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Heterogenous Job Separations and the Balassa-Samuelson Effect*

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Abstract

We incorporate different sectoral job separation rates into a two-sector small open economy model to investigate the Balassa-Samuelson (B-S) effect. While labour is mobile, unemployment occurs due to search frictions. In addition, unequal separation rates give rise to compensating wage differentials. When productivity grows in the tradeables sector, labour moves from the tradeables sector to the nontradeables sector if tradeables and nontradeables are complements in consumption. Nevertheless, unemployment always falls due to the positive income effect. We also find that the effect of productivity growth in the tradeables sector on the real exchange rate is reduced by almost 38 per cent when separation rates differ across sectors and tradeables and nontradeables are complements. Overall, an overvaluation of the real exchange rate in the basic B-S model can be explained by heterogeneous job separations.

Keywords: Balassa-Samuelson, unemployment, compensating wage differential, job matching
JEL classification: F31, F41, F66

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1. Introduction

Balassa (1964) and Samuelson (1964) showed that productivity growth in the tradeables sector rather than the nontradeables sector leads to a rise in the price of nontradeables because, with perfect labour mobility between sectors, a rise in the wage in the tradeables sector induces higher wages in the nontradeables sector. Since the tradeables price is determined in the world market, countries in which productivity grows faster in the tradeables sector tend to experience an appreciation of their real exchange rate (RER). The effect of tradeables productivity on the RER is called the Balassa-Samuelson (B-S) effect and has attracted considerable research. It has also interested policymakers since the B-S effect can indicate how purchasing power parity and the relative competitiveness across countries change over time.

In this paper, we theoretically investigate the B-S effect. While the key B-S prediction linking technological progress in export-oriented industries and an increasing RER seems to be borne out in some countries (for example, Japan according to Ito *et al.*, 1999 and for developing countries according to Choudhri and Khan, 2005), it lacks more widespread applicability. The basic B-S model predicts that a rise in productivity in the tradeables sector should raise the relative price of nontradeables proportionally. However, it is argued in many papers that the basic B-S model lacks empirical support.[1] For example, using panel data for 14 OECD countries, Cardi and Restout (2015) show that

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[1] The empirical literature on the B-S effect has grown enormously. Many papers fail to find evidence of the effect and then propose explanations. For example, Bergin *et al.* (2006) illustrate an essentially zero correlation between national price levels and incomes per capita. Their simulations show that allowing tradeability to be driven by technological developments can help explain the larger B-S effect evident in more recent decades. In a similar fashion, Bodart and Carpantier (2016) show that increasing cross-country differentials in skilled-unskilled labour ratios are a structural determinant of RER. Choudhri and Schembri (2010) parameterise a model in which a productivity improvement in the home tradeables sector increases the relative price of nontraded to traded goods but has an ambiguous effect on the terms of trade. If the productivity improvement lowers the terms of trade, the RER can appreciate or depreciate depending on whether the relative price effect outweighs the terms-of-trade effect. In Bordo *et al.* (2017) the terms-of-trade adjust in response to productivity changes. The two key determinants of the terms of trade response are the elasticity of substitution between home and foreign traded goods and the differential in the shares of home goods in domestic and foreign consumption of traded goods.

the relative price of nontradeables rises only by 0.78 per cent with a one per cent rise in relative productivity in the tradeables sector; i.e., not the one per cent rise in the relative price predicted by the basic B-S model. They also show that modifying the basic B-S model by removing the assumption of frictionless intersectoral labour mobility leads to a significant improvement in predictive ability. Specifically, the relative price and wage responses to changes in the productivity differential between tradeables and nontradeables are more muted.[2] Also, notable is the existence of persistent sectoral wage differentials. In Cardi and Restout (2015), the differentials are driven by intersectoral differences in search costs and imperfect labour mobility.

In contrast, in our model labour is mobile across sectors. In other words, we treat the unemployed as belonging to one pool of job seekers and not being tied to a specific industry. Supporting this view, Gomes (2015, p.1427) argues that the unemployed search across all sectors of an economy. When separated from an employer, workers enter the pool of unemployed and do not confine their search for work to just one sector. If unemployed workers can move freely between industries, at least in the medium-run, a *sectoral* unemployment rate is a redundant concept. Moreover, since we are interested in the determination of unemployment in the steady state, a sectoral unemployment rate is not a natural way to formulate our model. In our model, workers weigh the probabilities of finding a job, the wages on offer and the separation rates. In this respect, the setup is closer to the basic B-S model.

The difference between our model and those in existing studies is the presence of sectoral differences in job separation rates. These differences are readily apparent in the data. Figure 1 shows industry separation rates in the United States for 2018. It is obvious

[2] Sheng and Xu (2011) show that a change in the price of nontradeables may be higher or lower than what is predicted by the B-S model, depending on the relative market matching efficiency between the two sectors. Further, if the relative labour market matching efficiency in the tradeables sector is very low, an increase in tradeables productivity may be more than offset by relatively high frictional costs, thus operating against the standard B-S effect. Using panel data for Japan, the United Kingdom and United States, Sheng and Xu (2011) also show that labour market frictions are important for understanding the impact of productivity on the RER. Using a larger sample of countries, they find that countries in which hiring and firing costs are higher, that the B-S effect is significantly smaller.

that the rates differ across sectors. Moreover, the separation rate in the nontradeables sector is likely to be lower than that for the tradeables sector if the public sector is included (14.7 per cent for the Federal government and 19.2 per cent for the State and Local governments), and higher if it is not. Davis and Haltiwanger (1992) find that quarterly average job creation and job destruction flow rates vary widely among industries. Even if workers can readily move between sectors, an intersectoral difference in separation rates can generate wage dispersion because sectors with higher job separations cannot attract and hire a sufficient number of workers without offering higher wages. We refer to this wage dispersion as the "compensating wage differential". Consequently, when labour moves across sectors as exogenous shocks occur, changes in the relative supply of tradeables and nontradeables affect the magnitude of the B-S effect.

--- Insert Figure 1 here ---

We consider rising productivity in the tradeables sector caused by labour-augmenting technological progress. Sectoral shifts are affected by an appreciation of the RER, i.e., the price of domestically-produced, nontradeable goods and services rising faster than the price of internationally tradeable goods and services. Since many services, particularly those provided by the public sector, are not traded internationally, an economy with relatively low unemployment and a growing public and nontradeables goods and services sector draws labour from the internationally-exposed sectors of the economy, i.e., export-oriented industries and import-competing industries. While this effect may be attenuated by the presence of high unemployment, the sector in which technological progress occurs will have different effects on unemployment and sectoral wage differentials. The result will be shown to depend on the substitutability in consumption between tradeables and nontradeables. If tradeables and nontradeables are more substitutable, a rise in the nontradeables price leads to a significant decrease in the demand for nontradeables. As a result, the tradeables sector may draw labour from the nontradeables sector.

Labour movement across sectors affects the B-S effect. Given that the tradeables price is exogenously determined in the world market, the RER is positively associated with the nontradeables price. If the nontradeables sector draws labour from the tradeables sector and the supply of nontradeables increases, the nontradeables price decreases and the RER depreciates. Therefore, the model with heterogeneous job separations can explain an under- or over-valuation of the B-S effect because sectoral labour movement offsets or amplifies the basic B-S effect. For our baseline case in which the separation rate in the tradeables sector exceeds the rate in the nontradeables sector and tradeables and nontradeables are complements (i.e., if the public sector is a significant part of the nontradeables sector), we show that the usual effect of higher productivity in the tradeables sector is offset by almost 38 per cent by a labour reallocation effect. This is quantitatively consistent with the finding by Cardi and Restout (2015).

This paper also has implications for a contemporary concern in the literature about whether globalisation, and particularly international trade openness, affects unemployment. In a combination of theoretical and empirical work, Davidson *et al*. (1999), Kee and Hoon (2005), Moore and Ranjan (2005), Wälde and Weiss (2006), Dutt *et al*. (2009), Mitra and Ranjan (2010), Helpman and Itskhoki (2010), Felbermayr *et al*. (2011), Gaston and Rajaguru (2013), Xu and Sheng (2014) and Cardi and Restout (2015) incorporate equilibrium unemployment into general equilibrium trade models to show how the unemployment rate is affected by a range of international and domestic factors.[3] While the overall impact on unemployment is not uniform across theoretical models, a common theme is that labour market frictions worsen unemployment. We discuss how different job separation rates affect unemployment.

The next section introduces the model and Section 3 derives the main theoretical propositions. One feature of the model is that equilibrium wages are higher in the sector with higher separations. We then explicitly examine the B-S effect and show that the RER increases (decreases) when there is productivity growth in the tradeables (nontradeables) sector. However, the impact on unemployment depends on the change in labour demand in the two sectors. Section 4 calibrates the functional forms and parameters and overviews how the equilibrium values are determined. We also show that labour moves between

[3] In their review of research on trade and labour markets, Davidson and Matusz (2011) discuss how a variety of labour market frictions can be introduced into general equilibrium settings to generate unemployment. Implicit contract models, efficiency wage models, bargaining models and search or matching models all yield a variety of relationships between relative commodity prices and relative factor prices and between trade and unemployment.

sectors so that the marginal contribution of labour over wage cost is equalised. One feature of the model is that the economy-wide unemployment rate is higher or lower depending on which sector is larger. In Section 5, we simulate how productivity improvements affect unemployment and the RER. We show that whether both sectors expand or one sector expands and the other contracts, unemployment always falls if productivity growth occurs in the tradeables sector. Which sector expands depends on the substitutability of tradeables and nontradeables in consumption. Interestingly, we also show that unemployment may rise with productivity growth in the nontradeables sector when tradeables and nontradeables are complements. For the B-S effect, we show that heterogeneous separations can reduce the over-valuation of the B-S effect . Finally, Section 6 concludes.

2. The model

Our model is based on the B-S model with tradeables and nontradeables sectors. Since we incorporate unemployment it is natural to use the Diamond-Mortensen-Pissarides model to study how workers are matched to jobs in either sector, or possibly left unmatched. Wälde and Weiss (2006), Dutt *et al*. (2009) and Xu and Sheng (2014) use a similar approach to study the unemployment effects of international competition. Unlike these papers, we explicitly introduce different job separation rates in the tradeables and nontradeables sectors. The B-S effect is captured by the differential impact of sectoral productivity growth. The model shows that heterogeneous separation rates change the effect of relative productivity growth on the RER and unemployment.

2.1. Search and matching

There are two sectors in the economy, we let N denote the nontradeables sector and T the tradeables sector. Consider a representative firm i in sector I $(I = N, T)$, where the evolution of its labour stock is given by

$$\dot{L}_i = \frac{m_I}{V_I} V_i - s_I L_i, \qquad i \in I. \tag{1}$$

Here, L_i is the labour stock, V_i represents the vacancies posted by the firm, s_I is the sector-specific separation rate and m_I is the number of job matches. We allow for different sectoral separation rates. Other authors introduce heterogeneity by assuming unemployment to be sector-specific and the cost of posting vacancies to differ (e.g., Cardi and Restout, 2015). We eschew this approach. Arguably differences in sectoral demand are likely to be manifested in voluntary and involuntary separations. Moreover, the duration of implicit and explicit contracting relationships is likely to differ across sectors.[4] On the other hand, as pointed out by Davidson and Matusz (2004) and Gomes (2015), there is one pool of unemployed workers searching for jobs across sectors. V_I is the sum of vacancies posted by all firms in sector I, i.e., $V_I = \int_{i \in I} V_i di$. Each firm is small so that its behaviour affects neither sector- nor economy-wide variables such as V_N and V_T.

We assume that the number of matches in either sector is determined in the following manner:

$$m_I = \frac{V_I}{V} M(U, V), \tag{2}$$

where A is a positive constant, U denotes the unemployed labour force and V denotes economy-wide vacancies given as the sum of vacancies in both sectors, i.e., $V = V_N + V_T$. $M(U, V)$ is the economy-wide likelihood of matches and takes the following Cobb-Douglas form (Petrongolo and Pissarides, 2001):

$$M(U, V) = AU^{\alpha} V^{1-\alpha}, \ \alpha \in (0,1),$$

Equation (2) indicates that the number of matches in each sector increases when (a) the number of vacancies in the sector becomes larger compared to the economy-wide vacancies, or (b) the matching efficiency improves. Channel (a) implies that an increase in the number of vacancies in one sector decreases the matches formed in the other sector since unemployed workers are mobile across sectors. Channel (b) indicates that the number of matches in each sector is positively associated with the economy-wide matching likelihood.

We define, respectively, economy-wide and sectoral inverse Beveridge ratios as

$$\theta = \frac{V}{U} \quad \text{and} \quad \theta_I = \frac{V_I}{U}. \tag{3}$$

[4] For the tractability of the model, we treat separation rates as exogenous. It should be noted that separation rates and job stability are related and distinct from layoff or job loss rates. Job stability typically incorporates information about both (involuntary) layoff rates and (voluntary) quit rates. The stability of average job durations or retention rates may reflect the fact that higher layoff rates tend to be associated with lower quit rates over the business cycle. Historically, and particularly in recessions, the incidence of job loss has been higher in U.S. manufacturing industries. However, the growing importance of the service sector has seen a growing share of separations in nontradeable industries. See Farber (1993; 1997). However, it remains the case that private sector employment is far less stable than public sector employment, at least for the United States (Farber, 2010).

Therefore, the economy-wide inverse Beveridge ratio is $\theta = \theta_N + \theta_T$. Therefore, the number of matches in either sector can be rewritten as

$$m_I = V_I A \theta^{-\alpha}. \tag{4}$$

Hence, the total number of matches in the economy corresponds to $M(U, V)$ as

$$m_N + m_T = M(U, V). \tag{5}$$

2.2. Vacancy posting

Instantaneous profits are total revenue net of labour costs and the cost of posting job vacancies

$$\pi_i = p_i F_i(a_i, L_i) - w_i L_i - \gamma V_i,$$

where γ is the vacancy cost, p_i is the price, w_i is the wage and a_i is the productivity of each firm.[5] Note that the price of tradeables, p_T, is exogenous in a small open economy, but that the price of nontradeables, p_N, is endogenous. The firm's optimisation problem is[6]

$$\max_{V_i} \int_0^\infty \pi_i(t) e^{-rt}\, dt,$$

where r is the discount rate. The current value Hamiltonian is

$$H = \pi_i + \Lambda_i \dot{L}_i,$$

where Λ_i is the co-state variable, which represents the evaluation of hiring an additional worker. Using (1), the first-order conditions are obtained as

$$\Lambda_I m_I = \gamma V_I \tag{6}$$

$$\Lambda_I = \frac{p_I F'_{IL} - w_I}{r + s_I} + \frac{1}{r + s_I} \dot{\Lambda}_I. \tag{7}$$

Note that firms are homogeneous within sectors and the equilibrium values of the endogenous variables depend only on the sector to which firms belong. The transversality condition is $\lim_{t \to \infty} e^{-rt} \Lambda_I L_I = 0$. Combining (4) and (6), we have

$$\Lambda_I = \frac{\theta^\alpha \gamma}{A}, \tag{8}$$

so that the co-state variable is the same for both sectors in the equilibrium ($\Lambda_N = \Lambda_T$).

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[5] In the internet age, the cost of posting vacancies is unlikely to significantly vary across sectors.

[6] In this formulation, L_i is the state variable and V_i the control variable.

2.3. Steady state condition

In the steady state, the number of entrants to, and exits from, each sector is offsetting, i.e.,[7]

$$m_I = s_I L_I. \tag{9}$$

The labour market equilibrium condition is

$$\bar{L} = L_N + L_T + U. \tag{10}$$

Using (3)-(5), (9) and (10), we obtain the following relation

$$A\theta^{1-\alpha}(\bar{L} - L_N - L_T) = s_N L_N + s_T L_T. \tag{11}$$

In the steady state, $\dot{\Lambda} = 0$. Therefore (7) becomes

$$\Lambda_I = \frac{p_I F'_{IL} - w_I}{r + s_I}. \tag{12}$$

2.4. Wage determination

The probabilities of being matched to sectors N and T are, respectively, m_N/U and m_T/U. Let z be the value of unemployment benefits, and E_N and E_T the discounted returns of being employed in each sector. In equilibrium, a job seeker's permanent payoff, rE_U, must equal the sum of unemployment benefits and the gain from changing the employment status occurring with the probability of being matched, $M(U,V)/U$. Therefore,

$$rE_U = z + \frac{M(U,V)}{U}\left[\frac{m_N}{M(U,V)}(E_N - E_U) + \frac{m_T}{M(U,V)}(E_T - E_U)\right]. \tag{13}$$

The returns to being employed differ across sectors but are the same for being employed by any firm in a sector. In equilibrium, a job seeker's permanent payoff for being employed, rE_I, must equal the sum of the wage income and the expected gain from being separated. That is,

$$rE_I = w_I + s_I(E_U - E_I). \tag{14}$$

[7] An alternative and less restrictive assumption is that the total number of entrants to and exits from the unemployment pool are equal in the steady state, i.e. $m_N + m_T = s_N L_N + s_T L_T$. However in this case, there is a possibility that a sector disappears as a result that all the workers move into the other sector in the steady state because we employ the constant elasticity of substitution consumption where the Inada (1963) condition does not generally hold.

Using (13) and (14), we have

$$E_I - E_U = \frac{1}{r + s_I} w_I - \frac{r}{r + s_I} E_U. \tag{15}$$

Given the worker's payoff $E_I - E_U$ and the steady state value of hiring an additional worker Λ_I, the Nash product S_i is

$$S_I = (E_I - E_U)^\beta \Lambda_I^{1-\beta},$$

where $\beta \in (0,1)$ is the worker's bargaining power. The first order condition with respect to w_I implies

$$E_I - E_U = \frac{\beta}{1-\beta} \Lambda_I. \tag{16}$$

Here, we used the assumption that agents are small and act nonstrategically so that the negotiated wage does not affect E_U, i.e.,

$$\frac{\partial E_U}{\partial w_I} = 0.$$

(16) indicates that the worker captures a larger payoff compared to the producer when its bargaining power is high. (3), (4), (8), (13), (15) and (16) jointly imply the following sectoral wage setting equation

$$w_I = z + \frac{\beta\gamma}{1-\beta}\left\{\frac{\theta^\alpha (r + s_I)}{A} + \theta\right\}. \tag{17}$$

Wages are increasing in the inverse Beveridge ratio, θ. More significantly, wages differ across sectors due to heterogeneity in s. In the standard compensating differentials sense, wages are higher in whichever sector has higher separations. A firm cannot attract or retain workers without raising wages if its separation rate is higher. This wage difference arises due to the mobility of labour across sectors. The following Proposition summarises the finding for sectoral wage differences.

Proposition One (Compensating wage differentials). *Wages are higher in the sector that has higher separations.*

Proof. $\widetilde{W} = w_T - w_N = \frac{\beta\gamma\theta^\alpha (s_T - s_N)}{A(1-\beta)}$. ■

In addition, the wage differential, the absolute value of $\widetilde{W}$, is increasing in γ, the cost of filling vacancies, and β, worker bargaining power. In the former case, if the transactions costs of hiring workers were zero, wages are equalised. If not, this makes worker turnover costlier for firms, which increases wages at firms in high turnover industries. In the latter case, if all the bargaining power resides with employers, then all workers are driven down to their reservation wage.[8]

2.5. The price of nontradeables

Assume that the representative consumer has constant elasticity of substitution preferences given by

$$C = \left[\psi^{\frac{1}{\rho}} c_N^{1-\frac{1}{\rho}} + (1-\psi)^{\frac{1}{\rho}} c_T^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}}, \qquad \psi \in (0,1), \quad \rho > 0$$

where c_N and c_T are the consumption of nontradeables and tradeables, respectively. Consumer's cost minimisation leads to the following demand functions

$$c_N = \psi \left(\frac{p_N}{P}\right)^{-\rho} C, \qquad c_T = (1-\psi)\left(\frac{p_T}{P}\right)^{-\rho} C.$$

The price index is defined as $P \equiv \left[\psi p_N^{1-\rho} + (1-\psi) p_T^{1-\rho}\right]^{\frac{1}{1-\rho}}$. With homothetic preferences the income expansion path is linear, with the relative demand for nontradeables given by

$$\left(\frac{\Psi c_T}{c_N}\right)^{\frac{1}{\rho}} = \frac{p_N}{p_T} \equiv p, \quad \text{where} \quad \Psi \equiv \frac{\psi}{1-\psi},$$

where ρ is the elasticity of substitution between tradeables and nontradeables. As ρ approaches infinity (zero) tradeables and nontradeables become perfect substitutes (complements) in consumption.

The ratio of the price of nontradeables to the price of tradeables, p, is customarily referred to as the RER.[9] Market clearing conditions of tradeables and nontradeables are

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[8] Several studies provide empirical support for compensating wage differentials. See Moretti (2000), Böckerman et al. (2011), and Bassanini et al. (2013), for instance.

[9] In some empirical studies the RER is defined as the ratio of the domestic price level to the foreign price level. Our definition is consistent with a small open economy setup. The prices of tradeables and

$\bar{L}c_N = F_N$ and $\bar{L}c_T = F_T$, respectively.[10] We treat tradeables as the numeraire (i.e., $p_T = 1$). Economy-wide, the demand for nontradeables ($\bar{L}c_N$) is written as

$$\bar{L}c_N = \frac{\Psi Y}{\Psi p + p^{\rho}},$$

where national income, Y, is the sum of all factor income - profits plus wage income. It follows that $Y = pF_N + F_T$. Note that unemployment benefits, zU, are simply a transfer of income to the unemployed (e.g., these transfers are funded by a lump-sum tax on factor income). Moreover, the cost of posting vacancies, γV_I, is modelled by Mortensen and Pissarides (1999, p.2574) as a flow cost (e.g., recruitment costs).[11]

Since the total consumption and the domestic production of nontradeables are equal, i.e., $\bar{L}c_N = F_N$, then the price of nontradeables is

$$p = \left(\frac{\Psi F_T}{F_N}\right)^{\frac{1}{\rho}}. \tag{18}$$

In other words, p is determined by *domestic* demand and supply. The primary effect of relative productivity growth in the tradeables sector is to raise the production of tradeables, increase national income and the demand for all goods and services. Hence, the RER increases. This feature is one of the key predictions of the B-S model. In an aggregate production function model, Kohli and Natal (2014) show that, with balanced trade, unemployment *must* fall. This occurs even if the nontradeables sector contracts. We show below that there is a second effect which may reinforce or counteract the effect on the RER as well as amplify or dampen the effect on unemployment.

2.6. Equilibrium

nontradeables consumed in the foreign country are exogenous to the home country, thus the only important element in the RER is the ratio of the nontradeables price to the tradeables price.

[10] We ignore international borrowing and lending for present purposes, i.e., we assume balanced trade. See Kohli and Natal (2014) for a discussion of the basic comparative statics with, and without, the balanced trade assumption in an aggregate production framework.

[11] A *flow* cost is neither a one-time or lump-sum payment, nor is it a deadweight loss which would reduce national income. In the present case, it is a payment at rate γ until each vacancy is filled.

From (8), (12) and (17),

$$p_I F'_{IL} = z + \frac{\gamma\theta^{\alpha}(r+s_I)}{(1-\beta)A} + \frac{\beta\gamma\theta}{1-\beta}. \tag{19}$$

Conditions (11), (18) and (19) jointly determine the equilibrium levels of L_N, L_T, θ and p (note $p_T = 1$ and $p = p_N$). As an aside, note from (12) and $\Lambda_N = \Lambda_T$ that

$$\frac{F'_{TL}}{r+s_T} - \frac{pF'_{NL}}{r+s_N} = \frac{w_T}{r+s_T} - \frac{w_N}{r+s_N}. \tag{20}$$

Hence, since r, s_T and s_N are exogeneous, if the marginal productivity of labour in the tradeables sector increases, then wages must rise relatively more in the tradeables sector to both retain and attract workers.

3. Unemployment and the Balassa-Samuelson effect

The B-S model is used to examine the effects of productivity growth in the tradeables sector. When the labour market is frictionless, real wages are equalised across sectors and the RER is proportional to the sectoral productivity differential. Specifically, since the price of tradeables are determined internationally, productivity growth in the tradeables sector increases wages in all sectors. Consequently, the RER and the national price level both increase.

The B-S model was not designed to explain sectoral wage differentials. It also cannot explain why changes in relative sectoral productivities and changes in the RER may not be proportional. Cardi and Restout (2015) show that a two-sector general equilibrium model with labour market frictions can accommodate these features. They also show that the role of relative wages is crucial for determining how the RER responds to sectoral productivity shocks. In their model, the labour market frictions arise because of different search costs across sectors. In our model, these costs are assumed equal, but the sectors are assumed to differ in the types of implicit contracts offered to workers. In turn, this is reflected in different sectoral separation rates.

3.1. Equilibrium dynamics

Linearising (11) and (19), we have (noting that we consider changes in a_N and a_T below):[12]

[12] The definitions of positive coefficients Qs, Hs and Gs are given in Appendix A.

$$d\theta = R_N dL_N + R_T dL_T \tag{21}$$

$$F'_{NL}dp + pF''_{NL}dL_N + pF''_{NLa}da_N = Q_N d\theta \tag{22}$$

$$F''_{TL}dL_T + F''_{TLa}da_T = Q_T d\theta. \tag{23}$$

(18) is linearised as

$$\frac{dp}{p} = \underbrace{\eta^{-1}\left(\varepsilon_{Ta}\frac{da_T}{a_T} - \varepsilon_{Na}\frac{da_N}{a_N}\right)}_{\text{(i)}Relative\ Productivity\ Effect} + \underbrace{\eta^{-1}\left(\varepsilon_{TL}\frac{dL_T}{L_T} - \varepsilon_{NL}\frac{dL_N}{L_N}\right)}_{\text{(ii)}Relative\ Labour\ Effect}, \tag{24}$$

where $\eta \equiv \rho p^{\rho-1}$. $\varepsilon_{IL} = F'_{IL}L_I/F_I > 0$ is the elasticity of output with respect to labour and $\varepsilon_{Ia} = F'_{Ia}a_I/F_I > 0$ denotes the elasticity of output with respect to productivity. Expression (i) captures the standard B-S effect, i.e., if technology grows in the tradeables sector, then the RER appreciates. Expression (ii) is absent in the standard B-S framework.[13] In a model with full employment, an expansion of the tradeables sector can only occur by drawing labour from the nontradeables sector. With the existence of unemployment, this need not necessarily occur. Clearly, the rise in the RER could raise national income sufficiently for the nontradeables sector to also expand.

Using (21) to eliminate $d\theta$, (22)-(24) can be rearranged as

$$G_{11}dp - G_{12}dL_N - G_{13}dL_T = -H_{11}da_N \tag{25}$$

$$-G_{22}dL_N - G_{23}dL_T = -H_{22}da_T \tag{26}$$

$$\eta dp + G_{32}dL_N - G_{33}dL_T = -H_{31}da_N + H_{32}da_T. \tag{27}$$

(25)-(27) jointly provide comparative statics of p, L_N and L_T for productivity changes in both sectors.

3.2. The effect of productivity growth

As shown in Appendix B, it is proved that the effect of productivity growth in the tradeables sector on the RER is always positive, i.e.,[14]

$$\frac{dp}{da_T} > 0. \tag{28}$$

[13] Cardi and Restout (2015) show that imperfect labour mobility can result in wage dispersion across industries, moderating the standard B-S effect. Our model has mobile labour across both sectors, but compensating differentials exist. If wages were equalised, it is easy to show that effect (ii) disappears. So, from Proposition One, effect (ii) arises due to different separation rates across industries. We show below that effect (ii) can take *either* sign if there is productivity growth in either sector.

[14] The details of other derivatives are also given in Appendix B.

(28) indicates that the RER always rises with tradeables productivity.

It should be noted that the effects of tradeables productivity growth on labour demand in each sector are ambiguous (see (B3) and (B4) in Appendix B). Therefore, the sign of the Relative Labour Effect in (24) is indeterminate. However, from (28), since tradeables productivity growth increases the RER, the Relative Productivity Effect is never fully offset by the Relative Labour Effect. This occurs because the latter effect is a secondary or feedback impact and is always smaller than the primary former impact, even if the correlation between the two effects is negative.

In the next section, we show that in the baseline case in which the separation rate is higher in the tradeables sector and where tradeables and nontradeables are complements in consumption, with productivity growth in the tradeables sector the Relative Labour Effect is negative, and the Relative Productivity Effect is positive. However, tradeables productivity growth increases *both* sectoral wages. The increase in wages serves to moderate any increase in the size of each sector, limiting the size of the Relative Labour Effect. This is the reason why the secondary impact is always smaller in absolute value than the primary impact.

It is also shown that the inverse Beveridge ratio rises with tradeables productivity, as an improvement in the marginal productivity of labour makes the labour market relatively tighter ($d\theta/da_T > 0$). We also obtain the following derivatives for wages: $dw_N/da_T > 0$ and $dw_T/da_T > 0$. As mentioned previously, wages in both sectors rise with tradeables productivity. This is because a tighter labour market places upward pressure on wages. Therefore,

$$\frac{d\widetilde{W}}{da_T} = \frac{\beta\gamma\alpha\theta^{\alpha-1}(s_T - s_N)}{(1-\beta)A}\frac{d\theta}{da_T}.$$

Thus, $d\widetilde{W}/da_T$ becomes more positive (negative) if $s_T > s_N$ ($s_T < s_N$).

As shown in Appendix B, the effects on the sectoral labour demands are indeterminate. Also, the sign of the effect on unemployment is indeterminate without additional assumptions on the functional form of production functions and the elasticity of output with respect to labour in the two sectors as shown in (B5).

As shown in Appendix C, the derivative with respect to productivity growth in the nontradeables sector is

$$\frac{dp}{da_N} < 0. \tag{29}$$

Thus, the RER falls with nontradeables productivity. From (24), this occurs independently of whether separation rates differ between the sectors. The effects of nontradeables productivity growth on labour demand in both sectors are also ambiguous (see (C2) and (C3) in Appendix C). Notwithstanding, as shown in Appendix C, the impact on nontradeables labour demand is of opposite sign to labour demand in the tradeables sector. We show below in simulations of the model that the changes in labour demand can be positive or negative. Based on (28) and (29), we can state the following Proposition.

Proposition Two (Real exchange rate). *The real exchange rate, p, appreciates (depreciates) with improvements in technological progress in the tradeables (nontradeables) sector.*

Proof. (28) and (29) take positive and negative signs, respectively. ■

Proposition Two indicates that the Relative Labour Effect can moderate the Relative Productivity Effect but never fully offset it. To examine the relative impacts of these effects, we perform numerical simulations. The sign of the effects of nontradeables productivity growth on the inverse Beveridge ratio, labour demand, wages and unemployment are not uniquely determined.

4. Numerical simulations

Simulations are used to illustrate the key features of the model and to obtain quantitative implications.

4.1. Preliminaries

Equations (21)-(24) can be combined to eliminate p and θ to obtain

$$\left(\frac{\Psi F_T}{F_N}\right)^{\frac{1}{\rho}} F'_{NL} - \frac{r+s_N}{1-\beta}\frac{\gamma}{A}\left[\frac{s_N L_N + s_T L_T}{A(\bar{L}-L_N-L_T)}\right]^{\frac{\alpha}{1-\alpha}} - z$$

$$- \frac{\beta\gamma}{1-\beta}\left[\frac{s_N L_N + s_T L_T}{A(\bar{L}-L_N-L_T)}\right]^{\frac{1}{1-\alpha}} = 0, \tag{30}$$

$$F'_{TL} - \frac{r+s_T}{1-\beta}\frac{\gamma}{A}\left[\frac{s_N L_N + s_T L_T}{A(\bar{L}-L_N-L_T)}\right]^{\frac{\alpha}{1-\alpha}} - z$$

$$- \frac{\beta\gamma}{1-\beta}\left[\frac{s_N L_N + s_T L_T}{A(\bar{L}-L_N-L_T)}\right]^{\frac{1}{1-\alpha}} = 0. \tag{31}$$

These two equations jointly determine the steady-state values of L_N and L_T. First, consider the homogeneous case ($s_N = s_T = s$). Based on this version of the equations, we can show that L_N and L_T are determined so that

$$\left(\frac{\Psi F_T}{F_N}\right)^{\frac{1}{\rho}} F'_{NL} = F'_{TL}, \tag{32}$$

or $pF'_{NL} = F'_{TL}$. Also, recall that wages are equalised across sectors in the homogeneous case. We can also state the following Corollary about the allocation of labour across sectors.

Corollary (Labour allocation). *Labour moves across sectors so that the marginal contribution of labour over wage cost weighted by the inverse of the sum of the discount rate and the sectoral separation rate is equalised.*

Proof. Using (20), the steady-steady state condition can be rewritten as: $\frac{F'_{TL}-w_T}{r+s_T} = \frac{pF'_{NL}-w_N}{r+s_N}$. ■

With homogeneous separation rates, wages are equal and (32) is obtained. Regarding the stability of equilibrium, if $\frac{F'_{TL}-w_T}{r+s_T} < \frac{pF'_{NL}-w_N}{r+s_N}$, then firms in the nontradeables sector post vacancies and labour moves to that sector, thereby restoring equilibrium. Thus, the steady state is stable. (From (20), note that a shock creating such a temporary inequality results in the labour movement reducing the gap between the sectoral marginal products. Alternatively, if $\frac{F'_{TL}-w_T}{r+s_T} < \frac{pF'_{NL}-w_N}{r+s_N}$ occurs as a result of a temporary shock, from (12) the benefit of hiring an additional worker (Λ) becomes higher in the nontradeables sector. Then, the returns to being employed in this sector improves as implied in (16). As a result, the sector draws labour and (20) is restored.)

4.2. Calibrations

We examine the case where productivity enters multiplicatively in the production technology, as it is an obvious and tractable choice for investigating the B-S effect. Specifically, consider the constant relative risk aversion production function $F_I = \nu_I^{-1} a_I L_I^{\nu_I}$ where $\nu_I \in (0,1)$, the elasticity of output with respect to labour, $\varepsilon_{IL} = F'_{IL} L_I / F_I = \nu_I$ and the elasticity of output with respect to productivity, $\varepsilon_{Ia} = F'_{Ia} a_I / F_I = 1$.

Figure 2 plots equations (30) and (31) to illustrate the determination of L_T and L_N for the homogeneous case ($s_N = s_T$). The labour constraint, i.e., $L_N + L_T \leq \bar{L}$, is also depicted in Figure 2. For (31), L_T and L_N are inversely related inside the labour constraint. For (30), the relationship between L_T and L_N is not unambiguous. As Figure 2 makes clear, for high (low or moderate) levels of unemployment, when both labour demands are low, L_T and L_N are positively (inversely) related. In the simulation depicted in Figure 2, $\varepsilon_{TL} = \varepsilon_{NL}$ and, L_T and L_N are inversely related in (30) because the intersection of (30) and (31) is closer to the labour constraint than the origin, so that the unemployment rate is not unrealistically high. We show below that the substitutability of tradeables and nontradeables in consumption is also crucial.[15]

--- Insert Figure 2 here ---

According to the U.S. Bureau of Labor Statistics, the annual total separation rate for the private sector is approximately 0.4 and that for the government sector is 0.2. The average annual rate of separation is approximately 0.3.[16] We simulate the model for monthly data, thus the separation rate is set at 0.025 ($s_N = s_T = s = 0.025$) in Figure 2.

[15] The importance of substitutability for the B-S effect is also emphasised by Hamano (2014).

[16] See, e.g., https://www.bls.gov/news.release/jolts.t16.htm.

For the simulations, to focus on the heterogeneity in separation rates, we divide consumption demand equally between tradeables and nontradeables, i.e., $\psi = 0.5$. For the elasticity of substitution between tradeables and nontradeables, we set $\rho = 0.5$ as a baseline. We show that this parameter is crucial for understanding the effects of technological change on unemployment.[17]

For the parameters in the matching and bargaining functions, we follow Gertler and Trigari (2009) in setting $\alpha = 0.5$, $A = 1$, $\nu_N = \nu_T = 0.66$ and $\beta = 0.5$. For the discount factor, $r = 0.003$ so that annual rate of return is 3.7 per cent, which approximates the return on government bonds for developed countries. We set the total labour supply and productivity in both sectors to 1 for analytical convenience ($\bar{L} = 1$ and $a_N = a_T = 1$). For the unemployment benefit, or the flow value of not working, $z = 0.71$, which follows Hall and Milgrom (2008). Although Gertler and Trigari (2009) use a quadratic cost of posting vacancies function, we employ their parameter value of $\gamma = 2.5$. This has the advantage of generating a realistic value for the unemployment rate of approximately 5 per cent.[18] Parameter and equilibrium values are shown in Table 1.

--- Insert Table 1 here ---

[17] Gomes (2015) points to a lack of consensus about the substitutability between private and public goods. If the nontradeables include public goods, then the more generic public goods (such as law and order) are substitutes and merit goods (like education and health) tend to be complements in private consumption. Thinking about tradeables as being largely comprised of goods and nontradeables as public and private services led us to choose a relatively low value of ρ as our benchmark, i.e., to reflect a relatively low degree of substitutability. We examine this assumption in greater detail below.

[18] The value of γ varies extremely widely across papers. In a well-known paper, Hagedorn and Manovskii (2008) use $\gamma = 0.584$ and $z = 0.955$. The latter choice was driven by a view that z should include the utility of leisure time as well as the unemployment benefit replacement rate. Moreover, Hagedorn and Manovskii argue for an extremely low bargaining weight, $\beta = 0.052$. However, we chose to follow the larger part of the literature by setting $\beta = 0.5$. As a check, we note that our chosen z is approximately 57 per cent of the wage in the homogeneous case, discussed in more detail below. The net unemployment benefit replacement rate in 2010 was 34 per cent for the United States, 23 per cent for Japan and 41 per cent for Australia (statistics taken from https://www.cesifo-group.de/ifoHome/facts/DICE/Labour-Market/Labour-Market/Unemployment-Benefit-Schemes/unemployment-benefit-replacement-rates/fileBinary/unemployment-benefit-replacement-rates.xls).

Note that there are two intersections for the equations determining the equilibrium in the homogeneous case, i.e., (30) and (31). Of the two equilibria, we can ignore the upper right intersection since it violates the labour constraint. Hence, $L_T = 0.4738$ and $L_N = 0.4738$, so that $U = 0.0524$.[19] For the other endogenous variables, $w_T = w_N = 1.2512$, $p = 1$ and $\theta = 0.2038$.[20] Economy-wide homogeneity of all parameters values, the identical production functions and the fact that consumption demand is equally divided between the two sectors yields equal sectoral labour demands.

The Figure also shows that the equilibria are stable. For example, if the value of the marginal productivity of labour in the nontradeables sector is lower than that in the tradeables sector, i.e. $pF'_{NL} < F'_{TL}$, (so that the implied value of (31) is less than that of (30)), labour moves to the tradeables sector. Hence, L_T and L_N converge to their steady-state levels.

Now consider the heterogeneous case. Figure 3 shows the case in which $s_N = 0.02 < s_T = 0.03$.[21] Once again, the equilibrium that violates the labour constraint is ignored. Hence, $L_T = 0.4711$ and $L_N = 0.4761$. Therefore, $U = 0.0528$. For the other endogenous variables, $w_T = 1.2482$, $w_N = 1.2370$, $\widetilde{W} = 0.0112$, $p = 0.9860$ and $\theta = 0.2005$.

---

[19] We use the Trust-Region Dogleg method and the *fsolve* command in MATLAB. Initial values are set at $L_T = L_N = 0.33$.

[20] As robustness checks, as suggested by Hagedorn and Manovskii (2008), note the following. First, the calculated value of θ is less than one, as required. Secondly, the marginal productivity of labour in both sectors, 1.2880 > the wage in each sector, 1.2512. Thirdly, as discussed, both wages > *z*. Finally, *z* as a proportion of the marginal productivity of labour is 55 per cent.

[21] In most of the related literature, the public sector is ignored (although see Burdett, 2012 and Gomes, 2015). Public sector economic activity is mostly non-tradeable in nature and employment in that sector is usually far more secure and continuous, compared to private sector employment. In this paper, we treat both cases, $s_T > s_N$ and $s_T < s_N$, although for expository purposes we treat the former as the baseline. Based on data from the U.S. Bureau of Labor Statistics, the annual total separation rate is approximately 0.4 for the private sector while it is 0.2 for the government sector; the latter being a large part of the nontradeables sector. For Japan, workers leaving the manufacturing sector have higher average probabilities of unemployment, while those leaving the government sector have lower probabilities of unemployment. We thank Tomoko Kishi for providing the facts for Japan.

--- Insert Figure 3 here ---

The lower separations nontradeables sector is now slightly larger and the RER is lower. While the economy-wide weighted separation rate is unchanged, $\bar{s} = 0.025$, the unemployment rate is slightly higher.[22] Because p is lower, the economy-wide average real wage is higher in the heterogeneous case and this explains the slightly higher unemployment rate (and lower value of θ).[23]

5. Simulating the Balassa-Samuelson effect

In this section, we use the functional form and parameter settings used in section 4 to obtain further quantitative predictions. Doing so also serves to resolve the theoretical indeterminacies in the previous section and to highlight the crucial role played by the substitutability of tradeables and nontradeables in consumption.

5.1. Productivity growth in the tradeables sector

For the homogeneous case, with $s_T = s_N = 0.025$, we obtained $L_T = L_N = 0.4738$. Thus, from (24), the productivity gap *alone* determines the size and direction of the B-S effect. The RER rises if technology grows relatively more in the tradeables sector. In the Cobb-Douglas case $(\rho \to 1)$, $dp/p = da_T/a_T - da_N/a_N$. With even productivity growth across sectors, $(da_T/a_T = da_N/a_N)$, then $dp/p = 0$.

In the heterogeneous case, where the separation rates differ (with $s_T = 0.03$ and $s_N = 0.02$), the labour demands are $L_N = 0.4761$ and $L_T = 0.4711$. Therefore, the B-S effect is

$$\frac{dp}{p} = \underbrace{1.9860\left(\frac{da_T}{a_T} - \frac{da_N}{a_N}\right)}_{\text{(i)}Relative\ Productivity\ Effect} + \underbrace{1.9860 \times 0.6667\left(\frac{dL_T}{L_T} - \frac{dL_N}{L_N}\right)}_{\text{(ii)}Relative\ Labour\ Effect}.$$

The Relative Labour Effect is important in the heterogeneous case. For example, if we assume a ten per cent change in productivity in both sectors (i.e., $da_T/a_T = da_N/a_N = 0.1$), then

[22] The economy-wide weighted separation rate is $\bar{s} \equiv (s_N L_N + s_T L_T)/(L_N + L_T)$.

[23] The economy-wide weighted average real wage is $\bar{w} \equiv ((w_N/p)L_N + w_T L_T)/(L_N + L_T)$. In the homogenous case, $\bar{w}$ =1.2512, while it is $\bar{w}$ =1.2514 in the heterogeneous case.

$$\frac{dp}{p} = \underbrace{1.9860(0.1-0.1)}_{\text{(i)}Relative\ Productivity\ Effect} + \underbrace{1.9860 \times 0.6667\left(\frac{dL_T}{L_T} - \frac{dL_N}{L_N}\right)}_{\text{(ii)}Relative\ Labour\ Effect}$$

$$= -0.00026482 < 0.$$

Even in the absence of a Relative Productivity Effect, the Relative Labour Effect means that p decreases. This occurs due to the relative expansion of the sector with low separations and a nontradeables supply-side effect.

While the magnitude seems negligible, it is more significant if the tradeables sector grows faster than the nontradeables sector (i.e., the 'usual' B-S thought experiment). If $da_T/a_T = 0.1$ and $da_N/a_N = 0$, we have

$$\frac{dp}{p} = \underbrace{1.9860 \times 0.1}_{\text{(i)}Relative\ Productivity\ Effect} + \underbrace{1.9860 \times 0.6667\left(\frac{dL_T}{L_T} - \frac{dL_N}{L_N}\right)}_{\text{(ii)}Relative\ Labour\ Effect} \quad (33)$$

$$= 0.19860 - 0.0763 = 0.1223 > 0.$$

Hence, $p = 1.1066$ $(= 1.1223 \times 0.986)$.

With respect to the Relative Productivity Effect, productivity growth in the tradeables sector leads to an increase in the RER, i.e., the B-S effect. Note that the Relative Labour Effect offsets nearly 38 per cent of the Relative Productivity Effect. As mentioned in Section 1, Sheng and Xu (2011) find that the effect of relative productivity growth in the tradeables sector is overstated in the basic B-S model and that introducing labour market frictions improves the predictive ability of the model. The simulation result in (33) implies that introducing intersectoral differences in separation rates, an empirical reality, mutes the basic B-S effect and improves the predictive ability of the model.

The wage in the tradeables sector rises faster than that in the nontradeables sector. Therefore, the wage gap increases, $d\widetilde{W}/\widetilde{W} = 0.1226 > 0$. Since the tradeables price is exogenous, tradeables producers decrease employment to absorb the increased wage cost. The positive income effect overcomes the negative effect of a relative rise in the nontradeables price, leading to higher labour demand in the nontradeables sector. The rise in nontradeables employment exceeds the lower tradeables labour demand, so that unemployment falls, $dU/U = -0.1089 < 0$.

5.2. The role of substitutability

Next, we examine how the effect of productivity growth in the tradeables sector on the shift in labour demand, and the concomitant effect on unemployment, depends on the substitutability of tradeables and nontradeables in consumption. To provide a stark contrast to our findings above, consider the case in which tradeables and nontradeables are more substitutable. Suppose that $\rho = 2$. We now obtain $dL_T/L_T = 0.0406 > 0$ and $dL_N/L_N = -0.0309 < 0$, i.e., labour moves from the nontradeables sector to the tradeables sector. Perhaps, a more intuitive result.

When a_T rises, $(\Psi F_T/F_N)^{1/\rho} = p$ and F'_{TL} both increase. When ρ is sufficiently small, labour demand in the nontradeables sector expands more compared to the tradeables sector and the nontradeables sector draws labour from the tradeables sector. This is because, while both marginal products increase, the value of the marginal product in the nontradeables sector increases relatively more. Clearly, the domestic substitutability in consumption of tradeables and nontradeables is crucial. With greater substitutability, i.e., a higher ρ, a rise in the nontradeables price leads to a significant shift in demand from nontradeables to tradeables. As a result, the tradeables sector draws labour from the nontradeables sector. This amplifies the magnitude of the B-S effect through a rise in the Relative Labour Effect. With respect to unemployment, $dU/U = -0.0782$, i.e., the tradeables sector attracts more labour than the nontradeables sector sheds.

In the following, we numerically calculate the critical value of ρ, which reverses the flow of labour between the two sectors. In Figure 4, we show dL_T/L_T, dL_N/L_N and dU/U for the range $\rho \in [0,10]$.

--- Insert Figure 4 here ---

Interestingly, there is a range in which labour demand in both sectors increases. Included in this small range is the Cobb-Douglas case (i.e., $\rho = 1$). In this range, the positive income effect increases labour demand in both sectors. Also notable is that the improvement in dU/U declines in ρ, although the sign of dU/U is always negative. This occurs because the sector with the higher separations rate is becoming a relatively larger part of the economy, i.e., the average labour turnover is higher. Notwithstanding, unemployment unambiguously falls.

Figure 5 shows the relationship between the wage gap and the elasticity of substitution when $a_T = 1$ and $a_T = 1.1$. The wage gap increases with a_T independently of the value of ρ. While the gap between the blue dotted line and the red solid line narrows as ρ increases, the gap is higher when a_T increases. To attract workers to the higher separations sector, higher wages are needed.

--- Insert Figure 5 here ---

5.3. Higher separation rate in nontradeables

To date, we have assumed that the nontradeables sector is the lower separations sector. We have done so based on a view that a major part of the nontradeables sector in contemporary economies is a large (and growing) public sector with its high job security and very low rates of labour turnover. Almost a quarter of workers are employed in the public sector of developed country labour markets, see Burdett (2012). If we were to ignore the public sector and treat the private services sector as the only nontradeables sector, then this assumption may be untenable. To sketch the effects of what we have assumed about sectoral separation rates, we reverse the inequality in separation rates.

Consider the case in which $s_T = 0.02 < s_N = 0.03$. Again, equilibrium values are shown in Table 1. Once again, the equilibrium that violates the labour constraint is ignored, so that $L_T = 0.4765$ and $L_N = 0.4715$. Therefore, $U = 0.0520$. For the other endogenous variables, $w_T = 1.2541, w_N = 1.2655$, $\widetilde{W} = -0.0114$, $p = 1.0142$ and $\theta = 0.2072$.

Compared to the results in section 3, the nontradeables sector is now smaller and the RER is higher. While the economy-wide weighted separation rate is unchanged, $\bar{s} = 0.025$, the unemployment rate is smaller, which occurs because p is higher and the economy-wide average real wage is lower ($\overline{w}$ $=1.2510$).

When we increase a_T from 1.0 to 1.1, the B-S effect is 0.1214 ($= (1.1373 - 1.0142)/1.0142$).[24] Of the correct sign, but slightly smaller than the 0.1223 in the previous section. Notwithstanding, nearly 38 per cent of what we term the Relative

[24] The values of the endogenous variables are: $L_T = 0.4655$, $L_N = 0.4876$, $U = 0.0469$, $w_T = 1.3900$, $w_N = 1.4028$, $\widetilde{W} = -0.0128$, $p = 1.1373$ and $\theta = 0.2603$.

Productivity Effect is offset by the Relative Labour Effect (cf. equation (24)). Wages in both sectors rise as before but the wage in the nontradeables sector rises faster than that in the tradeables sector. The reduction in unemployment is slightly larger (-0.0981 versus -0.0782). As in the previous section, labour moves from the tradeables sector to the nontradeables sector. Once again, an important contributor to the direction and size of these effects is the assumed substitutability between tradeables and nontradeables. Figure 6 shows that the result does not change quantitatively by reversing the relationship between s_T and s_N. This is because the impact of changes in the wage gap on labour movement between sectors is negligible compared to the effect of a rise in the nontradeables price. As in the case with $s_T > s_N$ examined in Section 5.1, a rise in the nontradeables price improves the value of marginal productivity in this sector, which draws labour from the tradeables sector.

--- Insert Figure 6 here ---

5.4. Productivity growth in the nontradeables sector

The nontradeable sector has been expected innovate more slowly compared to the tradeables sector. Notwithstandingly, many researchers have investigated nontradeables productivity as an important potential driver of economic growth (e.g., Kakkar, 2003, Morikawa, 2011). Accordingly, we examine the effects of productivity growth in the nontradeables sector by increasing a_N from 1.0 to 1.1. Sectoral labour demand responses still depend on the value of ρ. Moreover, there is a critical value of ρ which reverses the signs of dL_T/L_T, dL_N/L_N and, more importantly, dU/U. Specifically, $dL_T/L_T = 0.0278$, $dL_N/L_N = -0.0293$ and $dU/U = 0.0159$. For the wages, we obtain $dw_N/w_N = -0.0090$ and $dw_T/w_T = -0.0091$. Figure 7 shows how ρ is related to the dynamics of labour demand and unemployment assuming that the separation rate is higher in the tradeables sector than the nontradeables sector ($s_T > s_N$). There are two important differences compared to the case of growth in tradeables productivity. First, there is *no* value of ρ for which both sectors expand employment. This indicates that a fall of the nontradeables price mitigates the positive impact of productivity growth on national income. In other words, the overall income effect is smaller in the case of the nontradeables productivity growth. However, the most significant difference is that there is a range of ρ for which unemployment *grows*. When $\rho < 1$, labour demand in the tradeables sector grows, but not by enough to compensate for the contraction in nontradeables labour demand. Again, the income effect is the key to understanding this phenomena. When ρ is small (i.e., the price elasticity of demand is small), the nontradeables price significantly falls to increase demand. This falling nontradeables price leads to a significant reduction in the income effect. As a result, the overall income effect is small when ρ is small, and a fall of nontradeables labour demand cannot be fully offset by higher tradeables labour demand. Therefore, if tradeables and nontradeables are complements in consumption (as in the benchmark case), unemployment expands with productivity growth in the nontradeables sector.[25] As shown in Figure 8, this result does not qualitatively change when the separation rate is higher in the nontradeables sector ($s_T < s_N$).

--- Insert Figures 7 and 8 here ---

6. Concluding comments

Can different sectoral job separation rates improve the predictive ability of the B-S model? What affects the equilibrium unemployment rate in a small open economy with tradeables and nontradeables sectors? The nontradeables sector includes the public sector as well as the private nontradeable services sector. Therefore, what is the impact of the job separation rate being relatively higher in the tradeables sector? To answer these questions, we develop a B-S model that combined the search matching frictions of the Diamond-Mortensen-Pissarides model, in which workers are matched to jobs in either sector or possibly left unmatched, and industry-specific job separations. Compared to the extant literature, an important feature of the model is that the unemployed are treated as belonging to a single pool of job seekers and not as being tied to a specific sector. Search costs are assumed to be equal across industries. In another departure from the bulk of the

[25] The fact that productivity growth in the tradeables sector is more effective than productivity growth in the nontradeables sector for lowering unemployment also arises in the context of a three-sector Dutch disease model (exports, import-competing and services) (Gaston and Rajaguru, 2013). In other words, productivity growth in the nontradeables sector (which includes the government sector) may increase unemployment. In the present model, this result is sensitive to the value of ρ.

previous literature, a compensating sectoral wage differential arises due to differences in separations between each sector.

We examined the main B-S predictions about rising productivity in the tradeables sector caused by labour-augmenting technological progress. There is an appreciation of the RER and an increase in the sectoral wage differential. It was found that heterogeneous job separations help explain the poor quantitative performance of the basic B-S framework.

The overall effect on unemployment depended on the size and direction of change in labour demand in the two sectors. While unemployment falls, it is possible that one sector expands and the other contracts. This was shown to depend on the substitutability of tradeables and nontradeables in consumption. According to our simulation results, national unemployment rises with productivity growth in the nontradeables sector when tradeables and nontradeables are complements. Another possible policy implication is that, in order to reduce unemployment, the government should encourage or facilitate productivity improvement in those nontradeables industries whose output is more substitutable for the output of the tradeables sector.

Appendix

A Definition of coefficients

The definitions of coefficients for $I = N, T$ are

$$Q_I = \frac{\gamma}{1-\beta}\left(\frac{\alpha\theta^{\alpha-1}(r+s_I)}{A} + \beta\right), \qquad R_I = \frac{A\theta^{1-\alpha} + s_I}{A(1-\alpha)\theta^{-\alpha}U},$$

$$G_{11} = F'_{NL}, \qquad G_{12} = -pF''_{NL} + Q_N R_N,$$

$$G_{13} = Q_N R_T, \qquad G_{22} = Q_T R_N,$$

$$G_{23} = -F''_{TL} + Q_T R_T, \qquad G_{32} = \frac{p\varepsilon_{NL}}{L_N},$$

$$G_{33} = \frac{p\varepsilon_{TL}}{L_T}, \qquad H_{11} = pF''_{NLa},$$

$H_{22} = F''_{TLa}$, $\qquad H_{31} = \dfrac{p\varepsilon_{Na}}{a_N}$,

$H_{32} = \dfrac{p\varepsilon_{Ta}}{a_T}$.

B The effect of changes in tradeables productivity

Focusing on changes in the tradeables productivity, (25)-(27) are rewritten as

$$\mathbf{GX} = \mathbf{HK},$$

where

$$\mathbf{G} = \begin{bmatrix} G_{11} & -G_{12} & -G_{13} \\ 0 & -G_{22} & -G_{23} \\ \eta & G_{32} & -G_{33} \end{bmatrix}, \mathbf{X} = \begin{bmatrix} dp/da_T \\ dL_N/da_T \\ dL_T/da_T \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 0 \\ -H_{22} \\ H_{32} \end{bmatrix}, \tag{B1}$$

$$\mathbf{X} = \mathbf{G}^{-1}\mathbf{H}.$$

Denoting the determinant of $\mathbf{G}$ by Δ, then

$$\begin{aligned} \Delta = G_{11}[G_{22}G_{33} + G_{23}G_{32}] + \eta[G_{12}G_{23} - G_{13}G_{22}] \\ = G_{11}[G_{22}G_{33} + G_{23}G_{32}] \\ + \eta[pF''_{NL}F''_{TL} - pF''_{NL}Q_TR_T - F''_{TL}Q_NR_N] > 0. \end{aligned}$$

Using Cramer's rule, we obtain the following derivatives

$$\begin{aligned} \frac{dp}{da_T} &= \Delta^{-1}\{H_{22}(G_{12}G_{33} + G_{13}G_{32}) + H_{32}(G_{12}G_{23} - G_{13}G_{22})\} \\ &= \Delta^{-1}\{pF''_{NL}F''_{TL} - pF''_{NL}Q_TR_T - F''_{TL}Q_NR_N\} > 0 \end{aligned} \tag{B2}$$

$$\frac{dL_N}{da_T} = \Delta^{-1}\{G_{11}(G_{33}H_{22} + G_{23}H_{32}) - H_{22}\eta G_{13}\} \tag{B3}$$

$$\frac{dL_T}{da_T} = \Delta^{-1}\{G_{11}(-G_{22}H_{32} + G_{32}H_{22}) + H_{22}\eta G_{12}\}. \tag{B4}$$

(B2) corresponds to (28). From (21), it is shown that

$$\begin{aligned} \frac{d\theta}{da_T} = \Delta^{-1}\{R_NH_{22}G_{11}G_{33} + R_TH_{22}G_{11}G_{32} - F''_{TL}R_NH_{32}G_{11} \\ - pF''_{NL}R_T\eta H_{22}\} > 0. \end{aligned}$$

From (10), we can also show that

$$\frac{dU}{da_T} = -\frac{dL_N}{da_T} - \frac{dL_T}{da_T}$$
$$= -\Delta^{-1}[(H_{22}G_{33} + H_{32}G_{23})G_{11} - \eta H_{22}G_{13} + (-H_{32}G_{22} + H_{22}G_{32})G_{11} + \eta H_{22}G_{12}]. \tag{B5}$$

The sign of this effect is indeterminate without additional assumptions on the different functional forms (and the elasticity of output with respect to labour in the two sectors).

C The effect of changes in nontradeables productivity

Focusing on changes in the nontradeables productivity, **X** and **H** in (B1) are replaced by

$$\mathbf{X} = \begin{bmatrix} dp/da_N \\ dL_N/da_N \\ dL_T/da_N \end{bmatrix}, \mathbf{H} = \begin{bmatrix} -H_{11} \\ 0 \\ -H_{31} \end{bmatrix}.$$

We obtain the following derivatives:

$$\frac{dp}{da_N} = -\Delta^{-1}\{H_{11}(G_{22}G_{33} + G_{23}G_{32}) + H_{31}(G_{12}G_{23} - G_{13}G_{22})\}$$
$$= -\Delta^{-1}\{H_{11}(G_{22}G_{33} + G_{23}G_{32}) - H_{31}(F''_{TL}[-pF''_{NL} + Q_N R_N] + pF''_{NL}Q_T R_T)\} < 0 \tag{C1}$$

$$\frac{dL_N}{da_N} = \Delta^{-1}G_{23}\{-G_{11}H_{31} + \eta H_{11}\} \tag{C2}$$

$$\frac{dL_T}{da_N} = \Delta^{-1}G_{22}\{G_{11}H_{31} - \eta H_{11}\} = -\frac{G_{22}}{G_{23}}\frac{dL_N}{da_N}. \tag{C3}$$

(C1) corresponds to (29). In addition,

$$\frac{d\theta}{da_N} = \Delta^{-1}F''_{TL}R_N[G_{11}H_{31} - \eta H_{11}],$$

indicating that the effect of nontradeables productivity growth on the inverse Beveridge ratio is ambiguous. For wages

$$\frac{dw_N}{da_N} = \frac{\beta\gamma}{1-\beta}\left\{\frac{\alpha\theta^{\alpha-1}(r+s_N)}{A} + 1\right\}\frac{d\theta}{da_N}$$
$$\frac{dw_T}{da_N} = \frac{\beta\gamma}{1-\beta}\left\{\frac{\alpha\theta^{\alpha-1}(r+s_T)}{A} + 1\right\}\frac{d\theta}{da_N},$$

indicating that wages in both sectors are positively correlated with the inverse Beveridge ratio. Regarding the wage gap,

$$\frac{d\widetilde{W}}{da_N} = \frac{\beta\gamma\alpha\theta^{\alpha-1}(s_T - s_N)}{(1-\beta)A}\frac{d\theta}{da_N}.$$

The effect on unemployment is

$$\frac{dU}{da_N} = \frac{1}{G_{23}}\left(\frac{s_N - s_T}{A(1-\alpha)\theta^{-\alpha}U}Q_T + F''_{TL}\right)\frac{dL_N}{da_N}.$$

Therefore, the sign of the effect of nontradeables productivity growth on unemployment is indeterminate.

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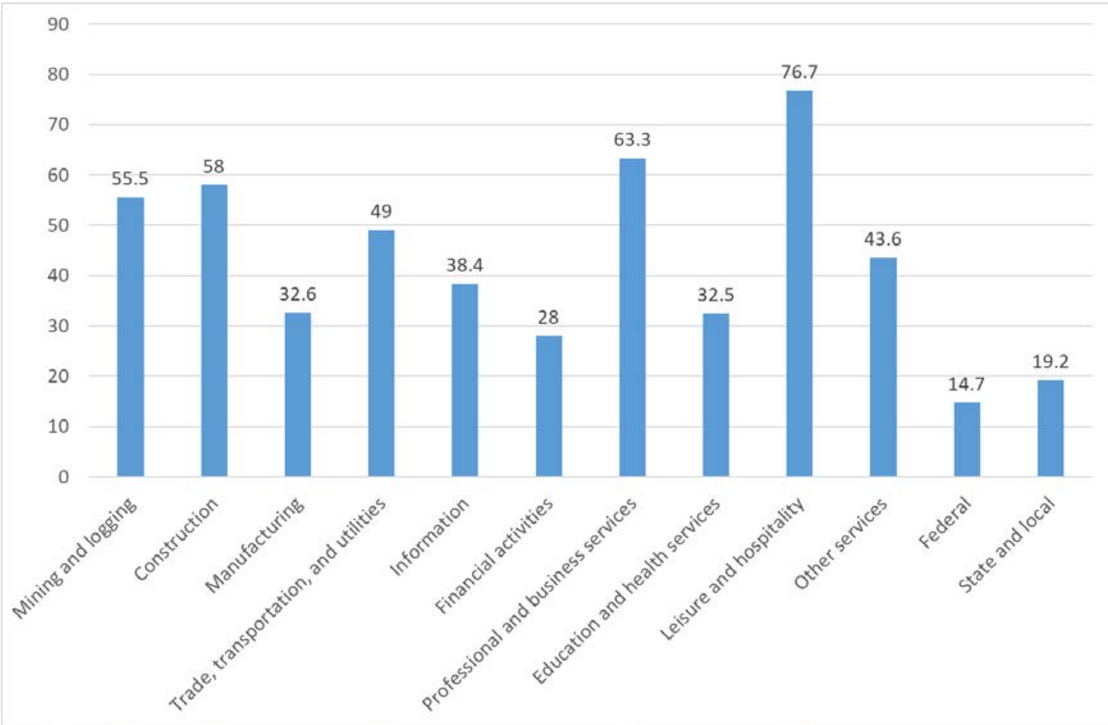
Table 1. Parameter and equilibrium values

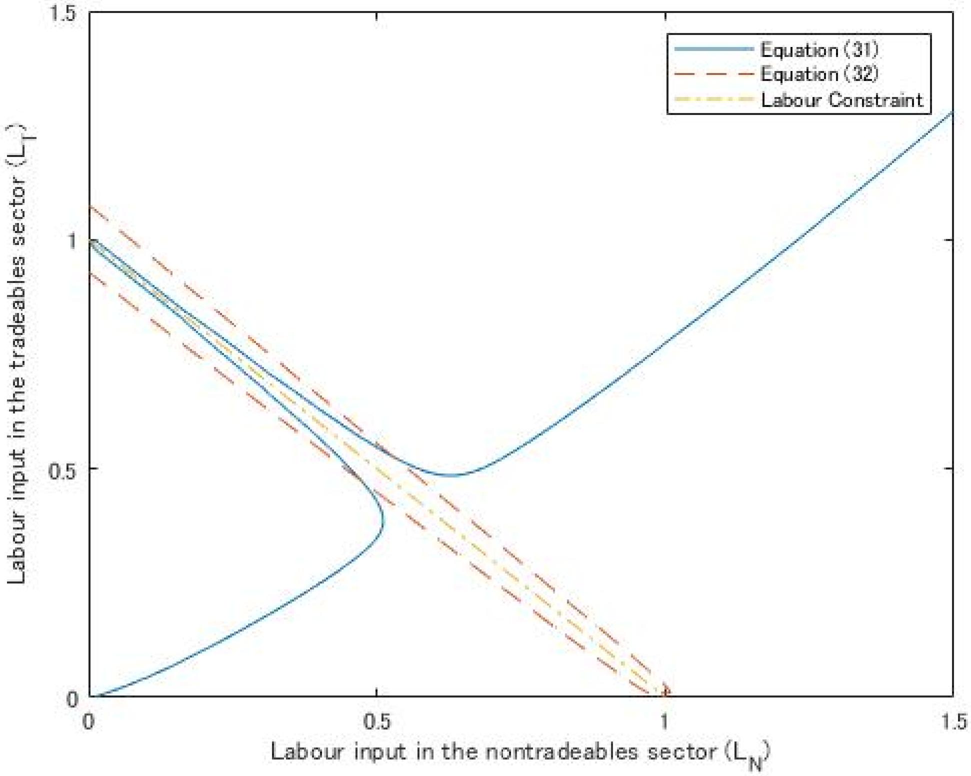
A. Homogeneous case ($s_T = s_N$)									
Parameter values									
s_T	0.025	ρ	0.5	ν_T	0.66	r	0.003	a_T	1
s_N	0.025	α	0.5	ν_N	0.66	$\bar{L}$	1	a_N	1
ψ	0.5	A	1	β	0.5	z	0.71	γ	2.5
Equilibrium values									
L_T	0.4738	w_T	1.2512	U	0.0524	θ	0.2038	$\bar{s}$	0.025
L_N	0.4738	w_N	1.2512	p	1	$\widetilde{W}$	0	$\overline{w}$	1.2512
B. Heterogeneous case ($s_T > s_N$)									
Parameter values									
s_T	0.03	ρ	0.5	ν_T	0.66	r	0.003	a_T	1
s_N	0.02	α	0.5	ν_N	0.66	$\bar{L}$	1	a_N	1
ψ	0.5	A	1	β	0.5	z	0.71	γ	2.5
Equilibrium values									
L_T	0.4711	w_T	1.2482	U	0.0528	θ	0.2005	$\bar{s}$	0.025
L_N	0.4761	w_N	1.2370	p	0.9860	$\widetilde{W}$	0.0112	$\overline{w}$	1.2514
C. Heterogeneous case ($s_T < s_N$)									
Parameter values									
s_T	0.02	ρ	0.5	ν_T	0.66	r	0.003	a_T	1
s_N	0.03	α	0.5	ν_N	0.66	$\bar{L}$	1	a_N	1
ψ	0.5	A	1	β	0.5	z	0.71	γ	2.5
Equilibrium values									
L_T	0.4765	w_T	1.2541	U	0.0520	θ	0.2072	$\bar{s}$	0.025
L_N	0.4715	w_N	1.2655	p	1.0142	$\widetilde{W}$	-0.0114	$\overline{w}$	1.2510

Figure 1. Sectoral Separation Rates (%) in the United States, 2018



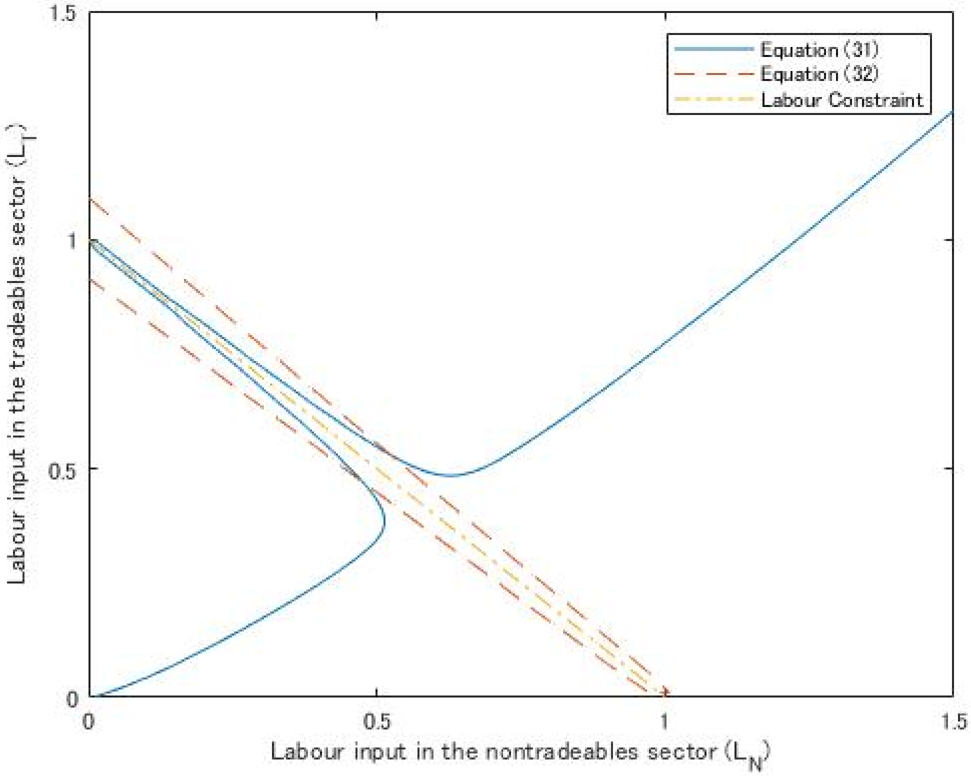
Source: U.S. Bureau of Labor Statistics (https://www.bls.gov/news.release/jolts.t16.htm)

Figure 2. Determination of L_T and L_N in the homogeneous case ($s_N = s_T$)



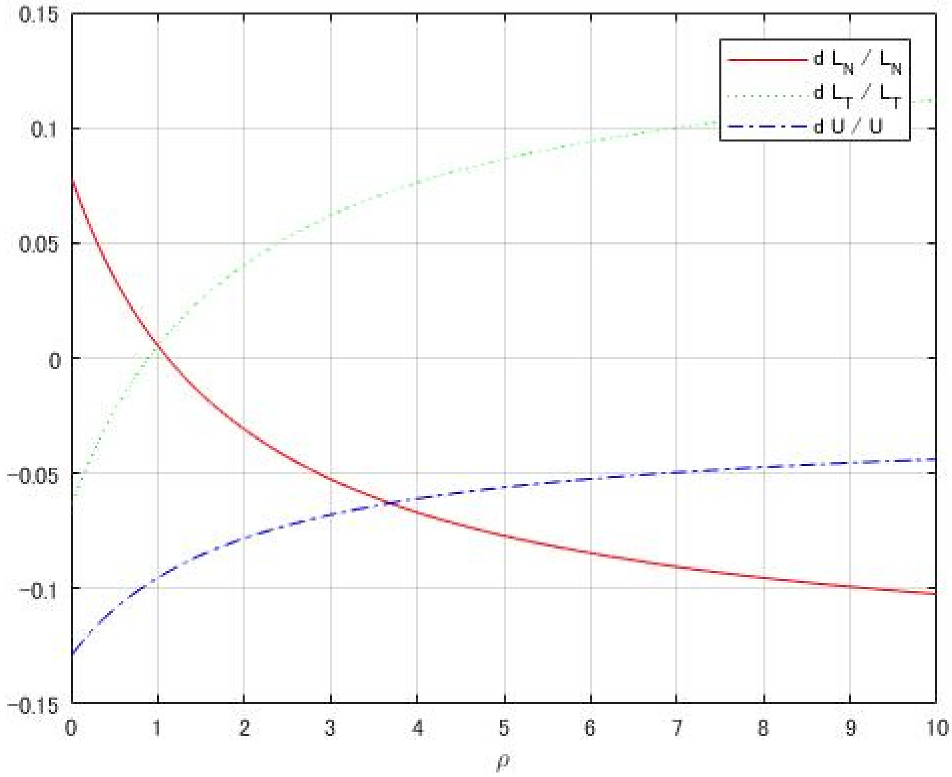
Note: Computation uses the parameter values in panel A of Table 1.

Figure 3. Determination of L_T and L_N in the heterogeneous case ($s_T > s_N$)



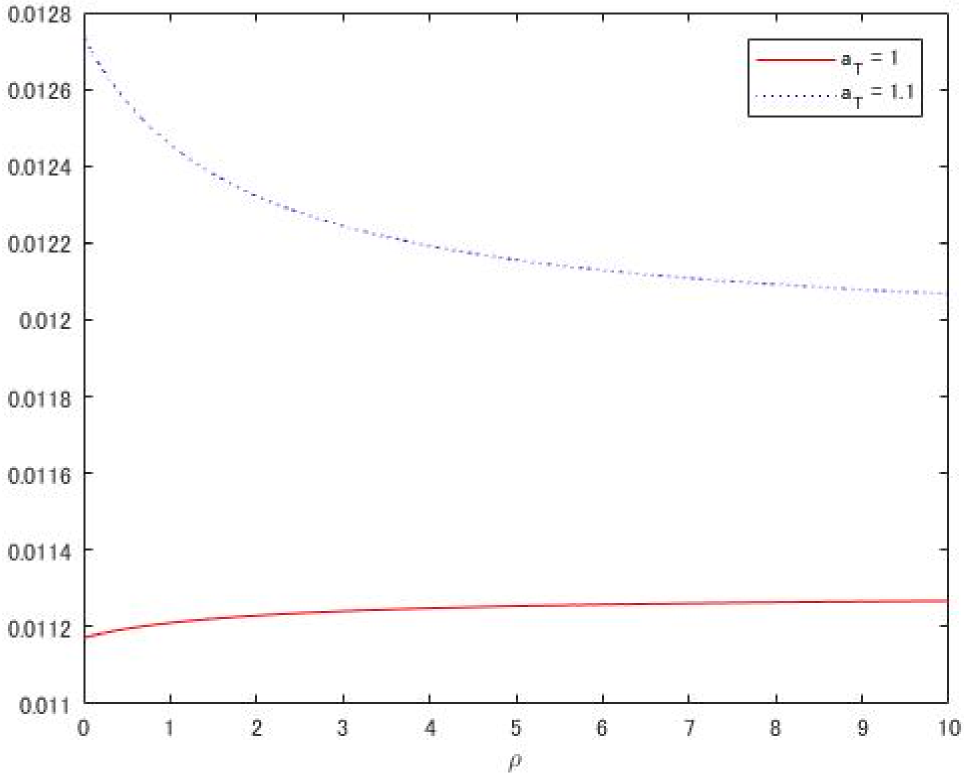
Note: Computation uses the parameter values in panel B of Table 1.

Figure 4. Response of labour demand and unemployment to productivity growth in the tradeables sector ($s_T > s_N$)



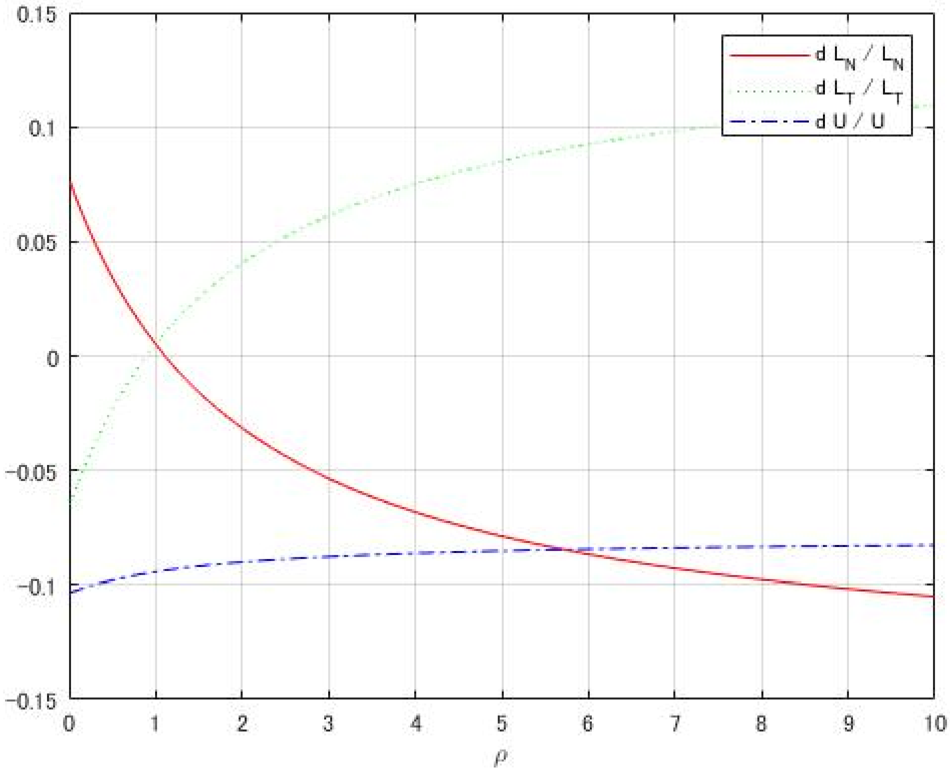
Note: Computation uses the parameter values in panel B of Table 1.

Figure 5. Response of the wage gap to productivity growth in the tradeables sector for alternative degrees of substitutability ($s_T > s_N$)



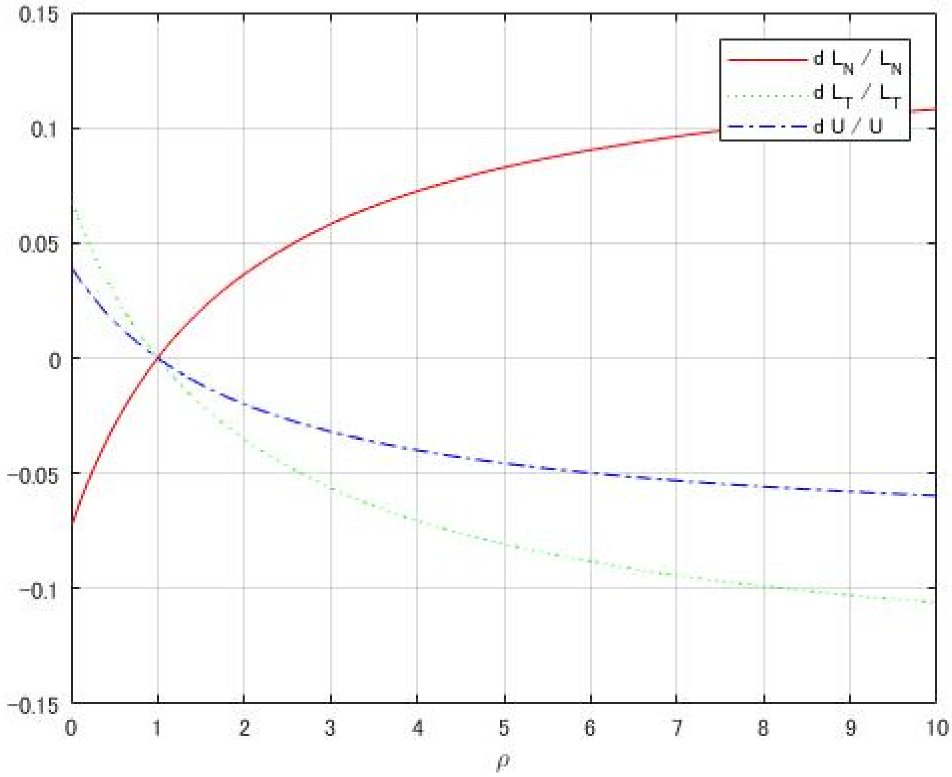
Note: Computation uses the parameter values in panel B of Table 1.

Figure 6. Response of labour demand and unemployment to productivity growth in the tradeables sector ($s_T < s_N$)



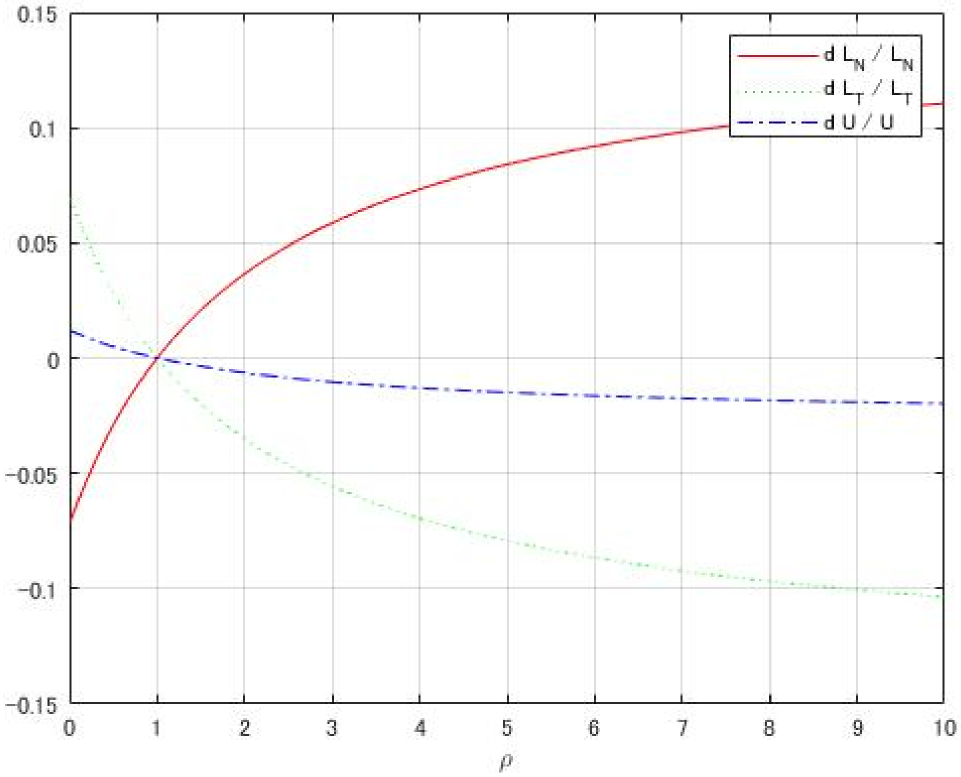
Note: Computation uses the parameter values in panel C of Table 1.

Figure 7. Response of labour demand and unemployment to productivity growth in the nontradeables sector ($s_T > s_N$)



Note: Computation uses the parameter values in panel B of Table 1.

Figure 8. Response of labour demand and unemployment to productivity growth in the nontradeables sector ($s_T < s_N$)



Note: Computation uses the parameter values in panel C of Table 1.