Online Appendix of
"Misallocation under the Shadow of Death"

Daisuke Miyakawa Koki Oikawa Kozo Ueda

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A Model Details

A.1 Proofs

Proof of Proposition 1 Fix s at $t=0$. Consider two arbitrary R&D strategy sequences $\{\chi_{hdt}\}_{h=0}^{\infty}$ and $\{\chi'_{hdt}\}_{h=0}^{\infty}$, where $\chi_0=1$, $\chi'_0=0$, and $\chi_{hdt}=\chi'_{hdt}$ for $h=1,2,...$. Let ω be a sample path for $t\in\{dt,2dt,...\}$ having events of non-R&D, success in R&D, or failure in R&D in each timing.

Define

$$A \equiv \mathbb{E}\left[\sum_{h=1}^{H} e^{-rhdt}\frac{s_{hdt}}{s_{dt}}\frac{1}{\sigma}\right], \qquad A' \equiv \mathbb{E}\left[\sum_{h=1}^{H'} e^{-rhdt}\frac{s'_{hdt}}{s'_{dt}}\frac{1}{\sigma}\right],$$

where the expectations are taken over sequences of s and the exit timings $T\equiv Hdt$ and $T'\equiv H'dt$. Note that $\frac{s_{hdt}}{s_{dt}}$ and $\frac{s'_{hdt}}{s'_{dt}}$ are identical for any h if they share a common path of R&D strategies and realizations of R&D success and failure except at $h=0$.

$\chi_0=1$ is desirable for a given sequence of $\{\chi_t\}_{t\geq dt}$ if

$$\begin{aligned}
0 \leq& A s_{dt} - A' s'_{dt} - \kappa_r w_0 dt\\
=& A\left((1+\gamma_\sigma-\theta dt)s\lambda dt + (1-\theta dt)s(1-\lambda dt)\right) - A'(1-\theta dt)s - \kappa_r w_0 dt\\
=& A\lambda\gamma_\sigma s dt + (A-A')(1-\theta dt)s - \kappa_r w_0 dt,
\end{aligned}$$

implying that

$$A\lambda\gamma_\sigma s - \kappa_r w_0 + \frac{(A-A')(1-\theta dt)s}{dt} \geq 0.$$

The third term in the left-hand side of the above inequality depends on the marginal increase in A with respect to dt, which is represented by

$$e^{-rT'}\frac{s'_{T'}}{s_{dt}}\frac{1}{\sigma} \times \text{marginal survival time},$$

for small dt. The marginal survival time comes from: (i) the initial gap, $s_{dt}-s'_{dt}$, which provides additional survival time of $s\gamma_\sigma\lambda dt/\theta$ in expectation; and (ii) productivity jumps during the additional survival time, which is negligible as $dt\to 0$. Hence,

$$\lim_{dt\to 0}\frac{(A-A')(1-\theta dt)s}{dt} = s\lim_{dt\to 0}\frac{d(A-A')}{d(dt)} = 0,$$

implying that for an arbitrary $\{\chi_t\}_{t>0}$, $\chi_0 = 1$ is the best choice if $A\lambda\gamma_\sigma s \geq \kappa_r w_0$, where A depends on $\{\chi_t\}_{t>0}$. Thus, there exists a threshold of s, conditional on $\{\chi_t\}_{t>0}$, above which R&D is done. The optimal future sequence of R&D strategy is a distribution over feasible $\{\chi_t\}_{t>0}$. The optimal threshold $\hat{s}$ is determined by the expected value of A with the same distribution.

$\hat{s}$ is determined as follows. Because R&D is an endogenous option, we have the smooth-pasting condition in the value function, namely, v_s is continuous at $\hat{s}$. Equations (11) and (12) imply that R&D investment is done when a firm has s satisfying

$$v_s(s, \theta_{it}, w_t)\, \mathbb{E}_t\left[\dot{s}|_{\chi=1}\right] - v_s(s, \theta_{it}, w_t)\, \mathbb{E}_t\left[\dot{s}|_{\chi=0}\right] \geq \kappa_r w_t$$
$$\Leftrightarrow \quad v_s(s, \theta_{it}, w_t)\, s\lambda\gamma_\sigma \geq \kappa_r w_t.$$

And equality holds at $s = \hat{s}$.

Proof of Proposition 2 First, define the firm value, $v^N(s, \theta, w)$, for the firms that commit not to do R&D in the current and future periods. It satisfies $v^N(s, \theta, w) = 0$ for $s \leq \bar{s}$ and

$$v^N(s, \theta, w) = \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}_i}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \kappa_o w\right) dt$$
$$= \frac{s}{\sigma(r+\theta)} - \frac{\kappa_o w}{r} + \underbrace{\left(\frac{\kappa_o w}{r} - \frac{\bar{s}}{\sigma(r+\theta)}\right)\bar{s}^{\frac{r}{\theta}} s^{-\frac{r}{\theta}}}_{\text{value from the exit option (positive)}}, \qquad \text{for } s \geq \bar{s}. \tag{A.1}$$

The optimal $\bar{s}_i$ should satisfy $v^N_s(s, \theta, w) = 0$ from the smooth-pasting condition, which suggests

$$\frac{\bar{s}}{\sigma} - \kappa_o w = 0 \quad \Rightarrow \quad \bar{s} = \sigma\kappa_o w \qquad \forall i. \tag{A.2}$$

For $\hat{s}$, we impose the smooth-pasting condition at $\hat{s}$. Using equations (11) and (A.1), we should have

$$\hat{s}v^N_s(\hat{s}, \theta, w) = \frac{\hat{s}}{\sigma(r+\theta)} - \frac{r}{\theta}\left(\frac{\kappa_o w}{r} - \frac{\bar{s}}{\sigma(r+\theta)}\right)\bar{s}^{\frac{r}{\theta}}\hat{s}^{-\frac{r}{\theta}} = \frac{\kappa_r w}{\lambda\gamma_\sigma}. \tag{A.3}$$
$$\Leftrightarrow \quad \frac{1}{r+\theta}\left(\frac{\hat{s}}{\bar{s}} - \left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}}\right) = \frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma}. \tag{A.4}$$

The left-hand side of equation (A.4) is strictly increasing and strictly concave in $\hat{s}/\bar{s}$ for any given $\theta < \infty$. Moreover, it takes zero at $\hat{s}/\bar{s} = 1$ and goes to infinity as $\hat{s}/\bar{s} \to \infty$, we have a unique ratio of $\hat{s}/\bar{s}$. Combining with equation (A.2), we have unique $\hat{s}$ above which firms invest in R&D. In addition, the left-hand side of the equation is decreasing in θ, implying that $\hat{s}$ increases with θ.

Proof of Proposition 3 In the social planner's problem, the ratio of $\hat{s}^*$ to $\bar{s}^*$ is

$$\frac{\hat{s}^*}{\bar{s}^*} = \frac{\rho\kappa_r/\kappa_o}{\lambda\gamma(\sigma-1)}.$$

On the other hand, in the market equilibrium,

$$\frac{\hat{s}}{\bar{s}} = \frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma\sigma v_s(\hat{s},\theta,w)} = \frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma\sigma v_s^N(\hat{s},\theta,w)},$$

where the second equality is from smooth-pasting at $\hat{s}$. Hence the relative size of the shadows of death is

$$\frac{\hat{s}^*/\bar{s}^*}{\hat{s}/\bar{s}} = \rho\sigma v_s^N(\hat{s},\theta,w) = \frac{\rho}{r+\theta}\left(1-\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}-1}\right) < 1,$$

where we use $r = \rho + \frac{\theta}{\sigma-1}$ to have $\rho < r+\theta$.

Proof of Proposition 4 Suppose that a firm obtains additional flow of K per unit of time. The value of firm that commits not to do R&D is

$$\begin{aligned} v^N(s,\theta,w) &= \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} + K - \kappa_o w\right)dt \\ &= \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \tau\kappa_o w\right)dt. \\ &= \frac{s}{\sigma(r+\theta)} - \frac{\tau\kappa_o w}{r} + \left(\frac{\tau\kappa_o w}{r} - \frac{\bar{s}}{\sigma(r+\theta)}\right)\left(\frac{s}{\bar{s}}\right)^{-\frac{r}{\theta}} \end{aligned} \tag{A.5}$$

Smooth pasting at both thresholds implies that $\bar{s}_\tau = \tau\sigma\kappa_o w$ and

$$\frac{1}{r+\theta}\left(\frac{\hat{s}}{\bar{s}} - \left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}}\right) = \frac{1}{\tau}\frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma}. \tag{A.6}$$

Because the left hand side of equation (A.6) is strictly increasing in $\hat{s}/\bar{s}$, an increase in τ reduces $\hat{s}/\bar{s}$ at intersection. Further, the total differentiation of equation (A.6) implies that

$$\frac{d\hat{s}}{d\tau} = \frac{\sigma\kappa_o w\left(\frac{\hat{s}}{\bar{s}} - \frac{r+\theta}{\tau}\frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma}\right)}{1 - \frac{r}{r+\theta}\frac{\frac{r+\theta}{\tau}\frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma}}{\hat{s}/\bar{s}}} > 0.$$

The inequality is because, again from equation (A.6), we have

$$\frac{\hat{s}}{\bar{s}} > \frac{r+\theta}{\tau}\frac{\kappa_r/\kappa_o}{\lambda\gamma_\sigma}.$$

Therefore, an increase in τ leads to increases in $\bar{s}_\tau$ and $\hat{s}_\tau$, and a decrease in $\hat{s}_\tau/\bar{s}_\tau$.

Outside Option

Proposition A.1. *The outside option value of ξ/r that a firm receives just after exit has the same structure of Proposition 4 by setting $\tau = 1 + \frac{\xi}{\kappa_o w}$.*

Proof. When a firm receives $\frac{\xi}{r}$ at exit, the value of non-R&D firm ($s \in [\bar{s}, \hat{s}]$) is

$$
\begin{aligned}
v^N(s,\theta,w) &= \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \kappa_o w\right)dt + e^{-\frac{r}{\theta}\ln\frac{s}{\bar{s}}}\frac{\xi}{r} \\
&= \frac{1}{r+\theta}\frac{s}{\sigma}\left(1 - \left(\frac{s}{\bar{s}}\right)^{-\frac{r+\theta}{\theta}}\right) - \frac{\kappa_o w}{r}\left(1 - \left(\frac{s}{\bar{s}}\right)^{-\frac{r}{\theta}}\right) + \left(\frac{s}{\bar{s}}\right)^{-\frac{r}{\theta}}\frac{\xi}{r} \\
&= \frac{s}{\sigma(r+\theta)} - \frac{\kappa_o w}{r} + \left(\frac{\tau\kappa_o w}{r} - \frac{\bar{s}}{\sigma(r+\theta)}\right)\left(\frac{s}{\bar{s}}\right)^{-\frac{r}{\theta}}.
\end{aligned}
$$

Only the difference from the value of non-R&D firm under uniform subsidy, equation (A.5), is the second term, which is independent of s. Hence, this difference does not matter in the smooth-pasting conditions and the thresholds. □

A.2 Size-dependent Subsidy

We formalize a size-dependent subsidy such that a firm receives subsidy flow of K if $s \leq \tilde{s}$. It is equivalent to the uniform subsidy when $\tilde{s} \to \infty$. Roughly speaking, such a size-dependent subsidy has the same effect on $(\bar{s}, \hat{s})$ if $\tilde{s}$ is sufficiently large. On the other hand, it has no impact if $\tilde{s}$ is too small. We focus on the middle range of $\tilde{s}$ in the main manuscript. The next proposition summarizes the impact of the size-dependent policy on individual firm's $\bar{s}$, $\hat{s}$, and $\hat{s}/\bar{s}$ when this subsidy policy is offered to it, taking the aggregate situation as given.

Proposition A.2. *Let $(\bar{s}_0, \hat{s}_0)$ be the stationary state values of the thresholds without distortions. Let $(\bar{s}_1, \hat{s}_1)$ be the individual firm's thresholds when it receives uniform subsidy of $\tau < 1$, with keeping the aggregate variables as in the distortion-free stationary state. Suppose that a firm receives the size-dependent subsidy of $(\tau, \tilde{s})$. If $\tilde{s} \geq \hat{s}_1$, then the distorted firm chooses $(\bar{s}, \hat{s})$ equivalent to the pair under uniform subsidy for any $\tau \in (0,1)$. If $\tilde{s} \in (\underline{s}, \hat{s}_1)$, where*

$$
\underline{s} \equiv \max\left\{\tau, \left(\frac{\theta}{r+\theta}\frac{1-\tau^{\frac{r}{\theta}+1}}{1-\tau}\right)^{\frac{\theta}{r}}\right\} \times \sigma\kappa_o w, \tag{A.7}
$$

the firm chooses $\bar{s} = \tau\sigma\kappa_o w$ and

$$
\frac{d(\hat{s}/\bar{s})}{d\tau} < 0, \quad \frac{d\hat{s}}{d\tau} < 0.
$$

If $\tilde{s} \leq \underline{s}$, then the firm chooses $(\bar{s}_0, \hat{s}_0)$.

Proof. First, suppose that $\tilde{s} \geq \hat{s}_1$, where $\hat{s}_1 > \hat{s}_0$ from Proposition 4. In this case, the decisions about exit and R&D are equivalent to the case under uniform subsidy because the values of firm that commits not to do R&D are identical between uniform and size-dependent subsidies.

Second, suppose that $\tilde{s} \leq \bar{s}_1 (< \bar{s}_0)$. In this case, the decisions about exit and R&D follows the case without subsidy because a firm exits at $\bar{s}_1$ even though it receives subsidy $K = (1-\tau)\kappa_o w$.

Third, we consider $\tilde{s} \in (\bar{s}_1, \bar{s}_0]$. A firm exits before s reaches $\tilde{s}$ if the firm value conditional on exiting before reaching $\tilde{s}$ is greater than that conditional on waiting for $\tilde{s}$, or equivalently,

$$\begin{aligned}\int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}_0}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \kappa_o w\right)dt > & \int_0^{\frac{1}{\theta}\log\frac{s}{\tilde{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \kappa_o w\right)dt \\ & + \int_{\frac{1}{\theta}\log\frac{s}{\tilde{s}}}^{\frac{1}{\theta}\log\frac{s}{\bar{s}_1}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \tau\kappa_o w\right)dt \\ \Leftrightarrow \quad \tilde{s} < & \left[\frac{\theta}{r+\theta}\frac{1-\tau^{\frac{r}{\theta}+1}}{1-\tau}\right]^{\frac{\theta}{r}} \sigma\kappa_o w.\end{aligned}$$

Therefore, if $\tilde{s} \leq \underline{s}$, defined in equation (A.7), the distorted firm exits before s reaches $\tilde{s}$ and, thus, the thresholds $(\bar{s}, \hat{s}) = (\bar{s}_0, \hat{s}_0)$.

Finally, we consider $\tilde{s} \in (\underline{s}, \hat{s}_1)$. If $\hat{s} \leq \tilde{s}$, the firm value at s close but smaller than $\hat{s}$ must satisfy

$$v(s,\theta,w) = \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \tau\kappa_o w\right)dt.$$

Then, $\tilde{s} \geq \hat{s} = \hat{s}_1 > \tilde{s}$, which is a contradiction. Hence, $\hat{s} > \tilde{s}$. Given this relation, the firm value is

$$v(s,\theta,w) = \begin{cases} \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \tau\kappa_o w\right)dt - (1-\tau)\kappa_o w \int_0^{\frac{1}{\theta}\log\frac{s}{\tilde{s}}} e^{-rt}dt & \text{for } s \in [\tilde{s}, \hat{s}], \\ \int_0^{\frac{1}{\theta}\log\frac{s}{\bar{s}}} e^{-rt}\left(\frac{se^{-\theta t}}{\sigma} - \tau\kappa_o w\right)dt & \text{for } s \in [\bar{s}, \tilde{s}), \end{cases}$$

which gives $\bar{s} = \tau\sigma\kappa_o w$ and the condition for $\hat{s}$ such that

$$\begin{aligned}\hat{s}v_s(\hat{s},\theta,w) &= \frac{\bar{s}}{\sigma(r+\theta)}\left[\frac{\hat{s}}{\bar{s}} - \left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}} - \frac{r+\theta}{\theta}\frac{1-\tau}{\tau}\left(\frac{\tilde{s}}{\bar{s}}\right)^{-\frac{r}{\theta}}\right] = \frac{\kappa_r w}{\lambda\gamma_\sigma} \\ \Leftrightarrow \quad \frac{\hat{s}}{\bar{s}} - h(\tau)\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}} &= \frac{r+\theta}{\tau}\frac{\kappa_r w}{\lambda\gamma_\sigma},\end{aligned}$$

where

$$h(\tau) \equiv 1 + \frac{r+\theta}{\theta}\frac{1-\tau}{\tau}\left(\frac{\tilde{s}}{\bar{s}}\right)^{-\frac{r}{\theta}}.$$

From the total differentiation,

$$\frac{d(\hat{s}/\bar{s})}{d\tau} = -\frac{1}{\tau^2}\frac{\frac{r+\theta}{\theta}\left(1 + \frac{(1-\tau)(r+\theta)}{\theta}\right)\left(\frac{\tilde{s}}{\bar{s}}\right)^{\frac{r}{\theta}}\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}} + \frac{(r+\theta)\kappa_r w}{\lambda\gamma_\sigma}}{1 + \frac{r}{\theta}h(\tau)\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}-1}} < 0.$$

Further,

$$\frac{d\hat{s}}{d\tau} = \frac{1}{\tau}\frac{\frac{\hat{s}}{\bar{s}} + \left[\tau h'(\tau) + \frac{r}{\theta}h(\tau)\right]\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}} - \frac{1}{\tau}\frac{(r+\theta)\kappa_r w}{\lambda\gamma_\sigma}}{1 + \frac{r}{\theta}h(\tau)\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}-1}}.$$

Since the denominator is strictly positive, $d\hat{s}/d\tau < 0$ if and only if

$$\begin{aligned}\frac{\hat{s}}{\bar{s}} + \left[\tau h'(\tau) + \frac{r}{\theta}h(\tau)\right]\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}} &< \frac{1}{\tau}\frac{(r+\theta)\kappa_r w}{\lambda\gamma_\sigma} = \frac{\hat{s}}{\bar{s}} - h(\tau)\left(\frac{\hat{s}}{\bar{s}}\right)^{-\frac{r}{\theta}} \\ \Leftrightarrow \quad \tau h'(\tau) + \frac{r}{\theta}h(\tau) &< -h(\tau) \\ \Leftrightarrow \quad \tilde{s} &> \bar{s},\end{aligned}$$

which is true under the current supposition:$\tilde{s} \in (\underline{s}, \hat{s}_1)$. Therefore,

$$\frac{d\hat{s}}{d\tau} < 0 \qquad \text{for } \tilde{s} \in (\underline{s}, \hat{s}_1).$$

□

A.3 Socially Optimal Stationary State

A.3.1 Social Planner's Problem

Here we derive the socially optimal stationary state. For notational simplicity, we redefine the productivity indices as

$$\zeta_{jt} \equiv z_{jt}^{\sigma-1}, \qquad \mathcal{Z}_t \equiv Z_t^{\sigma-1},$$

where we drop industry subscript to consider the symmetric industry case. With the above definition, we have

$$\zeta_{jt} \equiv s_{jt}\mathcal{Z}_t \quad \text{and} \quad \theta_t \equiv \frac{\dot{\mathcal{Z}}_t}{\mathcal{Z}_t}.$$

The first step of the social planner's problem is the choice of exit threshold, $\bar{\zeta}_t$ ($\equiv \bar{s}_t\mathcal{Z}_t$). For given n_t and L_{Xt}, the social planner shuts down a firm's operation if its ζ_{jt} is not commensurate with its fixed costs. Denoting ω_t as the value of labor in terms of utility, the exit threshold should satisfy

$$\frac{1}{\sigma-1}\frac{\bar{\zeta}_t}{\mathcal{Z}_t} = \omega_t\kappa_o \quad \Rightarrow \quad \bar{\zeta}_t = (\sigma-1)\kappa_o\omega_t\mathcal{Z}_t. \tag{A.8}$$

Next, suppose that $V(\mathcal{Z}_t, n_t)$ is the social value of $\mathcal{Z}_t$ and n_t. We consider the following dynamic programming problem:

$$\begin{aligned}\rho V(\mathcal{Z}_t, n_t) = \max_{L_{Xt},\mu_t,\hat{\zeta}_t} \; & \varepsilon \ln n_t + \frac{1}{\sigma-1}\ln \mathcal{Z}_t + \ln L_{Xt} \\ & + \omega_t\left[L - L_{Xt} - n_t\left[\kappa_o + \kappa_r\left(1 - F_{\zeta t}\left(\hat{\zeta}_t\right)\right) + \kappa_e\mu_t\right]\right] \\ & + \omega_t^{\mathcal{Z}}\dot{\mathcal{Z}}_t + \omega_t^n\dot{n}_t,\end{aligned}$$

subject to

$$\dot{\mathcal{Z}}_t = n_t \left[\lambda \gamma_\sigma \int_{\hat{\zeta}_t}^{\infty} \frac{\zeta}{\mathcal{Z}_t} f_t\left(\frac{\zeta}{\mathcal{Z}_t}\right) d\zeta + \mu_t \int_{\bar{\zeta}_t}^{\infty} \frac{\zeta}{\mathcal{Z}_t} f_e\left(\frac{\zeta}{\mathcal{Z}_t}\right) d\zeta - \dot{\bar{\zeta}}_t \frac{\bar{\zeta}_t}{\mathcal{Z}_t} f_t\left(\frac{\bar{\zeta}_t}{\mathcal{Z}_t}\right) - \bar{\delta} \int_{\bar{\zeta}_t}^{\infty} \frac{\zeta}{\mathcal{Z}_t} f\left(\frac{\zeta}{\mathcal{Z}_t}\right) d\zeta \right] \tag{A.9}$$

$$\dot{n}_t = n_t \left[\mu_t \left(1 - F_e\left(\frac{\bar{\zeta}_t}{\mathcal{Z}_t}\right)\right) - \dot{\bar{\zeta}}_t \frac{1}{\mathcal{Z}_t} f_t\left(\frac{\bar{\zeta}_t}{\mathcal{Z}_t}\right) - \bar{\delta} \right] \tag{A.10}$$

$$\dot{\bar{\zeta}}_t = (\sigma - 1)\kappa_o \left[\dot{\omega}_t \mathcal{Z}_t + \omega_t \dot{\mathcal{Z}}_t \right],$$

where $\hat{\zeta}_t$ ($\equiv \hat{s}_t \mathcal{Z}_t$) is the R&D threshold,

$$\omega_t^{\mathcal{Z}} \equiv \frac{\partial V}{\partial \mathcal{Z}_t}, \quad \omega_t^n \equiv \frac{\partial V}{\partial n_t},$$

and $\bar{\delta}$ is the exogenous exit rate.

The first-order conditions about control variables are

$$\frac{1}{L_{Xt}} = \omega_t, \tag{A.11}$$

$$\omega_t \kappa_e = \omega_t^{\mathcal{Z}} \int_{\bar{\zeta}_t}^{\infty} \frac{\zeta}{\mathcal{Z}_t} f_e\left(\frac{\zeta}{\mathcal{Z}_t}\right) d\zeta + \omega_t^n \left(1 - F_e\left(\frac{\bar{\zeta}_t}{\mathcal{Z}_t}\right)\right), \tag{A.12}$$

$$\left(\omega_t^{\mathcal{Z}} \mathcal{Z}_t\right) \frac{\hat{\zeta}_t}{\mathcal{Z}_t} = \frac{\kappa_r \omega_t}{\lambda \gamma_\sigma}. \tag{A.13}$$

The marginal values of the state variables, $\omega_t^{\mathcal{Z}}$ and ω_t^n, satisfy:

$$\rho \omega_t^{\mathcal{Z}} - \dot{\omega}_t^{\mathcal{Z}} = \frac{1}{\sigma - 1} \frac{1}{\mathcal{Z}_t} + \omega_t n_t \kappa_r \frac{\partial}{\partial \mathcal{Z}_t} F_{\zeta t}\left(\hat{\zeta}_t\right) + \omega_t^{\mathcal{Z}} \frac{\partial\left(\dot{\mathcal{Z}}_t\right)}{\partial \mathcal{Z}_t} + \omega_t^n \frac{\partial\left(\dot{n}_t\right)}{\partial \mathcal{Z}_t}, \tag{A.14}$$

$$\rho \omega_t^n - \dot{\omega}_t^n = \frac{\varepsilon}{n_t} - \frac{\omega_t \left(L - L_{Xt}\right)}{n_t} + \omega_t^{\mathcal{Z}} \frac{\partial\left(\dot{\mathcal{Z}}_t\right)}{\partial n_t} + \omega_t^n \frac{\partial\left(\dot{n}_t\right)}{\partial n_t}. \tag{A.15}$$

A.3.2 Optimal Stationary State

In a stationary state (balanced growth path) with a stationary distribution of s, we have

$$\begin{aligned} \theta &= \frac{\dot{\mathcal{Z}}_t}{\mathcal{Z}}, \\ \bar{\zeta}_t &= \bar{s}\mathcal{Z}_t \quad \text{and} \quad \hat{\zeta}_t = \hat{s}\mathcal{Z}_t, \\ \dot{\bar{\zeta}}_t &= \bar{s}\theta\mathcal{Z}_t, \\ \mu\left(1 - F_e(\bar{s})\right) &= \delta = \theta \bar{s} f(\bar{s}) + \bar{\delta}, \\ \omega_t &= \dot{\omega}_t^n = 0, \\ \frac{\dot{\omega}_t^{\mathcal{Z}}}{\omega_t^{\mathcal{Z}}} &= -\theta. \end{aligned}$$

Then, equations (A.11)-(A.13) in a stationary state become

$$\frac{1}{L_X} = \omega \tag{A.16}$$

$$\omega\kappa_e = \omega_t^{\mathcal{Z}} \mathcal{Z}_t \int_{\bar{s}}^{\infty} s f_e(s)\, ds + \omega^n \left(1 - F_e(\bar{s})\right) \tag{A.17}$$

$$\left(\omega_t^{\mathcal{Z}} \mathcal{Z}_t\right) \hat{s} = \frac{\kappa_r \omega}{\lambda \gamma_\sigma} \tag{A.18}$$

The optimal thresholds, $\bar{s}$ and $\hat{s}$ Equations (A.8) and (A.16) implies that

$$\bar{s} = \frac{\bar{\zeta}_t}{\mathcal{Z}_t} = \frac{(\sigma-1)\kappa_o}{L_X}. \tag{A.19}$$

Equation (A.13) in a stationary state implies that the optimal $\hat{s}$ depends on $\omega_t^{\mathcal{Z}} \mathcal{Z}_t$, which is pinned down by equation (A.14) in the stationary state such that

$$\rho\omega_t^{\mathcal{Z}} - \dot{\omega}_t^{\mathcal{Z}} = \frac{1}{\sigma-1}\frac{1}{\mathcal{Z}_t} + \omega n \kappa_r \underbrace{\frac{\partial}{\partial \mathcal{Z}_t} F_{\zeta t}\left(\hat{\zeta}_t\right)}_{\to 0} + \omega_t^{\mathcal{Z}} \underbrace{\frac{\partial\left(\dot{\mathcal{Z}}_t\right)}{\partial \mathcal{Z}_t}}_{\to \theta} + \omega^n \underbrace{\frac{\partial\left(\dot{n}_t\right)}{\partial \mathcal{Z}_t}}_{\to 0}$$

$$\Rightarrow \quad \omega_t^{\mathcal{Z}} \mathcal{Z}_t = \frac{1}{\sigma-1}\frac{1}{\rho}. \tag{A.20}$$

Hence,

$$\hat{s} = \frac{\rho(\sigma-1)\kappa_r}{\lambda\gamma_\sigma L_X}. \tag{A.21}$$

The social value of a unit of firm, ω^n, in the stationary state is derived from equation (A.15) with the stationarity conditions,

$$\rho\omega^n = \frac{\varepsilon}{n} - \frac{L - L_X}{nL_X} + \omega_t^{\mathcal{Z}} \underbrace{\frac{\partial\left(\dot{\mathcal{Z}}_t\right)}{\partial n_t}}_{\to \frac{\theta \mathcal{Z}_t}{n}} + \omega^n \underbrace{\frac{\partial\left(\dot{n}_t\right)}{\partial n_t}}_{\to 0},$$

leading to

$$\omega^n = \frac{1}{\rho n}\left[\frac{\theta}{\rho(\sigma-1)} - \frac{L-L_X}{L_X} + \varepsilon\right]. \tag{A.22}$$

Socially Optimal Solution Productivity growth, θ, in the stationary state can be described as a function of L_X and n, where L_X affects through $\bar{s}$ and $\hat{s}$ such that

$$\left[\frac{1}{n} + \bar{s}^2 f(\bar{s})\right]\left(\theta + \bar{\delta}\right) = \lambda\gamma_\sigma \int_{\hat{s}}^{\infty} s f(s)\, ds + \frac{\theta \bar{s} f(\bar{s})}{1 - F_e(\bar{s})} \int_{\bar{s}}^{\infty} s f_e(s)\, ds$$

$$\Rightarrow \quad \theta(n, L_X) = \left[\frac{1}{n} - \bar{s} f(\bar{s})\left\{\int_{\bar{s}}^{\infty} \frac{s f_e(s)}{1 - F_e(\bar{s})} ds - \bar{s}\right\}\right]^{-1} \lambda\gamma_\sigma \int_{\hat{s}}^{\infty} s f(s)\, ds - \bar{\delta} \tag{A.23}$$

Given this $\theta(n, L_X)$, the resource constraint:

$$L = L_X + n\left[\kappa_o + \kappa_r\left(1 - F\left(\hat{s}\right)\right) + \kappa_e \frac{\theta(n, L_X)\bar{s}f(\bar{s}) + \bar{\delta}}{1 - F_e(\bar{s})}\right], \tag{A.24}$$

and the optimal entry condition:

$$\frac{\kappa_e}{L_X} = \frac{1}{\rho}\frac{1}{\sigma - 1}\int_{\bar{s}}^{\infty} sf_e\left(s\right)ds + \left(1 - F_e\left(\bar{s}\right)\right)\omega^n, \tag{A.25}$$

give us the optimal combination of working labor, L_X, and the measure of firms, n, in the optimal stationary state.

The red dashed line of Figure B.3 shows the socially optimal state based on simulations. The horizontal axis is the degree of size-dependent subsidy $1 - \tau$, which changes from -0.2 to 0.2. The figure indicates that R&D threshold $\hat{s}$ decreases, shortening the length of the shadow of death. Real growth g increases by one percentage point. Further, the entry rate increases. Consequently, welfare improves by around 0.06 in units of consumption.

A.3.3 R&D and Entry Subsidy

We consider R&D subsidy, $\alpha_r \in (0, 1]$, to reduce the R&D cost to $\alpha_r\kappa_r w$. The required subsidy is financed by lump-sum tax on the households. To make $\hat{s}/\bar{s} = \hat{s}^*/\bar{s}^*$, the R&D subsidy should be set to satisfy

$$\underbrace{\frac{1}{\alpha_r}}_{\text{subsidy effect}} \times \underbrace{\sigma v_s(\hat{s}, \theta, w)}_{\text{private marginal value}} = \underbrace{\frac{1}{\rho}}_{\text{social marginal value}},$$

which implies

$$\alpha_r^* = \frac{\rho}{r(\theta^*) + \theta^*}\left(1 - \left(\frac{\hat{s}^*}{\bar{s}^*}\right)^{-\frac{r(\theta^*)}{\theta^*} - 1}\right), \tag{A.26}$$

where θ^* is the R&D intensity in the socially optimal allocation , and $r(\theta) = \rho + \theta/(\sigma - 1)$ on a balanced growth path.

The entry subsidy can be defined as $\alpha_e \in (0, 1]$ that reduces entry cost in the market equilibrium. From equation (15) in the main text and $w = \frac{\sigma-1}{\sigma L_X}$ in equilibrium, we can write the free-entry condition with the entry subsidy such as

$$\frac{\alpha_e\kappa_e}{L_X} = \frac{\sigma}{\sigma - 1}\int_{\bar{s}}^{\infty} v(s, \theta, w)dF_e.$$

The socially optimal condition for entry is given by equation (A.25) with the social value of an additional firm, equation (A.22). To meet the private value of entry with the social value of entry, α_e should be set to satisfy

$$\frac{1}{\alpha_e}\frac{\sigma}{\sigma - 1}\int_{\bar{s}^*}^{\infty} v\left(s, \theta^*, \frac{\sigma - 1}{\sigma L_X^*}\right)dF_e = \frac{1}{\rho}\frac{1}{\sigma - 1}\int_{\bar{s}^*}^{\infty} sdF_e + \left(1 - F_e\left(\bar{s}^*\right)\right)\omega^{n*}.$$

B TSR Data and Further Estimation Results

B.1 Descriptive Statistics

Table B.1 summarizes the dataset we use for the estimation of firms' pre-exit dynamics. Here, we show two summary statistics accounting for the unbalanced and balanced (to be precise, firms surviving for at least 10 years) data. Table B.2 summarizes the dataset we use for the estimation of firms' pre- and post-R&D termination dynamics. Here, we show two summary statistics in the case of $h' + 1 = 1$ and $h' + 1 = 2$. Each consists of 11 statistics for $h = -5, -4, \cdots, 4, 5$. Table B.3 summarizes the dataset we use for the estimation of the relationship between distortions and firm exit. For this estimation, we use the unbalanced panel data. Table B.4 summarizes the dataset we use for the estimation of the relationship between distortions and the termination of R&D investment. For this estimation, we use the unbalanced panel data.

B.2 R&D Investment

In Table B.5, we calculate descriptive statistics from the TSR data to examine whether R&D investment is associated with a lower likelihood of firm exit and a higher level of sales growth. The number of firms we observe is 4,236,113 firms, among which we can observe the level of R&D investment (including the case of zero expenditure) for 701,763 firms.

To measure R&D investment, we use three kinds of definitions: (i) R&D investment, (ii) selling, general, and administrative (SGA) expenses, and (iii) sales promotion, advertising, entertainment, and other selling expenses. The first definition is the most straightforward and narrowest one, which makes 41,856 firms (only 6%) spend strictly positive R&D investment at least once. The second definition could be the most broadest one, which may contain expenditures other than R&D investment. In the third definition, expenses related to advertising are interpreted as R&D investment. By construction, the fraction of firms that conduct strictly positive R&D investment increases in the second and third definitions, which amounts to 85% and 73%, respectively.

Irrespective of the definition of R&D investment, Table B.5 shows that the firms that make R&D investment more frequently are less likely to exit the market. To see this, we divide firms into five groups as follows. First, we calculate the probability of positive R&D investment for each firm (i.e., the length of periods over which a firm shows positive R&D divided by the total length of periods in which the firm is recorded). Second, based on this probability of positive R&D investment, we split 701,763 firms into two groups: firms with no R&D investment and those with positive R&D probability. Third, we further split the firms in the latter group to four groups: 0 to 25 percentile, 25 to 50, 50 to 75, and 75 to 100 percentile by the probability of positive R&D investment. Next, we calculate the voluntary exit rate (the number of voluntary exit firms divided by the total number of firms) for each group. The table shows that the voluntary exit rate tends to decrease as the probability of positive R&D investment increases. For example, for the first R&D definition, the voluntary exit rate is 3.2% for the firms with no R&D investment, which decreases to 1.4% for the firms that make R&D investment most frequently. The result is similar when we use the second and third R&D definitions: the voluntary exit rate is 4.5% and 4.4%, respectively, for the firms with no R&D investment, which decreases to 1.4% and 1.4% for the firms that make R&D investment most frequently.

Table B.5 further shows that the firms that make R&D investment more frequently are more likely to grow their sales. We use the same grouping based on the the probability of positive R&D investment and calculate the fraction of firms with positive average sales growth for each group. The table shows that firms are more likely to grow as the probability of positive R&D investment increases. For example, for the first R&D definition, the fraction of firms with positive sales growth is 46.9% for the firms with no R&D investment, which increases to 61.4% for the firms that make R&D investment most frequently.

These relationships are consistent with the implications of our model.

B.3 Further Estimation Results

Table B.6 summarizes the estimated coefficients accounting for the relative sales of an exiting firm as of $|h|$ years prior to the exit conditional on that the age of owner is between 15 to 65. Table B.7 summarizes the estimated coefficients accounting for the relative sales of the firm that terminates R&D as of $|h|$ years before or after the termination. Table B.8 summarizes the estimated coefficients of the probit estimation for firm exit and R&D termination.

Figure B.1 depicts the probability of firm exit (right axis) and R&D termination (left axis) conditional on the level of firm sales, which is plotted on the horizontal axis. Figure B.2 shows a dispersion in the sales of R&D termination (data-based $\hat{s}$) relative to fixed costs by industries.

C Numerical Simulations of the Model Incorporating Heterogenous Subsidy

For data fitting, we add the firm exit rate due to exogenous shocks, $\bar{\delta}$.

C.1 Equilibrium

We consider how equilibrium is determined. Endogenous variables are $\bar{s}_K$, $\hat{s}_K$, n, θ, w, δ, μ, r, g, C, and U, with value function $v_K(s)$ and distribution $F(K,s)$, under the distortion of τ_K.

The non-R&D firm value $\tilde{v}_N(s)$ when $s \in [\bar{s}_K, \hat{s}_K]$ is given by

$$\tilde{v}_N(s) = \int_0^{\frac{1}{\theta}\ln(s/\bar{s}_K)} e^{-\left(r+\bar{\delta}\right)t}\left(\frac{s_t}{\sigma} - \kappa_o w + K\right)dt.$$

The firm changes its behavior as if the fixed cost κ_o changes to $(1-\tau_K)\kappa_o$. Since $\tilde{v}_N(s) = 0$ when $s = \bar{s}_K$, we have

$$\bar{s}_K = (1-\tau_K)\sigma\kappa_o w. \tag{A.27}$$

From the smooth-pasting condition at $\hat{s}_K$, we have

$$\hat{s}_K \tilde{v}'_N(\hat{s}_K) = \frac{\kappa_r w}{\lambda\gamma_\sigma}.$$

$$\Leftrightarrow \quad \frac{1}{r+\bar{\delta}+\theta}\left(\frac{\hat{s}_K}{\bar{s}_K} - \left(\frac{\hat{s}_K}{\bar{s}_K}\right)^{-\frac{r+\bar{\delta}}{\theta}}\right) = \frac{\kappa_r/\{(1-\tau_K)\kappa_o\}}{\lambda\gamma_\sigma}. \tag{A.28}$$

The other endogenous variables are obtained by the following conditions. The real interest rate is given by

$$r = \rho + g. \tag{A.29}$$

The real growth rate is given by

$$g = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{Z}_t}{Z_t} = \frac{\theta}{\sigma-1}, \tag{A.30}$$

where

$$\theta = n\left[\lambda\gamma_\sigma \int_K \int_{\hat{s}_K}^{\infty} s dF(K,s) + \mu \int\int_{\bar{s}_K}^{\infty} s dF_0(s)dG(K) - (\delta - \bar{\delta})\int_K \bar{s}_K \int_{\bar{s}_K}^{\infty} dF(K,s)\right] - \bar{\delta}. \tag{A.31}$$

The free entry condition:

$$\int\int_{\bar{s}_K}^{\infty} v(s_K)dF_0(s)dG(K) = \kappa_e w. \tag{A.32}$$

The exit and entry rates equate in each industry as

$$\begin{aligned}\delta dt &= \bar{\delta}dt + \int_K dF(K,\bar{s}_K) \\ &= \mu dt \int_K \left[1 - F_0\left(\bar{s}_K\right)\right] dG(K),\end{aligned} \tag{A.33}$$

by choosing dt according to the grid size setting used in simulation as explained in Section C.2 below.

The labor market:

$$L = n\left[\frac{\sigma-1}{\sigma w}\left(\int_K \int_{\bar{s}_K}^{\infty} sdF(K,s)\right) + \kappa_o + \kappa_r \int_K \int_{\hat{s}_K}^{\infty} dF(K,s) + \kappa_e \mu\right]. \tag{A.34}$$

The representative households' welfare[1]

$$U = \frac{\ln C_0}{\rho} + \frac{g}{\rho^2}, \tag{A.35}$$

where $C_0 = Y_0 - n\int KdF(K,s)$, so that

$$\begin{aligned}\ln\left\{C_0 + n\int_K \int_{\bar{s}_K}^{\infty} \tau_K \kappa_o w dF(K,s)\right\} &= \ln\frac{\sigma-1}{\sigma w} + \ln Z_0 + \varepsilon \ln n \\ &= \ln\frac{\sigma-1}{\sigma w}.\end{aligned} \tag{A.36}$$

Note that $\varepsilon \ln n + \ln Z_0$ equals zero when $\varepsilon = -1/(\sigma-1)$, $z = 1$, and $s = 1/n$.

The value function in discrete time is given by

$$\begin{aligned}v_K(s) &= \max\left\{0,\ \{\pi(s) - (1-\tau_K)\kappa_o w\}\, dt\right. \\ &\quad + \max\left\{e^{-(r+\bar{\delta})dt} v_K((1-\theta)s),\right. \\ &\quad \left.-\kappa_r w dt + e^{-(r+\bar{\delta})dt}(1-e^{-\lambda dt}) v_K((1-\theta+\gamma_s)s) + e^{-(r+\bar{\delta}+\lambda)dt} v_K((1-\theta)s)\right\}.\end{aligned} \tag{A.37}$$

Given K, stationary density distribution with respect to s, $f(s)$, should satisfy the following condition. For example, if $s \geq (1+\gamma_\sigma)\hat{s}_K$, stationary density distribution is given by

$$\begin{aligned}f(s)ds &= f(s_l)ds(1-e^{-\lambda dt})e^{-\bar{\delta}dt} + f(s_h)dse^{-\lambda dt}e^{-\bar{\delta}dt} \\ &\quad + (1-e^{-\bar{\mu}dt})\bar{f}_0(s)ds - (1-e^{-\bar{\delta}dt})f(s)ds,\end{aligned} \tag{A.38}$$

$$\begin{aligned}s_l(dt) &= s\cdot\frac{Z_{t+dt}^{\sigma-1}/Z_t^{\sigma-1}}{(1+\gamma)^{\sigma-1}} = s\cdot\frac{1+\theta dt}{1+\gamma_\sigma} \\ s_h(dt) &= s\cdot\frac{Z_{t+dt}^{\sigma-1}}{Z_t^{\sigma-1}} = s\left(1+\theta dt\right),\end{aligned}$$

where ds and dt represents grid intervals for s and t, respectively.

Finally, we normalize s and w by the measure of firms:

$$A \equiv n\int_K \int_{\bar{s}_K}^{\infty} sdF(K,s). \tag{A.39}$$

For example, $\bar{s}$, $\hat{s}$, and w are normalized as $\bar{s}/A$, $\hat{s}/A$, and w/A, respectively.

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[1] In simulations, we evaluate welfare changes by how much the level of consumption C_0 should change when the growth rate g is fixed. When U and g changes to $U^{'}$ and $g^{'}$, respectively, this corresponds to $\ln C_0^{''} - \ln C_0 = \rho(U^{'} - U)$.

C.2 Numerical Solutions for the Equilibrium

Given the infinitesimal grid size of $d\log s$, we set dt at $d\log s/\theta$, so that a firm's market share decreases by $d\log s$ or one grid if the firm makes no R&D investment or fails in improving quality. The firm's market share increases by $d\log s_+$ or the grid size of floor$\{(1 - e^{\theta dt - \gamma_\sigma})/d\log s\}$ if the firm succeeds in improving quality by R&D.

We denote density distribution by $f_i(s)$ that is calculated at the i-th number of iteration. Stationary density distribution is given by

$$f_{i+1}(\log s) = \begin{cases} 0 & \text{for } \log s < \log \bar{s}, \\ f_i(\log s + d\log s)e^{-\bar{\delta}dt} & \\ \quad +(1-e^{-\mu dt})f_0(\log s) & \text{for } \log\bar{s} \le \log s < \log\hat{s}, \\ f_i(\log s + d\log s)e^{-\lambda dt}e^{-\bar{\delta}dt} & \\ \quad +(1-e^{-\mu dt})f_0(\log s) & \text{for } \log\hat{s} \le \log s < \log\hat{s} + d\log s_+, \\ f_i(\log s + d\log s)e^{-\lambda dt}e^{-\bar{\delta}dt} & \\ \quad +f_i(\log s - d\log s_+)(1-e^{-\lambda dt})e^{-\bar{\delta}dt} & \\ \quad +(1-e^{-\mu dt})f_0(\log s) & \text{for } \log\hat{s} + d\log s_+ \le \log s < \log s^{max} \\ f_i(\log s)e^{-\lambda dt}e^{-\bar{\delta}dt} & \\ \quad +\Sigma_{d\log s_j=1}^{d\log s_+} f_i(\log s - d\log s_j)(1-e^{-\lambda dt})e^{-\bar{\delta}dt} & \\ \quad +(1-e^{-\mu dt})f_0(\log s) & \text{for } \log s = \log s^{max}. \end{cases} \tag{A.40}$$

C.3 Model-based Variables

Sum of sales share: $n\int_K \int_{\bar{s}_K}^{\infty} sdF(K,s)$

Mean of sales share: $\int_K \int_{\bar{s}_K}^{\infty} sdF(K,s)$

Sum (mean) of entrants' sales share: $\int_K \int_{\bar{s}_K}^{\infty} sdF^e(K,s)$,

where $dF^e(K,s) \equiv (dF_0(s)dG(K)) \,/\, \left(\int\int_{\bar{s}_K}^{\infty} dF_0(s)dG(K)\right)$ for $s \ge \bar{s}_K$.

Entry rate: $\bar{\mu} = \mu \int\int_{\bar{s}_K}^{\infty} dF_0(s)dG(K)$

R&D cost share over sales for R&D firms: $\left(\int_K \int_{\hat{s}_K}^{\infty} w\kappa_r dF(K,s)\right) / \left(\int_K \int_{\hat{s}_K}^{\infty} sdF(K,s)\right)$

Fixed cost share over sales: $\left(\int_K \int_{\bar{s}_K}^{\infty} w\kappa_o dF(K,s)\right) / \left(\int_K \int_{\bar{s}_K}^{\infty} sdF(K,s)\right)$

The probability that R&D firms increase their sales share minus the probability that non-R&D firms increase their sales share: λ

Exit rate for R&D firms: $\bar{\delta}$

Speed of sales share change for non-R&D firms: $-\theta$

The ratio of R&D threshold to exit threshold: $\left(\int_K \int_{\bar{s}_K}^{\infty} \hat{s}_K dF(K,s)\right) / \left(\int_K \int_{\bar{s}_K}^{\infty} \bar{s}_K dF(K,s)\right)$

Fraction of R&D firms: $\left(\int_K \int_{\hat{s}_K}^{\infty} dF(K,s)\right)$

Sales share of R&D firms: $\left(\int_K \int_{\hat{s}_K}^{\infty} sdF(K,s)\right) / \left(\int_K \int_{\bar{s}_K}^{\infty} sdF(K,s)\right)$

Profit: $\int_{\bar{s}_K}^{\infty} sdF(K,s)/\sigma$, which is equal to $1/(\sigma n)$

Markup rate: $(p - w/z)/(w/z) = 1/(\sigma - 1)$

Labor share: $wl/(wl+\pi) = (\sigma-1)/\sigma$
HHI: $\int_{\bar{s}_K}^{\infty}(s/n)^2 dF(K,s)$

C.4 Data-based Variables

For calibration, we calculate the following variables based on the TSR data. We identify non-R&D firms when the firms record zero or missing R&D investment in the last three years. Entrants are identified when the firms are recorded for the first time in the TSR data and firm ages are three (five) years or less.

The probability of positive sales growth for R&D firms relative to non-R&D firms: We calculate the probability that the sales share increases for R&D and non-R&D firms as 51.03% and 47.33%, respectively. Their difference is 0.037. This is equivalent to λ in the model.

The exit rate of R&D firms: For R&D firms, the exit rate, including not just voluntary exit but also bankruptcy, equals 0.0028. This is equivalent to $\bar{\delta}$ in the model.

The entry rate: We calculate the number of entrants in one year divided by the number of existing firms in the previous year and take the mean over time. It is 0.006 (0.015) when entrant ages are three (five) years or less. Moreover, we calculate the average annual entry rate of establishments from 1980 to 2018 by using the Annual Report on Employment Insurance by the Ministry of Health, Labour and Welfare. The value is 0.051.

The share of fixed costs in sales: Fixed costs are the sum of selling, general, and administrative (SGA) expenses that consist of directors' remuneration, salaries and allowances, provision for bonuses, retirement benefits, welfare expenses, depreciation and amortization, advertising expenses, utilities expenses, taxes and dues, rent, and insurance premiums. For the firms that record SGA expenses, we calculate the sum of the fixed costs as well as the sum of sales. The share of fixed costs in sales is 0.050. This value is related to κ_o.

The share of R&D costs in sales for R&D firms: For the firms that record positive R&D costs, which are one item in SGA expenses, we calculate the sum of R&D costs as well as the sum of sales. The share of R&D costs in sales for R&D firms is 0.028. This value is related to κ_r.

The ratio of the median of log sales for R&D threshold to that for exit threshold: This value is equivalent to $\hat{s}/\bar{s}$. To calculate $\bar{s}$, we take the firms that record a non-missing value for R&D costs and calculate the median sales one year before their voluntary exit, which is 34,854 thousand yen. To calculate $\hat{s}$, we take the firms that experience voluntary exit and calculate the median sales of the firms when they record positive R&D costs in the current year but zero in the following year, which is 142,304 thousand yen. The ratio $\hat{s}/\bar{s}$ is 4.08.

The ratio of the mean of sales for all firms to that for entrants: It is 0.978 (1.431) when entrant ages are three (five) years or less.

The ratio of the standard deviation of sales for all firms to that for entrants: It is 0.534 (0.703) when entrant ages are three (five) years or less.

The speed of sales change for non R&D firms: We estimate the following equation:

$$\log\left(\text{sales}_{i,t}\right) = \alpha + \sum_{h=1}^{H} \beta_h \mathbb{1}\left(\text{exit}_{i,t+h}\right) + \eta_t + \varepsilon_{i,t},$$

for firm i and year t. The explanatory variable $\mathbb{1}\left(\text{exit}_{i,t+h}\right)$ takes one if firm i exits in year $t+h$ and zero otherwise. We calculate the yearly change in sales as $(\beta_1 - \beta_6)/5$. This value is equivalent to $-\theta$ in the model.

Table B.1: Summary Statistics of the Dataset for Pre-exit Firm Dynamics Estimation

Variables	Unbalanced			Firms surviving for at least 10 years		
	No. of obs.	Mean	S.D.	No. of obs.	Mean	S.D.
$\log(\text{sales}_{i,t})$	16,491,841	11.700	1.758	2,620,854	11.948	1.939
$\mathbb{1}(\text{exit}_{i,t+1})$	16,491,841	0.006	0.079	2,620,854	0.009	0.094
$\mathbb{1}(\text{exit}_{i,t+2})$	16,491,841	0.007	0.082	2,620,854	0.009	0.093
$\mathbb{1}(\text{exit}_{i,t+3})$	16,491,841	0.007	0.083	2,620,854	0.008	0.088
$\mathbb{1}(\text{exit}_{i,t+4})$	16,491,841	0.007	0.083	2,620,854	0.007	0.082
$\mathbb{1}(\text{exit}_{i,t+5})$	16,491,841	0.007	0.082	2,620,854	0.006	0.074
$\mathbb{1}(\text{exit}_{i,t+6})$	16,491,841	0.007	0.081	2,620,854	0.004	0.066
$\mathbb{1}(\text{exit}_{i,t+7})$	16,491,841	0.007	0.080	2,620,854	0.003	0.056
$\mathbb{1}(\text{exit}_{i,t+8})$	16,491,841	0.006	0.079	2,620,854	0.002	0.044
$\mathbb{1}(\text{exit}_{i,t+9})$	16,491,841	0.006	0.077	2,620,854	0.001	0.027
$\mathbb{1}(\text{exit}_{i,t+10})$	16,491,841	0.005	0.071			
$\mathbb{1}(\text{exit}_{i,t+11})$	16,491,841	0.004	0.066			
$\mathbb{1}(\text{exit}_{i,t+12})$	16,491,841	0.004	0.062			
$\mathbb{1}(\text{exit}_{i,t+13})$	16,491,841	0.003	0.057			
$\mathbb{1}(\text{exit}_{i,t+14})$	16,491,841	0.003	0.052			
$\mathbb{1}(\text{exit}_{i,t+15})$	16,491,841	0.002	0.046			
$\mathbb{1}(\text{exit}_{i,t+16})$	16,491,841	0.002	0.041			

Notes: The table shows the summary statistics of the dataset we use for the estimation of firms' pre-exit dynamics.

Table B.2: Summary Statistics of the Dataset for Pre- and Post-R&D Termination Firm Dynamics Estimation

Variables	$h'+1=1$			$h'+1=2$		
	No. of obs.	Mean	S.D.	No. of obs.	Mean	S.D.
$h=-5$						
$\log(\text{sales}_{i,t})$	79,698	14.498	2.300	75,287	14.626	2.267
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	79,698	0.290	0.454	75,287	0.319	0.466
$h=-4$						
$\log(\text{sales}_{i,t})$	84,112	14.526	2.323	79,610	14.656	2.290
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	84,112	0.286	0.452	79,610	0.315	0.465
$h=-3$						
$\log(\text{sales}_{i,t})$	88,793	14.548	2.345	83,912	14.682	2.312
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	88,793	0.285	0.451	83,912	0.313	0.464
$h=-2$						
$\log(\text{sales}_{i,t})$	93,617	14.563	2.369	90,348	14.674	2.325
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	93,617	0.282	0.450	90,348	0.312	0.463
$h=-1$						
$\log(\text{sales}_{i,t})$	101,852	14.527	2.379	90,348	14.681	2.325
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	101,852	0.284	0.451	90,348	0.312	0.463
$h=0$						
$\log(\text{sales}_{i,t})$	101,852	14.532	2.384	90,348	14.686	2.327
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	101,852	0.284	0.451	90,348	0.312	0.463
$h=1$						
$\log(\text{sales}_{i,t})$	88,293	14.658	2.412	90,348	14.691	2.333
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	88,293	0.275	0.447	90,348	0.312	0.463
$h=2$						
$\log(\text{sales}_{i,t})$	78,151	14.760	2.430	78,366	14.813	2.356
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	78,151	0.270	0.444	78,366	0.306	0.461
$h=3$						
$\log(\text{sales}_{i,t})$	69,144	14.861	2.451	69,017	14.919	2.373
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	69,144	0.267	0.442	69,017	0.301	0.459
$h=4$						
$\log(\text{sales}_{i,t})$	61,088	14.970	2.468	60,259	15.034	2.387
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	61,088	0.260	0.438	60,259	0.293	0.455
$h=5$						
$\log(\text{sales}_{i,t})$	53,557	15.096	2.477	52,257	15.152	2.392
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	53,557	0.250	0.433	52,257	0.282	0.450

Notes: The table shows the summary statistics of the dataset we use for the estimation of firms' pre- and post-R&D termination dynamics.

Table B.3: Summary Statistics of the Dataset for the Estimation of Distortions and Exit

Variables	$h=1$			$h=3$		
	No. of obs.	Mean	S.D.	No. of obs.	Mean	S.D.
Distortion: Net subsidy/Value-added						
$\log(\text{sales}_{i,t})$	9,064,930	11.609	1.725	6,983,006	11.706	1.698
$\mathbb{1}(\text{exit}_{i,t+1})$	9,064,930	0.006	0.079	6,983,006	0.007	0.081
Distortion	9,064,930	-0.073	0.088	6,983,006	-0.069	0.072
Distortion: Capital investment on used assets/Total capital investment						
$\log(\text{sales}_{i,t})$	4,756,232	11.776	1.736	3,577,931	11.885	1.726
$\mathbb{1}(\text{exit}_{i,t+1})$	4,756,232	0.006	0.076	3,577,931	0.006	0.080
Distortion	4,756,232	0.180	0.132	3,577,931	0.194	0.133

Notes: The table shows the summary statistics of the dataset we use for the estimation of the relationship between distortions and firm exit.

Table B.4: Summary Statistics of the Dataset for the Estimation of Distortions and R&D Investment $(h'+1=1)$

Variables	$h=1$			$h=3$		
	No. of obs.	Mean	S.D.	No. of obs.	Mean	S.D.
Distortion: Net subsidy/Value-added						
$\log(\text{sales}_{i,t})$	80,344	14.560	2.482	70,021	14.702	2.511
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	80,344	0.277	0.447	70,021	0.267	0.443
Distortion	80,344	-0.075	0.109	70,021	-0.077	0.114
Distortion: Capital investment on used assets/Total capital investment						
$\log(\text{sales}_{i,t})$	49,401	14.904	2.483	43,321	15.040	2.506
$\mathbb{1}(R\&D_{i,t-h,t-h+h'}=0)$	49,401	0.255	0.436	43,321	0.247	0.431
Distortion	49,401	0.150	0.134	43,321	0.149	0.136

Notes: The table shows the summary statistics of the dataset we use for the estimation of the relationship between distortions and the termination of R&D investment in the case of $h'+1=1$.

Table B.5: Relations between the R&D Frequency and the Exit Probability and Sales Growth

	Definition of R&D		
	R&D	Selling, general, and administrative (SGA) expenses	Sales promotion, advertising, entertainment, and other selling expenses
Number of firms			
All	4,236,113	4,236,113	4,236,113
R&D expenditure is not NA (A)	701,763	701,763	701,763
Zero R&D expenditure throughout	659,815	105,027	190,182
R&D expenditure is positive at least once (B)	41,948	596,736	511,581
(fraction, B/A)	(0.060)	(0.850)	(0.729)
Voluntary exit rate (the number of voluntary exit firms divided by the total number of firms)			
Probability of positive R&D			
Zero	0.032	0.045	0.044
Positive and 0 to 25%	0.012	0.045	0.041
25% to 50%	0.021	0.034	0.030
50% to 75%	0.017	0.022	0.020
75% -	0.014	0.014	0.014
Fraction of firms with positive average sales growth			
Probability of positive R&D			
Zero	0.469	0.460	0.440
Positive and 0 to 25%	0.516	0.419	0.429
25% to 50%	0.544	0.450	0.462
50% to 75%	0.584	0.485	0.488
75% -	0.614	0.554	0.573

Notes: NA represents not available. The probability of positive R&D is defined as the ratio of the periods in which R&D investment is positive to the periods in which sales are recorded. By the probability of positive R&D, we divide firms into five groups, that is, zero, under 25% (among firms with a positive probability of positive R&D), 25-50%, 50-75%, and over 75%.

Table B.6: Pre-exit Firm Dynamics: Dependence on Owner Ages

Pre-exit dynamics						
	Unbalanced			Firms surviving for at least 10 years		
	Coef.	s.e.		Coef.	s.e.	
β_1	-1.344	0.009	***	-1.623	0.022	***
β_2	-1.212	0.008	***	-1.54	0.021	***
β_3	-1.151	0.007	***	-1.491	0.021	***
β_4	-1.101	0.007	***	-1.469	0.022	***
β_5	-1.052	0.007	***	-1.436	0.023	***
β_6	-1.005	0.007	***	-1.404	0.025	***
β_7	-0.968	0.006	***	-1.381	0.029	***
β_8	-0.936	0.006	***	-1.373	0.036	***
β_9	-0.913	0.006	***	-1.424	0.058	***
β_{10}	-0.899	0.007	***			
β_{11}	-0.879	0.007	***			
β_{12}	-0.869	0.008	***			
β_{13}	-0.851	0.008	***			
β_{14}	-0.828	0.009	***			
β_{15}	-0.806	0.01	***			
β_{16}	-0.769	0.011	***			
Fixed-effect						
Year×Industry		yes			yes	
Number of observations		9,397,770			1,349,493	
Adj R-squared		0.1763			0.1771	

Notes: Coefficient β_h captures the relative sales of a firm terminating R&D as of $|h|$ years prior to exit.

Table B.7: Pre- and Post R&D Termination Firm Dynamics

Pre-R&D termination dynamics for $h' = 1$	$h=-5$			$h=-4$			$h=-3$			$h=-2$			$h=-1$			$h=0$		
	Coef.	s.e.		Coef.	s.e.		Coef.	s.e.		Coef.	s.e.		Coef.	s.e.		Coef.	s.e.	
δ_h	-0.819	0.015	***	-0.812	0.015	***	-0.818	0.015	***	-0.82	0.015	***	-0.8	0.014	***	-0.81	0.014	***
Fixed-effect																		
Year×Industry		yes			yes			yes			yes			yes			yes	
Number of observations	79,698			84,112			88,793			93,617			101,852			101,852		
Adj R-squared	0.3171			0.3256			0.3372			0.3471			0.3558			0.3541		
Pre-R&D termination dynamics for $h' = 2$																		
δ_h	-0.715	0.015	***	-0.719	0.015	***	-0.731	0.015	***	-0.722	0.014	***	-0.728	0.014	***	-0.735	0.014	***
Fixed-effect																		
Year×Industry		yes			yes			yes			yes			yes			yes	
Number of observations	75,287			79,610			83,912			90,348			90,348			90,348		
Adj R-squared	0.3212			0.3323			0.3439			0.3546			0.3537			0.3512		

Post-R&D termination dynamics for $h' = 1$	$h=1$			$h=2$			$h=3$			$h=4$			$h=5$		
	Coef.	s.e.		Coef.	s.e.		Coef.	s.e.		Coef.	s.e.		Coef.	s.e.	
δ_h	-0.85	0.015	***	-0.877	0.016	***	-0.91	0.018	***	-0.92	0.019	***	-0.981	0.021	***
Fixed-effect															
Year×Industry		yes			yes			yes			yes			yes	
Number of observations	88,293			78,151			69,144			61,088			53,557		
Adj R-squared	0.3586			0.3602			0.3623			0.3606			0.3592		
Post-R&D termination dynamics for $h' = 2$															
δ_h	-0.737	0.014	***	-0.753	0.016	***	-0.779	0.017	***	-0.803	0.018	***	-0.895	0.02	***
Fixed-effect															
Year×Industry		yes			yes			yes			yes			yes	
Number of observations	90,348			78,366			69,017			60,259			52,257		
Adj R-squared	0.3484			0.3491			0.3484			0.3468			0.3476		

Notes: Coefficient δ_h captures the relative sales of the firm that terminates R&D as of $|h|$ years before or after the termination. The negative (positive) number for h indicates $|h|$ years before (after) the termination.

Table B.8: Probit Estimations for Exit and R&D termination

Exit and R&D termination dynamics						
	Exit			R&D termination		
	Coef.	s.e.		Coef.	s.e.	
log (sales$_{i,t}$)	-0.186	0.001	***	-0.194	0.001	***
sales growth$_{i,t}$	-0.162	0.005	***	0.076	0.005	***
profit/sales$_{i,t}$	.00007	.00003	**	0.00002	0.00001	
Fixed-effect						
Year		yes			yes	
Industry		yes			yes	
Number of observations	6,793,163			4,015,461		
Pseudo R-squared	0.0812			0.1323		

Notes: The probit estimation for R&D termination is done for the data that include R&D records.

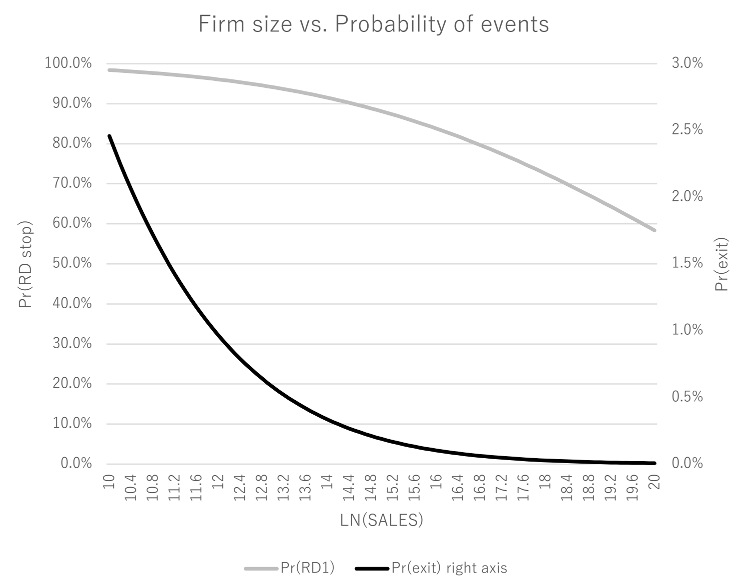

Figure B.1: Probabilities for exit and R&D termination conditional on firm sales

Note: The horizontal axis indicates the probability of firm exit (right axis) and R&D termination (left axis) conditional on the level of firm sales plotted on the horizontal axis.

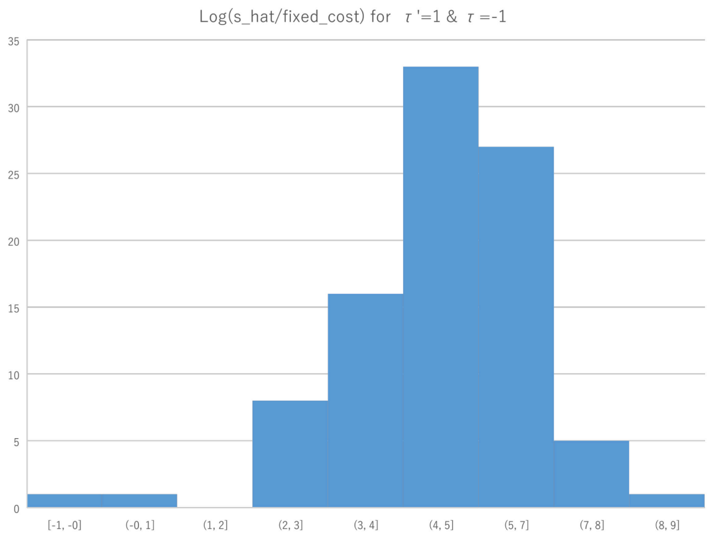

Figure B.2: Distribution of $\hat{s}$ over Fixed Costs

Note: The horizontal axis indicates $\hat{s}$ over fixed costs, where $\hat{s}$ is calculated as $\exp(\delta_{-1} + \gamma)$ for the regression of equation on R&D termination. The vertical axis is the number of industries.

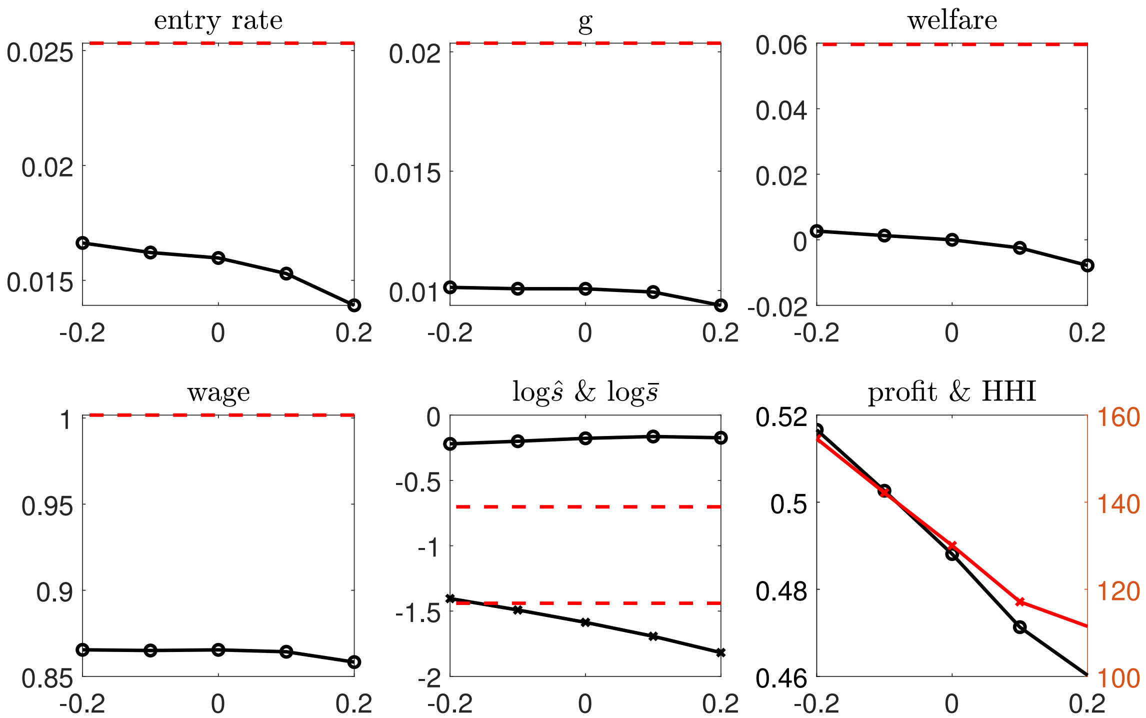

Figure B.3: socially optimal State and the Effects of a Size-Dependent Subsidy

Note: The horizontal axis represents subsidy $1 - \tau$; $\bar{s}$ and $\hat{s}$ are expressed in logarithm as the line with crosses and the line with circles, respectively; and the HHI is indicated as the red line with crosses on the right axis. The red dashed line represents the socially optimal state.