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Explaining Declining Business Dynamism: A Monetary Growth-Theoretic Approach¹

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Abstract

This study offers a monetary growth theory that can explain the declining business dynamism observed over the past few decades in some developed countries by developing a new R&D-based growth model; the main departure from existing models is the introduction of an entry cost after innovation and an endogenous survival investment that is subject to a cash-in-advance constraint. Due to this, in our model, the entry/exit rates and firm age distribution are all endogenous. The core finding is that, theoretically, the nature of business dynamism at the macro level essentially depends on nominal factors. Specifically, lower inflation leads to declining business dynamism, characterized by lower entry and exit rates and a maturity bias in the firm age distribution, if the entry cost is sufficiently high. Empirically, we also find supportive evidence that, among a set of European countries, firm entry/exit rates are higher in countries with higher inflation rates. Then, calibrating the model to the E.U. economy, we verify that lower inflation leads to declining business dynamism under empirically plausible values of entry, exit, and inflation rates.

Keywords: Cash-in-advance constraint, inflation, firm dynamics, R&D-based growth JEL classification: E40, O31, O34, O41

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1 Introduction

Recently, most developed economies, including the U.S. and some European economies, have more or less experienced a significant declining trend in business dynamism—the process by which firms continuously enter, exit, and survive.¹Accompanied by a decreasing entry rate and an increasing share of older firms, this downtrend can hurt the stable long-term growth of these economies.² Although recent literature has begun to identify an explanatory factor for and a plausible economic mechanism underlying this phenomenon (e.g., Akcigit and Ates, 2021), there is no concrete consensus; the declining trend of business dynamism continues to be a puzzle. In this study, we will formally explain declining business dynamism as an endogenous equilibrium phenomenon by focusing on nominal factors such as inflation.

To this end, we propose a new monetary growth theory that is equipped to capture the effects of an inflation rate change on the dynamics of firms at the macro level. Specifically, we consider a research-and-development (R&D)-based growth model with expanding varieties, based on Romer (1990). To incorporate money demand, we impose a cash-in-advance (CIA) constraint on firms' R&D investments, following Chu and Cozzi (2014). The essential departure from existing models is as follows. First, to examine the entry stage in detail, we introduce an entry cost for facilitating production after firms successfully innovate new products. Second, to consider an endogenous distribution of firm age, we introduce the investment activity of firms for market survival after the success of innovation to delay the obsolescence of their innovated goods. This survival investment also faces a CIA constraint, following extant empirical evidence (Musso and Schiavo, 2008).³ Due to these two features, the entry/exit rates and a firm age distribution are all endogenous in our model.

The core finding of our study is that, theoretically, the nature of business dynamism at the macro level essentially depends on nominal or inflationary factors. Specifically, we identify a mechanism through which lower inflation leads to declining business dynamism. A decrease in the inflation rate, determining the opportunity cost of cash holdings, lowers the cost for R&D and survival investments; the direct effect of lower inflation on both investments is positive. However, since it changes the relative profitability between R&D and survival ambiguously, lower inflation may encourage or discourage innovation and survival investments, potentially. Our analysis identifies the role of entry cost in determining the direction of effect of inflation on innovation and survival. If the entry cost is higher than some threshold value, a lower inflation rate discourages entry but encourages survival, thus, leading to a lower entry and exit rate for firms. This, in turn, results in a maturity bias in the firm-age distribution. Thus, if entries are costlier, lower inflation—or a deflationary trend—leads to declining business dynamism.

We also test our theoretical predictions concerning firm entry/exit empirically by using cross-country data in Europe. Specifically, we examine whether inflation rates are

¹See, for example, Calvino, Criscuolo, and Verlhac (2020), who demonstrate that many other countries face downtrends in business dynamism.

 $^{^{2}}$ For other characteristics, see Akcigit and Ates (2021), who document ten stylized facts relating to declining business dynamism. Here, we pick up two crucial characteristics from the ten.

³Evidence reveals that such investments in market survival are also financially constrained in several circumstances. From a broader context, financing investments in intangible assets, such as R&D, is essentially sensitive to cash/financial constraints because of the firm-specific and inalienable nature of intangible assets. For a helpful review on the 3

empirical literature, see, for instance, Chen (2014) and Morikawa (2015).

positively related to firm entry/exit rates, where firm entry/exit rates are taken from Eurostat's Structural Business Statistics and inflation rates are taken from the World Bank. Using generalized method of moments (GMM), we find supportive evidence that firm entry/exit rates are higher in countries with higher inflation rates. In terms of economic significance, a 1% increase in inflation rate from the mean is associated with a 0.7-0.8 percentage point increase in firm entry rate and a 0.5-0.6 percentage point increase in firm exit rate. By calibrating our theoretical model to the E.U. data, we further verify that our model can explain, approximately, a 15% to 30% of the empirically observed increase in the entry rate, mentioned above, and 50% of the empirically observed increase in the exit rate in terms of percentage.

Our study closely relates to a traditional macroeconomic issue. Specifically, the macroeconomic literature, stemming from Tobin (1965), explored the relationship between inflation and real investment activities. For instance, Stockman (1981) and Abel (1985) initiated an influential literature stream by focusing on incorporating a CIA framework for money demand into neoclassical capital accumulation models, which was pioneered by Lucas (1980), following Clower (1967).

More recently, a breakthrough occurred in this area, as Chu and Cozzi (2014) extended the analysis to a more recent class of growth models, that is, R&D-based models.⁴ They achieved this by incorporating the aforementioned empirical evidence that R&D investments face significantly severe cash/financial constraints (Hall, 2008). Since then, this literature has examined the role of nominal interest rates or inflation in innovation, innovation-driven growth, and other economic phenomena such as unemployment and income inequality both theoretically and empirically. Such studies include those by Chu et al. (2015, 2017, 2019a, 2019b, 2020), He and Zou (2016), He (2018), Hori (2020), Huang et al. (2020), and Zheng et al. (2020a).⁵ We complement these studies by examining the effects of the CIA constraint on R&D firm dynamics, not only at the entry stage but also for survival, thereby identifying the essential role of inflation as a determinant of business dynamism. Our approach differs from those of existing studies, which consider neither firm exit nor survival investment, in that firms' survival activity, exit rate, and distributions for age and size are all endogenous, depending on the different degrees of CIA severity for R&D and market survival.

In different contexts, several studies also examined the relationships between inflation, innovation, and growth; as recent examples, see Arawatari et al. (2018) and Zheng et al. (2020b). In particular, as in this study, Miyakawa et al. (2020) examined the effects of inflation on firm dynamics, but with a different source of money demand (i.e., endogenous price revisions in a new Keynesian with menu costs). Further, their focus is on a resource reallocation role of monetary policy. We complement their pioneering analysis by explicitly focusing on the causes of declining business dynamism and explaining the mechanism through which lower inflation leads to declining linkage based on a CIA approach.

Some recent studies in endogenous growth theory explicitly address declining business dynamism. For example, Akcigit and Ates (2021) document ten stylized facts on declining business dynamism, and explain them by developing a new endogenous growth model featuring product market competition with strategic interaction between competing firms.

 $^{{}^{4}}$ Romer (1990) first developed the R&D-based growth model with an expanding variety of goods. Meanwhile, Segerstrom et al. (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) developed the Schumpeterian growth model with quality improvement.

⁵See Chu (2020) for an extensive review.

Haruyama (2021) shows declining business dynamism can explain increasing trends in income inequality developing a new Schumpeterian growth model. Unlike these models, we focus on monetary factors as a fundamental cause of declining business dynamism. Therefore, we complement the existing literature by finding an essential linkage between inflation and business dynamism.

In modeling a firm's dynamic optimization for survival, we follow the growth models with endogenous firm survival developed by Dinopoulos and Syropoulos (2007) and Eicher and García-Peñalosa (2008). Subsequent studies, such as Grieben and Sener (2009) and Davis and Sener (2012), further extended the analysis to other dimensions. Further, Akiyama and Furukawa (2009), Furukawa (2013), Furukawa and Yano (2014), and Niwa (2018) considered endogenous firm survival under a so-called variety expansion growth model, similar to the present study. We contribute to this literature stream by identifying a new role of endogenous firm survival in the relationship between inflation and long-run growth, which has long been one of the most important topics in macroeconomics.

The remainder of the paper proceeds as follows. Section 2 proposes a new monetary R&D-based growth model with endogenous firm survival facing a CIA constraint. Section 3 examines the effects of inflation on the nature of business dynamism. Section 4 provides supportive empirical evidence acquired from a set of European countries. Section 5 calibrates the model to the E.U. data and provides the quantitative results. Finally, Section 6 draws conclusions.

2 A Monetary R&D-Based Growth Model with Endogenous Business Dynamism

2.1 Consumption

We consider a variety expansion model of endogenous growth by referencing Romer (1990) and Rivera-Batiz and Romer (1991). In this model, time is continuous and extends from 0 to ∞ . There is a single final good, taken as numeraire. An infinitely lived representative consumer supplies inelastically one unit of labor and consumes c_t units of final goods at each time point, t. The utility function is:

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt,\tag{1}$$

where $\rho > 0$ is the subjective discount rate.

Following Chu and Cozzi (2014), money is introduced by assuming a CIA constraint on R&D firms' investment; the asset accumulation constraint in real terms is

$$\dot{a}_t + \dot{m}_t = r_t a_t + i_t b_t + w_t L + \tau_t - c_t - \pi_t m_t, \tag{2}$$

where a_t denotes the real value of financial assets (i.e., the equity of monopolistic firms), m_t the real value of cash holdings, i_t the nominal interest rate, $r_t \equiv i_t - \pi_t$ the real interest rate, $b_t \leq m_t$ the real-term amount of cash borrowed by firms, w_t the real wage rate, τ_t the real value of transfers from the government, and $\pi_t \equiv \dot{P}_t/P_t$ the inflation rate. By solving the standard dynamic optimization, we obtain the following Euler equation:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{3}$$

2.2 Production

The market for a final good is perfectly competitive. Firms convert differentiated intermediate inputs, each indexed by j, into Y_t units of final goods. We then consider a Cobb-Douglas production function:

$$Y_t = L_t^{1-\alpha} \int_0^{N_t} x_t(j)^\alpha dj,\tag{4}$$

where L_t is the amount of labor input, N_t the number of intermediate inputs, and $x_t(j)$ the amount of intermediate input j used. The factor share of intermediate inputs, α , satisfies $\alpha \in (0, 1)$. Static optimization yields the demand function for good j as:

$$x_t(j) = \alpha^{1/(1-\alpha)} L_t p_t(j)^{-1/(1-\alpha)}, \tag{5}$$

where $p_t(j)$ is the real price of final good j.

The market for differentiated intermediates is monopolistically competitive. A monopolistic firm that originally innovates good j (or the firm that purchases the patent on good j from the original inventor) manufactures each input j. We assume that manufacturing one unit of good j requires one unit of final good as input and the marginal cost is equal to unity. Because the price elasticity of demand for any good j is $1/(1-\alpha)$ from (5), monopolistic pricing yields $p_t(j) = 1/\alpha$. Then, substituting this equilibrium price into (5) yields

$$x_t(j) = \alpha^{2/(1-\alpha)} \equiv x \tag{6}$$

when using the labor market clearing condition, $L_t = 1$, which denotes the profit for good j. The equilibrium profit is:

$$\Pi_t(j) = \alpha^{\frac{1+\alpha}{1-\alpha}} \left(1-\alpha\right) \equiv \Pi. \tag{7}$$

2.3 Endogenous R&D and Entry

At each date t, there is a continuum of perfectly competitive potential R&D firms in the economy. As in Romer (1990), we assume the number of potential firms, K_t , is equal to the current number of innovations, N_t (as a proxy of the cumulative knowledge that is not yet obsolete), that is, $K_t = N_t$. Each R&D firm can innovate one new technology to produce a new consumption good with probability $\psi_t dt$ during a short time interval dt. Here, the R&D firm invests $(k_t/\kappa)dt$ units of final goods during the same time interval, where $\kappa > 0$ denotes their productivity. That is, ψ_t denotes a Poisson arrival rate for innovation. We consider a concave production function for R&D, $\psi_t = (k_t)^{\gamma}$, where $\gamma \in (0, 1)$.

In order to consider the R&D firm's life cycle as an endogenous phenomenon, we observe the role of an entry fee, departing from Chu and Cozzi (2014).⁶ Thus, we assume that after a successful innovation, the R&D firm needs to pay $\varpi/\kappa > 0$ units of the final good for entry. We denote by v_t the real value of an innovation after entry (without entry, their benefit is zero). Then, perfectly competitive firms face the following optimization problem:

$$\max_{k_t: \text{ s.t. } \psi_t = (k_t)^{\gamma}} (\psi_t dt) \left(v_t - \varpi/\kappa \right) - \left(1 + \zeta i_t \right) (k_t/\kappa) dt.$$
(8)

⁶This is more in line with Chu et al. (2017), who identify the critical role of entry cost in explaining the nonmonotonic effects of inflation on innovation and growth. On the contrary, we focus on idetifying a relationship between inflation and R&D firm dynamics.

Here, we follow Chu and Cozzi (2014) in assuming that R&D investments face a CIA constraint. Specifically, in paying k_t/κ , a firm has to prepare $\zeta k_t/\kappa$ units of cash in advance. To do so, the R&D firm has to borrow from households because it is a startup company with no cash. Parameter $\zeta \in [0, 1]$ represents the severity of the financial constraint for startup firms trying to innovate.⁷ By solving (8), we obtain the optimal levels for R&D investment and its success probability:

$$k_t^* = \left(\frac{\gamma(\kappa v_t - \varpi)}{1 + \zeta i_t}\right)^{\frac{1}{1 - \gamma}} \text{ and } \psi_t^* = \left(\frac{\gamma(\kappa v_t - \varpi)}{1 + \zeta i_t}\right)^{\frac{\gamma}{1 - \gamma}},\tag{9}$$

assuming $v_t \geq \varpi$ (which will be ensured to hold in equilibrium).

2.4 Endogenous Survival and Exit

To examine the role of inflation in business dynamism, in addition to the introduction of an entry cost, we further depart from Chu and Cozzi (2014) by endogenizing the exit rate for R&D firms, which is a novel approach in the literature.⁸ We thus assume that each innovation (or the relevant R&D firm) faces a risk of obsolescence. Obsolescence occurs with probability $\delta_t dt$ during a short time interval dt, forcing firms to exit the market. Therefore, a firm would invest resources in survival by marketing, advertisement, or protecting intellectual property rights. In describing the dynamic process of firm survival, we assume that, when the firm invests z_t/κ units of a final good, the hazard (Poisson arrival) rate for obsolescence is $\delta_t \equiv \overline{\delta} - z_t$. Here, $\overline{\delta}$ gives the natural upper bound for the exit rate.

There are two critical considerations for the survival of innovation. The first is that, when making a survival investment, the firm also faces a CIA constraint: to facilitate the payment of z_t/κ , it has to prepare $\xi z_t/\kappa$ amount of cash by borrowing from consumers, where $\xi \in [0, 1]$. As we already mentioned in the introduction, this consideration is supported empirically. Another critical factor is that, different from R&D investment, the firm already earns profit Π_t and thereby holds some cash. In reality, firms typically use their own internal reserves for investment covering survival purposes. Naturally, we assume that the firm can use fraction $(1 - \theta)$ of profit Π_t (i.e., $(1 - \theta) \Pi_t$) for survival investment (in terms of final goods) at each date t, where $\theta \in [0, 1]$ is a parameter capturing corporate culture.⁹ As a result, there are two types of equilibrium. If the internal reserve can cover the cash required for survival investment, there is no need to borrow cash from consumers (i.e., no borrowing case, labeled as $\omega = 0$); otherwise, the firm borrows ($\xi z_t/\kappa - (1 - \theta) \Pi_t$) units of cash from consumers at cost i_t (i.e., borrowing case, labeled as $\omega = 1$). For simplicity, we introduce an indicator function, λ^{ω} , such

⁷Some can interpret ζ as an inverse measure of the quality of a payment system or a cash market; see Yano (2019) for a new theory of money based on the market quality economics proposed by Yano (2009).

⁸The novelty for the growth literature on endogenous survival of R&D firms (Dinopoulos and Syropoulos, 2007, and Eicher and García-Peñalosa, 2008) is incorporating the money demand via the CIA constraint on R&D investments.

⁹One can consider ratio θ to be endogenously determined through any maximization. In our model, $\theta = 0$ is optimal. Therefore, we consider θ to be exogenous because, in reality, firms do not typically invest all profit into a single plan. We are also interested in the role of corporate culture in the use of profits.

that $\lambda^0 = 0$ (i.e., no borrowing for survival) and $\lambda^1 = 1$ (i.e., borrowing for survival).¹⁰ Therefore, the total payment for the survival investment becomes:

$$I_t \equiv \frac{z_t}{\kappa} + i_t \max\left\{\frac{\xi z_t}{\kappa} - (1-\theta)\Pi_t, 0\right\} \equiv \frac{z_t}{\kappa} + \lambda^{\omega} i_t \left(\frac{\xi z_t}{\kappa} - (1-\theta)\Pi_t\right).$$
(10)

Now, we can express the dynamic optimization for existing firms as:¹¹

$$\max_{z_t: \text{ s.t. } \delta_t \equiv \overline{\delta} - z_t} W_t \equiv (\Pi_t - I_t) dt + (1 - \delta_t dt) \dot{v}_t dt - (\delta_t dt - \eta dt) v_t, \tag{11}$$

where η denotes an exogenous growth factor for firms, whose interpretation is provided in Section 2.6. Solving (11),¹² we obtain the following condition:

$$v_t = \frac{1 + \lambda^\omega \xi i_t}{\kappa} \equiv v, \tag{12}$$

which holds unless $z_t = 0$ (i.e., a trivial case with no survival activity, which we ignore in the primary analysis).

2.5 Monetary Authority

Following the literature (Chu and Cozzi, 2014), the monetary authority exogenously sets $i_t = i$ as a stationary policy instrument. The Fisher equation is $i = \pi_t + r_t$. Denoting the aggregate *nominal* money balance as M_t , its growth rate is given by:

$$\mu_t \equiv \frac{M_t}{M_t} = \pi_t + \frac{\dot{m}_t}{m_t} = i - r_t + \frac{\dot{m}_t}{m_t} = i - \rho - \frac{\dot{c}_t}{c_t} + \frac{\dot{m}_t}{m_t},\tag{13}$$

where the last equality uses the Euler equation (3). Given a stationary nominal interest rate, *i*, real consumption c_t and aggregate real money balance m_t grow at the same rate on a balanced growth path, $\mu = i - \rho$. Then, the monetary authority returns the seigniorage revenue as a lump-sum transfer $\tau_t = \dot{m}_t + \pi_t m_t$. See Chu and Cozzi (2014) for details.

2.6 Dynamic General Equilibrium

Here, we characterize the equilibrium dynamics for the aggregate economy under three conditions. First, given that the benefit of owning a bond of price v_t over the small time interval of dt is $(r_t dt)v_t$, the standard Bellman equation is:

$$r_t v_t = (1 + \lambda^{\omega} i (1 - \theta)) \Pi_t - (\overline{\delta} - \eta) v_t$$
(14)

$$W_t^n + \pi v_t^n = (\Pi_t^n - I_t^n) dt + (1 - \delta_t dt) \dot{v}_t^n dt - (\delta_t dt - \eta dt) v_t^n,$$

where variables with superscript n denotes nominal values. Given that $\dot{v}_t^n = \dot{P}_t v_t + P_t \dot{v}_t$. This implies that W_t is defined as the real value of innovation *minus* a nominal benefit of inflation.

¹²As a standard argument, we can ignore term $(dt)^2$.

¹⁰We implicitly assume that firms have to pay out the remainder of their available net internal reserves for survival investment $(1 - \theta) \Pi_t - \lambda^{\omega} \xi z_t / \kappa$ if it exists.

¹¹See Furukawa (2013) and Niwa (2018) for a discrete-time version of a similar setup. For more details, during the short time interval dt, the firm obtains $(\dot{v}_t + \eta v_t) dt$ with probability $(1 - \delta_t dt)$ and loses the current value v_t with probability $\delta_t dt$. Further, in nominal terms, (11) is written as

from using (11) and (12).

Second, the firm dynamics at the aggregate level can be summarized by the following differential equation:

$$\dot{N}_t = \psi_t K_t - \left(\overline{\delta} - z_t\right) N_t + \eta N_t, \tag{15}$$

which uses $\delta_t = \overline{\delta} - z_t$.

We now explain the meaning and role of an exogenous growth factor, η . First, we need $\eta > 0$ to ensure positive growth (and then have an endogenous evolution of firm size distribution), since our model is even more restrictive than the standard endogenous growth models, given the presence of borrowing and survival investment. Then, we facilitate positive growth by following Anderlini et al. (2013). Specifically, from (11) and (15), we use the exogenous growth factor, $\eta > 0$.¹³ Firms introduce new goods in the market under a profit-motivated R&D investment and through "invention by accident," which sometimes occurs in reality.¹⁴ Without any intended investment or effort, firms can create new ideas by accident, or even by mistake, as a byproduct of regular activities (in our case, production or survival).¹⁵ Such an accident occurs with a Poisson arrival rate of $\eta > 0$. Further, the expected value for invention by accident is $(\eta dt) v_t$ during a short time interval. When this happens, the existing firm innovates another good and obtains an additional value of v_t (i.e., $2v_t$ in total).¹⁶

Third, the final good market equilibrium condition is:

$$Y_t = c_t + N_t x_t + \left(\frac{k_t + \varpi}{\kappa}\right) K_t + \frac{z_t}{\kappa} N_t, \qquad (16)$$

where the supply of final goods is Y_t and demand results from consumption c_t , production $N_t x_t$, innovation $k_t K_t / \kappa$, and survival $z_t N_t / \kappa$.

By incorporating (3) and (12) into (14) and $K_t \equiv N_t$ into (9), we can derive the law of motion for (c_t, N_t) :

$$\frac{\dot{c}_t}{c_t} = \frac{1 + \lambda^{\omega} \left(1 - \theta\right) i}{1 + \lambda^{\omega} \xi i} \kappa \Pi - (\overline{\delta} + \rho - \eta) \tag{17}$$

and

$$\frac{N_t}{N_t} = \psi_t^* + z_t + \eta - \overline{\delta}.$$
(18)

We then solve (16) for z_t :

$$z_t = \frac{1+\alpha}{\alpha} \kappa \Pi - \left(\kappa \frac{c_t}{N_t} + (k_t^* + \varpi)\right),\tag{19}$$

which uses (4), (6), (7), and (9) with $K_t \equiv N_t$. The following lemma determines the equilibrium values for R&D investment.

¹³Exogenous growth factors are often considered for a deeper understanding of the role of technological progress in various phenomena. For instance, see Lucas and Moll (2014) and Benhabib et al. (2017). Our model also includes endogenous growth factors of innovation and survival. In this sense, it is closer to Anderlini et al. (2013), who considered both endogenous and exogenous growth factors.

¹⁴See, for example, Middendorf (1981) for more details on this innovation type.

¹⁵The representative consumer also has ownership for this sort of accidental innovation/existing firms. ¹⁶For simplicity, we assume that the firm sells the ownership right for the new "invention by accident" to a randomly chosen firm from a pool of potential manufacturing firms the representative consumer owns.

Lemma 1 For any $t \ge 0$, the equilibrium R & D investment and success rate are given by:

$$k_t^* = k^* \equiv \left(\frac{\gamma(1 - \varpi + \lambda^{\omega}\xi i)}{1 + \zeta i}\right)^{\frac{1}{1 - \gamma}} \text{ and } \psi_t^* = \psi^* \equiv \left(\frac{\gamma(1 - \varpi + \lambda^{\omega}\xi i)}{1 + \zeta i}\right)^{\frac{\gamma}{1 - \gamma}}.$$
 (20)

Proof. Substituting (12) into (9) yields (20). \blacksquare

We define $(\eta_0, \overline{\delta}_0, \kappa_0)$ as the threshold values of $(\eta, \overline{\delta}, \kappa)$. Then, we have the following lemma on the uniqueness and global stability of a balanced growth path, which allows us to restrict our analysis to a nontrivial equilibrium with borrowing for survival.

Lemma 2 Assume that $\eta > \eta_0$, $\overline{\delta} > \overline{\delta}_0$, and $\kappa > \kappa_0$ hold. Then, the economy immediately jumps onto a unique balanced-growth path at t = 0 and stays there permanently. On a balanced growth path, the existing firms borrow cash ($\omega = 1$) if and only if $\xi > (1 - \theta)$; the growth rate is:

$$g^* = \frac{1 + (1 - \theta)i}{1 + \xi i} \kappa \Pi - (\overline{\delta} + \rho - \eta) > 0$$

$$\tag{21}$$

and the firm exit rate is

$$\delta^* = \overline{\delta} + \rho + \psi^* - \frac{1 + (1 - \theta)i}{1 + \xi i} \kappa \Pi.$$
(22)

Proof. See the Appendix.

These two lemmas completely characterize the equilibrium behavior of R&D firms and, thereby, the dynamic general equilibrium of our model. In the subsequent section, from these two theoretical results, we will draw various economic insights concerning the effects of inflation on firm dynamics at the macro level.

3 Inflation and Business Dynamism

Our ultimate goal is to identify the essential role of inflation as a determinant of business dynamism at the macro level. As mentioned in the Introduction, business dynamism is typically characterized by a firm's exit rate, entry rate, and its average age. Therefore, in this section, we examine the effects of inflation on these three factors.

Before proceeding, it is beneficial to verify the positive relationship between the interest rate *i*, as a policy instrument, and the inflation rate π . In our model, the inflation rate follows the Fisher equation $\pi = i - r$. As shown in the Appendix, in our model, π and *i* are always positively related under the assumption we will impose in what follows, that is, $1 - \theta < \xi$. Therefore, in the equilibrium we focus on, an increase in *i* means an increase in π ; such a positive relationship is supported by empirical studies such as those of Mishkin (1992) and Booth and Ciner (2001). Following the literature (e.g., Chu et al. 2017), in the analysis below, we relate an increase in *i* to a higher inflation rate π .

From Lemmas 1 and 2 with (19), an increase in the nominal interest rate, i, as a monetary policy lever, affects the equilibrium level of innovation survival through changes

in the profitability of R&D and survival investments. In the analysis below, we naturally focus on an equilibrium where existing firms borrow cash for survival with $\xi > (1 - \theta)$ (see Lemma 2). We also consider the types of firms that face more stringent CIA constraints. There, thus, exists clear empirical evidence that startup firms in R&D-intensive sectors face more stringent financial constraints (Hall, 2008). Therefore, we should assume that the CIA severity, ζ , for startup firms engaging in R&D is higher than that for older, existing firms, ξ . In summary, we proceed with $1 - \theta < \xi < \zeta$.

A higher nominal interest, i, has two opposite effects on survival and firm exit. As a direct effect, a higher i hurts both R&D and survival investment because it increases the interest payment (total cost) of these investments due to the CIA constraint with severities ζ and ξ , respectively. This discourages both investment in innovation and survival, decreasing firm entry and increasing firm exit. However, whether the negative effect on entry is stronger or weaker than that on exit is ambiguous. On the one hand, since financial constraints are more stringent for startup firms engaging in R&D than existing firms investing in market survival ($\zeta > \xi$), a higher i tends to cause more damage to firms investing in R&D for entry. On the other hand, if the net benefit, $v - \varpi/\kappa$, of R&D is very low, R&D investment k^* is originally very small; thus, an additional damage by an increase in i on k^* is not so significant (as shown in (20)). Consequently, if $v_t - \varpi/\kappa$ is very low (or if the entry cost ϖ is very high), an increase in i tends to cause more damage to incumbent firms' survival investment. This implies that the effect of a higher i on the exit rate δ^* depends on the size of an entry cost.

The following formally characterizes the effect of a higher *i* on firm exit δ^* . We define $\hat{\Pi} \equiv (1 - \gamma) (\kappa \Pi) / (\gamma^{1/(1-\gamma)})$ for capturing the potential profitability of incumbent firms.

Proposition 1 Assume that $1 - \theta < \xi < \zeta$. If the entry cost is large such that $\varpi \ge 1 - \xi/\zeta$, a higher nominal interest rate, *i*, leads to a smaller investment in survival, z^* , and, thereby, a higher exit rate of firms, δ^* , in the long run. Thus, there is a positive relationship between inflation π and firm exit δ^* .

Proof. See the Appendix.

Proposition 1 identifies a critical role of entry $\cot \varpi$ in determining the relationship between the nominal interest *i* and the exit rate δ^* . As mentioned, a higher entry \cot , ϖ , implies a lower net benefit, $v - \varpi/\kappa$, of innovation. Thus, when ϖ is higher, R&D investment is originally very small; an increase in the higher nominal interest rate, *i*, causes insignificant damage on R&D. The damage on survival is more vital, which leads to a shift of labor resource from survival to R&D. This explains why the high entry cost is a condition under a higher nominal interest, discouraging survival investment, leads to a higher exit rate.

Another factor determining the effect of i on δ^* is the profitability $\hat{\Pi}$ of incumbents. Given that the profit rate of incumbent firms, $(1 + (1 - \theta) i) \Pi/v = (1 + (1 - \theta) i) \kappa \Pi/(1 + \xi i)$, is a function decreasing in i (reflecting the cost-pressure effect), a higher interest rate, i, leads to a higher equilibrium firm value, v (via (12)), which in turn decreases the profit rate. This effect of reducing the profit rate becomes more important as the gross profit Π increases. This explains why the high profitability $\hat{\Pi}$ of existing firms is another condition for the positive effect of i on δ^* .

Next, we examine the effects on firm entry. Intuitively, under $\zeta > \xi$, a higher interest rate, *i*, discourages R&D investment for firm entry, which, in turn, has a negative effect on the entry rate, $\varepsilon^* \equiv \psi^* K_t / N_t = \psi^*$ (since we assume that $K_t = N_t$). However, as mentioned above, this entry-discouraging effect of higher *i* can be insignificant if the net benefit, $v_t - \varpi/\kappa$, of R&D is very small. Therefore, the effect of higher *i* on entry can be positive or negative, depending on the size of an entry cost, ϖ . The following formally proves this.

Proposition 2 Assume that $1-\theta < \xi < \zeta$. If the entry cost is large such that $\varpi \ge 1-\xi/\zeta$, a higher nominal interest, i, leads to a higher firm entry rate, ε^* , in the long run. Thus, there is a positive relationship between inflation π and firm entry ε^* .

Proof. It is straightforward from (20), with $\xi < \zeta$.

We have shown that inflation, caused by higher nominal interest, encourages both firm entry and exit in the case with a higher entry cost, $\varpi \ge 1 - \xi/\zeta$. Notably, the positive relationship between inflation and entry/exit seems consistent with data; see Section 4 for an empirical analysis with some evidence supporting our theoretical findings.

To end this section, we will formally verify that the higher entry and exit rates caused by a higher interest rate i (Propositions 1 and 2) lead to a smaller share of old firms, noting that a larger share of old firms is the third factor of business dynamism. In doing this, we first derive the density function of firm age. To do so, we denote a firm's birth date as $b \ge 0$. Then, firms with b have age t - b. Additionally, we denote the number of firms of age t - b as $n_t(t - b)$. Keeping b constant, the law of motion for the evolution of $n_t(t - b)$ is:

$$\dot{n}_t(t-b) = -\delta^* n_t(t-b) \tag{23}$$

with initial condition $n_b(0) = \psi^* M_b + \eta N_b = (g^* + \delta^*) N_b = (g^* + \delta^*) (N_0 e^{g^* b})$. Deriving a particular solution to this differential equation from (23), we have $n_t(t-b) = (g^* + \delta^*) N_b e^{-\delta^*(t-b)}$. Dividing both sides by $N_t = N_0 e^{g^* t}$ yields a density function for firms of age $t - b \equiv a > 0$:

$$f_t(a) \equiv \frac{n_t(a)}{N_t} = (\varepsilon^* + \eta) e^{-(g^* + \delta^*)a},$$
 (24)

which is free from t, since firm age, a, is fixed. Consistent with empirical evidence,¹⁷ the firm age distribution in our economy obeys an exponential function. We can therefore show that the average firm age is $1/(g^* + \delta^*) = 1/(\varepsilon^* + \eta)$.

The inspection of (24) reveals how a higher nominal interest rate, i affects the firm age distribution. In the case with a higher entry cost ϖ , a higher i leads to an increase both entries and exits (Propositions 1 and 2). This implies that more young firms are more likely to enter the market while older firms are more likely to exit. Therefore, a higher i tends to decrease the average firm age. Given the positive relationship between i and π , a lower inflation rate π , or a deflationary trend, implies a maturity bias in the firm age distribution, which is one of the characteristics of declining business dynamism.

As shown in the Appendix, we can formally prove that if and only if the entry cost is large such that $\varpi \ge 1 - \xi/\zeta$ (under $1 - \theta < \xi < \zeta$), a lower nominal interest, *i*, increases the average firm age, causing a maturity bias in the firm age distribution. Together with Propositions 1 and 2, given the positive relationship between *i* and π , we have shown that, under a higher entry cost, a decrease in the inflation rate π —a deflationary trend is accompanied by declining business dynamism, characterized by lower entry/exit rates.

 $^{^{17}}$ See, for example, Coad (2010).

4 Cross-country Evidence

This section presents some cross-country evidence to support our theoretical predictions. While cross-country inflation data are readily available from, for instance, the World Bank, corresponding data are less available for firm entry/exit rates, and less so for firm age distributions. Due to data limitation, our empirical analysis below focuses on verifying whether there is a positive the relationship between inflation rates and firm entry/exit rates (as shown in Proposition 1 and Proposition 2) for a selected sample of European countries.

4.1 Data

Our main data for firm entry and exit come from Eurostat's Structural Business Statistics.¹⁸ This data set provides standardized data about business dynamism for a number of European countries. Specifically, we consider the following two key variables:

- 1. Firm entry rate (*entry*), defined as the number of enterprise births divided by the number of active enterprises.
- 2. Firm exit rate (exit), defined as the number of enterprise deaths divided by the number of active enterprises.

Data for inflation rates (π) (and GDP, which is to control for country size) are extracted from the World Bank.

The raw sample contains an unbalanced panel of 409 observations with non-missing birth and death rates for 34 countries over 2004-2018. These countries include Austria, Belgium, Bulgaria, Croatia, Cyprus, Czechia, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, North Macedonia, Norway, Poland, Portugal, Romania, Serbia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Turkey, and United Kingdom. Table 1 reports the summary statistics of the variables. In a typical year, the average inflation rate is about 2.1%, and the average firm entry and exit rates are about 10.5% and 8.9% respectively.

Table	1:	Summary	statistics
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	Mean	S.D.	1st Quartile	Median	3rd Quartile
Firm entry rate (<i>entry</i> , in percentage)	10.505	3.692	7.980	10.010	12.220
Firm exit rate $(exit, in percentage)$	8.928	3.551	6.790	8.340	10.580
Inflation rate (π , in percentage)	2.123	2.221	0.697	1.823	2.895
GDP (in constant 2010 billion US\$)	612.786	895.330	51.921	243.604	537.421

Note: N = 409. Each observation is a country by year over 2004-2018. Data come from Eurostat's Structural Business Statistics (for firm entry and exit rates) and the World Bank (for inflation and GDP).

For the regression analysis below, we collapse the time-series data into 3-year means.¹⁹ This results in another unbalanced panel of 150 country-period observations. Figure 1

¹⁸See https://ec.europa.eu/eurostat/web/structural-business-statistics.

¹⁹We do not collapse these data into 5-year means (as many other empirical macro studies do) mainly because we only have a relatively short sample period.

shows the scatter plots between inflation rate and firm entry and exit rates. We can see that, unconditionally, inflation rate is positively associated with firm entry and exit rates at the country level.



Figure 1: Scatter plots

Note: N = 150. Each observation is a country by 3-year period over 2004-2018. Data come from Eurostat's Structural Business Statistics (for firm entry and exit rates) and the World Bank (for inflation).

4.2 Regression Analysis

We first estimate the following simple ordinary least squares (OLS) regressions:

$$y_{ct} = \alpha_c + \alpha_t + \beta \pi_{ct-1} + \gamma X_{ct-1} + \varepsilon_{ct}, \qquad (25)$$

where c is a country, t is a 3-year period, y_{ct} is the dependent variable, which is either $entry_{ct}$ (firm entry rate) or $exit_{ct}$ (firm exit rate), π_{ct-1} is the inflation rate lagged by one period, X_{ct-1} includes lagged GDP (in log constant 2010 US\$), α_c and α_t are country and period fixed-effects, and ε_{ct} is the error term.

Table 2 reports the OLS regression results. In these regressions, we cluster the standard errors by country. After controlling for country and period fixed-effects and lagged log GDP, there is still a positive association between inflation rate and firm entry rate (in Column (3)) but there is no significant association between inflation rate and firm exit rate (in Column (6)).

Note that β from (25) can only inform us of the *correlation* between y and π . Standard arguments about potential endogeneity apply here. For instance, there may be time-varying country-specific policies that affect both inflation and firm entry and exit, leading to omitted-variable bias. Ideally, we would like to find an instrument that is correlated with the inflation rate but not the error term so that we can estimate (25) by two stage least squares (2SLS). In practice, an "external" instrument can be difficult to identify. Instead, we attempt to address the potential endogeneity issue by using the generalized method of moments (GMM) approach (see Arellano and Bond, 1991; Blundell and Bond, 1998). This approach employs lagged values of the right hand side variables as the "internal" instruments and has been used in the recent empirical growth literature (such as Murtin and Wacziarg, 2014).

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:		entry			exit	
π_{t-1}	0.446***	0.252^{**}	0.252^{**}	0.334***	-0.030	-0.027
	(0.069)	(0.096)	(0.097)	(0.088)	(0.062)	(0.062)
$\log \text{GDP}_{t-1}$			0.154			2.480
			(3.132)			(1.793)
Country fixed-effects	No	Yes	Yes	No	Yes	Yes
Period fixed-effects	No	Yes	Yes	No	Yes	Yes
Observations	150	150	150	150	150	150
R^2	0.117	0.867	0.867	0.076	0.840	0.842

Table 2: OLS regression results

Note: Each observation is a country by 3-year period over 2004-2018. Standard errors, clustered at the country level, are reported in parentheses. *: significance at 10% level; **: significance at 5% level; ***: significance at 1% level.

We follow Roodman (2009)'s suggestions to estimate (25) by two-step system GMM. Specifically, when we choose the set of instruments, we pay attention to the following "rules of thumb:" First, the number of instruments should be strictly less than the number of countries in the sample. Second, the *p*-value for Arellano-Bond test for AR(2) in differences should be greater than 0.1, so that we cannot reject the null hypothesis that "the error terms are not serially correlated." Third, the *p*-value for Hansen test of joint validity of instruments should also be greater than 0.1 but should not be too large, so that we cannot reject the null hypothesis that "all instruments are jointly exogenous."

Table 3 reports the GMM regression results. For each outcome variable, we try different combinations of instruments. Let $Z = [\pi, \log \text{ GDP}]$. In Columns (1) and (5), the instruments for π_{t-1} are Z_{t-2} and Z_{t-3} . In Columns (2) and (6), the instruments for π_{t-1} are Z_{t-3} In Columns (3) and (7), the instruments for π_{t-1} are Z_{t-3} and Z_{t-4} . In Columns (4) and (8), the instruments for π_{t-1} are Z_{t-2} , Z_{t-3} , and Z_{t-4} . All these instruments are collapsed.

Table 3: GMM regression results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable:	entry			exit				
π_{t-1}	0.745^{**}	0.838^{***}	0.525^{**}	0.702^{**}	0.594***	0.565^{**}	0.528^{**}	0.573^{***}
	(0.356)	(0.223)	(0.196)	(0.327)	(0.181)	(0.219)	(0.202)	(0.191)
$\log \text{GDP}_{t-1}$	-0.037	-0.955	-1.182	-0.137	0.078	-0.896	-0.816	0.035
	(1.006)	(0.777)	(0.730)	(0.970)	(0.659)	(0.561)	(0.515)	(0.684)
Observations	150	150	150	150	150	150	150	150
No. of countries	34	34	34	34	34	34	34	34
No. of instruments	11	9	11	13	11	9	11	13
Nature of instruments	Z_{t-2}	Z_{t-3}	Z_{t-3}	Z_{t-2}	Z_{t-2}	Z_{t-3}	Z_{t-3}	Z_{t-2}
	$\& Z_{t-3}$	[collapsed]	$\& Z_{t-4}$	to Z_{t-4}	$\& Z_{t-3}$	[collapsed]	$\& Z_{t-4}$	to Z_{t-4}
	[collapsed]		[collapsed]	[collapsed]	[collapsed]		[collapsed]	[collapsed]
AB2 p -value	0.384	0.442	0.429	0.376	0.091	0.108	0.094	0.086
Hansen p -value	0.305	0.367	0.425	0.512	0.128	0.644	0.880	0.203

Note: Each observation is a country by 3-year period over 2004-2018. $Z = [\pi, \log \text{ GDP}]$ is the vector of instruments. All regressions include country and period fixed-effects. "AB2 *p*-value" is the *p*-value for Arellano-Bond test for AR(2) in differences. "Hansen *p*-value" corresponds is the *p*-value for Hansen test of joint validity of instruments. Standard errors, clustered at the country level, are reported in parentheses. *: significance at 10% level; **: significance at 5% level; ***:

In these specifications, we can see that π is positively and significantly related to *entry* and *exit*. If we strictly follow Roodman (2009)'s suggestions, the results in Columns (5), (7), and (8) should be interpreted with caution; it is because the *p*-values for Arellano-Bond test for AR(2) in differences are less than 0.1, so that serial correlation of the error terms could be a concern. Literally, these regression results suggest that a 1% increase in inflation rate from the mean is associated with a 0.7-0.8 percentage point increase in firm entry rate and a 0.5-0.6 percentage point increase in firm exit rate.

To summarize, using the GMM approach to address the potential endogeneity issue, we do find consistent evidence at the country level to support Proposition 1 and Proposition 2.

5 Quantitative Exercises

In this section, we calibrate our model to the E.U. data and quantitatively evaluate how our model explains the observed positive effects of inflation on firm entry and exit rates, which we empirically characterized in Section 4.20

The model features the structural parameters $\{\alpha, \gamma, \kappa, \rho, \zeta, \xi, \eta, \theta, \varpi, \overline{\delta}\}$ and a policy instrument π . For a benchmark model, we normalize $\gamma = 0.5$, which implies a squareroot production function for innovation. We then follow Acemoglu and Akcigit (2012) in setting the discount rate ρ to 0.05.²¹ We set the labor intensity to an empirically reasonable value, $1 - \alpha = 0.56$, as in Chu et al. (2019a); for related empirics, see Karabarbounis and Neiman (2014). We also set the share $(1 - \theta)$ of internal cash available for survival investment in the total profit to 0.294, in line with data on propensities to pay dividend and repurchase shares in 2005 for the E.U. (von Aije and Megginson, 2008). We follow Chu et al. (2019a) to consider the case with $\zeta = 0.65$. In what follows, we compare several cases with different values of a natural exit rate, $\overline{\delta}$.

$\overline{\delta}$	$\overline{\omega}$	ξ	η	κ	i
1	0.806	0.367	0.00093	15.6	0.1005
0.23	0.833	0.632	0.00093	4.63	0.1005

 Table 4: Calibrated parameters

We calibrate the four remaining structural parameters, $\{\xi, \eta, \kappa, \varpi\}$, by matching theoretical moments to the data. We target the entry and exit rates in 2005, 10.826% and 8.419%,²² respectively. Using these values, we calibrate the entry cost ϖ and the R&D productivity κ . We calibrate the exogenous growth rate η to match the data on the long-run growth rate, set to 2.5%, which falls within the standard range.²³ We, in turn, calibrate the CIA parameter ξ to match the data on the average money-output, m_t/Y_t , which is around 7.83% in the E.U. (in 2005, base money per GDP).²⁴. Here, we note that

 $^{^{20}}$ We can show a similar quantitative result using U.S. data; see Furukawa and Niwa (2021).

²¹This value falls within the conventional range of ρ .

 $^{^{22}}$ Based on our own calculations from the data we use for our empirical analysis; see Section 4 for details on the data.

 $^{^{23}}$ This value is also consistent with a three-year average of the growth rate from 2014-2016 in the E.U.

²⁴The data are from the Statistical Data Warehouse of the European Central Bank.

our evidence-based assumption $\zeta > \xi$ holds only if the natural exit rate, $\overline{\delta}$, is higher than a value between 0.22 and 0.23. Thus, we may take two polar examples with $\overline{\delta} \in \{0.23, 1\}$. Using a 2005 inflation rate of 2.547% (for the countries covered in Section 4), we calibrate these parameters, as summarized in Table 4.

In the calibrated model, we find that as long as $0.23 \leq \overline{\delta} \leq 1$, the relationships of the inflation rate, π , to the firm entry and exit rates, ε^* and δ^* , are positive. Thus, our model is quantitatively equipped to explain the empirical fact shown in Section 4—that lower inflation rates imply lower entry and exit rates—under realistic values of the entry, exit, and inflation rates. More specifically, in the calibrated model with $\overline{\delta} = 0.23$ ($\overline{\delta} = 1$), a 1% increase in inflation π —from the benchmark level—causes a 0.23% (0.11%) increase in the entry rate ε^* and a 0.32% (0.18%) increase in the exit rate δ^* . This indicates that our model can explain, approximately, 15% to 30% of the observed increase in firm exit (0.5 - 0.6%).

6 Concluding Remarks

We offer a new growth theory that explains what causes declining business dynamism, by developing a new a monetary growth model. Specifically, based on the seminal monetary R&D-based growth model developed by Chu and Cozzi (2014), we propose two new considerations: (a) firms successfully innovating new goods need to further pay an entry cost, (b) after entry, incumbent firms invest in market survival, facing a CIA constraint. These two new features make entry/exit rates and the firm distribution in age endogenous. Using this model, we theoretically find that if the entry cost is sufficiently large, a decrease in the inflation rate, or a deflationary trend, can be accompanied by declining business dynamism, which is featured by lower entry and exit rates and a higher average firm age (or a larger share of older firms). Empirically, we also find evidence that firm entry/exit rates are higher in countries with higher inflation rates among a set of European countries, supporting our theoretical prediction. Calibrating the model to the E.U. data, we show that a decrease in inflation decreases both entry and exit rates under empirically plausible values of the entry, exit, and inflation rates.

Our model abstracts several important aspects of reality. First, the firm size distribution does not follow a Pareto distribution, which is inconsistent with some empirical evidence (e.g., Axtell, 2001) and related theories (e.g., Luttmer, 2007). Instead, the model is equipped to understand the impacts of inflation on the different stages of firms' dynamics—entry, exit, and survival. We achieve this by considering a relatively stylized equilibrium form of firm size distribution. Alternatively, the model can explain the mechanism behind an empirically observed form of the exponential age distribution. Second, consumers are homogeneous in the model. Therefore, there is no direct contribution to the vast literature on growth and income/wealth inequality (e.g., Acemoglu and Cao, 2015, and Jones and Kim, 2018).²⁵ Our model could be extended by assuming that different consumers are heterogeneous in their interest incomes. Third, in our model, firm growth is driven by an exogenous factor. This could be extended by introducing in-

 $^{^{25}}$ See, for example, Haruyama (2021) for a recent contribution to this literature, who develops a Schumpeterian growth model in which a double Pareto distribution of income emerges in equilibrium as a result of entrant (drastic) and incumbent innovations.

house R&D (e.g., Peretto 1996). Forth, our results should depend on the setting where when firms get cash, they have no choice but to borrow from consumers. Introducing an alternative way to raise fund from, for example, a venture capital market would be interesting. We leave these possible extensions for future research.

Finally, we discuss the theoretical insights and policy implications of our study. Theoretically, we offer an explanation for the mechanism through which a decrease in the inflation—or a deflationary trend—can be a cause of declining business dynamism via the CIA channel, which is a novel contribution to the literature. As in existing models, a lower inflation rate yields a lower cost of money holding and, thereby, money borrowing because of the lower nominal interest, which encourages R&D investment for entry. However, in our model, firms are also financially constrained in the survival and exit stage in the equilibrium where incumbent firms borrow cash. As a result, a lower inflation rate directly encourages survival investment and decreases firm exit, due to lower interest payment. Notably, it further has an indirect, general-equilibrium effect that emerges from a shift of labor resources between innovation and survival investment. The direction of this indirect effect is potentially ambiguous, and depends on the size of the entry cost. The reason is as follows. If the entry cost after successful R&D investment is very high; the net payoff of R&D investment is very small. In this case, R&D activity is originally very weak; the positive effect of lower inflation on entry is insignificant. Therefore, a decrease in the inflation rate, i.e., a deflationary trend, causes a shift of resources from entry to survival, discouraging R&D for firm entry and encouraging survival to delay exit. This explains why a lower inflation rate leads to a lower entry and exit rate and a higher average firm age.

These findings deliver the important policy implication that monetary policy controlling the interest rate, or targeting the inflation rate, can essentially affect the nature of business dynamics. For example, a low-interest-rate policy encourages both innovation and survival investment, thereby, enhancing long-run growth as a standard effect. However, as an additional effect, it can also cause declining business dynamism, depending on the size of an entry cost. These findings suggest that the monetary authorities should consider not only standard macroeconomic variables, such as the growth rate, but also variables for firm demographics, in creating a desirable monetary policy.

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Appendix

Proof of Lemma 2. Using Lemma 1, we combine (17)-(19) to obtain

$$\frac{\dot{e}_t}{e_t} = \kappa e_t + \left(\frac{1 + \lambda^{\omega} \left(1 - \theta\right) i}{1 + \lambda^{\omega} \xi i} - \frac{1 + \alpha}{\alpha}\right) \kappa \Pi + \left(k^* - \psi^* + \varpi\right) - \rho.$$
(A1)

Because k_t^* and ψ_t^* are independent of t, (A1) is an autonomous dynamic system for e_t . Applying the standard argument using the transversality condition, (A1) has a globally saddle-point stable steady state, e^* ; at time 0, e_t jumps to:

$$e^* \equiv \left[\rho + (\psi^* - k^* - \varpi) + \left(\frac{1+\alpha}{\alpha} - \frac{1+\lambda^{\omega} (1-\theta)i}{1+\lambda^{\omega}\xi i}\right)\kappa\Pi\right]\frac{1}{\kappa}.$$
 (A2)

Along this unique balanced growth path, the growth rate for c_t and N_t are given as (21) from (17). Substituting (A2) into (19) yields (22) by using $\delta_t = \overline{\delta} - z_t$ and Lemma 1.

To ensure the positivity of g^* and δ^* , we first take κ_0 such that:

$$z^* = \frac{1 + \lambda^{\omega} \left(1 - \theta\right) i}{1 + \lambda^{\omega} \xi i} \kappa \Pi - \left(\rho + \psi^*\right) > 0 \text{ for any } \kappa > \kappa_0.$$
(A3)

Then, we can consider some sufficiently high $\overline{\delta}$ that can ensure $\delta^* > 0$; we label the threshold as $\overline{\delta}_0$. Finally, we can also find some threshold value of η , η_0 , such that $g^* > 0$ for $\eta > \eta_0$ because z^* and δ^* are free from η , proving the positivity of g^* and δ^* .

An equilibrium where the existing firms borrow uniquely occurs if and only if $\frac{\xi z^*}{\kappa}\Big|_{\omega=1} > (1-\theta) \Pi$ and $\frac{\xi z^*}{\kappa}\Big|_{\omega=0} > (1-\theta) \Pi$, which is equivalent to

$$\frac{\xi - (1 - \theta)}{\xi} \kappa \Pi > \max\left\{ \left(1 + \xi i\right) \left(\rho + \left(\frac{\gamma \left(1 + \xi i\right)}{1 + \zeta i}\right)^{\frac{\gamma}{1 - \gamma}}\right), \left(\rho + \left(\frac{\gamma}{1 + \zeta i}\right)^{\frac{\gamma}{1 - \gamma}}\right) \right\}$$
(A4)

based on (A3). Further, (A4) holds for $\xi - (1 - \theta) > 0$ and a sufficiently large κ .²⁶

Proof for the positive relationship between i and π . Substituting (17) into the Euler equation in (3), in the steady state, we have:

$$r = \underbrace{\left[\frac{1+\lambda^{\omega}\left(1-\theta\right)i}{1+\lambda^{\omega}\xi i}\kappa\Pi - \left(\bar{\delta}+\rho-\eta\right)\right]}_{\text{From (17)}} + \rho = \frac{1+\lambda^{\omega}\left(1-\theta\right)i}{1+\lambda^{\omega}\xi i}\kappa\Pi - \left(\bar{\delta}-\eta\right). \tag{A5}$$

Substituting the above into the Fisher equation $(\pi = i - r)$, we obtain:

$$\pi = i - \frac{1 + \lambda^{\omega} \left(1 - \theta\right) i}{1 + \lambda^{\omega} \xi i} \kappa \Pi + (\bar{\delta} - \eta).$$
(A6)

To verify whether i and π are positively related, we check whether $d\pi/di \ge 0$. It is obvious that when $\lambda^{\omega} = 0$, $d\pi/di = 1 > 0$. When $\lambda^{\omega} = 1$:

$$\frac{d\pi}{di} = 1 - \frac{d}{di} \left[\frac{1 + (1 - \theta)i}{1 + \xi i} \kappa \Pi \right] = 1 - \underbrace{\frac{(1 - \theta) - \xi}{(1 + \xi i)^2}}_{<0 \text{ when } 1 - \theta < \xi} \kappa \Pi.$$
(A7)

²⁶We implicitly exclude any extremely large i.

Therefore, as long as $1 - \theta < \xi$, it must be the case that $d\pi/di > 0$ when $\lambda^{\omega} = 1$.

Proof of Proposition 1. We differentiate (22) with respect to *i*, with $\omega = 1$, $d\delta^*/di > 0$ holds (implying $dz^*/di < 0$ by definition) if and only if:

$$\kappa \Pi > \frac{\gamma^{\frac{1}{1-\gamma}}}{1-\gamma} \frac{\zeta - \xi - \zeta \varpi}{\xi - (1-\theta)} \left(\frac{1-\varpi + \xi i}{1+\zeta i}\right)^{\frac{1}{1-\gamma}} \left(\frac{1+\xi i}{1-\varpi + \xi i}\right)^2 \equiv \vartheta(i) \tag{A8}$$

holds. Considering the definition of $\hat{\Pi}$, we can verify that (A8) holds for any $i \geq 0$ if $\varpi \geq 1 - \xi/\zeta$. Under the opposite inequality, $\varpi < 1 - \xi/\zeta$, $\vartheta' < 0$ holds. Therefore, (A8) holds for any $i \geq 0$ if and only if $\kappa \Pi > \vartheta(0)$ holds, or equivalently

$$\kappa \Pi > \frac{\gamma^{\frac{1}{1-\gamma}}}{1-\gamma} \frac{\zeta - \xi - \zeta \varpi}{\xi - (1-\theta)} \left(1 - \varpi\right)^{\frac{1-2(1-\gamma)}{1-\gamma}}.$$
(A9)

Otherwise, if (A9) is violated, there are two cases: (i) if

$$\frac{\zeta - \xi - \zeta \varpi}{\xi - (1 - \theta)} \left(\frac{\xi}{\zeta}\right)^{\frac{1}{1 - \gamma}} < \hat{\Pi} < \frac{\zeta - \xi - \zeta \varpi}{\xi - (1 - \theta)} \left(1 - \varpi\right)^{\frac{1}{1 - \gamma} - 2},\tag{A10}$$

 $d\delta^*/di > (<)0$ holds for higher (lower) *i* (a U shape), and (ii) if

$$\hat{\Pi} < \frac{\zeta - \xi - \zeta \varpi}{\xi - (1 - \theta)} \left(\frac{\xi}{\zeta}\right)^{\frac{1}{1 - \gamma}},\tag{A11}$$

 $d\delta^*/di < 0$ holds for any $i \ge 0$ (monotonically negative). Noting the positive relationship between i and π completes the proof.

Proof for the effect on the firm age distribution. Differentiating (24) yields:

$$\frac{d}{di}f_t(a) > 0 \iff \left(-(\varepsilon^*)'\right)\left(1 - a(\varepsilon^* + \eta)\right)e^{-(\varepsilon^* + \eta)a} < 0.$$
(A12)

As shown above, under $1 - \theta < \xi$, $(\varepsilon^*)' < 0$ holds. In this case, there necessarily exists some cutoff value of a, \tilde{a} , such that $(-(\varepsilon^*)')(1 - a(\varepsilon^* + \eta)) < 0$ for any $a > \tilde{a}$. Therefore, a higher *i* decreases the density of older firms.