Firm Exit during the COVID-19 Pandemic: Evidence from Japan

MIYAKAWA, Daisuke
Hitotsubashi University

OIKAWA, Koki
Waseda University

UEDA, Kozo
Waseda University

The Research Institute of Economy, Trade and Industry
https://www.rieti.go.jp/en/
Firm Exit during the COVID-19 Pandemic: Evidence from Japan*

Daisuke MIYAKAWA
Hitotsubashi University
Koki OIKAWA
Waseda University
Kozo UEDA
Waseda University

Abstract
Firms have exited the market since the start of the COVID-19 pandemic. To evaluate the number of firms exiting the market and their exit rate, we construct a simple model, in which firms optimally choose stopping time for their exit. We estimate the model using firm-level data on firm exits before the pandemic. Subsequently, using recent survey data on firm sales growth, we simulate potential firm exits during the pandemic under the condition that the institutional background, represented by activities such as bankruptcy procedures and government rescue plans, did not change the exit option value. Our main findings are as follows. First, we find sizable heterogeneity with respect to the number and rate of firm exits across industries and regions. Second, in aggregate, the pandemic potentially increased firm exits by around 20% compared to the previous year under the assumption that the recent reduction in firm sales is temporary and, thus, partially incorporated into firms’ expectations for future trend sales growth. In two extreme cases in which the recent sales reduction has a full or no impact on firms’ expectations for future sales, firm exits increased by 110% and 10%, respectively. Third, these increases are mainly due to the decrease in the expected sales growth rate, rather than the increase in uncertainty. Finally, we quantify the hypothetical amount of government subsidies needed to prevent excess increases in potential firm exits, which is around 10⁻³ of Japan’s GDP.

Keywords: Covid-19; company bankruptcy; firm exit; optimal stopping time
JEL classification: D2, G3, H3, L1

*We are grateful to Tokyo Shoko Research (TSR) for providing us with valuable data and to seminar participants at the Research Institute of Economy, Trade and Industry (RIETI) for comments. This study was conducted as a part of the project “Determinants of Firm Dynamics: Causal Inference Approach” undertaken at the RIETI. Ueda is grateful for financial support from the JSPS Grant-in-Aid (16KK0065). All errors are our own.
1 Introduction

In this study, we examine the effects of the COVID-19 pandemic on firm exits. Under the ensuing social confinement, some businesses closed or their continuity was threatened. For instance, in the United States, Neiman Marcus, J.Crew, J. C. Penney, Hertz, Brooks Brothers, and MUJI USA filed for bankruptcy. Surprisingly, Figure 1 shows that, so far, we have not seen a surge in the official number of bankruptcies but rather a decrease. This likely is caused by institutional background changes: specifically, the social confinement prevented practitioners and courts from processing insolvencies as usual and governments implemented financial rescue plans for affected companies. Therefore, a natural question is how many firms would have become potentially bankrupt without those changes in the institutional background. Moreover, particularly in an aging society such as Japan, many small-firm owners are old, have no successor, and have thus been reported to exit voluntarily from the market. Therefore, the COVID-19 pandemic may have accelerated the voluntary exit of such firms. Given that business continuity remains a top economic concern as, for example, Baldwin (2020) points out, it is especially important to understand the nature of firm exits under a large economic shock such as the COVID-19 pandemic.\footnote{The pandemic started to affect the Japanese economy in February 2020. In Japan, the first case was reported on January 16—a Chinese national returning from China—and the first domestic case on January 29. Subsequently, the number of cases increased steadily, reaching 100 in February. Furthermore, in the same month, the news of the Diamond Princess cruise ship increased anxiety about the COVID-19 pandemic day by day. On February 20 and 27, the government released guidelines to contain the spread, asking the public to work from home, cancel events, and close schools. A state of emergency was declared on April 7 and was lifted in all the prefectures on May 25.}

To study the effects of the COVID-19 pandemic on firm exits, we use firm-level data for Japan provided by Tokyo Shoko Research (TSR). On the one hand, the TSR data are novel, in that they cover a wide range of firms in Japan and include information on when and why firms exited from the market. On the other hand, a caveat is that recent TSR data for 2020 track only relatively large-sized bankruptcy cases, ignoring voluntary exits and small-sized bankruptcies. Therefore, it is not straightforward to examine recent firm exits.

Given this data constraint, to evaluate the number of firm exits during the COVID-19 pandemic, we first construct a simple model following Kwon (2010) (see also Luttmer (2007)). In this model, firms decide the optimal stopping time for their market exit. It is shown that firm exit is influenced by three main factors: the growth rate of firm size,
uncertainty about firm growth, and exit option value. Second, we estimate the model using the TSR data for the year before the pandemic (i.e., 2019), which cover not only large-sized bankruptcies but also voluntary exits and small-sized bankruptcies. Third, using the estimated model and a survey on recent firm sales conducted by TSR during the pandemic, we simulate how much firm exits would have increased during the pandemic. These results are useful as a benchmark to study the economic impacts of the pandemic.

Our simulation results show that, first, sizable heterogeneity in terms of exit numbers and rates exists across industries and regions. The hospitality industry (i.e., accommodation, eating, and drinking services) shows the largest increase in firm exit rate, while large cities, such as Osaka and Tokyo, suffered from the largest increase in the number of firm exits.

Second, in aggregate, the pandemic potentially increased firm exits by around 20% compared to the previous year. This number is obtained under the assumption that the recent reduction in firm sales is temporary and, thus, partially (i.e., 2%, meaning the pandemic would dampen sales growth over two years) incorporated into firms’ sales trend expectations. In an extreme case, in which firms assume the recent drop in sales is permanent and thus the sales reduction has a full impact (i.e., 100%) on their sales trend expectations, firm exits would increase by 110%. In another extreme case, in which firms consider the recent sales decreases as purely transitory, thus causing no impact on their sales trend expectations, firm exits would increase by 10%. Our decomposition study shows that the increase in firm exits is mainly due to the decrease in the expected growth rates of firm sales, rather than the increase in uncertainty.

Our model-based study could be useful for providing policy guidelines, specifically on rescue plans. Governments worldwide implemented various financial rescue measures for firms in the wake of the COVID-19 pandemic. However, these measures should be addressed to firms that have only temporary difficulties. Such a policy would have been inefficient if governments rescued unprofitable firms before the pandemic (called zombies by Caballero et al. (2008)). In this regard, it is essential to identify the causes of firm exits and the number of firms predicted to exit the market given the current economic downturn. In our simulation, we calculated the potential number and rate of firm exiting the market by keeping the exit option value unchanged. In reality, the exit option value likely changed because of institutional background changes, such as delayed bankruptcy procedures and government rescue measures, which would have decreased firm exits to
some extent. Although the lack of comprehensive data on recent firm exits prevents us from estimating a timely exit option value during the pandemic, our model enables us to quantify the hypothetical amount of government subsidies needed to prevent excess increases in potential firm exits. We find that subsidies amount to around $10^{-3}$ of Japan’s GDP.

Empirical studies on firm exit include Griliches and Regev (1995), Olley and Pakes (1996), and Golombek and Raknerud (2018). Griliches and Regev (1995) show that exiting firms experience poor performance for several years before exiting the market, which provides the basis for our model. For Japan, studies in this area are Caballero et al. (2008), Tsuruta (2019), and Xu (2019). Our study contributes to the literature in that we structurally estimate firms’ exit decisions by constructing a simple model of the optimal stopping time. Moreover, our study focuses on the recent COVID-19 outbreak. Despite mounting concerns about firm exits during the COVID-19 pandemic, few studies seem to exist on this topic. See Elenev et al. (2020) for a theoretical study on bailouts and Miyakawa et al. (2020) for a preliminary study on the effects of confinement on firm bankruptcy in Japan. Firm exits during the COVID-19 pandemic are also examined by Bartik et al. (2020), Bernstein et al. (2020), Bosio et al. (2020), and Sánchez et al. (2020).

The remainder of our study is organized as follows. Section 2 develops a model of firm exit. Section 3 explains the data and discusses our estimation and simulation strategy. Section 4 explains the estimation and simulation results. Section 5 concludes.

## 2 Simple Model of Firm Exit

Here, we express firm exit as an optimal stopping time problem. Our model is highly stylized to analytically identify the determinants of firm exit and the mechanism, although it does not explicitly incorporate important factors for firm exit such as firm age and credit constraints. After explaining the model, we discuss our estimation strategy to investigate the effects of the COVID-19 pandemic on firm exits.
2.1 Model Setup

Assume that firm size, $s_t$, follows a geometric Brownian motion:

$$ds_t = \mu s_t dt + \sigma s_t dW_t,$$

where $W_t$ is a Wiener process and $\mu$ and $\sigma_s (> 0)$ represent a drift and deviation (uncertainty), respectively. Based on Itō’s Lemma, this stochastic process is described as

$$\ln s_t = \ln s_0 + \left( \mu - \frac{\sigma_s^2}{2} \right) t + \sigma_s W_t. \quad (1)$$

We assume that a firm’s manager has a preference over $s_t$ in the form of $u(s_t) = \ln s_t$ and maximizes the expected present discounted value of the firm by choosing the timing of the exit. For expositional convenience, we write $x_t \equiv \ln s_t$ below. With $x \equiv x_0$ and time preference $\rho (> 0)$, the value of the firm is expressed as

$$V(x) = \sup_b E_x \left[ \int_0^{T(b)} x_t e^{-\rho t} dt \right], \quad (2)$$

where $b$ is the lower bound below which the manager quits the business, and $T(b) \equiv \inf \{ t \mid x_t \leq b \}$ represents the stopping time. We assume that the manager obtains an exogenous value of $F$, which can be positive or negative, at the exit. We refer to $F$ as the exit option value.

The value function should satisfy the Hamilton-Jacobi-Bellman equation for a given $b$, such that

$$\rho V(x) = x + \left( \mu - \frac{\sigma_s^2}{2} \right) V'(x) + \frac{\sigma_s^2}{2} V''(x) \quad \text{for } x > b. \quad (3)$$

The lower bound, $b$, is pinned down by the boundary and smooth-pasting conditions, so that

$$V(b) = F, \quad V'(b) = 0.$$  

Two remarks are worth making here. First, the exit option value, $F$, captures several factors that affect the exit decision. They include the reservation value for the manager when he/she exits the market, credit constraints, bankruptcy costs, fixed costs, and so forth. Second, although this model depicts firm exit as the voluntary decision of man-
agers, some actual firm exit is involuntary, such as that initiated by lenders, which occurs as bankruptcy or liquidation. If exit option value \( F \) is sufficiently high, it can capture this involuntary firm exit. The inability to repay debts leads to firm bankruptcy or liquidation initiated by lenders. However, we do not claim that all bankruptcy and liquidation are involuntary. Managers’ determination to continue running their businesses matters when they face bankruptcy or liquidation risks. If their determination is strong, such managers will likely try to renegotiate with lenders, find new lenders, and so on. If not, they will likely close their businesses.

### 2.2 Model Implications

**Proposition 1.** The optimal exit policy exists uniquely. The optimal policy is to exit when firm size \( x \) goes below threshold \( b \), such that

\[
b = \rho F + \frac{\sigma^2}{\mu - \frac{\sigma^2}{2}} - \sqrt{\left(\mu - \frac{\sigma^2}{2}\right)^2 + 2\rho \sigma^2},
\]

and firm value under the optimal policy is

\[
V(x) = \begin{cases} \\
\frac{1}{\rho} \left[ x + \frac{1}{\rho} \left( \mu - \frac{\sigma^2}{2} \right) \right] - \frac{1}{\rho \lambda} e^{\lambda(x-b)} & \text{for } x > b, \\
F & \text{for } x \leq b, \\
\end{cases}
\]

where

\[
\lambda = -\frac{1}{\sigma^2} \left[ \left( \mu - \frac{\sigma^2}{2} \right) + \sqrt{\left( \mu - \frac{\sigma^2}{2} \right)^2 + 2\rho \sigma^2} \right].
\]

All proofs are presented in Appendix A. The value function for \( x > b \) consists of two terms. The first is the expected present discounted value of the firm, under the condition that it commits not to exit. The non-exit value is increasing in \( \mu \) and decreasing in \( \sigma \). The second term is the expected return from the exit that compensates the loss from continuing to do business even with \( x \leq b \) by non-exiting. Because \( \lambda < 0 \) for any set of parameters, the return from exit is positive, and decreases as \( x \) moves away from \( b \) because the probability that the firm will reach \( b \) in the near future decreases with \( x - b \).

Threshold \( b \) also has two terms. The first indicates that the manager decides to exit earlier under a higher constraint \( F \). The second term is negative, which implies
$b/\rho < F$. In other words, the value from the fixed flow payoff of $b$ cannot reach $F$. The manager decides to continue even with such a low flow payoff if there exists a sufficiently high probability that the firm recovers and moves from $b$ in the future. This probability depends on drift $\mu$ and uncertainty $\sigma_s$.

The next proposition summarizes how exit threshold $b$ responds to changes in drift $\mu$ and uncertainty $\sigma_s$.

**Proposition 2.** Exit threshold $b$ is decreasing in $\mu$. If $\rho < \mu$, then $b$ is increasing in $\sigma_s$, and if $\rho > \mu$, it is decreasing in $\sigma_s$.

The impact of $\mu$ is quite natural. Even under a negative current status, the manager waits if higher growth is expected in the future. However, the impact of $\sigma_s$ depends on other parameters. When the firm exhibits a decreasing trend (i.e., $\mu < 0$), only uncertainty $\sigma_s$ yields the probability of positive growth. Therefore, the manager has more incentives to wait to exit under a greater $\sigma_s$. When the firm has a non-decreasing trend ($\mu \geq 0$), there are two cases that depend on the time preference ($\rho$). Because $\text{Var}[x_t - x] = t\sigma_s^2$ and due to the concavity of the manager’s preference, firm value decreases with increasing $\sigma_s$ when the manager is sufficiently patient (small $\rho$). Therefore, a greater $\sigma_s$ leads to early exit with sufficiently small $\rho$, and late exit otherwise.\footnote{Kwon (2010) analyzes a similar firm exit problem with Brownian motion for the profit sequence. He focuses on the case with negative drift and risk neutrality and shows that the threshold is decreasing in the variance. Luttmer (2007) also considers firm exit as a stopping problem with an exponential (convex) payoff function of firm size, where the firm size follows a Brownian motion.}

From the firm exit model, the reasons for firm exit can be attributed to three factors: the growth rate of firm size ($\mu$), uncertainty about the growth rate of firm size ($\sigma_s$), and exit option value ($F$). The number of firm exits increases when $\mu$ decreases or $F$ increases. During the pandemic, $\mu$ likely decreased in many industries, such as foodservice and hospitality. The model also shows that uncertainty $\sigma_s$ matters. Baker et al. (2020) find that uncertainty increased during the pandemic. According to the model, firm exit will decrease if $\mu < \rho$ and increase if $\mu > \rho$. There are two opposing forces that affect $F$. The value increases when the pandemic tightens firms’ credit constraints. However, it likely decreased because the COVID-19 decreased the reservation value for managers when exiting the market and governments provided generous rescue plans. Moreover, confinement seems to have increased the bankruptcy costs associated with filing, which in turn likely decreased $F$.\footnotemark
3 Data

3.1 Data Description

We use three types of firm-level datasets provided by TSR. The first dataset is the most comprehensive of the three, containing information on firm sales and exits for around three million firms every year. TSR identifies the reasons for firm exit among closure, dissolution, bankruptcy (default), merger, and others.\(^3\) In this study, we consider firm exit only when it occurs for the first three reasons, because merged firms likely continue their business. We use data for \(t = 2019\). A dummy for firm exit takes one if a firm exited from the market from January to December 2019, and zero otherwise. The latest accounting records (e.g., sales) we could obtain are those for \(t = 2018\).\(^4\) The data also include accounting records for \(t = 2016\) and 2017. To use the records of firm exits during the COVID-19 pandemic, we need to wait for the updated dataset with a lag of around six months.

Table 1 shows the descriptive statistics. The number of firms recorded in 2018 was 3.5 million. According to the Economic Census of 2016, the total number of firms in Japan was 3.9 million; thus, the TSR data cover almost all firms in Japan. In 2019, 49 thousand firms exited the market, which amounts to an exit rate of 1.40%. Despite the large coverage of the TSR data, around two thirds of firms reported no firm sales in recent years. We omit these firms from our estimation, which reduces the number of firms from 3.5 million to 1.3 million. Consequently, the exit rate in the data used for the estimation decreases slightly, from 1.40% to 1.05%.

This table also shows that the main reason for firm exit is dissolution, followed by closure and bankruptcy. Moreover, firms exiting the market in 2019 performed worse in the previous year in terms of sales, sales growth, and employment than non-exiting firms.

The second dataset is from a special survey conducted during the COVID-19 pan-

\(^3\) According to TSR, closure is defined as the stopping of business without officially declaring its dissolution when a firm is solvent (assets exceed debts) and dissolution is defined as a procedure of ending a corporate entity by declaring it at a legal bureau.

\(^4\) Here, we consider firms whose accounting year for \(t = 2018\) ends from April 2018 to July 2019. Many firms in Japan start their accounting year in April and end in March of the following year. For those firms, the accounting records for \(t = 2018\) are those from April 2018 to March 2019. For the firms that ended their accounting year in August 2018, the records for \(t = 2018\) are those from September 2017 to August 2018. For the firms that ended their accounting year in July 2019, the records for \(t = 2018\) are those from August 2018 to July 2019.
demic, which includes a question about changes in firm sales.\textsuperscript{5} We use the results of the four waves of the survey conducted in March, March to April, April to May, and May to June 2020. In each wave, TSR asked firms about the levels of firm sales (an integer from 0 to 999) in February, March, April, and May 2020, respectively, considering firm sales in the same month of the previous year to be 100. We exclude the answers above 200. Around 10,000 firms answered the survey in each wave.

The third dataset is monthly bankruptcy data. Specifically, the data show the firms that went bankrupt, until June 2020. The right-hand side panel of Figure 1 shows the recent developments in bankruptcy based on the third dataset. The number of firm bankruptcies in February, March, and April 2020 increased by around 10\% compared to those in the same months of the previous year. However, surprisingly, the number for May 2020 decreased by half compared with that in May 2019, being also the lowest value in the last half a century. TSR explains this was due to the government’s financial rescue plans and reduced operations of the courts.

A caveat is necessary for this third dataset, as it includes only bankruptcy cases in which firm liability was not below 10 million yen. That is, neither small-sized firm bankruptcy, voluntary dissolution, nor voluntary closure is reported, although Table 1 shows these reasons as common for firm exit in 2019. If many small firms exit voluntarily during the COVID-19 pandemic, the underestimation of firm exit in the third dataset will be serious. We should thus consider this limitation, especially when comparing the simulated number of firm exits, which accounts for both firm bankruptcies and voluntary exits, with the actual number of firm bankruptcies recorded in the third dataset.

We merge these three datasets using the firm identification numbers uniquely assigned to each firm.

3.2 Exit Dependence on Firm Characteristics

Before showing the estimation results based on the proposed model, we conduct reduced-form regressions for firm exit. This illustrates the determinants of firm exit and provides a basis for our model. Furthermore, we examine any changes in firm exit before and during the COVID-19 pandemic.

\textsuperscript{5}Given the agreement between TSR and the surveyed firms, the authors only accessed summarized survey data.
We estimate the following equation:

\[ y_{it} = Z'_{it-1} \beta + \zeta_j + \epsilon_{it}, \] (7)

where \( i \in \Omega_{t-1} \) represents a firm that exists in the market at time \( t-1 \). Variable \( y_{it} \) represents a dummy that takes one if firm \( i \) exits from the market at \( t \), and zero otherwise. Vector \( Z_{it-1} \) consists of control variables for firm \( i \) at \( t-1 \)—such as log sales, sales growth, and ages—while \( \zeta_j \) is an industry effect, where \( j \) denotes the industry to which firm \( i \) belongs. We estimate this equation using a probit model.

Three remarks are necessary. First, we estimate the above equation using cross-sectional data for period \( t \), rather than panel data, because we are interested in investigating the changes in coefficients before and during the COVID-19 pandemic. Second, \( t \) denotes a particular month (or 12 months) and \( t-1 \) a particular month (or 12 months) in the previous year. Third, \( Z_{it-1} \) are variables at \( t-1 \), not at \( t \). Since the majority of firms are no longer included in the data at \( t \) when they exit the market at \( t \), using \( Z_{it} \) may cause selection bias.\(^6\) Moreover, \( Z_{it} \) for \( t = 2019 \) is not yet available for many firms.

We estimate the equation for year 2019, February 2020, March 2020, and April 2020. Table 3 shows the estimation results for 2019. The first column shows that firms tend to exit with a higher frequency when they previously recorded smaller sales or lower sales growth. Furthermore, for 2019, we regress the same equation by dividing firms by their reasons for firm exit: closure, dissolution, bankruptcy, and bankruptcy with firm liability equal or above 10 million yen. The estimation results do not change much, although past sales are no longer significant when the reason for firm exit is bankruptcy.

Next, Table 4 shows the estimation results during the COVID-19 pandemic. As explained in Section 3.1, the coverage of firm exit is narrower for the data from February to April 2020 than for year 2019, in that the former period covers only bankruptcy with firm liability equal or above 10 million yen. It should also be noted that, for firm sales growth, we do not use the past growth of each firm, but the sales growth forecasts in each month of 2020 (the second dataset of the TSR survey). Since not many firms answered this survey, we use the mean sales growth forecasts for the industry-prefecture to which each firm belongs. The table shows that neither firm sales nor sales growth are significant for firm exit. This result suggests that the exit pattern could be different from the period

\(^6\)However, in several cases, firms continue running their businesses and data are recorded even after TSR assigns an exit flag to these firms.
prior to the COVID-19 pandemic.

3.3 Estimation and Simulation Strategy

Using the model we developed, we estimate parameters and simulate firm exit during the COVID-19 pandemic using the following four steps. In the first and second steps, we use the first TSR dataset to estimate the model for year 2019 and estimate the parameters on firm growth, $\mu$ and $\sigma_s$, and the exit option value, $F$. Firm exits due to bankruptcy only are reported in the third TSR dataset and, thus, it is unable to estimate unbiased exit option values for 2020. Therefore, we use the 2019 data to estimate $F$, which we use as a hypothetical exit option value. In the third step, we estimate the parameters on firm growth, $\mu'$ and $\sigma'_s$, during the COVID-19 pandemic using TSR’s monthly survey data (the second dataset). In the final step, we simulate the number and rate of firm exits during the pandemic by assuming $F$ is unchanged.

Our simulations for the number and rate of firm exits intend to provide a prediction based on the pre-COVID mechanism governing exit (i.e., $F$) and the sales process implied by recent sales records (i.e., $\mu'$ and $\sigma'_s$). Two remarks are in order. First, the value of $F$ during the pandemic likely is different from that before the pandemic. Since the difference between the estimated pre-COVID $F$ and actual $F$ causes a difference between the simulated and actual exits during the pandemic, we subsequently use the third TSR dataset to discuss the change in $F$. Second, estimating future trend sales growth is not obvious simply because we cannot observe firms’ expectations for their permanent sales trends. As it is impossible to precisely identify firms’ expectations, we need to introduce assumptions regarding $\mu'$ and $\sigma'_s$ during the pandemic.

3.3.1 Estimation of $\mu$ and $\sigma_s$ for Year 2019

We estimate $\mu$ and $\sigma_s$ for each industry in 2019 using the first TSR dataset on firm size growth. Specifically, after classifying firms by industry, we estimate the discrete-time version of equation (1) for firm $i$ in a particular industry in year $t$:

$$x_{it} - x_{it-1} = (\mu - \sigma_s^2/2) + \sigma_s \varepsilon_{it}^s,$$

which enables us to obtain $\mu$ and $\sigma_s$. For $t$, we use $t = 2017$ and 2018, because we can observe $x_{it}$ for the past three years.
We use these estimates of drift $\mu$ and uncertainty $\sigma_s$ for the next step to study firm exit in year $t = 2019$. This is based on the premise that firm managers form adaptive expectations on drift $\mu$ and uncertainty $\sigma_s$ using their experience.

3.3.2 Estimation of $F$ for Year 2019

The second step is estimating the firm-exit equation to obtain the parameters associated with exit option value $F$. We fix $\rho$ at 0.01 and use firm sales as a proxy for firm size $s_{it} = e^{x_{it}}$.

To improve the fit of the model, we make the probability that firms with $x$ exit from the market smooth, rather than taking either zero or one. To this end, we assume that exit option value $F$ follows an i.i.d. normal distribution $\mathcal{N}(F_0, \sigma_F^2)$. Then, from equation (4) in Proposition 1, the exit condition is given by

$$F > x - \bar{b}(\mu, \sigma_s^2) = b(\mu, \sigma_s^2) \equiv \frac{\sigma_s^2}{\mu - \sigma_s^2/2 - \sqrt{(\mu - \sigma_s^2/2)^2 + 2\rho\sigma_s^2}}.$$

The probability that firm $x$ exits is given by

$$1 - \Phi\left(\frac{(x - \bar{b})/\rho - F_0}{\sigma_F}\right),$$

where $\Phi$ denotes the cumulative distribution function (CDF) of the standard normal distribution.

Note that econometricians cannot observe firm size $x_{it}$, but $x_{it-1}$, when firm $i$ exits the market in period $t$. The survival rate, that is, the probability that a firm with $x_{it-1}$ does not exit in period $t$ equals the probability that $x_{it-1} + \mu - \sigma_s^2/2 + \sigma_s\varepsilon_{it}^s > \bar{b} + \rho F_{it}$. We assume that $x_{it-1}$, $F_{it}$, and $\varepsilon_{it}^s$ are independent. Then, the survival rate for a firm with $x_{it-1}$ is written as

$$\eta(x_{it-1}|\mu, \sigma_s^2, F_0, \sigma_F^2) = \Phi\left(\frac{x_{it-1} + \mu - \sigma_s^2/2 - \bar{b}(\mu, \sigma_s^2) - \rho F_0}{\sqrt{\rho^2\sigma_F^2 + \sigma_s^2}}\right).$$

Using the exit dummy in the data and $\mu$ and $\sigma_s$ obtained in the first step, we define

---

7While we may consider using firm profit or employment, we choose to use firm sales because it has the fewest missing observations. Another reason is that cash flow is important in influencing managers’ decisions to exit, and sales can capture the cash flow better than profit or employment.
the log-likelihood function among firms in $\Omega_{t-1}$, such that

$$L \left( F_0, \sigma_F^2 | \mu, \sigma_s^2 \right) = \sum_{i \in \Omega_{t-1} \cap \text{Exit}} \log \left( 1 - \eta \left( x_{it-1} | \mu, \sigma_s^2, F_0, \sigma_F^2 \right) \right) + \sum_{i \in \Omega_{t-1} \cap \text{No Exit}} \log \left( \eta \left( x_{it-1} | \mu, \sigma_s^2, F_0, \sigma_F^2 \right) \right)$$ (11)

The maximum likelihood yields the estimates for $F_0$ and $\sigma_F$.

Information on $x_{it-1}$ (i.e., firm $i$’s sales in year 2018) is missing for some firms. We set $x_{it-1}$ to $x_{it-2} + \mu - \sigma_s^2/2$ if we do not observe $x_{it-1}$, but $x_{it-2}$. We set $x_{it-1}$ to $x_{it-3} + 2(\mu - \sigma_s^2/2)$ if we do not observe $x_{it-1}$ or $x_{it-2}$, but $x_{it-3}$. If none of $x_{it-1}$, $x_{it-2}$, or $x_{it-3}$ are observed, we exclude the firm from the estimation.

3.3.3 Estimation of $\mu'$ and $\sigma'_s$ during the COVID-19 Pandemic

We estimate equation (1) for firm $i$ in a particular industry and region in month $t$ (i.e., $t$ can be either February, March, April, or May 2020). For the left-hand side of the equation, we use the results of the firm-level survey (the second TSR dataset) conducted in each month, $t$, which asks how much firm sales in $t$ are expected to change from the same month in the previous year. Then, we estimate $\mu'$ and $\sigma'_s$. Note that not all firms answered this question. We assume that all firms have the same values of $\mu'$ and $\sigma'_s$ for a given industry, region, and month.

In this estimation, we classify firms not only by industry but also by the region in which their headquarters are located. This aims to capture regional as well as industrial heterogeneities when considering the effects of the COVID-19 pandemic on the economy. For example, Hokkaido, Tokyo, and Osaka in the Kinki area seem to have been affected relatively more by the pandemic. Specifically, we categorize firms by either 47 prefectures or 10 areas (Hokkaido, north Kanto and Koshin, Hokuriku, Tokai, Kinki, Chugoku, Shikoku, Kyushu, south Kanto (excluding Tokyo), and Tokyo). Because we do not necessarily have sufficient observations based on the former classification, we mainly use the results based on the latter.

It should also be noted that the estimates of $\mu'$ and $\sigma'_s$ do not necessarily imply new values for $\mu$ and $\sigma_s$. In the model, $\mu$ and $\sigma_s$ reflect permanent components that will continue to influence firms even after the COVID-19 pandemic ends, while not all effects of the pandemic on firm sales are permanent. Therefore, we introduce parameter $\kappa$ ($0 \leq \kappa \leq 1$) to account for the degree of partial change in $\mu$ and $\sigma_s$ as $\mu'_\kappa = \mu + (\mu' - \mu)\kappa$. 

12
and \( \sigma'_{s_{\kappa}} = \sigma^2_s + (\sigma^2_s - \sigma^2_{\kappa}) \kappa \). The values of \( \mu \) and \( \sigma_s \) change by factor \( \kappa \).

We calibrate the value of \( \kappa \) based on the length of time over which the COVID-19 pandemic will dampen firms’ sales growth. Assume that the sales growth rate is \( \mu' \) for \( t' \) years and then returns to the original \( \mu \) value. As such, considering future discounts, the mean of the sales growth rate is given by \( (1 - e^{-\rho t'}) \mu' + e^{-\rho t'} \mu \), which equals \( \mu'_{\kappa} \). Thus, \( \kappa \) equals \( 1 - e^{-\rho t'} \). In the benchmark simulation, we set \( \kappa \) to 0.02. This value implies that the pandemic will likely dampen sales growth for \( t' = 2 \) years given \( \rho = 0.01 \). The value of \( t' = 2 \) may seem slightly high, while the value of \( \rho = 0.01 \) may be slightly low. If we assume that firm managers discount future sales more (e.g., \( \rho = 0.02 \) because they are old), a value of \( t' = 1 \) leads to the same value of \( \kappa \).

To infer the value of \( \kappa \), the TSR survey (the second dataset) conducted between April and May 2020 is also useful. In this survey, TSR asked firms whether their sales were expected to go below 50% of those in the same month of the previous year for at least one month from May to December 2020. Table 2 summarizes firms’ expectations (i.e., yes, no, or unknown), conditional on their recent sales growth in April 2020. This table shows that 97.5% of the firms recording their sales in April 2020 to be 50% or below those in April 2019 expect their monthly sales to go below 50% for at least one month compared to the same month in the previous year until the end of 2020. Although this pessimistic expectation becomes less apparent for firms experiencing a smaller reduction in their monthly sales, not a few firms answered that they might lose 50% of sales in at least one month until the end of 2020. Specifically, almost all firms that recorded a value between 50 and 60 for their sales in April (i.e., around 45% sales decrease) expected their sales to further decrease, reaching below 50 (i.e., further sales decrease by more than 5 percentage points). These results suggest that firms’ sales expectations are not completely temporary, that is, the decrease in the recent sales growth will have a persistent impact on future sales growth.

### 3.3.4 Simulation of Firm Exit during the COVID-19 Pandemic

In the fourth step, we calculate the survival rate for firm with \( x_{it-1} \) during the COVID-19 pandemic by modifying equation (10) as

\[
\eta_{it} = \Phi \left( \frac{x_{it-1} + \mu' - \sigma^2_s/2 - \bar{b} (\mu'_{\kappa}, \sigma^2_{s_{\kappa}}) - \rho F_0}{\sqrt{\rho^2 \sigma^2_F + \sigma^2_{s_{\kappa}}}} \right). 
\] (12)
Note that \( \kappa \) only affects the exit threshold through \( \bar{b} \), and firm sales \( x_{it} \) during the COVID-19 pandemic are assumed to have mean \( x_{it-1} + \mu' - \sigma_s^2/2 \) and variance \( \sigma_s^2 \), rather than mean \( x_{t-1} + \mu' - \sigma_s^2/2 \) and variance \( \sigma_s^2 \). From February to May, we assume that firm sales change as per firms’ expectations.

The aggregate firm exit rate at the industry \( \times \) region level in month \( t \) equals

\[
\delta_t = \frac{1}{N_{t-1}} \sum_{i \in \Omega_{t-1}} (1 - \eta_{it}) = 1 - \frac{1}{N_{t-1}} \sum_{i \in \Omega_{t-1}} \eta_{it},
\]

where \( N_{t-1} \) is the number of firms active at \( t - 1 \) (i.e., the sum of \( i \) for \( i \in \Omega_{t-1} \)). Then, we multiply 1/12 to transform the exit rate from an annual to monthly value.

We also calculate the number of firm exits in each industry, region, and month. In our data, \( N_{t-1} \delta_t \) indicates the number of firm exits. However, \( x_{it} \) is not observable for all firms in the TSR data, which leads to under-evaluation. Thus, we multiply the ratio of the number of firms in the TSR data (corresponding to column “Total” in Table 1) to the number of firms used for estimation (corresponding to column “Used for estimation” in Table 1) for each industry and region.

4 Results

4.1 Before the COVID-19 Pandemic

We estimate \( \mu, \sigma_s, F_0, \) and \( \sigma_F \) for 12 industries, such as construction; manufacturing; and accommodation, eating, and drinking services. The left-hand side columns of Table 5 show the estimation results for firm sales growth based on equation (8) and the right-hand side columns of Table 5 show the estimation results for exit option values based on equation (11).

The table shows a large heterogeneity of parameter estimates, reflecting heterogeneous growth and exit rates across industries. Some industries, such as construction and transport and postal activities, exhibit higher exit option values \( F_0 \) and, in turn, higher threshold \( b \) than other industries, such as accommodation, eating, and drinking services. This implies that the former industries tend to experience a higher exit rate than the latter if firm sales are the same. The pattern of exit also depends on the distribution of firm sales in each industry and, thus, the pattern of estimated \( F_0 \) does not necessarily explain the actual exit pattern.
Nevertheless, investigating what type of firm characteristics are likely to be associated with $F_0$ is informative, as well as necessary for validating our model. Specifically, we look at the following three main firm characteristics that may matter for $F_0$: fixed costs, working capital, and debts. Our hypotheses are as follows. First, when fixed costs are large, $F_0$ is likely to be high, causing more exits. Second, a larger working capital might imply a greater need for liquidity holdings due to the larger risk in the industry and, thus, larger $F_0$. However, the opposite may be true if a larger working capital may lead to a lower risk of exit and, thus, lower $F_0$ because larger working capital implies greater liquidity (short-term financial health). Third, large debts may lead to a higher risk of exit and, thus, higher $F_0$. However, large debts may lead to a lower risk of exit and, thus, lower $F_0$ because they imply that these firms have a greater ability to borrow and repay their debts.

To obtain the industry-level information associated with the aforementioned three characteristics based on the comprehensive data, we use the Financial Statements Statistics of Corporations by Industry provided by the Ministry of Finance, Japan. First, the variables related to fixed costs are the ratio of fixed costs to sales, log(fixed costs per firm), and labor share. Here, fixed costs are defined as the sum of depreciation costs, labor costs, and interest expenses, while the labor share is calculated as the ratio of labor costs to value added. Second, the variables related to working capital are the ratios of working capital to sales and to assets and log(working capital per firm). Here, the working capital is defined as notes and accounts receivable plus inventories minus notes and accounts payable (trade credit). Third, the variables related to debt are the ratios of liquid debts to sales, of short-term bank borrowings to sales, and of bank borrowings to sales. We collect these variables for the 10 industries shown in Table 5, excluding compound services and miscellaneous services, for year 2018.

Table 6 shows the correlation coefficients between the $F_0$ estimates and the variables associated with the three factors. We also calculate the Spearman rank correlation coefficients because they are robust to outliers. Note that the number of industries is only 10, which makes it difficult to assess the significance of the results. We obtain the following three results. First, we find positive correlation coefficients for fixed costs, suggesting that industries with larger fixed costs tend to exhibit a higher $F_0$. Second, working capital is positively correlated with $F_0$, which implies that industries with larger working capital face a greater need for liquidity holdings and a larger exit risk. Third, debts are negatively correlated with $F_0$, implying that industries with larger debts have
4.2 Sales Growth during the COVID-19 Pandemic

Table 7 shows the estimation results for equation (8) during the COVID-19 pandemic. We estimate sales growth \( \mu' \) and uncertainty \( \sigma'_s \) for each industry (in total 12), prefecture (in total 47), and month (February to May 2020). In the table, we report the quantiles and means of \( \mu' \) and \( \sigma'_s \) for industries\(\times\)prefectures for a given month. See also the top two panels in Figure 4, which we discuss in the following.

This table shows three important results. First, sales growth \( \mu' \) declined from February to April and remained at the almost same level in May. On average, April and May sales were expected to decrease by around 20% compared to the same months in the previous year.

Second, large heterogeneity exists and appears to have increased over time. While the top 10% group exhibited a decline of only around 5% in \( \mu' \) in May, the bottom 10% group answered that their sales decreased by half over the same month. This discrepancy increased over time, being driven by the firms in bottom groups. The bottom 10% group reported rapidly declining sales prospects from February (15% decrease) to May (50% decrease), whereas the top 10% group reported relatively stable sales prospects from February (5% increase) to May (5% decrease).

Third, uncertainty \( \sigma'_s \) increased during the COVID-19 pandemic. This suggests that firms face greater uncertainty about their sales prospects, even after controlling for industries and regions.

4.3 Simulating Firm Exit during the COVID-19 Pandemic

Using the above estimation results, we simulate the rate and number of firm exits during the COVID-19 pandemic. Hereafter, we report the results for eight industries, rather than 12 by omitting the following four industries: education, medical services, compound services, and miscellaneous services. Figures 2 and 3 show the main results.

**Firm Exit Rate** Figure 2 shows the changes in firm exit rates by industry (top panel) and region (bottom panel) for year 2019 and February to May 2020. Industry- and regional-level firm exit rates are calculated by the weighted average of exit rates, weights being based on the number of firms in each region and industry, respectively. The
top panel suggests that the hospitality (accommodation, eating, and drinking services) industry suffered the largest increase in the firm exit rate, while the information and communications industry was the least affected. Regional discrepancies are small compared with industrial discrepancies. Although the exit rate level differs across regions, the pandemic seems to have affected firm exit uniformly around Japan, shifting the exit rate curve upward almost in a parallel manner.

Number of Firm Exits Figure 3 shows the changes in the number of firm exits by industry (top panel) and region (bottom panel) for February to May 2020 relative to year 2019. Zero on the vertical axis indicates that the number of firm exits is unchanged from 2019. This figure conveys a slightly different picture from Figure 2 in that the top panel shows that not only hospitality but also construction exhibited the largest increases in firm exits. The construction industry is important because the number of firms in construction is by far the largest (see Table 5). The bottom panel shows that regions with large cities, such as Tokyo and Kinki, experienced a large increase in the number of firm exits.

Aggregate Effects Column (1) of Table 8 shows the effects of the COVID-19 pandemic on the rate and number of firm exits aggregated by industry and region. The number of firm exits increased by around 1,700 from February to May 2020 compared with the same months in the previous year. The rate and number of firm exits increased by around 20% compared to the previous year.

The impact on firm exit depends on how persistent firms expect the pandemic shock to be. Columns (2)–(5) of Table 8 show the simulation results when we use different values for $\kappa$. Assuming $\kappa = 1$ means that the sales changes during the COVID-19 are expected to continue forever in the future. This is the most pessimistic scenario. Column (5) shows that the rate and number of firm exits increased by 110% from the previous year. Assuming $\kappa = 0$ is the most optimistic scenario. The sales growth parameters ($\mu$ and $\sigma_s$) are expected to return to their original levels immediately in the following month. Column (2) shows that the rate and number of firm exits increased by around 10% compared to the previous year.

8The value of $\kappa$ could exceed one if the pandemic worsens and firms expect a further decrease in future trend sales growth.
Firm Exit Decomposition  To understand the determinants of the change in firm exits in Figures 2 and 3, we examine the changes in two driving factors: $\mu$ and $\sigma_s$.  Figure 4 shows the changes in firm size growth $\mu$, uncertainty $\sigma_s$, exit threshold $b$, and exit rate $\delta$ at the industry level. The horizontal axis represents time: year 2019 (pre-COVID), February to May 2020. The top left-hand side panel shows that hospitality as well as living-related and personal services and amusement services experienced the largest decrease in $\mu$. The top right-hand side panel indicates increases in $\sigma_s$, although the pattern of the increases is not monotonous. The bottom left-hand side panel shows that $b$ increased considerably for hospitality, but not much for living-related and personal services and amusement services, which led to a considerable increase in the firm exit rate in the former industry, as the bottom right-hand side panel shows.

Using the model, we can investigate which factors contributed to the changes in firm exits. To do this, we calculate the first-order Taylor approximation for the firm exit rate, denoted by $\delta$, around the values in 2019 and obtain

$$d\delta = d\delta^{\mu} + d\delta^{\sigma_s} + d\delta^{\text{residual}},$$  \hspace{1cm} (14)

where $d\delta^{\mu}$, $d\delta^{\sigma_s}$, and $d\delta^{\text{residual}}$ represent the contributions of the first-order effects of a change in $\mu$, the first-order effects of a change in $\sigma_s$, and the residuals, respectively. Appendix B presents details.

The results are shown in Figure 5. The most important factor is a change in $\mu$, specifically, its decrease during the COVID-19 pandemic. By contrast, the increase in uncertainty $\sigma_s$ seems to have affected the firm exit rate only modestly.

Comparison between the Simulated and Actual Number of Firm Exits  To check the validity of the model, we compare the simulated number of firm exits based on the model from February to May with the actual number of firm bankruptcies in the same months (based on the third TSR dataset) at the industry level. Note that the scope of firm exit is narrower in the latter case because it excludes firm closure and dissolution. Moreover, as stated in the Introduction, we expect the number of firm bankruptcies to be small, because the confinement curbed processing insolvencies.

The left-hand side panel of Figure 6 shows the scatter plot, in which the horizontal and vertical axes represent the number of firm exits based on the model and the actual number of firm bankruptcies, respectively, cumulated over four months (February to
May) in 2020. The line is the 45 degree line. The figure suggests a positive correlation between the two variables, with the slope lower than one, as expected.

The right-hand side panel shows the scatter plot, in which both the horizontal and vertical axes represent the change in the number of firm exits in the four months of 2020 compared to the same months of 2019. The actual number of firm bankruptcies decreased, rather than increased, during the COVID-19 pandemic. As a result, we find no positive correlation between the two variables.

This difference between the model and the data is partly explained by the difference in the scope of firm exit. In the 2019 data shown in Table 1, bankruptcies account for only 1/8 of all reasons for exit. Moreover, this difference can be explained by the change in exit option value $F$. The fact that firm exits based on the data are fewer than those based on the model implies that $F$ decreased, which prevented firms from exiting the market. The confinement that prevented practitioners and courts from processing insolvencies and government’s financial rescue plans are also possible causes for the decrease in $F$.

4.4 Subsidy to Prevent Excess Exit

In the previous subsection, we discussed the possibility that exit option value $F$ decreased. Here, we consider the change in $F$ from a different perspective. Specifically, we ask how much a policymaker needs to subsidize firms to keep the exit rate in 2019 unchanged. To answer this question, we calculate the amount of exit option value $F_0$.

Let us denote the firm exit rate before and during the COVID-19 pandemic by $\delta^{pre} = \delta(\mu, \sigma_s^2, F_0, \sigma_F^2)$ and $\delta^{covid} = \delta(\mu', \sigma_s'^2, F_0, \sigma_F'^2)$, respectively. Then, we search for $F_0'$, such that $\delta^{pre}' = \delta(\mu', \sigma_s'^2, F_0', \sigma_F'^2)$ for each industry. Since $F = \int_0^\infty \log(s) e^{-\rho t} dt = \log(s)/\rho$, the change in $F_0$, that is, $F_0 - F_0'$, amounts to government subsidies to a firm of as much as $\exp\{\rho(F_0 - F_0')\}$ every year. We calculate the total amount of government subsidies by multiplying the number of firms in each industry and summing it across industries.

We obtain 3.6, 4.0, 5.2, and 5.4 billion yen for February, March, April, and May, respectively. The average amount is 4.6 billion yen. The size of the subsidies may appear small, considering that they account for only $3 \cdot 10^{-6}$ of total firm sales or $1 \cdot 10^{-5}$ of Japan’s GDP. However, we consider subsidies paid to firms every year. If we instead consider one-time subsidies by multiplying with $1/\rho$, the size of the subsidies becomes a hundred times larger, that is, 460 billion yen ($1 \cdot 10^{-3}$ of GDP), which is by no means small.
4.5 Robustness Checks

4.5.1 Adjusting Imbalanced Observations for Firm Exits and Non-Exits

As explained in Section 3.1, some firms in the TSR dataset do not include firm sales data and, thus, cannot be used for estimation. Consequently, the exit rate in the data that was used for estimation is 1.05%, which is slightly lower than that based on all TSR data, that is, 1.40%. Because this imbalance between firm exits and non-exits may underestimate the probability of firms exiting the market, we adjust this effect by attaching specific coefficients to the likelihood function of equation (11) following Manski and Lerman (1977) and King and Zeng (2001). For example, we use Table 1 and multiply the ratio of the number of firms in all TSR data to that in the data used for estimation, that is, 0.486/0.139 = 3.50 for exiting firms and 3.43/1.31 = 2.62 for non-exiting firms, to the first and second terms of equation (11), respectively. Specifically, we estimate the model for each industry using differing adjustment coefficients. This adjustment is based on the premise that the distribution of firm sales for the firms that do not have a sales record is the same as that used for our estimation, once we divide firms by whether they exit or not.

When the imbalanced observations for firm exits and non-exits are adjusted, the estimate of exit option value $F_0$ increases for all industries, which increases threshold $b$ and, thus, the likelihood of exit (Table 9). Consequently, from Column (6) in Table 8, the number of firm exits increased in the four months (February to May) in 2020 compared to the same months in 2019, becoming 2,100 as opposed to 1,700 in the benchmark case. The firm exit rate is 0.14%, whereas it is 0.11% in the benchmark. However, the size of the increases in the rate and number of firm exits compared to the previous year is unchanged at 18%.

4.5.2 Firm Size and Growth in 2020

Another scenario is possible when we consider firm size in year 2020 and onward. As a robustness check, we assume that expected log sales during the COVID-19 are expressed as $x_{it-1} + \left( \mu - \frac{\sigma_i^2}{2} \right) + \left( \mu' - \frac{\sigma_i'^2}{2} \right)$ rather than $x_{it-1} + \left( \mu' - \frac{\sigma_i'^2}{2} \right)$. This assumption is made for the following reason. April 2020 is the starting month of accounting year 2020 for many firms in Japan. In the TSR data, the most recent records on firm sales, $x_{it-1}$, are mostly for year 2018. Therefore, the expected log sales in year 2020 should be extrapolated as $x_{t-1} + \left( \mu - \frac{\sigma_i^2}{2} \right) + \left( \mu' - \frac{\sigma_i'^2}{2} \right)$. We confirm that our results are almost
the same after making this change (Column (7) in Table 8).

5 Concluding Remarks

We found that the COVID-19 pandemic caused sizable heterogeneity in the rate and number of firm exits across industries and regions, which led to an overall sizable increase in firm exits. The size of this increase depends on how firms incorporate the reduction in their current sales to their future sales prospects. While firm exit can increase by 110% in the most pessimistic case, that is, when firms consider the reduction of recent sales growth as permanent, firm exit will increase by only 10% if firms think their sales growth reduction is transitory. Under a more moderate assumption, in which the reduction in firms’ recent sales growth is partially (i.e., 2%) incorporated into their future growth expectations, firm exit will increase by around 20%. Given that we have observed a “reduction” in bankruptcy, these results suggest that potential exits have been avoided so far through decrease in the exit option value.

In the future, we are hoping to extend our work mainly in two directions. First, new comprehensive TSR data, which will be available with a lag of around six months, will incorporate nearly all firm exit cases during the COVID-19 pandemic. Although the current study assumed no change in the exit option value, this must have changed due to several reasons, including government rescue plans. Using new comprehensive TSR data allows us to estimate and compare the exit option value to that before the COVID-19 pandemic.

The second direction is the deeper investigation of firm exit. Because the TSR data we used are extremely valuable, we are hoping to provide constructive stylized facts on how firms are exiting the market. As such, it is also important to characterize firms actually exiting and not-exiting, which allows us to discuss the efficiency of the exit mechanism. To this end, studying particular episodes, such as the global financial crisis of 2008, would also be useful. While the model we constructed is simple and useful, it misses many important features such as population aging and financial constraints. A deeper investigation of firm exit would help refine our model.
References


Baldwin, R. (2020): “Keeping the lights on: Economic medicine for a medical shock,” VoxEU, CEPR.


**A Proofs of Propositions**

**A.1 Proof of Proposition 1**

To find the solution for equation (3), we consider the second-order differential equation:

\[
\frac{\sigma^2_s}{2} V''(x) + \left( \mu - \frac{\sigma^2_s}{2} \right) V'(x) - \rho V(x) = x.
\]

The characteristic function is

\[
\frac{\sigma^2_s}{2} \lambda^2 + \left( \mu - \frac{\sigma^2_s}{2} \right) \lambda - \rho = 0,
\]
yielding

\[ \lambda_1 = -\frac{1}{\sigma_s^2} \left[ \left( \mu - \frac{\sigma_s^2}{2} \right) + \sqrt{\left( \mu - \frac{\sigma_s^2}{2} \right)^2 + 2\rho\sigma_s^2} \right] < 0, \]  

\[ \lambda_2 = -\frac{1}{\sigma_s^2} \left[ \left( \mu - \frac{\sigma_s^2}{2} \right) - \sqrt{\left( \mu - \frac{\sigma_s^2}{2} \right)^2 + 2\rho\sigma_s^2} \right] > 0. \]  

The general solution can be expressed as \( A_1 e^{\lambda_1 x} + A_2 e^{\lambda_2 x} \) with constants \( A_1 \) and \( A_2 \). Further, let the particular solution take form \( v^*(x) = Ax + C \) with constants \( A \) and \( C \). Substituting \( v^*(x) \) into (3), we have

\[ \rho (Ax + C) - \left( \mu - \frac{\sigma_s^2}{2} \right) A = x \implies A = \frac{1}{\rho}, \quad C = \frac{1}{\rho^2} \left( \mu - \frac{\sigma_s^2}{2} \right). \]

Therefore, the solution for \( V(x) \) has the following form:

\[ V(x) = A_1 e^{\lambda_1 x} + A_2 e^{\lambda_2 x} + \frac{x}{\rho} + \frac{1}{\rho^2} \left( \mu - \frac{\sigma_s^2}{2} \right). \]

However, \( A_2 = 0 \) because of the following. For a given \( b \), \( A_2 < 0 \) implies \( V(x) \) is decreasing for sufficiently large \( x \) and diverges to \(-\infty\) because \( \lambda_2 > 0 \), which contradicts the fact that \( V(x) > F \) for any \( x > b \). To eliminate \( A_2 > 0 \), we consider a case with \( x \) so distant from \( b \) that the probability that \( x_t \) reaches \( b \) in a finite time is negligible. Then, \( V(x) \approx \frac{x}{\rho} + \frac{1}{\rho^2} \left( \mu - \frac{\sigma_s^2}{2} \right) \), ignoring the stopping time. However, \( A_2 > 0 \) implies \( V(x) \gg \frac{x}{\rho} + \frac{1}{\rho^2} \left( \mu - \frac{\sigma_s^2}{2} \right) \) for sufficiently large \( x \), which is, again, a contradiction. Hence, we have \( A_2 = 0 \).

To find \( A_1 \) and \( b \), we apply the boundary and smooth-pasting conditions, which imply

\[ A_1 e^{\lambda_1 b} + \frac{b}{\rho} + \frac{1}{\rho^2} \left( \mu - \frac{\sigma_s^2}{2} \right) = F, \quad A_1 \lambda_1 e^{\lambda_1 b} + \frac{1}{\rho} = 0, \]

leading to

\[ b = \rho F + \frac{\sigma_s^2}{\mu - \frac{\sigma_s^2}{2} - \sqrt{\left( \mu - \frac{\sigma_s^2}{2} \right)^2 + 2\rho\sigma_s^2}}, \]

\[ A_1 = -\frac{1}{\rho \lambda_1} e^{\lambda_1 b}. \]
Finally, we obtain

\[ V(x) = \frac{1}{\rho} \left[ x + \frac{1}{\rho} \left( \mu - \frac{\sigma^2}{2} \right) \right] - \frac{1}{\rho \lambda_1} e^{\lambda_1 (x-b)} \quad \text{for } x > b. \]

In the main text, we refer to \( \lambda_1 \) as \( \lambda \).

### A.2 Proof of Proposition 2

For convenience, we use the following notation:

\[ m \equiv \frac{1}{\sigma^2} \left( \mu - \frac{\sigma^2}{2} \right), \quad r \equiv \frac{\rho}{\sigma^2}, \]

and rewrite equations (6) and (4) as

\[ \lambda = -m - \sqrt{m^2 + 2r}, \]
\[ b = \rho F + \frac{1}{m - \sqrt{m^2 + 2r}}. \]

First, \( b \) is decreasing in \( \mu \) because

\[ \frac{\partial b}{\partial \mu} = \frac{-1}{(m - \sqrt{m^2 + 2r})^2} \left[ 1 - \frac{m}{\sqrt{m^2 + 2r}} \right] \frac{\partial m}{\partial \mu} \]
\[ = \frac{1}{\sigma^2} \frac{1}{\sqrt{m^2 + 2r} (m - \sqrt{m^2 + 2r})} < 0. \]

Second, we assume \( \mu < 0 \). \( b \) is decreasing in \( \sigma^2 \) because

\[ \frac{\partial b}{\partial \sigma^2} = \frac{-1}{(m - \sqrt{m^2 + 2r})^2} \left[ \frac{\partial m}{\partial \sigma^2} - \frac{1}{\sqrt{m^2 + 2r}} \left( \frac{m \partial m}{\partial \sigma^2} + \frac{\partial r}{\partial \sigma^2} \right) \right] \]
\[ = \frac{1}{(m - \sqrt{m^2 + 2r})^2} \frac{1}{\sigma^2} \left[ \left( m + \frac{1}{2} \right) - \frac{m \left( m + \frac{1}{2} \right) + r}{\sqrt{m^2 + 2r}} \right] < 0. \]

Third, we assume \( \mu \geq 0 \). Then,

\[ \frac{\partial b}{\partial \sigma^2} > 0 \iff \left( m + \frac{1}{2} \right) \sqrt{m^2 + 2r} > m \left( m + \frac{1}{2} \right) + r. \quad (16) \]
Suppose that the RHS of the last inequality is nonnegative. Then,

\[ \frac{\partial b}{\partial \sigma^2} > 0 \iff \left( m + \frac{1}{2} \right)^2 \left( m^2 + 2r \right) > \left[ m \left( m + \frac{1}{2} \right) + r \right]^2 \]

\[ \iff m + \frac{1}{2} > r \iff \rho < \mu. \]

Because (16) automatically holds if RHS < 0, we have \( \frac{\partial b}{\partial \sigma^2} > 0 \) if \( \rho < \mu \), independent of the sign of the RHS. The remaining case is RHS < 0 and \( \mu \leq \rho \). In this case, we have \( m + \frac{1}{2} \leq r < -m \left( m + \frac{1}{2} \right) \). If \( \mu = 0 \), it implies \( 0 < 0 \). If \( \mu > 0 \), \( 1 \leq \frac{\rho}{\mu} < -\frac{\mu}{\sigma^2} + \frac{1}{2} \Rightarrow \frac{\mu}{\sigma^2} < -\frac{1}{2} \), in contradiction to assumption \( \mu \geq 0 \). Therefore, there is no range of parameters that supports RHS < 0 and \( \mu \leq \rho \). Therefore, under \( \mu \geq 0 \), we can conclude that \( \frac{\partial b}{\partial \sigma^2} > 0 \) if and only if \( \rho < \mu \).

**B Firm Exit Decomposition**

We consider the parameter changes from \( \{ \mu, \sigma_s^2, F_0, \sigma_F^2 \} \) to \( \{ \mu', \sigma_s'^2, F'_0, \sigma_F'^2 \} \) with \( \mu' = \mu' (\mu) = \mu + (\mu' - \mu) \kappa \) and \( \sigma_s'^2 = \sigma_s'^2 (\sigma_s'^2) = \sigma_s^2 + (\sigma_s'^2 - \sigma_s^2) \kappa \).

Recall that the survival rate for a firm with \( x_{it-1} \) is expressed as

\[ \eta_{it} = \Phi \left( \frac{x_{it-1} + A}{B} \right), \]

where

\[ A \equiv \mu - \frac{\sigma_s^2}{2} - \bar{b} (\mu_\kappa, \sigma_{sk}^2) - \rho F_0, \]

\[ B \equiv \sqrt{\rho^2 \sigma_F^2 + \sigma_s^2}, \]

\[ \bar{b} (\mu_\kappa, \sigma_{sk}^2) \equiv \frac{\sigma_{sk}^2}{\mu_\kappa - \frac{\sigma_{sk}^2}{2} - \sqrt{\left( \mu_\kappa - \frac{\sigma_{sk}^2}{2} \right)^2 + 2\rho \sigma_{sk}^2}}. \]

For convenience, we also define

\[ C \equiv \left( \mu_\kappa - \frac{\sigma_{sk}^2}{2} \right)^2 + 2\rho \sigma_{sk}^2, \]

\[ \phi_{hi} \equiv \left( \frac{x_{it-1} + A}{B} \right)^h \phi \left( \frac{x_{it-1} + A}{B} \right), \quad (\text{we use } h = 0, 1) \]

26
and
\[ \bar{\phi}_h = \frac{1}{N_{t-1}} \sum_{i \in \Omega_{t-1}} \phi_{hi}. \]

Note that \( \phi'(z) = -z\phi(z) \).

The aggregate firm exit rate at \( t \) is \( \delta_t = 1 - \frac{1}{N_{t-1}} \sum_{i \in \Omega_{t-1}} \eta_{it} \), implying
\[ d\delta_t = -\frac{1}{N_{t-1}} \sum_{i \in \Omega_{t-1}} d\eta_{it}. \] (17)

We are interested in \( d\eta_{it} = \eta(x_{it-1}|\mu', \sigma_s'^2, F_0', \sigma_F'^2) - \eta(x_{it-1}|\mu, \sigma_s^2, F_0, \sigma_F^2) \). The first-order Taylor expansion of \( \eta_{it} \) around the original parameter values yields
\[ d\eta_{it} \approx \frac{\partial \eta_{it}}{\partial \mu} d\mu + \frac{\partial \eta_{it}}{\partial \sigma_s^2} d\sigma_s^2 + \frac{\partial \eta_{it}}{\partial F_0} dF_0 + \frac{\partial \eta_{it}}{\partial \sigma_F^2} d\sigma_F^2. \] (18)

The first-order derivatives are

\[
\begin{align*}
\frac{\partial \eta_{it}}{\partial \mu} &= \phi_{0i} \frac{\partial A}{B \partial \mu}, \\
\frac{\partial \eta_{it}}{\partial \sigma_s^2} &= \phi_{0i} \frac{\partial A}{B \partial \sigma_s^2} - \phi_{1i} \frac{\partial B}{B \partial \sigma_s^2}, \\
\frac{\partial \eta_{it}}{\partial F_0} &= -\rho \phi_{0i}, \\
\frac{\partial \eta_{it}}{\partial \sigma_F^2} &= -\phi_{1i} \frac{\partial B}{B \partial \sigma_F^2},
\end{align*}
\]

where

\[
\begin{align*}
\frac{\partial A}{\partial \mu} &= 1 - \kappa \frac{\bar{b}}{\mu} = 1 - \kappa \frac{\bar{b}}{\sqrt{C}}, \\
\frac{\partial A}{\partial \sigma_s^2} &= \frac{1}{2} - \kappa \frac{\bar{b}}{\sigma_s^2} = \frac{1}{2} - \kappa \frac{\bar{b} (\mu + \rho \bar{b})}{\sigma_s^2 \sqrt{C}}, \\
\frac{\partial B}{\partial \sigma_s^2} &= \frac{1}{2B}, \\
\frac{\partial B}{\partial \sigma_F^2} &= \frac{\rho^2}{2B}.
\end{align*}
\]

Specifically, let us define \( d\mu = \mu' - \mu \) and \( d\sigma_s^2 = \sigma_s'^2 - \sigma_s^2 \) and set \( dF_0 = d\sigma_F^2 = 0 \).
Using equations (17) and (18), we decompose the change in exit rate, \(d\delta\), as

\[
d\delta = d\delta^\mu + d\delta^{\sigma_s} + d\delta^{\text{residual}},
\]

where \(d\delta^\mu\) and \(d\delta^{\sigma_s}\) are the first-order effects of the changes in \(\mu\) and \(\sigma_s\), respectively, and \(d\delta^{\text{residual}}\) is the sum of the higher-order effects:

\[
d\delta^\mu = -\frac{1}{N_t-1} \sum_{i \in \Omega_{t-1}} \left\{ \frac{\partial \eta_{it}}{\partial \mu} d\mu \right\},
\]

\[
d\delta^{\sigma_s} = -\frac{1}{N_t-1} \sum_{i \in \Omega_{t-1}} \left\{ \frac{\partial \eta_{it}}{\partial \sigma_s^2} d\sigma_s^2 \right\},
\]

\[
d\delta^{\text{residual}} = d\delta - d\delta^\mu - d\delta^{\sigma_s}.
\]
Table 1: Descriptive Statistics of the TSR Data

<table>
<thead>
<tr>
<th></th>
<th>Number of firms</th>
<th>LN(sales)</th>
<th>Sales growth</th>
<th>LN(employment)</th>
<th>Firm ages</th>
</tr>
</thead>
<tbody>
<tr>
<td>The first dataset for 2019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Active at the end of 2018</td>
<td>3,479,995</td>
<td>1,320,427</td>
<td>11.279</td>
<td>0.007</td>
<td>1.738</td>
</tr>
<tr>
<td>(2) Active at the end of 2019 given (1)</td>
<td>3,431,386</td>
<td>1,306,540</td>
<td>11.293</td>
<td>0.008</td>
<td>1.745</td>
</tr>
<tr>
<td>(3) Exited in 2019 given (1)</td>
<td>48,609</td>
<td>13,887</td>
<td>9.984</td>
<td>-0.099</td>
<td>1.040</td>
</tr>
<tr>
<td>(4) Reasons for exit</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Closure</td>
<td>9,564</td>
<td>4,659</td>
<td>9.402</td>
<td>-0.131</td>
<td>0.719</td>
</tr>
<tr>
<td>Dissolution</td>
<td>32,951</td>
<td>7,047</td>
<td>9.899</td>
<td>-0.092</td>
<td>1.044</td>
</tr>
<tr>
<td>Bankruptcy</td>
<td>6,094</td>
<td>2,181</td>
<td>11.501</td>
<td>-0.051</td>
<td>1.707</td>
</tr>
</tbody>
</table>

The units of sales, employment, and age are a thousand yen, a person, and a year, respectively. Sales, employment, and age are for year 2018. Sales growth is the change in sales from 2017 to 2018.

Table 2: Survey Results for Future Firm Sales during the COVID-19 Pandemic

<table>
<thead>
<tr>
<th>Monthly sales growth</th>
<th>Q. Expect $x \leq 50$ from May to Dec for at least one month?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x=(\text{Apr '20/ Apr '19})\times 100$</td>
<td>Yes</td>
</tr>
<tr>
<td>$x &lt; 50$</td>
<td>1,866</td>
</tr>
<tr>
<td>50 $\leq x &lt; 60$</td>
<td>896</td>
</tr>
<tr>
<td>60 $\leq x &lt; 70$</td>
<td>835</td>
</tr>
<tr>
<td>70 $\leq x &lt; 80$</td>
<td>976</td>
</tr>
<tr>
<td>80 $\leq x &lt; 90$</td>
<td>907</td>
</tr>
<tr>
<td>90 $\leq x &lt; 100$</td>
<td>592</td>
</tr>
<tr>
<td>100 $\leq x &lt; 110$</td>
<td>644</td>
</tr>
<tr>
<td>110 $\leq x &lt; 120$</td>
<td>106</td>
</tr>
</tbody>
</table>

Note: This survey was conducted between April and May 2020.
Table 3: Reduced-Form Regression of Firm Exits for 2019

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Dependent vars =</th>
<th>Coef.</th>
<th>SE.</th>
<th>Coef.</th>
<th>SE.</th>
<th>Coef.</th>
<th>SE.</th>
<th>Coef.</th>
<th>SE.</th>
<th>Coef.</th>
<th>SE.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(sales) in 2018</td>
<td>2019 All exits</td>
<td>-0.170</td>
<td>0.002</td>
<td>***</td>
<td>-0.183</td>
<td>0.005</td>
<td>***</td>
<td>-0.190</td>
<td>0.003</td>
<td>***</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>Closure</td>
<td>-0.262</td>
<td>0.014</td>
<td>***</td>
<td>-0.221</td>
<td>0.023</td>
<td>***</td>
<td>-0.152</td>
<td>0.016</td>
<td>***</td>
<td>-0.371</td>
</tr>
<tr>
<td></td>
<td>Dissolution</td>
<td>0.002</td>
<td>0.000</td>
<td>***</td>
<td>0.003</td>
<td>0.000</td>
<td>***</td>
<td>0.003</td>
<td>0.000</td>
<td>***</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>Bankruptcy</td>
<td>-0.607</td>
<td>0.046</td>
<td>***</td>
<td>-0.954</td>
<td>0.081</td>
<td>***</td>
<td>-0.666</td>
<td>0.059</td>
<td>***</td>
<td>-2.820</td>
</tr>
<tr>
<td></td>
<td>Bankruptcy with</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>large-debt</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of firms (y=0 or 1)</td>
<td>10,192 (0.99%)</td>
<td>2,214 (0.22%)</td>
<td>6,139 (0.60%)</td>
<td>1,839 (0.18%)</td>
<td>1,803 (0.18%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of exit (y=1)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 4: Reduced-Form Regression of Firm Exits for 2020

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Dependent vars = default</th>
<th>Coef.</th>
<th>SE.</th>
<th>Coef.</th>
<th>SE.</th>
<th>Coef.</th>
<th>SE.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(sales) in 2018</td>
<td>February</td>
<td>-0.003</td>
<td>0.010</td>
<td>0.011</td>
<td>0.008</td>
<td>0.007</td>
<td>0.009</td>
</tr>
<tr>
<td>Sales growth based on</td>
<td>Exit month =</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>survey (industry &amp;</td>
<td>March</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>prefecture-level</td>
<td>April</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in each month)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ages in 2018</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>-3.042</td>
<td>0.184</td>
<td>***</td>
<td>-3.365</td>
<td>0.161</td>
<td>***</td>
</tr>
<tr>
<td># of firms (y=0 or 1)</td>
<td>854,104</td>
<td>871,198</td>
<td>880,288</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td># of exits (y=1)</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Different from the previous table, firm exit is considered only in the case of large-sized bankruptcy. ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.
Table 5: Estimation Results of Firm Growth and Exits for 2019

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\mu$</th>
<th>$\sigma_s$</th>
<th># of firms</th>
<th>$F_0$</th>
<th>S.E.</th>
<th>$\sigma_F$</th>
<th>SE</th>
<th>$b$</th>
<th># of firms</th>
<th>Exit</th>
<th>No-exit</th>
<th>Exit rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction</td>
<td>0.037</td>
<td>0.231</td>
<td>264,455</td>
<td>253.26</td>
<td>(21.00)</td>
<td>432.30</td>
<td>(9.09)</td>
<td>0.277</td>
<td>3,107</td>
<td>356,338</td>
<td>1,805</td>
<td>0.009</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.020</td>
<td>0.151</td>
<td>112,673</td>
<td>125.58</td>
<td>(33.31)</td>
<td>512.00</td>
<td>(15.14)</td>
<td>-0.331</td>
<td>272</td>
<td>32,849</td>
<td>152,832</td>
<td>0.012</td>
</tr>
<tr>
<td>Information and communications</td>
<td>0.067</td>
<td>0.238</td>
<td>22,426</td>
<td>-2.48</td>
<td>(111.77)</td>
<td>644.25</td>
<td>(47.81)</td>
<td>-4.538</td>
<td>260</td>
<td>36,747</td>
<td>39,024</td>
<td>0.008</td>
</tr>
<tr>
<td>Transport and postal activities</td>
<td>0.035</td>
<td>0.141</td>
<td>26,348</td>
<td>314.92</td>
<td>(78.99)</td>
<td>466.08</td>
<td>(31.15)</td>
<td>0.282</td>
<td>260</td>
<td>36,747</td>
<td>39,024</td>
<td>0.008</td>
</tr>
<tr>
<td>Wholesale and retail trade</td>
<td>0.006</td>
<td>0.161</td>
<td>192,150</td>
<td>-180.54</td>
<td>(44.55)</td>
<td>632.79</td>
<td>(20.84)</td>
<td>-2.642</td>
<td>3,962</td>
<td>262,164</td>
<td>89,992</td>
<td>0.009</td>
</tr>
<tr>
<td>Real estate agencies and goods rental</td>
<td>0.048</td>
<td>0.275</td>
<td>59,035</td>
<td>-264.61</td>
<td>(75.30)</td>
<td>636.35</td>
<td>(32.28)</td>
<td>-5.160</td>
<td>775</td>
<td>89,992</td>
<td>10,693</td>
<td>0.006</td>
</tr>
<tr>
<td>and leasing</td>
<td>0.006</td>
<td>0.169</td>
<td>17,484</td>
<td>-894.18</td>
<td>(276.80)</td>
<td>921.08</td>
<td>(126.72)</td>
<td>-9.803</td>
<td>361</td>
<td>26,157</td>
<td>9,935</td>
<td>0.008</td>
</tr>
<tr>
<td>Accommodation, eating, and drinking</td>
<td>0.014</td>
<td>0.168</td>
<td>17,580</td>
<td>-277.68</td>
<td>(259.81)</td>
<td>691.74</td>
<td>(122.36)</td>
<td>-3.961</td>
<td>428</td>
<td>26,334</td>
<td>9,935</td>
<td>0.008</td>
</tr>
<tr>
<td>services</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Living-related and personal services</td>
<td>0.037</td>
<td>0.165</td>
<td>7,100</td>
<td>166.83</td>
<td>(134.92)</td>
<td>479.49</td>
<td>(55.96)</td>
<td>-1.194</td>
<td>64</td>
<td>10,693</td>
<td>10,693</td>
<td>0.006</td>
</tr>
<tr>
<td>and amusement services</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td>0.038</td>
<td>0.181</td>
<td>65,166</td>
<td>-339.36</td>
<td>(41.00)</td>
<td>418.35</td>
<td>(16.88)</td>
<td>0.641</td>
<td>538</td>
<td>89,168</td>
<td>89,168</td>
<td>0.006</td>
</tr>
<tr>
<td>Medical services</td>
<td>0.047</td>
<td>0.261</td>
<td>8,011</td>
<td>-911.44</td>
<td>(338.18)</td>
<td>920.84</td>
<td>(144.36)</td>
<td>-11.733</td>
<td>93</td>
<td>10,935</td>
<td>10,935</td>
<td>0.008</td>
</tr>
<tr>
<td>Compound services</td>
<td>0.054</td>
<td>0.309</td>
<td>82,786</td>
<td>-1686.98</td>
<td>(71.63)</td>
<td>1229.59</td>
<td>(31.63)</td>
<td>-19.404</td>
<td>1,191</td>
<td>117,777</td>
<td>117,777</td>
<td>0.010</td>
</tr>
<tr>
<td>Miscellaneous services</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unweighted mean (sum)</td>
<td>0.049</td>
<td>0.227</td>
<td>939,338</td>
<td>-251.49</td>
<td></td>
<td>665.40</td>
<td></td>
<td>-4.797</td>
<td>12,856</td>
<td>1,211,986</td>
<td>1,211,986</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Table 6: Correlations between the Estimated $F_0$ and Firm Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Fixed costs</th>
<th>Working capital</th>
<th>Debts</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ratio to sales</td>
<td>log(costs per firm)</td>
<td>Labor share</td>
<td>Ratio to sales</td>
<td>Ratio to assets</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.236</td>
<td>0.316</td>
<td>0.155</td>
<td>0.102</td>
<td>0.461</td>
</tr>
<tr>
<td>Spearman rank correlation</td>
<td>0.236</td>
<td>0.176</td>
<td>0.273</td>
<td>0.309</td>
<td>0.418</td>
</tr>
</tbody>
</table>

The figures represent the correlation coefficients at the industry level. The number of industries is 10.
Table 7: Survey Results on Firm Sales Growth during the COVID-19 Pandemic

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
<th># of obs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>February</td>
<td>March</td>
<td>April</td>
<td>May</td>
<td>February</td>
<td>March</td>
<td>April</td>
</tr>
<tr>
<td></td>
<td>-0.059</td>
<td>-0.108</td>
<td>-0.223</td>
<td>-0.248</td>
<td>Mean</td>
<td>0.182</td>
<td>0.218</td>
</tr>
<tr>
<td>10%</td>
<td>-0.165</td>
<td>-0.283</td>
<td>-0.561</td>
<td>-0.575</td>
<td>10%</td>
<td>0.058</td>
<td>0.071</td>
</tr>
<tr>
<td>25%</td>
<td>-0.100</td>
<td>-0.148</td>
<td>-0.258</td>
<td>-0.299</td>
<td>25%</td>
<td>0.106</td>
<td>0.143</td>
</tr>
<tr>
<td>50%</td>
<td>-0.052</td>
<td>-0.089</td>
<td>-0.180</td>
<td>-0.209</td>
<td>50%</td>
<td>0.170</td>
<td>0.204</td>
</tr>
<tr>
<td>75%</td>
<td>-0.011</td>
<td>-0.034</td>
<td>-0.117</td>
<td>-0.139</td>
<td>75%</td>
<td>0.231</td>
<td>0.279</td>
</tr>
<tr>
<td>90%</td>
<td>0.048</td>
<td>0.037</td>
<td>-0.047</td>
<td>-0.060</td>
<td>90%</td>
<td>0.317</td>
<td>0.371</td>
</tr>
<tr>
<td># of obs</td>
<td>8,462</td>
<td>11,018</td>
<td>14,092</td>
<td>12,268</td>
<td># of obs</td>
<td>8,462</td>
<td>11,018</td>
</tr>
</tbody>
</table>

We calculate sales growth $\mu$ and uncertainty $\sigma_s$ for each industry, prefecture, and month, and then take the quantiles and means for industries and prefectures in a given month.

Table 8: Simulation Results on the Number and Rate of Firm Exits in Different Cases

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark</td>
<td>No effect</td>
<td>No effect</td>
<td>Large effect</td>
<td>Imbalance</td>
<td>Firm size</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\kappa = 0.02$</td>
<td>on growth (optimistic)</td>
<td>$\kappa = 0$</td>
<td>$\kappa = 0.1$</td>
<td>adjusted</td>
<td>greater by $\mu$</td>
<td></td>
</tr>
<tr>
<td>Firm exit rate (%)</td>
<td>Ave '19</td>
<td>0.094</td>
<td>0.094</td>
<td>0.094</td>
<td>0.094</td>
<td>0.094</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>Feb '20</td>
<td>0.101</td>
<td>0.097</td>
<td>0.099</td>
<td>0.114</td>
<td>0.177</td>
<td>0.128</td>
</tr>
<tr>
<td></td>
<td>Mar '20</td>
<td>0.106</td>
<td>0.100</td>
<td>0.103</td>
<td>0.126</td>
<td>0.189</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td>Apr '20</td>
<td>0.118</td>
<td>0.106</td>
<td>0.113</td>
<td>0.152</td>
<td>0.212</td>
<td>0.149</td>
</tr>
<tr>
<td></td>
<td>May '20</td>
<td>0.121</td>
<td>0.108</td>
<td>0.115</td>
<td>0.159</td>
<td>0.218</td>
<td>0.154</td>
</tr>
<tr>
<td></td>
<td>Ave '20</td>
<td>0.112</td>
<td>0.103</td>
<td>0.107</td>
<td>0.138</td>
<td>0.199</td>
<td>0.141</td>
</tr>
</tbody>
</table>

# of firm exits

|                      | Ave '19  | 2318  | 2318  | 2318  | 2318  | 2318  | 2949  | 2314 |
|                      | Feb '20  | 2486  | 2401  | 2444  | 2800  | 4367  | 3160  | 2481 |
|                      | Mar '20  | 2623  | 2474  | 2550  | 3099  | 4655  | 3313  | 2618 |
|                      | Apr '20  | 2916  | 2622  | 2775  | 3752  | 5236  | 3679  | 2910 |
|                      | May '20  | 2990  | 2654  | 2829  | 3933  | 5366  | 3788  | 2984 |
| Increase from '19    | 1741    | 878   | 1325  | 4310  | 10350 | 2143  | 1737  |      |

Change from 2019 (%)

|                      | 18.8   | 9.5   | 14.3  | 46.5  | 111.6 | 18.2  | 18.8  |

Note: The simulation is conducted only for the following eight industries: construction; manufacturing; information and communications; transport and postal activities; wholesale and retail trade; real estate agencies and goods rental and leasing; accommodation, eating, and drinking services; and living-related and personal services and amusement services.
Table 9: Estimation Results on Exit Option Value when Adjusting the Number of Observations for Firm Exits and Non-Exits

<table>
<thead>
<tr>
<th></th>
<th>Imbalance adjusted</th>
<th></th>
<th>Benchmark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_0$</td>
<td>$\sigma_F$</td>
<td>$b$</td>
</tr>
<tr>
<td>Construction</td>
<td>385.33</td>
<td>407.70</td>
<td>1.60</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>250.76</td>
<td>488.14</td>
<td>0.92</td>
</tr>
<tr>
<td>Information and</td>
<td>246.76</td>
<td>595.88</td>
<td>-2.05</td>
</tr>
<tr>
<td>communications</td>
<td>472.59</td>
<td>435.39</td>
<td>1.86</td>
</tr>
<tr>
<td>Transport and</td>
<td>-122.79</td>
<td>621.80</td>
<td>-2.06</td>
</tr>
<tr>
<td>postal activities</td>
<td>-126.02</td>
<td>610.61</td>
<td>-3.77</td>
</tr>
<tr>
<td>Wholesale and</td>
<td>-480.01</td>
<td>730.49</td>
<td>-5.98</td>
</tr>
<tr>
<td>retail trade</td>
<td>-803.06</td>
<td>903.34</td>
<td>-8.89</td>
</tr>
<tr>
<td>Real estate agencies</td>
<td>-370.70</td>
<td>440.86</td>
<td>0.84</td>
</tr>
<tr>
<td>and goods rental</td>
<td>-358.37</td>
<td>414.55</td>
<td>0.83</td>
</tr>
<tr>
<td>and leasing</td>
<td>-705.23</td>
<td>884.84</td>
<td>-9.67</td>
</tr>
<tr>
<td>Accommodation, eating,</td>
<td>-1424.96</td>
<td>1178.99</td>
<td>-16.78</td>
</tr>
<tr>
<td>and drinking services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Living-related and</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>personal services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>and amusement services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Medical services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compound services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous services</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unweighted mean</td>
<td>-131.46</td>
<td>642.72</td>
<td>-3.60</td>
</tr>
</tbody>
</table>
Figure 1: Firm Bankruptcies

Note: Data sources are the Epiq AACER for the United States, Insolvency Service for England and Wales, and Tokyo Shoko Research for Japan. The shaded areas indicate the period in which the COVID-19 affected the economy (from February 2020 for Japan and from March 2020 for the rest).
Figure 2: Effects of the COVID-19 Pandemic on Firm Exit Rates by Industry and Region

Con refers to construction; Man to manufacturing; ICT to information and communications; Tr to transport and postal activities; WR to wholesale and retail trade; RE to real estate agencies and goods rental and leasing; Hos (hospitality) to accommodation, eating, and drinking services; and Ser to living-related and personal services and amusement services. H/T stands for Hokkaido and Tohoku, NK/K for north Kanto and Koshin, H for Hokuriku, T for Tokai, Kin for Kinki, Ch for Chugoku, Sh for Shikoku, Ky for Kyushu, SK for south Kanto (excluding Tokyo), and Tok for Tokyo.
Figure 3: Effects of the COVID-19 Pandemic on the Number of Firm Exits by Industry and Region

See the notes of Figure 2 for the abbreviations.
Figure 4: Background for Changes in Firm Exits

See the notes of Figure 2 for the abbreviations.

Figure 5: Decomposition of the Reasons for Firm Exit Changes

The circles indicate the sum of three factors (i.e., changes in the exit rate). See the notes of Figure 2 for the abbreviations.
For the left-hand side panel, the horizontal axis represents the simulated number of firm exits from February to May 2020 and the vertical axis represents the actual number of firm bankruptcies from February to May 2020. For the right-hand side panel, the horizontal axis represents the increase in the simulated number of firm exits from February to May 2020 compared to the same four months in 2019 and the vertical axis represents the actual change in the number of firm bankruptcies from February to May 2020 compared to the same four months in the previous year. See the notes of Figure 2 for the abbreviations.