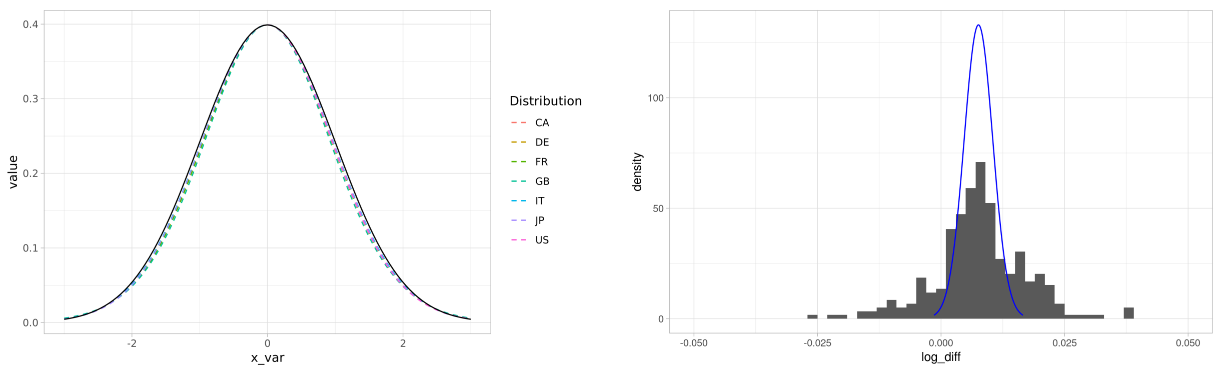
Figure 9: Comparison of the cumulant generating function. The cumulant generating functions of a Gaussian and Laplace distributions, and K_Z with the empirical weights are plotted.

becomes closer to Gaussian. To see whether the distribution of Z is close to Gaussian or Laplace, **Figure 9** compares the cumulant generating functions of Gaussian, Laplace, and Z for the US case. This figure shows that the cumulant generating function of Z is very close to that of a Gaussian. This closeness to Gaussian becomes clearer when the kurtosis is considered. Through the summation, the kurtosis of the distribution of the standardized Z is reduced to

$$\kappa_4(Z^*) = \kappa_j(\epsilon^*) \sum_i w_i^{*4} = 3 \times 0.05 = 0.15$$

where $w_i^* = w_i/\|w\|_2$ and $\epsilon_i^* = \epsilon_i/\sigma_\epsilon$. That is, the kurtosis, which is 3 when the single firm dominates the economy, is reduced to 0.15, which is very close to 0, i.e., that of the Gaussian distribution. This indicates that given the empirical granularity, the averaging effect works substantially, and thus, the corresponding distribution of Z is close to a Gaussian.

Given these backgrounds, I apply the saddlepoint approximation. The left panel of **Figure 10** shows the saddlepoint approximations of the density function of the normalized Z (i.e., Z^*) for G7 countries. In the



(a) For Z^* **(b)** For Z

Figure 10: Saddlepoint approximation of probability density function. In Panel (b), I also plot the histogram of the GDP growth rates for comparison.

same figure, I plot the standard Gaussian distribution for comparison. This figure shows that the all density approximations for G7 countries lie on almost the same curve and are very close to that of Gaussian. That is, although there are some differences in the weights and their norms between G7 countries, there is no differences when it comes to the distribution shape; that is, the differences in the weights and their norms only affect the variance of Z. Put differently, the empirical granularity for G7 countries are low enough to let the averaging effect work substantially, resulting to the distribution of aggregate output close enough to Gaussian.

The right panel of **Figure 10** shows the approximation of the density of Z at the original scale and compares it with the histogram of the GDP growth rates. This figure shows why the micro-originated aggregate variance is economically significant but the contribution to the tail probability of $Z$is negligible. That is, microeconomic shocks generate relatively small fluctuations of Z around its mean and generate non-negligible aggregate variance. In contrast, as seen in Section 2.1, the distribution of the GDP growth rates deviates from Gaussian, and thus, the difference between the empirical and approximated density is large in the tail region. This suggests that microeconomic shocks would explain only small fluctuations and not a large deviation in the GDP growth rate.

Finally, I approximate the distribution tail of Z. **Figure 11** shows that the saddlepoint approximation of the tail probability of Z. Consistent with the approximated probability density, the approximated tail probability decays rapidly (i.e., the Gaussian decay). This rapid decays means that microeconomic shocks with the empirical granularity cannot explain the observed tail probability of the GDP growth rates. As mentioned above, the point is the closeness to the Gaussian distribution: although the CLT does not hold and the distribution is rigorously not Gaussian, the averaging effect makes the distribution of Z close enough to Gaussian. Thus, I conclude that the empirical granularity is not large enough to explain the characteristic features of the GDP growth rates.

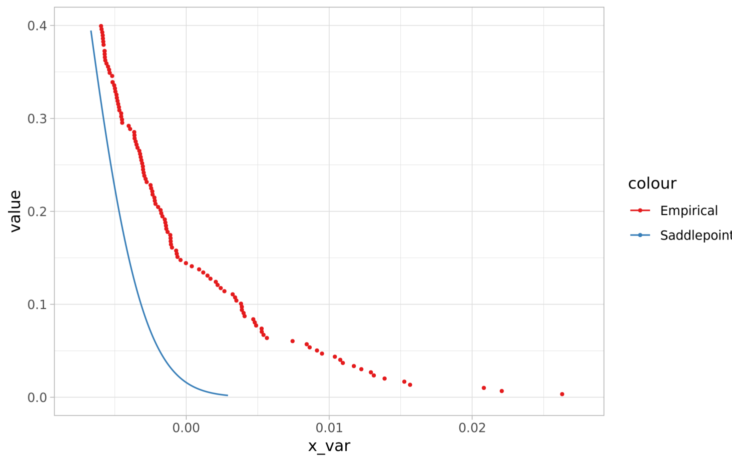


Figure 11: Saddlepoint approximation of the distribution function of Z. In Figure, $1 - F(x)$ is plotted. I also plot the CCDF of the GDP growth rates for comparison.

4.2 Two-dimensional distribution

Suppose that there are two countries denoted by A and B and that each firm in country A (B) exports its product to country B (A) by the fraction α_i (α_j) in its sales. Assume that when a microeconomic shock ϵ_i hits firm i with sale s_i in A, its impact on aggregate output is proportional to $(1-\alpha_i)s_i\epsilon_i$ for country A and to $\alpha_i s_i \epsilon_i$ for country B. This model can be interpreted that a firm with size $(1-\alpha_i)s_i$ in country A has its *branch* in country B with size $\alpha_i s_i$, and they share the common microeconomic shock ϵ_i. In this model, the micro-level dependence between a firm and its branch, that is, an export/import relation, leads to the dependence of aggregate outputs between the two countries. Thus, the relation between microeconomic shocks and aggregate output is given by

$$\begin{pmatrix} Z_A \\ Z_B \end{pmatrix} = \begin{pmatrix} \frac{1}{\mathrm{GDP}_A}\left(\sum_{i \in A} s_i(1-\alpha_i)\epsilon_i + \sum_{j \in B} s_j \alpha_j \epsilon_j\right) \\ \frac{1}{\mathrm{GDP}_B}\left(\sum_{i \in A} s_i \alpha_i \epsilon_i + \sum_{j \in B} s_j (1-\alpha_j)\epsilon_j\right) \end{pmatrix}$$

This equation corresponds to Eq.(1) in Section 3.

For simplicity, assume that $\alpha_i = \alpha_A$ for all $i \in A$. Thus, the variances of Z_A and Z_B are given by

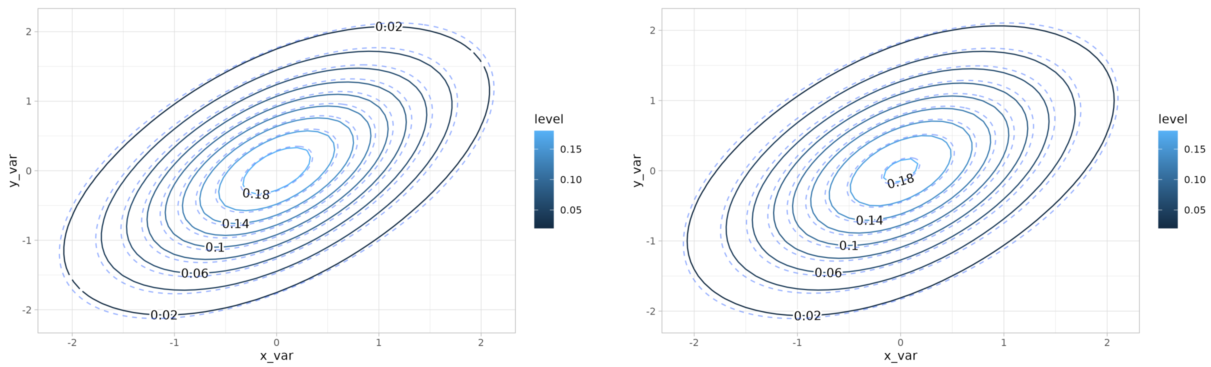
$$\sigma_A^2 = \frac{\sigma_\epsilon^2}{\mathrm{GDP}_A^2}\left((1-\alpha_A)^2 \|s\|_{2,A}^2 + \alpha_B^2 \|s\|_{2,B}^2\right)$$

$$\sigma_B^2 = \frac{\sigma_\epsilon^2}{\mathrm{GDP}_B^2}\left(\alpha_A^2 \|s\|_{2,A}^2 + (1-\alpha_B)^2 \|s\|_{2,B}^2\right)$$

and its covariance is

$$\sigma_{AB} = \frac{\sigma_\epsilon^2}{\mathrm{GDP}_A \mathrm{GDP}_B}\left(\|s\|_{2,A}^2 \alpha_A(1-\alpha_A) + \|s\|_{2,B}^2 \alpha_B (1-\alpha_B)\right)$$

For example, if $\alpha_A = \alpha_B = \alpha$ and $\|s\|_{2,A} = \|s\|_{2,B}$ (i.e., the two countries are symmetry), the correlation



(a) US and Canada

(b) France and Italy

Figure 12: Approximated copula with normal scores. The dotted line represents the Gaussian copula with correlation coefficient equal to the one used in the saddlepoint approximation.

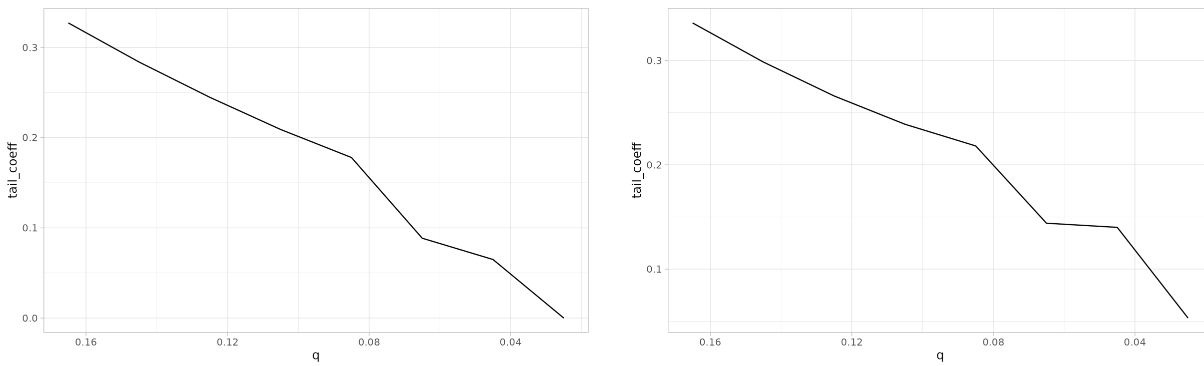
coefficient ρ is given by

$$\rho = \frac{2\alpha(1-\alpha)}{(1-\alpha)^2 + \alpha^2} \tag{5}$$

Note that ρ is uniquely determined by the export ratio α and does not depend on $\|s\|_2$. That is, ρ is independent of the granularity of firm size. However, as discussed in Section 2.1, the dependence structure (i.e., the copula function) is not determined by the correlation coefficient. In the following, I analyze how the degree of the granularity of two economies affects the dependence of aggregate outputs for two countries by using the saddlepoint approximation.

Given the firm-level data (i.e., sales for each country), I approximate the bivariate distribution of aggregate output under Assumption 3.1 and compute the resultant copula function that describe their dependence structure. **Figure 12** shows the approximated copula with normal scores for the US and Canada and for France and Italy, which are theoretical counterparts of **Figure 6** in Section 2.1. In **Figure 12**, I plot the Gaussian copula with the same correlation coefficient as empirical one for comparison. Both figures show that the approximated bivariate copula shows the elliptic shape and is very close to Gaussian copula. The logic behind this result is the same as in the univariate case in Section 4.1. That is, even when the CLT does not hold, the averaging effect does not cease to work. Through the summation of microeconomic shocks, the bivariate distribution of aggregate outputs becomes close to Gaussian, and therefore, the copula function becomes close to Gaussian copula as well. Similar to the univariate case, when it comes to non-asymptotic properties, the empirical granularity is not large enough to compensate the averaging effect of microeconomic shocks.

As discussed in Section 2.1, the closeness to Gaussian copula has an important implication for the tail dependence. Given the empirical weights, the copula of Z is sufficiently close the Gaussian copula and thus has no tail dependence, while empirical copula has significant tail dependence as shown by **Figure 6**. Indeed, the coefficient of the tail dependence numerically calculated from **Figure 12** is given in **Figure 13**.



(a) The US and Canada

(b) France and Italy

Figure 13: The coefficient of the tail dependence.

As expected, the coefficient decreases to 0 as x becomes large, that is, no tail dependence. Note that no tail dependence does not contradict the positive correlation coefficient; in fact, the positive export ratio generate the positive correlation of the approximated bivariate distribution. My finding shows that microeconomic shocks with the empirical granularity of the weights generate the positive correlation but no tail dependence because the averaging effect makes the copula close enough to the Gaussian copula.

To summarize, my analysis shows that the empirical granularity is not large enough to cease the averaging effect. Even when the CLT does not hold, most of microeconomic shocks are cancelled each other out, and thus, the distribution of Z both in one- or two-dimensional settings turns out to be close to Gaussian. The closeness to Gaussian immediately implies that the tail probability and tail dependence induced by microeconomic shocks would be very small. On the other hand, the empirical GDP growth rates deviate from the Gaussian distribution in the tail region. Therefore, given the empirical granularity observed in data, the granular hypothesis cannot explain the important features of the empirical aggregate fluctuations.

5 Conclusion

What drives aggregate fluctuations? Regarding this fundamental question, the recent literature proposes the granular view: because of the high heterogeneity of firm sizes, microeconomic shocks to giant firms in an economy do not die out but contribute substantially to aggregate fluctuations. Although this idea has been widely accepted in the literature, the size of the micro-originated aggregate fluctuations consistent with empirical granularity has not fully investigated. This paper aims to measure the micro-originated aggregate fluctuations using firm-level data in G7 countries.

The main contribution of this paper is to focus on the non-asymptotic properties of the distribution of aggregate output rather than the asymptotic properties. My analysis shows that the difference between these two properties is crucial. On the one hand, I find that because of the high granularity observed in data, the CLT does not hold, that is, the distribution of aggregate output does not converge to Gaussian as $n \to \infty$.

This means that the largest firm accounts for a significant part of the economy even at the limit, and thus, aggregate fluctuations are affected by microeconomic shocks to the largest firm. On the other hand, given the empirical granularity (i.e., fixed weights), I also find that the distribution of aggregate output induced by microeconomic shocks is very close to Gaussian. This result does not contradict the asymptotic result that the CLT does not hold because the closeness to Gaussian is non-asymptotic property of aggregate output. Even when the CLT does not hold, the averaging effect does not cease work, making the distribution of aggregate output closer to Gaussian. In other words, the empirical granularity is not large enough to compensate this averaging effect. Since the distribution of the GDP growth rate deviates from Gaussian (both in one- and two-dimension case), microeconomic shocks cannot explain the distribution properties of the GDP growth rates in the tail region.

Finally, it is worth mentioning the limitations of my analysis. Since my analysis is based on Hulten's theorem, other mechanisms that are not considered in Hulten's theorem are also beyond the scope of my analysis. For example, when an extensive margin such as entry/exit is introduced, microeconomic shocks can be amplified and contribute to aggregate fluctuations even more. My negative result about the granular idea should be viewed that such another amplification mechanism is necessary to explain the observed large deviation in aggregate output. Indeed, recent studies (e.g., Baqaee (2018)) theorize such extensive margins and analyze the impact of microeconomic shocks on aggregate output. I believe that my analysis contributes to this recent literature by showing the limitation of Hulten's theorem with intensive margins only.

6 Appendix

Section 6.1 provides the robustness of my findings in Section 4 by considering the distribution of microeconomic shocks with a fatter tail than that of a Laplace distribution. Section 6.2 describes data on the exporting share in sales for French firms.

6.1 Robustness

To check the robustness of my results in Section 4, I consider the distribution of microeconomic shocks with a heavier tail than an exponential tail considered in Assumption 3.1. Because of this heavier tail, the corresponding moment generating function does not exist. Since the saddlepoint approximation relies on the moment generating function to recover the true distribution, it cannot be applied to this case. However, by using the property of the heavier tail, I can estimate the tail probability of Z with given weights.

The class of distributions having a heavier tail than an exponential tail is called subexponential distributions.[14] This class includes many examples encountered in applications, such as lognormal, Pareto,

[14] For details about the subexponential distribution, see the monograph by Foss et al. (2011). See also Arata (2021), in which I use the property of the subexponential distributions in the analysis of aggregate fluctuations.

and Weibull distributions. One of the most important properties of this distribution class is *the principle of a single jump*. More precisely, let $X_1, ..., X_n$ be independent random variables drawn from a common subexponential distribution F. Then, one can show that the tail probability of the sum of $X_1, ..., X_n$ is asymptotically equivalent to the sum of the tail probability of $X_1, ..., X_n$:

$$P(\sum_i X_i < -x) \sim \sum_i P(X_i < -x)$$

Intuitively, the above equation means that the large deviation of the sum is driven by the large deviation of a single component X_i. In other words, the probability of the large deviation of the sum caused by the combination of many components with middle size can be ignored in the asymptotics.

This property can be extended to the case of weighted sum as in Z and enables us to calculate the tail probability of Z in a simple way. That is, I approximate the tail probability of Z by the following equation:

$$P(Z < -x) \sim \sum_i P(w_i \epsilon_i < -x) \tag{6}$$

For an empirical exercise, I consider the distributions with a Weibull tail and Pareto tail as that of microeconomic shocks in the following.

First, consider the two-sided Weibull distribution defined as follows:

$$P(\epsilon_i \leq -x) = \frac{1}{2} \exp\left(-\left(\frac{x}{b}\right)^\tau\right), \; b > 0, \; 0 < \tau < 1$$

where β is the scale parameter and τ is the shape parameter that controls the heaviness of the distribution tail. Note that the tail of the distribution becomes arbitrary close to exponential as $\tau \to 1$, that is, the Laplace distribution can be seen as the limit of the distributions as $\tau \to 1$. The variance of this distribution is given by

$$\sigma_\epsilon^2 = 2b^2 \left[\Gamma\left(1 + \frac{2}{\tau}\right) - \left(\Gamma\left(1 + \frac{1}{\tau}\right)\right)^2\right]$$

where Γ is Gamma function. In my exercise, I fix the scale parameter b to the estimated one used in Section 4 and change the shape parameter τ from 0.9 to 0.6. Then, I analyze how the tail probability of Z changes according to the shape parameter τ.

As the second example of subexponential distributions, I consider a distribution with Pareto tail. More precisely, suppose that there exists some constant x^* such that for $x \geq x^*$,

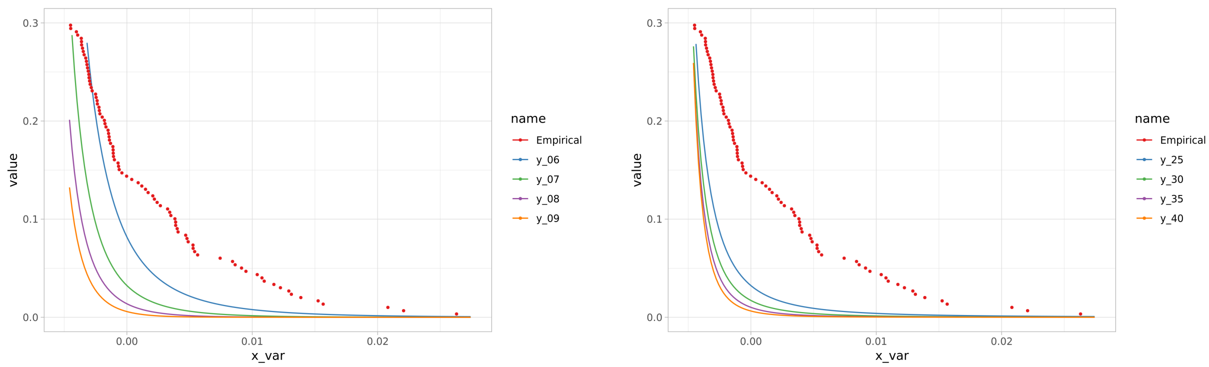
$$P(\epsilon_i \leq -x) = Kx^{-\tau}, \; \tau > 0$$

where K and τ are some positive constants. In my exercise, I set x^* equal to the first quantile x_{q_1}, that is,

$$P(\epsilon_i \leq -x) = \frac{1}{4}\left(\frac{x_{q_1}}{x}\right)^\tau, \; \tau > 0$$

In particular, I consider the shape parameter τ ranging from 2.5 to 4.0 so that the growth rates have finite variance. Then, I approximate the tail probability of Z using Eq.(6).[15]

[15] Since the quantile of the sales growth rates at the quarterly basis is not available, I use the property of the subexponential distribution again. That is, if the annual growth rate consists of iid quarterly growth rates, the tail probability of annual growth rates is



(a) Two-sided Weibull tail

(b) Pareto tail

Figure 14: Tail probability of Z. For comparison, the CCDF of the GDP growth rates for the US is plotted.

The tail probability under these two distributions for microeconomic shocks is given in **Figure 14**. For comparison, the CCDF of the empirical GDP growth rates for the US is also plotted. Both figures shows that as the tail of the distributions of microeconomic shocks becomes heavier, the resultant tail probability of Z become larger. However, in both cases, the tail probability of Z still underestimates the empirical counterpart. Note that as Eq.(6) suggests, only the tail probability for large firms contribute to the tail probability of Z. Thus, this result shows that the sizes of such large firms are not large enough to generate the large deviation of aggregate output. Consistent with the finding in Section 4, through the summation of microeconomic shocks, the tail probability of Z decays much faster than the aggregate variance, reflecting the power of the averaging effect.

6.2 Exporting ratio in sales

In my data, exporting shares in sales are not available for all countries but are available for about half of firms in France. I use the exporting share for these French firms to infer parameter α in Section 4.2. **Table 5** give their summary statistics, in which I exclude firms which report the exporting share of a negative value. **Figure 15** depicts the average of the exporting shares according to its firm size. Consistent with the literature on international economics, this figure shows that the exporting share is positively associated with the firm size, that is, larger firms exports their products more. Since large firms play a crucial role in the analysis of the distribution of aggregate output, I focus on exporting share for large firms only. More concretely, I focus on firms with sale larger than 100 million dollar and use their sample average of exporting shares, 0.168, as parameter α.

asymptotically equivalent to the sum of the tail probability of the quarterly growth rates.

$$P(\epsilon_{i,a} < -x) \sim 4P(\epsilon_{i,q} < -x)$$

where $\epsilon_{i,a}$ and $\epsilon_{i,a}$ are the annual and quarterly growth rates, respectively. Since the sample quantile of the annual growth rates is available from my date, I can calculate the tail probability of the quarterly growth rates.

count	mean	sd	q1	median	q3	max	min
349726	4.233371	16.47377	0	0	0	100	0

Table 5: Summary statistics of exporting shares in sales for firms in France. Firms which report a negative value of exporting share are excluded.

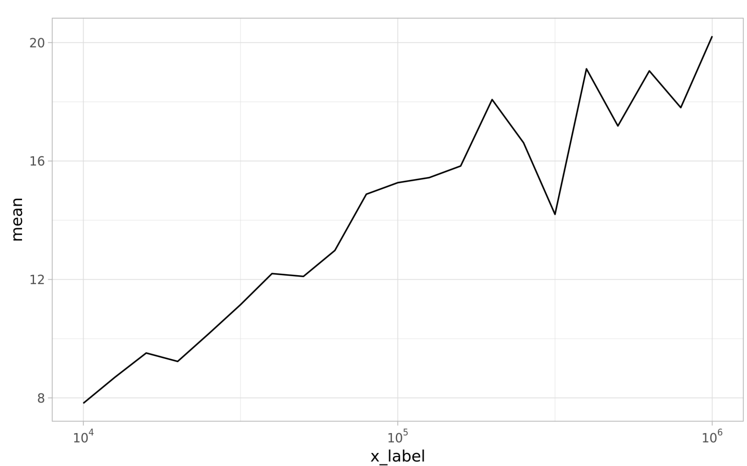


Figure 15: Sample average of exporting shares according to firm sizes.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A. E., and Tahbaz-Salehi, A. (2012). The Network Origins of Aggregate Fluctuations. *Econometrica*, 80(5):1977–2016.

Acemoglu, D., Ozdaglar, A., and Tahbaz-Salehi, A. (2017). Microeconomic Origins of Macroeconomic Tail Risks. *American Economic Review*, 107(1):54–108.

Adrian, T., Boyarchenko, N., and Giannone, D. (2019). Vulnerable growth. *American Economic Review*, 109(4):1263–89.

Arata, Y. (2019). Firm growth and laplace distribution: The importance of large jumps. *Journal of Economic Dynamics and Control*, 103:63–82.

Arata, Y. (2021). The Role of Granularity in the Variance and Tail Probability of Aggregate Output. *RIETI Discussion Paper*.

Arata, Y. and Miyakawa, D. (2022). The Size of Micro-originated Aggregate Fluctuations: An analysis of firm-level input-output linkages in Japan. *RIETI Discussion Paper*.

Axtell, R. L. (2001). Zipf distribution of us firm sizes. *science*, 293(5536):1818–1820.

Baqaee, D. R. (2018). Cascading Failures in Production Networks. *Econometrica*, 86(5):1819–1838.

Bottazzi, G. and Secchi, A. (2006). Explaining the distribution of firm growth rates. *The RAND Journal of Economics*, 37(2):235–256.

Butler, R. W. (2007). *Saddlepoint approximations with applications*, volume 22. Cambridge University Press.

Carvalho, V. M. (2014). From Micro to Macro via Production. *Journal of Economic Perspectives*, 28(4):23–47.

Carvalho, V. M. and Gabaix, X. (2013). The Great Diversification and its Undoing. *American Economic Review*, 103(5):1697–1727.

Carvalho, V. M. and Tahbaz-Salehi, A. (2019). Production Networks: A Primer. *Annual Review of Economics*, 11:635–663.

Clark, T. E. and Ravazzolo, F. (2015). Macroeconomic forecasting performance under alternative specifications of time-varying volatility. *Journal of Applied Econometrics*, 30(4):551–575.

Coad, A. (2009). *The growth of firms: A survey of theories and empirical evidence*. Edward Elgar Publishing.

Cúrdia, V., Del Negro, M., and Greenwald, D. L. (2014). Rare shocks, great recessions. *Journal of Applied Econometrics*, 29(7):1031–1052.

Di Giovanni, J. and Levchenko, A. A. (2012). Country size, international trade, and aggregate fluctuations in granular economies. *Journal of Political Economy*, 120(6):1083–1132.

Di Giovanni, J., Levchenko, A. A., and Mejean, I. (2014). Firms, Destinations, and Aggregate Fluctuations. *Econometrica*, 82(4):1303–1340.

Di Giovanni, J., Levchenko, A. A., and Mejean, I. (2018). The Micro Origins of International Business-Cycle Comovement. *American Economic Review*, 108(1):82–108.

Dosi, G., Pereira, M. C., and Virgillito, M. E. (2017). The footprint of evolutionary processes of learning and selection upon the statistical properties of industrial dynamics. *Industrial and Corporate Change*, 26(2):187–210.

Fagiolo, G., Napoletano, M., and Roventini, A. (2008). Are output growth-rate distributions fat-tailed? some evidence from oecd countries. *Journal of Applied Econometrics*, 23(5):639–669.

Foss, S., Korshunov, D., and Zachary, S. (2011). *An Introduction to Heavy-Tailed and Subexponential Distributions*.

Gabaix, X. (2009). Power laws in economics and finance. *Annu. Rev. Econ.*, 1(1):255–294.

Gabaix, X. (2011). The Granular Origins of Aggregate Fluctuations. *Econometrica*, 79(3):733–772.

Kolassa, J. E. (2006). *Series approximation methods in statistics*, volume 88. Springer Science & Business Media.

Magerman, G., Bruyne, K. D., Dhyne, E., and Hove, J. V. (2017). Heterogeneous Firms and the Micro Origins of Aggregate Fluctuations. *Working Paper*.

Miranda-Pinto, J. (2021). Production network structure, service share, and aggregate volatility. *Review of Economic Dynamics*, 39:146–173.

Petrov, V. V. (1995). *Limit Theorems of Probability Theory: Sequences of Independent Random Variables*. Clarendon Press.

Stella, A. (2015). Firm Dynamics and the Origins of Aggregate Fluctuations. *Journal of Economic Dynamics and Control*, 55:71–88.

Zolotarev, V. M. (1997). *Modern theory of summation of random variables*. de Gruyter.