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Reserve Requirements and Bubbles*

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Abstract

This study investigates the effectiveness of the reserve requirement policy as a preventive measure against economic bubbles. In the existing literature, it has been pointed out that the expansion of bubbles can be prevented by raising the required reserve ratio. The present study demonstrates this may not be the case. If the ratio is below a certain threshold, the conventional policy prediction fails or in other words, raising the required reserve ratio expands a bubble. If, in contrast, the ratio is above the threshold, it prevents the expansion of the bubble (or the conventional prediction holds). In either case, a policy of raising the reserve requirement is welfare reducing in our model. This implies that if the ratio is below the threshold, the optimal policy is to cut the required reserve ratio, which will increase welfare while at the same time that it will reduce the bubble.

Keywords: Bubbles, Reserve requirement

JEL classification: E40, E58

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1 Introduction

In many developing countries in which money markets are underdeveloped, policymakers believe that they can suppress bubbles by controlling the required reserve ratio[1]. In 2010, the Central Bank of Brazil raised the required reserve ratio on term deposits to prevent the formation of asset price bubbles. Raising the required reserve ratio curbs overheating in consumer credit loans and disables investment. OECD (2011) points out that the policy has an effect of preventing the expansion of bubbles[2]. In macroeconomic theory, however, no study analyzes the effect of the reserve requirement policy in a bubble economy[3].

This study demonstrates the possibility that the reserve requirement policy is ineffective in getting out of the bubble economy. In particular, the policy stimulates a bubble if the required reserve ratio is lower than a certain threshold. This result suggests that Brazil's policy may not be appropriate. If the ratio is higher than the threshold, the policy has the effect of suppressing the bubble. In either case, the policy reduces social welfare. Therefore, if the ratio is below the threshold, the optimal policy is to cut the required reserve ratio. To show these results, we introduce a banking sector that can lend portfolios of loans to firms and has a cash reserve under the reserve requirement; we incorporate these features into a monetary model. Moreover, we provide a necessary and sufficient condition for the existence of the bubble in a steady-state equilibrium in a monetary model that includes the banking sector.

Our study is related to Chari, Jones, and Manuelli (1995); Roubini and Sala-i-Martin (1995); Haslag (1998); and Basu (2001). Chari, Jones, and Manuelli (1995), Roubini and Sala-i-Martin (1995), and Haslag (1998) develop a model with a banking sector under reserve requirements and show that an increase in the required reserve ratio increases the cost of financial intermediation and deteriorates the efficiency of the production sector and economic growth. In particular, our model is based on the framework of Chari, Jones, and Manuelli (1995). In their model, consumers hold deposits,

[1] The Bank of Japan points out this.

[2] In China, the central bank made a surprisingly aggressive cut in the reserve requirement to induce economic booms in 2015. Thus, there was a concern that the money released by the reserve requirement ratio cut could lead to a speculative bubble in the stock market (The Wall Street Journal, 2015).

[3] The reserve requirement in Tirole (1985) represents a lower limit for consumers to invest in financial assets. Moreover, Tirole (1985) does not accurately describe the commercial banking system. The reserve requirement in this study represents the restrictions imposed by the government on commercial banks.

and banks offer loans from deposits and hold cash reserves according to a reserve requirement. In the production sector, firms take up loans from the banking sector to purchase capital. However, Basu (2001) suggests the possibility that raising the required reserve ratio accelerates economic growth through public spending. He considers a continuous-time version of Chari, Jones, and Manuelli's (1995) model with endogenous growth that derives from the spillover effect of government spending[4], and they show that a Laffer curve-type relationship is obtained between the required reserve ratio and economic growth[5].

In our model, we introduce an asset into the continuous-time version of Chari, Jones, and Manuelli's model. Specifically, consumers hold an asset in addition to deposits, and a bubble emerges in the pricing of the asset. In order to show our results, we incorporate the asset and deposits in a utility function. In other words, consumers obtain utility from not only consumption goods but also the social status gained by accumulating wealth, which is composed of deposits and the asset, as in Kamihigashi (2008) and Zhou (2016)[6]. These studies analyze the existence of a bubble by including wealth in the utility function, but our model differs from theirs. Kamihigashi's and Zhou's models do not focus on deposits; instead, wealth is composed of assets and capital. In our model, because capital is supplied to firms through the banking sector via deposits, we can analyze the effect of the reserve requirement policy.

Our main finding is that a reserve requirement policy does not always prevent bubbles. More specifically, the size of a bubble has an inverted U-shaped relationship with the required reserve ratio in a steady state. Raising the required reserve ratio accelerates the expansion of the bubble until the ratio reaches a certain threshold. When the required reserve ratio is beyond the threshold, the reserve requirement policy can prevent the expansion of the bubble. Moreover, we show that raising of the required reserve ratio always reduces the social welfare in the steady state with the bubble and provides a condition where the social welfare in the bubble-less economy is higher than that in the bubble economy. Therefore, when the bubble economy is undesirable from the viewpoint of the social welfare, the reduction

[4] The endogenous growth settings are developed by Barro (1990) and Barro and Sala-i-Martin (1995).

[5] Recently, Oh (2011) shows that raising the required reserve ratio has a negative effect on the financial sector and a growth-enhancing effect through public spending on the real sectors.

[6] Zhou (2018) shows that a higher money growth rate leads to a larger bubble in wealth in the utility function model.

of the required reserve ratio is an effective policy to rise from the bubble economy if the required reserve ratio is below the threshold.

The rest of this chapter is organized as follows. Section 2 develops an infinite horizon model with a banking sector. Section 3 obtains the equilibrium dynamics using parameterized production sector functions. Then, we demonstrate the existence of steady-state equilibria with and without a bubble. We also discuss the effects of the reserve requirement policy on the steady state with the bubble. Section 4 concludes the paper. Finally, we provide some proofs in the Appendix.

2 Model

This section describes the structure of our model, which is based on the model developed by Chari, Jones, and Manuelli (1995) and Basu (2001) with an asset. First, we provide the structure of the production side. Second, we formulate the representative household's problem. Third, we consider the banking aspect, following Chari, Jones, and Manuelli (1995) and Basu (2001). Fourth, we set the monetary authority. Finally, we obtain equilibrium dynamics.

2.1 Production Sector

We use the structure provided by Chari, Jones, and Manuelli (1995) and Basu (2001) to reflect the production side of the economy. A representative firm produces output goods, Y_t, using capital, K_t, and labor, N_t. Capital is the intermediate good produced by the banking sector in our model. The representative firm takes up new loans from banks to purchase capital. Thus, the production maximization problem is as follows:

$$\max \int_0^\infty e^{-\int_0^t \left(R_j^L - \pi_j\right)dj} \left[p_t Y_t - p_t w_t N_t - p_t \dot{K}_t + p_t \dot{l}_t - \left(R_t^L - \pi_t\right) p_t l_t\right] dt,$$

subject to

$$l_t \geq K_t, \tag{1}$$

where r is the firm's discount factor, l_t is the real bank loans received at time t, p_t is the nominal price, R_t^L is the nominal interest rate on loans, w_t is the real wage, and $\pi_t := \dot{p}_t/p_t$ is the inflation rate. Then, $R_t^L - \pi_t$ is the real interest rate on loans. The constraint, (1), implies that the firm receives new loans from the bank, which constrains its purchase of new capital. We can rewrite this problem as follows:

$$\max Y_t - w_t N_t - R_t^L K_t + \pi_t K_t.$$

Let us define the production function as $Y_t = F(K_t, N_t)$

$$\begin{aligned} R_t^L - \pi_t &= F_1(K_t, N_t), \quad (2) \\ w_t &= F_2(K_t, N_t). \quad (3) \end{aligned}$$

Then, $R_t^L - \pi_t$ is the real rental rate on loans.

2.2 Households

We assume that there is no population growth and normalize the number of households to be equal to one. The representative household maximizes the following optimization problem:

$$\max_{c_t} \int_0^{+\infty} e^{-\rho t} u(c_t, a_t)\, dt,$$

subject to

$$\dot{a}_t = \left(R_t^D - \pi_t\right) d_t + \dot{v}_t s_t - c_t + w_t + \tau_t,$$

where c_t is consumption, d_t is the real deposit, R_t^D is the nominal interest rate on the deposit, s_t indicate shares of a zero-dividend asset, v_t is the real price of the asset, τ_t is real government transfers, $a_t := d_t + v_t s_t$ is the real value of wealth, ρ is the rate of time preference, and $u(\cdot)$ is the utility function. We introduce a real wealth term, a_t, to the utility function, as in Kamihigashi (2008) and Zhou (2016). We specify the utility function as $u(c_t, a_t) = \alpha \log c_t + \beta \log a_t$, where $\alpha > 0$ and $\beta > 0$. Moreover, $R_t^D - \pi_t$ is the real interest rate on deposits.

In our model, we assume no dividend, and the asset does not contribute to production and does not necessarily contribute to utility. Thus, the fundamental value of the asset is zero. In this study, a bubble is defined as the asset that contributes to utility when the price is positive. Moreover, as in the standard literature on bubbles, the total supply of the asset is normalized to one. Later, we discuss conditions of the existence of the bubble.

Next, we obtain the necessary conditions for this problem. We define the current value Hamiltonian function as follows:

$$H = u(c_t, a_t) + \lambda_t \left(\left(R_t^D - \pi_t\right) d_t + \dot{v}_t s_t - c_t + w_t + \tau_t\right) + \eta_t (a_t - d_t - v_t s_t),$$

where λ_t is the co-state variable of a_t, and η_t is the Lagrange multiplier. The necessary and transversality conditions are given as follows:

$$\begin{aligned}
\lambda_t &= u_1(c_t, a_t), && (4)\\
\lambda_t\left(R_t^D - \pi_t\right) &= \eta_t, && (5)\\
\lambda_t \dot{v}_t &= \eta_t v_t, && (6)\\
\dot{\lambda}_t &= \rho\lambda_t - u_2(c_t, a_t) - \eta_t, && (7)\\
\lim_{t\to\infty} e^{-\rho t}\lambda_t d_t &= 0, && (8)\\
\lim_{t\to\infty} e^{-\rho t}\lambda_t v_t s_t &= 0. && (9)
\end{aligned}$$

Finally, we consider the relationship between the existence of a bubble and the form of the utility function. In our study, we introduce the bubble by using the method advocated by Zhou (2016)[7]. In the method, we can define the bubble with the transversality condition holding. When v_t is positive, then we say that there exists a bubble. From (5) and (6), the expansion rate of the bubble is $R_t^D - \pi_t$ at time t. Here, the asset market clearing condition is $s_t = 1$. Thus, we need the condition that the growth rate of $\lambda_t v_t$ is less than ρ for the transversality condition (9) to not rule out the bubble in equilibrium. Using (4), (6), and (7), the growth rate of $\lambda_t v_t$ equals to $\rho - u_2/u_1$. Therefore, under the condition of

$$\lim_{t\to\infty} \frac{u_2}{u_1} > 0, \qquad (10)$$

the growth rate of $\lambda_t v_t$ is less than ρ, and the transversality condition (9) holds. Thus, the condition of (10) is necessary for the bubble to exist. In our model, the condition of (10) is given by

$$\lim_{t\to\infty} \frac{\beta c_t}{\alpha a_t} > 0.$$

On the contrary, when the marginal utility of wealth equals zero, the transversality condition (9) is violated if there exists a bubble. Thus, the wealth in the utility function is the key assumption that the bubble exists with the transversality condition (9) holding.

2.3 Banking Sector

In our model, the structure of the banking sector is the same as that in the models of Chari, Jones, and Manuelli (1995) and Basu (2001). We assume

[7] The method is similar to the methods of credit-driven bubble models of Kocherlakota (2009) and Miao and Wang (2013).

that this economy includes competitive banks. These banks take deposits from households and choose portfolios of loans to firms and cash reserves. In setting these portfolios, they face a reserve requirement. Thus, competitive banks solve the following maximization problem:

$$\max R_t^L L_t - R_t^D D_t,$$

subject to

$$\begin{aligned} M_t + L_t &= D_t, && (11) \\ M_t &\geq \epsilon D_t, && (12) \end{aligned}$$

where M_t is the nominal cash reserve, L_t is the nominal bank loan, D_t is the nominal deposit, and ϵ is the required reserve ratio. Thus, $d_t = D_t/p_t$ and $l_t = L_t/p_t$. We focus on the case in which (12) is binding. Moreover, the zero-profit condition holds because this banking sector is competitive. Therefore, the reserve requirement binds, yielding

$$(1-\epsilon) R_t^L = R_t^D. \tag{13}$$

2.4 Money Growth

Next, we consider the monetary authority. Let μ be the constant monetary growth rate. We assume that the government faces the following budget constraint:

$$\dot{M}_t = \mu M_t = p_t \tau_t.$$

Real money, $m_t := M_t/p_t$, behaves as

$$\frac{\dot{m}_t}{m_t} = \mu - \pi_t. \tag{14}$$

2.5 Complete Dynamics

We derive the dynamic systems in equilibrium, where the good, asset, and labor markets are clear:

$$\begin{aligned} s_t &= 1, \\ \dot{K}_t + c_t &= Y_t, \\ N_t &= 1. \end{aligned} \tag{15}$$

Taken together, equations (11), (12), and (1) yield

$$K_t = (1-\epsilon)\, d_t, \tag{16}$$

$$K_t = \frac{1-\epsilon}{\epsilon} m_t. \tag{17}$$

Combining (13) and (2), we obtain

$$R_t^D = (1-\epsilon)\left(F_1(K_t, 1) + \pi_t\right). \tag{18}$$

Moreover, using (14), (15), and (17), we obtain the inflation rate as

$$\pi_t = \mu - \frac{Y_t}{K_t} + \frac{c_t}{K_t}. \tag{19}$$

Then, using (18) and (19), we can express $R_t^D - \pi_t$ as

$$R_t^D - \pi_t = -\epsilon \frac{c_t}{K_t} + \epsilon \frac{F(K_t, 1)}{K_t} + (1-\epsilon) F_1(K_t, 1) - \epsilon\mu =: R(c_t, K_t). \tag{20}$$

From (4), (5), (7), (16), and (20), we obtain

$$\frac{\dot{c}_t}{c_t} = R(c_t, K_t) - \rho + \frac{\beta}{\alpha} \frac{c_t}{\frac{1}{1-\epsilon} K_t + v_t}. \tag{21}$$

In addition, using (5), (6), and (20), we find that the real price of the asset changes according to

$$\frac{\dot{v}_t}{v_t} = R(c_t, K_t). \tag{22}$$

Thus, differential equations (15), (21), and (22) describe this economy.

3 Dynamic Equilibrium

In this section, we examine the existence of a steady-state equilibrium with a bubble and analyze the effect of banking sector policies on the bubble. We assume the production function as $F(K_t, N_t) = AK_t^{\gamma} (N_t)^{1-\gamma}$, where $A > 0$ and $\gamma \in (0,1)$. Thus, we consider the neoclassical growth economy. First, we derive the equilibrium dynamics of the model presented in Section 2 and provide a necessary and sufficient condition for the existence of a steady-state equilibrium with a bubble. Next, we analyze banking sector policies at the steady state with the bubble.

We specify the production function as $Y_t = AK_t^{\gamma} (N_t)^{1-\gamma}$, where $A > 0$ and $\gamma \in (0,1)$. Then, we obtain

$$F_1(K_t, 1) = \gamma A K_t^{\gamma-1}. \tag{23}$$

Substituting (23) for (15), (20), (21), and (22), we obtain the equilibrium dynamics as follows:

$$\begin{aligned}
\frac{\dot{K}_t}{K_t} &= AK_t^{\gamma-1} - \frac{c_t}{K_t}, && (24)\\
\frac{\dot{c}_t}{c_t} &= R(c_t, K_t) - \rho + \frac{\beta}{\alpha}\frac{c_t}{\frac{1}{1-\epsilon}K_t + v_t}, && (25)\\
\frac{\dot{v}_t}{v_t} &= R(c_t, K_t), && (26)
\end{aligned}$$

where

$$R(c_t, K_t) = -\epsilon\frac{c_t}{K_t} + AK_t^{\gamma-1}((1-\epsilon)\gamma + \epsilon) - \epsilon\mu.$$

3.1 The Existence of the Bubble

We demonstrate the existence of a steady-state equilibrium with a bubble. This economy has two steady states: one without the bubble, in which the value of the asset equals zero, and one with the bubble, in which the value of the asset is some positive constant.

This economy is described by the nonlinear differential equations (24), (25), and (26). The locus of $\dot{K}_t = 0$ and $\dot{c}_t = 0$ is given by

$$\begin{aligned}
AK^\gamma - c &= 0, && (27)\\
R(c, K) - \rho + \frac{\beta}{\alpha}\frac{c}{\frac{1}{1-\epsilon}K + v} &= 0. && (28)
\end{aligned}$$

The locus of $\dot{v}_t = 0$ is given by the values without the bubble

$$v = 0, \tag{29}$$

or by those with the bubble

$$R(c, K) = 0. \tag{30}$$

X^* and X^{**} denote the values of X on a steady state both without the bubble and with the bubble, respectively. From (27), (28), and (29), we obtain a steady state without the bubble as follows:

$$(K^*, c^*, v^*) = \left(\left(\frac{\rho + \epsilon\mu}{\left(\frac{\beta}{\alpha} + \gamma\right)(1-\epsilon)A}\right)^{\frac{1}{\gamma-1}}, AK^{*\gamma}, 0\right). \tag{31}$$

Then, from (27), (28), and (30), we obtain a steady state with the bubble as follows:

$$(K^{**}, c^{**}, v^{**}) = \left(\left(\frac{\epsilon\mu}{(1-\epsilon)\gamma A} \right)^{\frac{1}{\gamma-1}}, \left(\frac{\epsilon\mu}{(1-\epsilon)\gamma} \right) K^{**}, \left(\frac{\beta\epsilon\mu}{(1-\epsilon)\alpha\gamma\rho} - \frac{1}{1-\epsilon} \right) K^{**} \right). \tag{32}$$

Thus, $\epsilon > (\alpha\gamma\rho)/(\beta\mu)$ is the condition that guarantees $v^{**} > 0$.

We summarize the above results with the following proposition.

Proposition 1 1. *In this economy, there is always a unique steady-state equilibrium without the bubble, given by (31).*

2. *Assume that*

$$1 > \epsilon > \frac{\alpha\gamma\rho}{\beta\mu}. \tag{33}$$

Then, there exists a unique steady-state equilibrium with the bubble, given by (32).

Next, we examine the dynamic properties around the steady state in our model. The steady state without the bubble is a local saddle point. On the contrary, the steady state with the bubble may be a local saddle point or a local source point. We can summarize these results as follows:

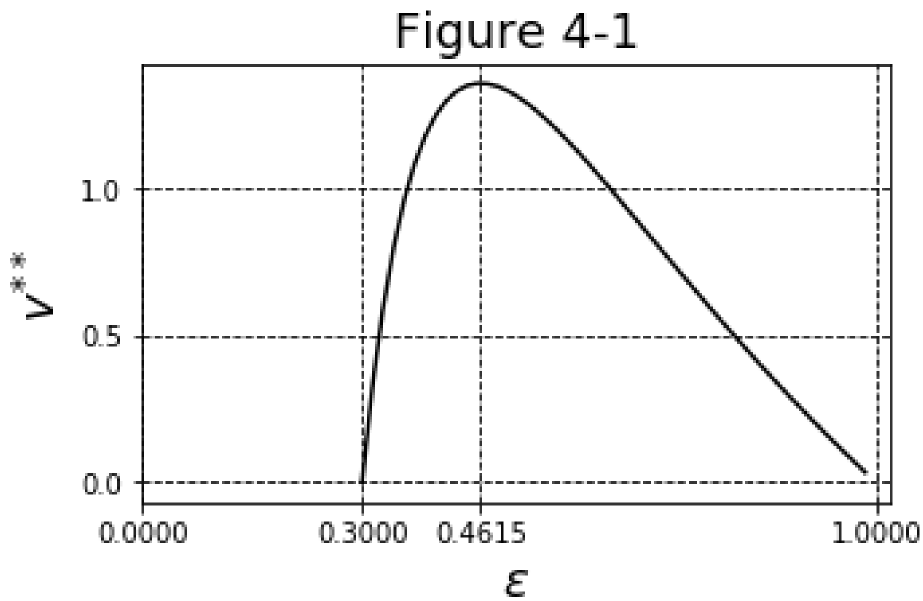
Proposition 2 1. *If $\epsilon < (\alpha\gamma\rho)/(\beta\mu)$, the steady state without the bubble is saddle stable. If $\epsilon > (\alpha\gamma\rho)/(\beta\mu)$, the steady state without the bubble is locally indeterminate.*

2. *Assume that $\epsilon > (\alpha\gamma\rho)/(\beta\mu)$. The steady state with the bubble is saddle stable if $(\beta\mu)/\alpha < \rho + \mu(1-\gamma)$.*

Proof. See Appendix I. ■

3.2 The Reserve Requirement Policy

First, we analyze the use of the reserve requirement policy to prevent the expansion of the bubble. We consider that a central bank controls the required reserve ratio under a given growth rate of the money supply. Next, we show that raising the required reserve ratio always reduces the social welfare, and the social welfare in the bubble-less economy is higher than in the bubble economy. Finally, we discuss the effect of the reserve requirement policy in the bubble economy.



We assume that there exists the steady state with the bubble and the steady state is saddle stable, that is, $(\alpha\gamma\rho)/(\beta\mu) < 1$ and $(\beta\mu)/\alpha < \rho + \mu(1-\gamma)$ hold. From (32), we can derive

$$\frac{\partial v^{**}}{\partial \epsilon} \gtreqless 0 \Leftrightarrow \frac{1}{\frac{\beta\mu}{\alpha\rho} + 1 - \gamma} \gtreqless \epsilon. \tag{34}$$

Moreover, we obtain $v^{**} \to 0$ as $\epsilon \to 1$ and $v^{**} \to 0$ as $\epsilon \to (\alpha\gamma\rho)/(\beta\mu)$. When we assume that the government cannot control the money supply, from the necessary and sufficient condition of the existence of the bubble, (33), we obtain the following condition:

$$\frac{\alpha\gamma\rho}{\beta\mu} < \frac{1}{\frac{\beta\mu}{\alpha\rho} + 1 - \gamma} < 1. \tag{35}$$

(34) and (35) imply that the bubble expands when the reserve requirement is below the threshold $1/((\beta\mu)/(\alpha\rho) + 1 - \gamma)$. Thus, a raising of the reserve requirement accelerates the expansion of the bubble if $(\alpha\gamma\rho)/(\beta\mu) < \epsilon < 1/((\beta\mu)/(\alpha\rho) + 1 - \gamma)$. On the contrary, the reserve requirement tends to reduce the bubble only when it is above the threshold, $1/((\beta\mu)/(\alpha\rho) + 1 - \gamma) < \epsilon < 1$. Thus, increasing the reserve requirement does not always prevent the

expansion of the bubble. We can interpret this result as follows. The interest rate on deposits is relatively low compared to the value of the asset as the required reserve ratio rises until $1/\left(\left(\beta\mu\right)/\left(\alpha\rho\right)+1-\gamma\right)$. Thus, the asset becomes more attractive to households. However, as the required reserve ratio approaches one, the fraction of deposits that are transformed into loans becomes high. Thus, the output, capital, and wage decrease, and, as a result, households cannot afford to hold the asset.

Figure 4-1 shows an example of the relationship between v^{**} and ϵ. The example assumes $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0.5$, $\mu = 0.5$, $\rho = 0.3$, and $A = 1$. Then, $\left(\alpha\gamma\rho\right)/\left(\beta\mu\right) = 0.3$ and $1/\left(\left(\beta\mu\right)/\left(\alpha\rho\right)+1-\gamma\right) = 0.4615....$ Moreover, the example satisfies the saddle stable condition of Proposition 2, that is, $0.5 = \left(\beta\mu\right)/\alpha < \rho + \mu\left(1-\gamma\right) = 0.55$.

Next, we show that raising the required reserve ratio always reduces the social welfare in the bubble economy. u^* and u^{**} denote the value of the utility on the steady state both without the bubble and with the bubble. Thus, we have

$$\begin{aligned} u^* &= \alpha \log c^* + \beta \log \frac{1}{1-\epsilon} K^* \\ &= \frac{\alpha\gamma+\beta}{\gamma-1} \log \frac{\rho+\epsilon\mu}{\left(\frac{\beta}{\alpha}+\gamma\right)(1-\epsilon)} + \beta \log \frac{1}{1-\epsilon} + \frac{\alpha+\beta}{\gamma-1} \log \frac{1}{A}, \end{aligned} \tag{36}$$

$$\begin{aligned} u^{**} &= \alpha \log c^{**} + \beta \log \left(\frac{1}{1-\epsilon} K^{**} + v^{**} \right) \\ &= \frac{\alpha\gamma+\beta}{\gamma-1} \log \frac{\epsilon\mu}{(1-\epsilon)\gamma} + \beta \log \frac{1}{1-\epsilon} + \beta \log \frac{\beta\epsilon\mu}{\alpha\gamma\rho} + \frac{\alpha+\beta}{\gamma-1} \log \frac{1}{A}. \end{aligned} \tag{37}$$

Raising the required reserve ratio crowds out investment, and decrease outputs. As a result, raising the required reserve ratio reduces the social welfare. In practice, from (37), we obtain $\partial u^*/\partial\epsilon < 0$ and $\partial u^{**}/\partial\epsilon < 0$. Moreover, from (36) and (37),

$$\left(\frac{\frac{\beta}{\alpha}\mu + \mu\gamma}{\rho\gamma + \mu\gamma} \right)^{\frac{\alpha\gamma+\beta}{1-\gamma}} > \left(\frac{\beta\mu}{\alpha\gamma\rho} \right)^{\beta}, \tag{38}$$

if and only if $u^* > u^{**}$ for all $\left(\alpha\gamma\rho\right)/\left(\beta\mu\right) < \epsilon < 1$ (see Appendix II)[8]. Therefore, social welfare is lower in the bubble economy than in the bubble-less economy under the condition of (38). Then, the bubble economy may

[8] As in Zhou (2016), we consider "catching up with the Joneses" utility functions that take the form of $u\left(c_t, a_t\right) = \alpha \log c_t + \beta \log\left(a_t/\bar{a}_t\right)$, where $\bar{a}_t$ denotes the average wealth level; thus, the social welfare is always lower in the bubble economy than in the bubble-less economy without parameter conditions. Since $u_2\left(c_t, a_t\right) = \beta/a_t$, the equilibrium dynamics

be undesirable from the viewpoint of social welfare. Parameters of Figure 4-1 satisfy the condition of (38)[9].

Finally, we discuss the effect of raising the required reserve ratio in the bubble economy. We assume the condition of (38). At this time, the bubble economy is undesirable from the viewpoint of the social welfare. In the bubble economy, policymakers control the required reserve ratio to rise from the bubble economy. If $(\alpha\gamma\rho)/(\beta\mu) < \epsilon < 1/((\beta\mu)/(\alpha\rho)+1-\gamma)$, the raising of the reserve requirement accelerates the expansion of the bubble and reduces the social welfare. Then, the reduction of the required reserve ratio is an effective policy. On the contrary, if $1/((\beta\mu)/(\alpha\rho)+1-\gamma) < \epsilon < 1$, a raising of the reserve requirement reduces the degree of the bubble and the social welfare.

4 Conclusion

We explore the effectiveness of the reserve requirement policy as a measure to prevent bubbles. To analyze this measure, we introduce wealth in the utility function and an asset into the model developed by Chari, Jones, and Manuelli (1995) and Basu (2001). Regarding the steady state with a bubble, raising the required reserve ratio accelerates the expansion of the bubble if the ratio is below a certain threshold. In contrast, the reserve requirement policy can prevent the expansion of the bubble if the required reserve ratio is beyond the threshold. Moreover, we show that raising the required reserve ratio always reduces the social welfare and provides a condition in which the social welfare in the steady state without the bubble is higher than that in the steady state with the bubble. Therefore, when a bubble economy is undesirable from the viewpoint of social welfare, the optimal policy is to cut the required reserve ratio, which will increase welfare while reducing the bubble.

This study focuses only on the reserve requirement and does not consider

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of this case are described by (24), (25), and (26). Thus, it holds the same results as in the case of $u(c_t, a_t) = \alpha \log c_t + \beta \log a_t$ without the welfare analysis. Since the total number of individuals is normalized to one, the aggregate wealth a_t must be equal to the average wealth $\bar{a}_t$. Thus, $u^* = \alpha \log c^* = \alpha \log AK^{*\gamma}$ and $u^{**} = \alpha \log c^{**} = \alpha \log AK^{**\gamma}$. Then, the steady-state values are given by (31) and (32). We obtain $K^* > K^{**}$ if $\epsilon > (\alpha\gamma\rho)/(\beta\mu)$. Therefore, $u^* > u^{**}$ always holds for all $\epsilon > (\alpha\gamma\rho)/(\beta\mu)$. Moreover, $\partial u^{**}/\partial\epsilon < 0$ holds.

[9] Under $\alpha = 0.5$, $\beta = 0.5$, $\gamma = 0.5$, $\mu = 0.5$, $\rho = 0.3$, and $A = 1$, we obtain $\left(\frac{\frac{\beta}{\alpha}\mu+\mu\gamma}{\rho\gamma+\mu\gamma}\right)^{\frac{\alpha\gamma+\beta}{1-\gamma}} = 2.567...$ and $\left(\frac{\beta\mu}{\alpha\gamma\rho}\right)^{\beta} = 1.825....$

other monetary policies. Many studies that analyze the effects of monetary policies do not consider reserve requirements. Therefore, clarifying the relationship between the effect of the reserve requirement and that of other monetary policies on bubbles deserves further examination.

Appendix

Appendix I

This section derives Proposition 2. We obtain partial derivatives of the dynamic system (24), (25), and (26) with respect to each variable as follows:

$$\begin{aligned}
\frac{\partial \dot{K}_t}{\partial K_t} &= \gamma A K_t^{\gamma-1}, \\
\frac{\partial \dot{K}_t}{\partial c_t} &= -1, \\
\frac{\partial \dot{K}_t}{\partial v_t} &= 0,
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \dot{c}_t}{\partial K_t} &= c_t \left(R_2(c_t, K_t) - \frac{\beta}{\alpha} \frac{\frac{1}{1-\epsilon} c_t}{\left(\frac{1}{1-\epsilon} K_t + v_t\right)^2} \right), \\
\frac{\partial \dot{c}_t}{\partial c_t} &= R(c_t, K_t) - \rho + \frac{\beta}{\alpha} \frac{c_t}{\frac{1}{1-\epsilon} K_t + v_t} + c_t \left(R_1(c_t, K_t) + \frac{\beta}{\alpha} \frac{1}{\frac{1}{1-\epsilon} K_t + v_t} \right), \\
\frac{\partial \dot{c}_t}{\partial v_t} &= -\frac{\beta}{\alpha} \frac{c_t^2}{\left(\frac{1}{1-\epsilon} K_t + v_t\right)^2},
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \dot{v}_t}{\partial K_t} &= R_2(c_t, K_t) v_t, \\
\frac{\partial \dot{v}_t}{\partial c_t} &= R_1(c_t, K_t) v_t, \\
\frac{\partial \dot{v}_t}{\partial v_t} &= R(c_t, K_t).
\end{aligned}$$

Around the Steady state Without Bubbles

We linearize the dynamic system at the steady state without bubbles. Then, the Jacobian matrix is given by

$$J = \begin{pmatrix} \gamma A K^{*\gamma-1} & -1 & 0 \\ J_{21} & J_{22} & -\frac{\beta}{\alpha}\frac{c^{*2}}{\left(\frac{1}{1-\epsilon}K^*\right)^2} \\ 0 & 0 & R(c^*, K^*) \end{pmatrix},$$

where

$$\begin{aligned} J_{21} \quad &: \quad = \left(\epsilon - \frac{\beta}{\alpha}(1-\epsilon) - (1-\gamma)\left((1-\epsilon)\gamma + \epsilon\right)\right) A^2 K^{*2\gamma-2}, \\ J_{22} \quad &: \quad = \left(-\epsilon + \frac{\beta}{\alpha}(1-\epsilon)\right) A K^{*\gamma-1}. \end{aligned}$$

The characteristic equation of matrix J is given by

$$F(\phi) = (R(c^*, K^*) - \phi)\left(\phi^2 - \phi\left(J_{22} + \gamma A K^{*\gamma-1}\right) + J_{22}\gamma A K^{*\gamma-1} + J_{21}\right).$$

Since

$$J_{22}\gamma A K^{*\gamma-1} + J_{21} = -(1-\gamma)(1-\epsilon)\left(\frac{\beta}{\alpha} + \gamma\right) A^2 K^{*2\gamma-2} < 0,$$

one characteristic root, ϕ_1, is negative, and one characteristic root, ϕ_2, is positive. The other characteristic root, $\phi_3 = R(c^*, K^*)$, is negative or positive. The dynamic system has only one predetermined variable, K_t. Therefore, since $\phi_3 < 0$ if $\epsilon > (\alpha\gamma\rho)/(\beta\mu)$, the steady state without the bubble is locally indeterminate. Since $\phi_3 > 0$ if $\epsilon < (\alpha\gamma\rho)/(\beta\mu)$, the steady state without the bubble is saddle stable.

Around the Steady state With the Bubble

We linearize the dynamic system at the steady state with the bubble. Then, the Jacobian matrix is given by

$$J = \begin{pmatrix} J_{11} & -1 & 0 \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & 0 \end{pmatrix},$$

where

$$J_{11} := \gamma A K^{**\gamma-1} = \frac{\epsilon\mu}{1-\epsilon},$$

$$J_{21} := c^{**}\left(R_2(c^{**},K^{**}) - \frac{\beta}{\alpha}\frac{\frac{1}{1-\epsilon}c^{**}}{\left(\frac{1}{1-\epsilon}K^{**}+v^{**}\right)^2}\right)$$

$$= -\gamma\left(\frac{\epsilon\mu}{(1-\epsilon)\gamma}\right)^2((1-\gamma)(1-\epsilon)-\epsilon) - \frac{\alpha\rho^2}{\beta(1-\epsilon)},$$

$$J_{22} := c^{**}\left(R_1(c^{**},K^{**}) + \frac{\beta}{\alpha}\frac{1}{\frac{1}{1-\epsilon}K^{**}+v^{**}}\right) = \rho - \frac{\epsilon^2\mu}{(1-\epsilon)\gamma},$$

$$J_{23} := -\frac{\beta}{\alpha}\frac{c^{**2}}{\left(\frac{1}{1-\epsilon}K^{**}+v^{**}\right)^2} = -\frac{\alpha}{\beta}\rho^2,$$

$$J_{31} := R_2(c^{**},K^{**})\,v^{**} = (-(1-\gamma)(1-\epsilon)+\epsilon)\,\gamma A K^{**\gamma-2}v^{**},$$

$$J_{32} := R_1(c^{**},K^{**})\,v^{**} = -\epsilon\frac{v^{**}}{K^{**}} = -\frac{\beta\mu\epsilon^2}{(1-\epsilon)\alpha\gamma\rho} + \frac{\epsilon}{1-\epsilon}.$$

The characteristic equation of the matrix J is given by

$$F(\phi) = -\phi^3 + (J_{22}+J_{11})\,\phi^2 + (-J_{11}J_{22}+J_{23}J_{32}-J_{21})\,\phi + J_{23}\,(-J_{31}-J_{11}J_{32}).$$

First, we have

$$-J_{31}-J_{11}J_{32} = \gamma A K^{**\gamma-2}v^{**}\,(1-\gamma)\,(1-\epsilon) > 0.$$

Since $J_{23}\,(-J_{31}-J_{11}J_{32}) < 0$, at least one characteristic root is negative, $\phi_1 < 0$. Moreover, $\phi_1\phi_2\phi_3 = J_{23}\,(-J_{31}-J_{11}J_{32}) < 0$. Thus, the steady state is a saddle point if one characteristic root is negative and two characteristic roots are positive, and it is a local sink point if all characteristic roots are negative. Next, we consider the conditions under which the steady state is a saddle point. We calculate $-J_{11}J_{22}+J_{23}J_{32}-J_{21}$ as follows:

$$-J_{11}J_{22}+J_{23}J_{32}-J_{21} = \frac{\epsilon\mu}{1-\epsilon}\left(-\rho+\frac{\epsilon\rho}{\gamma}+\frac{(1-\gamma)\,\epsilon\mu}{\gamma}\right)+\frac{\alpha\rho^2}{\beta}.$$

We explore the condition of $-J_{11}J_{22}+J_{23}J_{32}-J_{21} > 0$. If $(\beta\mu)/\alpha < \rho+\mu(1-\gamma)$, then $(\gamma\rho)/(\rho+(1-\gamma)\mu) < (\alpha\gamma\rho)/(\beta\mu)$. At the steady state with the bubble, $\epsilon > (\alpha\gamma\rho)/(\beta\mu)$ holds by Proposition 1. Then, we obtain

$$\frac{\gamma\rho}{\rho+(1-\gamma)\,\mu} < \epsilon \Longleftrightarrow 0 < -\rho+\frac{\epsilon\rho}{\gamma}+\frac{(1-\gamma)\,\epsilon\mu}{\gamma}.$$

Since $\phi_1\phi_2+\phi_2\phi_3+\phi_3\phi_1 = -\left(-J_{11}J_{22}+J_{23}J_{32}-J_{21}\right)$, $-J_{11}J_{22}+J_{23}J_{32}-J_{21}>0$ and $\phi_1\phi_2+\phi_2\phi_3+\phi_3\phi_1<0$ hold under $\left(\beta\mu\right)/\alpha<\rho+\mu\left(1-\gamma\right)$. Finally, we discuss ϕ_2 and ϕ_3 by considering two cases under $\left(\beta\mu\right)/\alpha<\rho+\mu\left(1-\gamma\right)$. One case is that ϕ_2 and ϕ_3 are both real numbers. Then, from $\phi_1\phi_2\phi_3<0$, $\phi_1\phi_2+\phi_2\phi_3+\phi_3\phi_1<0$, and $\phi_1<0$, ϕ_2 and ϕ_3 must be positive. The other is that ϕ_2 and ϕ_3 are one pair of complex numbers. We denote them by $\phi_2=a+bi$ and $\phi_3=a-bi$. Then, $\phi_1\phi_2+\phi_2\phi_3+\phi_3\phi_1=\phi_1 2a+a^2+b^2$. By $\phi_1\phi_2+\phi_2\phi_3+\phi_3\phi_1<0$ and $\phi_1<0$, a must be positive.

To summarize the above, under $\left(\beta\mu\right)/\alpha<\rho+\mu\left(1-\gamma\right)$ and $\epsilon>\left(\alpha\gamma\rho\right)/\left(\beta\mu\right)$, there is one negative characteristic root. Therefore, the steady state with the bubble is a local saddle point because the dynamic system has only one predetermined variable.

Appendix II

In this section, we show $u^*>u^{**}$, where u^* and u^{**} are defined in (36) and (37). From (36) and (37), we obtain

$$\begin{aligned} h\left(\epsilon\right) &:= u^*-u^{**} \\ &= \frac{\alpha\gamma+\beta}{\gamma-1}\log\left(\rho\gamma+\epsilon\mu\gamma\right)-\frac{\alpha\gamma+\beta}{\gamma-1}\log\left(\frac{\beta}{\alpha}\mu+\mu\gamma\right) \\ &\quad -\frac{\alpha\gamma+\beta}{\gamma-1}\log\epsilon-\beta\log\epsilon-\beta\log\frac{\beta\mu}{\alpha\gamma\rho}. \end{aligned}$$

Then,

$$h'\left(\epsilon\right)=\frac{\beta\epsilon\mu\gamma\left(1-\gamma\right)-\gamma^2\rho\left(\alpha+\beta\right)}{\left(\gamma-1\right)\epsilon\left(\rho\gamma+\epsilon\mu\gamma\right)}.$$

Therefore, we obtain

$$h'\left(\epsilon\right)\gtreqless 0 \Longleftrightarrow \epsilon \lesseqgtr \frac{\gamma\rho\left(\alpha+\beta\right)}{\beta\mu\left(1-\gamma\right)}.$$

For the steady state with the bubble to exist, $\left(\alpha\gamma\rho\right)/\left(\beta\mu\right)<\epsilon<1$ must hold. Then, we have $h\left(\left(\alpha\gamma\rho\right)/\left(\beta\mu\right)\right)=0$ and $\left(\alpha\gamma\rho\right)/\left(\beta\mu\right)<\left(\gamma\rho\left(\alpha+\beta\right)\right)/\left(\beta\mu\left(1-\gamma\right)\right)$. Thus, $h\left(\epsilon\right)>0$ for all $\left(\alpha\gamma\rho\right)/\left(\beta\mu\right)<\epsilon<\left(\gamma\rho\left(\alpha+\beta\right)\right)/\left(\beta\mu\left(1-\gamma\right)\right)$. Next, we consider the case of $\epsilon=1$. Then, $h\left(1\right)=\frac{\alpha\gamma+\beta}{\gamma-1}\log\left(\rho\gamma+\mu\gamma\right)-\frac{\alpha\gamma+\beta}{\gamma-1}\log\left(\frac{\beta}{\alpha}\mu+\mu\gamma\right)-\beta\log\frac{\beta\mu}{\alpha\gamma\rho}$, and we obtain

$$h\left(1\right)>0\Longleftrightarrow\left(\frac{\frac{\beta}{\alpha}\mu+\mu\gamma}{\rho\gamma+\mu\gamma}\right)^{\frac{\alpha\gamma+\beta}{1-\gamma}}>\left(\frac{\beta\mu}{\alpha\gamma\rho}\right)^{\beta}.$$

Thus, $h(\epsilon) > 0$ for all $(\gamma\rho(\alpha+\beta)) / (\beta\mu(1-\gamma)) \leq \epsilon < 1$ if and only if $\left(\frac{\frac{\beta}{\alpha}\mu+\mu\gamma}{\rho\gamma+\mu\gamma}\right)^{\frac{\alpha\gamma+\beta}{1-\gamma}} > \left(\frac{\beta\mu}{\alpha\gamma\rho}\right)^{\beta}$.

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