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Macroscopic Structure and Evolution in the Japanese Production Network*

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Abstract

We investigate the Japanese production network at firm-level, using the dataset of financial statements and supplier-customer relationship for one million firms in Japan collected by Tokyo Shoko Research Inc. from 2011 to 2016. Chakraborty et al. (2018) reported that the Japanese production network has a tightly-knit structure with a giant strongly connected component (GSCC) core surrounded by its upstream (IN) and downstream (OUT) components constituting two half shells. In this paper, we analyze its macroscopic structure and evolution. Generally, larger firms obey a power-law in size distribution (Pareto's law) and have a property that the variance in their growth rates does not depend on their size, (Gibrat's law). We focus on the relationship between the macroscopic network structure and firm size and growth rate measured by sales. Major results of this study are as follows. First, the firms obeying Pareto's law are mainly composed of GSCC, and the firms within the IN component tend to be smaller than the firms in the other components. Second, although about half of supplier-customer links are disrupted or reformed from 2011 to 2016, Japan's production network has stable firms that do not move between components of macroscopic structure. Third, we can observe Gibrat's law for such stable firms for each component, but the applicable region of firms located in the IN component is extremely small compared to the other components. The results obtained suggest that the macro hierarchy and stability of Japan's production network helps characterize the stability of firm growth.

Keywords: production network, hierarchical structure, firm growth rate, Pareto's law, Gibrat's law

JEL classification: D22, D85, L14, L16

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I. Introduction

One of the most important networks in the economy is a production network, which is the backbone of the economy and refers to the supply chain, and interactions between individual firms expect to play an important role in the economy. Although, traditionally, the industrial structure has been studied on the basis of input-output tables (Leontief, 1986), such classification of firms by industry may be too aggregated, and much research in recent years has begun to focus on the analysis of firm-level network based on a comprehensive data of supply chain (Fujiwara and Aoyama, 2010; Atalay et al., 2011; Acemoglu et al., 2012; Luo et al., 2012). The topology of firm-level production networks can expect to become an alternative approach to characterize not only industrial structure such as input-output tables but also the nation economy.

Recently, Chakraborty et al. (2018) have shown that the firm-level production networks in Japan form a walnut structure having a tightly knit structure with a giant strongly connected component surrounded by its upstream and downstream constituting two half shells, and they have a community structure including overexpressions of industrial and regional components. And Kichikawa et al. (2019) have provided a deep understanding of the macroscopic topology of the Japanese production network by analyzing the walnut structure in terms of hierarchical and circular flow, and suggested the needs to replace the conventional industrial classification scheme to a new one based on the actual transactions. However, the relation between the macroscopic structure and evolution of Japanese production network remains unclear. Since previous works have focused only on the topological aspect of the networks, we attempt to reveal the dynamics of individual firms in the networks, especially, how the macroscopic structure of Japanese production network relates to the firms' size and its growth.

For more than 100 years researchers have been investigating the relation between the size and its growth for the individuals, especially in the economy, the statistical feature in distributions with a power-law tail known as Pareto's law (Pareto, 1964) and the dynamics process of growth independently its size known as the law of proportionate effect or Gibrat's law (see Sutton (1997)) have importance. For example of application in Japan, Fujiwara et al. (2007) have investigated the largest database of Japanese small and mid-sized companies in terms of the breakdown of Gibrat's law, and confirmed that the firms' size happen the breakdown are consistent with the definition by the small and medium enterprise agency in Japan. However, they have not addressed them in terms of the macroscopic structure of the network. In fact, the definition of small and medium enterprises by the government depends on the industrial sector. In this work, we present the feature of Japanese production network in terms of macroscopic structure, and the relation to the evolution of individual firms.

The rest of this paper is organized as follow. First, we explain the methodology to investigate how macroscopic structure of the network relates to the growth of individuals in Section II. And Section III provides some results and discussion in terms of hierarchy and stability in Japanese production network. Finally, conclusion and research perspective will be given in Section IV.

II. Methodology

A. *Japanese Production Network*

Our data for the Japanese production network are based on a survey conducted by Tokyo Shoko Research (TSR), Inc., one of the leading credit research agencies in Tokyo, and was supplied to us through the Research Institute of Economy, Trade and Industry (RIETI). The survey inquires firms who the top 24 suppliers and customers for each, and this form of data collection can expect to avoid including data on a one-time trades. Although the replies from large firms that have many suppliers and customers become incomplete, these data could be supplemented with data on the other side of trade. In this work, therefore, we assume that our data provides a good approximation of the real complete picture of Japanese economy by combining all the submissions from both side of trade, which covers about one million firms and several millions supplier-customer relationships. In this work, we investigate the macroscopic structure and evolution of Japanese production networks by using four datasets collected in 2011, 2012, 2014 and 2016.

We represent the data as a directed network composed of active firms and supplier-customer links and not contained self-loops, where inactive and failed firms are excluded by using an indicator flag on the basic financial information. Let us denote that a directional link is present as $i \to j$ in the production network, where firm i is a supplier to another firm j, or equivalently, firm j is a customer of firm i. Although a production network should represent flows of products or services as a directed network with weights, our networks are unweighted networks because of the property of our data. The macroscopic structure mentioned later are independent of weights of the network, so allowing us to analyze them without additional data.[1] Moreover, the financial statements can provide each node several attributes, firm size, which is mea-

[1] This problem can be solved by using an apportionment based on input-output tables discussed in Inoue and Todo (2019).

sured as sales, profit, number of employees and the firm's growth, the major and minor classifications of industrial sectors, so on. For the sake of our study, we focus on firm size measured as sales and its growth rate.

B. *Bow-tie Decomposition*

In order to elucidate the relation between the economic dynamics at firm-level and the macroscopic flow of the production network, we focus on its hierarchy and circularity. In general, a directed network has the largest connected component when it is viewed as an undirected network, which is called the giant weakly connected components (GWCC). And GWCC can decompose as giant strongly connected components (GSCC), which is largest size of SCC in GWCC, and its upstream and downstream portions (IN and OUT), known as a bow-tie decomposition in the Web by Broder et al. (2000),

$$\mathrm{GWCC} = \mathrm{IN} + \mathrm{GSCC} + \mathrm{OUT} + \mathrm{TE} \;, \tag{1}$$

where we defined the components of GWCC not belonging to the GSCC, IN and OUT components as Tendril (TE). In terms of the evolution of the networks, we should note that there are firms disconnected or disappeared from GWCC in different two years. In this work, therefore, we focus on the firms having located on the GWCC at least once for our four observation points (2011, 2012, 2014, and 2016), and we define the disconnected/disappeared firms from GWCC at the point as DIS components.

As reported in (Chakraborty et al., 2018; Kichikawa et al., 2019), the Japanese production networks form the walnut structure, which are a quit different from the shape of bow-tie observed in the Web. Because the IN and OUT components are not as separated as the two wings of a bow-tie, they are more similar to two halves of a walnut shell, surrounding the central GSCC core. In the sense of industrial structure of Japanese production network, the IN components are mainly composed of Construction, Information & Communications, and Scientific Research, Professional & Technical Services. And Mining, Manufacturing, Transport & Postal, and Wholesale sectors are important constituents in the GSCC. Moreover, the OUT components mainly include Retail Trade, Finance & Insurance, Accommodations, Eating/Drinking Services, Living-related/Personal & Amusement Services, and Education, Learning Support. The bow-tie decomposition can help us understand the hierarchical and circular flow of the networks from a macroscopic point of view because each component by the decomposition in the production network has its own industrial characteristics.

C. *Measurement of Evolution of Japanese Production Network*

First, we can define the node-level evolution as the growth rate of node attributes with respect to firms' size such as sales and in/out-degree. Let $x_i(t)$ be a size of firm i at the point t, and we denote its growth rate $R_i(t,t') = x_i(t')/x_i(t)$, where $t < t'$. And we also express the rate in terms of its logarithm, $r_i = \log_{10} R_i$.

In order to examine the edge-level evolution of individual firms between networks at different years, we compute the similarity between the edge lists at different two years with the Jaccard index,

$$J_i(t,t') = \frac{|E_i(t) \cap E_i(t')|}{|E_i(t) \cup E_i(t')|} \ , \tag{2}$$

where $E_i(t)$ is the set of in-/out-going edges of firm i at certain year t. As investigated by Mizuno et al. (2014), $E_i(t')$ can be distinguished as

$$|E_i(t')| = |E_i(t) \cap E_i(t')| + |\Delta^+ E_i(t,t')| - |\Delta^- E_i(t,t')| \ , \tag{3}$$

where $E_i(t) \cap E_i(t')$ corresponds to survived edges and $\Delta^\pm E_i(t,t')$ are added and removed edges. The Jaccard index is given as the fraction of edges which appear in both different two years over the aggregated number of edges of two years, and it can be seen as a rewiring ratio to measure how the firm changed its suppliers and customers. If the firm i located in GWCC rewired completely or moved to DIS, the Jaccard index should be zero, $J_i = 0$. In contrast, when the edge list of firm i does not change, the Jaccard index becomes $J_i = 1$. We should note that $J_i = 1$ does not always mean that the firm i has not moved between the bow-tie components because the transaction can occur by the rewiring of the other firm. For example, when $J_i = 1$, the firm i located in IN (OUT) component can move to GSCC if its supplier (customer) would get new linkage to the firm having located in GSCC.

In order to investigate the evolution of macroscopic structure of the network, we measure the transaction rate between bow-tie components at different two years as,

$$F_{ab}(t,t') = \frac{|V_a(t) \cap V_b(t')|}{|V_a(t)|} \quad (t < t') \ , \tag{4}$$

where we use the subscripts a and b for the bow-tie components, and $V_a(t)$ is the set of nodes of firms located in the bow-tie component a at certain year t. Thus, the element of F_{ab} represents the fraction of the number of firms having moved from a to b component at certain year over the number of firms having located in component a at initial year, so that $\sum_b F_{ab} = 1$. Since the GSCC is the core of economic activities, transitions to the GSCC from the others seems to be important in terms of firms' growth.

D. *Pareto's Law, Gibra's Law and Detailed Balanced*

In terms of the stability of firms' growth in the networks, we investigate statistical law in the firms' size and its growth rate distribution. It has been known that the firms' size x follow the power-law distribution,

$$P_>(x) \propto x^{-\mu} \quad \text{for } x \to \infty \ , \tag{5}$$

where $P_>(x)$ is a complementary cumulative distribution function (CCDF) for x, which corresponds to the probability when the individual firm has a size greater than x. And Eq. 5 is well known as Pareto's law, which was first observed in the field of personal income (Pareto, 1964), the index μ is called the Pareto index. Moreover, the probability density function (PDF) for the growth rate $q(R|x)$ on condition that size x in an initial year is fixed becomes statistically independent of firm size in the initial year when the $x(t)$ becomes larger than certain size x_0,

$$q(R|x(t)) = q(R) \quad \text{for } x(t) > x_0 \ . \tag{6}$$

This is known as the Gibrat's law (Sutton, 1997). Fujiwara et al. (2003, 2004) have analytically shown that Pareto's law is derived from Gibrat's law under the detailed balance condition,

$$P_J(x(t), x(t')) = P_J(x(t'), x(t)) \ , \tag{7}$$

where detailed balance requires the symmetry in the joint PDF $P_J(x(t), x(t'))$ under the time reversal exchange $t \leftrightarrow t'$.

In terms of economy, it is highly important issue to distinguish whether the firm has enough large size to reach the Pareto's regime or not, because such firms hold Gibrat's law and such certain firms' size can expect to define small and mid-sized firms quantitatively. In this work, therefore, we estimate the conditional PDF for the growth rate using the method of Fujiwara et al. (2003, 2004), in addition, we attempt to reveal the relation to the macroscopic structure.

III. Results and Discussion

As discussed previously, the aim of our work was to reveal how the macroscopic structure of production network in Japan relates to the dynamics of individual firms. For the sake of our purpose, we focus on $1,181,566$ firms having been belonging to GWCC at least once for our four observation points (2011, 2012, 2014, and 2016). First, we show the number of firms belonging to each bow-tie component in different year in Table 1. As reported by Chakraborty et al. (2018), about half of firms are belonging to GSCC. Although the number of nodes and edges are increasing, the fractions of each component in GWCC do not so change.

Component	2011	2012	2014	2016
GWCC	991,118	1,014,494	1,066,476	1,066,037
IN	18.13 %	18.68 %	20.34 %	20.63 %
GSCC	49.71 %	49.04 %	49.48 %	49.73 %
OUT	28.51 %	28.54 %	26.49 %	26.16 %
TE	3.65 %	3.73 %	3.69 %	3.48 %
# of edges	4,459,205	4,558,494	4,897,050	4,974,826

Table 1. The number of firms belonging to each bow-tie component in different year.

Figure 1 shows the CCDF $P_>(x)$ for in- and out-degree, $x(t) = k_{\text{in,out}}(t)$, in terms of bow-tie decomposition for the GWCC in 2016. According to Chakraborty et al. (2018), one of the most remarkable features is the shortest distance from GSCC to IN/OUT, which is mostly one, and the max SCC size in IN and OUT is 5 and 6, respectively. This feature seems to appear in degree distributions. Compared with IN and OUT components in Figure 1, it is clear that the firms belonging to OUT component tend to have larger number of suppliers. This is because the number of firms located upstream of firms belonging IN component becomes consequently small. However, this kind of feature did not occur in the opposite case. The number of customers of firms belonging to IN component is at most one hundred.

In Figure 2, we show the CCDF $P_>(x)$ for firm size measured by sales and number of employees, $x(t) = S(t)$ and $N(t)$, in terms of bow-tie decomposition for the GWCC in 2016. As can be seen, similar behaviours were observed in both case, the Pareto's region is mostly covered by the firms belonging to GSCC and OUT components. And the firms belonging to IN component tend to be smaller than the firms in the other components. This suggests that the firm size may have relation to the hierarchy in Japanese production network.

In order to investigate how individual firms grow up in the production network, we estimate the change of supplier-customer linkages. Figure 3 shows the degree dependency of Jaccard index of individual firms between the networks in 2014 and 2016. As observed in degree distribution, similar behaviour between GSCC and OUT with respect to supplier links were observed. And the peaks in Figure 3 may indicate that about 80% of supplier-customer linkages of larger firms seems to be unchanged in two years. However, the mean values

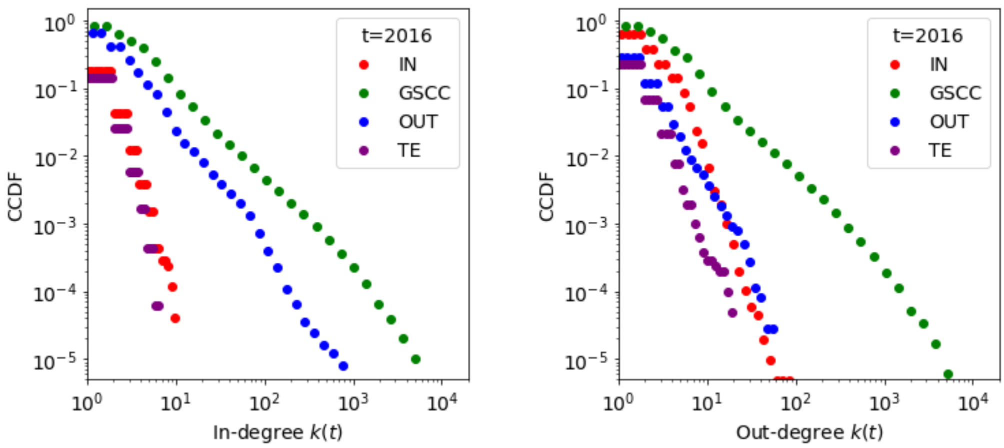

Figure 1. The CCDF for in- and out-degree in terms of bow-tie decomposition for the GWCC in 2016. The red, green, blue and purple represents IN, GSCC, OUT and TE component, respectively.

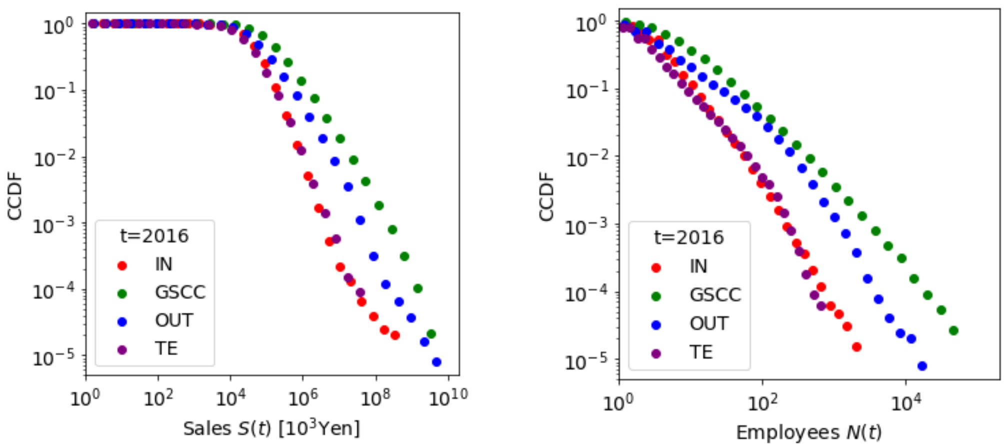

Figure 2. The CCDF for firm size measured by sales and the number of employees in terms of bow-tie decomposition for the GWCC in 2016. The red, green, blue and purple represents IN, GSCC, OUT and TE component, respectively.

for the Jaccard index are $\langle J_i^{\mathrm{in}} \rangle = 0.656$ and $\langle J_i^{\mathrm{out}} \rangle = 0.611$. This indicate that the change of supplier-customer relation may happen for small and mid-sized firms frequently, and the firms tend to change the customers more than suppliers. In addition, we summarized the statistics related to estimate the change of supplier-customer relations and Jaccard index in Table 2. Note that we did not distinguish in- and out-going edges for the simplicity, and the figures in parentheses in Table 2 indicate the ratio to the number of edges in the initial year. It is apparent that only 63% of the supplier-customer relations in 2011 are survived in five years, and in terms of the Jaccard index, more than half of relationships in production network are different. This is partially consistent with results obtained by Mizuno et al. (2014), however, our results suggest that Japanese production networks seem to be stable in the short term, but they were dynamically changing in our five-year observation of large-scale network.

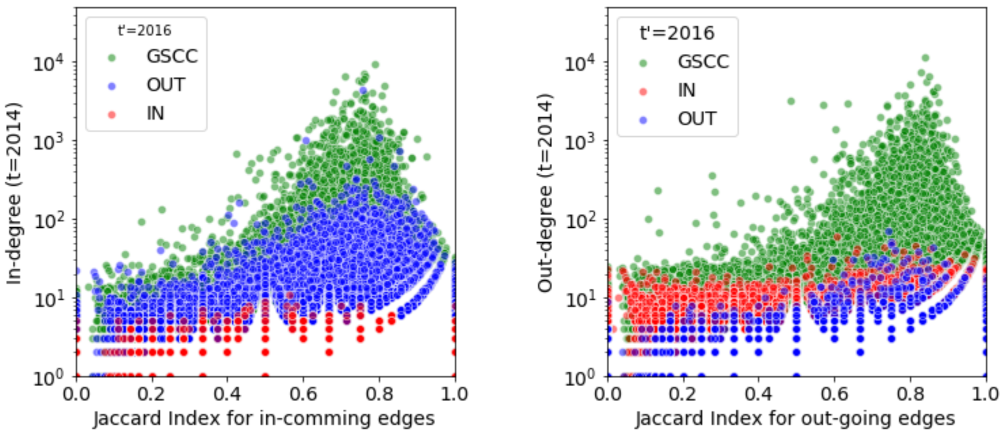

Figure 3. In- and out-degree dependency of Jaccard index between the network in 2014 and 2016. The red, green and blue represents IN, GSCC and OUT component, respectively.

We are now in a position to investigate the evolution of macroscopic structure under the dynamically changing supplier-customer relations. Figure 4 shows the transitions between bow-tie components from 2011 to 2016, where we ignored the transition between DIS components in two years, and we have observed high stability in Japanese production network under the bow-tie decomposition. The elements of transition matrix represent the fraction of the number of firms having moved between bow-tie components, and we also visualized them as Sankey diagrams. Compared the GSCC with the other components, the stability of firms belonging to GSCC is remarkable, about 80% of the firms have located in GSCC at 2011 continued to be the GSCC for five years. One may guess that the firms belonging to IN and OUT component could move to the GSCC as growing. For the firms having located in IN/OUT

Years (t, t')	#of survived $\lvert E(t) \cap E(t')\rvert$	#of added $\lvert\Delta^+ E(t, t')\rvert$	#of removed $\lvert\Delta^- E(t, t')\rvert$	J
$(2011, 2012)$	3,962,896 (0.889)	595,598 (0.134)	496,309 (0.111)	0.784
$(2012, 2014)$	3,652,297 (0.801)	1,244,753 (0.273)	906,197 (0.199)	0.629
$(2014, 2016)$	4,077,497 (0.833)	897,329 (0.183)	819,553 (0.167)	0.703
$(2011, 2016)$	2,810,152 (0.630)	2,810,152 (0.485)	2,164,674 (0.370)	0.424

Table 2. Change of supplier-customer relation. The figures in parentheses indicate the ratio to the number of edges in the initial year.

at 2011, firms less than 20% could join the GSCC as a core of economic activity, but the larger number of firms than them have disconnected or disappeared from the production network in Japan. And it seems to be more difficult for firms belonging to TE component to reach GSCC. In Figure 5, we summarized transitions between bow-tie components in two close observation points. As we have seen in the edge-level evolution, the stable structures each bow-tie component become remarkable in short term.

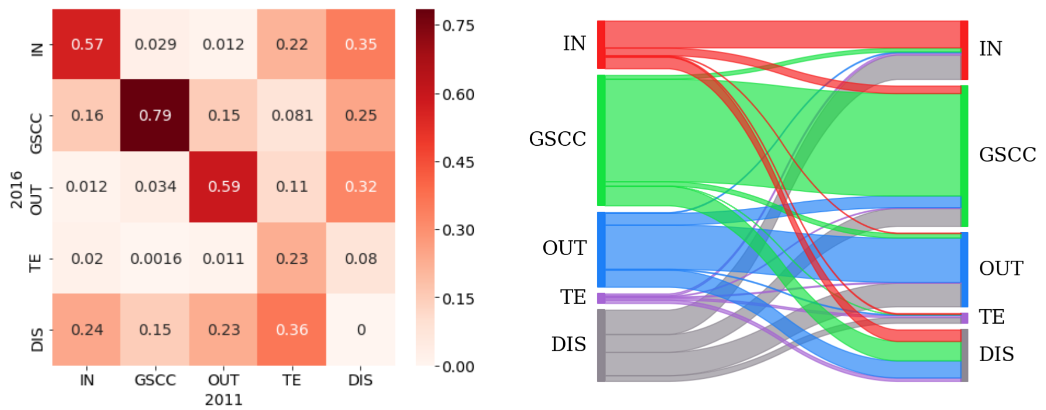

Figure 4. Transitions between bow-tie components from 2011 to 2016. The elements of transition matrix (left) represent the fraction of the number of firms having moved between bow-tie components. And right figure visualized left matrix as Sankey diagram.

We consider that the walnut structure of the supply chain network represents the maturity of the economy. For example, in the closed and undeveloped

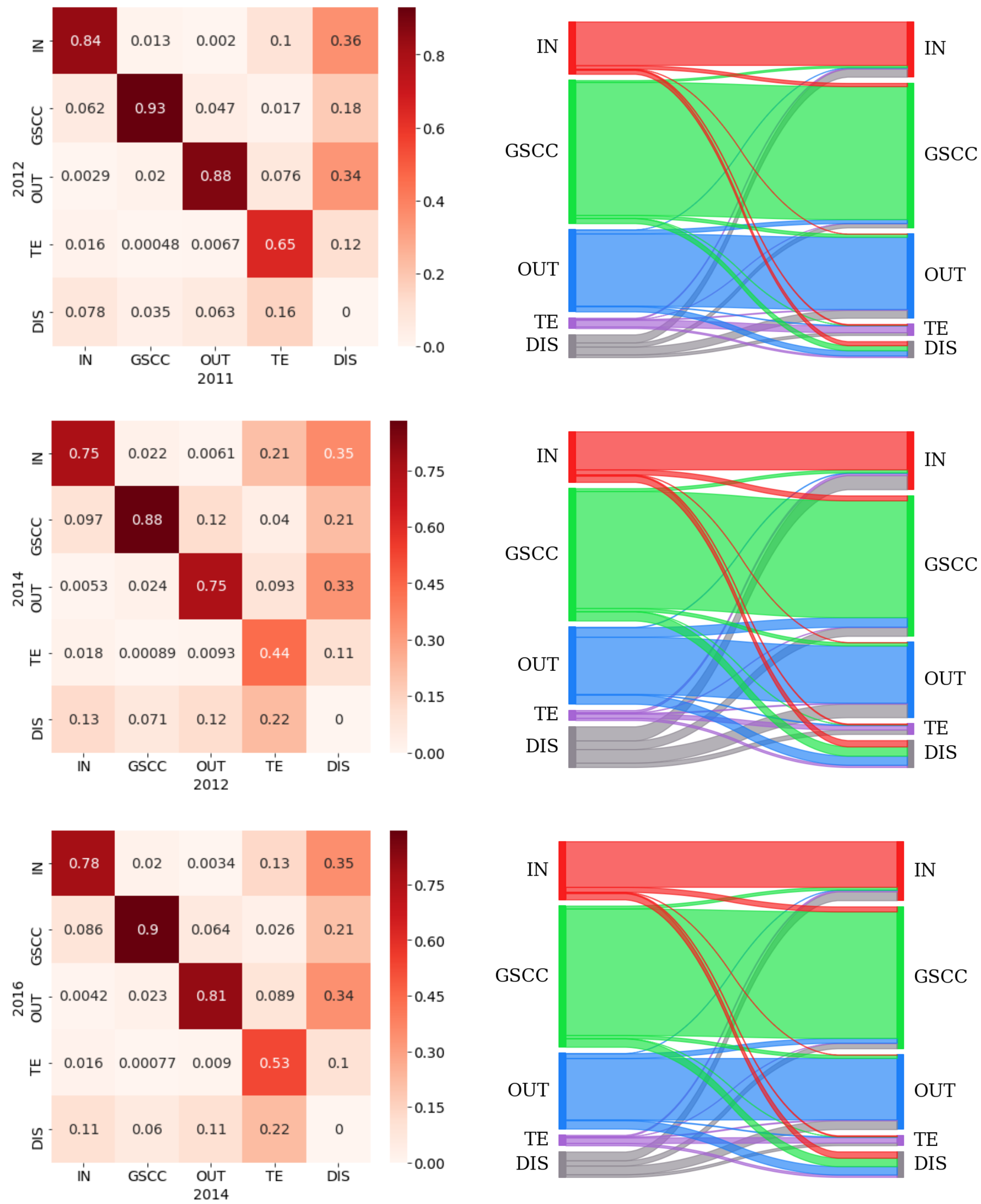

Figure 5. Transitions ransitions between bow-tie components in two close observation points, $(t, t') = (2011, 2012)$, $(2012, 2014)$, and $(2014, 2016)$, respectively.

country constructed from only the primary sector of the economy, the supply chain network is the bipartite network in which customers directly connect to producers of agricultural products, fish, meat, wood, etc. However, if some persons in that country begin to make processed foods and processed materials, these producers buy agricultural products, fish, meat, wood, etc. from persons working in the primary sector of the economy, and sell processed material to customers. If persons making processed goods consider to grow up productivity, they introduce machines. Thus, some persons in this country begin to make machines. Therefore, the GSCC emerges in the supply chain network, and the pursuit of efficiency and innovation make the GSCC giant and complex. As investigated by Hidalgo (2015), economic complexity is the highest in Japan. Thus, the maturity of the economy is highest in Japan, therefore, the structure of GSCC in Japan is stable.

Finally, we focus on the growth of firms' size in the production networks. The firms holding Gibrat's law can be expected to grow up stably, which means that the fluctuation of growth rate distribution does not depend of its size at initial year, but for the firms not holding Gibrat's law the groth-rate fluctuation become larger for smaller firms. Therefore, it is highly important to understand when the breakdown of Gibrat's law happens and what kind of firms can reach in Pareto's region in the production networks. Before moved to the discussion in terms of the macroscopic structure, we confirm above statements using our dataset. The joint PDF for firms' size measured by sales and number of employees at 2014 and 2016 is shown in Figure 6. And Figure 7 shows the conditional PDF $q(r|x(t))$ of logarithmic growth rate r of firms' size measured by sales and number of employees, $x(t) = S(t)$ and $N(t)$, and the change as the initial value, where we divided the range of initial value into logarithmically equal bins as

$$S(t) \in \left[10^{3+0.5n}, 10^{3+0.5(n+1)}\right] \quad \text{and} \quad N(t) \in \left[10^{0.5n}, 10^{0.5(n+1)}\right] \ . \qquad (8)$$

The conditional PDF for the firm size measured by sales has an explicit dependence $S(t)$ showing the breakdown of Gibrat's law. Figure 8 shows the firms' size dependency of fluctuation of growth rate in two close observation points, where we measured the standard deviation σ of the logarithmic growth rate r. It is evident that the results in the sales here are in good agreement with previous discussion. Hereafter, we focus on the growth-rate fluctuation in the sales in terms of bow-tie componetns.

It was suggested in the results of the evolution of bow-tie components that the firms in the production network grow up in each component in spite of the rewired for a half of supplier-customer links. In order to characterize each bow-tie component in terms of firms' growth, we focus on the firms not having

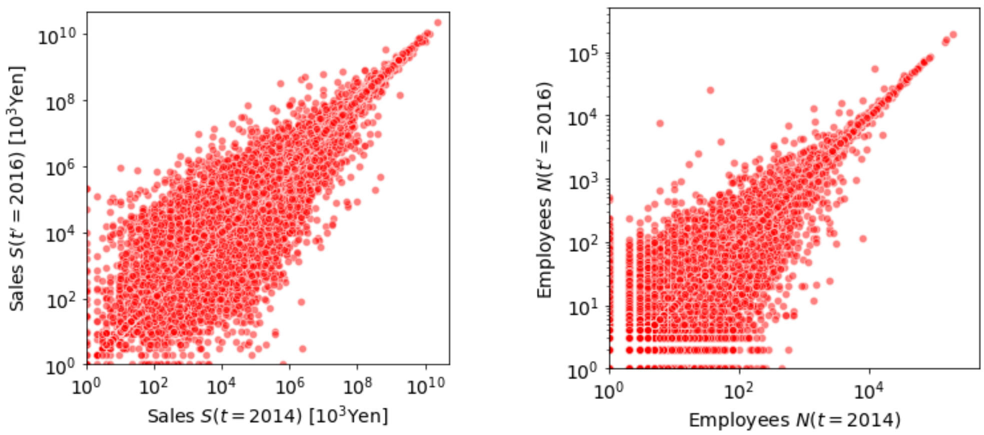

Figure 6. The joint PDF for frims' size at 2014 and 2016, measured by sales (left) and number of employees (right).

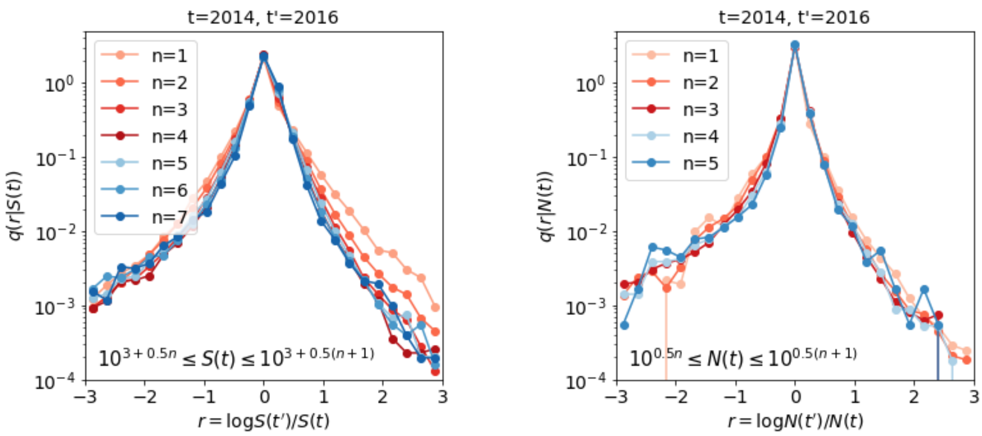

Figure 7. Conditional PDF of logarithmic growth rate of firms' size measured by sales (left) and number of employees (right). We divided the range of initial value into logarithmically equal bins as Eq. 8.

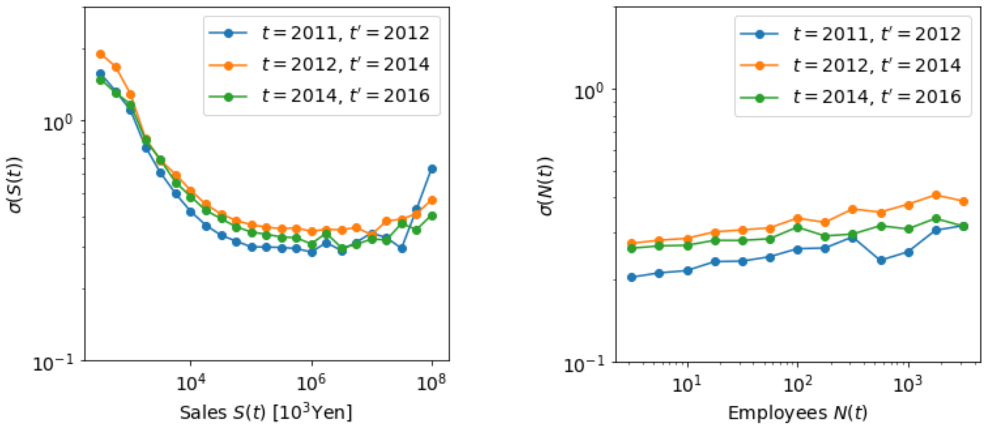

Figure 8. Firms' size dependency of fluctuation measured standard deviation of growth rate in two close observation points, for sales (left) and number of employees (right).

moved between components in two different years, here we used from 2014 to 2016. Figure 9 shows the joint PDF for the sales of the firms not having moved between components in 2014 and 2016, and conditional PDF of logarithmic growth rate of the corresponding sales. And we show how the standard deviations of the logarithmic growth rate for these firms behave as a function of the initial sales size in Figure 10 (left). We observed the sales region holding Gibrat's law in all cases, however, it becomes short range for the case of IN component because of a small number of large firms in IN component. As shown in Figure 10 (right), moreover, the difference of the growth rate distribution appear only on the decline region ($r < 0$), which seems to indicate that firms belonging to IN and OUT have more risk of declines than the firms categorized as GSCC. Therefore, this suggests the importance for the firms to locate on the GSCC in Japanese production network in terms of stable firms' growth. As shown in Figure 11, in fact, the firms moved to (from) GSCC tend to be larger (smaller), in particular, this property clearly appears in the transition between GSCC and IN. In general, the firms belonging to IN (OUT) component can move to GSCC if they could obtain new supplier (customer) the other component, especially GSCC. Compared the two cases, it is apparent that the firms belonging to IN have more difficulties to satisfy the situation than downstream firms in the production networks. This is because they need to be recognized by new supplier as the firm having ability to buy products or services. In other word, the trust based on the ability ties the supplier-customer relation and they are percolated in whole production networks. Needless to say that the degree of difficulty depends on not only a category of business but also country. Out characteristic results for the IN component, therefore, may suggest that the bow-tie decomposition can categorize the firms, which have small ability to gather the trust in the production network, as IN component.

IV. Conclusions

One of the most important networks in the economy is a production network, which is the backbone of the economy and interactions between individual firms expect to play an important role in the micro- and macro-economy. In order to understand the dynamics of this network and reach into the region of business cycles as well as firms' growth and decline, we need to understand the structure and relations with the economic variables based on actual data.

In order to reveal how the macroscopic structure of Japanese production network relates to the firms' size and its growth, we investigated the dynamics of firm-level production network in Japan from 2011 to 2016 analyzing over one million firms by using the bow-tie decomposition, which can categorize the firms in the networks as a giant core of economic activities, i.e. giant strongly connected component (GSCC), and its upstream (IN) and downstream (OUT). The edge-level evolution seems to be dynamically changing such that about half of supplier-customer relations were rewired in five years. In macroscopic evolution, on the other hand, we found that Japanese production network has stable firms not moving between bow-tie components. We also investigated the characteristics of each bow-tie component in terms of firms' size and growth rate. The firms obeying power-law in size distribution are mainly composed of GSCC and OUT component, and the firms categorized IN component tend to be smaller than the others. And characteristics of the growth rate distribution suggest the importance for the firms to locate on the GSCC in Japanese production network in terms of stable firms' growth. Moreover, our results may provide the framework to characterize the decline risks of the firms categorized as small and mid-sized firms in terms of macroscopic hierarchical structure of the production network.

References

Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The network origins of aggregate fluctuations," *Econometrica*, 2012, *80* (5), 1977–2016.

Atalay, Enghin, Ali Hortacsu, James Roberts, and Chad Syverson, "Network structure of production," *Proceedings of the National Academy of Sciences*, 2011, *108* (13), 5199–5202.

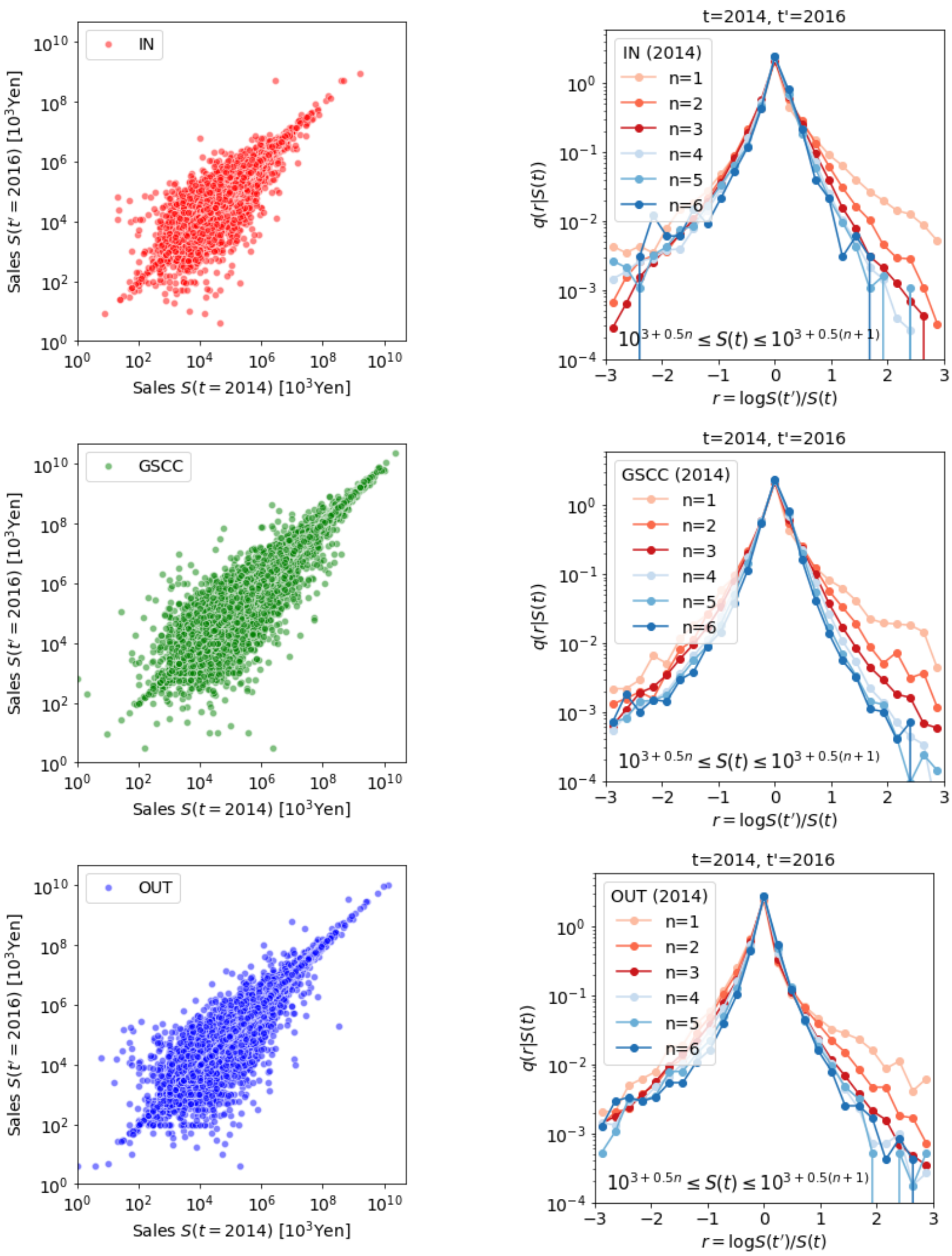

Figure 9. The joint PDF for sales of the firms not having moved between components in 2014 and 2016 (left) and conditional PDF of logarithmic growth rate of these sales (left). We divided the range of initial value into logarithmically equal bins as Eq. 8. The top, middle and bottom figure represent IN, GSCC and OUT component, respectively.

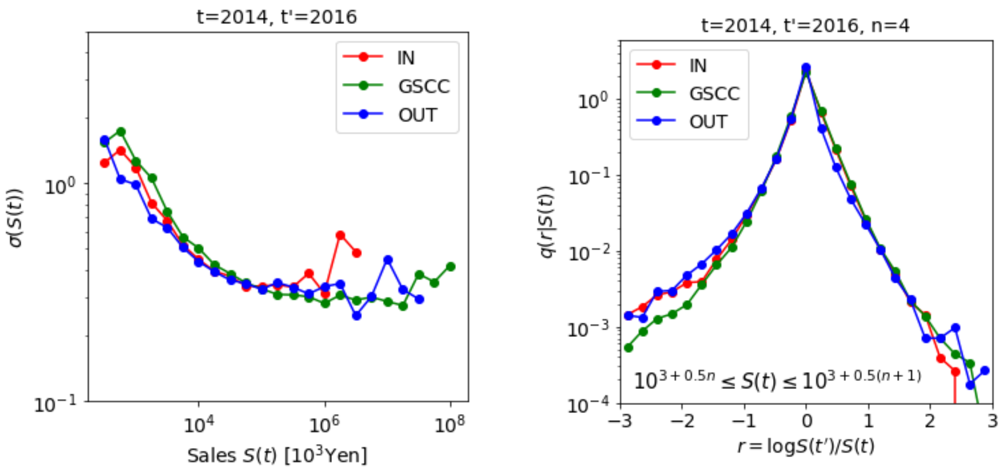

Figure 10. The initial sales dependency of standard deviation of growth rate in 2014 and 2016 for the firms not having moved between bow-tie components (left) and the conditional PDF of the growth rate in the sales range holding Gibrat's law (right).

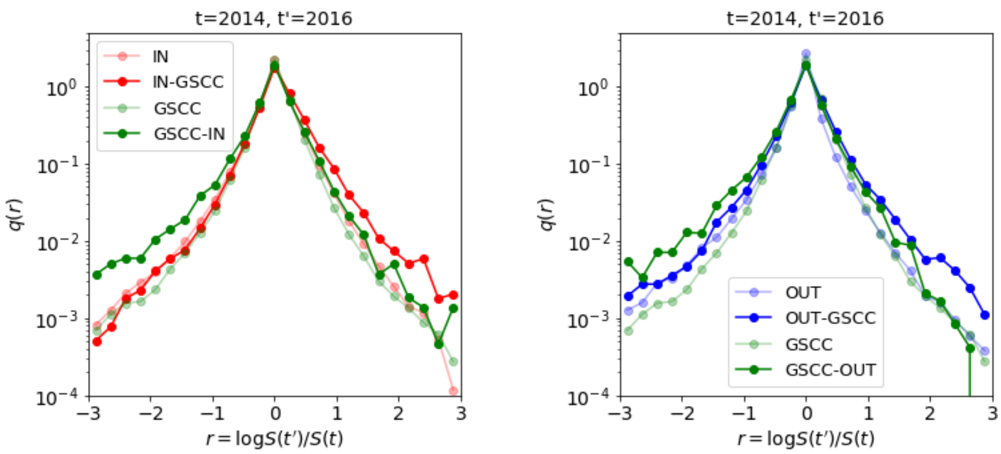

Figure 11. PDF of the sales growth rate in 2014 and 2016 for the firms having moved between IN and GSCC (left), between OUT and GSCC (right). For the comparison, we plotted also the PDF for the firms not having moved.

Broder, Andrei, Ravi Kumar, Farzin Maghoul, Prabhakar Raghavan, Sridhar Rajagopalan, Raymie Stata, Andrew Tomkins, and Janet Wiener, "Graph structure in the web," *Computer networks*, 2000, *33* (1-6), 309–320.

Chakraborty, Abhijit, Yuichi Kichikawa, Takashi Iino, Hiroshi Iyetomi, Hiroyasu Inoue, Yoshi Fujiwara, and Hideaki Aoyama, "Hierarchical communities in the walnut structure of the Japanese production network," *PloS one*, 2018, *13* (8).

Fujiwara, Yoshi and Hideaki Aoyama, "Large-scale structure of a nation-wide production network," *The European Physical Journal B*, 2010, *77* (4), 565–580.

– , Corrado Di Guilmi, Hideaki Aoyama, Mauro Gallegati, and Wataru Souma, "Do Pareto–Zipf and Gibrat laws hold true? An analysis with European firms," *Physica A: Statistical Mechanics and its Applications*, 2004, *335* (1-2), 197–216.

– , Hideaki Aoyama, and Wataru Souma, "Growth of firms and networks," *Econophysics and Sociophysics: Trends and Perspectives*, 2007.

– , Wataru Souma, Hideaki Aoyama, Taisei Kaizoji, and Masanao Aoki, "Growth and fluctuations of personal income," *Physica A: Statistical Mechanics and its Applications*, 2003, *321* (3-4), 598–604.

Hidalgo, Cesar, *Why information grows: The evolution of order, from atoms to economies*, Basic Books, 2015.

Inoue, Hiroyasu and Yasuyuki Todo, "Firm-level propagation of shocks through supply-chain networks," *Nature Sustainability*, 2019, *2* (9), 841–847.

Kichikawa, Yuichi, Hiroshi Iyetomi, Takashi Iino, and Hiroyasu Inoue, "Community structure based on circular flow in a large-scale transaction network," *Applied Network Science*, 2019, *4* (1), 92.

Leontief, Wassily, *Input-output economics*, Oxford University Press, 1986.

Luo, Jianxi, Carliss Y Baldwin, Daniel E Whitney, and Christopher L Magee, "The architecture of transaction networks: a comparative analysis of hierarchy in two sectors," *Industrial and Corporate Change*, 2012, *21* (6), 1307–1335.

Mizuno, Takayuki, Wataru Souma, and Tsutomu Watanabe, "The structure and evolution of buyer-supplier networks," *Plos one*, 2014, *9* (7).

Pareto, Vilfredo, *Cours d'économie politique*, Vol. 1, Librairie Droz, 1964.

Sutton, John, "Gibrat's legacy," *Journal of economic literature*, 1997, *35* (1), 40–59.