RIETI Discussion Paper Series 21-E-032

Competition, Productivity and Trade, Reconsidered

ARA, Tomohiro
Fukushima University

Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
https://www.rieti.go.jp/en/

RIETI Discussion Paper Series 21-E-032
April 2021

Competition, Productivity and Trade, Reconsidered*

Tomohiro ARA
Fukushima University

Abstract

We study unilateral trade liberalization and unilateral market expansion and their impact on optimal tariffs in a heterogeneous firm model with a general productivity distribution and an endogenous wage. We show that unilateral trade liberalization entails selection effects, but unilateral market expansion entails anti-selection effects in a country of origin. Conditional on the two sufficient statistics for welfare, the optimal level of import tariffs is the same across different trade models with a constant trade elasticity, but more generally the optimal level depends on the micro structure that makes the trade elasticity variable.

Keywords: Trade liberalization, country size, optimal tariff, variable trade elasticity
JEL classification: F12, F13, F16

*This study is conducted as a part of the Project "Economic Policy Issues in the Global Economy" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). The author would like to thank Svetlana Demidova and Takumi Naito for helpful comments on an earlier version of the manuscript. Financial support from the Japan Society of the Promotion of Science under grant numbers 19K01599, 20H01492 and 20H01498 is gratefully acknowledged.

1 Introduction

A growing body of empirical evidence using aggregate and firm-level data has demonstrated that country size has a critical impact on the domestic trade share, one of the sufficient statistics for welfare along with the trade elasticity (Arkolakis et al., 2012). For example using aggregate data on manufacturing for 25 countries, Eaton and Kortum (2011) show that larger countries tend to buy much more products from the domestic market than smaller countries. Using firm-level data on manufacturing, Bernard et al. (2007) and Mayer and Ottaviano (2008) show a similar trend in the United States and European countries respectively in that larger countries tend to have a larger fraction of firms that sell their products for the domestic market than smaller countries. These pieces of evidence indicate that country size has an opposite effect on the domestic trade share to trade liberalization (i.e., the share is higher, the larger and the *less* open are countries), even though both are associated with sources of competitive pressures on the domestic market.

In this paper, we explore the mechanism through which country size and trade liberalization work differently on firm selection, welfare gains and optimal policy in recent trade models with imperfect competition and heterogeneous firms. As in most of these models in the literature, we employ an asymmetric-country version of the Melitz (2003) model with monopolistic competition and CES preferences. One of the well-known drawbacks in this framework is that firms' markups are constant which implies under firm heterogeneity that country size has no selection effects. To achieve our goal mentioned above, we make three key departures from the existing models. First, we develop a general model without imposing specific parameterizations to a productivity distribution. Second, we drop the assumption of a freely traded outside good sector, which makes factoral terms-of-trade (i.e., wages) endogenous. Finally, we analyze not only iceberg trade costs but also import tariffs that raise government revenue. These distinctions jointly help understand the role of two competitive pressures in generating different effects in a single unified setting.

We show that unilateral trade liberalization entails selection effects but unilateral market expansion entails anti-selection effects in a country of origin. Unilateral reductions in trade costs lower (raise) expected profit in a liberalizing (non-liberalizing) country, which directly induce less (more) firms to enter there. With endogenous wages, such reductions worsen the terms-of-trade (i.e., lower the relative wage) and raise expected profit in a liberalizing country, which indirectly induce more (less) firms to enter in a liberalizing (non-liberalizing) country. In equilibrium, the indirect effect through the terms-of-trade outweighs the direct effect in a liberalizing country but the converse is true for a non-liberalizing country, and thus unilateral trade liberalization brings about intense competition in both countries, raising the cutoff at which least productive firms can survive. Unilateral expansions in market size, in contrast, do not directly affect expected profit due to the restrictive feature of monopolistic competition and CES preferences. Such expansions affect expected profit only through the terms-of-trade by raising wages as in Krugman (1980), which indirectly induce less (more) firms to enter in an expanding (non-expanding) country and hence unilateral market expansion entails anti-selection effects in a country of origin.

Our finding for the market size effect on selection contradicts that in Melitz and Ottaviano (2008). The reason stems from an outside good sector incorporated into their model in addition to a differentiated good sector, as shown by Demidova and Rodríguez-Clare (2013) for unilateral trade liberalization, and this claim applies to unilateral market expansion. With an outside good, the difference in country size allows for a home market effect on trade patterns by muting the factoral terms-of-trade so that a larger (smaller) country specializes in a differentiated (outside) good, stimulating firm entry in respective sectors. Without an outside good like ours, in contrast, country size has an endogenous effect on wages, changing firm entry as well as trade patterns. Thus it is not surprising that our model gives a different effect of market size on selection from Melitz and Ottaviano (2008), confirming that we have to be careful about under which conditions this popular assumption enables us to innocuously abstract from wage channels.[1] As for welfare, although a larger country exhibits lower productivity (associated with anti-selection effects), the country enjoys welfare gains in free trade because a negative impact on declined productivity is dominated by a positive impact on increased product variety.

Given the different effects of unilateral trade liberalization and unilateral market expansion on firm selection, what can we say about their policy implications? In the last part of the paper, we show that the effects are important for the characterization of optimal tariffs. In our model, the optimal tariff in a country is inversely related to its trading partner's export supply elasticity, which is composed of the domestic trade share and the trade elasticity, as in the existing models. See, for example, Gros (1987) for a homogeneous firm model and Felbermayr et al. (2013) for a heterogeneous firm model. In contrast to these papers, however, trade liberalization and country size do not always lead to higher optimal tariffs in this paper. From the policy point of view, the market size effect on optimal tariffs is of particular interest: a larger country does not necessarily benefit from setting higher tariffs. Our model predicts that a larger country accommodates more inefficient firms in the domestic market by anti-selection effects. While the larger country enjoys terms-of-trade gains by setting higher tariffs, this also accelerates welfare losses from protecting inefficient firms. We show that if the trade elasticity is constant, the optimal tariff increases with country size as in the previous literature, which means that the terms-of-trade effects dominate the anti-selection effects; however, if the trade elasticity is variable and differs across markets and levels of trade costs as found by empirical work (e.g., Helpman et al., 2008; Novy, 2013), the converse can be true identifying a potential importance to reconsider existing implications.

To appreciate the mechanism of our policy result, following Chaney (2008), let us decompose the trade elasticity into the intensive margin elasticity and the extensive margin elasticity where the former refers to the elasticity of each incumbent firm's shipment whereas the latter refers to the elasticity of new entrants' shipment. Since the intensive margin elasticity is constant under monopolistic competition and CES preferences, the variable nature of the trade elasticity comes from the extensive margin elasticity, which depends crucially on the micro structure of the model.

[1] Our setting also differs from Melitz and Ottaviano (2008) in preferences that generate constant/variable markups, but the absence of an outside good can reverse their result even with quadratic preferences; see Demidova (2017).

In a homogeneous firm model where all firms export, there is no adjustment margin from entry of new firms (i.e., the extensive margin elasticity is zero) and thus the trade elasticity is the same as the intensive margin elasticity. In a heterogeneous firm model with an untruncated Pareto distribution, the extensive margin elasticity is constant (Chaney, 2008) and the trade elasticity is constant as well. In these special cases, country size affects the optimal tariff only through the domestic trade share, so that the optimal tariff increases with country size (e.g., Gros, 1987; Felbermayr et al., 2013). The result however does not generally hold and, even with a slight generalization of this distribution to truncated Pareto with a finite upper bound, the extensive margin elasticity is variable and so is the trade elasticity. In this more general case, country size affects the optimal tariff not only through the domestic trade share but also through the trade elasticity. Due to this additional channel that previous work has not taken into account, we find that the optimal tariff does not necessarily increase with country size.

A number of papers have explored welfare and policy implications in the Melitz (2003) model. Regarding welfare implications, Arkorakis et al. (2012) derive a simple formula that can capture welfare gains by the two sufficient statistics, which applies to an important class of trade models, and followup papers have examined the extension/robustness of the welfare result. For example, Arkorakis et al. (2019) explore demand functions that yield variable markups, Felbermayr et al. (2015) introduce tariffs that raise government revenue, and Melitz and Redding (2015) employ a general productivity distribution that makes the trade elasticity variable. We demonstrate that the welfare formula by Arkolakis et al. (2012) can be used to reconsider conventional wisdom of optimal tariffs. In particular, conditional on the two sufficient statistics for welfare, the optimal level of import tariffs is the same across across different trade models with a constant trade elasticity, but more generally it depends on the micro structure that makes the trade elasticity variable. We also show that firm heterogeneity outside a Pareto distribution can impact welfare measurements as in Melitz and Redding (2015), but the scope of this paper differs from theirs since we consider different effects between iceberg trade costs and import tariffs, and provide an analytical solution of the optimal tariff without specifying a productivity distribution.

From policy perspectives, the literature has analyzed the optimal tariff in abridged versions of the Melitz (2003) model. Demidova and Rodríguez-Clare (2009) compute the optimal tariff for a small economy, Felbermayr et al. (2013) extend this result to a large country, and Demidova (2017) expands it by using quadratic preferences that generate variable markups. Although they find that the optimal level of import tariffs is strictly positive, the crucial assumption made by all of these papers is to constrain the distribution of firm productivity to be untruncated Pareto. As shown by Melitz and Redding (2015), welfare changes are highly sensitive to this assumption, and small deviations from this restriction lead to different welfare implications by making the trade elasticity variable. We highlight this caveat by characterizing analytically optimal tariffs with a general productivity distribution and an endogenous wage, adopting the technique known as the exact hat algebra in the literature.[2]

[2] See Ossa (2016) for a recent survey using this technique that applies for the analysis of optimal tariffs.

2 Model

2.1 Setup

Consider the Melitz (2003) model with two asymmetric countries i, j and one differentiated good sector. Country i is populated by a mass L_i of identical consumers whose preferences are

$$U_i = \left(\sum_{n=i,j} \int_{\omega \in \Omega_n} q_{ni}(\omega)^{\rho} d\omega \right)^{1/\rho}, \qquad 0 < \rho < 1,$$

where an elasticity of substitution between varieties is $\sigma = 1/(1-\rho) > 1$. Throughout this paper, we denote the exporting (importing) country by the first (second) subscript and hence $q_{ji}(\omega)$ is a quantity shipped from country j to country i. As is well-known, utility maximization subject to budget constraint yields the demand for variety ω:

$$q_{ji}(\omega) = R_i P_i^{\sigma-1} (p_{ji}(\omega))^{-\sigma},$$

where R_i is aggregate expenditure of consumers and P_i is an associated price index in country i. Defining an aggregate good $Q_i \equiv U_i$, these satisfy $P_i Q_i = R_i$.

To produce varieties, upon paying fixed entry costs f_i^e (measured in country i's labor units with wages w_i), a mass M_i^e of firms draw productivity φ from a distribution $G_i(\varphi)$ with support $(\varphi_{\min}, \varphi_{\max})$, where the upper bound is either finite ($\varphi_{\max} < \infty$) or infinite ($\varphi_{\max} = \infty$). If a firm from country j chooses to serve for country i, it pays variable trade costs $\theta_{ji} \geq 1$ (with $\theta_{jj} = 1$) and fixed trade costs f_{ji} (both measured in country j's labor units with wages w_j). A government in each country imposes import tariffs on foreign varieties and the above firm also pays ad valorem tariffs $\tau_{ji} = 1 + t_{ji}$, where $\tau_{ji} \geq 1$ (with $\tau_{jj} = 1$). Tariffs are assumed to be imposed before each firm sets markups, i.e., tariffs are modeled only as cost shifters thereby ignoring demand shifters (see Felbermayr et al. (2015) for these differences). Consequently country i's government collects tariff revenue $(\tau_{ji} - 1)p_{ji}(\omega)/\tau_{ji}$ per unit, so that the firm receives only $p_{ji}(\omega)/\tau_{ji}$ per unit.

Following Helpman et al. (2008) and Melitz and Redding (2015), it is useful to define

$$J_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] dG_i(\varphi),$$

$$V_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \varphi^{\sigma-1} dG_i(\varphi),$$

where $J_i(\varphi^*)$ and $V_i(\varphi^*)$ are strictly decreasing in φ^*.

2.2 Equilibrium Conditions

Under our preference assumption, a firm with productivity φ from country j to country i charges a constant markup $1/\rho$ over marginal cost $\theta_{ji} w_j/\varphi$ and tariffs τ_{ji}, and hence $p_{ji}(\varphi) = \tau_{ji}\theta_{ji} w_j/(\rho\varphi)$.

Combining the variety demand, firm revenue *net of tariffs* $r_{ji}(\varphi) = p_{ji}(\varphi)q_{ji}(\varphi)/\tau_{ji}$ is given by

$$r_{ji}(\varphi) = \sigma B_i \tau_{ji}^{-\sigma} (\theta_{ji} w_j)^{1-\sigma} \varphi^{\sigma-1},$$

where

$$B_i = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} R_i P_i^{\sigma-1}$$

is the index of market demand. Since firm variable profit is $r_{ji}(\varphi)/\sigma$, the productivity cutoff that satisfies zero profit ($\frac{r_{ji}(\varphi_{ji}^*)}{\sigma} = w_j f_{ji}$) is implicitly defined as

$$B_i \tau_{ji}^{-\sigma} (\theta_{ji} w_j)^{1-\sigma} (\varphi_{ji}^*)^{\sigma-1} = w_j f_{ji}, \tag{1}$$

which implies that

$$\left(\frac{\varphi_{ji}^*}{\varphi_{jj}^*}\right)^{\sigma-1} = \frac{\tau_{ji}^\sigma \theta_{ji}^{\sigma-1} f_{ji}}{f_{jj}} \frac{B_j}{B_i}.$$

To ensure export market selection ($\varphi_{ji}^* > \varphi_{jj}^*$), we assume that relative market demand B_i/B_j – proportional to relative country size L_i/L_j – is not too different by imposing the restriction such that trade costs are large enough to satisfy $\tau_{ji}^\sigma \theta_{ji}^{\sigma-1} f_{ji} > f_{jj}$ for i, j.

Free entry requires that the expected profits of entering the market in all operating countries equal the fixed entry costs ($\sum_n \int_{\varphi_{in}^*}^{\varphi_{\max}} (\frac{r_{in}(\varphi)}{\sigma} - w_i f_{in}) dG_i(\varphi) = w_i f_i^e$). Using the definition of $J_i(\varphi^*)$ in Section 2.1, the free entry condition in country i is

$$\sum_{n=i,j} f_{in} J_i(\varphi_{in}^*) = f_i^e. \tag{2}$$

Next, we look at the labor market clearing condition. Labor is used for entry and production ($L_i = M_i^e f_i^e + \sum_n L_{in}$). Using (1), (2) and the definition of $V_i(\varphi^*)$ in Section 2.1, the amount of labor used in country i is expressed as (see Appendix A.1)

$$L_i = \frac{R_i - T_i}{w_i},$$

where $R_i = \sum_n \tau_{ni} R_{ni}$ is aggregate expenditure and $T_i = (\tau_{ji} - 1) R_{ji}$ is aggregate tariff revenue (R_{ji} is aggregate expenditure of goods from country j to country i *net of tariffs*). Country i's wage is thus determined by equality between aggregate expenditure R_i and aggregate labor income $w_i L_i$ plus aggregate tariff revenue T_i as in usual general-equilibrium trade models. It is possible to show that the labor market clearing condition is equivalent with the trade balance condition ($R_{ij} = R_{ji}$) in that both conditions induce the same equality, $R_i = w_i L_i + T_i$.

Let $\lambda_{ji} \equiv \tau_{ji} R_{ji} / \sum_n \tau_{ni} R_{ni}$ denote the foreign trade share spent on goods from country j in country i. Using this share, we can define the corresponding foreign trade share *net of tariffs*:

$$\tilde{\lambda}_{ji} \equiv \frac{R_{ji}}{\sum_n R_{ni}} = \frac{\lambda_{ji}}{\tau_{ji}(1-\lambda_{ji}) + \lambda_{ji}},$$

Not surprisingly, we have $\tilde{\lambda}_{ji} = \lambda_{ji}$ if countries do not impose import tariffs ($\tau_{ji} = 1$). We also find it useful for our analysis to define a "tariff multiplier" (Felbermayr et al., 2015), i.e., the ratio of aggregate expenditure to aggregate labor income. Substituting λ_{ji} into $R_i = w_i L_i + (\tau_{ji} - 1) R_{ji}$,

$$\mu_i \equiv \frac{R_i}{w_i L_i} = \frac{\tau_{ji}}{\tau_{ji}(1 - \lambda_{ji}) + \lambda_{ji}},$$

where $\mu_i \geq 1$ as tariff revenue is redistributed to consumers and $\mu_i = 1$ in the absence of tariffs. Finally, using $w_i L_i = \sum_n R_{in}$ (labor income in country i consists of revenues earned by domestic firms and exporting firms from country i) and $R_{ij} = R_{ji}$ (trade is balanced between countries), the labor market clearing or trade balance condition is expressed as

$$w_i L_i = \sum_{n=i,j} \tilde{\lambda}_{in} w_n L_n. \tag{3}$$

Now, we are ready for the characterization of the important variables in general equilibrium. For given exogenous variables, an equilibrium in levels can be defined as a set of $\{\varphi_{ij}^*, B_i, w_i\}$ which are jointly characterized by (1), (2), and (3) for i, j, where wages in one of the countries are normalized to unity by setting labor there as a numeraire. Once these endogenous variables are determined, other endogenous variables are written as a function of the unknown variables. Using the definition of B_i in (1), welfare per worker is expressed as follows (see Appendix A.2):

$$W_i = \left(\frac{L_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma - 1}} (\mu_i)^{\frac{1}{\rho}} \rho \varphi_{ii}^*,$$

where μ_i enters the welfare expression because tariff revenue is rebated back to consumers.

3 Trade Liberalization

The previous section has defined the equilibrium conditions and equilibrium variables in *levels*. This section will define the equilibrium conditions and equilibrium variables in *changes*. We first examine the impact of changes in trade barriers, holding all other exogenous variables constant. Demidova and Rodríguez-Clare (2013) study a welfare effect of asymmetric trade liberalization in the Melitz (2003) model, dispensing with the assumption of an outside good. They show that unilateral reductions in trade barriers on either exports and imports always increase welfare in a liberalizing country, which stands in contrast to the presence of an outside good in the model with CES preferences (Demidova, 2008) and quadratic preferences (Melitz and Ottaviano, 2008). Here, with help of the exact hat algebra, we analytically show their result.[3] More importantly, we show in the next section that endogenous wages can reverse the impact of country size on productivity, just as in the impact of trade liberalization on welfare.

[3] While the result in this section is not entirely new, the optimal tariff cannot be characterized without analytical solutions using the exact hat algebra, which previous work has not computed in a general productivity distribution.

Suppose that country i unilaterally reduces trade costs of importing from country j. Below we mainly analyze the impact of variable trade costs θ_{ji} on the key equilibrium variables, but the impacts of fixed trade costs f_{ji} and ad valorem tariffs τ_{ji} are qualitatively similar. In contrast to variable and fixed trade costs, tariffs have a different effect on welfare through tariff revenue rebated back to consumers. Hence, the following analysis should be understood as the impact of exogenous changes in trade costs. We will characterize welfare-maximizing optimal tariffs after examining the impact of these exogenous changes.

Under the circumstance, denoting proportional changes of variables by a "hat" (i.e., $\hat{x} = dx/x$), and taking the log and differentiating the zero profit cutoff condition (1) with respect to θ_{ji},

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma \hat{w}_j + (\sigma - 1)\hat{\theta}_{ji}. \tag{4}$$

Similarly, differentiating the free entry condition (2) with respect to θ_{ji},

$$\sum_{n=i,j} f_{in} J'_i(\varphi^*_{in})\varphi^*_{in}\hat{\varphi}^*_{in} = 0. \tag{5}$$

Finally, taking the log and differentiating the trade balance condition (3) with respect to θ_{ji},

$$\hat{w}_i = \sum_{n=i,j} \delta_{in}(\hat{\tilde{\lambda}}_{in} + \hat{w}_n), \tag{6}$$

where

$$\delta_{ij} \equiv \frac{R_{ij}}{R_i} = \frac{\tilde{\lambda}_{ij} w_j L_j}{w_i L_i}.$$

Just like (1), (2) and (3) can be used to solve for the equilibrium in levels, (4), (5) and (6) can be used to solve for the equilibrium in changes. In the comparative statics considered here, for given changes in variable trade costs $\hat{\theta}_{ji}$, the equilibrium in changes is defined as a set of $\{\hat{\varphi}^*_{ij}, \hat{B}_i, \hat{w}_i\}$ which are jointly characterized by (4), (5), and (6) for i, j, where proportional changes of wages in one of countries are normalized to zero. As will be described shortly, changes in the foreign trade share net of tariffs $\hat{\tilde{\lambda}}_{ij}$ in (6) can be written as a function of changes in the domestic productivity cutoff $\hat{\varphi}^*_{ii}$.

In what follows, we show that the system of the equations in changes can be explicitly solved for the equilibrium variables in changes. First, rearranging (5) gives us the relationship between the domestic productivity cutoff and the export productivity cutoff in changes:

$$\hat{\varphi}^*_{ij} = -\alpha_i \hat{\varphi}^*_{ii}, \tag{7}$$

where

$$\alpha_i \equiv \frac{f_{ii} J'_i(\varphi^*_{ii})\varphi^*_{ii}}{f_{ij} J'_i(\varphi^*_{ij})\varphi^*_{ij}}.$$

The following lemma records some important properties of α_i (see Appendix A.3):

Lemma 1

(i) *From the definitions of $J_i(\varphi^*)$ and $V_i(\varphi^*)$ in Section 2.1,*

$$\alpha_i = \frac{f_{ii}(\varphi_{ii}^*)^{1-\sigma} V_i(\varphi_{ii}^*)}{f_{ij}(\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*)} = \frac{R_{ii}}{R_{ij}},$$

where $\alpha_i \alpha_j > 1$.

(ii) *From the definition of α_i and the trade balance condition,*

$$\lambda_{ji} = \frac{\tau_{ji}}{\alpha_i + \tau_{ji}}, \quad \tilde{\lambda}_{ji} = \frac{1}{\alpha_i + 1}, \quad \mu_i = \frac{\alpha_i + \tau_{ji}}{\alpha_i + 1}.$$

By definition, α_i is a function of φ_{ii}^* and φ_{ij}^*. Thus Lemma 1 means that once these cutoffs are endogenously determined by (1), (2) and (3), α_i in turn pins down λ_{ji}, $\tilde{\lambda}_{ji}$ and μ_i.

Next, applying (7) and $\tilde{\lambda}_{ji}$ in Lemma 1 to (6) gives us the relationship between the wages and the domestic productivity cutoffs in changes:

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}_{ii}^* + \beta_j \hat{\varphi}_{jj}^*, \tag{8}$$

where

$$\beta_i \equiv \frac{\alpha_i}{\alpha_i + 1}[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i],$$
$$\gamma_{ij} \equiv -\frac{d \ln V_i(\varphi_{ij}^*)}{d \ln \varphi_{ij}^*}.$$

Note that β_i is a function of φ_{ii}^* and φ_{ij}^* as in α_i, while γ_{ij} can be regarded as the extensive margin elasticity. The following lemma records some important properties of β_i (see Appendix A.4):

Lemma 2

(i) *From the definition of β_i,*

$$\frac{\beta_i}{\alpha_i} = \varepsilon_{ij} + \frac{\gamma_{ii} - \gamma_{ij}}{\alpha_i + 1},$$

where $\varepsilon_{ij} \equiv \sigma - 1 + \gamma_{ij}$ is the partial trade elasticity capturing only the direct effect of θ_{ij} on trade flows from country i to country j.[4]

(ii) *From the definitions of β_i and μ_i,*

$$\hat{\mu}_i = (\tau_{ji} - 1)\lambda_{ii} \frac{\beta_i}{\alpha_i} \hat{\varphi}_{ii}^*.$$

[4]Since we only model cost-shifting tariffs, ε_{ij} is the partial trade elasticity of τ_{ij} as well (Felbermayr et al., 2015).

The first part of Lemma 2 says that β_i/α_i can be greater or smaller than ε_{ij}, depending on the sign of $\gamma_{ii}-\gamma_{ij}$, i.e., the differences in the extensive margin elasticities between domestic and export markets. Further, β_i/α_i differs across markets and levels of trade costs, as the extensive margin elasticities γ_{ii}, γ_{ij} are variable in a general productivity distribution. These properties – absent in an untruncated Pareto distribution with a shape parameter k where $\beta_i/\alpha_i = \varepsilon_{ij} = k$ – plays an important role in characterizing the optimal tariff later.

The second part says that the domestic productivity cutoff φ_{ii}^* is a single sufficient statistic for welfare even with tariff revenue.[5] Any changes in variable trade costs always induce changes in the foreign trade share λ_{ji}, which affects redistribution of tariff revenue μ_i, but these changes are captured solely by changes in φ_{ii}^* from (7). As a result, changes in welfare are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji}-1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}_{ii}^*, \tag{9}$$

which shows that, to know what happens to welfare as a result of unilateral trade liberalization, we just need to see what happens to φ_{ii}^*. Importantly, the fact that β_i/α_i enters (9) implies that changes in welfare depend not only on ε_{ij} but also on $\gamma_{ii}-\gamma_{ij}$.

Now we can solve the system of seven equations ((4), (7), (8)) for seven unknowns ($\hat{\varphi}_{ij}^*, \hat{B}_i, \hat{w}_i$ for i,j) by setting $w_j = 1$ (hence $\hat{w}_j = 0$). Solving (4), (7) and (8) simultaneously yields

$$\begin{aligned} \hat{\varphi}_{ii}^* &= -\frac{\rho(\beta_j+\rho)}{\Xi}\hat{\theta}_{ji}, \\ \hat{\varphi}_{jj}^* &= -\frac{\rho(\beta_i-\rho\alpha_i)}{\Xi}\hat{\theta}_{ji}, \\ \hat{w}_i &= \frac{\rho^2(\beta_i+\alpha_i\beta_j)}{\Xi}\hat{\theta}_{ji}, \end{aligned} \tag{10}$$

where $\beta_i - \rho\alpha_i > 0$ (from the definitions of α_i and β_i) and

$$\Xi \equiv \sum_{n=i,j}(\beta_n+\rho) - \sum_{n=i,j}(\beta_n-\rho\alpha_n) > 0.$$

(10) shows that reductions in θ_{ji} increase $\varphi_{ii}^*, \varphi_{jj}^*$ and decrease w_i. From (9), these changes in turn mean that welfare rises not only in country j but also in country i because a decline in w_i is smaller than a decline in P_i (hence w_i/P_i rises) and tariff revenue rebated back to consumers μ_i rises (see Lemma 2(ii)).

The intuition behind the result is clearly seen by solving (4) and (7) first without (8):

$$\begin{aligned} \hat{\varphi}_{ii}^* &= \frac{1}{\alpha_i\alpha_j-1}\hat{\theta}_{ji} - \frac{\alpha_j+1}{\rho(\alpha_i\alpha_j-1)}\hat{w}_i, \\ \hat{\varphi}_{jj}^* &= -\frac{\alpha_j}{\alpha_i\alpha_j-1}\hat{\theta}_{ji} + \frac{\alpha_i+1}{\rho(\alpha_i\alpha_j-1)}\hat{w}_i, \end{aligned} \tag{11}$$

[5] This holds true for the case of variable trade costs that use real resources. In the case of tariffs that raise revenue, this revenue affects the welfare analysis (see Section 5).

where the first term captures the direct effect of reductions in θ_{ji}, while the second term captures the indirect effect of these reductions through changes in terms of trade. The direct effect lowers (raise) expected profit and induces less (more) firms to enter in a liberalizing (non-liberalizing) country with free entry. Thus, reductions in θ_{ji} decrease φ_{ii}^* but increase φ_{jj}^*. Note that the effect exists even when wages are exogenously fixed by a freely tradable outside good.[6] In such a case, (9) means that such reductions reduce (raise) welfare in country i (country j) due to a rise (a fall) in P_i (P_j). The welfare effect is in line with previous work where unilateral trade liberalization reduces welfare in a liberalizing country (Demidova, 2008; Melitz and Ottaviano, 2008).

If wages are endogenous, in contrast, the indirect effect also changes firms' expected profit. A decline in country i's relative wage improves (worsens) profitability in country i (country j), which leads more (less) firms to enter the domestic market in the respective country under free entry. Hence, if wages are endogenous, reductions in w_i (induced by reductions in θ_{ji}) increase φ_{ii}^* but decrease φ_{jj}^*, which works in the opposite direction to the direct effect. It follows from the equilibrium outcomes in (10) that the indirect effect outweighs the direct effect for φ_{ii}^* whereas the converse is true for φ_{jj}^*, and thus both cutoffs rise as a result of reductions in θ_{ji}.

The above finding implies that endogenous wages have a critical impact on the home market effect on trade patterns. Solving the price index P_i for the mass of entrants M_i^e yields

$$\frac{M_i^e}{M_j^e} = \left(\frac{w_i}{w_j}\right)^{\sigma-1} \frac{(P_i/P_j)^{1-\sigma} V_j(\varphi_{jj}^*) - \tau_{ji}^{-\sigma} \theta_{ji}^{1-\sigma} V_j(\varphi_{ji}^*)}{V_i(\varphi_{ii}^*) - \tau_{ij}^{-\sigma} \theta_{ij}^{1-\sigma} (P_i/P_j)^{1-\sigma} V_i(\varphi_{ij}^*)}.$$

If w_i is exogenous by an outside good, (7) and (11) reveal that M_i^e/M_j^e is decreasing in θ_{ji}, which means that trade liberalization in country i leads to redistribution of firms into the outside good (differentiated good) sector in country i (country j). Observing that φ_{ij}^* rises whereas φ_{ji}^* falls, the relative mass of exporting firms is decreasing in θ_{ji}. Further, firm export revenue satisfies

$$\frac{r_{ij}(\varphi)}{r_{ji}(\varphi)} = \frac{B_j}{B_i} \left(\frac{\tau_{ij}}{\tau_{ji}}\right)^{-\sigma} \left(\frac{\theta_{ij} w_i}{\theta_{ji} w_j}\right)^{1-\sigma}.$$

From (4) and (11), $r_{ij}(\varphi)/r_{ji}(\varphi)$ is also decreasing in θ_{ji}, which means that trade liberalization in country i changes the trade patterns in favor of country j, not only through firm entry (extensive margin) but also through firm revenue (intensive margin). As shown by Venables (1987), this can result in welfare losses (gains) in the liberalizing (non-liberalizing) country. If w_i is endogenous without an outside good, (10) shows that entry and revenue are not always decreasing in θ_{ji}, and thus the home market effect is not operative on the trade patterns. However, trade liberalization increases $\varphi_{ii}^*, \varphi_{jj}^*$ but decreases $\varphi_{ij}^*, \varphi_{ji}^*$. From Lemma 1, these changes increase the foreign trade share $\lambda_{ji}, \tilde{\lambda}_{ji}$, and hence reduce the domestic trade share $\lambda_{ii}, \tilde{\lambda}_{ii}$ for i, j, which ensures the welfare gains from trade liberalization in both countries (Arkolakis et al., 2012).

[6] To introduce an outside good, we require that L_i/L_j is not too different across countries to allow for complete specialization between countries (i.e., if L_i/L_j is too different, a small country can specialize in an outside good).

Though we have focused on the impact of variable trade costs on imports θ_{ji}, we can show that the impacts of *any* trade costs ($\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji}$) on the productivity cutoffs are qualitatively similar (see Appendix A.5). In the case of variable trade costs on exports θ_{ij}, for example,

$$\hat{\varphi}_{ii}^{*} = -\frac{\rho(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ij},$$
$$\hat{\varphi}_{jj}^{*} = -\frac{\rho(\beta_i + \rho)}{\Xi}\hat{\theta}_{ij},$$
$$\hat{w}_i = -\frac{\rho^2(\beta_j + \alpha_j\beta_i)}{\Xi}\hat{\theta}_{ij}.$$

Thus, reductions in export costs θ_{ij} increase the domestic productivity cutoffs in both countries as above. Only the difference is that reductions in *import* costs θ_{ji} reduce w_i, whereas reductions in *export* costs θ_{ij} raise w_i. This means that the direct effect outweighs the indirect effect for φ_{ii}^* whereas the converse is true for φ_{jj}^*, because the signs of (11) are opposite in this liberalization. The same claim applies not only to variable trade costs, but also to fixed trade costs and tariffs.

Finally, starting from a symmetric situation, the effect of trade liberalization is always greater in a liberalizing country than in a non-liberalizing country. In the case of variable trade costs on imports θ_{ji}, evaluating (10) at $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$ reveals that $\hat{\varphi}_{ii}^* > \hat{\varphi}_{jj}^*$. It follows immediately from (9) that welfare gains from trade liberalization are greater in country i than in country j. Clearly, a similar claim applies to the case of variable trade costs on exports θ_{ij} in that, starting from a symmetric situation, country j enjoys higher welfare gains from trade liberalization than country i.

Proposition 1 *Unilateral trade liberalization in variable and fixed trade costs on either exports or imports as well as tariffs has the following effects:*

(*i*) *The relative wage falls in a liberalizing country.*

(*ii*) *The domestic (export) productivity cutoff rises (falls), and the domestic (foreign) trade share falls (rises) in both countries.*

(*iii*) *Trade liberalization is unambiguously welfare-enhancing for both countries. Starting from a symmetric situation, the effect is always greater in a liberalizing country than in a non-liberalizing country.*

Proposition 1 is essentially the same as that in Demidova and Rodríguez-Clare (2013).[7] They find that endogenous wages can reverse the impact of asymmetric trade liberalization on welfare in a liberalizing country due to a failure of the home market effect on trade patterns without an outside good. While they graphically show the finding with a simple figure, we analytically show

[7] The result also relates to that in Felbermayr et al. (2013), though their analysis is less general than ours in that the model relies on an untruncated Pareto distribution, whereby variable and fixed trade costs are symmetric.

a similar result with the exact hat algebra. More important, however, is our tractability to study the impact of another competitive measure, i.e., country size, which can be examined in a parallel manner with trade liberalization without imposing a specific productivity distribution.

4 Country Size

Let us next consider changes in country size, holding all the other exogenous variables constant, which has been extensively examined in the literature. Melitz and Ottaviano (2008) are the first to show that a country with larger size entails higher productivity and welfare through tougher competition in the domestic market, reducing firms' average markups. Due to an outside good that gives rise to the home market effect on trade patterns, however, trade liberalization has an opposite impact from country size on welfare: a unilaterally liberalizing country can be worse off through the home market effect on trade patterns, which relocates production between countries (sometimes referred to as "firm delocation" in the literature).

We show that, in the absence of an outside good, endogenous wages can reverse the impact of country size, just as in the impact of trade liberalization: a country with larger size entails lower productivity (i.e., the domestic productivity cutoff decreases with country size), which stands in sharp contrast to Melitz and Ottaviano (2008) with an outside good. Although a larger country exhibits lower productivity (associated with anti-variety effects), the country nonetheless enjoys welfare gains in free trade because a negative impact on declined productivity is dominated by a positive impact on increased product variety.

Suppose that country i unilaterally expands market size L_i. Denoting proportional changes of variables by a "hat" once again, and taking the log and differentiating (1) with respect to L_i,

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{ij} = \sigma \hat{w}_i. \tag{12}$$

While (5) is the same as before, taking the log and differentiating (3) with respect to L_i,

$$\hat{w}_i + \hat{L}_i = \sum_{n=i,j} \delta_{in}(\hat{\hat{\lambda}}_{in} + \hat{w}_n) + \delta_{ii}\hat{L}_i. \tag{13}$$

The definition of the equilibrium in changes is similar to that in the previous section: for given changes in country size $\hat{L}_i$, the equilibrium in changes can be defined as a set of $\{\hat{\varphi}_{ij}, \hat{B}_i, \hat{w}_i\}$ which are jointly characterized by (5), (12), and (13) for i, j, where proportional changes in wages in one of the countries are zero. With help of Lemma 2(ii), changes in welfare are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}^*_{ii} + \frac{\hat{L}_i}{\sigma - 1}, \tag{14}$$

which shows that, to know what happens to welfare as a result of unilateral market expansion, we need to see what happens not only to φ^*_{ii} but also to L_i.

As in trade liberalization, we can explicitly solve the system of equations in changes below. While (7) remain the same, (13) is expressed as

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}^*_{ii} + \beta_j \hat{\varphi}^*_{jj} - \hat{L}_i, \tag{15}$$

where the definitions of α_i and β_i appearing in the equilibrium in changes are the same as those in the previous section. Noting that (7), (12), and (15) are seven equations with seven unknowns, and solving these equations with $w_j = 1$ simultaneously yields

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\rho(\alpha_j + 1)}{\Xi} \hat{L}_i, \\ \hat{\varphi}^*_{jj} &= \frac{\rho(\alpha_i + 1)}{\Xi} \hat{L}_i, \\ \hat{w}_i &= \frac{\rho^2(\alpha_i \alpha_j - 1)}{\Xi} \hat{L}_i. \end{aligned} \tag{16}$$

(16) shows that an increase in L_i decreases φ^*_{ii} but increases φ^*_{jj} as well as w_i. From (14), these changes mean that welfare rises in country j, whereas welfare can rise or fall in country i, depending on the magnitudes of a decline in φ^*_{ii} (declined productivity) and a rise in L_i (increased product variety).

The intuition is again clearly explained by solving (7) and (12) first without (15):

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\alpha_j + 1}{\rho(\alpha_i \alpha_j - 1)} \hat{w}_i, \\ \hat{\varphi}^*_{jj} &= \frac{\alpha_i + 1}{\rho(\alpha_i \alpha_j - 1)} \hat{w}_i. \end{aligned} \tag{17}$$

Simple comparison between (11) and (17) immediately reveals that the direct effect of increases in country size is absent in this case due to the peculiar and restrictive property of monopolistic competition and CES preferences, and there is only the indirect effect of these increases through changes in terms of trade. Therefore, if $\hat{w}_i = 0$ by a freely tradable outside good, (17) shows that country size has no impact on the domestic productivity cutoff ($\hat{\varphi}^*_{ii} = \hat{\varphi}^*_{jj} = 0$). From (14), this in turn means that country size raises welfare in country i due solely to increased product variety, as in a standard heterogeneous firm model with CES preferences (e.g., Melitz, 2003), let alone a homogeneous firm model (e.g., Krugman, 1980).

If wages are endogenous, in contrast, county size indirectly changes firms' expected profit. A rise in country i's relative wage worsens (improves) profitability in country i (country j), which leads less (more) firms to enter the domestic market in the respective country under free entry. Hence, if wages are endogenous, increases in w_i (induced by increases in L_i) decrease φ^*_{ii} but increase φ^*_{jj}. It is important to emphasize that the negative impact on φ^*_{ii} comes from the home market effect on w_i as in Krugman (1980). (The negative impact is absent in Krugman (1980) as productivity is exogenous.) This causes higher marginal cost and lower profitability, which leads to less competitive pressures on firms and makes it possible for less productive firms to survive there. Note also that, in contrast to Melitz and Ottaviano (2008) in which country size has no impact on the productivity cutoffs of a trading partner, country size does affect these cutoffs in the present paper through the relative wage that changes competitiveness across countries.

As with trade liberalization, the home market effect on trade patterns (induced by increases in country size) does not necessarily work in the presence of endogenous wages. From the labor market clearing condition, the mass of entrants is alternatively expressed as

$$\frac{M_i^e}{M_j^e} = \left(\frac{\sum_n f_{jn}(\varphi_{jn}^*)^{1-\sigma} V_j(\varphi_{jn}^*)}{\sum_n f_{in}(\varphi_{in}^*)^{1-\sigma} V_i(\varphi_{in}^*)} \right) \frac{L_i}{L_j}.$$

Furthermore, let $M_{ii} = [1 - G_i(\varphi_{ii}^*)]M_i^e$ and $M_{ij} = [1 - G_i(\varphi_{ij}^*)]M_i^e$ respectively denote the mass of domestic firms and that of exporting firms, which satisfy

$$\frac{M_{ii}}{M_{jj}} = \left(\frac{1 - G_i(\varphi_{ii}^*)}{1 - G_j(\varphi_{jj}^*)} \right) \frac{M_i^e}{M_j^e}, \quad \frac{M_{ij}}{M_{ji}} = \left(\frac{1 - G_i(\varphi_{ij}^*)}{1 - G_j(\varphi_{ji}^*)} \right) \frac{M_i^e}{M_j^e}.$$

If w_i is exogenous, country size has no impact on the values in the brackets above (see (17)). This means that the mass of entrants increases proportionately to country size in the current single differentiated good sector setting, and that both the mass of domestic firms and that of exporting firms increase proportionately to the mass of entrants. Therefore, market expansion in country i gives rise to the following pattern of firm entry:

$$\frac{M_i^e}{M_j^e} = \frac{M_{ii}}{M_{jj}} = \frac{M_{ij}}{M_{ji}}.$$[8]

From (12) and (17), we have that $r_{ij}(\varphi)/r_{ji}(\varphi)$ is not affected by country size. If w_i is endogenous, in contrast, country size has an impact on the values in the brackets above (see (16)). While the mass of entrants does not necessarily increase more than proportionately to country size and thereby the home market effect is not operative on trade patterns, the mass of domestic firms (exporting firms) increases more (less) than proportionately to the mass of entrants. Therefore, market expansion in country i gives rise to the following pattern of firm entry:

$$\frac{M_{ij}}{M_{ji}} < \frac{M_i^e}{M_j^e} < \frac{M_{ii}}{M_{jj}}.$$

Further, $r_{ij}(\varphi)/r_{ji}(\varphi)$ is also decreasing in L_i, which means that market expansion in country i changes the trade patterns in favor of country j through both extensive and intensive margins.

[8] In a multi-sector version of our model with an outside good, it can be easily shown that the mass of entrants rises more than proportionately to country size. This generates a home market effect on trade pattens in such a way that an increase in country size leads to disproportionately reallocations of labor to the differentiated good sectors, thereby allowing a larger country to enjoy higher welfare gains through increased product variety. Note, however, that the above entry pattern still holds even in this case.

Intuitively, country i with a relatively higher proportion of consumers has more incentive to save trade costs that are high enough to generate selection; thus firms find it less (more) profitable to export their products to a smaller (larger) country, allowing relatively less (more) exporting firms to exist in country i (country j). Just like unilateral trade liberalization has an impact on welfare by the shift in trade patterns, unilateral market expansion also has an impact on welfare by the same shift from an expanding country with higher wages to a non-expanding country with lower wages. In a model with additively separable indirect utilities, Bertoletti and Etro (2017) show that the shift can be thought of as business destruction (creation) where a richer (poorer) country with higher (lower) wages is characterized by concentration (expansion) of large exporting firms. In our model with CES preferences, the shift arises only when wages are endogenous.

The fact that country size affects selection also yields empirically consistent predictions that larger (smaller) countries tend to be less (more) open. At the aggregate level, the domestic trade share in total expenditure in country i can be expressed from Lemma 1 as

$$\lambda_{ii} = \frac{\alpha_i}{\alpha_i + \tau_{ji}}, \quad \tilde{\lambda}_{ii} = \frac{\alpha_i}{\alpha_i + 1}.$$

Since φ_{ii}^* is decreasing in L_i and α_i is decreasing in φ_{ii}^*, the share is increasing in country size: the domestic spending share is higher, the larger is country size, as documented by aggregate data from many countries (e.g., Eaton and Kortum, 2011). While the large domestic trade share would encourage firms to export those products for which they have the large domestic market (known as the Linder hypothesis), this is not the case in our model due to the feedback from country size to selection. At the firm level, on the other hand, the ratio of exporting firms to domestic firms in country i (which is less than unity with export market selection) can be expressed as

$$\frac{M_{ij}}{M_{ii}} = \frac{1 - G_i(\varphi_{ij}^*)}{1 - G_i(\varphi_{ii}^*)}.$$

It follows from (16) that the ratio is decreasing in country size, which implies that the share of exporting firms among operating firms is lower, the greater is country size. This is also in line with empirical evidence. Bernard et al. (2007) find that 18% of firms export in the United States, while Mayer and Ottaviano (2008) report that a much larger fraction of firms export in European countries.[9]

It remains to show the impact of country size on welfare in an expanding country. The impact depends on the magnitudes of a decline in φ_{ii}^* and a rise in L_i, where the former gives rise to a welfare loss by increasing the domestic trade share $\lambda_{ii}, \tilde{\lambda}_{ii}$ there. Applying (16) and rearranging, (14) can be expressed in terms of changes in φ_{ii}^* only (see Appendix A.6):

$$\hat{W}_i = \frac{1}{\sigma - 1}\left((\sigma - 1)(\beta_i + \rho) - \frac{\sigma\beta_i}{\mu_i} - (\beta_j - \rho\alpha_j)\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\right)\hat{\varphi}_{ii}^*.$$

[9] It should be noted that the analysis applies to trade among similar countries without technological differences.

As unilateral market expansion in country i decreases φ^*_{ii}, such expansion leads to welfare gains in that country if the value in the brackets is negative. Unfortunately this is not always the case, and we cannot say in general that the country gains from market expansion in the current model. It is possible to prove, however, that starting from a symmetric situation and free trade ($\mu_i = 1$), unilateral market expansion in country i unambiguously improves welfare in both countries.

Proposition 2 *Unilateral market expansion has the following effects:*

(i) *The relative wage rises in an expanding country.*

(ii) *The domestic (export) productivity cutoff falls (rises), and the domestic (foreign) trade share rises (falls) in an expanding country. The converse is true in a non-expanding country.*

(iii) *Starting from a symmetric situation and free trade, market expansion is unambiguously welfare-enhancing for both countries.*

The result in Proposition 2 has a noticeable difference from that in the existing literature.[10] In an influential study on allocation efficiency with VES preferences, Dhingra and Morrow (2019) find that market expansion provides welfare gains when preferences are "aligned," i.e., demand shifts alter private and social markups in the same directions. Their finding means that market expansion increases welfare in CES preferences, but this is not true in our model. The reason is that unilateral market expansion entails anti-selection effects that work to decline productivity in a country of origin. As shown by Dhingra and Morrow (2019), one of sufficient conditions for welfare gains is that productivity does not decline after market expansion, which is not satisfied here. Thus market expansion does not always lead to gains due to distortions from anti-selection effects in our setting, whereas distortions stem from variable markups in their setting.

5 Optimal Tariff

So far, we have examined the impact of exogenous changes in the two competitive measures on key endogenous variables without specifying a productivity distribution function. In this section, we show that the generality is important for the characterization of a country's optimal tariff.

Suppose that country i chooses a tariff rate on imports from country j to maximize welfare. For the moment, we focus on the effect of country i's tariffs τ_{ji} holding country j's tariffs τ_{ij} fixed. In country j that faces tariffs by country i, the effect of τ_{ji} is essentially the same as that of θ_{ji}, and changes in welfare per worker with respect to τ_{ji} are expressed as

$$\hat{W}_j = \left(\frac{(\tau_{ij} - 1)\lambda_{jj}}{\rho} \frac{\beta_j}{\alpha_j} + 1 \right) \hat{\varphi}^*_{jj}.$$

[10] In their appendix, Demidova and Rodríguez-Clare (2013) also show similar results like ours but the analysis of market size is confined to an untruncated Pareto distribution, whereby variable and fixed trade costs are symmetric.

From Proposition 1, an increase in τ_{ji} decreases the domestic productivity cutoff φ^*_{jj} which lowers welfare in country j. In country i that imposes tariffs on country j, there is an additional effect of τ_{ji} on welfare through changes in redistribution of tariff revenue. Using λ_{ji} in Lemma 1, changes in μ_i with respect to τ_{ji} in Lemma 2(ii) are given by

$$\hat{\mu}_i = (\tau_{ji} - 1)\lambda_{ii}\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii} + \lambda_{ji}\hat{\tau}_{ji}.$$

Changes in welfare per worker in country i corresponding to (9) and (14) are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i} + 1\right)\hat{\varphi}^*_{ii} + \frac{\lambda_{ji}}{\rho}\hat{\tau}_{ji},$$

where the first term is a welfare loss from tariffs (as inefficient firms are sheltered by tariffs), and the second term is a welfare gain from tariffs (as tariff revenue is rebated back to consumers). After rearranging, this can be written in terms of changes in φ^*_{ii} only (see Appendix A.7):

$$\hat{W}_i = \frac{\lambda_{ji}(\beta_i - \rho\alpha_i)}{\rho}\left(\frac{\beta_j - \rho\alpha_j}{\beta_j + \rho} - \frac{1}{\tau_{ji}}\right)\hat{\varphi}^*_{ii}. \tag{18}$$

Recall from Proposition 1 that an increase in τ_{ji} also decreases φ^*_{ii}. Setting $\tau_{ji} = 1$ in (18) then implies that a small import tariff τ_{ji} unambiguously improves welfare in country i (which comes at the expense of country j), and hence the optimal tariff is strictly positive for country i. Further, starting from a symmetric situation, country i's gains cannot compensate country j's losses and the effect of τ_{ji} on world welfare is always negative.

Before moving to characterizing the optimal tariff, it is useful to relate the expression in (18) to that in the existing literature. Using λ_{ii} and μ_i in Lemma 1, (18) is alternatively written as

$$\hat{W}_i = -\frac{\alpha_i}{\beta_i}\hat{\lambda}_{ii} + \left(\frac{\beta_i - \rho\alpha_i}{\rho\beta_i}\right)\hat{\mu}_i. \tag{19}$$

Welfare changes in (19) encompass the results in Arkolakis et al. (2012) without tariff revenue (i.e., $\mu_i = 1$ and hence $\hat{\mu}_i = 0$) and those in Felbermayr et al. (2015) with tariff revenue for the Melitz (2003) model with an untruncated Pareto distribution with a shape parameter k. In fact, noting that the extensive margin elasticity is constant at $k - (\sigma - 1)$ and β_i/α_i coincides with the partial trade elasticity $\varepsilon_{ij}(= k)$ under the specific distribution, (19) is expressed as

$$\hat{W}_i = -\frac{1}{\varepsilon_{ij}}\hat{\lambda}_{ii} + \left(1 + \frac{\eta}{\varepsilon_{ij}}\right)\hat{\mu}_i,$$

where $\eta \equiv \frac{k}{\sigma-1}(1 + \frac{1-\sigma}{k}) > 0$. The above expression shows that welfare changes can be captured solely by λ_{ii} and ε_{ij} without tariff revenue as indicated by the first term (Arkolakis et al., 2012), but their welfare formula requires qualification with tariff revenue if tariffs act as cost shifters as indicated by the second term (Felbermayr et al., 2015).

The results however depend critically on the assumption that the trade elasticity is constant, as stressed by Melitz and Redding (2015). To see this in our setting, using the general expression of β_i/α_i in Lemma 2(i) and rearranging, let us further express (19) as

$$\hat{W}_i = \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \left(\hat{M}_i^e - \hat{\lambda}_{ii} \right) + \left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \right) \hat{\mu}_i.$$

This is a counterpart to that in Melitz and Redding (2015, Eq. (33)), albeit the difference that we derive welfare changes by tariffs that raise government revenue. Note that, besides the domestic trade share λ_{ii} and the trade elasticity ε_{ij}, welfare changes also depend on the extensive margin elasticity differential between domestic and export markets $\gamma_{ii} - \gamma_{ij}$, which they refer to as the "hazard differential." It is clear that, if the extensive margin is more elastic for an export market than for a domestic market ($\gamma_{ii} - \gamma_{ij} < 0$), welfare changes by tariffs tend to be under-estimated relative to those without this differential. The converse is true for cases $\gamma_{ii} - \gamma_{ij} > 0$.[11]

Under which productivity distribution does the extensive margin elasticity differential exist? Obviously, $\gamma_{ii} = \gamma_{ij} (= k - (\sigma - 1))$ under an untruncated Pareto distribution. Consider a slight generalization from this distribution to truncated Pareto with a finite upper bound $\varphi_{\max}$ in $G_i(\varphi)$. In this more general case, the extensive margin elasticity from country i to country $n = i, j$ is expressed as follows (see Melitz and Redding (2015)):

$$\gamma_{in} = (k - (\sigma - 1)) \frac{\left(\frac{\varphi_{\min}}{\varphi_{in}^*} \right)^{k-(\sigma-1)}}{\left(\frac{\varphi_{\min}}{\varphi_{in}^*} \right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}} \right)^{k-(\sigma-1)}},$$

where γ_{in} is strictly increasing in the productivity cutoff φ_{in}^* (untruncated Pareto can be treated as a limit case in which $\lim_{\varphi_{\max} \to \infty} \gamma_{in} = k - (\sigma - 1)$). From this, it follows that (i) $\gamma_{ii} - \gamma_{ij} < 0$ under export market selection ($\varphi_{ii}^* < \varphi_{ij}^*$), and (ii) the trade elasticity $\varepsilon_{ij} = \sigma - 1 + \gamma_{ij}$ is variable. Using a truncated Pareto distribution, Melitz and Redding (2015) show that the micro structure matters for welfare beyond the domestic trade share and the trade elasticity in such a way that there are *larger* welfare gains from *reductions* in trade costs while *smaller* welfare losses from *increases* in trade costs for $\gamma_{ii} - \gamma_{ij} < 0$ than for $\gamma_{ii} - \gamma_{ij} = 0$. As the welfare-maximizing tariff is strictly positive in country i, this implies in our policy context that the government faces smaller welfare losses from increases in tariffs and therefore has more incentive to impose higher tariffs for $\gamma_{ii} - \gamma_{ij} < 0$ than for $\gamma_{ii} - \gamma_{ij} = 0$.

Although we will consider a truncated Pareto distribution to deliver the main point, the fact that the trade elasticity is variable is not specific to this distribution. For example, noting that a gravity equation with a constant trade elasticity is mis-specified under any distribution other than untruncated Pareto, Head et al. (2014) study welfare gains under a log-normal distribution, which also induces a variable trade elasticity as well as a negative differential.

[11] If $\gamma_{ii} - \gamma_{ij} \lesseqqgtr 0$, changes in the mass of entrants are given by $\hat{M}_i^e \gtreqqless 0$.

We now turn to characterizing the optimal tariff. Setting $\hat{W}_i = 0$ in (18) and solving for τ_{ji} yields the following expression for the optimal tariff for country i:[12]

$$\tau_{ji}^* = 1 + \underbrace{\frac{\rho}{\frac{\alpha_j}{\alpha_j+1}\left(\frac{\beta_j}{\alpha_j} - \rho\right)}}_{t_{ji}^*} = \frac{\beta_j + \rho}{\beta_j - \rho\alpha_j} > 1.$$

Further, using $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ from Lemma 1(ii) and substituting β_j/α_j from Lemma 2(i),

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}\left(\varepsilon_{ji} + \frac{\gamma_{jj}-\gamma_{ji}}{\alpha_j+1} - \rho\right)}. \tag{20}$$

Hence, the optimal tariff in country i is inversely related to country j's export supply elasticity, which is composed of the domestic trade share in country j ($\tilde{\lambda}_{jj}$) and the partial trade elasticity from country j to country i (ε_{ji}), as in the existing models. The crucial difference, however, is that the partial trade elasticity is not necessarily constant in this model.

It is worth stressing that the optimal tariff in (20) is a generalization of some of well-known results in the literature. If the underlying distribution is assumed to be untruncated Pareto with a shape parameter k, the extensive margin elasticity γ_{jj}, γ_{ji} is constant at $k - (\sigma - 1)$ and the partial trade elasticity ε_{ji} is constant at k. Thus, (20) reduces to

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}(k - \rho)}.$$

This expression is exactly the same as the optimal tariff derived by Felbermayr et al. (2013) in a heterogeneous firm model a la Melitz (2003) under an untruncated Pareto distribution. It is also possible to consider a homogeneous firm model as a special case with a degenerated productivity distribution (see Melitz and Redding (2015) for details). When all homogeneous firms can export, the extensive margin elasticity γ_{jj}, γ_{ji} is constant at zero and the partial trade elasticity ε_{ji} is constant at $\sigma - 1$. Thus, (20) reduces to

$$t_{ji}^* = \frac{1}{\tilde{\lambda}_{jj}(\sigma - 1)}.$$

This expression is exactly the same as the optimal tariff derived by Gros (1987) in a homogeneous firm model a la Krugman (1980).

At this standpoint, we need to mention two caveats for the optimal tariff. First, we cannot say that the optimal tariffs are smaller in a heterogeneous firm model than in a homogeneous firm model. Just like the two different models give us the different domestic trade shares $\tilde{\lambda}_{jj}$, these models also give us the different partial trade elasticities ε_{ji}. This means that the optimal tariffs

[12] Following Felbermayr et al. (2013), we use the F.O.C. of welfare maximization (18) assuming the sufficiency of it to be satisfied. Instead of using the F.O.C., Demidova (2017) looks at the direct impact of tariffs on aggregate quantity and finds the result that strongly resembles the one derived by Felbermayr et al. (2013).

in these two models are not directly comparable without taking account of the difference in the partial trade elasticity. Our general result in (20) is useful to shed light on this point. Plugging (20) in $\gamma_{jj} - \gamma_{ji} = 0$ that holds in the two models, we find that given the domestic trade share $\tilde{\lambda}_{jj}$ and the partial trade elasticity ε_{ji}, the optimal tariffs are the same between a homogeneous firm model and a heterogeneous model. Of course, the result is a direct implication of the insight by Arkolakis et al. (2012) for our optimal tariff setting: conditional on the two sufficient statistics for welfare $\tilde{\lambda}_{jj}, \varepsilon_{ji}$, welfare changes induced by tariffs are the same and, consequently, levels of the optimal tariffs are also the same.

Second, the equivalence of the optimal tariffs between the different trade models holds only if the extensive margin elasticity differential is zero, i.e., $\gamma_{jj} - \gamma_{ji} = 0$. If this does not hold, the optimal tariffs are different even after controlling for the two sufficient statistics for welfare. As seen above, welfare losses from tariffs under a truncated Pareto distribution with $\gamma_{jj} - \gamma_{ji} < 0$ are smaller than those under distributions with $\gamma_{jj} - \gamma_{ji} = 0$, which prompts country i's government to set higher tariffs. As a result, even conditional on the sufficient statistics, the optimal tariff is higher than in the above special cases. This means that the optimal tariff that does not control for the differential $\gamma_{jj} - \gamma_{ji}$ tends to be under-estimated since $\varepsilon_{ji} > \varepsilon_{ji} + (\gamma_{jj} - \gamma_{ji})/(\alpha_j + 1)$ in (20). The converse is true for cases $\gamma_{jj} - \gamma_{ji} > 0$ in that the optimal tariff tends to be over-estimated.

Proposition 3 *Conditional on the domestic trade share and the partial trade elasticity, the optimal tariff has the following properties:*

(*i*) *If the extensive margin elasticity is the same between domestic and export markets, levels of the optimal tariffs are the same across different trade models.*

(*ii*) *If the extensive margin is more (less) elastic for an export market than for a domestic market, levels of the optimal tariffs are greater (smaller) than those in the absence of this differential.*

Next, we examine the impacts of trade costs and country size on the optimal tariff in (20). Let us consider first a constant trade elasticity case (such as untruncated Pareto), so that exogenous changes affect the optimal tariff only through the domestic trade share. Proposition 1 says that reductions in *any* trade costs increase the domestic productivity cutoff φ^*_{jj}, which decrease $\tilde{\lambda}_{jj}$. Proposition 2 says that market expansions in country i (country j) increase (decrease) φ^*_{jj}, which decrease $\tilde{\lambda}_{jj}$ if market size in country i is relatively larger than that in country j. From these comparative statics, it follows that the optimal tariff in country i is higher, the lower are trade costs between countries or the larger is country i's relative size. In addition to this, the fact that the optimal tariff in country i decreases with country j's tariffs means that the best response functions are downward-sloping (i.e., tariffs are strategic substitutes). The result corresponds to the optimal tariff properties by Felbermayr et al. (2013) in an untruncated Pareto distribution. In our setting with a general productivity distribution, however, this is not necessarily the case since the exogenous variables affect the optimal tariff not only through the domestic trade share but also through the partial trade elasticity.

This additional channel for the optimal tariff can be shown more formally by making clear the relationship between the extensive margin elasticity differential and the partial trade elasticity in a general productivity distribution. Applying the comparative statics in Propositions 1 and 2 to Lemma 2(i), the following lemma is immediately obtained (see Appendix A.8).[13]

Lemma 3

(i) *If the extensive margin is more (less) elastic for an export market than for a domestic market, reductions in trade costs between countries decrease (increase) the partial trade elasticity.*

(ii) *If the extensive margin is more (less) elastic for an export market than for a domestic market, relative market expansions in country i decrease (increase) the partial trade elasticity.*

Lemma 3 implies that, if $\gamma_{jj} - \gamma_{ji} \neq 0$, the partial trade elasticity is not constant and therefore differs across markets and levels of trade costs. In the case of trade costs, for example,

$$\gamma_{jj} - \gamma_{ji} \lesseqqgtr 0 \implies \frac{d\varepsilon_{ji}}{d\theta_{ji}} \gtreqqless 0, \ \frac{d\varepsilon_{ji}}{d\theta_{ij}} \gtreqqless 0, \ \frac{d\varepsilon_{ji}}{df_{ji}} \gtreqqless 0, \ \frac{d\varepsilon_{ji}}{df_{ij}} \gtreqqless 0, \ \frac{d\varepsilon_{ji}}{d\tau_{ji}} \gtreqqless 0, \ \frac{d\varepsilon_{ji}}{d\tau_{ij}} \gtreqqless 0.$$

If the extensive margin elasticity differential is negative ($\gamma_{jj} - \gamma_{ji} < 0$), reductions in trade costs between countries decrease the partial trade elasticity. If the differential is positive ($\gamma_{jj} - \gamma_{ji} > 0$), such reductions shift the partial trade elasticity in the opposite direction. It is clear that, only in the case of no differential ($\gamma_{jj} - \gamma_{jj} = 0$), is the partial trade elasticity invariant to trade costs. The same claim also applies to increases in country i's relative country size.

It is then easily shown that exogenous changes have an additional effect on the optimal tariff. Consider reductions in variable trade costs θ_{ji}. If the differential is negative ($\gamma_{jj} - \gamma_{ji} < 0$), such reductions decrease the partial trade elasticity ($\frac{d\varepsilon_{ji}}{d\theta_{ji}} > 0$) as well as the domestic trade share in country j ($\frac{d\tilde{\lambda}_{jj}}{d\theta_{ji}} > 0$). It follows from (20) that, due to an extra adjustment margin through ε_{ji} that is absent in a constant trade elasticity case, the impact on the optimal tariff is reinforced. If the differential is positive, the converse is true in that the impact on the optimal tariff is attenuated. Only when there is no differential, is the partial trade elasticity constant and reductions in θ_{ji} affect the optimal tariff only through decreases in the domestic trade share. These highlight a potential bias in evaluating the optimal tariff without allowing for a variable trade elasticity that differs across markets and levels of trade costs. In other words, the optimal tariff that does not control for the differential $\gamma_{jj} - \gamma_{ji}$ tends to be under/over-estimated not only in terms of levels but also in terms of changes induced by exogenous shocks (see Appendix A.9).

[13] Strictly speaking, we require that the extensive margin elasticity γ_{ji} is a monotonic function in the productivity cutoff φ_{ji}^* for this lemma. With this restriction, the sign of $\gamma_{jj} - \gamma_{ji}$ is the same for a given productivity distribution and does not change with key parameters of the model so long as export market selection is ensured. As seen above, the property holds for a truncated Pareto distribution (Head et al. (2014) show this for a log-normal distribution), in which case the sign of the differential does not switch under the comparative static exercises here. We are not sure if this always holds for other popular productivity distributions and the discussion below simply bypasses the question.

Proposition 4 *Reductions in trade costs between countries or relative market expansions in country i have the following effects on the optimal tariff:*

(i) *If the extensive margin elasticity is the same between domestic and export markets, they increase the optimal tariff only through decreases in the domestic trade share.*

(ii) *If the extensive margin is more (less) elastic for an export market than for a domestic market, they reinforce (attenuate) the impact on the optimal tariff through decreases (increases) in the partial trade elasticity.*

One of interesting results in this proposition arises when $\gamma_{jj} - \gamma_{ji} > 0$ and increases in the trade elasticity (induced by exogenous shocks) are greater than decreases in the domestic trade share. In such a case, our model predicts that the optimal tariff in country i is lower, the lower are trade costs between countries and the larger is the relative size in country i. The impact of country size on the optimal tariff accords with recent research. For example, Naito (2019) finds a significant negative relationship between GDP and tariffs across countries, meaning that larger countries tend to set lower tariffs.[14] To account for this fact that is inconsistent with the existing optimal tariff theory, Naito (2019) develops a dynamic Ricardian model in which the long-run welfare effects of tariffs on revenue and economic growth jointly characterize the optimal tariff, which is shown to be decreasing in a country's absolute advantage parameter.

While our model also yields a similar prediction, the mechanism behind the result is different. Our model predicts that a large country accommodates relatively inefficient firms in the domestic market by lowering the domestic productivity cutoff, which has a negative impact on its welfare. If this large country is allowed to choose tariffs to maximize welfare, it can enjoy terms-of-trade gains by setting higher tariffs in our model as in the conventional optimal tariff theory. However this simultaneously accelerates welfare losses from protecting inefficient firms because a larger country faces an anti-selection effect on domestic firms. Taking these effects on welfare together, the optimal tariff is decreasing in country size if welfare losses from protecting inefficient firms are stronger than welfare gains from improving the terms-of-trade in which case a larger country does not always benefit from higher tariffs due to the feedback from country size to selection.

We have characterized the optimal tariff in country i, taking tariffs in country j as given. Now consider the situation in which both countries set tariffs so as to maximize respective welfare. From Lemma 3, if $\gamma_{jj} - \gamma_{ji} \leq 0$, country i's optimal tariff τ^*_{ji} is decreasing in country j's tariff τ_{ij}, since an increase in τ_{ij} decreases $\tilde{\lambda}_{jj}$ and ε_{ji}. As indicated earlier, the best response functions are downward-sloping and the optimal tariffs are strategic substitutes for one another. In contrast, if $\gamma_{jj} - \gamma_{ji} > 0$ and increases in ε_{ji} are greater than decreases in $\tilde{\lambda}_{jj}$, country i's optimal tariff τ^*_{ji} is increasing in country j's tariff τ_{ij}. In this case, the best response functions are upward-sloping and the optimal tariffs are strategic complements for one another. As usual, the optimal tariffs in Nash equilibrium $(\tau^*_{ji}, \tau^*_{ij})$ are determined at which the best response functions intersect in the

[14] From Section 4, the larger country i's size L_i, the larger its national income $w_i L_i + (\tau_{ji} - 1)R_{ji}$ (for a given τ_{ji}).

(τ_{ji}, τ_{ij}) space, but the variable trade elasticity alters the equilibrium properties of Nash tariffs. Further, the optimal tariff is bounded from above and below in Nash equilibrium. On the one hand, if tariffs are sufficiently high ($\tau_{ij} \to \infty$) that no firm exports from country i, the domestic trade share in country j approaches to unity ($\tilde{\lambda}_{jj} \to 1$) in (20). Thus

$$\underline{\tau}^*_{ji} = 1 + \frac{\rho\alpha_j}{\beta_j - \rho\alpha_j} = \frac{\beta_j}{\beta_j - \rho\alpha_j}.$$ [15]

On the other hand, if trade costs are sufficiently low that all surviving firms export ($\varphi^*_{jj} = \varphi^*_{ji}$), we have $\alpha_j = f_{jj}/f_{ji}$ (from the definition of α_j) and $\gamma_{jj} = \gamma_{ji}$ (from the definition of γ_{ji}) yielding $\beta_j/\alpha_j = \varepsilon_{ji}$. Using these and $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ in (20),

$$\overline{\tau}^*_{ji} = 1 + \frac{\rho\left(1 + \frac{f_{jj}}{f_{ji}}\right)}{\frac{f_{jj}}{f_{ji}}\left(\varepsilon_{ji} - \rho\right)} = \frac{\varepsilon_{ji} + \rho\frac{f_{ji}}{f_{jj}}}{\varepsilon_{ji} - \rho}.$$

Note that both of the bounds are variable in our setting with a general productivity distribution and vary endogenously with exogenous shocks.

To better appreciate the equilibrium properties of Nash tariffs, we follow Felbermayr et al. (2013) in assuming that two countries are symmetric and choose their tariff non-cooperatively. In Nash equilibrium, they impose the same optimal tariff $\tau^*_{ij} = \tau^*_{ji} \equiv \tau^*$ and wages are equalized across countries $w_i = w_j \equiv w = 1$. Exploiting the symmetry, let us also define

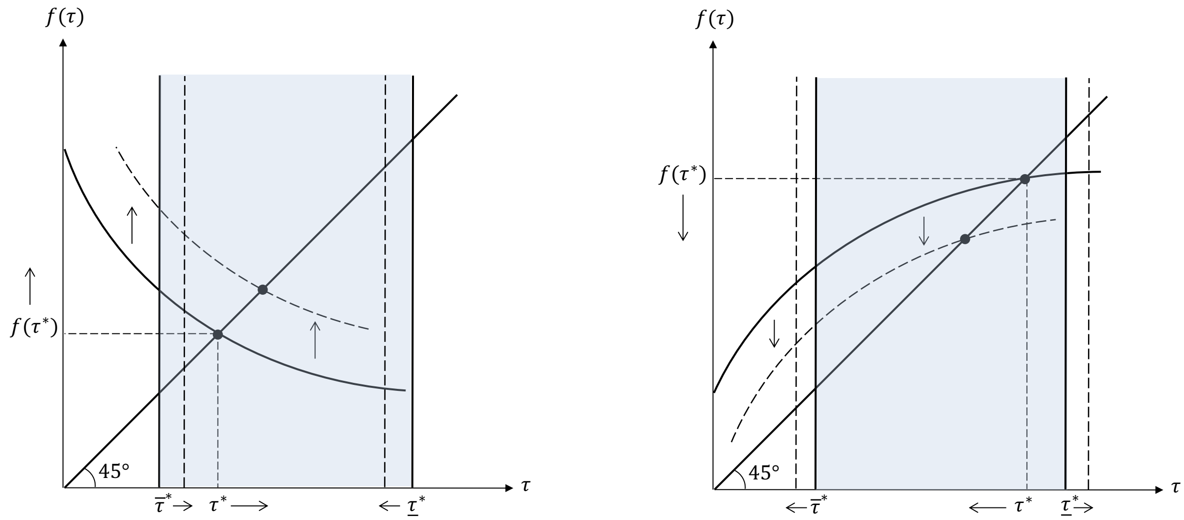
$$\begin{aligned}
&\theta_{ij} = \theta_{ji} \equiv \theta, \quad f_{ii} = f_{jj} \equiv f_d, \quad f_{ij} = f_{ji} \equiv f_x, \quad L_i = L_j \equiv L, \quad \tilde{\lambda}_{ii} = \tilde{\lambda}_{jj} \equiv \tilde{\lambda}, \\
&\varepsilon_{ij} = \varepsilon_{ji} \equiv \varepsilon, \quad \gamma_{ii} = \gamma_{jj} \equiv \gamma_d, \quad \gamma_{ij} = \gamma_{ji} \equiv \gamma_x, \quad \alpha_i = \alpha_j \equiv \alpha, \quad \beta_i = \beta_j \equiv \beta.
\end{aligned}$$

Then, finding Nash tariffs is equivalent to finding a solution to the fixed point problem $\tau = f(\tau)$ in (20) where the dependence of $f(\tau)$ on θ, f_x and L is understood:

$$f(\tau) = 1 + \frac{\rho}{\tilde{\lambda}\left(\varepsilon - \frac{\gamma_d - \gamma_x}{\alpha+1} - \rho\right)}.$$

Applying Proposition 4, $f(\tau)$ is decreasing in τ if $\gamma_d - \gamma_x \leq 0$ in which case tariffs are strategic substitutes; however, $f(\tau)$ is increasing in τ if $\gamma_d - \gamma_x > 0$ and increases in ε are greater than decreases in $\tilde{\lambda}$ in which case tariffs are strategic complements. Figure 1 depicts a 45-degree line plus a $f(\tau)$ curve for two possible cases – tariffs are strategic substitutes in Panel (a) while tariffs are strategic complements in Panel (b). In either panel, the optimal tariffs in Nash equilibrium τ^* is found at which a 45-degree line and a $f(\tau)$ curve intersect. Nash tariffs lie within the shaded area in the figure where the lower and upper bounds are respectively denoted by $\underline{\tau}^*_{ij} = \underline{\tau}^*_{ji} \equiv \underline{\tau}^*$ and $\overline{\tau}^*_{ij} = \overline{\tau}^*_{ji} \equiv \overline{\tau}^*$.

[15]This lower bound also represents the optimal tariff when country i is treated as a limit case of a small economy where country j has a sufficiently large domestic trade share (i.e., $\tilde{\lambda}_{jj} \to 1$).



(a) Negative differential

(b) Positive differential

Figure 1 – Effect of trade liberalization on Nash tariffs

Let us examine the impact of exogenous changes on the optimal tariffs in Nash equilibrium. Consider the impact of trade liberalization first. Reductions in trade costs (both variable θ and fixed f_x) always decrease the domestic trade share $\tilde{\lambda}$. On top of that, these reductions decrease the partial trade elasticity ε if the differential is negative ($\gamma_d - \gamma_x < 0$). In this case, $f(\tau)$ shifts up and Nash tariffs τ^* become higher, thereby narrowing the gap between the upper and lower bounds (i.e., tariffs tend to converge) as a result of trade liberalization, as depicted in Panel (a). The converse is true if the differential is positive ($\gamma_d - \gamma_x > 0$) and increases in ε are greater than decreases in $\tilde{\lambda}$, as depicted in Panel (b). Finally, if the differential is zero ($\gamma_d - \gamma_x = 0$), these reductions have no impact on ε and Nash tariffs τ^* become higher only through a decline in $\tilde{\lambda}$, thereby leaving the two bounds unaffected (see Appendix A.10).

Regarding the impact of country size, expansions in market size (i.e., increases in L) have no impact on the domestic trade share $\tilde{\lambda}$ as well as the partial trade elasticity ε, and therefore Nash tariffs τ^*. The gap between the two bounds $\overline{\tau}^*, \underline{\tau}^*$ also remain unchanged. The reason is that the indirect effect through the terms-of-trade is not operative in a symmetric situation that equalizes wages across countries $\hat{w}_i = \hat{w}_j \equiv \hat{w} = 0$ (see (17)). As a result, market expansions have no selection effects, leaving a $f(\tau)$ curve unaffected through either $\tilde{\lambda}$ or ε. In contrast to trade liberalization, this impact of country size on Nash tariffs necessarily holds irrespective of the sign of the differential $\gamma_d - \gamma_x$.

The key upshot of our argument above is that policy evaluations of Nash tariffs that do not control for the differential $\gamma_d - \gamma_x$ can lead to a bias even in an environment in which countries choose tariffs non-cooperatively. This is of particular importance for assessment of trade policy in globalization where reductions in transportation or communication costs are significant across countries. Our model reveals that, whenever $\gamma_d - \gamma_x \neq 0$, there is an additional channel through which trade costs endogenously affect Nash tariffs, i.e., a variable trade elasticity. In fact, recent work using firm-level data has identified empirical relevance of this aspect, paying attention to the extensive margin. For example, estimating trade flows in their gravity equation, Helpman et al. (2008) find substantial variations in the trade elasticities with respect to trade costs between country pairs. Calibrating their model into US firm-level data, Melitz and Redding (2015) also show that missing the variable nature of the trade elasticities can lead to a quantitatively large discrepancy from the true welfare gains from trade liberalization. In our optimal tariff setting, these insights suggest that the micro structure that makes the trade elasticity variable matters for evaluating Nash tariffs, since globalization always gives rise to higher (lower) Nash tariffs for $\gamma_d - \gamma_x < (>)0$ than for $\gamma_d - \gamma_x = 0$. As stressed by Melitz and Redding (2015), not only are the domestic trade share and the partial trade elasticity, but the differential can be also empirically examined if firm-level data are available, and we need to take fully into account these observable moments for trade policy evaluations.

Proposition 5 *Evaluating at a symmetric situation, Nash tariffs have the following equilibrium properties:*

(i) *If the extensive margin elasticity is the same between domestic and export markets, Nash tariffs rise by reductions in trade costs between countries only through decreases in the domestic trade share.*

(ii) *If the extensive margin is more (less) elastic for an export market than for a domestic market, such reductions have stronger (weaker) effects on Nash tariffs through decreases (increases) in the partial trade elasticity.*

(iii) *Regardless of the sign of the extensive margin elasticity differential, market expansions have no impact on Nash tariffs.*

We conclude this section by briefly mentioning the welfare effect in Nash equilibrium. It can be easily confirmed that welfare changes with respect to variable trade costs in (9) similarly hold in Nash equilibrium, and symmetrically scaling down variable trade costs improves welfare due to a rise in $\varphi_{ij}^* = \varphi_{ji}^* \equiv \varphi_d^*$. (The same also holds for fixed trade costs.) Next, welfare changes with respect to country size in (14) are given by $\hat{W} = \hat{L}/(\sigma - 1)$ where a fall in φ_d^* does not enter, since the cutoff is invariant to market size with equalized wages and, even with tariff revenue, symmetrically scaling up market size improves welfare due solely to increased product variety. Finally, welfare changes with respect to tariffs in (18) are expressed as

$$\hat{W} = \left(\frac{(\tau - 1)(\beta - \rho\alpha)}{\rho(\alpha + \tau)} \right) \hat{\varphi}_d^*,$$

and thus symmetrically scaling down tariffs (from $\tau \geq 1$) improves welfare by increasing in φ_d^*.

6 Conclusion

This paper presents a heterogeneous firm model of trade to study unilateral trade liberalization and unilateral market expansion and their impact on optimal tariffs. In order to circumvent the drawback in monopolistic competition and CES preferences and to provide better understanding of the role of country size in policy evaluations, we drop the assumption of an outside good sector in a general productivity distribution. Our key contributions are broadly summarized as follows. First, unilateral trade liberalization entails selection effects while unilateral market expansion entails anti-selection effects in a country of origin; however, more competitive pressures by these two measures are welfare-enhancing in free trade. Second, the optimal level of import tariffs is inversely related to the two empirically observable moments – domestic trade share and partial trade elasticity – where the second integrant can be variable depending on the micro structure of the model. In particular, if the trade elasticity is constant as in most previous work, the optimal level of import tariffs is the same between different trade models (conditional on the domestic trade share and a constant trade elasticity); however, if the trade elasticity is variable as found by empirical work, the optimal level of import tariffs tends to be under/over-estimated relative to a constant trade elasticity case. Similarly, the impact of trade liberalization and country size on optimal tariffs depends critically on the micro structure that makes the trade elasticity variable. These results go through for Nash trade policies in which welfare-maximizing governments can choose their tariffs non-cooperatively.

We have focused mainly on the qualitative aspect for policy implications to highlight a new role played by the micro structure in characterizing optimal tariffs throughout the paper, but it would be interesting to investigate the quantitative relevance of our results for optimal tariffs. Melitz and Redding (2015) quantitatively measure the discrepancies in welfare gains between a constant trade elasticity under an untruncated Pareto distribution (as in Arkolakis et al. (2012)) and a variable trade elasticity under a truncated Pareto distribution, and find big discrepancies ranging up to a factor of four. Given such substantial differences in welfare gains induced by the variable nature of the trade elasticity, we expect that evaluating optimal tariffs under a constant trade elasticity would result in a serious bias relative to those under a variable trade elasticity. Using a standard parameterization of heterogenous firm models employed by Felbermayr et al. (2013) and Melitz and Redding (2015), we conjecture that it is possible in principle to implement this kind of quantitative exercises in our setting. This would allow us not only to gain a sense of the magnitudes, but also to quantify a sensitivity of the strategic relationships across countries' optimal tariffs to exogenous changes. To examine this sensitivity, we need to replace a (truncated or untruncated) Pareto distribution by another one, since this distribution induces a non-positive differential and hence tariffs are always strategic substitutes. Unfortunately, we are not certain about which firm productivity distributions give a positive differential and whether the resulting quantitation can provide a good fit for aggregate and firm-level data. We leave this question and the relevant quantitative exercise to future work.

A Appendix

A.1 Labor Market Clearing Condition

We first show that the labor market clearing condition is given by

$$L_i = \frac{R_i - T_i}{w_i}.$$

With a linear cost function and cost-shifting tariffs that do not use real resources, we have

$$L_i = M_i^e f_i^e + M_i^e \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} \left(f_{in} + \frac{\theta_{in} q_{in}(\varphi)}{\varphi} \right) dG_i(\varphi).$$

From the free entry condition in (2), labor used for entry is expressed as

$$M_i^e f_i^e = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \frac{1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) - \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} \right\}.$$

On the other hand, noting from the pricing rule that $q_{ij}(\varphi) = \tau_{ij} r_{ij}(\varphi)/p_{ij}(\varphi) = \rho\varphi r_{ij}(\varphi)/\theta_{ij} w_i$, aggregate labor used for production is expressed as

$$M_i^e \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} \left(f_{in} + \frac{\theta_{in} q_{in}(\varphi)}{\varphi} \right) dG_i(\varphi) = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} + \frac{\sigma - 1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) \right\}.$$

Summing up these terms,

$$\begin{aligned} L_i &= \frac{M_i^e}{w_i} \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) \\ &= \frac{\sum_n R_{in}}{w_i}, \end{aligned}$$

where $R_{ij} = M_i^e \int_{\varphi_{ij}^*}^{\varphi_{\max}} r_{ij}(\varphi) dG_i(\varphi)$ is aggregate firm revenue (or consumer expenditure) of goods from country i to country j net of tariffs. The result follows from $R_i = \sum_n \tau_{ni} R_{ni}$ and $R_{ij} = R_{ji}$.

Next, we show that the labor market clearing condition is equivalent with the trade balance condition. On the one hand, aggregate labor income in country i consists of revenues by domestic firms and exporting firms of country i net of tariffs, $w_i L_i = \sum_n R_{in}$. On the other hand, aggregate expenditure in country i consists of expenditure on domestic goods in country i and imported goods from country j, $R_i = \sum_n \tau_{ni} R_{ni}$. From these, the trade balance condition is rearranged as

$$\underbrace{R_{ii} + R_{ij}}_{w_i L_i} = \underbrace{R_{ii} + \tau_{ji} R_{ji}}_{R_i} - \underbrace{(\tau_{ji} - 1) R_{ji}}_{T_i},$$

and hence both conditions are equivalent in that they induce the same equality, $R_i = w_i L_i + T_i$.

A.2 Welfare Expression

Welfare per worker is given by

$$\begin{aligned} W_i &\equiv \frac{U_i}{L_i} \\ &= \frac{R_i}{L_i P_i} \\ &= \frac{\mu_i w_i}{P_i} \end{aligned}$$

where the second equality follows from noting that $U_i \equiv Q_i$ and $P_i Q_i = R_i$, and the third equality follows from noting that $R_i = \mu_i w_i L_i$. Further, aggregate market demand is expressed as

$$B_i = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \mu_i w_i L_i P_i^{\sigma-1}.$$

Substituting this into (1) and rearranging, the real wage is given by

$$\frac{w_i}{P_i} = \left(\frac{\mu_i L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} \rho \varphi_{ii}^*,$$

which becomes the same as that in Demidova and Rodríguez-Clare (2013) without tariff revenue ($\mu_i = 1$). Finally, substituting w_i/P_i into above W_i establishes the result.

A.3 Proof of Lemma 1

We first show that

$$\begin{aligned} \alpha_i &\equiv \frac{f_{ii} J_i'(\varphi_{ii}^*)\varphi_{ii}^*}{f_{ij} J_i'(\varphi_{ij}^*)\varphi_{ij}^*} \\ &= \frac{f_{ii}(\varphi_{ii}^*)^{1-\sigma} V_i(\varphi_{ii}^*)}{f_{ij}(\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*)}. \end{aligned} \tag{A.1}$$

The definition of α_i follows immediately from solving (5) for $\hat{\varphi}_{ij}^*$ as in (7). On the other hand, the equality in (A.1) follows from differentiating $J_i(\varphi^*)$ with respect to φ^*:

$$\begin{aligned} J_i'(\varphi^*) &= -\left(\frac{\sigma-1}{\varphi^*}\right)[J_i(\varphi^*) + 1 - G_i(\varphi^*)] \\ &= -(\sigma-1)(\varphi^*)^{-\sigma} V_i(\varphi^*), \end{aligned}$$

where the second equality comes from the definitions of $J_i(\varphi^*)$ and $V_i(\varphi^*)$ that satisfy

$$J_i(\varphi^*) + 1 - G_i(\varphi^*) = (\varphi^*)^{1-\sigma} V_i(\varphi^*).$$

Substituting this equality into the definition of α_i gives us the result.

Next, we show several properties of α_i.

- The first property is that $\alpha_i\alpha_j > 1$. To show this, from (1), we have that

$$\left(\frac{\varphi^*_{ij}}{\varphi^*_{ii}}\right)^{\sigma-1} = \frac{\tau_{ij}^{\sigma}\theta_{ij}^{\sigma-1}f_{ij}}{f_{ii}}\frac{B_i}{B_j}. \tag{A.2}$$

  Substituting this equality into $\alpha_i\alpha_j$ that satisfies (A.1),

$$\alpha_i\alpha_j = (\tau_{ij}\tau_{ji})^{\sigma}(\theta_{ij}\theta_{ji})^{\sigma-1}\left(\frac{V_i(\varphi^*_{ii})V_j(\varphi^*_{jj})}{V_i(\varphi^*_{ij})V_j(\varphi^*_{ji})}\right) > 1.$$

  The inequality follows from $\varphi^*_{ij} > \varphi^*_{ii}$ and noting that $V_i(\varphi^*)$ is strictly decreasing in φ^*.

- The second property is that $\alpha_i = R_{ii}/R_{ij}$. Using (1), $R_{ij} = M_i^e\int_{\varphi^*_{ij}}^{\varphi_{\max}} r_{ij}(\varphi)dG_i(\varphi)$ is given by

$$R_{ij} = M_i^e\sigma w_i f_{ij}(\varphi^*_{ij})^{1-\sigma}V_i(\varphi^*_{ij}). \tag{A.3}$$

  The result follows from substituting (A.3) into the equality of (A.1).

- The third property is that $\lambda_{ji}, \tilde{\lambda}_{ji}$ and μ_i are written in terms of α_i. By definition,

$$\begin{aligned}\lambda_{ji} &= \frac{\tau_{ji}R_{ji}}{R_{ii}+\tau_{ji}R_{ji}} = \frac{\tau_{ji}R_{ij}}{R_{ii}+\tau_{ji}R_{ij}} = \frac{\tau_{ji}}{\alpha_i+\tau_{ji}},\\ \tilde{\lambda}_{ji} &= \frac{\lambda_{ji}}{\tau_{ji}(1-\lambda_{ji})+\lambda_{ji}} = \frac{1}{\alpha_i+1},\\ \mu_i &= \frac{\tau_{ji}}{\tau_{ji}(1-\lambda_{ji})+\lambda_{ji}} = \frac{\alpha_i+\tau_{ji}}{\alpha_i+1}.\end{aligned} \tag{A.4}$$

  This follows from the second property and the trade balance condition.

A.4 Proof of Lemma 2

We first show that $\varepsilon_{ij} = \sigma - 1 + \gamma_{ij}$. Following Melitz and Redding (2015) and using the domestic trade share $\lambda_{jj} = \alpha_j/(\alpha_j+\tau_{ij})$ from (A.4), this elasticity is defined as

$$\varepsilon_{ij} = -\frac{\partial\ln\left(\frac{1-\lambda_{jj}}{\lambda_{jj}}\right)}{\partial\ln\theta_{ij}} = \frac{\partial\ln\left(\frac{\alpha_j}{\tau_{ij}}\right)}{\partial\ln\theta_{ij}}.$$

Note, in our asymmetric-country setting, that ε_{ij} is defined as the elasticity of the import share relative to the domestic share in country j. Moreover, using (1) and (A.3), $\alpha_j = R_{jj}/R_{ji} = R_{jj}/R_{ij}$ and ε_{ij} is also defined as the elasticity of imports relative to domestic demand in country j where

$$\frac{\alpha_j}{\tau_{ij}} = \frac{M_j^e}{M_i^e}\left(\frac{\tau_{ij}\theta_{ij}w_i}{w_j}\right)^{\sigma-1}\frac{V_j(\varphi^*_{jj})}{V_i(\varphi^*_{ij})}.$$

Since the partial trade elasticity is estimated from a gravity equation where incomes and price indices are held constant (Arkolakis et al., 2012), reductions in θ_{ij} have no impact on M_i^e, M_j^e and w_i, w_j appearing in α_j/τ_{ij}. To show this in our model, we follow Melitz and Redding (2015) in holding the domestic productivity cutoffs $(\varphi_{ii}^*, \varphi_{jj}^*)$ constant. It then follows from (8) that the wage effects are muted as $\hat{w}_j = 0$. Further applying (A.3) to the labor market clearing condition,

$$L_i = M_i^e \sigma \sum_{n=i,j} f_{in}(\varphi_{in}^*)^{1-\sigma} V_i(\varphi_{in}^*).$$

Taking the log and differentiating this equality with respect to θ_{ij} and using (7),

$$\hat{M}_i^e = \frac{\alpha_i}{\alpha_i + 1}(\gamma_{ii} - \gamma_{ij})\hat{\varphi}_{ii}^*, \tag{A.5}$$

and the entry effects are muted so long as φ_{ii}^* is held constant. Taking the partial derivative of α_j/τ_{ij} with respect to θ_{ij} holding φ_{jj}^* constant,

$$\varepsilon_{ij} = \frac{\partial \ln(\alpha_j/\tau_{ij})}{\partial \ln \theta_{ij}} = (\sigma - 1) - \frac{\partial \ln V_i(\varphi_{ij}^*)}{\partial \ln \varphi_{ij}^*} \frac{\partial \varphi_{ij}^*}{\partial \theta_{ij}},$$

where $\partial \ln V_i(\varphi_{ij}^*)/\partial \ln \varphi_{ij}^* = d \ln V_i(\varphi_{ij}^*)/d \ln \varphi_{ij}^*$ from the definition of $V_i(\varphi^*)$ and $\partial \ln \varphi_{ij}^*/\partial \ln \theta_{ij} = 1$ from (A.2). In a similar vein, we can show that ε_{ij} is the partial trade elasticity of τ_{ij}.

Next, we show that φ_{ii}^* is a single sufficient statistic for welfare even with tariff revenue. Taking the log and differentiating μ_i in Lemma 1 with respect to θ_{ji},

$$\hat{\mu}_i = -\frac{(\tau_{ji} - 1)\alpha_i}{(\alpha_i + \tau_{ji})(\alpha_i + 1)}\hat{\alpha}_i.$$

Substituting $\hat{\alpha}_i = -[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i]\hat{\varphi}_{ii}^*$ and the definitions of β_i and λ_{ii} in Lemmas 1 and 2 into (9) gives us the result.

A.5 Proof of Proposition 1

We first show (10). From (4), (7), and (8), it follows that

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}_{ii}^* = \sigma \hat{w}_i, \tag{A.6}$$

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}_{jj}^* = \sigma \hat{w}_j, \tag{A.7}$$

$$\hat{B}_j + (\sigma - 1)\hat{\varphi}_{ij}^* = \sigma \hat{w}_i, \tag{A.8}$$

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}_{ji}^* = \sigma \hat{w}_j + (\sigma - 1)\hat{\theta}_{ji}, \tag{A.9}$$

$$\hat{\varphi}_{ij}^* = -\alpha_i \hat{\varphi}_{ii}^*, \tag{A.10}$$

$$\hat{\varphi}_{ji}^* = -\alpha_j \hat{\varphi}_{jj}^*, \tag{A.11}$$

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}_{ii}^* + \beta_j \hat{\varphi}_{jj}^*. \tag{A.12}$$

From (A.6), (A.9), (A.10), (A.12) and (A.7), (A.8), (A.11), (A.12) respectively,

$$
\begin{aligned}
(\rho+\beta_i)\hat{\varphi}_{ii}^* - (\beta_j - \rho\alpha_j)\hat{\varphi}_{jj}^* &= -\rho\hat{\theta}_{ji}, \\
-(\beta_i - \rho\alpha_i)\hat{\varphi}_{ii}^* + (\beta_j+\rho)\hat{\varphi}_{jj}^* &= 0,
\end{aligned}
$$

where

$$
\beta_i - \rho\alpha_i = \frac{\alpha_i}{\alpha_i+1}[\sigma - 1 - \rho + \gamma_{ii} + (\sigma - 1 - \rho + \gamma_{ij})\alpha_i] > 0.
$$

Solving for $\hat{\varphi}_{ii}^*$ and $\hat{\varphi}_{jj}^*$ and subsequently substituting them into (A.12) yields (10). Then,

$$
\frac{d\varphi_{ii}^*}{d\theta_{ji}} < 0,\ \frac{d\varphi_{jj}^*}{d\theta_{ji}} < 0,\ \frac{d\varphi_{ij}^*}{d\theta_{ji}} > 0,\ \frac{d\varphi_{ji}^*}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} > 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} > 0.
$$

Further, from (9), we have that $dP_i/d\theta_{ji} > 0$ and $dP_j/d\theta_{ji} > 0$. In contrast, if w_i is exogenous,

$$
\frac{d\varphi_{ii}^*}{d\theta_{ji}} > 0,\ \frac{d\varphi_{jj}^*}{d\theta_{ji}} < 0,\ \frac{d\varphi_{ij}^*}{d\theta_{ji}} < 0,\ \frac{d\varphi_{ji}^*}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} < 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} = 0,
$$

and, from (9), we have that $dP_i/d\theta_{ji} < 0$ and $dP_j/d\theta_{ji} > 0$. These differences imply that variable trade costs have different impacts on the extensive and intensive margins.

Next, we show that the impacts of fixed trade costs and tariffs are similar to those of variable trade costs. Following similar steps, we can derive the equilibrium in changes for f_{ji}:

$$
\begin{aligned}
\hat{\varphi}_{ii}^* &= -\frac{\beta_j+\rho}{\sigma\Xi}\hat{f}_{ji}, \\
\hat{\varphi}_{jj}^* &= -\frac{\beta_i - \rho\alpha_i}{\sigma\Xi}\hat{f}_{ji}, \\
\hat{w}_i &= \frac{\rho(\beta_i + \alpha_i\beta_j)}{\sigma\Xi}\hat{f}_{ji},
\end{aligned}
$$

and those for f_{ij}:

$$
\begin{aligned}
\hat{\varphi}_{ii}^* &= -\frac{\beta_j - \rho\alpha_j}{\sigma\Xi}\hat{f}_{ij}, \\
\hat{\varphi}_{jj}^* &= -\frac{\beta_i+\rho}{\sigma\Xi}\hat{f}_{ij}, \\
\hat{w}_i &= -\frac{\rho(\beta_j + \alpha_j\beta_i)}{\sigma\Xi}\hat{f}_{ij}.
\end{aligned}
$$

and those for τ_{ji}:

$$
\begin{aligned}
\hat{\varphi}_{ii}^* &= -\frac{\beta_j+\rho}{\Xi}\hat{\tau}_{ji}, \\
\hat{\varphi}_{jj}^* &= -\frac{\beta_i - \rho\alpha_i}{\Xi}\hat{\tau}_{ji}, \\
\hat{w}_i &= \frac{\rho(\beta_i + \alpha_i\beta_j)}{\Xi}\hat{\tau}_{ji},
\end{aligned}
\tag{A.13}
$$

and those for τ_{ij}:

$$\hat{\varphi}_{ii}^* = -\frac{\beta_j - \rho\alpha_j}{\Xi}\hat{\tau}_{ij},$$
$$\hat{\varphi}_{jj}^* = -\frac{\beta_i + \rho}{\Xi}\hat{\tau}_{ij},$$
$$\hat{w}_i = -\frac{\rho(\beta_j + \alpha_j\beta_i)}{\Xi}\hat{\tau}_{ij}.$$

Hence, reductions in any trade costs on exports and imports raise φ_{ii}^* and φ_{jj}^*, but starting from a symmetric situation (i.e., $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$), the effect of trade liberalization is always greater in a liberalizing country than in a non-liberalizing country. Only the difference is that reductions in *import* costs $\theta_{ji}, f_{ji}, \tau_{ji}$ reduce w_i, whereas reductions in *export* costs $\theta_{ij}, f_{ij}, \tau_{ij}$ raise w_i.

A.6 Proof of Proposition 2

We first show (16). From (12) and (15),

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}_{ii}^* = \sigma\hat{w}_i, \tag{A.14}$$
$$\hat{B}_j + (\sigma - 1)\hat{\varphi}_{jj}^* = \sigma\hat{w}_j, \tag{A.15}$$
$$\hat{B}_j + (\sigma - 1)\hat{\varphi}_{ij}^* = \sigma\hat{w}_i, \tag{A.16}$$
$$\hat{B}_i + (\sigma - 1)\hat{\varphi}_{ji}^* = \sigma\hat{w}_j, \tag{A.17}$$
$$\hat{w}_i - \hat{w}_j = -\beta_i\hat{\varphi}_{ii}^* + \beta_j\hat{\varphi}_{jj}^* - \hat{L}_i. \tag{A.18}$$

From (A.10), (A.14), (A.17), (A.18) and (A.11), (A.15), (A.16), (A.18) respectively,

$$(\beta_i + \rho)\hat{\varphi}_{ii}^* - (\beta_j - \rho\alpha_j)\hat{\varphi}_{jj}^* = -\hat{L}_i,$$
$$-(\beta_i - \rho\alpha_i)\hat{\varphi}_{ii}^* + (\beta_j + \rho)\hat{\varphi}_{jj}^* = \hat{L}_i.$$

Solving for $\hat{\varphi}_{ii}^*$ and $\hat{\varphi}_{jj}^*$ and substituting them into (A.18) yields (16).

Next, we show that (14) can be expressed in terms of the domestic productivity cutoff φ_{ii}^* only. Substituting $\hat{L}_i = -(\beta_i + \rho)\hat{\varphi}_{ii}^* + (\beta_j - \rho\alpha_j)\hat{\varphi}_{jj}^*$ derived above into (14),

$$\begin{aligned}
\hat{W}_i &= \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i} + 1\right)\hat{\varphi}_{ii}^* + \frac{1}{\sigma - 1}(-(\beta_i + \rho)\hat{\varphi}_{ii}^* + (\beta_j - \rho\alpha_j)\hat{\varphi}_{jj}^*) \\
&= \frac{1}{\rho}\left((1 - \lambda_{ii})\beta_i - \lambda_{ii}\frac{\beta_i}{\alpha_i} + \rho - \frac{\beta_i + \rho}{\sigma}\right)\hat{\varphi}_{ii}^* + \frac{1}{\sigma - 1}(\beta_j - \rho\alpha_j)\hat{\varphi}_{jj}^* \\
&= \frac{1}{\sigma - 1}\left((\sigma - 1)(\beta_i + \rho) - \sigma\beta_i\underbrace{\left(\frac{\alpha_i + 1}{\alpha_i + \tau_{ji}}\right)}_{\frac{1}{\mu_i}} - (\beta_j - \rho\alpha_j)\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\right)\hat{\varphi}_{ii}^*,
\end{aligned}$$

where the second equality comes from rewriting $\lambda_{ii} = \alpha_i/(\alpha_i + \tau_{ji})$ in (A.4) as $\tau_{ji}\lambda_{ii} = \alpha_i(1 - \lambda_{ii})$

and the third equality comes from rewriting the first two relationships in (16) as

$$\hat{\varphi}^*_{jj} = -\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\hat{\varphi}^*_{ii}.$$

Finally, we show that starting from a symmetric situation and free trade, market expansion unambiguously improves welfare for country i. Evaluating at $\alpha_i = \alpha_j, \beta_i = \beta_j$ and $\mu_i = 1$,

$$\hat{W}_i = -\frac{1}{\sigma - 1}\left(\beta_i - (\sigma-1)\rho + (\beta_i - \rho\alpha_i)\right)\hat{\varphi}^*_{ii},$$

where $\beta_i - (\sigma - 1)\rho > 0$. The desired result follows from $\hat{\varphi}^*_{ii} < 0$. Together with (7) and (12),

$$\frac{d\varphi^*_{ii}}{dL_i} < 0,\ \frac{d\varphi^*_{jj}}{dL_i} > 0,\ \frac{d\varphi^*_{ij}}{dL_i} > 0,\ \frac{d\varphi^*_{ji}}{dL_i} < 0,\ \frac{dB_i}{dL_i} > 0,\ \frac{dB_j}{dL_i} < 0,\ \frac{dw_i}{dL_i} > 0.$$

Further, from (14), we have that $dP_i/dL_i < 0$ and $dP_j/dL_i < 0$. In contrast, if w_i is exogenous,

$$\frac{d\varphi^*_{ii}}{dL_i} = 0,\ \frac{d\varphi^*_{jj}}{dL_i} = 0,\ \frac{d\varphi^*_{ij}}{dL_i} = 0,\ \frac{d\varphi^*_{ji}}{dL_i} = 0,\ \frac{dB_i}{dL_i} = 0,\ \frac{dB_j}{dL_i} = 0,\ \frac{dw_i}{dL_i} = 0,$$

and, from (14), $dP_i/dL_i < 0$ and $dP_j/dL_i = 0$.

A.7 Proof of Proposition 3

We first show the derivation of (18). Taking the log and differentiating W_i with respect to τ_{ji},

$$\begin{aligned}\hat{W}_i &= \frac{1}{\rho}(\tau_{ji} - 1)\underbrace{\left(\frac{\alpha_i}{\alpha_i + \tau_{ji}}\right)}_{\lambda_{ii}}\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii} + \frac{1}{\rho}\underbrace{\left(\frac{\tau_{ji}}{\alpha_i + \tau_{ji}}\right)}_{\lambda_{ji}}\hat{\tau}_{ji} + \hat{\varphi}^*_{ii} \\ &= \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i} + 1\right)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}\hat{\tau}_{ji}.\end{aligned}$$

Compared to (9), there is an additional term that captures changes in tariff revenue raised by changes in τ_{ji}. Taking the log and differentiating (1) with respect to τ_{ji} gives the counterparts to (A.6) and (A.9). Cancelling $\hat{B}_i$ out from these and using (7) and (8) that hold for changes in τ_{ji},

$$\hat{\tau}_{ji} = -(\beta_i + \rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}.$$

Further, noting that $\lambda_{ji} = 1 - \lambda_{ii}$ and substituting $\hat{\tau}_{ji}$ derived above,

$$\hat{W}_i = -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}(\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}. \quad \text{(A.19)}$$

Since an increase in tariffs decreases both φ^*_{ii} and φ^*_{jj}, (A.19) shows that tariffs in country i have a positive (negative) impact on welfare in country i by increasing (decreasing) consumption of domestic (imported) varieties. In fact, $\hat{\varphi}_{ii}^*$ and $\hat{\varphi}_{jj}^*$ have the following relationship from (A.13):

$$\hat{\varphi}_{jj}^* = \left(\frac{\beta_i - \rho\alpha_i}{\beta_j + \rho}\right)\hat{\varphi}_{ii}^*.$$

Substituting this into (A.19) and rearranging,

$$\hat{W}_i = \frac{\beta_i - \rho\alpha_i}{\rho}\left(-\frac{\lambda_{ii}}{\alpha_i} + \frac{\lambda_{ji}(\beta_j - \rho\alpha_j)}{\beta_j + \rho}\right)\hat{\varphi}_{ii}^*.$$

Further, substituting $\lambda_{ii}/\alpha_i = \lambda_{ji}/\tau_{ji}$ from (A.4) into the above, we obtain the expression in (18).

Next, we show that starting from a symmetric situation, country i's gains cannot compensate country j's losses. Adding $\hat{W}_i$ in (A.19) and $\hat{W}_j$ in the main text,

$$\begin{aligned}\hat{W}_i + \hat{W}_j &= -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i - \rho\alpha_i)\hat{\varphi}_{ii}^* + \left(\frac{(\tau_{ji}-1)\lambda_{jj}}{\rho}\frac{\beta_j}{\alpha_j} + 1 + \frac{\lambda_{ji}}{\rho}(\beta_j - \rho\alpha_j)\right)\hat{\varphi}_{jj}^* \\ &= \frac{\beta_i - \rho\alpha_i}{\rho\Xi}\left(\frac{\beta_j + \rho}{\alpha_i + \tau_{ji}} - \frac{(\tau_{ji}-1)\beta_j}{\alpha_j + \tau_{ij}} - \rho - \frac{\tau_{ji}(\beta_j - \rho\alpha_j)}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji},\end{aligned}$$

where the second equality follows from using (A.4) and (A.13). Notice that the first term in the brackets is positive and the others are negative, which means that changes in total welfare are in general ambiguous, as in changes in country i's welfare. However, evaluating at a symmetric situation where $\alpha_i = \alpha_j$, $\beta_i = \beta_j$ and $\tau_{ij} = \tau_{ji}$,

$$\hat{W}_i + \hat{W}_j = -\frac{\beta_i - \rho\alpha_i}{\rho\Xi}\left(\frac{(\tau_{ji}-1)(\beta_i + \rho + \beta_i - \rho\alpha_i)}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji},$$

where the value in the brackets is positive from observing that $\tau_{ji} - 1 \geq 0$. This establishes the desired result.

Finally, we show the derivation of (19). Taking the log and differentiating W_i with respect to τ_{ji}, welfare changes can be simply expressed as

$$\hat{W}_i = \frac{\hat{\mu}_i}{\rho} + \hat{\varphi}_{ii}^*.$$

To show that changes can be expressed in terms of changes in λ_{ii} and μ_i, we use the fact that $\lambda_{ii} \times \mu_i = \alpha_i/(\alpha_i + 1)$ from (A.4). Taking the log and differentiating this with respect to τ_{ji},

$$\hat{\lambda}_{ii} + \hat{\mu}_i = -\frac{\beta_i}{\alpha_i}\hat{\varphi}_{ii}^*. \tag{A.20}$$

Solving for $\hat{\varphi}_{ii}^*$ and substituting it into the above welfare changes gives us the expression in (19). Note that these changes in $\hat{W}_i$ and $\hat{\lambda}_{ii} + \hat{\mu}_i$ hold with respect to θ_{ji} and f_{ji}, and (19) also applies to variable and fixed trade costs. While (19) is expressed in terms of $\hat{\lambda}_{ii}$ and $\hat{\mu}_i$ only, it is possible to express (19) in terms of $\hat{M}_i^e$ as well. Using the general expression of β_i/α_i in Lemma 2(i), let us further express (19) as

$$\hat{W}_i = -\left(\frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\lambda}_{ii} + \left(\frac{1}{\rho} - \frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\mu}_i.$$

After rearranging, this can be rewritten as

$$\hat{W}_i = -\left(\frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}}\right)\hat{\lambda}_{ii} - \left(\frac{\alpha_i(\gamma_{ii} - \gamma_{ij})}{(\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})((\alpha_i + 1)\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})}\right)\hat{\lambda}_{ii} + \left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} - \frac{\alpha_i(\gamma_{ii} - \gamma_{ij})}{(\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})((\alpha_i + 1)\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij})}\right)\hat{\mu}_i.$$

Solving (A.5) for $\hat{\varphi}_{ii}^*$ that holds for changes in τ_{ji} and substituting this and β_i/α_i into (A.20),

$$\hat{\lambda}_{ii} = -\left(\frac{(\alpha_j + 1)\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}}{\alpha_i(\gamma_{ii} - \gamma_{ij})}\right)\hat{M}_i^e - \hat{\mu}_i.$$

Substituting this into the second $\hat{\lambda}_{ii}$ above yields the expression $\hat{W}_i$ mentioned as the counterpart to Melitz and Redding (2015), which becomes the same as theirs without tariff revenue ($\hat{\mu}_i = 0$).

A.8 Proof of Lemma 3

We first show that, if $\gamma_{jj} - \gamma_{ji}$ is negative (positive), ε_{ji} is increasing (decreasing) in trade costs. Let $\phi \in \{\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji}\}$ denote a set of trade costs between countries. From the definition of γ_{jn}, let us re-express this as a function of the productivity cutoff φ_{jn}^* for $n = i, j$:

$$\gamma_j(\varphi_{jn}^*) \equiv -\frac{d\ln V_j(\varphi_{jn}^*)}{d\ln \varphi_{jn}^*}.$$

If $\gamma_j(\varphi_{jn}^*)$ is strictly increasing (decreasing) in the productivity cutoff φ_{jn}^*, the extensive margin elasticity differential is negative (positive) under export market selection so that

$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \implies \gamma_{jj} - \gamma_{ji} \lesseqgtr 0.$$

With this restriction, the sign of the extensive margin elasticity differential is therefore the same for a given productivity distribution $G_j(\varphi)$. Then exploiting the fact that φ_{jj}^* is held constant for the derivation of the partial trade elasticity $\varepsilon_{ji} = \sigma - 1 + \gamma_{ji}$ (see Appendix A.4),

$$\frac{d\varepsilon_{ji}}{d\phi} = \gamma_j'(\varphi_{ji}^*)\frac{d\varphi_{ji}^*}{d\phi}.$$

Note that, if $\gamma_j(\varphi_{jn}^*)$ is constant, ε_{ji} is invariant to ϕ. Since $\frac{d\varphi_{ji}^*}{d\phi} > 0$ from Proposition 1,

$$\gamma_j'(\varphi_{jn}^*) \gtreqless 0 \implies \frac{d\varepsilon_{ji}}{d\phi} \gtreqless 0.$$

Next, we show that, if $\gamma_{jj} - \gamma_{ji}$ is negative (positive), ε_{ji} is decreasing (increasing) in market size in country i, while the converse is true for market size in country j. Differentiating ε_{ji} with respect to L_i and L_j respectively and noting that $\frac{d\varphi^*_{ji}}{dL_i} < 0$ and $\frac{d\varphi^*_{ji}}{dL_j} > 0$ from Proposition 2,

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{dL_i} \lesseqgtr 0,$$
$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{dL_j} \gtreqless 0.$$

The result follows immediately from the above expressions.

A.9 Proof of Proposition 4

We first show that, if $\gamma_{jj} - \gamma_{ji}$ is negative (positive), reductions in trade costs have the impact on t^*_{ji} not only by decreasing $\tilde{\lambda}_{jj}$ but also by decreasing (increasing) ε_{ji}. To show this, recall that the optimal tariff in (20) is rewritten as

$$t^*_{ji} = \frac{\rho}{\tilde{\lambda}_{jj}\left(\frac{\beta_j}{\alpha_j} - \rho\right)},$$

where reductions in $\phi \in \{\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji}\}$ necessarily decrease $\tilde{\lambda}_{jj}$ irrespective of the sign of $\gamma_{jj} - \gamma_{ji}$ from Proposition 1. For our purpose, it thus suffices to show that, if $\gamma_{jj} - \gamma_{ji}$ is negative (positive), β_j/α_j decreases (increases) with ϕ. From Lemma 2(i), $\beta_j/\alpha_j = \varepsilon_{ji} + (\gamma_{jj} - \gamma_{ji})/(\alpha_j + 1)$ and differentiating this with respect to ϕ,

$$\begin{aligned}\frac{d(\beta_j/\alpha_j)}{d\phi} &= \gamma'_j(\varphi^*_{ji})\frac{d\varphi^*_{ji}}{d\phi} + \frac{-\gamma'_j(\varphi^*_{ji})\frac{d\varphi^*_{ji}}{d\phi}(\alpha_j+1) - (\gamma_{ji} - \gamma_{jj})\frac{d\alpha_j}{d\phi}}{(\alpha_j+1)^2} \\ &= \frac{\alpha_j}{\alpha_j+1}\left(\frac{d\varepsilon_{ji}}{d\phi} - \left(\frac{\gamma_{jj} - \gamma_{ji}}{\alpha_j(\alpha_j+1)}\right)\frac{d\alpha_j}{d\phi}\right).\end{aligned}$$

Using the impact of ϕ on ε_{ji} in Lemma 3 and $\frac{d\alpha_j}{d\phi} > 0$ from Proposition 1,

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{d\phi} \gtreqless 0,$$

Note that, if $\gamma_j(\varphi^*_{jn})$ is constant, β_j/α_j is invariant to ϕ.

Next, we show that country size has a similar impact on t^*_{ji}. From the impact of market size on $\tilde{\lambda}_{jj}$ from Proposition 2, it suffices to see the impact of L_i, L_j on β_j/α_j:

$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{dL_i} \lesseqgtr 0,$$
$$\gamma'_j(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{dL_j} \gtreqless 0.$$

A.10 Proof of Proposition 5

We first show that, if $\gamma_d - \gamma_x$ is negative (positive), reductions in trade costs not only decrease $\tilde{\lambda}$ but also decrease (increase) ε, thereby increasing (decreasing) τ^* relative to that with $\gamma_d - \gamma_x = 0$ in Nash equilibrium. Evaluating (A.6)-(A.11) at a symmetric situation by defining $B_i = B_j \equiv B$, $\varphi^*_{ii} = \varphi^*_{jj} \equiv \varphi^*_d$ and $\varphi^*_{ij} = \varphi^*_{ji} \equiv \varphi^*_x$,

$$\begin{aligned}\hat{B} + (\sigma - 1)\hat{\varphi}^*_d &= 0,\\ \hat{B} + (\sigma - 1)\hat{\varphi}^*_x &= (\sigma - 1)\hat{\theta},\\ \hat{\varphi}^*_x &= -\alpha\hat{\varphi}^*_d,\end{aligned}$$

which can be solved for

$$\hat{\varphi}^*_d = -\frac{1}{\alpha + 1}\hat{\theta}, \quad \hat{\varphi}^*_x = \frac{\alpha}{\alpha + 1}\hat{\theta}.$$

Further, noting from (A.4) that $\tilde{\lambda} = \alpha/(\alpha + 1)$,

$$\hat{\tilde{\lambda}} = -\left(\frac{\sigma - 1 + \gamma_d + (\sigma - 1 + \gamma_x)\alpha}{\alpha + 1}\right)\hat{\varphi}^*_d.$$

Since φ^*_d is decreasing in θ, reductions in θ decrease $\tilde{\lambda}$ irrespective of the sign of $\lambda_d - \lambda_x$. As for the partial trade elasticity, differentiating $\varepsilon = \sigma - 1 + \gamma(\varphi^*_x)$ with respect to θ,

$$\frac{d\varepsilon}{d\theta} = \gamma'(\varphi^*_x)\frac{d\varphi^*_x}{d\theta}.$$

As φ^*_x is increasing in θ, if $\gamma(\varphi^*_h)$ is strictly increasing (decreasing) in φ^*_h for $h = d, x$ so that $\gamma_d - \gamma_x < (>)0$, reductions in θ decrease (increase) ε. Note that, if $\gamma(\varphi^*_h)$ is constant, ε is invariant to θ. Together with the impact on $\tilde{\lambda}$ above, if $\gamma_d - \gamma_x$ is negative (positive and increases in ε are greater than reductions in $\tilde{\lambda}$), reductions in θ shift up (down) $f(\tau)$, which increase (decrease) τ^*. A similar proof also applies to reductions in f_x.

Next, we show that, irrespective of the sign of $\gamma_d - \gamma_x$, market expansions have no impact on $\tilde{\lambda}$ and ε and hence τ^* in Nash equilibrium. Evaluating (A.14)-(A.17) at a symmetric situation,

$$\begin{aligned}\hat{B} + (\sigma - 1)\hat{\varphi}^*_d &= 0,\\ \hat{B} + (\sigma - 1)\hat{\varphi}^*_x &= 0,\\ \hat{\varphi}^*_x &= -\alpha\hat{\varphi}^*_d,\end{aligned}$$

which can be solved for

$$\hat{\varphi}^*_d = \hat{\varphi}^*_x = 0.$$

Since φ^*_d and φ^*_x are invariant to country size in a symmetric situation, neither $\tilde{\lambda}$ nor ε is affected by country size as well. Thus increases in L have no impact on $f(\tau)$ as well as τ^* regardless of the sign of $\gamma_d - \gamma_x$.

Finally, we show that, if $\gamma_d - \gamma_x$ is negative (positive), reductions in trade costs narrow (widen) the gap between $\bar{\tau}^*$ and $\underline{\tau}^*$, whereas expansions in market size have no impact on these bounds irrespective of the sign of $\gamma_d - \gamma_x$ in Nash equilibrium. Evaluating $\bar{\tau}^*$ at a symmetric situation and differentiating it with respect to θ,

$$\gamma'(\varphi_h^*) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\bar{\tau}^*}{d\theta} = \frac{\rho\left(1 - \frac{f_x}{f_d}\right)}{(\varepsilon - \rho)^2} \frac{d\varepsilon}{d\theta} \gtreqless 0,$$

where $1 - \frac{f_x}{f_d} < 0$. Thus, if $\gamma(\varphi_h^*)$ is strictly increasing (decreasing) in φ_h^*, the upper bound $\bar{\tau}^*$ is stritly increasing (decreasing) in θ. Note that, if $\gamma(\varphi_h^*)$ is constant, so is ε and the upper bound is not affected by reductions in θ. Regarding $\underline{\tau}^*$, on the other hand, let us rewrite it as

$$\underline{\tau}^* = \frac{\frac{\beta}{\alpha}}{\frac{\beta}{\alpha} - \rho}.$$

Differentiating this expression with respect to θ,

$$\gamma'(\varphi_h^*) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\underline{\tau}^*}{d\theta} = -\frac{\rho}{(\frac{\beta}{\alpha} - \rho)^2} \frac{d(\beta/\alpha)}{d\theta} \lesseqgtr 0.$$

Thus, if $\gamma(\varphi_h^*)$ is strictly increasing (decreasing) in φ_h^*, the lower bound $\underline{\tau}^*$ is strictly decreasing (increasing) in θ. Note that, if $\gamma(\varphi_h^*)$ is constant, so is β/α and the lower bound is not affected. While f_x has a similar impact on the two bounds, L has no impact on φ_d^*, φ_x^* and hence $\tilde{\lambda}$ as well as ε as shown above, which means that increases in L have no impact on the two bounds.

References

Arkolakis C, Costinot A, Donaldson D, Rodríguez-Clare A. 2019. The Elusive Pro-Competitive Effects of Trade. *Review of Economic Studies* 86, 46-80.

Arkolakis C, Costinot A, Rodríguez-Clare A. 2012. New Trade Models, Same Old Gains? *American Economic Review* 102, 94-130.

Bernard AB, Jensen JB, Redding SJ, Schott SK. 2007. Firms in International Trade. *Journal of Economic Perspectives* 21, 105-130.

Bertoletti P, Etro F. 2017. Monopolistic Competition When Income Matters. *Economic Journal* 127, 1217-1243.

Chaney T. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review* 98, 1707-1721.

Demidova S. 2008. Productivity Improvements and Falling Trade Costs: Boon or Bane? *International Economic Review* 49, 1437-1462.

Demidova S. 2017. Trade Policies, Firm Heterogeneity, and Variable Markups. *Journal of International Economics* 108, 260-273.

Demidova S, Rodríguez-Clare A. 2009. Trade Policy under Firm-Level Heterogeneity in a Small Economy. *Journal of International Economics* 78, 100-112.

Demidova S, Rodríguez-Clare A. 2013. The Simple Analytics of the Melitz Model in a Small Economy. *Journal of International Economics* 90, 266-272.

Dhingra S, Morrow J. 2019. Monopolistic Competition and Optimum Product Diversity under Firm Heterogeneity. *Journal of Political Economy* 127, 196-232.

Eaton J, Kortum S. 2011. Putting Ricardo to Work. *Journal of Economics Perspectives* 26, 65-89.

Felbermayr G, Jung B, Larch M. 2013. Optimal Tariffs, Retaliation, and the Welfare Loss from Tariff Wars in the Melitz Model. *Journal of International Economics* 89, 13-25.

Felbermayr G, Jung B, Larch M. 2015. The Welfare Consequences of Import Tariffs: A Quantitative Perspective. *Journal of International Economics* 97, 295-309.

Gros D. 1987. A Note on the Optimal Tariff, Retaliation and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade. *Journal of International Economics* 23, 357-367.

Head K, Mayer T, Thoenig M. 2014. Welfare and Trade without Pareto. *American Economic Review* 104, 310-316.

Helpman E, Melitz MJ, Rubinstein Y. 2008. Estimating Trade Flows: Trading Partners and Trading Volumes. *Quarterly Journal of Economics* 123, 441-487.

Krugman P. 1980. Scale Economies, Production Differentiation, and the Pattern of Trade. *American Economic Review* 70, 950-959.

Mayer T, Ottaviano GIP. 2008. The Happy Few: The Internationalisation of European Firms. *Intereconomics* 43, 135-148.

Melitz MJ. 2003. The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica* 71, 1695-1725.

Melitz MJ, Ottaviano GIP. 2008. Market Size, Trade, and Productivity. *Review of Economic Studies* 75, 295-316.

Melitz MJ, Redding SJ. 2015. New Trade Models, New Welfare Implications. *American Economic Review* 105, 1105-1146.

Naito T. 2019. A Larger Country Sets a Lower Optimal Tariff. *Review of International Economics* 27, 643-665.

Novy D. 2013. International Trade without CES: Estimating Translog Gravity. *Journal of International Economics* 89, 271-282.

Ossa R. 2016. Quantitative Models of Commercial Policy. *Handbook of Commercial Policy* Volume 1a, Elsevier: North Holland, 207-259.

Venables A. 1987. Trade and Trade Policy with Differentiated Products: A Chamberlian-Ricardian Model. *Economic Journal* 97, 700-717.