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FCPA and Market Quality in Emerging Economies*

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Abstract

In this paper we tried to explore the overall effects of FCPA in a corruption ridden emerging economy, where only the US firm is subject to FCPA regulations. We demonstrate the following.
(i) While an increase in fines under FCPA reduces overall corruption, it leads to a deterioration in the market quality in an emerging economy. (ii) If the products are substitutes (complements), overall corruption increases (decreases) and market quality in the emerging economy improves (deteriorates) with stricter enforcement of FCPA. (iii) An increase in the US firm's technological advantage unambiguously leads to a decrease in the market quality.

Keywords: FCPA, market quality, bribe, Cournot

JEL Classification: L11, L13, 016, 017

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1 Introduction

In emerging economies, transitioning from a traditional economy to a modern market economy, many economic impediments have existed since the pre-modern days. Examples are state-sanctioned monopolies and entry restrictions. These countries traditionally had incompetent and corrupt bureaucracies, which required businesses to deal with enormous amounts of red tape. In such economies, "political contributions," or, more negatively put, "bribes," were often effective tools to reduce economic impediments.

Even today, this feature exists and doing any kind of business without bribery in these countries is extremely difficult and this poses a significant legal and economic risk for corporations operating in these countries. The *Foreign Corrupt Practices Act* (FCPA) is an American act enacted in 1977 to curb such practices. It tries to prevent corrupt payments to foreign government officials and others considered to be 'instrumentalities of a foreign government'. FCPA applies to US companies and institutions, as well as foreign companies, institutions or persons with a nexus to the United States, and their affiliates.

However, in an emerging economy, laws such as the FCPA could delay the removal of remaining economic impediments, thereby preserving economic inefficiency and unfairness. In this paper, we study this aspect of foreign anti-corruption policy from the viewpoint of market quality theory, under which market quality is defined as a measure for efficiency in allocation and fairness in trading.

Provisions under FCPA The FCPA contains both antibribery prohibitions and accounting requirements. Under anti-bribery provisions the FCPA criminalizes providing anything of value to a foreign official corruptly to influence an official act or secure an improper advantage in order to obtain or retain business. Under accounting provisions the FCPA requires companies whose stock is traded on a U.S. exchange to make and keep accurate books and records and devise and maintain appropriate internal controls. This is designed to prevent accounting practices that hide corrupt payments and ensure that shareholders and the 'Securities and Exchange Commission' (SEC) have an accurate picture of a company's finances. The 'Department of Justice' (DOJ) and the 'Securities and Exchange Commission' (SEC) enforce the FCPA. The DOJ, SEC, and FBI all have specialized units dedicated to investigating and prosecuting FCPA violations. An entity that violates the FCPA can be subject to criminal charges by the DOJ, in addition to penalties by the SEC. Such criminal or civil penalties can be imposed on companies and individuals, including prison time for individuals.[1]

Does the FCPA work? A natural question that arises is the following. Does the FCPA work? As with any regulation, the answer to this question depends on the measuring stick used to evaluate the act. And, as expected, the jury is still out in this matter. Survey data and other evidences suggest that business opinions about the FCPA are quite divided regarding its impact (see Perlman and Sykes, 2018). From the perspective of curbing corruption, the FCPA seems to have worked (see Lippit, 2013). From the perspective of encouraging American business growth in foreign markets, it has probably been a hindrance (see Reich, 2016 and Miron, 2012).[2]

It may be noted that bribery in many developing economies does not just grease the palms, but also moves the wheels of commerce (without bribery things simply do not get done). Many economists are of the opinion that bans on bribery are effective only when they are more widespread. Unilateral enforcement laws, like the FCPA, loose their effect, when other countries fail to enforce similar bans.[3]

As a result, while U.S. multinationals cannot bribe (or they bribe to a much lesser extent) because they are scared of FCPA enforcement and the fines that come with enforcement, everyone else in the global market bribes. This puts, as many analysts claim, the U.S. firms at a competitive disadvantage because they are competing with companies in the global market place who can and often do bribe.

However, other scholars take a different view. For example, the findings in Lippit (2013) do not provide support to the hypothesis that FCPA has harmed U.S. business abroad. The paper by

[1]A violation of the FCPA consists of a payment, offer, authorization, or promise to pay money or anything of value: (i) to a foreign government official (including a party official or manager of a state-owned concern, or other instrumentality of a foreign government), or to any other person, knowing that the payment or promise will be passed on to a foreign official or instrumentality with a corrupt motive for the purpose of influencing any act or decision of that person, (ii) inducing such a person to do or omit any action in violation of his lawful duty, securing an improper advantage, or (iii) inducing such a person to use his influence to affect an official act or decision in order to assist in obtaining or retaining business for or with, or directing any business to, any person.

[2]Some scholars argue that FCPA harms US business interests, especially when it reduces bribes, because much bribery is an attempt to get around laws that make little sense in the first place. These laws may have good intentions, but they are frequently so onerous that their main effect is to discourage business activities and this leads to corruption. It is impossible to do business in some countries without paying bribes. (see Miron, 2012)

[3]Today, some of the biggest players in the global market, including BRICS nations like India, do not have effective anti bribery laws. Major global economic players like China and Russia have laws, but these laws are not enforced properly.

Perlman and Sykes (2018) suggests several reasons why the adverse effects of FCPA enforcement on U.S. business may be considerably smaller than some FCPA critics suggest, and why significant numbers of U.S. firms may actually benefit from enforcement.

Effects of FCPA in an emerging economy Surprisingly, analysis of the effects of FCPA has received scant attention in the theoretical economics literature (except Arbatskaya and Mialon, 2020). We would like to fill this gap and explore the overall effects of FCPA in an emerging economy which, due to historical and cultural factors, suffers from endemic corruption. More specifically, we will explore the consequences of the following: (i) an increase in FCPA penalty, (ii) more stringent enforcement of FCPA, (iii) an increase in US firm's technological advantage and (iv) deterioration in the quality of information.

We try to answer these questions by using a differentiated product two-stage duopoly model (with one US firm and one local firm). In our set-up only the US firm is subject to FCPA regulations. The local firm is not subject to FCPA as it has no US connections. This is often the case in many emerging economies (for example, Amazon and Big Basket in India).

Our analysis provide some definitive answers. (i) While an increase in fines under FCPA reduces overall corruption, it leads to a deterioration in the market quality in an emerging economy. (ii) If the products are substitutes (complements), overall corruption increases (decreases) and market quality in the emerging economy improves (deteriorates) with stricter enforcement of FCPA. (iii) In the presence of FCPA an increase in the US firm's technological advantage leads to a decrease in the market quality in an emerging economy. (iv) While, unlike the previous propositions, we do not have very definitive and sharp results regarding the effects of poorer quality of information on the market quality, we can show that if the market size is large enough then with substitutes (complements) expost market quality will increase (decrease) with poorer quality of information under some conditions. In short, our results indicate that while overall corruption may come down (in some cases) in the developing country; adverse effects on the market quality are quite likely.

We now proceed to say a few words on increasing penalties and enforcements under FCPA and technological advantage of the US firms.

1.1 FCPA enforcement and technological advantage of US firms: Some stylized facts

Since the launch of FCPA the U.S. enforcement authorities have charged and prosecuted a number of corporations for bribing non-U.S. officials especially in emerging and middle income countries. It is also to be noted that many US firms have huge technological advantages. We provide below some stylized facts.

1.1.1 FCPA Penalty

The magnitude of FCPA enforcement penalties has grown dramatically over time, with negligible monetary penalties imposed in the early years and a huge uptick beginning roughly a decade ago. Many firms now have to pay huge fines as a penalty for corrupt practices. Aggregate total sanctions are now approaching $11 billion (see Perlman and Skykes, 2018). [4][5]

The magnitude of FCPA enforcement penalties has grown dramatically over time, with negligible monetary penalties imposed in the early years and a huge uptick beginning roughly a decade ago. The table below provides a snapshot of the average penalty imposed under FCPA during the period

[4] See <<https://www.lexisnexis.com/communities/lexisnexis_biz/b/bizblog/archive/2018/01/11/six-lessons-from-2017-fcpa-enforcement-actions.aspx>>

[5] Even doing business with the government, though lucrative, can end up being an FCPA nightmare, too. Massachusetts-based medical manufacturer Alere paid a price because of this in 2017. The company's Indian subsidiary, in 2011, won a contract to provide malaria testing kits to a local governmental entity for a national disease control programme. But it came with an ask of 4 percent commission. Keen to get increased orders under the tender—from 200,000 to 1,000,000 testing kits—Alere India approved the commission, which led to an about $150,000 in profit for the parent. Consequently, Alere agreed to pay more than $13 million as settlement charges for accounting fraud and improper payments to government officials in several countries, including India. See <<https://www.bloombergquint.com/law-and-policy/how-us-companies-are-navigating-the-fcpa-risk-in-india>>

2015-2019.[6]

Year	Average Penalty
2015	$5,376,833
2016	$43,516,771
2017	$51,368,779
2018	$44,321,886
2019	$116,044,004

The top three FCPA fines of all time have been imposed in the last few years[7]:

1. Airbus SE (Netherlands/France): $2.09 billion in 2020
2. Telefonaktiebolaget LM Ericsson (Sweden): $1.06 billion in 2019
3. Petroleo Brasileiro S.A. – Petrobras (Brazil): $1.78 billion in 2018

1.1.2 Stringency of FCPA enforcement

Apart from the increase in fines, in recent years, the FCPA has been enforced more stringently. It may be noted that the first enforcement action under FCPA came in 1978, with only 52 actions by the end of 2000.

Enforcement efforts expanded rapidly thereafter. The substantial uptick in enforcement began during the George W. Bush administration and accelerated during the Obama administration, with 2016 seeing more enforcement actions by DOJ and SEC against companies and individuals (61) than any prior year except 2010.

Enforcement tailed off somewhat during the first year (2017) of the Trump administration (see Perlman and Skykes, 2018). However, there has been steady rise in enforcements since then. Enforcement actions initiated against companies and individuals by DOJ and SEC combined numbered 41 in 2017, 43 in 2018 and 49 in 2019.[8]

[6] Source:<<https://www.willkie.com/-/media/pwa/articles/cles/20200109-fcpa-year-in-review-2019/handout–fcpa-2019-year-in-review.pdf>>

[7] see <<https://www.refinitiv.com/perspectives/financial-crime/fcpa-fines-show-bribery-and-corruption-risk/>>

[8] See <<https://www.willkie.com/-/media/pwa/articles/cles/20200109-fcpa-year-in-review-2019/handout–fcpa-2019-year-in-review.pdf>>

1.1.3 Technological advantage of US firms

It may be noted that many US firms (like Apple, Google or Amazon) are very likely to have a very *substantial* and *growing* technological advantage (as compared to any local firm in an emerging economy).

The US firms, because of their size, innovation, and market position, have a number of competitive advantages, that translates into cost advantage. Scale enables these companies to have more data, attract more capital, enjoy stronger network effects, and reach tipping points, giving them an extraordinary advantage over rivals, especially those that are smaller in size. For example, during the COVID crisis Amazon reached dizzy heights.

We now proceed to say a few words on the concept of "market quality".

1.2 Market Quality

'Market quality' is a relatively new concept that has not been part of conventional economics. It is essentially a twenty-first century idea. It is not surprising that initially one may struggle to understand exactly what is meant by market quality. However, common sense suggests that there are bad markets and good markets. Yano (2008b, 2009, 2016) defines 'market quality' as a measure of "efficiency in allocation" and "fairness in pricing" in a market.[9]

Efficiency refers to Pareto efficiency. Fairness may be stated as fairness in dealing or in the process in which the terms of trade are formed. A price formed through fair dealing is a fair price.[10]

Fair dealing should be measured against a set of rules and laws imposed so as to maintain the well functioning of a market. According to Yano (2008a), one such rule may be the non-discriminatory treatment of actual and potential trading partners or, in other words, to ensure free entry and exit in the market. However, such fairness cannot be guaranteed when agents have the powers (for instance, the ability to use unfair means like payment of bribes) enabling them to change the payoff structure. This is often observed in emerging economies (see Dastidar and Yano, 2020).

[9] Dastidar and Dei (2014) provides a short introduction to "market quality economics". Dastidar (2017) provides more details and many theoretical results.

[10] "Actions in a particular market are competitively fair if they are conducted in compliance with the set of "generally accepted" rules. Moreover, a state of that market is competitively fair if it is formed through competitively fair actions and if there are no profit opportunities left available for competitively fair actions" (Yano, 2009).

Yano (2009, 2016) also conjectured that poorer quality of information and competition lead to a deterioration of market quality. Broad pattern of historical events seem to support this idea.[11]

In our exercise we define the total bribe in the system, $\mathbb{C}$, to be the 'corruption index'. Fairness, ϕ, is defined to be $\phi = -\mathbb{C}$. In other words, fairness is the opposite of the 'corruption index' (payment of bribes by one agent adversely affects its rivals and consequently, fairness is compromised). Following Yano (2009, 2016) we define market quality in the developing country to be a sum of 'total social surplus' and 'fairness'. That is, we define market quality in the developing country to be $\mathbb{Q} = [\textit{total social surplus}] - [\textit{corruption index}]$.

We now provide a brief review of the literature related to our analysis.

1.3 Related Literature

Surprisingly, there are very few papers in the economics literature that analyze the effects of FCPA. Most papers related to FCPA have been published in law journals where the analysis is from a legal viewpoint.

The paper by Geo-JaJa and Mangum (2000) analyzes the experience of foreign firms doing business in Nigeria and shows that there is no evidence that the enforcement of FCPA has impeded the growth of US trade.

Lippit (2013) undertakes an empirical analysis to analyze the relationship between the incidence of prosecuted FCPA violations, on the one hand, and corruption growth and U.S. foreign direct investment growth, on the other. The results in this paper appear to be consistent with the idea that the FCPA enforcement has been beneficial in the global fight against corruption. The empirical data seem to suggest that countries with greater numbers of prosecuted FCPA violations also tend to be those where people perceive corruption to be declining.

[11] One of the things that support this hypothesis is that a series of industrial revolutions and economic crises over the past two hundred years, tend to have a cyclical pattern and all such events were probably triggered by changes in market quality. The First Industrial Revolution gave rise to the exploitation of industrial workers, a major labor issue. The Second Industrial Revolution was followed by the formation of industrial monopolies, the Great Depression, and massive unemployment. The exploitation of workers and the monopolization of industries occurred because competition was imperfect, and the Great Depression occurred because information was not properly shared. The subprime loan crisis of 2008 was a result of poor quality information and greed that compelled people to take on debts they could never repay. There seems to be a common pattern of events. The advent of technological innovation is typically followed by a decline in the quality of competition and information, and this reduced market quality and this turn led to the economic crisis.

The article by Perlman and Sykes (2018) suggests several reasons why the adverse effects of FCPA enforcement on U.S. business may be considerably smaller than some FCPA critics suggest, and why significant numbers of U.S. firms may actually benefit from enforcement. This conclusion finds support in Congressional testimony, business surveys, and interviews with prominent FCPA practitioners and compliance officers.

To the best of our knowledge, the only theoretical paper in economics that analyses FCPA is Arbatskaya and Mialon (2020). This paper uses a contest model of competition between a U.S. multinational firm and a competitor for a government contract in a host country. Firms can increase their chances of winning the contract through two activities, productive investment and bribery. If the FCPA only applies to the U.S. firm, it reduces that firm's competitiveness and either increases bribery by the foreign firm or reduces overall investment. If the FCPA also applies to foreign firms, it reduces bribery, and in host countries with high corruption levels, it increases investment. It may be noted that our set-up FCPA applies to only the US firm. Our model and the results are very different from this paper and we address a different set of issues (namely how FCPA penalty, stringency of application of this act and US firm's technological advantage affect market quality).

We now proceed to discuss in brief the set-up and the main results of our exercise.

1.4 Our set-up

As noted earlier, we would like to explore whether FCPA adversely affect the 'market quality' and the profits of the firms in a corruption ridden emerging economy. To do this we analyze a differentiated product duopoly in an emerging economy. In our set-up firm 1 is from USA and firm 2 is a local firm that operates in this market. Only the US firm is subject to FCPA regulations. The local firm is not subject to FCPA as it has no US connections. This is often the case in many emerging economies (for example, Amazon and Big Basket in India). In our model γ is the differentiation parameter ($\gamma > 0$ means that products are substitutes and $\gamma < 0$ means that products are complements).

The US firm has a cost advantage (indexed by c) due to its superior technology. However, both firms can reduce their marginal costs by paying bribes. If a firm does not pay any bribe, its marginal cost will be high as a corrupt bureaucrat or politician will make life difficult for it. The bureaucrat or politician can create regulatory hurdles for firms in a market, including controls by licensing authorities, health and building inspectors and planning boards. This scenario is very common in a country like India.[12]

We capture this phenomenon in the following way. If the US firm pays a bribe $b_1 \geq 0$ its marginal cost is $\lambda(b_1)$, where $\lambda(.)$ is strictly decreasing. If the local firm pays a bribe $b_2 \geq 0$ its marginal cost is $[c + \lambda(b_2)]$. Note that in the absence of any bribe (i.e. $b_1 = b_2 = 0$), the local firm's marginal cost is higher than the US firm's marginal cost by an amount c. The term $c \geq 0$ is the index of the US firm's cost advantage.

Note that in order to constitute an FCPA violation, a payment must be intended to cause an official to take an action or make a decision that would benefit the payer's business interest. Clearly, a bribe to reduce costs is an example of such a payment. Because of FCPA, the US firm faces a positive probability of getting caught, ρ, and if it is caught it has to pay a total penalty, kB (as per FCPA regulation) in US. Here B is the basic penalty and k is the intensity with which FCPA rules will be applied. The probability of getting caught, ρ, is an increasing function of bribe that it pays. If the amount of bribe paid is higher, it becomes easier for the DOJ (or the SEC) to detect the corruption and prove the wrongdoing.

remark 1 *The magnitude of k depends on the institutional and political features of US system. Low k means that FCPA rules are not strictly enforced. Firm 1, being the US firm, has a much better knowledge (as compared to firm 2) of the US system and knows more about k. Consequently, the value of k is private information to firm 1.*[13]

Firm 2 is a local firm with no US connection. Hence, FCPA does not apply to this firm. Also, its corrupt practices (payment of bribes) does not get punished by the government in the emerging economy. This is due to extremely poor law enforcement in such an economy (for example, India, Pakistan, Bangladesh) and the dismal state of the criminal justice system there. Since the US firm is the only firm that pays the penalty (with probability ρ), it has a disadvantage vis-a-vis the local firm.

As noted before, in our model the value of k is private information to firm 1. 2 believes $k \in \left[\hat{k} - \varepsilon,\ \hat{k} + \varepsilon\right]$ with uniform distribution, where $\varepsilon \in \left(0, \hat{k}\right)$.

[12] Note that in emerging economies such bribery is often an attempt to get around antiquated laws. Such laws include barriers to entry, union protections that make firing or plant closures all but impossible, and excessive environmental, health and safety regulation.

[13] Another interpretation of k may be the following: ρk is the probability of getting caught. While ρ is publicly known, k is known only to firm 1 as it knows the US system better.

remark 2 *The basic penalty, B, is the index of FCPA and $\hat{k}$ is the average strictness with which FCPA rules will be applied. The term ε captures the uncertainty part. We may say that ε is an index of the 'quality of information'. Lower is ε, better is the 'quality of information'.*

We consider a two-stage game. In the first stage firms simultaneously choose bribes, b_1 and b_2. Since in this stage firm 1 knows k but firm 2 does not know it; we have an incomplete information game. In the second stage the firms first observe the choices made in the previous stage and then simultaneously choose quantities. Since the bribes paid are revealed, both firms come to know of each others' marginal costs. Consequently, firms in this stage play a complete information Cournot duopoly game.

1.5 Summary of the main results

We now provide a summary of the main results below.

1. In section 1.1 we noted that the amount of FCPA enforcement penalties has grown dramatically over the last decade. In our model B denotes the FCPA penalty. When goods are substitutes, we show that while a higher FCPA penalty (B) reduces corruption index ($\mathbb{C}$) in the emerging economy, the overall market quality ($\mathbb{Q}$) decreases especially when the US firm has a substantial cost advantage. Typically, the US firm is expected to have a large technological advantage. In such a scenario, with an increase in FCPA penalty, the US firm's (firm 1's) output and profit decreases but the local firm's (firm 2's) output and profit goes up. However, the increase in 2's output is not large enough and total output decreases. Consequently, consumer surplus and total surplus goes down. This results in a decrease in market quality. When goods are complements, the results are very similar. *This clearly shows that while an increase in fines under FCPA reduces overall corruption, it leads to a deterioration in the market quality in an emerging economy.*

2. In section 1.1 we also noted that in recent years, the FCPA has been enforced more stringently. In this context we investigate the effects of a higher $\hat{k}$ (i.e. stricter enforcement of the FCPA on an average). Unlike the previous case, the effects of an increase in $\hat{k}$ are sensitive to whether goods are substitutes or complements. With substitute goods we show the following. If $\hat{k}$ increases, firm 1 (the US firm) bribes less but firm 2 (the local firm) bribes more and total bribe goes up (the corruption index, $\mathbb{C}$, increases). Firm 1's output goes down while firm 2's output goes up. If the market size of the emerging economy is large enough (which is

the case in India or Brazil, for instance) total output sold in the emerging economy increases and this pushes up consumer surplus. If the market size of the emerging economy is large enough, then firm 1's profit decreases but 2's profit increases. This leads to an increase in market quality (market quality in the merging economy does not depend on firm 1's profit). Hence, when goods are substitutes, the impact of a stricter enforcement of FCPA (higher $\hat{k}$) is quite different from the impact of an increase in the FCPA penalty (higher B). We also demonstrate that results would be very different if goods are complements. In this case, an increase in $\hat{k}$ results in a decrease in overall corruption, $\mathbb{C}$, but market quality, $\mathbb{Q}$, also declines. *In short, if the products are substitutes (complements), overall corruption increases (decreases) and market quality in the emerging economy improves (deteriorates) with stricter enforcement of FCPA.*

3. We have noted before that the US firms, because of their size, innovation, and market position, have a number of competitive advantages, that translates into cost advantage. In our model, the technological advantage of the US firm is indexed by c. With substitute products we show that when the market size in the emerging economy is large enough, if c increases then firm 1 (the US firm) bribes more but firm 2 (the local firm) bribes less. The US firm's output and profit goes up, the local firm's output and profit goes down. If the degree of substitutability is small enough, then both overall corruption, $\mathbb{C}$, and total output decline. However, with a rise in c, the market quality unambiguously decreases. In fact, the decline in market quality will be more precipitous if the penalty, B, is higher. When goods are complements, the results are very similar. *In short, in the presence of FCPA (high penalty), any increment in the technological advantage of the US firm reduces the market quality in the developing country.*[14]

4. In our model, ε is the index of 'quality of information'. Higher is ε, poorer is the quality of information. However, our results regarding the effects of an increase in ε are not as sharp and as unambiguous as the earlier results. With some restrictions on the curvature of $\rho(.)$ and $\lambda(.)$ we demonstrate the following. With substitute products, if the market size 'a' is large enough, then expost overall corruption and total output sold will increase and market quality will improve with an increase in ε. The intuition behind this is as follows. Although corruption increases in the emerging economy with a decline in the quality of information,

[14] Note that our results hold true for any non-negative level of technological advantage, c. This means in the presence of FCPA, starting at c close to zero (almost no technological advantage of the US firm), any increase in c, will lead to a decrease in market quality.

total output increases and this leads to an increase in consumer surplus. Profit of the local firm also increases. The last two effects dominate the negative effect of a rise in corruption and this results in an improvement in expost market quality. This result goes somewhat against Yano (2009, 2016) who conjectured that poorer quality of information will lead to lower fairness (more corruption in our set-up) and lower market quality. Surprisingly, in our exercise, when products are substitutes, while poorer quality of information results in more corruption (lower fairness), it also leads to better market quality. However, when products are complements, poorer quality of information leads to lower corruption and also leads to an inferior market quality. With complements our findings support the Yano (2009, 2016) conjecture.

5. Our results seem to suggest the following intriguing phenomenon and this goes against conventional wisdom. *Higher (lower) corruption may lead to higher (lower) market quality.* For instance, any increase in FCPA penalty, B, reduces overall corruption index in the emerging economy but the market quality also deteriorates. A stricter enforcement of the FCPA (higher $\hat{k}$) pushes up overall corruption but market quality also improves. In the presence of FCPA, a higher technological advantage of the US firm (larger c) leads to a decrease in both overall corruption and the market quality. A possible intuition behind this phenomenon is as follows. Note that market quality, $\mathbb{Q} = [\textit{total social surplus}] - [\textit{corruption index}]$. The direct effect of a decrease in corruption is to improve the 'market quality' in the economy. However, there is also an indirect effect. Lower corruption (bribes) increases marginal costs and this leads to a decrease in a firm's output and profit. This reduces total surplus and this in turn leads to a decrease in market quality. If the indirect effect dominates, then lower corruption (as induced by a higher penalty, B) leads to a decrease in market quality. Similarly, higher overall corruption (as induced by a stricter enforcement of FCPA i.e. higher $\hat{k}$) leads to an improvement in market quality.

1.5.1 Plan of the paper

In section 2 we provide the model of our exercise. In section 3 we provide the solution to the two-stage duopoly game. Section 4 provides the preliminary results and section 5 gives the major results (for substitute products). In section 6 we extend our analysis to the case where the goods are complements. Section 7 provides some concluding remarks. There are two appendices at the end. In appendix A we report some equations and cross partials. This will help us to prove our results. Appendix B provides all the proofs.

We now proceed to provide the model and the details of all our findings.

2 Model

There are two firms, 1 and 2. Firm 1 is the foreign firm (from USA). Firm 2 is the local firm (in a developing country, say Philippines or India or Brazil). Only the US firm is subject to FCPA regulations. The local firm is not subject to FCPA as it has no US connections. Both firms operate in a differentiated product market in the developing country.

There is endemic corruption in the emerging economy and each firm can reduce its marginal cost by paying a bribe. Firm 1 pays b_1 and firm 2 pays b_2.

Firm 1 has cost advantage (denoted by c), due to its superior technology. Higher is c, higher is the cost advantage. Firm 1 faces a positive probability of getting caught, ρ, and if it is caught paying bribes it has to pay a total penalty, kB. (as per FCPA regulation) in US. Here B is the basic penalty and k is the intensity with which FCPA rules will be applied. The probability of getting caught, ρ, is an increasing function of bribe that it pays.

As noted before k is private information to firm 1. That is, firm 1 knows k but firm 2 does not. 2 knows that higher is k, stricter will be the enforcement of FCPA rules but does not know the exact value of k. 2 believes $k \in \left[\hat{k} - \varepsilon,\ \hat{k} + \varepsilon\right]$ with uniform distribution, where $\varepsilon \in \left(0, \hat{k}\right)$.

As noted before, $\hat{k}$ is the *average strictness* with which FCPA rules will be applied. The term ε captures the uncertainty part. ε is an index of the 'quality of information'. Lower is ε, better is the 'quality of information'. The basic penalty, B, can be interpreted as proxy for 'FCPA regulation'.

Firm 1's cost function is as follows.

$$\begin{aligned} C_1(q_1) &= \lambda(b_1) q_1 + \underbrace{b_1}_{\text{bribe paid by 1}} + \underbrace{\rho(b_1) kB}_{\text{expected total penalty}} \\ \rho(b_1) &= \text{probability of getting caught} \\ kB &= \text{total penalty to be paid if caught} \end{aligned}$$

Firm 2's cost function is as follows.

$$C_2(q_2) = [c + \lambda(b_2)] q_2 + \underbrace{b_2}_{\text{bribe paid by 2}}$$

As noted before, the term c captures firm 1's *technological advantage* vis-a-vis firm 2.

We assume the following:

Assumption 1 $\lambda'(.) < 0,\ \lambda''(.) > 0.$

Assumption 2 $\rho(0) = 0,\ \exists\bar{b}$ s.t. $\rho(b) = 1\ \forall b \geq \bar{b}$, $\forall b \in \left(0, \bar{b}\right), \rho'(b) > 0$ and $\rho''(.) \geq 0$.

remark 3 *Marginal cost for each firm is strictly decreasing in the amount of bribe it pays. However, the rate of decrease peters off with each additional unit of bribe. For firm 1 the probability of getting caught increases with increases in the bribe amount as it becomes easier for the DOJ (or SEC) to prove the corrupt deed. If bribe paid by firm 1 exceeds a certain amount,* $\bar{b}$*, then firm 1 will get caught with certainty. Hence, in equilibrium bribe paid by firm 1 will always remain below* $\bar{b}$*.*

In the emerging economy the firms compete in a differentiated product market. For this we consider a representative consumer's utility function based on Dixit (1979). Scores of papers in the literature have used this.[15]

On the demand side of the market, the representative consumer's utility function of two differentiated products, q_1 and q_2, and a numeraire good, q_0, is given by the following:

$$U = a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0.$$

The parameter γ measures the degree of product differentiation and $\gamma \in [-1, 1]$. When $\gamma < 0$ the goods are complements and when $\gamma > 0$ the goods are substitutes. Note that when γ is unity then the products are homogeneous (perfect substitutes) and when γ is zero the products are independent. We will consider cases where $\gamma \neq 0$.

The utility function generates the following system of inverse demand functions:

$$\begin{aligned} p_1 &= a - q_1 - \gamma q_2 \\ p_2 &= a - \gamma q_1 - q_2 \end{aligned}$$

Note that 'a' is a proxy for market size in the developing country. We assume that 'a' is high enough. In particular, we assume the following.

[15] A small sample of such papers is as follows: Singh and Vives (1984), Hackner (2000), Bester and Petrakis (1993), Zanchettin (2006), Pal (2010), Alipranti, Milliou and Petrakis (2014) and Dastidar (2015).

Assumption 3 $a > 2[c + \lambda(0)] + 2\frac{[\lambda'(b)]^2}{\lambda''(b)}$ for all $b \in (0, \bar{b})$.

That is, the market in the developing country is very lucrative and firm 1 (the US firm) continues business here despite the corrupt system. A country like India or Brazil would be an example of such a developing country. We now provide an example where all our assumptions are satisfied.

Example Let $\lambda(b) = \frac{1}{1+b}$ and $\rho(b) = hb$ where $h > 0$. Note that $\lambda'(b) = -\frac{1}{(1+b)^2}$ and $\lambda''(b) = \frac{2}{(1+b)^3}$. Clearly $\lambda'(b) < 0$, $\lambda'' > 0$, $\rho'(b) > 0$, $\rho''(b) = 0$. This means assumptions 1 and 2 are satisfied. Also, $\bar{b} = \frac{1}{h}$. Note that by choosing h small enough we can make $\bar{b}$ arbitrarily large. This will ensure that in any equilibrium the bribe chosen by firm 1 is strictly less than $\bar{b}$. Also note that any $a > 2c + 4$ will satisfy assumption 3.

2.1 Two stage game

We consider a two-stage game.

1st stage: Firms simultaneously choose bribes, b_1 and b_2. 1 knows k but 2 does not know it. It only knows that $k \in \left[\hat{k} - \varepsilon,\ \hat{k} + \varepsilon\right]$ with uniform distribution where $\hat{k} > 0$ and $\varepsilon \in \left(0, \hat{k}\right)$. Note that b_1 and b_2 determine the marginal costs of firms 1 and 2 respectively.

2nd stage: The firms observe the choices made in the previous stage and then choose quantities simultaneously. Bribes were paid in the first stage and these become known to the stakeholders. That is, both firms come to know each others' marginal costs. Firm 1's marginal cost is $\lambda(b_1)$ and firm 2's marginal cost is $[c + \lambda(b_2)]$. Firms play a complete information Cournot duopoly game in the second stage.

Note that since b_1 and b_2 are chosen in the first stage, $[b_1 + \rho(b_1) kB]$ is like a sunk cost for firm 1 in the second stage. Similarly, firm 2's sunk cost is b_2. When firms choose quantities in this stage, these sunk costs do not matter.

3 Analysis of the two stage game

We now proceed to solve the two-stage game.

3.1 2nd stage

Note that the firms play a complete information quantity choice game in the second stage. The firms equilibrium Cournot outputs are as follows:

$$\begin{aligned} q_1^c &= \frac{a(2-\gamma) - 2\lambda(b_1) + \gamma(c + \lambda(b_2))}{4-\gamma^2} \\ q_2^c &= \frac{a(2-\gamma) + \gamma\lambda(b_1) - 2(c + \lambda(b_2))}{4-\gamma^2} \end{aligned}$$

Our assumptions ensure that $q_1^c,\ q_2^c > 0$ (interior equilibrium).

Routine computations show that the firms equilibrium profits are as follows:

$$\begin{aligned} \pi^1 &= [q_1^c]^2 - b_1 - \rho(b_1) kB \\ &= \left[\frac{a(2-\gamma) - 2\lambda(b_1) + \gamma(c + \lambda(b_2))}{4-\gamma^2}\right]^2 - b_1 - \rho(b_1) kB \end{aligned}$$

$$\begin{aligned} \pi^2 &= [q_2^c]^2 - b_2 \\ &= \left[\frac{a(2-\gamma) + \gamma\lambda(b_1) - 2(c + \lambda(b_2))}{4-\gamma^2}\right]^2 - b_2 \end{aligned}$$

We also assume that in equilibrium $\pi^1,\ \pi^2 > 0$. It may be noted that if the market size, 'a', is sufficiently high, positive profits will be assured for both.

3.1.1 First stage

In this stage k is private information to firm 1. Firm 2 does not know that value of k. It only knows that $k \in \left[\hat{k}-\varepsilon,\ \hat{k}+\varepsilon\right]$ with uniform distribution where $\hat{k} > 0$ and $\varepsilon \in \left(0, \hat{k}\right)$. In this stage an incomplete information game is played where firms simultaneously choose bribes, b_1 and b_2. Firm 1's strategy is a mapping

$$b_1(.) : \left[\hat{k}-\varepsilon,\ \hat{k}+\varepsilon\right] \longrightarrow [0, \infty).$$

Firm 2's strategy is to choose $b_2 \geq 0$.

Bayesian-Nash Equilibrium Let the Bayesian-Nash equilibrium be given by: $\{b_1(k);\ b_2^*\}$. At this equilibrium, given b_2^*, firm 1 (for each type k) chooses b_1 to maximize π^1.

Given $b_1(k)$ firm 2 chooses b_2 to maximize its expected payoff

$$\begin{aligned} E^2 &= \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^2\right) \frac{1}{2\varepsilon} dk \\ &= \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left[\left\{\frac{a(2-\gamma) + \gamma\lambda(b_1(k)) - 2(c + \lambda(b_2))}{4-\gamma^2}\right\}^2 - b_2\right] \frac{1}{2\varepsilon} dk \end{aligned}$$

At the Bayesian-Nash equilibrium we have

$$\begin{aligned}
\pi^1_{b_1} &= \frac{\partial \pi^1}{\partial b_1} \\
&= -\frac{4\lambda'\left(b_1\left(k\right)\right)\left[a\left(2-\gamma\right)-2\lambda\left(b_1\left(k\right)\right)+\gamma\left(c+\lambda\left(b_2^*\right)\right)\right]}{\left[4-\gamma^2\right]^2}-1-\rho'\left(b_1\left(k\right)\right)kB=0
\end{aligned}$$

$$\begin{aligned}
E^2_{b_2} &= \frac{\partial E^2}{\partial b_2} \\
&= -\frac{4\lambda'\left(b_2^*\right)\left[a\left(2-\gamma\right)+\gamma\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\lambda\left(b_1\left(k\right)\right)\frac{1}{2\varepsilon}dk-2\left(c+\lambda\left(b_2\right)\right)\right]}{\left[4-\gamma^2\right]^2}-1=0
\end{aligned}$$

The 2OCs are as follows:

$$\begin{aligned}
\pi^1_{b_1b_1} &= \frac{\partial^2 \pi^1_{b_1}}{\partial b_1} \\
&= -\frac{4\lambda''\left(b_1\right)\left[a\left(2-\gamma\right)-2\lambda\left(b_1\right)+\gamma\left(c+\lambda\left(b_2\right)\right)\right]}{\left[4-\gamma^2\right]^2}+\frac{8\left[\lambda'\left(b_1\right)\right]^2}{\left[4-\gamma^2\right]^2}-\rho''\left(b_1\right)kB<0
\end{aligned}$$

$$\begin{aligned}
E^2_{b_2b_2} &= \frac{\partial E^2_{b_2}}{\partial b_2} \\
&= -\frac{4\lambda''\left(b_2\right)\left[a\left(2-\gamma\right)+\gamma\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\lambda\left(b_1\left(k\right)\right)\frac{1}{2\varepsilon}dk-2\left(c+\lambda\left(b_2\right)\right)\right]}{\left[4-\gamma^2\right]^2}+\frac{8\left[\lambda'\left(b_2\right)\right]^2}{\left[4-\gamma^2\right]^2}<0
\end{aligned}$$

Note that if the market size, 'a', is *high enough* then the 2OCs will hold. In fact, assumption 3 ensures that the 2OCs will always hold. This is shown in Appendix-A.

Some cross partials and an additional assumption Note that from equations $\pi^1_{b_1}=0$ and $E^2_{b_2}=0$ we get, implicitly, $b_1(k)$ and b_2^*. We will later use the implicit function theorem for a system of equations to conduct a series of comparative static exercises. In appendix A we report all the equations and the set of cross partials.

Since $\lambda''(.)>0$ and the absolute value of $\lambda(.)$ and $\lambda'(.)$ are bounded above by $|\lambda(0)|$ and $\left|\lambda'(0)\right|$ respectively, one can show that if the market size 'a' is high enough, then the absolute value of $\pi^1_{b_1b_1}$ is strictly greater than the absolute value of $\pi^1_{b_1b_2}$(compare equations 8 and 12a in Appendix-A). Similarly, the absolute value of $E^2_{b_2b_2}$ will be strictly greater than the absolute value of $\pi^1_{b_1b_2}$(compare equations 9 and 12a in Appendix-A). In fact, we will assume these conditions.

Assumption 4 $\left|\pi^1_{b_1b_1}\right|>\left|\pi^1_{b_1b_2}\right|$ and $\left|E^2_{b_2b_2}\right|>\left|\pi^1_{b_1b_2}\right|$.

3.2 Corruption Index and Market Quality in the developing country

We define the total bribe, $\mathbb{C} = [b_1 + b_2]$ to be the '*corruption index*'. It is the total bribe paid in country 2 (the developing country).

Note that total social surplus (in the developing country)

$$W = \ [consumer\ surplus] + [profit\ of\ firm\ 2] + [total\ bribe\ paid]$$

remark 4 *Since total bribe goes into the pockets of recipients in country 2 (the developing country), this forms a part of total surplus in that country.*

Market Quality We define market quality in the developing country to be

$$\mathbb{Q} = [total\ social\ surplus] - [corruption\ index]$$

We follow the notion of 'market quality' as defined by Yano (2009, 2016), who refers to 'market quality' as a measure of "efficiency in allocation" and "fairness" in a market. Here total surplus is taken as an indicator of efficiency and the corruption index (total bribe) to be the opposite of fairness in the market (higher is $\mathbb{C}$ lower is total fairness).

Note that on the demand side of the market, the representative consumer's utility function of two differentiated products, q_1 and q_2, and a numeraire good, q_0 is given by

$$U = a\left(q_1 + q_2\right) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0.$$

Here consumer surplus is

$$CS = U\left(q_1, q_2\right) - p_1 q_1 - p_2 q_2$$

The profit of the developing country firm (firm 2) is

$$\pi^2 = p_2 q_2 - b_2$$

Expost social surplus (for any realized value of k) is as follows:

$$\begin{aligned} W(k) &= CS(k) + \pi^2(k) + [b_1(k) + b_2] \\ &= a\left(q_1 + q_2\right) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0 - p_1 q_1 + b_1 \end{aligned}$$

Expost market quality is as follows:

$$\mathbb{Q}(k) = W(k) - \mathbb{C} = CS + \pi^2$$

Routine computations show that in equilibrium

$$\mathbb{Q}(k) = \frac{1}{2(4-\gamma^2)} \left[\begin{array}{c} 2\lambda(b_2^*)\{3c - 3a + a\gamma\} \\ +3(\lambda(b_2^*))^2 + 2\lambda(b_1(k))\{-a + a\gamma - c\gamma\} \\ +(\lambda(b_1(k)))^2 - 2\gamma\lambda(b_1(k))\lambda(b_2^*) \\ -6ac - 2a^2\gamma + 4a^2 + 3c^2 \\ +2ac\gamma - 2\left(4-\gamma^2\right) b_2^* \end{array} \right] + q_0$$

4 Preliminary results

We now provide some preliminary results in terms of lemmas. These results will be very useful in deducing our main results later. All proofs are provided in Appendix-B.

Lemma 1 $\frac{\partial b_1(.)}{\partial k} < 0$ *and* $\frac{\partial \lambda(b_1(.))}{\partial k} > 0$.

Lemma 2 *(i) If* $\lambda(b_1(k))$ *concave in* k *then* $\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k)) \frac{1}{2\varepsilon} dk\right] \leq 0$. *(ii) If* $\lambda(b_1(k))$ *is convex in* k *then* $\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k)) \frac{1}{2\varepsilon} dk\right] \geq 0$.

Lemma 3 $\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k)) \frac{1}{2\varepsilon} dk\right] > 0$.

Define the following:

$$\begin{aligned} X &= 3c - 3a + a\gamma - \gamma\lambda(b_1) + 3\lambda(b_2) \\ Y &= -a + a\gamma - c\gamma + \lambda(b_1) - \gamma\lambda(b_2) \end{aligned}$$

Lemma 4 *(i) For all* $\gamma \in [-1, 1]$ *and* $c \geq 0$, $X < 0$. *(ii) For all* $\gamma \in [-1, 1]$ *if* $c \geq \lambda(0)$ *then* $Y < 0$. *(iii) For all* $c \geq 0$ *if* $\gamma < 0$ *then,* $Y < 0$. *(iv) For all* $\gamma \in [-1, 1]$ *and* $c \geq 0$, $|X| - |Y| > 0$.

5 FCPA and market quality: main results

In this section we attempt to provide an answer to our main question. How does the Foreign Corrupt Practices Act (FCPA) affect market-quality in an emerging economy? Since in most oligopolistic markets firms' products are substitutes, we will concentrate mainly in the case where the goods are substitutes (i.e. $\gamma > 0$). In a separate section we will briefly discuss the results with complements ($\gamma < 0$).

A few notations:

1. The expost corruption index is $\mathbb{C}(k) = [b_1(k) + b_2^*]$ and the expected corruption index is $Exp.\mathbb{C} = \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{C}(k) \frac{1}{2\varepsilon} dk$.

2. The expost equilibrium outputs are q_1^c and q_2^c and total output (expost) is $Q^c = q_1^c + q_2^c$. The expected total equilibrium output is $Exp.Q^c = \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} [q_1^c + q_2^c] \frac{1}{2\varepsilon} dk$.

3. The expost equilibrium profits are π^1 and π^2. The expected profits are $E^1 = \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} (\pi^1) \frac{1}{2\varepsilon} dk$ and $E^2 = \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} (\pi^2) \frac{1}{2\varepsilon} dk$.

4. The expost equilibrium market quality is $\mathbb{Q}(k)$ and the expected equilibrium market quality is $Exp.\mathbb{Q} = \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}(k) \frac{1}{2\varepsilon} dk$.

We now proceed to provide our main results. As noted before, in all the following results we will assume that $\gamma \in (0, 1]$. That is, goods are substitutes. All proofs are given in the appendix.

5.1 FCPA index (B)

Does a higher penalty, B, adversely affect the market quality in the emerging economy? Our results suggest that this may indeed be the case.

Proposition 1 $\frac{\partial b_1(.)}{\partial B} < 0$, $\frac{\partial b_2^*}{\partial B} = 0$, $\frac{\partial \mathbb{C}(.)}{\partial B} < 0$ *and* $\frac{\partial}{\partial B}[Exp.\mathbb{C}] < 0$.

Proposition 2 $\frac{\partial q_1^c(.)}{\partial B} < 0$, $\frac{\partial q_2^c}{\partial B} > 0$, $\frac{\partial}{\partial B}[Q^c] < 0$ *and* $\frac{\partial}{\partial B}[Exp.Q^c] < 0$.

Proposition 3 *(i) If the market size 'a' is large enough then* $\frac{\partial \pi^1}{\partial B} < 0$ *and* $\frac{\partial}{\partial B}[E^1] < 0$. *(ii)* $\frac{\partial \pi^2}{\partial B} > 0$ *and* $\frac{\partial}{\partial B}[E^2] > 0$.

Proposition 4 *If* $c \geq \lambda(0)$ *then* $\frac{\partial}{\partial B}\mathbb{Q} < 0$ *and* $\frac{\partial}{\partial B}[Exp.\mathbb{Q}] < 0$.

Comment Unsurprisingly, higher FCPA penalty reduces corruption index, $\mathbb{C}$, in the emerging economy. Higher B acts as a disincentive for firm 1 to bribe and hence b_1 decreases. This leads to an increase in the marginal cost for firm 1. Since firm 2 has no fear of getting caught, its bribe remains the same. Consequently, total bribe, $\mathbb{C}$, comes down. Following Yano (2009) we may say that 'fairness' in the economy improves as there is less corruption. However, this is not the end of the story. Firm 1's output q_1^c decreases since its marginal cost increases. Since $\gamma > 0$ the products are strategic substitutes and this implies 2's output, q_2^c, increases. However, total output (both expost and expected) goes down. This leads to a decrease in consumer surplus. If the technological advantage of firm 1, indexed by c, is substantial (i.e. $c \geq \lambda(0)$), market quality (both expost and expected) goes down. Typically, the US firm is expected to have a large technological advantage. In such a case, while firm 2's output, q_2^c, goes up, the increase is not big enough and consequently, consumer surplus and total surplus goes down. This results in a decrease in market quality. *This clearly shows that when the US firm has a significant technological advantage, a higher penalty under FCPA adversely affects the market quality in the emerging economy.*

5.2 Average strictness of FCPA rules $\left(\hat{k}\right)$

Note that $\hat{k}$ is the *average strictness* with which FCPA rules will be applied. In the previous subsection we noted that a higher penalty brings down the market quality in the emerging economy. Does the same conclusion follow if the FCPA rules are enforced more stringently (on an average)? Our results indicate that this may not be the case.

Proposition 5 $\frac{\partial b_1(.)}{\partial \hat{k}} < 0$, $\frac{\partial b_2^*}{\partial \hat{k}} > 0$, $\frac{\partial}{\partial \hat{k}} \mathbb{C}(.) > 0$ *and* $\frac{\partial}{\partial \hat{k}} [Exp.\mathbb{C}] > 0$.

Proposition 6 *(i)* $\frac{\partial q_1^c}{\partial \hat{k}} < 0$ *and* $\frac{\partial q_2^c}{\partial \hat{k}} > 0$. *(ii) If the market size 'a' is large enough then for all* $\gamma \in (0,1]$, $\frac{\partial}{\partial \hat{k}} [Q^c] > 0$ *and* $\frac{\partial}{\partial \hat{k}} [Exp.Q^c] > 0$.

Proposition 7 *(i) If the market size 'a' is large enough then* $\frac{\partial \pi^1}{\partial \hat{k}} < 0$ *and* $\frac{\partial}{\partial \hat{k}} \left[E^1\right] < 0$. *(ii) If the market size 'a' is large enough then* $\frac{\partial \pi^2}{\partial \hat{k}} > 0$ *and* $\frac{\partial}{\partial \hat{k}} \left[E^2\right] > 0$.

Proposition 8 *If the market size 'a' is large enough then* $\frac{\partial}{\partial \hat{k}} \mathbb{Q} > 0$ *and* $\frac{\partial}{\partial \hat{k}} [Exp.\mathbb{Q}] > 0$.

Comment The effects of a stricter enforcement of FCPA depends on the market size in the emerging economy. Firm 1 bribes less as it apprehends getting caught. But firm 2 bribes more and total bribe (the corruption index, $\mathbb{C}$) increases. Following Yano (2009) we may say that 'fairness' in the economy deteriorates as there is more corruption. If the market size 'a' is large enough, firm 1's output goes up but firm 2's output goes down (as products are strategic substitutes). However, total output (both expost and expected) increases. This pushes up consumer surplus and this outweighs any increase in the corruption index. As a result market quality improves. *In short, if the market size is large enough and products are substitutes, market quality in the emerging economy improves with stricter enforcement of FCPA.* Clearly, the effects of a stricter enforcement are very different from the effects of a higher penalty.

5.3 Technological advantage of the foreign firm (c)

Typically the US firm (say Apple or Google) is expected to have a large technological advantage (see section 1.1). We would like to analyze as to what happens when this technological advantage increases. Our results indicate that the market quality in the developing country is likely to suffer.

Proposition 9 *(i)* $\frac{\partial b_1(.)}{\partial c} > 0$ *and* $\frac{\partial b_2^*}{\partial c} < 0$. *(ii) If* $|\gamma|$ *is small enough then* $\frac{\partial}{\partial c}\mathbb{C}(.) < 0$ *and* $\frac{\partial}{\partial c}[Exp.\mathbb{C}(.)] < 0$.

Proposition 10 *(i)* $\frac{\partial q_1^c}{\partial c} > 0$ *and* $\frac{\partial q_2^c}{\partial c} < 0$. *(ii) If market size'a' is large enough and* $|\gamma|$ *is small enough then* $\frac{\partial}{\partial c}[Q^c] < 0$ *and* $\frac{\partial}{\partial c}[Exp.Q^c] < 0$.

Proposition 11 *If market size'a' is large enough then* $\frac{\partial \pi^1}{\partial c} > 0$, $\frac{\partial}{\partial c}\left[E^1\right] > 0$, $\frac{\partial \pi^2}{\partial c} < 0$ *and* $\frac{\partial}{\partial c}\left[E^2\right] < 0$.

Proposition 12 *If the market size 'a' is large enough then* $\frac{\partial}{\partial c}\mathbb{Q} < 0$ *and* $\frac{\partial}{\partial c}[Exp.\mathbb{Q}] < 0$.

Comment If there is an increase in the technological advantage of the US firm, then the US firm will bribe more (as its marginal cost comes down it can take more risk by bribing), but firm 2 bribes less. Overall corruption, $\mathbb{C}$, will come down if the degree of substitutability is small enough. The US firm's output and profit will increase (because of lower costs). Since products are strategic substitutes, firm 2's output and profit will shrink. If the market size 'a' is large enough then market quality (both expost and expected) will unambiguously decrease.

In fact, it can be shown that under certain conditions, higher is the FCPA penalty, B, more precipitous will be the fall in market quality. The reason is as follows. Suppose that $c \geq \lambda(0)$ and $\rho''(.) > 0$. From lemma 4(ii) we get $Y < 0$. From equation 8 in Appendix A we get that absolute value of $\pi^1_{b_1 b_1}$ will be higher if B is higher. This means that the absolute value of Δ will also be higher if B is higher. From equation (36) in Appendix B we get the following: higher B implies that the absolute value of $\frac{\partial b_1}{\partial c}$ will be lower. In equation 44 in Appendix B the term $\lambda'(b_1)\frac{\partial b_1}{\partial c}Y$ is positive (as $\lambda'(.) < 0, \frac{\partial b_1}{\partial c} > 0$ and $Y < 0$). Clearly, higher B implies a lower absolute value of the term $\lambda'(b_1)\frac{\partial b_1}{\partial c}Y$. This means higher B implies a lower value of $\frac{\partial \mathbb{Q}}{\partial c}$ (see equation 44). Note that a country like India (or Brazil) is likely to have a large market and a US firm (Apple, Google or Amazon) is very likely to have a substantial and growing technological advantage (as compared to the domestic firm). In such a case with FCPA in place, *an improvement in the US firm's technology induces a decrease in the market quality in the developing country.*

5.4 Quality of information (ε)

As noted before, ε is an index of the 'quality of information'. Higher is ε, poorer is the quality of information. Yano (2009, 2016) conjectured that poorer quality of information should lead to a deterioration of market quality. However, in our model, we find that under some restrictions on the curvature of $\rho(.)$ and $\lambda(.)$, expost market quality improves with a decline in the quality of information. We proceed to provide a preliminary result in this regard.

Lemma 5 *If $\rho''(.) = 0$ and $\lambda'''(.) \geq 0$ then $\lambda(b_1(k))$ is convex in k.*

Now we provide some of the main results related to changes in the quality of information.

Proposition 13 *(i) If $\rho''(.) = 0$ and $\lambda'''(.) \geq 0$ then $\frac{\partial b_1(.)}{\partial \varepsilon} < 0$ and $\frac{\partial b_2^*(.)}{\partial \varepsilon} > 0$. (ii) $\frac{\partial}{\partial \varepsilon}\mathbb{C}(.) > 0$ but the sign of $\frac{\partial}{\partial \varepsilon}[Exp.\mathbb{C}]$ is ambiguous.*

Proposition 14 *If $\rho''(.) = 0$, $\lambda'''(.) \geq 0$ and the market size 'a' is large enough then $\frac{\partial q_1^c}{\partial \varepsilon} < 0$, $\frac{\partial q_2^c}{\partial \varepsilon} > 0$, $\frac{\partial}{\partial \varepsilon}[Q^c] > 0$ but the sign of $\frac{\partial}{\partial \varepsilon}[Exp.Q^c]$ is ambiguous.*

Proposition 15 *If $\rho''(.) = 0$, $\lambda'''(.) \geq 0$ and the market size 'a' is large enough then $\frac{\partial \pi^1}{\partial \varepsilon} < 0$. The sign of $\frac{\partial}{\partial \varepsilon}\left[E^1\right]$ is ambiguous. $\frac{\partial \pi^2}{\partial \varepsilon} > 0$ but the sign of $\frac{\partial}{\partial \varepsilon}\left[E^2\right]$ is ambiguous.*

Proposition 16 *If $\rho''(.) = 0$, $\lambda'''(.) \geq 0$ and the market size 'a' is large enough then $\frac{\partial}{\partial \varepsilon}\mathbb{Q} > 0$ but the sign of $\frac{\partial}{\partial \varepsilon}[Exp.\mathbb{Q}]$ is ambiguous.*

Comment Note that propositions 13 and 16 imply that if $\rho''(.) = 0$, $\lambda'''(.) \geq 0$ and if the market size 'a' is large enough then expost corruption index ($\mathbb{C}$) and total output (Q^c) sold will increase and market quality ($\mathbb{Q}$) will improve with an increase in ε. Although corruption increases in the emerging economy with a decline in the quality of information, total output increases and this leads to an increase in consumer surplus. Profit of the local firm also increases. The last two effects dominate the negative effect of an increase in corruption and this results in an improvement in expost market quality. This result goes somewhat against the Yano (2009, 2016) conjecture. The shape of $\rho(.)$ and $\lambda(.)$ play very important roles in determining the effect of an increase in ε. However, unlike the previous propositions, we do not have any definitive results regarding the effects of poorer quality of information (higher ε) on the *expected* market quality.

6 Extension (the case of complements)

In this section we report the effects of FCPA on market quality when goods are complements ($\gamma < 0$).[16]

1. Does a higher penalty, B, under FCPA result in a lower market quality in the emerging economy when goods are complements? The answer is an unambiguous "yes". The results are very similar to the case where goods are substitutes. That is, while overall corruption goes down with higher penalty, market quality (both ex-post and ex-ante) deteriorate.

2. When the FCPA is enforced more strictly on an average (higher $\hat{k}$), then the effects on market quality are very different. In this case, unlike the case where goods are substitutes, while overall corruption comes down, market quality decreases if the market size is large enough. Hence, the effects of an increase in $\hat{k}$ is sensitive to whether γ is positive or negative.

3. When the technological advantage of the US firm increases, then with complements, while overall corruption goes down with higher penalty, market quality (both ex-post and ex-ante) unambiguously deteriorate. The results are very similar to the case where goods are substitutes.

4. When the products are complements, the effects of a deterioration in the quality of information (higher ε) is exactly the opposite to the effect for substitutes. That is, if $\rho''(.) = 0$, $\lambda'''(.) \geq 0$ and if the market size 'a' is large enough then total corruption (ex-post) goes down, total output sold (ex-post) decreases and market quality (ex-post) diminishes. When products are complements, our result supports the Yano (2009, 2016) conjecture.

7 Conclusion

In many emerging economies, various market impediments impediments have existed since the pre-modern days. We carry our analysis in the backdrop of an economy, which, due to historical and cultural factors, suffers from endemic corruption. In such an economy even bribery can contribute to the enhancement of market quality by reducing economic impediments. Conventional economics regards the market free from economic impediments as the ideal benchmark and analyzes the various effects of moving away from the ideal benchmark. The novelty of this study is to emphasize

[16] The proofs are very similar to the ones for the substitute case and are available from the authors on request.

the importance of departing from this conventional approach in studying emerging economies and of setting the benchmark at the existing state of an economy.

In this paper, we focus on the economic impediments that persist in emerging economy and show that not only bribery but also activities to influence politicians and bureaucrats by lobbying and political contribution may raise market quality. In such an environment, anti-corruption rules such as the FCPA may actually reduce the quality of the market in the emerging economy.

This effect exists only when bribes and political contributions can remove remaining economic impediments. It does not emerge if bribery is simply to buy a favor from politicians and officials. Therefore, in enforcing the anti-corruption rule (through an act like FCPA), it is desirable to weigh positive and negative effects of a particular action, i.e., to take an approach like the rule of reason under the antitrust law. Clearly, more research is needed on this front.[17]

[17] "The FCPA is, in many ways, a law with long term goals. By throwing America's global economic heft behind more ethical business practices, the United States can push developing nations towards ultimately more stable, less corrupt governance. As more countries adopt similar laws one can be optimistic about the future, but time will tell whether the FCPA can live up to its promises" (Reich, 2016).

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Appendix-A

In this appendix we first report some equations which have already been discussed in the main body of the paper and then we report some cross partials as well (that has not been derived earlier). All these equations will be used in the proofs of our results. Appendix B provides all the proofs.

The firms equilibrium Cournot outputs are as follows:

$$
\begin{aligned}
q_1^c &= \frac{a(2-\gamma) - 2\lambda(b_1) + \gamma(c + \lambda(b_2))}{4-\gamma^2} ----(1) \\
q_2^c &= \frac{a(2-\gamma) + \gamma\lambda(b_1) - 2(c + \lambda(b_2))}{4-\gamma^2} ----(2)
\end{aligned}
$$

The firms' profits are as follows.

$$
\pi^1 = \left[\frac{a(2-\gamma) - 2\lambda(b_1) + \gamma(c + \lambda(b_2))}{4-\gamma^2}\right]^2 - b_1 - \rho(b_1) kB ----(3)
$$

$$
\pi^2 = \left[\frac{a(2-\gamma) + \gamma\lambda(b_1) - 2(c + \lambda(b_2))}{4-\gamma^2}\right]^2 - b_2 ----(4)
$$

Given $b_1(k)$ firm 2 chooses b_2 to maximize its expected payoff

$$
\begin{aligned}
E^2 &= \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^2\right) \frac{1}{2\varepsilon} dk \\
&= \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left[\left\{\frac{a(2-\gamma) + \gamma\lambda(b_1(k)) - 2(c + \lambda(b_2))}{4-\gamma^2}\right\}^2 - b_2\right] \frac{1}{2\varepsilon} dk ----(5)
\end{aligned}
$$

At the Bayesian-Nash equilibrium we have

$$
\begin{aligned}
\pi_{b_1}^1 &= \frac{\partial \pi^1}{\partial b_1} \\
&= -\frac{4\lambda'(b_1(k))\left[a(2-\gamma) - 2\lambda(b_1(k)) + \gamma(c + \lambda(b_2^*))\right]}{\left[4-\gamma^2\right]^2} - 1 - \rho'(b_1(k)) kB = 0 ----(6)
\end{aligned}
$$

$$
\begin{aligned}
E_{b_2}^2 &= \frac{\partial E^2}{\partial b_2} \\
&= -\frac{4\lambda'(b_2^*)\left[a(2-\gamma) + \gamma\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k)) \frac{1}{2\varepsilon} dk - 2(c + \lambda(b_2))\right]}{\left[4-\gamma^2\right]^2} - 1 = 0 ----(7)
\end{aligned}
$$

The 2OCs are as follows:

$$
\pi_{b_1 b_1}^1 = -\frac{4\lambda''(b_1)\left[a(2-\gamma) - 2\lambda(b_1) + \gamma(c + \lambda(b_2))\right]}{\left[4-\gamma^2\right]^2} + \frac{8\left[\lambda'(b_1)\right]^2}{\left[4-\gamma^2\right]^2} - \rho''(b_1) kB < 0 --(8)
$$

$$E^2_{b_2b_2} = -\frac{4\lambda''(b_2)\left[a(2-\gamma)+\gamma\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\lambda(b_1(k))\frac{1}{2\varepsilon}dk - 2(c+\lambda(b_2))\right]}{[4-\gamma^2]^2} + \frac{8\left[\lambda'(b_2)\right]^2}{[4-\gamma^2]^2} < 0 - -(9)$$

Note that if the market size, 'a', is *high enough* then the 2OCs will hold. In fact, assumption 3 ensures that the 2OCs will always hold. The reason is as follows. Since $\rho''(.) \geq 0$ from (8) we get that

$$\begin{aligned}
\pi^1_{b_1b_1} &= -\frac{4\lambda''(b_1)\left[a(2-\gamma)-2\lambda(b_1)+\gamma(c+\lambda(b_2))\right]+2\left\{\lambda'(b_2)\right\}^2}{[4-\gamma^2]^2} - \rho''(b_1)kB \\
&\leq -\frac{4\left[\lambda''(b_1)\left[a(2-\gamma)-2\lambda(b_1)+\gamma(c+\lambda(b_2))\right]-2\left\{\lambda'(b_2)\right\}^2\right]}{[4-\gamma^2]^2} - - - -(10)
\end{aligned}$$

Note that

$$\begin{aligned}
\lambda''(b_1)a(2-\gamma)-2\lambda(b_1)+\gamma(c+\lambda(b_2))-2\left\{\lambda'(b_2)\right\}^2 &> 0 \Longleftrightarrow \\
a-\frac{1}{(2-\gamma)}\left[-2\lambda(b_1)+\gamma(c+\lambda(b_2))+\frac{2\left[\lambda'(b_2)\right]^2}{\lambda''(b_1)}\right] &> 0 - - -(11)
\end{aligned}$$

We now report some cross partials.

$$\begin{aligned}
\pi^1_{b_1b_2} &= \frac{\partial^2\pi^1_{b_1}}{\partial b_2} = -\frac{4\gamma\lambda'(b_1)\lambda'(b_2)}{[4-\gamma^2]^2} - - - -(12a) \\
E^2_{b_2b_1} &= \frac{\partial E^2_{b_2}}{\partial b_1} = 0 - - - -(12b) \\
\pi^1_{b_1k} &= \frac{\partial^2\pi^1_{b_1}}{\partial k} = -\rho'(b_1)B - - - -(12c) \\
E^2_{b_2k} &= \frac{\partial E^2_{b_2}}{\partial k} = 0 - - - -(12d) \\
\pi^1_{b_1\varepsilon} &= \frac{\partial^2\pi^1_{b_1}}{\partial k} = 0 - - - -(12e) \\
E^2_{b_2\varepsilon} &= \frac{\partial E^2_{b_2}}{\partial \varepsilon} = -\frac{4\gamma\lambda'(b_2)}{[4-\gamma^2]^2}\frac{\partial}{\partial\varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\lambda(b_1(k))\frac{1}{2\varepsilon}dk\right] - - - -(12f)
\end{aligned}$$

We now report another set of cross partials. These will be used to prove our main results.

$$\begin{aligned}
\pi^1_{b_1B} &= \frac{\partial^2\pi^1_{b_1}}{\partial B} = -\rho'(b_1)k - - - -(13a) \\
E^2_{b_2B} &= \frac{\partial E^2_{b_2}}{\partial B} = 0 - - - -(13b)
\end{aligned}$$

$$\pi^1_{b_1c} = \frac{\partial^2 \pi^1_{b_1}}{\partial c} = -\frac{4\gamma\lambda'(b_1)}{[4-\gamma^2]^2} > 0 - - - - (13c)$$

$$E^2_{b_2c} = \frac{\partial E^2_{b_2}}{\partial c} = \frac{8\lambda'(b_2)}{[4-\gamma^2]^2} < 0 - - - - (13d)$$

$$\pi^1_{b_1a} = \frac{\partial^2 \pi^1_{b_1}}{\partial a} = -\frac{4\lambda'(b_1)}{[4-\gamma^2]^2}(2-\gamma) - - - - (13e)$$

$$E^2_{b_2a} = \frac{\partial E^2_{b_2}}{\partial a} = -\frac{4\lambda'(b_2)}{[4-\gamma^2]^2}(2-\gamma) - - - - (13f)$$

$$\pi^1_{b_1\hat{k}} = \frac{\partial^2 \pi^1_{b_1}}{\partial \hat{k}} = 0 - - - - (13g)$$

$$E^2_{b_2\hat{k}} = \frac{\partial E^2_{b_2}}{\partial \hat{k}} = -\frac{4\gamma\lambda'(b_2)}{[4-\gamma^2]^2}\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k))\frac{1}{2\varepsilon}dk\right] - - - - (13h)$$

Since $\pi^1_{b_1b_1} < 0,\ E^2_{b_2b_2} < 0$ (see 8 and 9) and $E^2_{b_2b_1} = 0$ (see 12b) , we have

$$\Delta = \det\begin{vmatrix} \pi^1_{b_1b_1} & \pi^1_{b_1b_2} \\ E^2_{b_2b_1} & E^2_{b_2b_2} \end{vmatrix} = \pi^1_{b_1b_1}E^2_{b_2b_2} > 0 - - - - (14)$$

We now use the implicit function theorem to report the following (we use the cross partials derived earlier):

$$\frac{\partial b_1}{\partial k} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1k} & \pi^1_{b_1b_2} \\ E^2_{b_2k} & E^2_{b_2b_2} \end{vmatrix} = -\frac{\pi^1_{b_1k}E^2_{b_2b_2}}{\Delta} - - - - (15a)$$

$$\frac{\partial b_2^*}{\partial k} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1b_1} & \pi^1_{b_1k} \\ E^2_{b_2b_1} & E^2_{b_2k} \end{vmatrix} = 0 - - - - (15b)$$

$$\frac{\partial b_1}{\partial B} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1B} & \pi^1_{b_1b_2} \\ E^2_{b_2B} & E^2_{b_2b_2} \end{vmatrix} = -\frac{\pi^1_{b_1B}E^2_{b_2b_2}}{\Delta} - - - - (16a)$$

$$\frac{\partial b_2^*}{\partial B} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1b_1} & \pi^1_{b_1B} \\ E^2_{b_2b_1} & E^2_{b_2B} \end{vmatrix} = 0 - - - - (16b)$$

$$\frac{\partial b_1}{\partial \hat{k}} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1\hat{k}} & \pi^1_{b_1b_2} \\ E^2_{b_2\hat{k}} & E^2_{b_2b_2} \end{vmatrix} = \frac{\pi^1_{b_1b_2}E^2_{b_2\hat{k}}}{\Delta} - - - - (17a)$$

$$\frac{\partial b_2^*}{\partial \hat{k}} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1b_1} & \pi^1_{b_1\hat{k}} \\ E^2_{b_2b_1} & E^2_{b_2\hat{k}} \end{vmatrix} = -\frac{\pi^1_{b_1b_1}E^2_{b_2\hat{k}}}{\Delta} - - - - (17b)$$

$$\frac{\partial b_1}{\partial c} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1c} & \pi^1_{b_1b_2} \\ E^2_{b_2c} & E^2_{b_2b_2} \end{vmatrix} = -\frac{1}{\Delta}\left[\pi^1_{b_1c}E^2_{b_2b_2} - \pi^1_{b_1b_2}E^2_{b_2c}\right] ----(18a)$$

$$\frac{\partial b_2^*}{\partial c} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1b_1} & \pi^1_{b_1c} \\ E^2_{b_2b_1} & E^2_{b_2c} \end{vmatrix} = -\frac{\pi^1_{b_1b_1}E^2_{b_2c}}{\Delta} ----(18b)$$

$$\frac{\partial b_1}{\partial \varepsilon} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1\varepsilon} & \pi^1_{b_1b_2} \\ E^2_{b_2\varepsilon} & E^2_{b_2b_2} \end{vmatrix} = \frac{\pi^1_{b_1b_2}E^2_{b_2\varepsilon}}{\Delta} ----(19a)$$

$$\frac{\partial b_2^*}{\partial \varepsilon} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1b_1} & \pi^1_{b_1\varepsilon} \\ E^2_{b_2b_1} & E^2_{b_2\varepsilon} \end{vmatrix} = -\frac{\pi^1_{b_1b_1}E^2_{b_2\varepsilon}}{\Delta} ----(19b)$$

$$\frac{\partial b_1}{\partial a} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1a} & \pi^1_{b_1b_2} \\ E^2_{b_2a} & E^2_{b_2b_2} \end{vmatrix} = -\frac{1}{\Delta}\left[\pi^1_{b_1a}E^2_{b_2b_2} - \pi^1_{b_1b_2}E^2_{b_2a}\right] ----(20a)$$

$$\frac{\partial b_2^*}{\partial a} = -\frac{1}{\Delta}\det\begin{vmatrix} \pi^1_{b_1b_1} & \pi^1_{b_1c} \\ E^2_{b_2b_1} & E^2_{b_2c} \end{vmatrix} = -\frac{\pi^1_{b_1b_1}E^2_{b_2a}}{\Delta} ----(20b)$$

Routine computations show that in equilibrium

$$\mathbb{Q}(k) = \frac{1}{2(4-\gamma^2)}\left[\begin{array}{c} 2\lambda(b_2^*)\{3c-3a+a\gamma\} \\ +3(\lambda(b_2^*))^2 + 2\lambda(b_1(k))\{-a+a\gamma-c\gamma\} \\ +(\lambda(b_1(k)))^2 - 2\gamma\lambda(b_1(k))\lambda(b_2^*) \\ -6ac-2a^2\gamma+4a^2+3c^2 \\ +2ac\gamma-2\left(4-\gamma^2\right)b_2^* \end{array}\right] + q_0 ----(21)$$

Define the following:

$$X = 3c-3a+a\gamma-\gamma\lambda(b_1)+3\lambda(b_2) ----(22a)$$

$$Y = -a+a\gamma-c\gamma+\lambda(b_1)-\gamma\lambda(b_2) ----(22b)$$

Appendix-B

Proof of lemma 1 Note that using (14) and (15a) we get that $\frac{\partial b_1}{\partial k} = -\frac{\pi^1_{b_1 k}}{\pi^1_{b_1 b_1}}$. Since $\pi^1_{b_1 k} = -\rho'(b_1) B < 0$ (see 12c) and $\pi_{b_1 b_1} < 0$ (see 8), we have $\frac{\partial b_1}{\partial k} < 0$. Now $\frac{\partial \lambda(b_1(k))}{\partial k} = \lambda'(b_1(k)) b_1'(k) > 0$ as $\lambda'(.) < 0$ (see assumption 1).■

Proof of lemma 2 (i) Suppose first that $\lambda(b_1(k))$ is concave in k. Note that

$$\begin{aligned}\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k)) \frac{1}{2\varepsilon}\right] dk &= \left[\lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right)\frac{1}{2\varepsilon} + \lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right)\frac{1}{2\varepsilon}\right] - \frac{1}{2\varepsilon^2}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k))\, dk \\ &= \frac{1}{2\varepsilon^2}\left[\varepsilon\left\{\lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right) + \lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right)\right\} - \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k))\, dk\right] -- (23)\end{aligned}$$

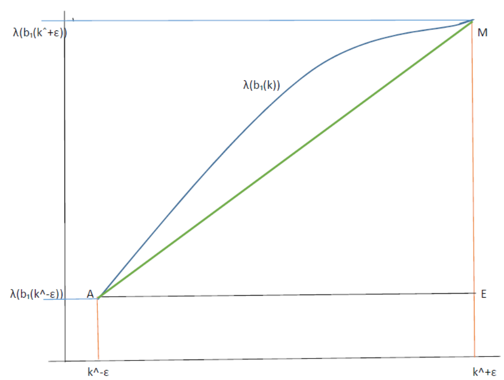

Figure 1

Now note that $\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k))\, dk$ is the area under the graph of $\lambda(b_1(k))$ in figure 1. Since $\lambda(b_1(k))$ is strictly increasing and *concave* (as shown in figure 1 above),

$$\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda(b_1(k))\, dk \geq [\text{the area of the triangle } AEM] + [\text{the area of the rectangle under the line } AE].$$

Now

$$\begin{aligned}[\text{the area of the triangle } AEM] &= \frac{1}{2}(base)(altitude) \\ &= \frac{1}{2}(2\varepsilon)\left[\lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right) - \lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right)\right] \\ &= \varepsilon\left[\lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right) - \lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right)\right]\end{aligned}$$

Also

$$[\text{the area of the rectangle under the line } AE] = (base)\,(altitude) = (2\varepsilon)\,\lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right).$$

Hence,

$$\begin{aligned}
&[\text{the area of the triangle } AEM] + [\text{the area of the rectangle under the line } AE] \\
= \;& \varepsilon\left[\lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right) - \lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right)\right] + 2\varepsilon\lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right) \\
= \;& \varepsilon\left[\lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right) + \lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right)\right]
\end{aligned}$$

Since

$$\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda\left(b_1(k)\right) dk \geq \varepsilon\left[\lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right) + \lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right)\right]$$

we have (using 23)

$$\frac{\partial}{\partial\varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda\left(b_1(k)\right)\frac{1}{2\varepsilon}dk\right] \leq 0.$$

(ii) Proof similar.■

Proof of lemma 3 Note that since $\lambda(b_1(k))$ is strictly increasing in k we have

$$\frac{\partial}{\partial\hat{k}}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \lambda\left(b_1(k)\right)\frac{1}{2\varepsilon}dk = \lambda\left(b_1\left(\hat{k}+\varepsilon\right)\right)\frac{1}{2\varepsilon} - \lambda\left(b_1\left(\hat{k}-\varepsilon\right)\right)\frac{1}{2\varepsilon} > 0.■$$

Proof of lemma 4 Note that from (22a) and (22b) we have the following.

$$\begin{aligned}
X &= 3c - 3a + a\gamma - \gamma\lambda(b_1) + 3\lambda(b_2) \\
Y &= -a + a\gamma - c\gamma + \lambda(b_1) - \gamma\lambda(b_2)
\end{aligned}$$

(i) Now since $\gamma \in [-1,1]$, we have

$$X \leq 3c - 3a + a + \lambda(b_1) + 3\lambda(b_2) < 0 \iff a > \frac{1}{2}\left[3c + \lambda(b_1) + 3\lambda(b_2)\right]$$

Assumption 3 implies that $a > \frac{1}{2}\left[3c + 4\lambda(0)\right]$. Since $\lambda'(.) < 0$, using the above we get that for all $\gamma \in [-1,1]$ and $c \geq 0$, $X < 0$.

(ii) Note that since $\gamma \in [-1,1]$ and $a - c > 0$ (assumption 3)

$$Y = -a + \gamma(a-c) + \lambda(b_1) - \lambda(b_2) < -c + \lambda(b_1) - \lambda(b_2).$$

Now $c \geq \lambda(0) \Longrightarrow c > \lambda(b_1) - \lambda(b_2)$. And this implies for all $\gamma \in [-1,1]$ and $c \geq \lambda(0)$, $Y < 0$ (see the above condition).

(iii) Now suppose $\gamma < 0$. Since $a - c > 0$ we have

$$Y \leq -a + \lambda(b_1) - \lambda(b_2).$$

From assumption 3 we have $a > \lambda(0)$. Since $\lambda'(.) < 0$, using the above we get that for all $c \geq 0$ if $\gamma < 0$ then, $Y < 0$.

(iv) We have to show that for all $\gamma \in [-1, 1]$ and $c \geq 0$, $|X| - |Y| > 0$. We will take two cases.

Case (i): $Y < 0$. In this case,

$$\begin{aligned} |X| - |Y| &= 3a - 3c - a\gamma - 3\lambda(b_2) + \gamma\lambda(b_1) - a + a\gamma - c\gamma + \lambda(b_1) - \gamma\lambda(b_2) \\ &= 2a - c(3+\gamma) - \lambda(b_2)(3+\gamma) + \lambda(b_1)(1+\gamma) - - - - (24) \end{aligned}$$

Note that if $a > \frac{1}{2}(3+\gamma)(c + \lambda(b_2))$ then $2a - c(3+\gamma) - \lambda(b_2)(3+\gamma) + \lambda(b_1)(1+\gamma) > 0$. Now $a > 2(c + \lambda(0)) \Longrightarrow a > \frac{1}{2}(3+\gamma)(c+\lambda(b_2))$. Assumption 3 ensures that $a > 2(c + \lambda(0))$. This means (by using 24) we get $|X| - |Y| > 0$.

Case (ii): $Y \geq 0$. In this case

$$\begin{aligned} |X| - |Y| &= 3a - 3c - a\gamma - 3\lambda(b_2) + \gamma\lambda(b_1) + a - a\gamma + c\gamma - \lambda(b_1) + \gamma\lambda(b_2) \\ &= (2-\gamma)[a - c - \lambda(b_2) - \lambda(b_1)] + [(2-\gamma)a - c - \lambda(b_2) + \lambda(b_1)] - - - - (25) \end{aligned}$$

Note that from assumption 3 we have $[a - c - \lambda(b_2) - \lambda(b_1)] > 0$. Also since $\gamma \in [-1, 1]$, we get $(2-\gamma)a - c - \lambda(b_2) + \lambda(b_1) \geq a - c - \lambda(b_2) + \lambda(b_1) > 0$. Using (25) this implies $|X| - |Y| > 0$.$\blacksquare$

Proof of Proposition 1 Using (14) and (16a) we get that $\frac{\partial b_1(.)}{\partial B} = -\frac{\pi^1_{b_1 B}}{\pi^1_{b_1 b_1}}$. Now from 8 we get $\pi^1_{b_1 b_1} < 0$ and from (13a) we get $\pi^1_{b_1 B} < 0$. Hence, $\frac{\partial b_1(.)}{\partial B} < 0$. From (16b) we have $\frac{\partial b_2^*}{\partial B} = 0$. Since $\mathbb{C}(k) = [b_1(k) + b_2^*]$ from the previous result it is trivial to show that $\frac{\partial \mathbb{C}(.)}{\partial B} < 0$. Note that

$$\frac{\partial}{\partial B}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{C}(k)\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\frac{\partial \mathbb{C}(.)}{\partial B}dk < 0.\blacksquare$$

Proof of Proposition 2 Note that since $\lambda'(.) < 0$ using (1) and the fact that $\frac{\partial b_1(.)}{\partial B} < 0$ (see proposition 1) we get

$$\frac{\partial q_1^c}{\partial B} = \frac{1}{4-\gamma^2}\left[-2\lambda'(b_1)\frac{\partial b_1}{\partial B}\right] < 0.$$

Using (2) and a similar logic we get

$$\frac{\partial q_2^c}{\partial B} = \frac{1}{4-\gamma^2}\left[\gamma\lambda'(b_1)\frac{\partial b_1}{\partial B}\right] > 0 \Longleftrightarrow \gamma > 0.$$

Now since $\gamma \in (0,1]$ we have $(-2+\gamma) < 0$. This means

$$\frac{\partial}{\partial B}\left[q_1^c + q_2^c\right] = \frac{(-2+\gamma)}{4-\gamma^2}\lambda'(b_1)\frac{\partial b_1}{\partial B} < 0.$$

It is easy to check that

$$\frac{\partial}{\partial B}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(q_1^c + q_2^c\right)\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\frac{\partial}{\partial B}\left[q_1^c + q_2^c\right]dk < 0. \blacksquare$$

Proof of Proposition 3 (i) Since $\pi^1 = \left[q_1^c\right]^2 - b_1 - \rho(b_1)kB$, we have

$$\frac{\partial \pi^1}{\partial B} = 2q_1^c\frac{\partial q_1^c}{\partial B} - \frac{\partial b_1}{\partial B}\left[1 + \rho'(b_1)kB\right] - \rho(b_1)k.$$

From propositions 1 and 2 we have $\frac{\partial b_1(.)}{\partial B} < 0$ and $\frac{\partial q_1^c}{\partial B} < 0$. This means if '$a$' is large enough (implying q_1^c is large enough) we get $\frac{\partial \pi^1}{\partial B} < 0$. Also,

$$\frac{\partial}{\partial B}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\pi^1\right)\frac{1}{2\varepsilon}dk\right] = \left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\frac{\partial \pi^1}{\partial B}\right)\frac{1}{2\varepsilon}dk\right] < 0.$$

(ii) Since $\pi^2 = \left[q_2^c\right]^2 - b_2^*$, we have

$$\frac{\partial \pi^2}{\partial B} = 2q_2^c\frac{\partial q_2^c}{\partial B} - \frac{\partial b_2^*}{\partial B}$$

From propositions 1 and 2 we have $\frac{\partial b_2^*}{\partial B} = 0$ and $\frac{\partial q_2^c}{\partial B} > 0$. Hence, $\frac{\partial \pi^2}{\partial B} > 0$. Also,

$$\frac{\partial}{\partial B}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\pi^2\right)\frac{1}{2\varepsilon}dk\right] = \left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\frac{\partial \pi^2}{\partial B}\right)\frac{1}{2\varepsilon}dk\right] > 0. \blacksquare$$

Proof of Proposition 4 From (21) we have

$$\mathbb{Q}(k) = \frac{1}{2\left(4-\gamma^2\right)}\left[\begin{array}{c} 2\lambda(b_2^*)\{3c - 3a + a\gamma\} + 3\left(\lambda(b_2^*)\right)^2 \\ +2\lambda(b_1(k))\{-a + a\gamma - c\gamma\} + \left(\lambda(b_1(k))\right)^2 \\ -2\gamma\lambda(b_1(k))\lambda(b_2^*) - 6ac - 2a^2\gamma + 4a^2 \\ +3c^2 + 2ac\gamma - 2\left(4-\gamma^2\right)b_2^* \end{array}\right]$$

From (22b) we have

$$Y = -a + a\gamma - c\gamma + \lambda(b_1) - \gamma\lambda(b_2).$$

From proposition 1 we know that $\frac{\partial b_2^*}{\partial B} = 0$. Using this and some routine computations we get

$$\frac{\partial \mathbb{Q}}{\partial B} = \frac{\lambda'(b_1)}{\left(4-\gamma^2\right)}\frac{\partial b_1}{\partial B}Y - - - - (26)$$

From lemma 4 we know that for all $\gamma \in [-1,1]$ if $c \geq \lambda(0)$ then $Y < 0$ and from proposition 1 we have $\frac{\partial b_1(.)}{\partial B} < 0$. Since $\lambda'(b_1) < 0$ using (26) we get that for all $\gamma \in (0,1]$ if $c \geq \lambda(0)$ then $\frac{d\mathbb{Q}}{dB} < 0$. Now

$$\frac{\partial}{\partial B}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[\frac{\partial \mathbb{Q}}{\partial B}\right]dk ----(27)$$

This means for all $\gamma \in (0,1]$ if $c \geq \lambda(0)$ then $\frac{\partial}{\partial B}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}\frac{1}{2\varepsilon}dk\right] < 0$ (see 27).■

Proof of Proposition 5 (i) We know that $\Delta > 0$ (see 14). From (17a) we get

$$\frac{\partial b_1}{\partial \hat{k}} = \frac{\pi^1_{b_1 b_2} E^2_{b_2 \hat{k}}}{\Delta}$$

From (12a) we have $\gamma > 0 \Longrightarrow \pi^1_{b_1 b_2} < 0$ and from (13h) and lemma 3 we have $\gamma > 0 \Longrightarrow E^2_{b_2\hat{k}} > 0$. These two together means that $\gamma > 0 \Longrightarrow \frac{\partial b_1(.)}{\partial \hat{k}} < 0$.

From (17b) we have

$$\frac{\partial b_2^*}{\partial \hat{k}} = -\frac{\pi^1_{b_1 b_1} E^2_{b_2 \hat{k}}}{\Delta}.$$

Since $\pi^1_{b_1 b_1} < 0$ (see 8), by using a similar logic as above we get that $\gamma > 0 \Longrightarrow \frac{\partial b_2^*}{\partial \hat{k}} > 0$.

Note that

$$\frac{\partial \mathbb{C}(.)}{\partial \hat{k}} = \frac{E^2_{b_2\hat{k}}}{\Delta}\left[\pi^1_{b_1 b_2} - \pi^1_{b_1 b_1}\right] ----(28)$$

Since $\pi^1_{b_1 b_1} < 0$ we have $\left|\pi^1_{b_1 b_1}\right| = -\pi^1_{b_1 b_1}$. From assumption 4 we get that $\left|\pi^1_{b_1 b_1}\right| > \left|\pi^1_{b_1 b_2}\right|$. This means $\pi^1_{b_1 b_2} - \pi^1_{b_1 b_1} > 0$. From (13h) and lemma 3 we have $\gamma > 0 \Longrightarrow E^2_{b_2\hat{k}} > 0$. Using (28) we get that $\gamma > 0 \Longrightarrow \frac{\partial \mathbb{C}(.)}{\partial \hat{k}} > 0$.

Note that

$$\frac{\partial}{\partial \hat{k}}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{C}(k)\frac{1}{2\varepsilon}dk = \frac{1}{2\varepsilon}\mathbb{C}\left(\hat{k}+\varepsilon\right) - \frac{1}{2\varepsilon}\mathbb{C}\left(\hat{k}-\varepsilon\right) + \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[\frac{\partial \mathbb{C}(.)}{\partial \hat{k}}\right]dk ----(29)$$

From the previous result we get that $\gamma > 0 \Longrightarrow \frac{\partial \mathbb{C}(.)}{\partial \hat{k}} > 0$. Now this also means that $\mathbb{C}\left(\hat{k}+\varepsilon\right) - \mathbb{C}\left(\hat{k}-\varepsilon\right) > 0 \Longleftrightarrow \gamma > 0$. Using both these together in (29) we get that

$$\gamma > 0 \Longrightarrow \frac{\partial}{\partial \hat{k}}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{C}(k)\frac{1}{2\varepsilon}dk > 0 \Longleftrightarrow \gamma > 0.■$$

Proof of Proposition 6 (i) Note that using (1) we get

$$\frac{\partial q_1^c}{\partial \hat{k}} = \frac{1}{4-\gamma^2}\left[-2\lambda'(b_1)\frac{\partial b_1}{\partial \hat{k}} + \gamma\lambda'(b_2)\frac{\partial b_2^*}{\partial \hat{k}}\right] ----(30)$$

If $\gamma > 0$ then from proposition 4 we have $\frac{\partial b_1}{\partial \hat{k}} < 0$ and $\frac{\partial b_2^*}{\partial \hat{k}} > 0$. Since $\lambda'(.) < 0$ from 30 we get that $\gamma > 0 \Longrightarrow \frac{\partial q_1^c}{\partial \hat{k}} < 0$.

Note that using (2) we get

$$\frac{\partial q_2^c}{\partial \hat{k}} = \frac{1}{4-\gamma^2}\left[\gamma\lambda'(b_1)\frac{\partial b_1}{\partial \hat{k}} - 2\lambda'(b_2)\frac{\partial b_2^*}{\partial \hat{k}}\right] ----(31)$$

Using a logic very similar to the one used in the previous proof we can easily show the following. $\gamma > 0 \Longrightarrow \frac{\partial q_2^c}{\partial \hat{k}} > 0$.

Note that

$$\frac{\partial}{\partial \hat{k}}\left[q_1^c + q_2^c\right] = \frac{(-2+\gamma)}{4-\gamma^2}\left[\lambda'(b_1)\frac{\partial b_1}{\partial \hat{k}} + \lambda'(b_2)\frac{\partial b_2^*}{\partial \hat{k}}\right] ----(32)$$

From proposition 4 we know that if $\gamma > 0$ we have $\frac{\partial b_1(.)}{\partial \hat{k}} < 0$ and $\frac{\partial b_2^*}{\partial \hat{k}} > 0$. Since $\lambda'(.) < 0$ we get that if $\gamma > 0$ then $\lambda'(b_1)\frac{\partial b_1}{\partial \hat{k}} > 0$ and $\lambda'(b_2)\frac{\partial b_2^*}{\partial \hat{k}} < 0$. Now it may be noted that $\frac{\partial b_1}{\partial \hat{k}} = \frac{\pi^1_{b_1b_2}E^2_{b_2\hat{k}}}{\Delta}$ and $\frac{\partial b_2^*}{\partial \hat{k}} = -\frac{\pi^1_{b_1b_1}E^2_{b_2\hat{k}}}{\Delta}$ (see the proof of proposition 4). Using assumption 4 we get that $\left|\pi^1_{b_1b_1}\right| - \left|\pi^1_{b_1b_2}\right| > 0$ and this means $\left|\frac{\partial b_2^*}{\partial \hat{k}}\right| - \left|\frac{\partial b_1}{\partial \hat{k}}\right| > 0$. Note that in equilibrium $b_1 \in \left[0, \bar{b}\right]$ and this means $\lambda''(b_1)$ is bounded above and below. Since $\lambda'(.) < 0$ and $\lambda''(.) > 0$, we get $|\lambda(b_1)| \leq |\lambda(0)|$ and $\left|\lambda'(b_1)\right| \leq \left|\lambda'(0)\right|$. Similarly, $|\lambda(b_2)| \leq |\lambda(0)|$ and $\left|\lambda'(b_2)\right| \leq \left|\lambda'(0)\right|$. From (8) and (12a) we get that if the market size 'a' is large enough then $\left|\pi^1_{b_1b_1}\right| - \left|\pi^1_{b_1b_2}\right|$ and $\left|\frac{\partial b_2^*}{\partial \hat{k}}\right| - \left|\frac{\partial b_1}{\partial \hat{k}}\right|$ will also be large enough This shows that if 'a' is large enough then $\left[\lambda'(b_1)\frac{\partial b_1}{\partial \hat{k}} + \lambda'(b_2)\frac{\partial b_2^*}{\partial \hat{k}}\right] < 0$. Since $\gamma \in (0,1]$ we have $\left[\frac{-2+\gamma}{4-\gamma^2}\right] < 0$. This means that if $\gamma > 0$ and market size 'a' is large enough then $\frac{\partial}{\partial \hat{k}}\left[q_1^c + q_2^c\right] > 0$ (see 32). Note that

$$\begin{aligned}
&\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[q_1^c + q_2^c\right]\frac{1}{2\varepsilon}dk\right] \\
= \;& \frac{1}{2\varepsilon}\left[q_1^c\left(\hat{k}+\varepsilon\right) + q_2^c\left(\hat{k}+\varepsilon\right)\right] - \frac{1}{2\varepsilon}\left[q_1^c\left(\hat{k}-\varepsilon\right) + q_2^c\left(\hat{k}-\varepsilon\right)\right] + \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\frac{\partial}{\partial \hat{k}}\left[q_1^c + q_2^c\right]dk --(33)
\end{aligned}$$

We have already demonstrated that if $\gamma > 0$ and market size 'a' is large enough then $\frac{\partial}{\partial \hat{k}}\left[q_1^c + q_2^c\right] > 0$. This means if $\gamma > 0$ and market size 'a' is large enough then $\left[q_1^c\left(\hat{k}+\varepsilon\right) + q_2^c\left(\hat{k}+\varepsilon\right)\right] > \left[q_1^c\left(\hat{k}-\varepsilon\right) + q_2^c\left(\hat{k}-\varepsilon\right)\right]$. Using this in (33) we get that if $\gamma > 0$ and market size 'a' is large enough then $\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[q_1^c + q_2^c\right]\frac{1}{2\varepsilon}dk\right] > 0.$■

Proof of Proposition 7 (i) Since $\pi^1 = \left[q_1^c\right]^2 - b_1 - \rho(b_1)kB$, we have

$$\frac{\partial \pi^1}{\partial \hat{k}} = 2q_1^c\frac{\partial q_1^c}{\partial \hat{k}} - \frac{\partial b_1}{\partial \hat{k}}\left[1 + \rho'(b_1)kB\right]$$

From propositions 5 and 6 we have $\frac{\partial b_1(.)}{\partial B} < 0$ and $\frac{\partial q_1^c}{\partial \hat{k}} < 0$. This means if '$a$' is large enough (implying q_1^c is large enough) we get $\frac{\partial \pi^1}{\partial \hat{k}} < 0$.

Note that from lemma 1 we have $\frac{\partial b_1}{\partial k} < 0$ and $\frac{\partial}{\partial k}\left[\lambda\left(b_1\right)\right] > 0$. This means $\frac{\partial q_1^c}{\partial k} = -\frac{2}{(4-\gamma^2)}\frac{\partial}{\partial k}\left[\lambda\left(b_1\right)\right] < 0$.

$$\frac{\partial \pi^1}{\partial k} = 2q_1^c \frac{\partial q_1^c}{\partial k} - \frac{\partial b_1}{\partial k}\left[1 + \rho'\left(b_1\right) kB\right] - \rho\left(b_1\right) B.$$

From above, we get if 'a' is large enough $\frac{\partial \pi^1}{\partial k} < 0$. Also,

$$\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\pi^1\right)\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\left[\pi^1\left(\hat{k}+\varepsilon\right) - \pi^1\left(\hat{k}-\varepsilon\right)\right] + \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\frac{\partial \pi^1}{\partial \hat{k}}\right)\frac{1}{2\varepsilon}dk < 0.$$

Since $\frac{\partial \pi^1}{\partial k} < 0$, we have $\pi^1\left(\hat{k}+\varepsilon\right) - \pi^1\left(\hat{k}-\varepsilon\right) < 0$. We have already shown that $\frac{\partial \pi^1}{\partial \hat{k}} < 0$. Hence, for '$a$' is large enough we get $\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\pi^1\right)\frac{1}{2\varepsilon}dk\right] < 0$.

(ii) Since $\pi^2 = \left[q_2^c\right]^2 - b_2^*$, we have

$$\frac{\partial \pi^2}{\partial \hat{k}} = 2q_2^c \frac{\partial q_2^c}{\partial \hat{k}} - \frac{\partial b_2^*}{\partial \hat{k}}$$

From propositions 5 and 6 we have $\frac{\partial b_2^*}{\partial \hat{k}} > 0$ and $\frac{\partial q_2^c}{\partial \hat{k}} > 0$. This means if '$a$' is large enough (implying q_2^c is large enough) we get $\frac{\partial \pi^2}{\partial \hat{k}} > 0$. Note that from lemma 1 we have $\frac{\partial}{\partial k}\left[\lambda\left(b_1\right)\right] > 0$ and from (15b) we have $\frac{\partial b_2^*}{\partial k} = 0$. This means $\frac{\partial q_2^c}{\partial k} = \frac{\gamma}{(4-\gamma^2)}\frac{\partial}{\partial k}\left[\lambda\left(b_1\right)\right] > 0$.

$$\frac{\partial \pi^2}{\partial k} = 2q_2^c \frac{\partial q_2^c}{\partial k} - \frac{\partial b_2^*}{\partial k}$$

From above, $\frac{\partial \pi^2}{\partial k} > 0$. Also,

$$\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\pi^2\right)\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\left[\pi^2\left(\hat{k}+\varepsilon\right) - \pi^2\left(\hat{k}-\varepsilon\right)\right] + \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\frac{\partial \pi^1}{\partial \hat{k}}\right)\frac{1}{2\varepsilon}dk < 0.$$

Since $\frac{\partial \pi^2}{\partial k} > 0$, we have $\pi^2\left(\hat{k}+\varepsilon\right) - \pi^2\left(\hat{k}-\varepsilon\right) > 0$. We have already shown that $\frac{\partial \pi^2}{\partial \hat{k}} > 0$. Hence, for '$a$' is large enough we get $\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left(\pi^1\right)\frac{1}{2\varepsilon}dk\right] < 0$.

Proof of Proposition 8 (i) Note that from (21) we get the expression for $\mathbb{Q}$ and from (22a) and (22b) we get the expressions for X and Y. Routine computations show that

$$\frac{\partial \mathbb{Q}}{\partial \hat{k}} = \frac{1}{4-\gamma^2}\left[\lambda'\left(b_1\right)\frac{\partial b_1}{\partial \hat{k}}Y + \lambda'\left(b_2\right)\frac{\partial b_2^*}{\partial \hat{k}}X - \left(4-\gamma^2\right)\frac{\partial b_2^*}{\partial \hat{k}}\right] - - - - (34)$$

From proposition 4 we know that $\gamma > 0 \Longrightarrow \frac{\partial b_1}{\partial \hat{k}} < 0$ and $\frac{\partial b_2^*}{\partial \hat{k}} > 0$. This implies $\lambda'\left(b_2\right)\frac{\partial b_2^*}{\partial \hat{k}}X > 0$. Note that in lemma 4 we have shown that $X < 0$ and $|X| - |Y| > 0$. Since $\lambda'(.) < 0$, we get

$|\lambda(b_1)| \leq |\lambda(0)|$ and $|\lambda(b_2)| \leq |\lambda(0)|$. This means if the market size 'a' is large enough then $|X|$ will also be large enough (see 21). Using assumption 4 we get that $\left|\pi^1_{b_1b_1}\right| - \left|\pi^1_{b_1b_2}\right| > 0$ and this means $\left|\frac{\partial b_2^*}{\partial \hat{k}}\right| - \left|\frac{\partial b_1}{\partial \hat{k}}\right| > 0$. In the previous proof we have demonstrated that large enough market size 'a' implies large enough value of $\left|\pi^1_{b_1b_1}\right| - \left|\pi^1_{b_1b_2}\right|$ and this implies that large enough value of $\left|\frac{\partial b_2^*}{\partial \hat{k}}\right| - \left|\frac{\partial b_1}{\partial \hat{k}}\right|$. That is, for large enough 'a', $\gamma > 0 \Longrightarrow \left[\lambda'(b_1)\frac{\partial b_1}{\partial \hat{k}}Y + \lambda'(b_2)\frac{\partial b_2^*}{\partial \hat{k}}X - \left(4-\gamma^2\right)\frac{\partial b_2^*}{\partial \hat{k}}\right] > 0$. This in turn implies $\frac{\partial \mathbb{Q}}{\partial \hat{k}} > 0$ (see 34).

Note that

$$\frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}(k)\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\mathbb{Q}\left(\hat{k}+\varepsilon\right) - \frac{1}{2\varepsilon}\mathbb{Q}\left(\hat{k}-\varepsilon\right) + \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[\frac{\partial \mathbb{Q}}{\partial \hat{k}}\right]\frac{1}{2\varepsilon}dk - - - - (35)$$

$\frac{\partial \mathbb{Q}}{\partial \hat{k}} > 0 \Longrightarrow \mathbb{Q}\left(\hat{k}+\varepsilon\right) - \mathbb{Q}\left(\hat{k}-\varepsilon\right) > 0$. This means for large enough 'a', $\gamma > 0 \Longrightarrow \frac{\partial}{\partial \hat{k}}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}(k)\frac{1}{2\varepsilon}dk\right] > 0$ (see 34).■

Proof of Proposition 9 (i) Using (18a), (13c) and (13d) we get that

$$\begin{aligned}\frac{\partial b_1}{\partial c} &= -\frac{1}{\Delta}\left[\pi^1_{b_1c}E^2_{b_2b_2} - \pi^1_{b_1b_2}E^2_{b_2c}\right]\\ &= -\frac{4}{\Delta\left(4-\gamma^2\right)^2}\left[-\gamma E^2_{b_2b_2}\lambda'(b_1) - 2\pi^1_{b_1b_2}\lambda'(b_2)\right] - - - - (36)\end{aligned}$$

Note that $\lambda'(.) < 0$, $E^2_{b_2b_2} < 0$ (see 9), $\Delta > 0$ (see 14) and $\pi^1_{b_1b_2} < 0 \Longleftrightarrow \gamma > 0$ (see 12a). Using all these together in (35) we get that $\gamma > 0 \Longrightarrow \frac{\partial b_1}{\partial c} > 0$.

Using (18b), (13d) and (8)we get that

$$\frac{\partial b_2^*}{\partial c} = -\frac{\pi^1_{b_1b_1}E^2_{b_2c}}{\Delta} = -\frac{8}{\Delta\left(4-\gamma^2\right)^2}\pi^1_{b_1b_1}\lambda'(b_2) < 0 - - - - (37).$$

That is, for all $\gamma \in (0,1]$, $\frac{\partial b_2^*}{\partial c} < 0$.

(ii) Note that by using (36) and(37) we get

$$\frac{\partial \mathbb{C}(.)}{\partial c} = -\frac{4}{\Delta\left(4-\gamma^2\right)^2}\left[-\gamma E^2_{b_2b_2}\lambda'(b_1) - 2\lambda'(b_2)\left\{\pi^1_{b_1b_2} - \pi^1_{b_1b_1}\right\}\right] - - - - (38)$$

Since $\pi^1_{b_1b_1} < 0$ we have $\left|\pi^1_{b_1b_1}\right| = -\pi^1_{b_1b_1}$. From assumption 4 we get that $\left|\pi^1_{b_1b_1}\right| > \left|\pi^1_{b_1b_2}\right|$. This means $\pi^1_{b_1b_2} - \pi^1_{b_1b_1} > 0$. If $\gamma > 0$ but $|\gamma|$ is small enough then $\left[-\gamma E^2_{b_2b_2}\lambda'(b_1) - 2\lambda'(b_2)\left\{\pi^1_{b_1b_2} - \pi^1_{b_1b_1}\right\}\right] > 0$. That is, from (38) we get that if $\gamma > 0$ and $|\gamma|$ is small enough then $\frac{\partial}{\partial c}\mathbb{C}(.) < 0$.

Now

$$\frac{\partial}{\partial c}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{C}(.)\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\mathbb{C}\left(\hat{k}+\varepsilon\right) - \frac{1}{2\varepsilon}\mathbb{C}\left(\hat{k}-\varepsilon\right) + \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[\frac{\partial \mathbb{C}(.)}{\partial c}\right]\frac{1}{2\varepsilon}dk < 0 - - (39)$$

Note that if $\gamma > 0$ and $|\gamma|$ is small enough then $\frac{\partial}{\partial c}\mathbb{C}(.) < 0$. This implies that $\frac{1}{2\varepsilon}\mathbb{C}\left(\hat{k}+\varepsilon\right) - \frac{1}{2\varepsilon}\mathbb{C}\left(\hat{k}-\varepsilon\right) < 0$. Using (38) we get that if $\gamma > 0$ and $|\gamma|$ is small enough then $\frac{\partial}{\partial c}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\mathbb{C}(.)\frac{1}{2\varepsilon}dk\right] < 0$.■

Proof of Proposition 10 (i) Note that from (1) we get

$$\frac{\partial q_1^c}{\partial c} = \frac{1}{4-\gamma^2}\left[-2\lambda'(b_1)\frac{\partial b_1}{\partial c} + \gamma\lambda'(b_2)\frac{\partial b_2^*}{\partial c} + \gamma\right] - - - - (40)$$

From proposition 7 we know that $\gamma > 0 \Longrightarrow \frac{\partial b_1}{\partial c} > 0$ and $\frac{\partial b_2^*}{\partial c} < 0$. Since $\lambda'(.) < 0$, using (40) it is routine to show that $\gamma > 0 \Longrightarrow \frac{\partial q_1^c}{\partial c} > 0$.

Note that from (2) we get

$$\frac{\partial q_2^c}{\partial c} = \frac{1}{4-\gamma^2}\left[\gamma\lambda'(b_1)\frac{\partial b_1}{\partial c} - 2\lambda'(b_2)\frac{\partial b_2^*}{\partial c} - 2\right] - - - - (41)$$

Using (41) and proposition 7 it is easy to demonstrate that $\gamma > 0 \Longrightarrow \frac{\partial q_2^c}{\partial c} < 0$.

(ii) Note that using (40), (41) (16a), (16b), (36) and (37) we get

$$\begin{aligned}\frac{\partial}{\partial c}\left[q_1^c + q_2^c\right] &= \left(\frac{-2+\gamma}{4-\gamma^2}\right)\left[\lambda'(b_1)\frac{\partial b_1}{\partial c} + \lambda'(b_2)\frac{\partial b_2^*}{\partial c} + 1\right] \\ &= \left[\frac{-4(-2+\gamma)}{(4-\gamma^2)^3}\right]\left[\begin{array}{c} -\gamma\left(\lambda'(b_1)\right)^2 E_{b_2b_2}^2 - 2\pi_{b_1b_2}^1\lambda'(b_1)\lambda'(b_2) \\ +2\left(\lambda'(b_2)\right)^2\pi_{b_1b_1}^1 + 1 \end{array}\right] - - - - (42)\end{aligned}$$

First note that since $\gamma \in (0,1]$ we have $-4\left(\frac{-2+\gamma}{4-\gamma^2}\right) > 0$. Using assumption 4 we get that $\left|\pi_{b_1b_1}^1\right| - \left|\pi_{b_1b_2}^1\right| > 0$. In the proof of proposition 5 we demonstrated that if the market size 'a' is large enough then the value of $\left|\pi_{b_1b_1}^1\right| - \left|\pi_{b_1b_2}^1\right|$ will also be large enough. This implies that if 'a' is large enough then $-2\pi_{b_1b_2}^1\lambda'(b_1)\lambda'(b_2) + 2\left(\lambda'(b_2)\right)^2\pi_{b_1b_1}^1 < 0$. Also, for for large enough 'a' the expression $\left[-2\pi_{b_1b_2}^1\lambda'(b_1)\lambda'(b_2) + 2\left(\lambda'(b_2)\right)^2\pi_{b_1b_1}^1\right]$ will dominate the number 1. This means from (42) we get that if $\gamma > 0$, market size 'a' is large enough and $|\gamma|$ is small enough then $\frac{\partial}{\partial c}\left[q_1^c + q_2^c\right] < 0$.

Note that

$$\frac{\partial}{\partial c}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[q_1^c + q_2^c\right]\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[\frac{\partial}{\partial c}\left(q_1^c + q_2^c\right)\right]dk - - - - (43)$$

Hence, from (42) and the discussion above we get that if $\gamma > 0$, market size 'a' is large enough and $|\gamma|$ is small enough then $\frac{\partial}{\partial c}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[q_1^c + q_2^c\right]\frac{1}{2\varepsilon}dk\right] < 0$.■

Proof of Proposition 11 Since $\pi^1 = \left[q_1^c\right]^2 - b_1 - \rho(b_1)kB$, we have

$$\frac{\partial\pi^1}{\partial c} = 2q_1^c\frac{\partial q_1^c}{\partial c} - \frac{\partial b_1}{\partial c}\left[1 + \rho'(b_1)kB\right].$$

From propositions 9 and 10 we have $\frac{\partial b_1(.)}{\partial c} > 0$ and $\frac{\partial q_1^c}{\partial c} > 0$. This means if '$a$' is large enough (implying q_1^c is large enough) we get $\frac{\partial \pi^1}{\partial c} > 0$. Also,

$$\frac{\partial}{\partial c}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^1\right) \frac{1}{2\varepsilon} dk\right] = \left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\frac{\partial \pi^1}{\partial c}\right) \frac{1}{2\varepsilon} dk\right] > 0.$$

Since $\pi^2 = \left[q_2^c\right]^2 - b_2^*$, we have

$$\frac{\partial \pi^2}{\partial c} = 2q_2^c \frac{\partial q_2^c}{\partial c} - \frac{\partial b_2^*}{\partial c}$$

From propositions 1 and 2 we have $\frac{\partial b_2^*}{\partial c} < 0$ and $\frac{\partial q_2^c}{\partial c} < 0$. Hence, $\frac{\partial \pi^2}{\partial c} > 0$. Also,

$$\frac{\partial}{\partial c}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^2\right) \frac{1}{2\varepsilon} dk\right] = \left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\frac{\partial \pi^2}{\partial c}\right) \frac{1}{2\varepsilon} dk\right] < 0. \blacksquare$$

Proof of Proposition 12 Note that from (21) we get the expression for $\mathbb{Q}$ and from (22a) and (22b) we get the expressions for X and Y. Routine computations show that

$$\frac{\partial \mathbb{Q}}{\partial c} = \frac{1}{4-\gamma^2}\left[\lambda'\left(b_1\right) \frac{\partial b_1}{\partial c} Y + \lambda'\left(b_2\right) \frac{\partial b_2^*}{\partial c} X + X - \left(4-\gamma^2\right) \frac{\partial b_2^*}{\partial c}\right] ----(44)$$

From lemma 4 we get that $X < 0$ and $|X| - |Y| > 0$. Also, as shown in the proof of proposition 6, if the market size 'a' is large enough, the values of $|X|$ and $|X| - |Y|$ will also be large enough. From proposition 7 we know that $\frac{\partial b_2^*}{\partial c} < 0$. Hence, $\lambda'\left(b_2\right) \frac{\partial b_2^*}{\partial c} X + X < 0$. Consequently, for large enough 'a' the expression $\left[\lambda'\left(b_2\right) \frac{\partial b_2^*}{\partial c} X + X\right]$ will dominate the expression $\left[\lambda'\left(b_1\right) \frac{\partial b_1}{\partial c} Y - \left(4-\gamma^2\right) \frac{\partial b_2^*}{\partial c}\right]$. This means that for large enough 'a' we get (by using 44) $\frac{\partial \mathbb{Q}}{\partial c} < 0$.

Again, if 'a' is large enough

$$\frac{\partial}{\partial c}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q} \frac{1}{2\varepsilon} dk\right] = \frac{1}{2\varepsilon} \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left[\frac{\partial \mathbb{Q}}{\partial c}\right] dk < 0. \blacksquare$$

Proof of lemma 5 Note that

$$\frac{\partial^2\left[\lambda\left(b_1(k)\right)\right]}{\partial k^2} = \lambda''\left(b_1(k)\right)\left\{\frac{\partial b_1(k)}{\partial k}\right\}^2 + \lambda'\left(b_1(k)\right) \frac{\partial^2\left(b_1(k)\right)}{\partial k^2} --(45)$$

From (12c), (14) and (15a) we get

$$\frac{\partial b_1(.)}{\partial k} = \frac{\rho'\left(b_1(k)\right) B}{\pi_{b_1 b_1}^1} ----(46)$$

Since $\rho"(.) = 0$, using (46) we get

$$\frac{\partial^2\left[b_1(.)\right]}{\partial k^2} = -\frac{\rho'\left(b_1(k)\right) B}{\left[\pi_{b_1 b_1}^1\right]^2}\left[\frac{\partial\left(\pi_{b_1 b_1}^1\right)}{\partial k}\right] ----(47)$$

Since ρ" $(.) = 0$, using (10) we get

$$\pi^1_{b_1b_1} = -\frac{4\left[\lambda''(b_1)\left[a(2-\gamma) - 2\lambda(b_1) + \gamma(c + \lambda(b_2))\right] - 2\left\{\lambda'(b_2)\right\}^2\right]}{\left[4-\gamma^2\right]^2}$$

Hence,

$$\frac{\partial\left(\pi^1_{b_1b_1}\right)}{\partial k} = -\frac{4\frac{\partial b_1(.)}{\partial k}}{\left[4-\gamma^2\right]^2}\left[\begin{array}{c}\lambda'''(b_1)\left[a(2-\gamma) - 2\lambda(b_1) + \gamma(c+\lambda(b_2))\right] \\ -6\lambda''(b_1)\lambda'(b_1)\end{array}\right] - -(48)$$

Since $\frac{\partial b_1(.)}{\partial k} < 0$, $\lambda'(.) < 0$, $\lambda''(.) > 0$ and $\lambda'''(.) \geq 0$, using (48) above we get $\frac{\partial\left(\pi^1_{b_1b_1}\right)}{\partial k} \geq 0$. Since $\rho'(.) > 0$, from (47) we have $\frac{\partial^2[b_1(.)]}{\partial k^2} \leq 0$. This means $\frac{\partial^2[\lambda(b_1(k))]}{\partial k^2} > 0$ (see (45)). That is, $\lambda(b_1(k))$ is convex in k.■

Proof of Proposition 13 (i) From (19a) and (19b) we get

$$\begin{aligned}\frac{\partial b_1}{\partial \varepsilon} &= \frac{\pi^1_{b_1b_2}E^2_{b_2\varepsilon}}{\Delta} - - - - (49a) \\ \frac{\partial b_2^*}{\partial \varepsilon} &= -\frac{\pi^1_{b_1b_1}E^2_{b_2\varepsilon}}{\Delta} - - - - (49b)\end{aligned}$$

From (12f) and lemma 2 we know that if $\lambda(b_1(k))$ is convex in k then $E^2_{b_2\varepsilon} > 0$. From lemma 5 we get that if $\rho''(.) = 0$ and $\lambda'''(.) \geq 0$ then $\lambda(b_1(k))$ is convex in k. From (12a) we get that $\gamma > 0 \Longrightarrow \pi^1_{b_1b_2} < 0$. All these put together in (45a) and (45b) demonstrate that if $\rho''(.) = 0$ and $\lambda'''(.) \geq 0$ then $\frac{\partial b_1(.)}{\partial \varepsilon} < 0$ and $\frac{\partial b_2^*(.)}{\partial \varepsilon} > 0$.

(ii) Note that

$$\frac{\partial}{\partial \varepsilon}\mathbb{C}(.) = \frac{E^2_{b_2\varepsilon}}{\Delta}\left[\pi^1_{b_1b_2} - \pi^1_{b_1b_1}\right] - - - - (50)$$

Using assumption 1 we get that $\left[\pi^1_{b_1b_2} - \pi^1_{b_1b_1}\right] > 0$. Since $\Delta > 0$, the sign of $\frac{\partial}{\partial \varepsilon}\mathbb{C}(.)$ is same as that of $E^2_{b_2\varepsilon}$. From (49b) we get that the sign of $E^2_{b_2\varepsilon}$ is same as the sign of $\frac{\partial b_2^*}{\partial \varepsilon}$. This implies $\frac{\partial}{\partial \varepsilon}\mathbb{C}(.)$ has the same sign as $\frac{\partial b_2^*(.)}{\partial \varepsilon}$. That is, if $\rho''(.) = 0$ and $\lambda'''(.) \geq 0$ then $\frac{\partial}{\partial \varepsilon}\mathbb{C}(.) > 0$.

Now note that

$$\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\mathbb{C}(.)\frac{1}{2\varepsilon}dk\right] = \frac{1}{2\varepsilon}\left[\mathbb{C}\left(\hat{k}+\varepsilon\right) + \mathbb{C}\left(\hat{k}-\varepsilon\right)\right] - \frac{1}{2\varepsilon^2}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\mathbb{C}(.)\,dk + \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\frac{\partial \mathbb{C}(.)}{\partial \varepsilon}dk - -(51)$$

Note that $\left[\mathbb{C}\left(\hat{k}+\varepsilon\right) + \mathbb{C}\left(\hat{k}-\varepsilon\right)\right] > 0$, $-\frac{1}{2\varepsilon^2}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\mathbb{C}(.)\,dk < 0$ and $\frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\frac{\partial \mathbb{C}(.)}{\partial \varepsilon}dk > 0$. From (51) it is clear that the sign of $\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\mathbb{C}(.)\frac{1}{2\varepsilon}dk\right]$ is ambiguous.■

Proof of Proposition 14 Note that using (1), (49a) and (49b) we get

$$\begin{aligned}\frac{\partial q_1^c}{\partial \varepsilon} &= \frac{1}{4-\gamma^2}\left[-2\lambda'(b_1)\frac{\partial b_1}{\partial \varepsilon}+\gamma\lambda'(b_2)\frac{\partial b_2^*}{\partial \varepsilon}\right] \\ &= \frac{E^2_{b_2\varepsilon}}{\Delta(4-\gamma^2)}\left[-2\lambda'(b_1)\pi^1_{b_1b_2}-\gamma\lambda'(b_2)\pi^1_{b_1b_1}\right] ----(52)\end{aligned}$$

From assumption 4 we get $\left|\pi^1_{b_1b_1}\right|-\left|\pi^1_{b_1b_2}\right|>0$. We have already demonstrated that if 'a' is large enough then $\left|\pi^1_{b_1b_1}\right|-\left|\pi^1_{b_1b_2}\right|$ will also be large enough. Since $\lambda''(.)>0$, $\left|\lambda'(.)\right|$ is bounded above by $\left|\lambda'(0)\right|$. All these together implies that if $\gamma>0$ and if the market size 'a' is large enough then $\left[-2\lambda'(b_1)\pi^1_{b_1b_2}-\gamma\lambda'(b_2)\pi^1_{b_1b_1}\right]<0$. From (49b) we get that the sign of $E^2_{b_2\varepsilon}$ is same as the sign of $\frac{\partial b_2^*}{\partial \varepsilon}$. That is if $\gamma>0$ and market size 'a' is large enough then $\frac{\partial q_1^c}{\partial \varepsilon}$ has the opposite sign of $\frac{\partial b_2^*(.)}{\partial \varepsilon}$ (see 52). Now we know that if $\rho''(.)=0$ and $\lambda'''(.)\geq 0$ then $\frac{\partial b_2^*(.)}{\partial \varepsilon}>0$. That is, $\frac{\partial q_1^c}{\partial \varepsilon}<0$.

Note that using (2), (49a) and (49b) we get

$$\begin{aligned}\frac{\partial q_2^c}{\partial \varepsilon} &= \frac{1}{4-\gamma^2}\left[\gamma\lambda'(b_1)\frac{\partial b_1}{\partial \varepsilon}-2\lambda'(b_2)\frac{\partial b_2^*}{\partial \varepsilon}\right] \\ &= \frac{E^2_{b_2\varepsilon}}{\Delta(4-\gamma^2)}\left[\gamma\lambda'(b_1)\pi^1_{b_1b_2}+2\lambda'(b_2)\pi^1_{b_1b_1}\right] ----(53)\end{aligned}$$

Using (53) and a logic similar to the one used in the previous proof we can easily show that if $\rho''(.)=0$, $\lambda'''(.)\geq 0$ and the market size 'a' is large enough then $\frac{\partial q_2^c}{\partial \varepsilon}$ has the same sign as $\frac{\partial b_2^*(.)}{\partial \varepsilon}$. That is, $\frac{\partial q_2^c}{\partial \varepsilon}>0$.

Note that

$$\begin{aligned}\frac{\partial}{\partial \varepsilon}\left[q_1^c+q_2^c\right] &= \frac{-2+\gamma}{4-\gamma^2}\left[\lambda'(b_1)\frac{\partial b_1}{\partial \varepsilon}+\lambda'(b_2)\frac{\partial b_2^*}{\partial \varepsilon}\right] \\ &= \frac{E^2_{b_2\varepsilon}(-2+\gamma)}{\Delta(4-\gamma^2)}\left[\lambda'(b_1)\pi^1_{b_1b_2}-\lambda'(b_2)\pi^1_{b_1b_1}\right] ----(54)\end{aligned}$$

From (49b) we get that the sign of $E^2_{b_2\varepsilon}$ is same as the sign of $\frac{\partial b_2^*}{\partial \varepsilon}$. Also, $(-2+\gamma)<0$. Using (54) and a logic similar to used in some proofs above it is routine to demonstrate that if the market size 'a' is large enough then $\frac{\partial}{\partial \varepsilon}\left[q_1^c+q_2^c\right]$ has the same sign as $\frac{\partial b_2^*(.)}{\partial \varepsilon}$. That is, $\frac{\partial}{\partial \varepsilon}\left[q_1^c+q_2^c\right]>0$. Using a logic used in proposition 13 we can show that the sign of $\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon}\left[q_1^c+q_2^c\right]\frac{1}{2\varepsilon}dk\right]$ is ambiguous.■

Proof of Proposition 15 Since $\pi^1=\left[q_1^c\right]^2-b_1-\rho(b_1)kB$, we have

$$\frac{\partial \pi^1}{\partial \varepsilon}=2q_1^c\frac{\partial q_1^c}{\partial \varepsilon}-\frac{\partial b_1}{\partial \varepsilon}\left[1+\rho'(b_1)kB\right].$$

From proposition 14 we get that if $\rho''(.) = 0$, $\lambda'''(.) \geq 0$ and if the market size 'a' is large enough then $\frac{\partial q_1^c}{\partial \varepsilon} < 0$. This means if '$a$' is large enough (implying q_1^c is large enough) we get $\frac{\partial \pi^1}{\partial \varepsilon} < 0$. Also,

$$\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^1\right) \frac{1}{2\varepsilon} dk\right] = \frac{1}{2\varepsilon}\left[\pi^1\left(\hat{k}+\varepsilon\right) + \pi^1\left(\hat{k}-\varepsilon\right)\right] + \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\frac{\partial \pi^1}{\partial \varepsilon}\right) \frac{1}{2\varepsilon} dk - \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^1\right) \frac{1}{2\varepsilon^2} dk$$

From above it is clear that the sign of $\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^1\right) \frac{1}{2\varepsilon} dk\right]$ is ambiguous.

Since $\pi^2 = \left[q_2^c\right]^2 - b_2^*$, we have

$$\frac{\partial \pi^2}{\partial \varepsilon} = 2q_2^c \frac{\partial q_2^c}{\partial \varepsilon} - \frac{\partial b_2^*}{\partial \varepsilon}$$

From proposition 14 we get that $\frac{\partial q_2^c}{\partial \varepsilon} > 0$.. This means if '$a$' is large enough (implying q_2^c is large enough) we get $\frac{\partial \pi^2}{\partial \varepsilon} > 0$. Also,

$$\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^2\right) \frac{1}{2\varepsilon} dk\right] = \frac{1}{2\varepsilon}\left[\pi^2\left(\hat{k}+\varepsilon\right) + \pi^2\left(\hat{k}-\varepsilon\right)\right] + \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\frac{\partial \pi^2}{\partial \varepsilon}\right) \frac{1}{2\varepsilon} dk - \int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^2\right) \frac{1}{2\varepsilon^2} dk.$$

From above it is clear that the sign of $\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \left(\pi^2\right) \frac{1}{2\varepsilon} dk\right]$ is ambiguous.■

Proof of Proposition 16 Routine computations show that

$$\frac{\partial \mathbb{Q}}{\partial \varepsilon} = \frac{1}{4-\gamma^2}\left[\lambda'(b_1)\frac{\partial b_1}{\partial \varepsilon} Y + \lambda'(b_2)\frac{\partial b_2^*}{\partial \varepsilon} X - \left(4-\gamma^2\right)\frac{\partial b_2^*}{\partial \varepsilon}\right] - - - - (55)$$

From (49a) and (49b) we get that $\left|\frac{\partial b_2^*}{\partial \varepsilon}\right| > \left|\frac{\partial b_1}{\partial \varepsilon}\right|$. Note that from lemma 4 we get $X < 0$ and $|X| - |Y| > 0$. Also, we have shown before that if the market size 'a' is large enough the value of $|X| - |Y|$ is also large enough. Note that $\lambda'(b_2)\frac{\partial b_2^*}{\partial \varepsilon} X$ has the same sign as $\frac{\partial b_2^*}{\partial \varepsilon}$. This means that for '$a$' large enough the term $\lambda'(b_2)\frac{\partial b_2^*}{\partial \varepsilon} X$ will dominate $\left[\lambda'(b_1)\frac{\partial b_1}{\partial \varepsilon} Y - \left(4-\gamma^2\right)\frac{\partial b_2^*}{\partial \varepsilon}\right]$. This implies for '$a$' large enough $\frac{\partial}{\partial \varepsilon}\mathbb{Q}$ has the same sign as that of $\frac{\partial b_2^*}{\partial \varepsilon}$. That is, f $\rho''(.) = 0$, $\lambda'''(.) \geq 0$ and if the market size 'a' is large enough then $\frac{\partial}{\partial \varepsilon}\mathbb{Q} > 0$.

Note that

$$\frac{\partial}{\partial \varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}\frac{1}{2\varepsilon} dk = \frac{1}{2\varepsilon}\left[\mathbb{Q}\left(\hat{k}+\varepsilon\right) + \mathbb{Q}\left(\hat{k}-\varepsilon\right)\right] - \frac{1}{2\varepsilon^2}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}\frac{1}{2\varepsilon} dk + \frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \frac{\partial \mathbb{Q}}{\partial \varepsilon} dk - - (56)$$

Note that $\left[\mathbb{Q}\left(\hat{k}+\varepsilon\right) + \mathbb{Q}\left(\hat{k}-\varepsilon\right)\right] > 0$, $-\frac{1}{2\varepsilon^2}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}\frac{1}{2\varepsilon} dk < 0$ and $\frac{1}{2\varepsilon}\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \frac{\partial \mathbb{Q}}{\partial \varepsilon} dk > 0$. Using (56) we get that the sign of $\frac{\partial}{\partial \varepsilon}\left[\int_{\hat{k}-\varepsilon}^{\hat{k}+\varepsilon} \mathbb{Q}\frac{1}{2\varepsilon} dk\right]$ is ambiguous.■