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Dual Labor Market and the "Phillips Curve Puzzle"*

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Abstract

Low inflation was once a welcome to both policy makers and the public. However, Japan's experience during the 1990's changed the consensus view on price held by economists and central banks around the world. Facing deflation and the zero-interest-rate bound at the same time, BOJ had difficulty in conducting effective monetary policy. It unusually prolonged Japan's stagnation. The excessively low inflation which annoys central banks today is translated into the "Phillips curve puzzle". In the US and Japan, in the course of recovery from the Great Recession after the 2008 global financial crisis, the unemployment rate had steadily declined to the level which was commonly regarded as lower than the natural rate or NAIRU; and yet, inflation stayed low. In this paper, we consider a minimal model of labor market. The essential assumption is that the labor market is dual. In this model, we explore what kinds of changes in the economy flatten the Phillips curve. The level of bargaining power of workers, the elasticity of the supply of labor to wages in the secondary market and the composition of the workforce are the main factors in explaining the flattening of the Phillips curve. We argue that the changes we consider in the model, in fact, have plausibly flattened the Phillips curve in recent years.

Keywords: bargaining power; secondary workers

JEL classification: C60; E31

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1 Introduction

Low inflation was once a welcome to both policy makers and the public. However, Japan's experience during the 1990's changed the consensus view on price of economists and central banks around the world; After the financial bubble burst at the beginning of the 1990's, Japan lapsed into deflation. During the course, the Bank of Japan (BOJ) kept cutting the nominal interest rate down to zero. Facing deflation and zero interest bound at the same time, BOJ had difficulty in conducting effective monetary policy. It made Japan's stagnation unusually prolonged.

The "Japan problem" made economists aware of long-forgotten danger of deflation. In the prewar period, deflation was a menace to the economy, and its danger was emphasized by famous economists such as Keynes 1931 and Fisher 1933. To prevent deflation, central bank must seek low inflation rather than zero inflation or stable price level. Today, following this idea, many central banks including The US Federal Reserve, BOJ, and European Central Bank target at two percent inflation of consumer price index. However, few central banks have been successful in achieving this goal in any satisfactory way.

Too low inflation which annoys central banks today is translated into the Phillips curve puzzle. The bench mark Phillips curve is as follows (Phillips 1958; Friedman 1968):

$$\pi_t = a\,(u - u^*) + b\,\pi_t^*, \tag{1}$$

where π and u are inflation and the unemployment rate, respectively. π^* is either inflationary expectation or inertia of past inflation. u^* is the natural rate of unemployment or the NAIRU (Non-Accelerating Inflation Rate of Unemployment). According to conventional macroeconomics, Eq. (1) or the Phillips curve determines inflation.

In the US and Japan, in the course of recovery from the Great Recession after the 2008 global financial crisis, the unemployment rate had steadily declined to the level which was commonly regarded as lower than NAIRU, u^*. And yet, inflation stayed low; 0.5% for Japan, and 1.5% for the US.

In terms of the Phillips curve, Eq. (1), two things have been pointed out. First, coefficient b for inflationary expectations or lagged inflation declined significantly almost to zero in recent years. Blanchard 2018, Figure 7, for example, found that b which was almost zero in the early 1960's, rose sharply to one in the late 1960's, had stayed there for thirty years, and then declined suddenly to zero at the beginning the 2000's. While the decline in inflation from the '90s had been often attributed to better policy management, and even the tern "Great Moderation" was coined Clarida et al., 2000, more recent analyses identify the anchoring of inflation expectations for the change in the trade-off between inflation and unemployment Barnichon and Mesters, 2020; Blanchard, 2016. Greenspan 2001 left the following remark: "Price stability is best thought of as an environment in which inflation is so low and stable over time that it does not materially enter into the decisions of households and firms."

Second is a change of coefficient a for the unemployment rate in Eq. (1).

A decline of a entails lower inflation than otherwise when the unemployment rate declines. Some researchers even argue that unemployment no longer has an effect on inflation, at least over some unemployment and inflation range.

Figure 1 (a) is Japan's Phillips curve, namely, the quarterly relation between the unemployment rate and nominal wage growth for 1980.II-2019.II. We can indeed observe that the Phillips curve has flattened in recent years.

These are the "Phillips curve puzzle". In this paper, we leave the first problem untouched: We simply take b as zero, and focus on the second problem, namely why a gets smaller in Eq. (1). For this purpose, we consider a minimal model of labor market.

The essential assumption is that labor market is dual (McDonald and Solow, 1981, 1985; Gordon, 2017), In this model, we explore what kind of change in the economy makes the Phillips curve flat. In the model, the level of bargaining power of workers, the elasticity of the supply of labor to wage in the secondary market, and the composition of the workforce are the main factors in explaining the flattening of the Philips curve. We argue that the changes we consider in the model, in fact, have plausibly made the Phillips curve flat in recent years.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical model. Sections 3 and 4 presents the analytical and numerical results, respectively. Finally, section 5 provides some concluding remarks.

2 The Model

Labor market is dual consisting of primary labor and secondary labor (McDonald and Solow 1981, 1985; Gordon 2017). As in Di Guilmi and Fujiwara 2020, we identify as secondary workers all the employees without a permanent contract (agency, temporary, and part-time).

Firms are heterogeneous in size and efficiency, but adopt the same production function. Each firm produces a homogeneous goods by employing only labor, composed by primary and secondary workers. Specifically, the firm j ($j = 1, \cdots, N$) employs $L_{1,j}$ primary workers with wage $w_{1,j}$ and $L_{2,j}$ secondary workers with wage w_2. There are $L_1 = \sum_{j=1}^{N} L_{1,j}$ primary workers and $L_2 = \sum_{j=1}^{N} L_{2,j}$ secondary workers employed.

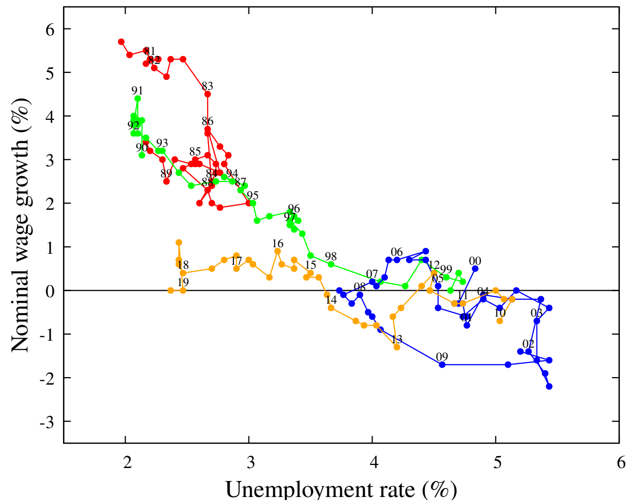
Primary workers are a fixed endowment for each firm. The Japanese firm rarely lays off its primary workers. In 2020, for example, in the mid of Covid–19 recession, real GDP fell by unprecedented 28.1%, and yet, the unemployment rate rose slightly only to 2.8%. We take $L_{1,j}$ as given in the model. In contrast, firm freely changes the level of secondary workers. Following the empirical findings of Munakata and Higashi 2016, the wage of secondary workers is determined in market and is uniform across firms.

Output of the firm j is as follows:

$$Y_j = A_j(L_{1,j} + c\,L_{2,j})^{\alpha}, \tag{2}$$

where $\alpha \in (0, 1)$. A_j is firm-specific total factor productivity (TFP). Parameter

(a)



(b)

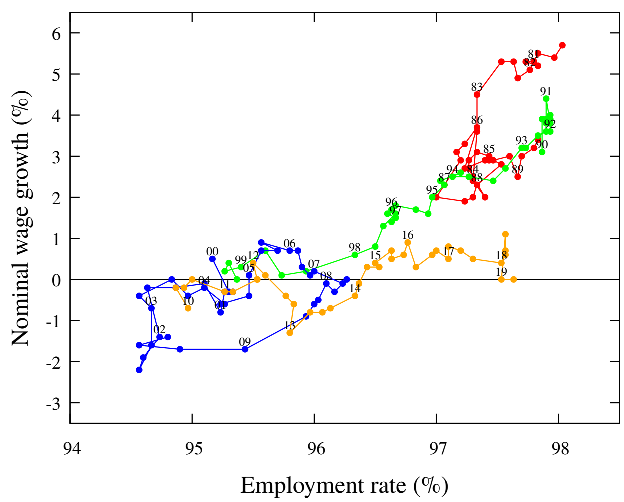


Figure 1: **Japan's Phillips curve: Unemployment rate and nominal wage growth of 157 quarters from 1980.II to 2019.II.** (a) The abscissa is the unemployment rate in %. (b) The abscissa is reversed and is the employment rate (=100-unemployment rate (%)). Colors: 1980s (red), 1990s (green), 2000s (blue), 2010s (orange). The last several years' data (in yellow) show definite deviation from those of the earlier years, with nominal wage growth is low in spite of the low unemployment. Sources are Ministry of Health, Labor and Welfare, Japan 2020 and Statistics Bureau of Japan 2020.

c ($\in (0,1)$)) is productivity of secondary labor relative to that of primary labor (Di Guilmi and Fujiwara, 2020; Fukao and Ug Kwon, 2006).

The profit is given by

$$\Pi_j = Y_j - L_{1,j} w_{1,j} - L_{2,j} w_2. \tag{3}$$

In order to mimic the firm-level bargaining process that is prevailing in Japan, the primary workers' wage is set in a two-step process for each employer. Assuming a fixed endowment of primary workers (or *insiders*) $L_{1,j}$ for each firm, in the first stage firm and primary workers determine the number of secondary workers (or *outsiders*) $L_{2,j}$ to be hired. Assuming a perfectly competitive market for secondary workers, firms take the secondary wage w_2 as given. Once the number of secondary workers is determined, firms and insiders share the surplus defined by revenue less the wages paid to secondary workers through a Nash bargaining (Mortensen and Pissarides, 1994).

Maximization 1

Firm maximizes its profit (3) by choosing the number of secondary workers or outsiders:

$$\max_{L_{2,j}} \left[\Pi_j\right]. \tag{4}$$

This determines demand for secondary workers $L_{2,j}^{(\mathrm{d})}$ of firm j as follows:

$$L_{2,j}^{(\mathrm{d})} = \frac{1}{c}\left[-L_{1,j} + \left(\frac{\alpha c A_j}{w_2}\right)^{1/(1-\alpha)}\right]. \tag{5}$$

The total demand for secondary workers in the economy as a whole is then:

$$L_2^{(\mathrm{d})} = \frac{1}{c}\left[-L_1 + \left(\frac{\alpha\, c\, A}{w_2}\right)^{1/(1-\alpha)}\right], \tag{6}$$

where A is the following nonlinear sum of A_j:

$$A = \left(\sum_{j=1}^{N} A_j^{1/(1-\alpha)}\right)^{(1-\alpha)}. \tag{7}$$

The level of output of firm j is

$$Y_j = A_j \left(\frac{\alpha c A_j}{w_2}\right)^{\alpha/(1-\alpha)}. \tag{8}$$

We assume the following supply function of secondary workers L_2:

$$L_2^{(\mathrm{s})} = B\, w_2^{\beta}. \tag{9}$$

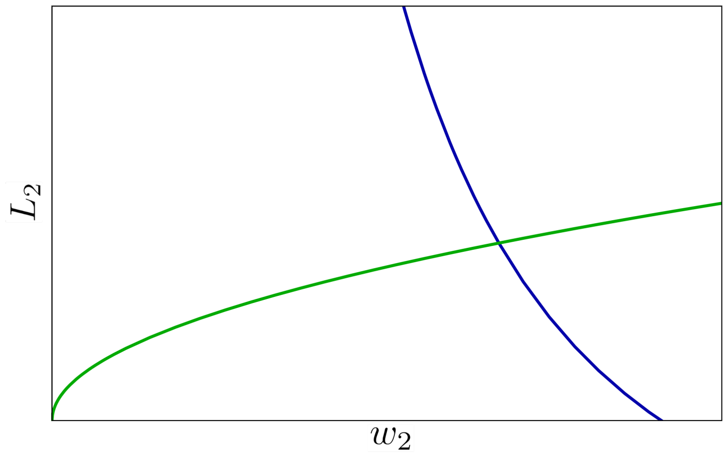


Figure 2: **Demand function $L_2^{(\mathrm{d})}$ (Eq.(6)) in blue and supply function $L_2^{(\mathrm{s})}$ (Eq.(9)) in green.**

Matching demand for and supply of secondary workers, $L_2^{(\mathrm{s})} = L_2^{(\mathrm{d})}$, we obtain the following nonlinear equation for w_2:

$$B\, w_2^{\beta} = \frac{1}{c}\left[-L_1 + \left(\frac{\alpha\, c\, A}{w_2}\right)^{1/(1-\alpha)}\right], \tag{10}$$

Demand for and supply of secondary labor as functions of w_2 are shown on (L_2, w_2) plane in Fig. 2. It can be shown that the solution of Eq. (10) always exists.

Maximization 2

Firm and primary workers (insiders) determine the wage of primary workers $w_{1,j}$ through Nash bargaining.

$$\max_{w_{1,j}}\left[(L_{1,j}w_{1,j})^{\gamma}(\Pi_j)^{(1-\gamma)}\right], \tag{11}$$

where $\gamma \in (0,1)$ indicates bargaining power of primary workers.

The Nash bargaining determines $w_{1,j}$ as follows:

$$w_{1,j} = \gamma\frac{Y_j - L_{2,j}w_2}{L_{1,j}}. \tag{12}$$

By combining Eqs.(5) and (12), we find that

$$\Pi_j = \frac{1-\gamma}{\gamma}L_{1,j}w_{1,j}. \tag{13}$$

In the following, we study the relationship between the total employment of workers,

$$L = L_1 + L_2 = L_1 + Bw_2^{\beta}, \tag{14}$$

and the average wage,

$$\bar{w} = \frac{W}{L}. \tag{15}$$

In Eq.(15), W is the total earnings of all the workers:

$$\begin{aligned} W &= \sum_{j=1}^{N} (L_{1,j} w_{1,j}) + L_2 w_2 \\ &= \gamma \sum_{j=1}^{N} Y_j + (1-\gamma) L_2 w_2 \\ &= \gamma A \left(\frac{\alpha c A}{w_2} \right)^{\alpha/(1-\alpha)} + (1-\gamma) B w_2^{1+\beta}. \end{aligned} \tag{16}$$

We used the result of the second maximization, Eq.(12). Because L and $\bar{w}$ are functions of A, we obtain functional relationship between L and $\bar{w}$ by eliminating A, while keeping other parameters $\{L_1, c,\ \alpha,\ \ B,\ \beta, \gamma\}$ fixed. We shall call the curve so defined $w(L)$ as the "pseudo-Phillips curve". This relation is shown in Fig.1(b).

Though the Phillips curve relates the rate of change of wage/price to employment, given the level of wage/price in the previous period, it is essentially the relationship between the current level of wage/price to unemployment. This is the "pseudo-Phillips cureve" we explore in the following.

The parameters and variables of this model are listed in Table 1. The model has parameters $A, L_1, c, \alpha, B, \beta, \gamma$. The variables $L_2, w_{1.j}, w_2$ are determined as functions of these parameters. The model is very simple to the extent that it is justifiably called minimal. However, because of nonlinearity, solving it is not trivial at all.

Output	
A	Nonlinear sum of the total factor productivity A_j (Eq.(7))
L_1	Total number of primary workers
L_2	Total number of secondary workers employed (Eq.(5))
c	Secondary workers' productivity coefficient
α	Output exponent
Supply of secondary workers	
B	Coefficient of supply function of secondary workers
β	Wage elasticity
Nash Bargaining	
$w_{1,j}$	The wage of the primary workers at firm j (Eq. (12))
w_2	The wage of secondary workers (Eq. (10))
γ	Bargaining power of primary workers

Table 1: **List of the parameters and variables of the model.** Those with equation numbers are variables determined by the equation.

3 Solving the Model

Toward the goal of solving the model, we first rewrite Eq.(10) as follows:

$$\frac{w_2}{\alpha\, c\, A} = \left(L_1 + c\, B\, w_2^{\beta}\right)^{-(1-\alpha)}. \tag{17}$$

By introducing the following scaled variable v;

$$v \equiv \frac{L_1^{\beta(1-\alpha)}}{(\alpha\, c\, A)^{\beta}}\, w_2^{\beta}. \tag{18}$$

we can rewrite Eq.(17) as follows:

$$v = (1 + g\, v)^{-\beta(1-\alpha)}, \tag{19}$$

where

$$g \equiv c\, B\, (\alpha\, c\, A)^{\beta}\, L_1^{-1-\beta(1-\alpha)}. \tag{20}$$

Recall that solving the model amounts to finding the equilibrium w_2 which is equivalent to v. Thus, we focus on Eq.(19). We note here that both v and g are dimensionless quantities in Eq.(19) (see Appendix A). This makes the following analysis straightforward.

With variable v, Eqs.(14) and (16) are written as follows:

$$L = L_1\left[1 + \frac{g}{c} v\right], \tag{21}$$

$$W = \frac{g^{1/\beta} L_1^{1+1/\beta}}{\alpha c (cB)^{1/\beta}} \left[\gamma\, v^{-\alpha/(\beta(1-\alpha))} + (1-\gamma)\, \alpha\, g\, v^{1+1/\beta}\right]. \tag{22}$$

This leads to the following average wage:

$$\bar{w} = \frac{W}{L} = \left(\frac{L_1}{B}\right)^{1/\beta} Z(\alpha, c, \beta, \gamma, g, v). \tag{23}$$

The coefficient $(L_1/B)^{1/\beta}$ is the *only* factor with the same dimension as $\bar{w}$. The function $Z(\alpha, c, \beta, \gamma, g, v)$ is the following dimensionless function of the dimensionless parameters $\alpha, c, \beta, \gamma, g$ and $v = v(g)$:

$$Z(\alpha, c, \beta, \gamma, g, v) = \\ \frac{g^{1/\beta}}{\alpha\, c^{1+1/\beta}} \left[\gamma\, v^{-\alpha/(\beta(1-\alpha))} + (1-\gamma)\, \alpha\, g\, v^{1+1/\beta}\right] \Big/ \left[1 + \frac{g}{c} v\right]. \tag{24}$$

The pseudo-Phillips curve which is the relationship between the average wage and employment, $\bar{w}(L)$, is obtained by eliminating g (and $v = v(g)$) from Eq. (21) and Eq. (23).

Now, the second term in the parenthesis of the right-hand-side of Eq.(19), gv is the ratio cL_2/L_1. Therefore, if L_1 and L_2 measured in efficiency unit are different in order, we may approximate it around the larger term. In other words, if $L_1 \gg cL_2$, namely, if the primary workers dominate in efficiency in productio, we expand the right hand side around for $gv \ll 1$ or equivalently $g \ll 1$ (note that large L_1 implies small g because of Eq.(20)).

The small-g perturbative solution of Eq.(19) is the following:

$$v = 1 - \sigma g + \frac{1}{2}\sigma(1 + 3\sigma)g^2 + \cdots , \tag{25}$$

where $\sigma \equiv \beta(1 - \alpha)$. By substituting Eq.(25) into Eqs.(21), (23), and (24), we obtain the followings:

$$L = L_1 \left[1 + \frac{g}{c} - \sigma \frac{g^2}{c} + \cdots \right] \tag{26}$$

$$\bar{w} = \left(\frac{L_1}{B} \right)^{1/\beta} \frac{g^{1/\beta}}{\alpha\, c^{1+1/\beta}} \left[\gamma + \left(\alpha - \frac{\gamma}{c} \right) g + \cdots \right] . \tag{27}$$

In order to eliminate g from these two equations and obtain a relationship between L and $\bar{w}$, we first solve Eq.(26) for small g:

$$g = c \left(\frac{L}{L_1} - 1 \right) + c^2\sigma \left(\frac{L}{L_1} - 1 \right)^2 + \cdots . \tag{28}$$

Note that this is a perturbative series in $L/L_1 - 1 = L_2/L_1 \ll 1$. By substituting this expression into Eq.(27), we obtain the following expression of $\bar{w}$:

$$\bar{w} = \frac{\gamma}{\alpha\, c} \left(\frac{L_1}{B} \right)^{1/\beta} \left(\frac{L}{L_1} - 1 \right)^{1/\beta} \left[1 + \frac{c(\alpha + \gamma - \alpha\gamma) - \gamma}{\gamma} \left(\frac{L}{L_1} - 1 \right) + \cdots \right] . \tag{29}$$

Thus, the average wage $\bar{w}$ is a monotonically increasing function of total employment, L. Namely, the pseudo-Phillips curve has the expected sign of slope; it is upward sloping on employment–wage plane, therefore, is downward sloping on wage–unemployment plane. We find from the leading term of this expression that the slope of the curve depends on two key factors, $\gamma/(c\,\alpha B^{1/\beta})$ and $1/\beta$.

The above analysis assumes $L_1 \gg cL_2$. We can make similar analysis in the case of $L_1 \ll cL_2$. It is given in the Appendix B.

4 Comparative Statistics

For the numerical calculations, the parameters are estimated as follows: $c = 0.5$ following Di Guilmi and Fujiwara, 2020; $\gamma = 0.5$ Carluccio and Bas, 2015; $\beta \in [0.7, 0.1]$ Kuroda and Yamamoto, 2007, while other parameters are calibrated. The pseudo-Phillips curve is plotted in Fig. 3, where the solid curve is the numerical solution of Eqs. (19), (21), (23) and the dashed and dotted curves are the analytical solutions Eq.(29) and Eq.(42), respectively. We find that the

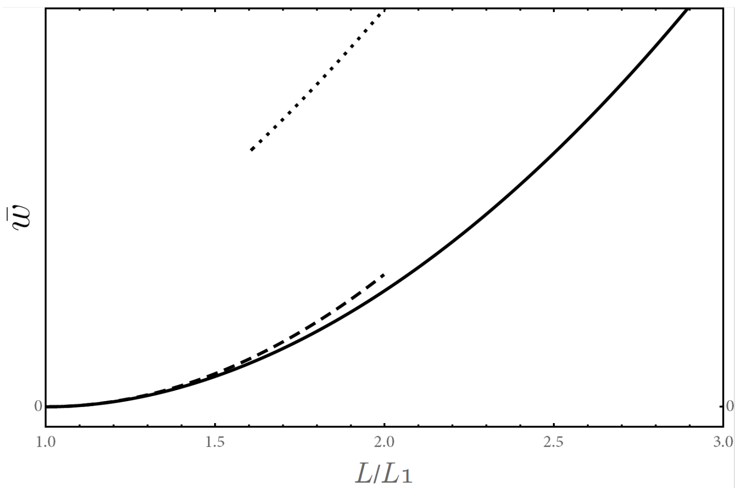


Figure 3: **The exact and approximate pseudo-Phillips curve.** The solid curve: the exact solution $\bar{w}(L)$ (Eqs. (19), (21), (23)), the dashed curve: the analytical solution in case primary workers dominate (the leading term of Eq.(29)), and the dotted curve: the analytical solution in case secondary workers dominate (the leading term of Eq.(42)). The last approximation is not valid in this range of L/L_1. The parameters are chosen to be $\alpha = c = \beta = \gamma = 0.5$.

analytical solution (29) for the case when when primary workers are dominant provides a reliable approximation for it mimics well the original function in this range of L/L_1.

The wages of primary and secondary workers,

$$\bar{w}_1 \equiv \frac{1}{L_1} \sum_{j=1}^{N} L_{1,j} w_{1,j} \tag{30}$$

and w_2 as functions of total employment L are shown in Fig. 4. It is interesting to observe that the wages of primary workers determined by Nash bargaining increase more than market–determined wages of secondary workers when the total employment increases.

Our goal is to find the answer for the question why the Phillips curve flattened in recent years. For this purpose, we explore how the slope of pseudo-Phillips curve depends on parameters c, B, γ, and β.

Change of parameter β, however, needs careful consideration: In the supply function Eq.(9), the coefficient B has a dimension that depends on β. Therefore, when changing the value of β, keeping the same numerical value of B does not make sense. One way of making clear the dimensional consideration is to set $B = L_1/w_0^{\beta}$ so that

$$L_2^{(\mathrm{s})} = L_1 \left(\frac{w_2}{w_0}\right)^{\beta}. \tag{31}$$

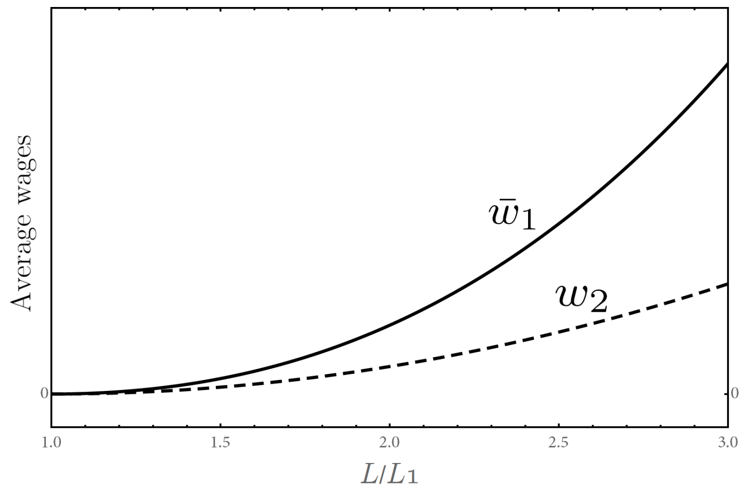


Figure 4: **The behavior of the wages $\bar{w}_1$ (solid curve) and w_2 (dashed curve).** The parameters are chosen to be the same with Fig. 3

This way, the supply function is parametrized by w_0 and β instead of B and β. In this parametrization, Eq.(23) is written as follows:

$$\bar{w} = w_0\, Z(\alpha, c, \beta, \gamma, g, v). \tag{32}$$

which makes the dimensionality trivial. Using this, when varying β, we keep w_0 constant and vary the value of β in $Z(\alpha, c, \beta, \gamma, g, v)$ in the above equation. This situation is illustrated in Fig. 5.

Now, the pseudo-Philips curve with slight change of the parameters c, γ, B and β in the manner explained above are illustrated in Fig. 6 in comparison with the pseudo-Phillips curve in Fig. 3.

Combining all the effects of changes of four parameters c, γ, B and β shown in Fig. 6, we obtain Fig. 7, where we observe quite flattening of the pseudo-Phillips curve.

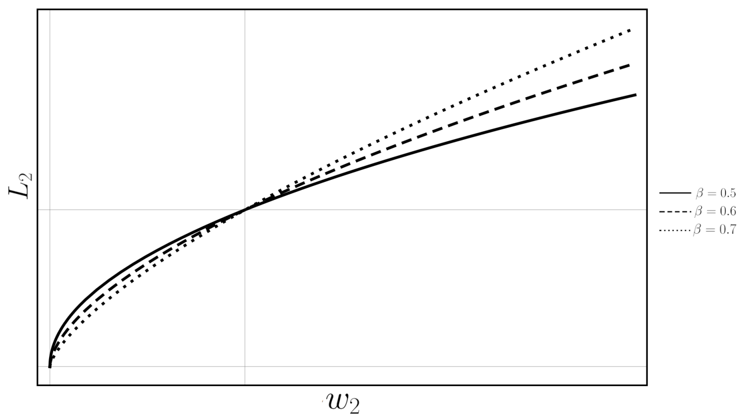


Figure 5: **The meaning of the parametrizaion of the supply function with varying β.**

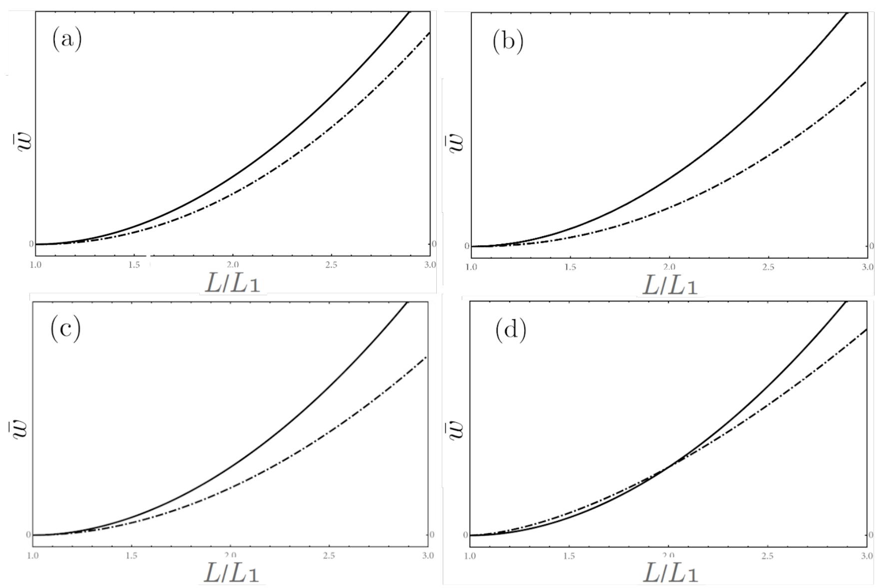


Figure 6: **The parameter dependence of the pseudo-Phillips curve.** The Solid curve is the same as in Fig. 3, while the dot-dashed curve is with the following parameter change. (a) c is increased from 0.5 to 0.9. (b) γ is decreased from 0.5 to 0.2. (c) B is increased by 20%. (d) β is increased from 0.5 to 0.6.

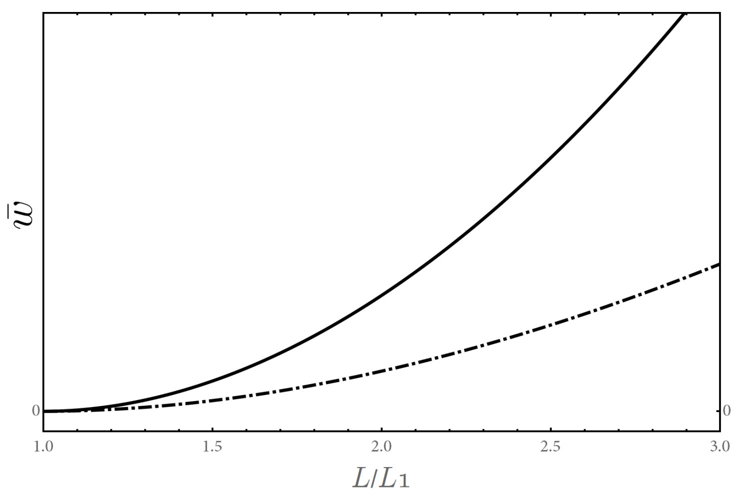


Figure 7: **Cumulative Effect of the changes of the parameters.** The parameters are $\alpha = c = \beta = \gamma = 0.5$ for the solid curve and $\alpha = 0.5,\ c = 0.9,\ \beta = 0.6,\ \gamma = 0.2$ with 20% increase in B for the dot–dashed curve. The combined effect of small parameter changes accumulate and flatten the pseudo-Phillips curve very much.

5 Concluding Remarks: Structural Changes in the Economy

The changes of parameters in the model which make the Phillips curve flatter are (1) an increase of productivity of secondary workers relative to primary workers, (2) weaker bargaining power of primary workers, (3) an increase of supply of secondary workers, and (4) an increase of wage elasticity of supply of secondary workers. These are indeed changes which occurred in the Japanese economy over thirty years.

The share of secondary or irregular workers in Japan was 15–16% during the late 1980's, but has steadily increased since then to almost 40% in 2020 (Kawaguchi and Ueno 2013; Gordon 2017). Most important is that after bubble bursted at the beginning of the 1990's, Japanese firms facing unprecedented difficulties had attempted to cut labor cost by replacing highly paid primary workers with low-wage secondary workers. More recently, post-war baby boomers had reached the age of retirement leaving primary jobs and entering secondary labor market. At the same time, more female workers had entered labor market. For these reasons, supply of secondary workers increased with high wage elasticity.

Meanwhile, after bubble bursted at the beginning of the 1990's, labor union faced a tough choice between employment and wages. It entailed weak bargaining power of primary workers. Weak bargaining power of labor union is not confined to Japan, but seems global; Stansbury and Summers 2020 reports the case for the U.S. economy.

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A Dimensional Analysis

In this appendix, we discuss the dimensions of various parameters and variables in our model.

Dimensions play important role in various fields of natural science. Basic dimensions in natural science are, Length, Weight, Time and Charge. In any equation that deals with natural quantities, the dimension of the left-hand side has to be equal to the dimension on the right-hand side. For example, "1 [in meter] = 1 [in kilogram]" does not make sense. For this reason, we often learn a lot by simply looking at the dimensions of the constants and variables. This is called "dimensional analysis".

Also, dimensionless quantities play important roles in analysis: The most famous dimensionless constant is the fine structure constant $\alpha = e^2/\hbar c = 1/137.035...$ (in cgs units), where e is the unit of electric charge, $\hbar$ is the reduced Planck's constant and c is the speed of light. As this quantity is dimensionless, α has this value, regardless of whether length is measured in meters or feet, or whether weight is measured in kilogram or pounds, and so on.

Out analysis of the model benefits greatly by the dimensional analysis also Let us examine dimensional properties of quantities in our model. We denote the dimension of the number of workers by $\mathbf{H}$, unit of value, like the dollar or yen, by $\mathbf{V}$, and time by $\mathbf{T}$.

First, the parameters c, α, β and γ are dimensionless by their definitions. Dimensions of the fundamental variables are the following:

$$\dim Y = \mathbf{V}\,\mathbf{T}^{-1}, \tag{33}$$

$$\dim L_{1,2} = \mathbf{H}, \tag{34}$$

$$\dim w_{1,2} = \mathbf{H}^{-1}\,\mathbf{V}\,\mathbf{T}^{-1}, \tag{35}$$

as Y is value created per a unit of time (yen per year, for example), $L_{1,2}$ are number of workers, and $w_{1,2}$ are value per person per time. From these, we find the following dimensions of the parameters:

$$\dim A = \mathbf{H}^{-\alpha}\,\mathbf{V}\,\mathbf{T}^{-1}, \tag{36}$$

$$\dim B = \mathbf{H}^{1+\beta}\,\mathbf{V}^{-\beta}\,\mathbf{T}^{\beta}. \tag{37}$$

The former is obtained by the requirement that dimensions of the right-hand side and the left-hand side of Eq.(2) matches, and the latter similarly from Eq.(9). From these, we find that the scaled variable v (Eq.(18)) and the parameter g (Eq.(20)) are dimensionless. For this reason, the nonlinear equation Eq.(10), which plays a central role in our model but is rather complicated, is simplified to a form much simpler and easier to analyse, Eq.(19).

B Domination of secondary workers

Large-g perturbative solution for Eq.(19) is the following:

$$v = g^{-\sigma/(1+\sigma)} \left[1 - \frac{\sigma}{1+\sigma}\, g^{-1/(1+\sigma)} + \frac{\sigma}{2(1+\sigma)^2}\, g^{-2/(1+\sigma)} + \cdots \right] \tag{38}$$

This leads to,

$$L = \frac{L_1}{c}\, g^{1/(1+\sigma)} \left[1 + \left(c - \frac{\sigma}{1+\sigma} \right) g^{-1/(1+\sigma)} + \cdots \right], \tag{39}$$

$$\bar{w} = \left(\frac{L_1}{B} \right)^{1/\beta} \frac{\alpha + \gamma - \alpha\gamma}{\alpha\, c^{1/\beta}} g^{1/(\beta(1+\sigma))} \left[1 + a_1\, g^{-1/(1+\sigma)} + \cdots \right]. \tag{40}$$

The coefficient a_1 of the non-leading term in $\bar{w}$ is a complicated function of α, β, c and γ, which is not essential for our discussion and is not given here.Perturbative solution of Eq.(39) for g is the following:

$$g = \left(c\, \frac{L}{L_1} \right)^{1+\sigma} \left[1 - \left(1 + \sigma - \frac{\sigma}{c} \right) \frac{L_1}{L} \cdots \right]. \tag{41}$$

Note that current assumption is that $L/L_1 = 1 + L_2/L_1 \gg 1$. Therefore, we find that

$$\bar{w} = \left(\frac{L}{B} \right)^{1/\beta} \frac{\alpha + \gamma - \alpha\gamma}{\alpha} \left[1 + \cdots \right], \tag{42}$$

where the non-leading term '$\cdots$' is of order of L_1/L ($\ll 1$). Again we obtain a monotonically increasing pseudo-Phillips curve, whose gradient is determined by B.

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