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The Effects of Emulation in the Reward System on Relative Deprivation, Selective Incentive, and Gender Inequality[1]

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Abstract

Except for Raymond Boudon's study (Boudon 1981), which explains the cause of paradoxical increases in people's dissatisfaction in a society or in an organization where social opportunities are more abundant, and a study by Ishikawa (1981) on labor incentive in the workplace, social consequences of emulation, which is defined as competition under the presence of rewards for relative standing in position, have not been systematically analyzed in sociological theory based on rational choice models. This paper attempts to fill this niche by introducing a formal comparison of a reward system without emulation and two distinct reward systems with emulation, effort-based emulation and performance-based emulation, to clarify how the presence of emulation changes people's choices and how those choices generate certain macrosocial consequences. First, the study shows that effort-based emulation and performance-based emulation have a commonality in causing people to expend more effort at work.

At the same time, the paper also shows that effort-based emulation and performance-based emulation have very different effects on who receives stronger incentives to raise effort levels, and, as a result of this, the two kinds of emulation have contrasting effects on (a) the tendency for certain groups of people to experience relative deprivation due to their reduced benefits despite their increase in the amount of effort at work that the emulation has provoked, and (b) gender inequality in the presence of a gender gap in salary/wages or in the presence of a traditional household division of labor. As in Boudon's theory, all theoretical implications are derived through deductive reasoning based on the models of reward systems.

Keywords: emulation, reward system, relative position, relative deprivation, selective incentive, gender inequality

JEL classification: M52, J01, D90, J71

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I. INTRODUCTION

This paper presents a theory that gives a distinct theoretical linkage between two groups of related theories, one about relative deprivation and the other about relative standing in position. As I review in this introduction, there have been studies that are related to both groups of theories, but as is typically the case in studies on happiness, relative deprivation often implies a feeling of relative inferiority in position, so that the functional difference of these two theoretical concepts has not been very clear. On the other hand, in this paper, relative deprivation and relative position appear on opposite sides of causality, one as a consequence and the other as a cause of social actions. This paper introduces this new linkage through the modeling of *emulation*, which implies in this paper competition for rewards based on *relative standing in position* (Ishikawa 1981).[2] For example, if school grades are based on absolute test scores, grades are not the results of emulation. On the other hand, if grades are “curved” according to the percentile of the test scores among students who stook the examination, grades are the results of emulation. College admission is also an emulation if the number of students to be admitted is fixed, and it is not an emulation if the number of students to be admitted can be adjusted in accordance with the number of qualified applicants by a fixed criterion of qualification. Competitions involve emulation when awards are given only to a fixed number of the top few. The International Mathematical Olympiad among high school students, however, is not an emulation, because the numbers of gold, silver, and bronze medalists are not fixed but are based on individual performance on the test. Promotion in the internal labor market is an emulation because it is a vacancy-chain competition among a small number of qualified people who are

[2] Ishikawa, in a research article written in Japanese, used the term “emulation” in his game-theoretic models of work incentives in team work to show how a reward for relative labor input compared with others affects workers’ behavior. Although his game-theoretic models differ greatly from the models introduced in this paper, one finding is common: emulation in work effort induces workers to expend more effort in work.

already employed in the firm. On the other hand, if wages are equated with the marginal productivity of labor, which neoclassical economics theory considers rational wage allocation, then wage competition is not an emulation.

Relative deprivation has been considered important in sociological and economic theories from various different theoretical viewpoints. However, we can classify theories related to relative deprivation by two dimensions. One dimension is the distinction between theories that are mainly related to consequences of relative deprivation and theories that are mainly related to causes of relative deprivation. Another dimension is the theory's substantive content. On group of studies is concerned with the role that relative deprivation plays in explaining the relationship among income, social inequality, and happiness. The second group is concerned with an explanation for the paradoxical negative relationship between social opportunity and people's satisfaction with the social system. The third group is concerned with the role that the avoidance of relative deprivation, or, more generally, the seeking of better relative standing in position, plays as a cause of social choices and actions. Table 1 presents a cross-classification of theories with names of the major contributors whose studies are briefly reviewed below, except for the ultimatum game, which is replaced by a category rather than by the names of contributors.

(Table 1 About Here)

Regarding the first substantive area, Runciman's (1966) primary contribution is his micro conception of the causes of relative deprivation as the results of social comparisons within one's own group as well as between groups. By extending this conception, he also considers status inconsistency to be a cause of relative deprivation (Runciman and Bagley 1969).

A theoretical interest in the consequence of relative deprivation in this area of studies is exemplified by the Easterlin paradox. From the comparison of the average life satisfactions of people across time in each nation as well as across nations, Easterlin (1974, 1995) observed that (1) the happiness of people within a nation was positively correlated with individual income within each nation, and (2) the average happiness of people was positively correlated with the average income among nations, and yet (3) the average happiness of people in nations did not change significantly over time even if nations experienced growth in the average income.

The third observation was considered paradoxical, given the first two observations: if a higher income increases individual happiness, it should also hold that the average happiness of people should increase when the average income becomes higher. This is one of the earliest cases where the notion of relative deprivation was introduced to explain a paradox, by the argument that people's happiness may depend on the *relative* standing in income, which may not change despite growth in the average income.

However, the explanation was incomplete, because Easterlin did not provide a clear explanation of the mechanism by which individual-level relative deprivation is determined, and how it affects the aggregate-level distribution of happiness. In particular, Easterlin's theory does not explain "the China puzzle" (Brockmann et al. 2008). Brockmann et al. (2008) showed that during 1990-2000 the People's Republic of China (PRC) experienced a rapid increase in the average income, but the level of life satisfaction steadily decreased during that time. Easterlin's theory may have provided an explanation for why people's average level of satisfaction remains the same despite growth in the average income, but it cannot explain why it decreases when the average income is growing

A theoretical explanation for the China puzzle can be derived, however, from Yitzhaki's study (1979). Yitzhaki first defines the individual level relative deprivation measure as the average of dyadic measures based on the comparison of each person's income with each of all other persons in society, by defining the dyadic measure as the difference in income when the person's income is smaller than that of another person, and as a constant zero when the person's income is greater than that of another person. Then, he showed that the societal average of these individual-level relative deprivation measures becomes mathematically proportional to the Gini coefficient of the income distribution in the population (Ishida et al. 2014, Ishida, 2015). Hence, this theory considers social inequality the major cause of the macro distribution of the extent of relative deprivation. Indeed, Ravallion and Chen (2007) showed that the Gini coefficient increased significantly during 1990-2000 in the PRC, and, therefore, Yitzhaki's theory offers an explanation for the China puzzle.

Another major theory of the causes of relative deprivation is introduced by Boudon (1981). Boudon's theory on relative deprivation is also related to an explanation for paradoxical empirical findings called the Tocqueville paradox and the Stouffer paradox. Tocqueville in his *Democracy in America* (Tocqueville 1835) observed that greater equality of opportunity tended to make individuals find their inequality harder to bear. Similarly, in their study *The American Soldier* (Stouffer et al. 1949), Stouffer et al. observed that soldiers who worked in the air force department with greater promotion opportunities than those in the military police department were more discontent with their promotion system. Similarly, Stouffer et al. also found soldiers with higher education were less content with the promotion system despite the fact they had greater promotion opportunities. In both Tocqueville's and Stouffer et al.'s observations, a greater social opportunity makes peoples less satisfied with the society or the system, which is indeed paradoxical. What Boudon has shown in explaining the mechanism of these paradoxes in his rational choice model is a theoretically derived fact that when the extent of social opportunity becomes greater, starting from a relatively low level of social opportunity, and if the attainment of a desirable status depends on individuals' human-capital investment in themselves, such as investment in college education for the attainment of professional and managerial occupations or increased effort in the workplace, then the growth rate in the number of people who make an investment for status attainment tends to exceed the growth rate in the number of desirable positions available for those who make the investment. It follows that when social opportunity increases through an increase in the number of desirable positions for status attainment, there will be many more people who cannot attain the desirable status despite their investment to obtain the status, thereby generating a greater number of people who are relatively deprived. Boudon's theory is elaborated by Kosaka (1986), Yamaguchi (1988), and Manzo (2011). In particular, by introducing population heterogeneity in the costs and benefits of actions and in the probability of status attainment given the actor's investment into Boudon's model, Yamaguchi (1988) showed that one of the factors that causes a higher rate of increase in the proportion of people who are relatively deprived is a higher average incentive to make human capital investment because of the higher expected benefit of the status attainment by the investment. Hence, a modification of the individual incentive changes the macrosocial situation of relative deprivation, and this fact is reproduced in a distinct form in the present paper.

While Boudon's theory and its extensions may not explain well the China puzzle, because rapid growth of college attendance rates in the PRC started only in late 1990s, it is consistent with the case of South Korea, which has attained the world highest college admission rate, about 80%, but also has one of the highest rates among nations regarding the rate of unemployment among young people with college degrees.

Boudon's theory is related to the theory introduced in this paper, because both Boudon's theory and the one introduced here assume a behavioral model of rational choice. In addition, although emulation is more explicitly modeled in the theory introduced in this paper, Boudon's theory also assumed competition for a desired status which is fixed in number in society. The major difference, however, is the fact the model in this paper is concerned with the effects of distinct reward systems with and without emulation on the amount of work effort people expend to attain higher rewards and therefore more closely related to organizational study and management science. However, it has a similarity with Boudon's theory on the cause of relative deprivation, because the theory introduced in this paper shows that that some people get a reduced reward despite their increase in effort under emulation and thereby become relatively deprived.

The third group of theories, which I refer to as theories of relative standing in position, is primarily concerned with the consequences of relative deprivation, or more generally the consequence of preferring relative standing to absolute standing in position in the choice of action. Gurr's study (Gurr 1970) of relative deprivation as the major cause of rebellion is an example. In behavioral economics, people's apparent choices to weigh equity with rationality, and to reject an offer below certain percentage of share in the benefit allocation has become well known through the experimental study of the ultimatum game. More recently, Frank and

Sunstein (2011) developed a "relative position theory" to emphasize that people's choices often aim at improving their relative standing in position, rather than their absolute standing. Frank (2012) extends the theory to distinguish between "positional consumption," for which the relative standing in the particular context matters, and "nonpositional consumption," for which the absolute standing independent of the context matters, and observes consumption as positional "in proportion to the extent to which additional investment in the category makes individual reproductive success more likely" (page 72). He shows that people's choices depend on relative standing for a wide range of issues and claims that the reason for those choices is similar to the logic of evolutionary advantage in the theory of Charles Darwin, so that the attainment of relative, rather than absolute, advantage in the context increases chances of better survival and success over generations. While whether this evolutionary reason is the real cause of preferring relative rather than absolute standing in position is still hypothetical, Frank's theory is important, not only because it is concerned with regularities of social choices that differ from the expected choices of rational utility-maximizing actors, as in other theories of behavioral economics, but also because a specific motivation for social actions, namely, the seeking of higher relative position, or the combination of the avoidance of relative deprivation and seeking of relative gratification, is considered one of the major causes of those regularities, and those regularities bring distinct social consequences.

Jasso's theory (Jasso 1980) also has elements of both causes and consequences of relative deprivation and relative gratification. As in the third group of theories, her theory is concerned with choices and actions based on the avoidance of relative deprivation and seeking of relative gratification. Hence, her theory is concerned with relative deprivation as a cause of social action. At the same time, as in the first group of theories, her theory explicitly assumes that relative deprivation and relative gratification arise from social comparisons where "actual rewards" are compared with "just rewards," thereby incorporating a social psychological cognition of justice about how much reward actors believe they deserve as a basis for assessing relative deprivation and gratification. Jasso's theory thus provides a linkage between the first and third groups of theories by "justice evaluation." My reservation about her theory is that although the hypothetical examples she gives as applications of her theory assume that the maximization of relative gratification is the sole principle of social choices and actions, I consider it only one element of the determinants of actors' social choices.

The theory introduced in this paper provides a distinct linkage between two groups of theories, the second and the third groups. A unique aspect of this theory is that the role that relative position plays greatly differs from the role that relative deprivation plays. Seeking the improvement of relative position is a cause of actions, as in the third group of theories. Unlike those theories, however, the theory introduced in this paper assumes that the seeking of improvement in relative position is not one's preference, but is externally imposed by the presence of a reward system with emulation. Unlike the third group of theories, the theory introduced in this paper assumes that relative deprivation is not a cause, but a consequence of social actions, as in the second group of theories, represented by Boudon's study. The behavioral model is the same as the model of utility maximizers, as in Boudon's theory. The substantive focus, however, differs and is on how two distinct forms of emulation, which are referred to as *effort-based emulation* and *performance-based emulation*, bring different outcomes in relative deprivation, selective incentives, and gender inequality. As in Boudon's theory, the theory developed in this paper is deductive and is based only on mathematical and logical reasoning.

II. Models of the Reward System with and without Emulation

II-1. Reward System without Emulation: The Baseline Model

First, the paper describes a model without emulation, which is called the baseline model. The purpose of this baseline model is to provide a standard against the properties of which we can derive some theorems on the role of emulation when emulation is introduced into the baseline model.

We assume the following baseline model of performance-based reward without emulation for the utility for person *i* who holds job *j*:

$$U_{i,j}(x_i) = \lambda_j y(x_i, \theta_i) - \alpha_i z(x_i),$$

$$\text{where } \lambda_j > 0,\ \alpha_i > 0, \theta_i > 0, y_i(0, \theta_i) = 0, z(0) = 0, \text{and } U'(0) > 0. \tag{1}$$

In this equation, x_i indicates the amount of work effort expended by person *i*, $y(x_i, \theta_i)$ indicates the work performance of person *i* for a given amount of work effort x_i and a given extent of person *i*'s work-related ability θ_i, and λ_j is the reward for job *j*, given for each unit level of performance.

Function $z(x_i)$ indicates the opportunity costs of expending effort for the job, and α_i is a parameter for its individual heterogeneity. Note that what $\alpha_i z(x_i)$ represents here is the opportunity costs of expending effort in employment and is not the opportunity costs of choosing nonemployment. The former is the costs of sacrificing private and family lives due to time and effort spent for employment; the latter is the costs of forgone income by choosing nonemployment or by leaving employment.

Regarding the work performance function $y_i(x_i, \theta_i)$, we make the following three assumptions, (A1), (A2), and (A3). Note that the total derivative of y with respect to x, dy/dx, is equal to its partial derivative for a given θ, because θ is not a function of effort x. We denote the first and the second derivatives of y with respect to x, dy/dx and $d^2y/(dx)^2$, by y' and y'', respectively, below.

(A1) $y(x_i, \theta_i)$ is a monotonically increasing function of x, that is, $y'>0$. In other words, if work-related ability is the same, more effort generates better performance.

(A2) $y'(x_i, \theta_i)$ is a monotonically decreasing function of x, that is, $y''<0$. In other words, while an additional unit of effort increases performance, the marginal return of the additional effort in raising performance declines with the amount of work effort.

(A3) $y_i(x_i, \theta_i)$ is a monotonically increasing function of θ, such that $\partial y(x, \theta)/\partial \theta > 0$ when $x>0$. In other words, if the amount of work effort is the same, a person with a higher ability performs better at work.

In addition, how the growth rate of performance $y'(x_i, \theta_i)$ behaves as a function of θ, the sign of $\partial y'(x, \theta)/\partial \theta$ in particular, becomes crucial in understanding the characteristics of the model, as shown below. However, this issue is also related to what kinds of work-related ability we are concerned with, because there can be more than one kind of work-related ability that affects work performance. We will come back to this discussion later.

Regarding the opportunity cost function $z(x_i)$, we make the following two assumptions, (A4) and (A5):

(A4) $z(x_i)$ is a monotonically increasing function of x, that is, $z'>0$, for $x>0$. In other words, a greater amount of work-related effort generates greater opportunity costs.

(A5) $z'(0)=0$ and $z'(x_i)$ is a monotonically increasing function of *x*, that is, $z''>0$. In other words, the growth rate for the opportunity costs as a function of effort is initially zero, but each additional effort generates a higher rate of growth in opportunity costs.

Assumption (A5) means that $z(x)$ is unbounded as a function of *x* and can be very large as *x* increases. Assumption (A5) is made because we consider the amount of work effort to be partly a function of time spent for work effort, but the time which can be used to expend additional effort at work is limited, and, therefore, each additional effort to raise work performance becomes more costly in opportunity. This is because amounts of time spent for work, family and private life, and subsistence (sleeping, eating, etc.) for each day are to some extent complementary, and, therefore, it becomes increasing costly to reduce time for the latter two activities in order to increase time for work-related activities. Note that the combination of assumptions (A2) and (A5) makes $U''(x)<0$. In order to distinguish otherwise confusing quantities, we refer to $\alpha_i z(x_i)$ as the opportunity costs, α_i as the coefficient of the opportunity costs, and $\alpha_i z'(x_i)$ as the growth rate of the opportunity costs.

In addition, we assume that actors behave rationally to maximize their utility. In equation (1), utility is maximized when $U'(x)=0$. Hence, the optimal amount of *x*, denoted by *x**, satisfies $\lambda_j y'(x_i^*, \theta_i) = \alpha_i z'(x_i^*)$, or

$$\frac{y'(x_i^*, \theta_i)}{z'(x^*)} = \frac{\alpha_i}{\lambda_j}. \qquad (2)$$

Note that, since $y'>0$ monotonically decreases with *x* and $z'>0$ monotonically increases with *x*, starting from value 0, in accordance with assumptions (A2) and (A5), the ratio y'/z' monotonically decreases with *x*, starting from near infinity, and approaches 0 as *x* becomes very large, and, therefore, equation (2) has a single solution for *x**. When the ratio of α_i/λ_j becomes smaller, the optimal amount of work effort x* becomes larger. In other words, the quantity of equation (2) gives a measure of *disincentive* for raising the effort level, or, equivalently, the inverse of the quantity of equation (2) is a measure of incentive for raising the effort level. Thus, as the property of the baseline model, the following two theorems hold

(B1) Other things being equal, people who have a greater coefficient (α) of the opportunity costs expend a smaller amount of work effort.

(B2) Other things being equal, people with a higher reward for work performance (λ) expend a larger amount of work effort.

The effect of ability θ on the optimal amount of effort is more complicated. Generally, it is useful to distinguish two kinds of work-related abilities, namely, ability to attain high work quality and ability to work efficiently, which we model formally as follows:

We consider the expression of performance to be a function of two parameters of work-related abilities which we call *work quality* and *work efficiency*, denoted by θ^Q and θ^E, respectively, and assume the following model of work performance as the additional assumption (A6) of the baseline model.

(A6) The work performance function y can be expressed as a function of work effort and two parameters for ability:

$$y(x_i, \theta_i^Q, \theta_i^E) = \theta_i^Q f(\theta_i^E x_i), \theta_i^Q > 0, \theta_i^E > 0, \qquad (3)$$

where a smooth function f(.) satisfies conditions $f(0) = 0,\ f'(x) > 0,$ and $f''(x) < 0$ in accordance with assumptions (A1) and (A2).

We can regard $f(\theta^E x)$ as the quantity of work output by effort x for a given level of θ^E, θ^Q as the quality of each unit of work output, and performance as the product of these two elements. Thus, we refer to θ^Q as *work quality*. Under assumption (A6), we obtain

$y'(x_i, \theta_i^Q, \theta_i^E) = \theta_i^Q \theta_i^E f'(x\theta_i^E) > 0$, and therefore, y' linearly increases with work quality θ^Q. A specification of the work-related ability in equation (3) causes equation (1) to be expressed as $U_{i,j}(x_i) = \lambda_j \theta_i^Q f(\theta_i^E x_i) - \alpha_i z(x_i)$, and, therefore, parameter θ^Q appears in the product $\lambda_j \theta_i^Q$. This may seem to make parameters λ_j and θ_i^Q indistinguishable. One may also argue that, since the reward for performance may already reflect the quality of work, parameter θ^Q is redundant. However, parameter λ_j is a characteristic of jobs, and parameter θ_i^Q is a characteristic of individuals, and, therefore, they are conceptually distinct. Indeed, the actual reward, such as salary, may consist of a job-specific element and an individual-specific element, and differences among individuals in the quality of work in the same job may still affect individual differences in salary, while the average salary among people also depends on their jobs. The distinction is important, because even though the effects of these two parameters on the optimal work effort are not distinguishable in the baseline model, their effects become quite different in one of two emulation models described later.

Since a greater value of $y'(x_i, \theta_i^Q, \theta_i^E) = \theta_i^Q \theta_i^E f'(\theta_i^E x)$ due to a greater value of θ^Q requires more effort to satisfy equation (2), it follows that

(B3) Other things being equal, people whose work quality (θ^Q) is higher expend a greater amount of work effort.

For another parameter θ^E that affects the quantity of the output of a given amount of work effort, it is expressed as the multiplier of the effort in the quantity of output $f(\theta_i^E x_i)$. Suppose, for simplicity of explanation, that effort is measured by time of input. Then if $\theta^E = 2$ for a person, this person can finish in one full day a task that a person with $\theta^E = 1$ needs two full days to finish. Hence, θ^E indicates the relative speed of task performance, or the *work efficiency*.

However, whether $y'(x_i, \theta_i^Q, \theta_i^E) = \theta_i^Q \theta_i^E f'(\theta_i^E x)$ increases or decreases as a function of θ^E is not certain. This is because, since $\partial y'(x_i, \theta_i^Q, \theta_i^E) \big/ \partial \theta_i^E = \theta_i^Q \left\{ f'(x_i, \theta_i^E) + (\theta_i^E)^2 f''(x_i, \theta_i^E) \right\}$, it is positive when θ^E is close enough to 0, because $f'(x_i, \theta_i^E)$ >0. However, since $f''(x_i, \theta_i^E)$ <0, the sign of $\partial y'(x_i, \theta_i^Q, \theta_i^E) \big/ \partial \theta_i^E$ cannot be determined. An additional formal investigation given in the appendix that considers two concrete representative functions for *f*(.), which satisfies $f(0) = 0$, *f′*>0, and *f″*<0, indicates, however, the following property:

(B4) It is highly likely that except for an increase from a very small value of work efficiency θ^E, for which an increase in θ^E has a positive effect on the rate of growth in performance ($y'(x_i, \theta_i^Q, \theta_i^E)$), a difference in work efficiency has an increasingly smaller impact on change in the value of $y'(x_i, \theta_i^Q, \theta_i^E)$ as work efficiency increases, and the impact becomes negligible as θ^E becomes large.

We refer to characteristics (B1), (B2), (B3), and (B4) given above as the four properties of the baseline model below. Now I clarify the characteristics of models that introduce into the baseline model one of the two alternative forms of an emulation-based reward system.

II-2. Reward System That Introduces Effort-Based Emulation

The reward system with effort-based emulation implies that in addition to a reward based on individual performance, there is an additional reward or penalty depending on the amount of work effort compared with the work effort of others having the same job type defined below. Then, the model is specified as follows:

$$U_{i,j}(x_i) = \lambda_j y(x_i, \theta_i) - \alpha_i z(x_i) + \beta_{k(j)}^X (x_i - \bar{x}_k), \qquad (4)$$

where $\bar{x}_k$ is the average amount of work effort made among *other* people having a job type *k* that is the same as that of person *i*, either within the work organization or in a collectivity, depending on where the emulation occurs, and β_k^X indicates the reward received or the penalty imposed, depending on how much one's effort is more than or less than the average effort expended among these comparable others. Here, a "job type" indicates a group of jobs within each of which emulation is assumed to occur. For example, a job type may indicate an occupation, whereas distinct jobs with which reward λ_j varies within each occupation may reflect distinctions of jobs by job rank and employment status such as a distinction between parttime and fulltime workers, and differences in job tenure.

Then, when this utility function is maximized, *U′*(*x*)=0, and that makes the optimal amount of work effort *x** satisfy $\lambda_j y'(x^*, \theta_i) - \alpha_i z'(x^*) + \beta_{k(j)}^X = 0$, or

$$\frac{y'(x_i^*, \theta_i)}{z'(x^*)} = \frac{\alpha_i - \dfrac{\beta_{k(j)}^X}{z'(x^*)}}{\lambda_j} < \frac{\alpha_i}{\lambda_j}. \qquad (5)$$

Since *y′*/*z′* monotonically decreases with *x* and the right-hand side of equation (5) is smaller than that of the baseline model, the optimal amount of work effort under the effort-based emulation increases for everybody compared to the amount under the baseline model without emulation. Hence, the major property of the effort-based emulation is that

(E1) Under the presence of effort-based emulation, the optimal amount of work effort increases for everybody.

As equation (4) indicates, the average reward/penalty associated with emulation is assumed to be zero in the population of reward recipients, and therefore, there are no net average benefits/costs of emulation either for those who pay those rewards β_k^X for the emulation, or for those who receive them in the population. Hence, although collectivities, work organizations, or societies that provide rewards/penalties for emulation may benefit because people expend more effort and thereby increase the collective performance to that extent, individuals will have to expend more effort with emulation than the optimal amount of work effort they would expend without emulation, and therefore the average utility in the population of reward recipients becomes smaller. Hence, we may conclude that effort-based emulation brings negative externalities for reward recipients—though I discuss this issue further in the concluding section.

Equation (4) also indicates that the basic properties of the baseline model, (B1), (B2), (B3), and (B4), regarding who expends more effort as a result of rational choice, are maintained under the effort-based emulation model as well.

Although the reward with effort-based emulation increases the optimal amount of work effort for everybody, it does not give the incentive to increase the amount of effort uniformly. Some are given more incentive to increase the effort level more than others. This is an issue of the *selective incentive* of the reward system with emulation.

. A theoretical question here is what the indicator of the extent of the relative incentive to expend more effort under emulation should be. In particular, should it be the difference or the ratio of the incentives to expend effort between the two reward systems? The answer is that it should be the ratio rather than the difference. There are two reasons. First, while the disincentive measures, indicated by equation (2) for the baseline model and by equation (5) for the effort-based emulation model, are expressed as the ratio of two factors, it does not make much sense to take the difference of the two ratios if each factor's unit of measurement is not specified, but the ratio of the two ratios retains the meaning of the impact of the relative increase/decrease of each factor for a given unit. Second, when the work performance function

$y = \theta^Q f(\theta^E x)$ has the logarithmic form for *f*(.), then, *y*′, which is the numerator of equations (2) and (5), is highly correlated to the inverse of *x*, and, therefore, the ratio of *y*′ is also highly correlated with the ratio of *x*, as shown in the appendix, and the ratio of *x* is the indicator of relative extent of expending effort.

On the basis of this reasoning, and since each of quantity of equations (2) and (5) gives measures of the disincentive of expending effort, the measure of the *relative incentive* to expend effort under the emulation system becomes the disincentive measure of the baseline model divided by the disincentive measure of the emulation model, given as follows:

$$\left(\frac{y'(x_i^B*,\theta_i)}{z'(x_i^B*)}\right)\Big/\left(\frac{y'(x_i^E*,\theta_i)}{z'(x_i^E*)}\right) = \frac{1}{1-\dfrac{\beta_k^X}{\alpha_i z'(x_i^E*)}}, \qquad (6)$$

where x_i^B* and x_i^E* indicate the optimal amount of effort to be expended under the baseline model and the effort-based emulation model, respectively. Note that the denominator of equation (6) never becomes 0 or negative, because, from the equilibrium condition of $\lambda_j y'(x_i,\theta_i) - \alpha_i z'(x_i) + \beta_k^X = 0$, $\alpha_i z'(x_i) - \beta_k^X = \lambda_j y'(x_i,\theta_i) > 0$ holds. Hence, the relative incentive to expend more effort becomes greater when $\beta_k^E / \alpha_i z'(x_i^E*)$ is greater. From this, we obtain the following properties of effort-based emulation on selective incentive:

(S1) Other things being equal, the relative incentive to expend additional effort under effort-based emulation is greater when the reward from emulation for the job type (β_j^X) is greater.

(S2) Other things being equal, the relative incentive to expend additional effort under effort-based emulation is greater for people for whom the coefficient α_i of opportunity costs is smaller.

(S3) The relative incentive to expend additional effort due to a smaller coefficient of opportunity costs is reduced to the extent that the amount of work effort expended under the baseline system is greater.

Property (S3) holds because $z'(x_i^E*)$ is a monotonically increasing function of the effort amount, and the relative incentive is smaller for people with greater $z'(x_i^E*)$, which is highly correlated with $z'(x_i^B*)$ because of the commonalities of the major determinants of the amount of work effort between the two reward systems. Findings (S2) and (S3) indicate the following:

(E2) Effort-based emulation works toward more inequality in the amount of work effort people expend by selectively motivating people with a smaller coefficient of opportunity costs, and who are thereby already expending greater effort than others on average under the baseline system, to expend more effort. However, the tendency to generate greater inequality in the amount of work effort is smaller among people who already expend a large amount of work effort in the baseline system than among people who expend a small amount of work effort.

Equation (6) also shows the following property:

(S4) Reward for performance for jobs (λ_j) does not affect the relative incentive under effort-based emulation, and, therefore, people with different amounts of job-specific rewards are equally motivated to expend additional effort under effort-based emulation.

Regarding the effects of work-related ability, the work quality does not affect the relative incentive at all, because, since $y'(x,\theta^Q,\theta^E) = \theta^Q\theta^E f'(\theta^E x)$, the left-hand side of equation (6) does not include parameter θ^Q, because it is canceled out between the numerator and the denominator of the equation. Hence, (S5) Work quality θ_i^Q does not affect the relative incentive under effort-based emulation, and, therefore, people with different work quality are equally motivated to expend additional effort under effort-based emulation.

Boudon (1981) considers that people make investments on their own human capital to attain a desired position but fail to attain the status experience "relative frustration," which we can call relative deprivation. Similarly, we may consider that people who increases the amount of their effort under the new reward system with emulation *as much as others, or even more than others*, but get penalized as a result of their relatively inferior standing in the emulation will experience relative deprivation. Then, the combination of properties (B2) and (S4) indicates the following.

(R1) Other things being equal, those who have a lower job-related reward (λ_j) than others with the same job type *k* will be more likely to be relatively deprived.

Similarly, the combination of properties (B3) and (S5) indicates the following.

(R2) Other things being equal, those who have lower work quality (θ^Q) will be more likely to be relatively deprived.

On the other hand, those with a larger coefficient for opportunity costs α_i who expend less effort than others and are therefore likely to be penalized under the effect-based emulation may not be relatively deprived nonetheless because they do not expend as much additional amount of effort as others under effort-based emulation.

An implication of property (R1) on gender inequality is important.

(G1) In a society, such as Japan and Korea, where the gender inequality in salary and wages for the same occupation arises partly from gender differences in employment status and the duration of employment for the same employer, women are more likely to be relatively deprived than men under the effort-based emulation system because job reward λ_j is smaller for the same job type *k*, on average, for women than for men.

Regarding the effects of work efficiency on relative incentive, equation (6) can be rewritten as

$$\frac{f'(\theta_i^E x_i^B *)}{f'(\theta_i^E x_i^E *)}\frac{z'(x_i^E *)}{z'(x_i^B *)} = \frac{1}{1-\dfrac{\beta_k^E}{\alpha_i z'(x_i^E *)}}. \qquad (7)$$

Since *f′* > 0 monotonically decreases with θx, and the optimal amount of work effort $x_i^E *$ under emulation is greater than the optimal amount of work effort $x_i^B *$ under the baseline model, $\frac{f'(\theta_i^E x_i^B *)}{f'(\theta_i^E x_i^E *)}$ >1, and when θ_i^E approaches 0, this ratio approaches 1.0. A question is whether this ratio monotonically increases with θ_i^E. Although this cannot be determined just from the assumptions we made for the model, a further analysis of concrete cases of function *f*(.) with either an exponential decay or a hyperbolic decay in $f'(x)$, indicates, as explained in the appendix, that the ratio monotonically increases with θ_i^E. Hence, we obtain the following property of effort-based emulation.

(S6) It is highly likely that higher work efficiency gives a greater relative incentive to expend more effort under effort-based emulation.

The combination of property (B4) concerning the effect of work efficiency on the optimal amount of work effort and property (S6) on the selective incentive indicates that

(E3) Effort-based emulation generates more inequality in the amount of work effort to be expended among people with different work efficiency.

Properties (E2) and (E3) reveal the following characteristic:

(E4) Effort-based performance not only increases the optimal amount of work effort to be expended by everybody, but also generates greater inequality among people in the optimal amount of work effort to be expended.

II-3. Reward System with Performance-Based Emulation

When the emulation is performance based, we have an alternative model:

$$U_{i,j}(x_i) = \lambda_j y(x_i, \theta_i) - a_i z(x_i) + \beta^Y_{k(j)}(y(x_i, \theta_i) - \bar{y}_k) , \quad (8)$$

where $\bar{y}_k$ is the average work performance of people with comparable job type *k*, and β^Y_k indicates the reward received or the penalty imposed depending on how much one's performance is better than or worse than the average performance of these comparable others[3]. This utility is

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[3] One aspect of the generality of the finding is worth mentioning. Although I characterized reward/penalty of emulation as a linear function of difference between own performance and the average performance of comparable others, one may argue that in the real world, the reward, or the award, of performance-based emulation may be typically given only to a few top people, as in sport competitions. Those who get rewarded/penalized may depend on a threshold, such as above versus below the mean or only to a top few. When a dichotomous distinction between those who "win" and "lose" in emulation has to be made with a threshold such as the mean, we have to replace $(y_i - \bar{y})$ in the emulation component with the probability of performing better than average $P(y_i - \bar{y} > 0)$, by assuming a monotonic link function, such as $P(y - \bar{y} > 0) = \exp\left\{c(y - \bar{y})/(1 + c(y - \bar{y}))\right\}$. Then, even though the mathematical formula of equations (9) described below become different, all qualitative properties of the emulation model remain the same.

Whether the threshold of reward/penalty is the mean or is different from the mean does not make any difference in the qualitative, or formal, properties of the emulation models, but quantitatively, it gives different results on relative deprivation. Assuming the average net benefit from the emulation is zero in the population, the size and severity of the relative deprivation among those who get penalized because of their failure in emulation changes. The smaller the number of people who win in the emulation, the greater the number of the relatively deprived, but the less the degree of relative deprivation for each of those persons. Conversely, the smaller the number of people who win in the emulation, the smaller the number of the relatively gratified, but the more the degree of relative gratification for each of those persons.

maximized when U'(x)=0, that is, when $\lambda_j y'(x_i, \theta_i) - a_i z'(x_i) + \beta^Y_{k(j)} y'(x_i, \theta_i) = 0$. It follows that the optimal amount of work effort under the performance-based emulation model satisfies

$$\frac{y'(x^*, \theta_i)}{z'(x^*)} = \frac{\alpha_i}{\lambda_j + \beta^Y_{k(j)}} < \frac{\alpha_i}{\lambda_j}. \qquad (9)$$

Since y'/z' monotonically decreases with *x*, the optimal amount of work effort to be expended under performance-based emulation also increases for everybody compared with the amount of effort to be expended under the baseline system. Hence,

(E5) Like effort-based emulation, performance-based emulation increases the optimal amount of work effort for everybody.

Equation (9) indicates that four basic properties of the baseline system, (B1), (B2), (B3), and (B4), regarding who expends less effort as results of rational choice, are maintained under performance-based emulation.

Regarding the relative incentive of the performance-based emulation compared with the baseline model, its indicator is given as

$$\left(\frac{y'(x_i^{B}*, \theta_i)}{z'(x_i^{B}*)}\right) \Bigg/ \left(\frac{y'(x_i^{E}*, \theta_i)}{z'(x_i^{E}*)}\right) = 1 + \frac{\beta^Y_{k(j)}}{\lambda_i}. \qquad (10)$$

This formula indicates that the following characteristics hold for the selective incentive of performance-based emulation:

(S7) Other things being equal, the relative incentive to expend additional effort under performance-based emulation is greater when the reward from emulation for the job (β^Y_j) is greater.

(S8) Unless the reward for work performance and the reward for emulation are perfectly correlated, the reward for work performance affects the relative incentive such that the relative incentive to increase work effort under performance-based emulation is, on average, higher for people with a lower job reward (λ_i) for work performance. This tendency becomes stronger as the correlation between the reward for performance and the reward for emulation becomes weaker.

Properties (S8) indicates that, unlike effort-based emulation, performance-based emulation tends to equalize the amount of work effort to be expended among people with different amounts of job rewards.

(E6) Performance-based emulation tends to equalize the amount of work effort to be expended among people with different job-related rewards, and this equalization is stronger when the correlation between performance-based reward and emulation-based reward is smaller.

On the other hand, equation (10) indicates that

(S9) Heterogeneity in the coefficient of opportunity costs (α_i) as well as the optimal amount of work effort under the baseline model does not affect the relative incentive under performance-based emulation, and, therefore, people with different opportunity costs are equally motivated to expend additional effort under performance-based emulation.

Regarding the effects of work-related ability under performance-based emulation, equations (6) and (10) show that the functional form of the left-hand side of the equation that includes the parameter for work-related ability does not differ between the two forms of emulation. Hence, characteristics (S5) and (S6) also hold under performance-based emulation as well.

(S10) Work quality θ_i^Q does not affect the relative incentive under performance-based emulation, and, therefore, people with different work quality are equally motivated to expend additional effort under performance-based emulation.

(S11) It is highly likely that higher work efficiency gives a greater relative incentive to expend more effort under performance-based emulation.

Since performance depends on work efforts controlling for work-related ability, those who expend less effort than others, and perform less well than others as a result are still penalized under the performance-based emulation, even when they increased their effort as much as, or even more than others. This implies that the characterizations of who, other things being equal, is more likely to be relatively deprived under effort-based emulation, that is, properties (R1) and (R2) of the effort-based emulation system, also holds under the performance-based emulation system.

(R3) Regarding who is more likely to be relatively deprived, performance-based emulation shares the characteristics (R1) and (R2) of effort-based emulation.

In addition, from property (S9) of performance-based emulation, the following also holds:

(R4) Other things being equal, those who have a greater coefficient (α_i) of opportunity costs (α_i) than others will be more likely to be relatively deprived under performance-based emulation.

On the other hand, those with lower work efficiency who, on average, perform less well than others and are, as a result, more likely to be penalized under performance-based emulation may not be relatively deprived nonetheless because as property (S11) indicates, they do not expend additional efforts as much as others under the emulation.

A comparison of the relative incentive for effort-based emulation and the relative incentive for performance-based emulation reveals a sharp contrast in the effects of the opportunity costs and job-specific rewards.

(E7) The effects of selectivity in inducing more effort in work contrast markedly between effort-based emulation and performance-based emulation: (a) differences in the opportunity costs affect the relative incentive under effort-based emulation, but they do not affect the relative incentive under performance-based emulation, and (b) differences in the job-related reward affect the relative incentive under performance-based emulation, but they do not affect the relative incentive under effort-based emulation.

This contrast has important implications for gender inequality.

(G2) Under the presence of gender inequality in salary/wages, the introduction of performance-based emulation will decrease the gender gap in the amount of work effort between women and men.

In a society where the traditional division of household labor is strong, married women, who are expected to be mainly responsible for childrearing and household care, have greater opportunity costs of expending additional work effort than married men. It follows that:

(G3) Under the presence of the traditional division of labor in society that makes married women's opportunity costs of expending effort in work greater than married men's, the introduction of effort-based emulation will increase the gender gap in the amount of work effort between married women and married men.

These two properties have the following effect on gender inequality:

(G4) Other things being equal, there will be more gender equality in the average work performance between men and women under performance-based emulation than under effort-based emulation. Hence, other things being equal, there will be more gender equality in status attainment under performance-based emulation than under effort-based emulation.

However, there is also a drawback for women under performance-based emulation. Since characteristic (R4) on relative deprivation holds only under performance-based emulation, and not under effort-based emulation, it follows that:

(G5). Under the presence of the traditional division of labor in society that makes married women's opportunity costs of expending effort in work greater than married men's, married women are more likely to be relatively deprived than married men under the performance-based emulation system.

CONCLUSION AND DISCUSSION

The major deductively derived findings about the consequences of effort-based and performance-based emulation are summarized by properties (E1) through (E7), and the effects of these emulation systems on relative deprivation, selective incentive, and gender inequality are summarized in properties (R1) through (R4), (S1) through (S11), and (G1) through (G5), respectively.

As reviewed in the introduction, many existing theories related to relative standing consider the seeking of improvement in the relative position to be an alternative to rational choice. On the other hand, this paper models the situation where the seeking of improvement in the relative position is socially induced by the presence of emulation-based reward systems, but it retains the rational choice model for actions. As is Boudon's theory, the theory is concerned with relative deprivation as a consequence of social actions under emulation, and not as a cause of social actions.

One interesting question is concerned with the apparently exploitative nature of emulation, shown in the fact that under emulation, people work more than they optimally would under the reward system without emulation while the average benefits/costs from the rewards/penalties for the emulation are zero in the population. Expending more work effort "now than later," however, may involve an element of investment for the future, and, therefore, an inducement by emulation to expend more effort may bring long-term benefits, especially among myopic people who may not expend effort for their work, even though the deferred gratification of expending more effort at work may bring long-term benefits. However, the model introduced in this paper is for one-time decision making, and, therefore, an investigation of those considerations for lifetime benefits remains to be seen in a future study.

On the other hand, since emulation among workers gives selective incentives to expend more effort to certain workers more than to others, employers' motivation in an internal labor market (Doenger and Piore 1971) to introduce emulation systems will depend on which group of workers employers wish to expend more effort. This paper has demonstrated that the distinction between effort-based emulation and performance-based emulation is important because of their almost contrastive impacts on selective incentives, including their implications for gender inequality under the presence of a gender gap in salary/wages and the presence of the traditional division of household labor. It is noteworthy, however, that while performance-based emulation generates more gender equality in performance as well as in the expected status attainment as its consequence, while effort-based emulation generates more gender inequality, the former tends to generate more women who are relatively deprived. This paradox, more relative deprivation with more equality, resembles cases that Boudon analyzed, while the underlying mechanism is not exactly the same.

In particular, internal labor markets developed under the "lifetime employment system" for regular employees in large and medium-sized firms in Japan (Cole 1973) have built-in emulation among those employees because the appointment to managerial positions is usually possible only for those who have already been employed in the firm for a long period. The emulation there has a characteristic of effort-based emulation because only white-collar employees who are willing to work overtime regularly are considered primary candidates for promotions to managerial positions (Yamaguchi 2019). The theory introduced in this paper suggests that this effort-based emulation is highly likely to be among the causes of the persistent large gender inequality in wages and promotion in the Japanese workplace.

The presence versus absence of emulation-based rewards in different forms and different social contexts in a society is thus among important social-structural elements in understanding what we would expect for people's social choices and the macrosocial consequences of those social choices, and a formal theory on this topic such as the one introduced in this paper will offer guidance in understanding such mechanisms of the macro-macro linkage in society through macro-micro and micro-macro linkages (Coleman 1990).

REFERENCES

Brockman, Hilke, Jan Delhey, Christian Welzel, and Hao Yuan. 2008. "The China Puzzle: Falling Happiness in a Rising Economy." *Journal of Happiness Studies* 10:387-405.

Boudon, Raymond. 1982. Unintended Consequences of Social Action. London: McMillan Press.

Cole, Robert E. 1973. "Functional Alternatives and Economic Development: An Empirical Example of Permanent Employment in Japan." *American Sociological Review* 38: 424-437.

Coleman, James S. 1990. *Foundations of Social Theory*. Cambridge, MA: Harvard University Press.

Doeringer, Peter B., and Michael Piore. 1971. "Internal Labor Market and Manpower Analysis."

Cambridge, MA: Harvard University Press.

Easterlin, Richard A. 1974. “Does Empirical Growth Improve the Human Lot? Some Empirical Evidence.” P.A. David and M.W. Reder (eds.), *Nations and Households in Economic Growth: Essays in Honor of Moses Abramovits*. New York, NY: Academic Press.

Easterlin, Richard D. 1995. “Will Rising the Income for All Increase the Happiness of All?” *Journal of Economic Behavior and Organization*, 27: 35-47.

Frank, Robert H. 2011. *The Darwin Economy: Liberty, Competition, and the Common Good.* Princeton, NJ, Princeton University Press.

Frank, Robert H., and Cass R. Sunstein. 2011. “Cost-Benefit Analysis and Relative Position.” *The University of Chicago Law Review*, 68:

Gurr, Ted Robert. 1970. *Why Men Rebel*? Princeton, NJ: Princeton University Press.

石田淳. 2015. 『相対的剥奪の社会学』東京大学出版会（Ishida, Atsushi. 2015. *Sociology of Relative Deprivation.* Tokyo: University of Tokyo Press.

Ishida, Atsushi, Kenji Kosaka, and Hiroshi Hamada. 2014. “A Paradox of Economic Growth and Relative Deprivation.” *Journal of Mathematical Sociology* 38:269-84.

石川経夫. １９８１. 「労働意欲の決定要因としてのエミューレーション効果について」『経済学論集』第 47 巻第一号 1－25 頁。（Ishikawa, Tsuneo. 1981. “Emulation as a determinant of labor incentive.” *Economics Theories*, 47(1):1-25）

Jasso, Guillermina, 1980. “A New Theory of Distributive Justice.” *American Sociological Review*: 45:3-32.

Kosaka, Kenji. 1986. “A Model of Relative Deprivation.” *Journal of Mathematical Sociology* 12:35-48.

Manzo, G. 2011. "Relative Deprivation in Silico: Agent-based Models and Causality in Analytical Sociology," Pp. 266-308 in P. Demeulenaere (ed.), *Analytical Sociology and Social Mechanisms*, Cambridge, Cambridge University Press.

Ravallion, Martin, and Shaohua Chen. 2007. "China's (Uneven) Progress against Poverty." *Journal of Development Economic*s 82:1-42.

Runciman, Walter G. 1966, *Relative Deprivation and Social Justice: A Study of Social Attitude to Social Inequality in Twentieth-Century England*. London, UK. Routledge & Kegan Paul.

Runciman, W.G., and C.R. Bagley. 1969. "Stats Inconsistency, Relative Deprivation, and Attitude toward Immigrants." *Sociology* 3:359-375.

Stouffer, Samuel A., Edward A. Suchman, Leland C. DeVinney, Shirley A Star, and Robin A. Williams, Jr. 1949. *The American Soldier: Adjustment in Army Life*. Princeton, NJ, Princeton University Press.

Tocqueville, Alexis de. 1983. *De la Démocratie en Amérique.* Paris, France: Michel Lévy.

Yamaguchi, Kazuo. 1998. "Rational-Choice Theory of Anticipatory Socialization and Anticipatory Non-socialization." *Rationality and Society.* 10:163-99.

Yamaguchi, Kazuo. 2019. *Gender Inequality in Japanese Workplace and Employment: Theories and Empirical Evidence.* Singapore, Springer.

Yitzhaki , Shlomo. 1979. "Relative Deprivation and the Gini Coefficient." *Quarterly Journal of Economics,* 93: 321-24.

Table 1. A Classification of Theories of Relative Deprivation

	Causes of Relative Deprivation	Consequences of Relative Deprivation
Wealth, Social Inequality, and Happiness	Runciman, Yitzhaki	Easterlin
Social Opportunity and Dissatisfaction with the Social System	Stouffer et al., Boudon	
Relative Standing in Position and Social Action	Jasso, Frank	Gurr, Jasso, Frank, (Ultimatum game)

APPENDIX

This appendix tries to show that (1) the property (B4) of the baseline model regarding the effects of work efficiency on the optimal amount of work effort and properties (S6) and (S11) regarding the effects of work efficiency on the relative incentives of the emulation models hold when we assume either of two specific functions for performance.

Generally, we assume $y(x,\theta^Q,\theta^E)=\theta^Q f(\theta^E x),$ where we assume $f(0)=0,\ f'(\theta^E x)>0,\ f''(\theta^E x)<0$, as indicated by assumption (A6) in section II. In order to model a decaying function for f', we examine below both the case where the decay function is hyperbolic and the case where the decay function is exponential.

Case 1

Suppose we assume a model of $y=a\theta^Q\log(b\theta^E x_i+1)$, where *a* and *b* are positive constants, that represents a hyperbolic decline in *y′*, as a function of the product of work efficiency and effort. For this model $y'=ab\theta^Q\theta^E/(b\theta^E x+1)$ holds, and since $\partial y'/\partial\theta^E=ab\theta^Q/(b\theta^E x+1)^2$ >0, the rate of growth in performance as a function of effort increases as work efficiency becomes higher. However, this increase in the rate of growth in performance becomes smaller as θ^E becomes larger, and it approaches 0 as θ^E approaches infinity. Hence, differences in work efficiency have a declining impact on the amount of work effort as efficiency becomes higher except for a range of very small θ^E values. Hence, property (B4) holds for the baseline model.

Regarding the effect on the relative incentive,

$$\frac{y'(x^B*,\theta^Q,\theta^E)}{y'(x^E*,\theta^Q,\theta^E)}=\frac{f'(x^B*,\theta^Q,\theta^E)}{f'(x^E*,\theta^Q,\theta^E)}=\frac{b\theta^E x^{E*}+1}{b\theta^E x^{B*}+1}>1,$$

because $x^E*>x^B*$ holds. Hence, properties (S6) and (S11) hold.

Case 2

Suppose we assume an alternative model of $y = a\theta^Q(1-\exp(-b\theta^E x))$, where *a* and *b* are positive constants, that represents an exponential decline in *y*′, as a function of the product of work efficiency and effort. For this model $y' = ab\theta^Q\theta^E \exp(-b\theta^E x))$ holds, and since $\partial y'/\partial\theta^E = ab\theta^Q(1-b\theta^E x)\exp(-b\theta^E x)$, the rate of growth in performance as a function of effort is initially positive when work efficiency is nearly 0. However, for a given amount of work effort *x*, this increase in the rate of growth in performance becomes smaller as work efficiency becomes larger, reaches 0 at the value of θ^E that satisfies $1-b\theta^E x = 0$, and will become negative for a further increase in work efficiency. However, this negative effect on the rate of growth also approaches 0 as work efficiency further increases toward infinity. Hence, property (B4) also holds for this case.

Regarding the effect on the relative incentive,

$$\frac{y'(x^B*,\theta^Q,\theta^E)}{y'(x^E*,\theta^Q,\theta^E)} = \frac{f'(x^B*,\theta^Q,\theta^E)}{f'(x^E*,\theta^Q,\theta^E)} = \exp(b\theta^E(x^{E*}-x^{B*})) > 1,$$

because $x^E* > x^B*$ holds. Hence, properties (S6) and (S11) also hold.