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Optimal wealth taxation in the Schumpeterian growth model with unemployment

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Abstract

In this paper, we construct a Schumpeterian endogenous growth model, taking into account unemployment, and study the effect of wealth tax policy on the employment and the growth rate. In our model, the final good firms use labor and intermediate goods as input. The firms search and match with workers in a frictional labor market. The wage rate is determined by Nash bargaining. We first show that there may be one or two balanced growth paths in the model. We next show that when the equilibrium path is uniquely determined, the reduction of the bargaining power of the worker reduces the balanced growth rate and raises unemployment. We finally show that the wealth tax can enhance innovation, reduce unemployment and raise the economic growth rate. The welfare-maximizing wealth tax rate is generally non-zero, and for some plausible parameter values, the rate is strictly positive.

Keywords: Endogenous growth; unemployment; wealth tax

JEL classification: E5;

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1 Introduction

Optimal taxation has been one of the most important topics in macroeconomics. A pioneering work of Chamley (1986) studies the continuous time neoclassical growth model and finds that the optimal tax rate on capital is zero. Some authors investigate the optimal tax policy in the endogenous growth framework. Nuno (2011) derives the optimal capital and R&D subsidy rates in a Schumpeterian endogenous growth model. Long and Pelloni (2017) and Gross and Klein (2021) study the optimal labor and capital income tax in a variety expansion model which is based on Rivera-Batiz and Romer (1991). However, they do not study unemployment and then it is not clear at this point that how the labor market frictions affects the optimal tax policy.

In this paper, we construct the Schumpeterian endogenous growth model with search unemployment. The basic set-up is very close to the general equilibrium model with unemployment constructed by Blanchard and Gali (2010) and Tesfaselassie and Wolters (2014), but here we incorporate Schumpeterian process of creative destruction into the model. Using the model, we first study the relationship between growth and unemployment. We then study the effect of tax policy, especially wealth tax policy on the employment and the growth rate. In our model, the final good firms uses labor and the intermediate good as input. The firms search and match with workers in the frictional labor market. The wage rate is determined by the Nash bargaining. The intermediate good is differentiated and the intermediate good firm has a monopoly power over the price determination. The entrepreneurs make costly investment and tries to improve the efficiency of the intermediate goods production. If the entrepreneur succeed in innovation, the entrepreneur becomes a monopoly supplier of the differentiated intermediate good.

We first show that there may be one or two balanced growth paths in the model. We next show that when the equilibrium path is uniquely determined, the reduction of the bargaining power of the worker reduces the balanced growth rate and raises unemployment. This conclusion is consistent with a recent paper of Stansbury and Summers (2020) who claims that decline in the power of the workers is one of the main causes of the recent stagnation in the United States.

We finally study the optimal taxation and show that the wealth tax can enhance innovation, reduce unemployment and raise economic growth rate. Here we focus on the wealth tax instead of the labor income tax because the wealth tax does not affect the bargaining process between workers and firms directly. Welfare-maximizing wealth tax rate is generally non-zero, and that for some plausible parameter values, the rate is strictly positive. As far as we know, our paper is the first to study the welfare maximizing tax policy in the endogenous growth model with unemployment.

There are a lot of existing literature that investigates the relationship between unemployment and growth in endogenous growth models. The pioneering work is Aghion and Howitt (1994) who incorporates search and matching in the labor market a la Mortensen and Pissarides (1999) into the Schumpeterian growth model of Aghion and Howitt (1992). Some authors study the OLG models. Bean and Pissarides (1994) work with AK model, while Hashimoto and Im (2014) study the effect of bubbles on growth and unemployment in the variety expansion models. However, none of them works with welfare-maximizing fiscal policy.

Some authors study monetary and fiscal policy in neoclassical models with labor market frictions. Shi and Wen (1999) study the neoclassical growth model and show that labor income tax may be more costly than capital income tax. Domeij (2005) and Arseneau, and Chugh (2012) study the Ramsey optimal taxation and find that in the steady state, the optimal capital tax is not zero. However, none of them study the endogenous growth models. In this paper, we study the endogenous growth model and show that wealth tax may be growth-enhancing and also welfare-improving. This paper is also related to Schubert and Turnovsky (2017) who numerically investigates the effect of tax and debt policy on growth and unemployment, but they do not study the welfare effect of these tax policies. With respect to monetary policy, Heer (2003) studies the neoclassical monetary growth model with unemployment and show that a deviation from the Friedman rule is welfare-improving. Chu (2020) studies the welfare effect of inflation in a monetary Schumpeterian endogenous growth model. However, they do not work with tax policy.

Wealth taxation has been intensively studied recently. Jakobsen et al. (2020) proves

the benefits of wealth taxation in a simple life cycle model with utility from bequest. On the other hand, a seminal paper of Straub and Werning (2020) who investigate the capitalist-worker model and show that the optimal wealth tax is nonzero. Here we show that the wealth tax is welfare-improving in the endogenous growth model with unemployment.

This paper is organized as follows. Section 2 describes the basic structure of the model. Section 3 investigates the balanced growth paths. Section 4 considers the welfare effect of wealth taxation. Section 5 concludes the paper.

2 The Model

In this section, we describe the set-up of the model.

2.1 Households

The model is close to Blanchard and Gali (2010). While the economic growth rate in Blanchard and Gali (2010) is exogenous, here we consider innovation and then the economic growth rate is endogenously determined. The process of innovation is based on Benigno and Fornaro (2020). Time is discrete and has infinite horizons. The model consists of the final good, intermediate good, workers, firms and entrepreneurs. There is a representative family that consists of continuum of individuals with measure one. An individual matches with firms with some probability, and supplies labor in the labor market, receives wage income, consumes and saves. Total level of labor supply is one.

The representative family maximizes the following intertemporal quasi-linear utility:

$$U = \sum_{t=0}^{\infty} \beta^t \{\ln C_t - \psi N_t\}, \quad (1)$$

subject to the following constraints

$$X_{t+1} = (1 - \tau)R_t X_t + W_t N_t + T_t - C_t, \quad (2)$$

$$N_t = (1 - \delta)N_{t-1} + p_t u_t, \quad (3)$$

$$u_t = 1 - (1 - \delta)N_{t-1}, \quad (4)$$

where $\beta \in (0, 1)$ is the discount factor, C_t is the consumption level of the family in period t , N is a time spent on working in period t , X_t is the asset of the individual in period t , R_t is the gross interest rate in period t , τ is the wealth tax rate, W_t is the wage income in period t , T_t is the government transfer, p_t is the probability of finding firms, $\delta > 0$ is the job separation rate, $\psi > 0$ is the parameter on the labor disutility, u_t is the unemployment at the beginning of time t . A similar wealth tax policy is studied by Straub and Werning (2020). In a later section, we describe the process in which the matching probability p_t is determined.

The consumption Euler equation is

$$\frac{C_{t+1}}{C_t} = \beta R_{t+1}(1 - \tau). \quad (5)$$

In this paper, to simplify the analysis, we suppose that there is a perfect separation between worker and the firms in each period, that is, $\delta = 1$. In that case, every individual search for the firms at the beginning of each period and then $u_t = 1$. Therefore $N_t = p_t u_t$. In this paper, we measure the unemployment U_t by the number of unemployed after matches. Since the total number of workers is equal to one, $U = 1 - N_t$.

2.2 Final good firms

The final good is produced by a large number of homogeneous firms. The final good market is competitive. The final good firm can produce the good by using the labor and the differentiated intermediate goods as input. The final good firm first enters the market by paying a fixed cost including the set-up cost and vacancy posting cost. We follow Aghion and Howitt (1994) and also Blanchard and Gali (2010), we assume that the entry cost is proportional to the aggregate productivity level.

In the labor market, the final good firm meets with a worker with probability $q_t > 0$. With probability $1 - q_t$, the firm fails in matching with a worker and produce nothing in that period. Here every employment relationship is broken at the end of each period. Therefore every firm must re-enter the market in the next period for producing the good in that period.

When the final good firm meets with a worker, the firm then produce the good by using the intermediate good. The intermediate good is differentiated and the total variety of the intermediate good is one. The production function of the final good firm is given by

$$Y_t^F = \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha di, \quad (6)$$

where A_{it} is the technology level of intermediate good i , and x_{it} is the level of intermediate good i at time t .

The intermediate good firm produces one unit of intermediate good by using one unit of the final good. The firm of the each variety of the intermediate good has monopoly power for production. Let the price of intermediate good i for the final good firm be $p_{it} > 0$. Then the profit of the final good firm (except for the wage payment) is

$$\Pi_t^F = \int_0^1 \{A_{it}^{1-\alpha} x_{it}^\alpha - p_{it} x_{it}\} di. \quad (7)$$

The first order conditions on the choice of x_i is given by

$$\alpha A_{it}^{1-\alpha} x_{it}^{\alpha-1} = p_{it}. \quad (8)$$

In equilibrium, the number of the final good firm is equal to the total labor supply N_t , since each firm matches with one worker and each worker supplies one unit of labor.

2.3 Intermediate good firm

The total profit of the intermediate good firm i , $\Pi_i^I = N_t(p_{it}x_{it} - x_{it})$ is expressed as $\Pi_{it}^I = N_t\{\alpha A_{it}^{1-\alpha} x_{it}^\alpha - x_{it}\}$. The first order condition on x are $\alpha^2 A_{it}^{1-\alpha} x_{it}^{\alpha-1} = 1$. Thus the price of the intermediate good is simply a constant mark-up:

$$p_{it} = 1/\alpha. \quad (9)$$

If we let $\epsilon = \alpha^{2/(1-\alpha)}$, the quantities are $x_{it} = \epsilon A_{it}$. The profit of the intermediate good i is therefore expressed as

$$\Pi_{it}^I = (\alpha^{-1} - 1)\epsilon N_t A_{it}.$$

In that case, the gross outputs of the final good firm is

$$Y_t = \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha di = \epsilon^\alpha A_t, \quad (10)$$

where $A_t = \int_0^1 A_{it} di$ is the average productivity index. We call A_t the aggregate productivity level. We suppose that the initial level of the technology level $A_0 > 0$ is given.

The profit of the final good firm is respectively written as

$$\Pi_t^F = \int_0^1 \left(\epsilon^{\alpha-1} - \frac{1}{\alpha} \right) \epsilon A_{it} di = \rho A_t, \quad (11)$$

where $\rho = (1 - \alpha)\epsilon^\alpha = (1 - \alpha)\alpha^{2\alpha/(1-\alpha)} = (1/\alpha - 1)\alpha^{2\alpha/(1-\alpha)} > 0$ is a constant.

Let v_t denote the number of final goods firms which enters the market. Then the matches between firms and workers at time t is equal to $N_t = q_t v_t$. Therefore total amount of output at time t , $Y_t = N_t Y_t$ is equal to

$$Y_t = q_t v_t \cdot \epsilon^\alpha A_t = \epsilon^\alpha N_t A_t. \quad (12)$$

Also the profit of the producer of the intermediate good i is

$$\Pi_{it}^I = \omega N_t A_{it}, \quad (13)$$

where $\omega = (\alpha^{-1} - 1)\epsilon > 0$ is a constant.

Finally, since one unit of the final good is needed to produce one unit of the intermediate good, the amounts of final goods the firm A and the firm A use to produce one unit of the final good are respectively $\int_0^1 x_{it} di = \epsilon A_t$. Thus the total amount of final goods in this economy used as the input to the intermediate goods production is $\epsilon N_t A_t$. Therefore the net output is

$$Y_t - \epsilon N_t A_t = \Psi N_t A_t, \quad (14)$$

where $\Psi = \epsilon^\alpha - \epsilon > 0$. This equation implies that when the employment N is constant, the net output is proportional to the aggregate technology level.

2.4 Innovation

The process of innovation is exactly the same as Benigno and Fornaro (2020). There is a large number of entrepreneurs. When an entrepreneur spends I_{jt} units of the final

good to improve the productivity of product j , she succeeds in productivity improvement with probability $\mu_j = \chi I_{jt}/A_{jt}$, where the parameter $\chi > 0$ shows the efficiency of the innovation. When the entrepreneur succeed in innovating product j at time t , the entrepreneur becomes a monopolist only at time $t + 1$. After time $t + 1$, then the patent of the intermediate good production is randomly allocated to other entrepreneur. Here we assume that the productivity of product j becomes γA_{jt} at time $t + 1$.

The present value of the expected profit the entrepreneur gets after she innovates is

$$\mu_{jt} \cdot \frac{\gamma \cdot \Pi_{j,t+1}^I}{R_{t+1}}. \quad (15)$$

Under the free entry condition, it must be equal to cost of innovation. This cost is equal to I_{jt} units of final goods. Since the profit of the intermediate good firm that produces the variety j at time $t + 1$ is $\Pi_{j,t+1}^I = \omega N_{t+1} A_{j,t+1}$, the free entry condition is simplified as

$$\frac{\omega \gamma \chi N_{t+1}}{R_{t+1}} = 1.$$

Following Benigno and Fornaro (2020), we restrict our attention on the symmetric equilibrium in which the probability of success μ_t , is the same across sectors.

From the law of large numbers, the measure of firms who succeed in innovation is equal to the probability of success μ_t . Therefore in the symmetric equilibrium, the aggregate productivity evolves according to $A_{t+1} = \mu_t \gamma A_t + (1 - \mu_t) A_t$. Thus the productivity growth rate is equal to

$$g_t^A = \frac{A_{t+1}}{A_t} = 1 + \mu_t(\gamma - 1). \quad (16)$$

As Benigno and Fornaro (2020) shows, the total investment which entrepreneurs spends at time t is

$$\int_0^1 I_{jt} dj = \frac{1}{\chi} \mu_t A_t = \frac{A_t (g_t^A - 1)}{\chi(\gamma - 1)}. \quad (17)$$

Along the balanced growth path, the investment is proportional to the aggregate technology level.

2.5 Frictions in the labor market

Here we closely follow Blanchard and Gali (2010) and describe the process of wage determination. Let J_t^e denote the value of employment at time t . Also, let J^v denote the value

of vacancy at time t . The Bellman equations that determine the value functions J_t^e and J^v are given by

$$J_t^v = -\kappa_t + q_t J_t^e + (1 - q_t) R_{t+1}^{-1} J_{t+1}^v, \quad (18)$$

$$J_t^e = \Pi_t^F - W_t + R_{t+1}^{-1} J_{t+1}^v, \quad (19)$$

where κ_t is the market entry cost at time t and q_t is the probability of meeting with the unemployed individual. Following the existing literature such as Blanchard and Gali (2010), We assume that the entry cost of the final good firm, κ_t , is proportional to the aggregate productivity level. Let κ_t denote $\kappa_t = \kappa A_t$, where $\kappa > 0$ is constant.

From the free entry condition, the value of vacancy is equal to zero: $J^v = 0$. Moreover, the profits of the final good firm is proportional to the aggregate technology level: $\Pi_t^F = \rho A_t$. Thus the Bellman equation is simplified as

$$\kappa A_t = q J_t^e, \quad (20)$$

$$J_t^e = \rho A_t - W_t. \quad (21)$$

Let V^N denote the marginal value of the employed worker. Similarly, let V^U denote the marginal value of unemployed worker. If the probability of matching with the firm is p , then these values satisfy

$$V_t^N = W_t - \psi C_t + R_{t+1}^{-1} \{p_t V_{t+1}^N + (1 - p_t) V_{t+1}^U\}, \quad (22)$$

$$V_t^U = R_{t+1}^{-1} \{p_t V_{t+1}^N + (1 - p_t) V_{t+1}^U\}. \quad (23)$$

Here the term $W_t - \psi C_t$ shows the surplus of the worker from the match, which depends on labor disutility. The first term W_t shows the marginal increase of final good by supplying one unit of labor. Here the utility function is quasi-linear, then the marginal increase of labor disutility from one unit of labor supply is equal to a constant ψ . On the other hand, marginal utility from consuming one unit of the final good is equal to $1/C_t$. Thus labor disutility ψ is interpreted as the ψC_t unit of consumption good. Thus the surplus from the match is equal to $W_t - \psi C_t$.

From the four value functions obtained above, the surplus of the households $S_t^H = V_t^N - V_t^U$ and the one of the firms $S_t^F = J_t^e - J_t^v = J_t^e$ are respectively written as

$$\begin{aligned} S_t^H &= W_t - \psi C_t, \\ S_t^F &= \rho A_t - W_t. \end{aligned}$$

Following the existing literature, we assume the real wage W_t is determined so as to maximize the Nash product $(S_t^F)^\theta (S_t^H)^{1-\theta}$, where θ is the bargaining power of the firm. Therefore the surplus of the household S_t^H is proportional to the one of the firm S_t^F and we have the following equality:

$$\theta S^H = (1 - \theta) S^F. \quad (24)$$

Here the total surplus is $S_t^H + S_t^F = \rho A_t - \psi C_t$. Thus

$$S_t^F = \theta(\rho A_t - \psi C_t). \quad (25)$$

Similarly $S_t^H = (1 - \theta)(\rho A_t - \psi C_t)$. Then the free entry condition $\kappa A_t = q_t J_t^e$ is re-expressed as

$$\kappa A_t = q\theta(\rho A_t - \psi C_t). \quad (26)$$

The real wage is determined by $W_t = \theta\psi C_t + (1 - \theta)\rho A_t$.

Now consider the Cobb-Douglas type matching function $\Omega(u, v) = \Omega u^\phi v^{1-\phi}$ in which where the parameter $\phi \in (0, 1)$ is the elasticity of the unemployment rate on the matching function and the parameter $\Omega > 0$ is the efficiency of the matching process. We assume that efficiency parameter $\Omega > 0$ is sufficiently small to ensure that the matching probability p_t and q_t are strictly less than one.

3 Equilibrium

In this section, we characterize the equilibrium path.

3.1 Equilibrium conditions

Here we assume that the probability of separation δ is equal to one. Thus Eq. (3) implies that the measure of unemployed u_t at the beginning of period t is equal to one. Therefore

we have $N_t = \Omega v_t^{1-\phi}$. The probability of matching for the firm $q_t = \frac{\Omega_t}{v}$ and the one for the worker $p_t = \frac{\Omega_t}{u_t}$ are respectively given by

$$\begin{aligned} q_t &= \Omega v^{-\phi}, \\ p_t &= \Omega v^{1-\phi}. \end{aligned}$$

The resource constraint on the final goods is given by

$$Y_t^F = \kappa_t v_t + C_t + \int_0^1 I_{jt} dj + N_t \int_0^1 x_{it} di,$$

where the first term $\kappa_t v_t$ is the entry cost, the second term C_t is the consumption, the third term $\int_0^1 I_{jt} dj$ is the investment for R&D, and the fourth term $N_t \int_0^1 x_{it} di$ is the amount of the intermediate good used for final good production.

Using Eqs. (14) and (17), we can simplify the resource constraint as

$$\Psi N_t A_t = \kappa_t v_t + C_t + A_t \frac{A_{t+1}/A_t - 1}{\chi(\gamma - 1)},$$

where $\Psi = \epsilon^\alpha - \epsilon > 0$.

The next proposition normalize the constraints above by the aggregate technology level A_t and characterizes the equilibrium allocation. In equilibrium, the effective measure of firms are $N_t = \Omega v_t^{1-\phi}$.

Proposition 1 *We let $c_t = C_t/A_t$. Given the initial level of aggregate productivity A_0 , the equilibrium sequence $\{v_t, c_t, A_{t+1}\}$ is determined by*

$$\kappa = \theta(\rho - \psi c_t)q(v_t), \quad (27)$$

$$A_{t+1}/A_t = \omega \gamma \chi \beta (1 - \tau) N(v_{t+1}) \frac{c_t}{c_{t+1}}, \quad (28)$$

$$\Psi N(v_t) = \kappa v_t + c_t + \frac{A_{t+1}/A_t - 1}{\chi(\gamma - 1)}, \quad (29)$$

where $q(v) = \Omega v^{-\phi}$ and $N(v) = \Omega v^{1-\phi}$.

Proof. See the appendix. ■

In Proposition 1, Eq. (27) is the free entry conditions of the final good firm, Eq. (28) is the research arbitrage equation and Eq. (29) represents the resource constraint.

3.2 Balanced growth paths

We now characterize the balanced growth path. Let $g = A_{t+1}/A_t > 1$ denote the technological growth rate along the balanced growth path, output and consumption grows at the rate g , and the normalized consumption level c_t and the number of the vacancy v is constant. The next proposition characterizes the balanced growth paths.

Proposition 2 *Along the balanced growth path, the normalized consumption c and the vacancy v is determined by the following equations.*

$$\kappa = \theta(\rho - \psi c)q(v), \quad (30)$$

$$\sigma N(v) = \kappa v + c - \frac{1}{\chi(\gamma - 1)}, \quad (31)$$

where $\sigma = \Psi - \xi(1 - \tau)$, $\xi = \frac{\omega\gamma\beta}{\gamma-1}$, $q(v) = \Omega v^{-\phi}$ and $N(v) = \Omega v^{1-\phi}$.

Proof. See the appendix. ■

The first equation shows the negative relationship between vacancy v and consumption c . On the other hand, the second equation shows a possibly non-monotone relationship between c and v . This implies that the balanced growth rates may be multiple. The balanced growth rate is determined by

$$g(v) = \omega\gamma\chi\beta(1 - \tau)N(v)$$

Substitution of the first equation into the second equation yields

$$\sigma N(v) + \frac{\kappa}{\psi\theta q(v)} = \kappa v + \frac{\rho}{\psi} - \frac{1}{\chi(\gamma - 1)}, \quad (32)$$

where $\sigma = \Psi - \xi(1 - \tau)$. Substitution of the functional forms for the matching functions into the above equation, Thus we have a nonlinear equation on v :

$$\Omega\sigma v^{1-\phi} + \frac{1}{\Omega} \frac{\kappa}{\theta\psi} v^\phi = \kappa v + \eta \quad (33)$$

where $\eta = \frac{\rho\chi(\gamma-1)-\psi}{\psi\chi(\gamma-1)}$. The next proposition characterizes the uniqueness and the multiplicity of the balanced growth path.

Proposition 3 *If $\rho\chi(\gamma - 1) - \psi \leq 0$, then there is unique balanced growth path. On the other hand, If $\rho\chi(\gamma - 1) - \psi > 0$, there are two balanced growth paths.*

Proof. See the Appendix. ■

In the following, we assume that the inequality $\rho\chi(\gamma - 1) < \psi$ or equivalently $\eta < 0$ holds. In that case, the balanced growth path is unique. The uniqueness condition holds as long as the parameter on the degree of innovation γ is sufficiently low.

Unfortunately, there is no closed form solution path to Eq. (33). Blanchard and Gali (2010) argue that the empirical estimate of ϕ is close to 1/2. If we follow Blanchard and Gali (2010) and Tesfaselassie and Wolters (2018) and assume that the coefficient on the matching function ϕ equals to 0.5, Eq. (33) has a explicit solution.¹ First of all, when $\phi = 0.5$, if we let $x = \sqrt{v}$, then the equation is re-written as the quadratic equation on x : $(\Omega\sigma + \frac{1}{\Omega} \frac{\kappa}{\theta\psi})x = \kappa x^2 + \eta$. The value of v that solves Eq. (33) is

$$v^* = \left\{ \alpha_0 + \sqrt{(\alpha_0)^2 + 4 \frac{\eta}{\kappa}} \right\}^2, \quad (34)$$

where $\alpha_0 = \frac{\Omega}{\kappa}\sigma + \frac{1}{\Omega} \frac{1}{\theta\psi}$.

3.3 Bargaining power and growth

In a recent paper, Stansbury and Summers (2020) argues that the current secular stagnation of the advanced countries including the United States is due to a decline in the worker's power relative to firms. In our paper, the bargaining power of the worker $1 - \theta$ can be interpreted as the power of workers. In this section, we investigate how the balanced growth rate is affected by the parameter θ . Let $F(v) = \Omega\sigma v^{1-\phi} + \frac{1}{\Omega} \frac{\kappa}{\theta\psi} v^\phi$ denote the left hand side of Eq. (33). Then Eq. (33) can be simplified as $F(v) = \kappa v - \eta$.

Let v^* be the solution to the equation. Since $F(0) = 0$ and $F''(v) < 0$, $F'(v^*) < \kappa$. Moreover, $\frac{\partial F}{\partial \theta} < 0$. Thus

$$\frac{\partial v^*}{\partial \theta} = \frac{1}{\kappa - F'(v^*)} \frac{\partial F}{\partial \theta} < 0.$$

¹Tesfaselassie and Wolters (2018) cite Blanchard and Gali (2010) and argue that this assumption is empirically plausible.

This implies that the increases in the bargaining power of firm or equivalently the decline in the bargaining power of worker actually reduces the equilibrium vacancy. Therefore we obtain the following proposition.

Proposition 4 *Reduction of the bargaining power of worker increases unemployment and decreases the balanced growth rate.*

If the bargaining power of the firm increases, the profit of the firm increases and then the more firms try to enter the labor market for production. However, This also raises the entry cost and as a result, less resources are devoted to the innovation. The proposition shows that this is harmful for the unemployment rate and also the balanced growth rate decreases.

4 Wealth tax

In this section, we investigate the impact of the wealth tax on the employment, the balanced growth rate and welfare along the balanced growth path.

4.1 Growth effect

If wealth tax rate τ goes up, the left hand side F shifts upward. On the other hand, the right hand side is unchanged. Thus the equilibrium level of vacancy always increases and unemployment is reduced. However, growth effect of the wealth taxation is ambiguous because the balanced growth rate $g(v) = \omega\gamma\chi\beta(1 - \tau)N(v)$ is an increasing function of v but is a decreasing function of the wealth tax.

It is well-known that the wealth tax generates the distortion on the intertemporal saving decision and this has negative impact on the economic growth. General analysis is not easy and then in the following, we focus on the case where the intensity parameter of the matching function ϕ is 0.5, and the parameter η is zero. We have the following proposition on the relationship between wealth tax and the economic growth rate and the unemployment rate.

Proposition 5 *The introduction of the wealth tax always reduces unemployment rate. If $\phi = 1/2$, $\psi = \rho\chi(\gamma - 1)$ and $\frac{\Psi}{\xi} + \frac{\kappa}{\theta\psi\Omega^2\xi} < 2$, then the balanced growth rate is inverted-U shaped of the wealth tax rate τ . The growth maximizing wealth tax rate $\tau^* \in (0, 1)$ exists.*

Proof. See the Appendix. ■

4.2 Welfare effect

In this section, we derive the social welfare along the balanced growth path and investigate the welfare impact of wealth tax. Since $C_t = cA_t$ and $A_t = A_0g^t$, the intertemporal utility along the balanced growth path is written as

$$U = \sum_{t=0}^{\infty} \beta^t \{\ln C_t - \psi N_t\} = \frac{\ln c - \psi N}{1 - \beta} + \ln g \sum_{t=0}^{\infty} \beta^t t + \bar{C}$$

where $\bar{C} = \ln A_0$ is a policy-independent constant. For simplicity, here we multiply the intertemporal utility by $(1 - \beta)^2$. Since $\sum_{t=0}^{\infty} \beta^t t = 1/(1 - \beta)^2$, if we define the social welfare as $V = (1 - \beta)^2 U$, this equation is simplified as

$$V = (1 - \beta)(\ln c - \psi\Omega v^{1-\phi}) + \beta \ln g.$$

Here we ignore the policy irrelevant constant terms. As the discount factor converges to 1, the welfare U converges to $\ln g(v)$. In this case, welfare-maximizing wealth tax rates is equal to the growth-maximizing tax rate. As Proposition 2 shows, the rate is positive for some parameter.

Proposition 6 *If $\phi = 1/2$, $\eta = 0$, $\frac{\Psi}{\xi} + \frac{\kappa}{\theta\psi\Omega^2\xi} < 2$ and the discount rate β is sufficiently closed to one, then the welfare maximizing wealth tax rate exists and is positive.*

Proof. See the Appendix. ■

Usually, the optimal wealth tax rate or capital income tax rate are either zero or negative because the subsidization of savings stimulates capital accumulation or innovation. However, here the positive wealth tax has two advantages. First, positive wealth tax reduces unemployment and raises the profit of entrepreneurs. Second, positive wealth tax deters the socially excessive entry of the final good firm and this reduces the total entry/vacancy cost. This implies that more resources are allocated to innovative activities.

5 Conclusion

In this paper, we construct the Schumpeterian endogenous growth model with search unemployment. We study the effect of the reduction of the bargaining power of the worker and the effect of wealth tax policy on the employment and the growth rate. In our model, the final good firms uses labor and the intermediate good as input. The firms search and match with workers in the frictional labor market. The wage rate is determined by the Nash bargaining. We first show that there may be one or two balanced growth paths in the model. We next show that when the equilibrium path is uniquely determined, the reduction of the bargaining power of the worker reduces the balanced growth rate and raises unemployment. We finally show that the wealth tax can enhance innovation, reduce unemployment and raise economic growth rate.

Appendix

The Appendix provides proofs for propositions.

A Proof of Proposition 1

If we divide both sides of the resource constraint by the aggregate technology level A_t , we get

$$\Psi N_t = \kappa v_t + c_t + \frac{g_t^A - 1}{\chi(\gamma - 1)}.$$

where $g_t^A = A_{t+1}/A_t$. On the other hand, the Euler equation is re-expressed as $\frac{1}{R_{t+1}} = \frac{1}{g_t^A} \beta(1 - \tau) \frac{c_t}{c_{t+1}}$. Thus the free entry condition for the entrepreneur is written as $\omega \gamma \chi \beta (1 - \tau) N_{t+1} \frac{c_t}{c_{t+1}} = g_t^A$. This completes the proof. ■

B Proof of Proposition 2

In the steady state, $v_t = v$ and $c_t = c$ are constant and then Eq (27) obviously implies Eq (30). When we substitute Eq. (28) into Eq. (29), we have

$$\Psi N(v_t) = \kappa v_t + c_t + \xi(1 - \tau) N(v_{t+1}) \frac{c_t}{c_{t+1}} - \frac{-1}{\chi(\gamma - 1)}$$

where $\xi = \frac{\omega \gamma \beta}{\gamma - 1}$. In the steady state, $\frac{c_t}{c_{t+1}} = 1$. Thus we have Eq (31). This completes the proof. ■

C Proof of Proposition 3

The left hand side is a strictly concave function of the vacancy v that passes the origin. On the other hand, the right hand side is a linear function with a strictly positive slope. Therefore the solutions to the above equation is either one or two. The number depends on the constant term η . ■

D Proof of Proposition 4

In this case, Eq. (33) is simplified as $\frac{\Omega}{\kappa}\{\Psi - \xi(1 - \tau)\} + \frac{1}{\Omega} \frac{1}{\theta\psi} = \sqrt{v}$, and the balanced growth rate is now becomes a quadratic function of the wealth tax rate τ :

$$g = \frac{\Omega^2}{\kappa} \omega \gamma \chi \beta (1 - \tau) \left[\frac{\Psi}{\xi} + \frac{\kappa}{\theta \psi \Omega^2 \xi} - (1 - \tau) \right].$$

Definitely it is a non-monotonic function of τ . The growth rate is maximized when the wealth tax rate is equal to

$$\tau = 1 - \frac{1}{2} \left(\frac{\Psi}{\xi} + \frac{\kappa}{\theta \psi \Omega^2 \xi} \right).$$

This completes the proof. ■

E Proof of Proposition 6

In the limit case where the discount factor is equal to one, social welfare $V = \ln g$ coincides with the logarithm of the economic growth rate g . As we show in Proposition 4, if $\phi = 1/2$, $\psi = \rho \chi (\gamma - 1)$ and $\frac{\Psi}{\xi} + \frac{\kappa}{\theta \psi \Omega^2 \xi} < 2$, then the variable g is inverted-U shaped of the wealth tax rate τ . The growth maximizing wealth tax rate $\tau^* \in (0, 1)$ also maximizes the welfare along the balanced growth rate. ■

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