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Economic Black Holes and Labor Singularities in the Presence of Self-replicating Artificial Intelligence*

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Abstract

This study is motivated by the widely-held view that self-replicating artificial intelligence may approach "some essential singularity . . . beyond which human affairs, as we know them, could not continue" (von Neumann). It investigates what state this process would lead to in an economy with frictionless markets. We demonstrate that if the production technologies, too, are frictionless, all workers will eventually be pulled into the most labor friendly sector (economic black hole). If, instead, they are subject to a friction created by congestion, it will give rise to, within a finite span of time, a state in which all workers will be unemployed (total job destruction).
JEL Codes: E13; E24; O33.

Keywords: Artificial intelligence; economic black holes; labor singularity; general equilibrium models
JEL classification: E13; E24; O33

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1 Introduction

This study is motivated by the widely-held view that artificial intelligence (AI) may someday become capable of improving and replicating itself. The news has been received enthusiastically that such a machine has been created for neural network (see Chang and Lipson (2018)). The idea of self-replicating AI is, however, not new. In the 1950s, von Neumann started working on self-reproducing automata (von Neumann (1966)). In the *Bulletin of the American Mathematical Society* tribute to him, Ulam (1958, p. 5) recalls a remark by von Neumann that "centered the ever accelerating progress of technology and changes in the mode of human life, which gives the appearance of approaching some essential singularity in the history of the race beyond which human affairs, as we know them, could not continue." As self-replicating AI has come closer to realization, what von-Neumann's sigularity might be like has become a more important question. The present study makes the first approach to this question under the assumption that the market is frictionless.

Our first result is that if the production technologies, too, are frictionless, in the presence of self-replicating AI, the most labor friendly sector functions like a black hole, pulling workers into a small set of slightly less labor friendly sectors and eventually absorbing all workers in an infinitely far future. As the gravity around a black hole increases to infinity, the labor density over the segment of sectors employing workers will become infinity. For this reason, we may call the most labor friendly sector an economic black hole. The simplicity of our model shows that an economy with high quality markets without any frictions has a natural tendency to create an economic black hole if AI starts self-replicating.

If an economic black hole exists, all workers are fully employed at any moment of time. The main concern of people may, however, be the possibility that self-replicating AI might take away all human jobs.[1] Our first result shows that in order to capture such a state, either the market or the production technology must be subject to some frictions. Our result demonstrates that if the production technologies are subject to a friction created by worker congestion, all the workers may be replaced within a finite span of time by AI, which we call a labor singularity.[2]

[1]For example, a renowned physicist Stephen Hawking states, "I fear that AI may replace humans altogether. If people design computer viruses, someone will design AI that improves and replicates itself. This will be a new form of life that will outperform humans;" see Hawing's interview for WIRED magazin (Medeiros, 2017).

[2]See Aghion, Benjamin, Jones and Jones (2019) for a study on singularity in a different context.

Over the long course of history, humans have discovered many new technologies. Each time a fundamental technology was discovered, it pulled workers out of the existing workplaces and shifted them into more capital-intensive workplaces created by the new technology. At the level of elementary textbooks, this process may be described by a standard isoquant between labor and capital; if capital increases with a fixed amount of labor, the wage rate will rise relative to the rate of return to capital, thereby benefitting workers. Although this process has progressed very slowly until now, self-replicating AI is expected drastically to change this picture. We incorporate such a technological progress so as to describe von Neumann's singularity, which cannot be explained in the elementary textbook mechanism. Economic black holes and labor singularities are among them.

This study is related to the literature on automation, which is concerned with the process of endogenous innovation through which the existing labor intensive production processes are replaced by more capital intensive processes (see Zeira (1998), Boldrin and Levine (2002), Peretto and Seater (2013), Acemoglu and Restrepo (2018), and Chu, Cozzi, Furukawa, and Liao (2018)). This study differs from that literature in focusing on self-replicating AI, which improves and replicates itself independently of the human factor, as envisioned by von Neumann (1966).

In what follows, in Section 2, we will demonstrate the existence of an economic black hole in a model in which neither the market nor the production process is subject to frictions. In Section 3, we will demonstrate that congestion in labor intensive sectors will eventually give rise to a labor singularity. Section 4 is for concluding remarks.

2 Black Hole Absorbing All Workers

In this study, we focus on artificial intelligence (AI) that is self-replicating; such artificial intelligence is a physical production factor that improves and replicates itself and will eventually overwhelm workers. That is, we say that artificial intelligence, denoted as X, is self-replicating at the rate of γ if it grows at a rate faster than labor resources, H;

$$\dot{X} = \gamma X. \tag{1}$$

and

$$\dot{H} = \kappa H \tag{2}$$

with $\gamma > \eta$.

In order to describe the process in which workers shifts more labor intensive sectors by the self-replicating AI, X, assume that there is a continuum of differentiated middle products, $s \in [0,1]$, each of which can be produced from X only, labor only, or both. In order to assume that the higher s, the more difficult it is to be replaced by the self-replicating AI. Following Zeira (1999) and, more specifically, Acemoglu and Restrepo (2018), we model this process by a linear production function

$$y(s) = a_H(s)h(s) + a_X(s)x(s), \tag{3}$$

where $h(s)$, $x(s)$, and $y(s)$, respectively, denote the labor output for s, the artificial intelligence input for s, and the output of s. Let $a_H(s) > 0$ and $a_X(s) > 0$. Assume that the larger s, the more labor "friendly" sector s, $a'_H > 0$, and that the smaller s, the more AI "friendly" sector s ($a'_X < 0$). This assumption implies that the larger s, the more labor intensive relative to AI sector s is, i.e., $\alpha'(s) > 0$ where $\alpha(s) = a_H(s)/a_X(s)$.

Assume that the final consumption good is produced from the middle products only. The production function is CES,

$$Y = \left(\int_0^1 y(s)^{\frac{\varepsilon-1}{\varepsilon}} ds \right)^{\frac{\varepsilon-1}{\varepsilon}}, \tag{4}$$

where $\varepsilon > 0$ is the elasticity of substitution for middle products. Assume that both the final good production sector and intermediate good production sectors are perfectly competitive.

Let w be the wage rate and p_X be the price of AI. Denote as $\omega = w/p_X$ the factor price ratio. Then, sector s employs both labor and AI if and only if

$$\omega = \alpha(s) \tag{5}$$

Let $S \equiv S(\omega)$ be the s satisfying (5) for ω. Then, $S(\omega)$ captures the critical value the sectors below which employ only AI in the case in which the factor price ratio is ω. If sectors $s > S(\omega)$ employ only labor. Since $\alpha' > 0$ by assumption, so is S, i.e., $S' > 0$.

The demands for AI and labor depend on the factor price ratio, ω, and are expressed as

$$X^d \equiv \int_0^{I(\omega)} x(s)ds \text{ and } H^d \equiv \int_{I(\omega)}^1 h(s)ds, \tag{6}$$

respectively, where

$$I(\omega) = \begin{cases} 0 & \text{if } \omega < \alpha(0) \\ S(\omega) & \text{if } \alpha(0) \le \omega \le \alpha(1) \\ 1 & \text{if } \alpha(1) < \omega \end{cases} . \tag{7}$$

This implies that the density of workers per sector is $H/(1-I(\omega))$, $I(\omega) < 1$. Suppose that $I(\omega) \to 1$ as $t \to \infty$ and that $I(\omega) < 1$ for all t. Then, all workers, H, are employed in the sectors between $[I(\omega), 1]$. As $t \to \infty$, the density of workers per sector, $H/(1 - I(\omega))$, will become infinity. For this reason, we call sector 1 an economic black hold if $I(\omega) \to 1$ as $t \to \infty$ and that $I(\omega) < 1$ for all t.

The final good sector's demand for intermediate input is given by

$$y(s) = p(s)^{-\varepsilon} Y, \tag{8}$$

where $p(s)$ is the price of intermediate input s. Since $p(s)$ must be equal to the marginal cost of production for each sector, it holds that $r/a_X(s) = p(s)$ for $s < I(\omega)$ and $w/a_H(s) = p(s)$ for $s > I(\omega)$. The input demand for $s < I(\omega)$ is $x(s) = y(s)/a_X(s)$ whereas that for $s > I(\omega)$ is $h(s) = y(s)/a_H(s)$. Thus, by using (6) and (8), the aggregate demands for AI and labor are

$$h(s) = \frac{Y}{w^{\varepsilon}} a_H(s)^{\varepsilon-1} \text{ for } s > s(\omega) \tag{9}$$

and

$$x(s) = \frac{Y}{r^{\varepsilon}} a_X(s)^{\varepsilon-1} \text{ for } s < s(\omega). \tag{10}$$

By (6), (9), and (10), the relative demand for labor is

$$\frac{H^d}{X^d} = \frac{\int_{I(\omega)}^{1} a_H(s)^{\varepsilon-1} ds}{\int_0^{I(\omega)} a_X(s)^{\varepsilon-1} ds} \omega^{-\varepsilon} = d(\omega). \tag{11}$$

In equilibrium, it must hold that

$$X^d = X \text{ and } H^d = H. \tag{12}$$

The next theorem shows the existence of an economic black hole.

Theorem 1 (Economic Black Hole) *Assume $\gamma > \kappa$ and $a_X(1) > 0$. If $\gamma > \eta$, then, sector 1 is an economic black hole.*

Proof. Since $\theta'_H < 0$, $\theta_H(\omega) = 0$ if $\omega \geq \alpha(1)$, and $\theta_H(\omega) > 0$ if $\omega < \alpha(1)$. Moreover, by $\theta'_X > 0$, $\theta_X(\omega) = 0$ if $\omega \leq \alpha(0)$, and $\theta_X(\omega) > 0$ if $\omega > \alpha(0)$. Thus, $d(\omega) \to \infty$ as $\omega \to \alpha(0)$, and $d(\omega) \to 0$ as $\omega \to \alpha(1)$. Since $d(\omega) = H/X = e^{(\kappa-\gamma)t}$ and $\kappa < \gamma$, $d(\omega) \to 0$ and $\omega \to \alpha(1)$ as $t \to \infty$. ∎

3 Job Destruction

As is shown above, if the market and the production process are both frictionless, the most labor friendly sector acts like a black hole, gradually pulling all the workers into the sector. As the time passes, the labor density on sectors employing workers become larger and converge to infinity in the "end." If labor density will increase indefinitely, that itself should be no problem for the workers, all of whom will continue to be employed.

As is shown in Hawking's WIRED interview (Medeiros, 2017), what people are concerned with is not the presence of such a black-hole-like sector but the possibility that all workers, replaced by AI, will lose workplaces. The above analysis reveals that such a cause may be found at least at two places: Markets and production processes. Since we are concerned with an economy with completely frictionless markets, we shall focus on frictions that may exist in production processes.

As a source of a friction in the production sector, we focus on the congestion that reduces the productivity of workers if too many workers are employed in a single sector. In what follows, we illustrate that this friction may give rise to the total unemployment of workers.

In order to incorporate this congestion factor, we now explicitly think of an economy with two productive sectors: AI-intensive and labor-intensive. In order to produce output, the AI-intensive sector uses only AI to whereas the labor-intensive sector use only. In order to introduce congestion. We assume that the marginal cost of labor is equal to $m_H(s) = a_H(s)$ up to a fixed level of production, $\bar{h} \geq h(s)$. Once labor employment exceeds $\bar{h}$, however, workers loses their productivity completely due to congestion, i.e., $m_H(s) = 0$ for $h(s) > \bar{h}$. The marginal product of AI in each sector, $m_X(s)$, is, as before, constant at $m_X(s) = a_X(s)$. In the case in which $\bar{h} = \infty$, the model in this section coincides with that of the previous section. This captures one of the two polar cases on the elasticity of labor employment. The other polar case is that labor employment can be enlarged indefinitely without reducing the marginal productivity, as assumed in the previous section.

Because AI and labor freely moves between the two sectors, in equilibrium, the AI-intensive sector's output is given by

$$y(s) = a_X(s)x(s) > 0 \text{ if } r = p(s)a_X(s) \text{ and } w > p(s)a_H(s). \qquad (13)$$

Since $y(s) = a_H(s)h(s)$ for $h(s) \leq \bar{h}$ and $y(s) = a_H(s)\bar{h}$ for $h(s) > \bar{h}$, a corner solution arises at $h(s) = \bar{h}$. This implies the labor-intensive sector's output is given by

$$y(s) = \begin{cases} a_H(s)h(s) > 0 & \text{if } h(s) < \bar{h},\ w = p(s)a_H(s) \text{ and } r > p(s)a_L(s) \\ a_H(s)\bar{h} > 0 & \text{if } h(s) \geq \bar{h},\ w \leq p(s)a_H(s) \text{ and } r > p(s)a_L(s). \end{cases} \tag{14}$$

We will first prove that even if the amount of AI, X, is arbitrarily small, the labor input of each AI sector does not hit the upper bound $\bar{h}$. $h(s)$, $s < s(\omega)$, the size of the AI sector is always positive. In order to prove this result, the size if X is sufficiently small, $s = s(\omega) > 0$ in equilibrium. In order to illustrate the possibility of unemployment in this model, we transform condition (11) into the following system of equilibrium conditions:

$$\omega = \left(\frac{X}{H}\right)^{1/\varepsilon} \left(\frac{\int_s^1 a_H(s)^{\varepsilon-1} ds}{\int_0^s a_X(S)^{\varepsilon-1} dS}\right)^{1/\varepsilon} = \omega_1(s, X) \tag{15}$$

and

$$\omega = \alpha(s). \tag{16}$$

Lemma 1 *Let* (s, ω) *be the solution to the equilibrium condition,* (15) *and* (16)*. For any small* $X > 0$, $s = s(\omega) > 0$ *in equilibrium, and* $h(s) < \bar{h}$ *for all* $s < s(\omega)$.

Proof. This is because $\alpha' > 0$, $\partial\omega_{1X}/\partial s < 0$ and $\partial\omega_{1X}/\partial X > 0$ for any (s, ω) satisfying (15) and (16). This implies that in equilibrium, $s > 0$ and $s \to 0$ and $X \to 0$. ■

If the upper bound is imposed on labor employment, (10) remains valid. Thus, it holds that

$$\omega = X^{1/\varepsilon} \frac{w}{Y^{1/\varepsilon}} \left(\frac{1}{\int_0^{s(\omega)} a_X(S)^{\varepsilon-1} dS}\right)^{1/\varepsilon}. \tag{17}$$

In contrast, if the upper bound is imposed, (9) no longer holds. Instead, the following holds:

Lemma 2 *Let* $y(I) = a_H(I)h(I)$, $h(I) = \bar{h}$, *and* $w = p(I)a_H(I)$*. Then,* $y(s) = a_H(s)\bar{h}$ *if* $s > I$.

Proof. Since $y(I) = p(I)^{-\varepsilon} Y$, $p(I) = \left(\frac{a_H(I)\bar{h}}{Y}\right)^{-1/\varepsilon}$. Thus,

$$w = \left(\frac{a_H(I)\bar{h}}{Y}\right)^{-1/\varepsilon} a_H(I) = \left(\frac{Y}{a_H(I)\bar{h}}\right)^{1/\varepsilon} a_H(I) = \frac{a_H(I)}{a_H(I)^{1/\varepsilon}} \left(\frac{Y}{\bar{h}}\right)^{1/\varepsilon}$$

This implies, by $\varepsilon > 1$ and $a'_H > 0$,

$$w = a_H(I)^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{Y}{\bar{h}}\right)^{1/\varepsilon} < a_H(s)^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{Y}{\bar{h}}\right)^{1/\varepsilon}$$

for all $s > I$. ■

The next lemma follows from the above lemma.

Lemma 3 *Let $w = a_H(I)^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{Y}{\bar{h}}\right)^{1/\varepsilon}$. Then, $h(I) \leq \bar{h}$ and $h(s) = \bar{h}$ for $s > I$.*

Lemmas 1, 2, and 3 imply that a dynamic equilibrium can be in the following four phases:

Phase 1: $h(s) < \bar{h}$ for all s, $s(\omega) < s < 1$.
Phase 2: $y(s) = a_H(s)\bar{h}$ for all s, $s(\omega) < I < s < 1$, and $y(s) = a_H(s)h(s)$ for all s, $s(\omega) < s < I$.
Phase 3: $y(s) = a_H(s)\bar{h}$ for all s, $s(\omega) < s < 1$.
Phase 4: $y(s) = 0$ for all s.

Since the equilibrium allocation in Phase 1 is characterized by (15) and (16), it suffices to focus on Phases 2 and 3. Let us first start with Phase 3, which is easier.

In Phase 3, we may assume $I = s(\omega)$ in Lemma 3. Thus, by Lemma 3, we have

$$\frac{w}{Y^{1/\varepsilon}} = a_H(s(\omega))^{\frac{\varepsilon-1}{\varepsilon}} \left(\frac{1}{\bar{h}}\right)^{1/\varepsilon}. \tag{18}$$

Thus, by (17) and (18), the equilibrium allocation in Phase 3 is characterized by the system of equations

$$\omega = \left(\frac{X}{\bar{h}}\right)^{1/\varepsilon} \left(\frac{a_H(s)^{\varepsilon-1}}{\int_0^s a_X(S)^{\varepsilon-1} dS}\right)^{1/\varepsilon} = \omega_3(s, X) \tag{19}$$

and (5).

The determination of the equilibrium allocation in Phase 2 is somewhat more complicated. In this phase, $y(s) = a_H(s)\bar{h}$ for all s, $s(\omega) < I < s < 1$, and $y(s) = a_H(s)h(s)$ for all s, $s(\omega) < s < I$; note that I is an endogenous variable. Then, the following holds:

Lemma 4 *If* $s(\omega) < s < I$,

$$h(s) = a_H(s)^{\varepsilon-1}\frac{Y}{w^{\varepsilon}},\ \ x(s) = 0,\ \ \textit{and}\ y(s) = a_H(s)h(s).$$

By Lemmas 3 and 4, the full employment condition in the labor market can be written as

$$H = \frac{\bar{h}}{a_H(I)^{\varepsilon-1}}\int_s^I a_H(S)^{\varepsilon-1}dS + \int_I^1 \bar{h}dS \tag{20}$$

where $s = s(\omega)$. Thus, the I satisfying (20) can be expressed as a function of s, $I = I(s)$. Moreover, since, by Lemma 3, $\frac{w}{Y^{1/\varepsilon}} = a_H(I)^{\frac{\varepsilon-1}{\varepsilon}}\left(\frac{1}{\bar{h}}\right)^{1/\varepsilon}$ in Phase 2, by (17), the equilibrium allocation in Phase 2 may be written by the system of equation

$$\omega = \left(\frac{X}{\bar{h}}\right)^{1/\varepsilon}\left(\frac{a_H(I(s))^{\varepsilon-1}}{\int_0^s a_X(S)^{\varepsilon-1}dS}\right)^{1/\varepsilon} = \omega_2(s, X). \tag{21}$$

and (5).

In our model, the equilibrium allocation can be illustrated by the intersection between demand and supply curves of AI sectors. See Figure 1. Because ω is the inverse of the relative price of AI, the demand curve is upward-sloping whereas the supply curve is downward-sloping. That is, curve $\omega = \alpha(s)$ may be thought of as the demand for AI sectors. Curves ω_1, ω_2 and ω_3 illustrate the graphs of functions $\omega = \omega_1(s, X)$, $\omega = \omega_2(s, X)$ and $\omega = \omega_2(s, X)$, respectively. As (15), (21), and (19) show, curve ω_i shifts upwards as X increases. This reflects the fact that more AI sectors will be born in response to an increase in the amount of AI; this may be interpreted as an upward shift of the supply of AI sectors. In each phase, an equilibrium allocation is determined at the intersection between demand curve $\alpha(s)$ and supply curve ω_i.

We are now ready to characterize the process of automation in the presence of self-replicating AI, $\dot{X}_t > 0$. In Figure 1, the graph of $\omega = \alpha(s)$ is illustrated by curve α, which is upward-sloping by assumption. By definition, $1/\omega = r/w$ is the relative price of AI in terms of labor. This implies that curve α may be interpreted as the inverse demand function for the AI-using sector; as $1/\omega$ increases, the demand for the AI sector, $s = \alpha^{-1}(\omega)$, increases. Moreover, we illustrates the graphs of $\omega = \omega_1(s, X)$, $\omega = \omega_2(s, X)$ and $\omega = \omega_3(s, X)$ by curves ω_1, ω_2 and ω_3, respectively. These curves may be thought of as the inverse supply function of the AI-using sector; as (15), (19), and (21) show, these curves shift upwards as AI self-replicates, or X increases.

First, Lemma 1 implies that when X is sufficiently small, the equilibrium allocation (s_t, ω_t) is determined at the intersection of curves α and ω_1; $\omega_t = \omega_1(s_t, X_t) = \alpha(s_t)$. As is noted in the proof of Lemma 1, curve ω_1 is downward-sloping for each given X_t and shifts upwards as X_t increases. In Phase 1, no labor sector hits the upper bound of employment; or in other words, $h_t(1) < \bar{h}$.

When the most labor-friendly sector, 1, hits the upper bound, $h_t(1) = \bar{h}$, Phase 2 starts. In Phases 2 and 3, in general, supply curves ω_2 and ω_3 could be upward-sloping. For the sake of simplicity, we focus on the normal case, in which they are downward-sloping. In Phase 2, the equilibrium allocation (s_t, ω_t) is determined at the intersection of curves α and ω_2; $\omega_t = \omega_2(s_t, X_t) = \alpha(s_t)$. In our model, the size of labor sectors that hit the upper bound of employment, $\bar{h}$, is given by $1-I_t$, where $I_t = I(s_t)$. At the time at which Phase 2 starts, only sector 1 hits the upper bound, i.e., $I_t = 0$. As X_t increases, the equilibrium allocation shifts to the right. With this shift, we may prove that $I_t - S(\omega_t)$ decreases until $I_t - S(\omega_t) = 0$.

In Phase 3, the equilibrium allocation (s_t, ω_t) is determined at the intersection of curves α and ω_3; $\omega_t = \omega_3(s_t, X_t) = \alpha(s_t)$. This phase starts when $I_t = s(\omega_t)$ holds. By (20), the labor demand becomes

$$H^d = \bar{h}[1 - S(\omega_t)]. \tag{22}$$

If $H > H^d$, $H - H^d$ workers are employed. Thus, as soon as $S(\omega_t) > 1 - H/\bar{h}$, some workers will be unemployed; the distance between $S(\omega_t)$ and $1 - H/\bar{h}$ measures the scale of labor unemployment. At this point, partial job destruction starts. Note

$$\omega_3(1, X) = \left(\frac{X}{\bar{h}}\right)^{1/\varepsilon} \left(\frac{a_H(1)^{\varepsilon-1}}{\int_0^1 a_X(S)^{\varepsilon-1} dS}\right)^{1/\varepsilon} > 0. \tag{23}$$

This implies that X_t increases, the vertical intercept at $s = 1$ of $\omega_3(s, X)$ shifts upwards. Thus, as is shown in Figure 1, once this vertical intercept hits the vertical intercept at $s = 1$ of curve α, i.e., once $\omega_3(1, X_t) = \alpha(1)$ holds, $s_t = 1$, which implies all the workers will be unemployed. After this point, Phase 4 starts, in which total job destruction occurs within a finite span of time.

Theorem 2 (Total Job Destruction) *Let $\alpha(1) < \infty$. If all sector's are subject to an upper bound of labor employment, $\bar{h}$, there is t' such that $S(\omega_t) = 1$ for all $t > t'$.*

4 Concluding Remarks

This paper demonstrates that if not only the market but also the production technologies are frictionless, self-replication AI will eventually push all the workers into the most labor friendly sector (economic black hole). If, in contrast, the production technology is subject to a congestion factor that prevents an individual sector from employing too large a number of workers, the economy will eventually reach a state in which all the workers will be unemployed in an infinitely far future (labor singularity). Our results suggest that in order to avoid the emergence of an economic black hole and/or a labor singularity, labor productivity must increase basically at the same speed as self-replicating AI. This might be possible because self-replicating AI would produce more and more output. If that output is invested into education and/or innovation, it may be possible to raise labor productivity at a sufficiently high speed.[3]

[3]See Yano and Furukawa (2019) for the role of education in the presence of self-replicating AI. For the role of innovation in enhancing the productivity of workers, see Acemoglu and Restrepo (2018), who focus on a different type of AI.

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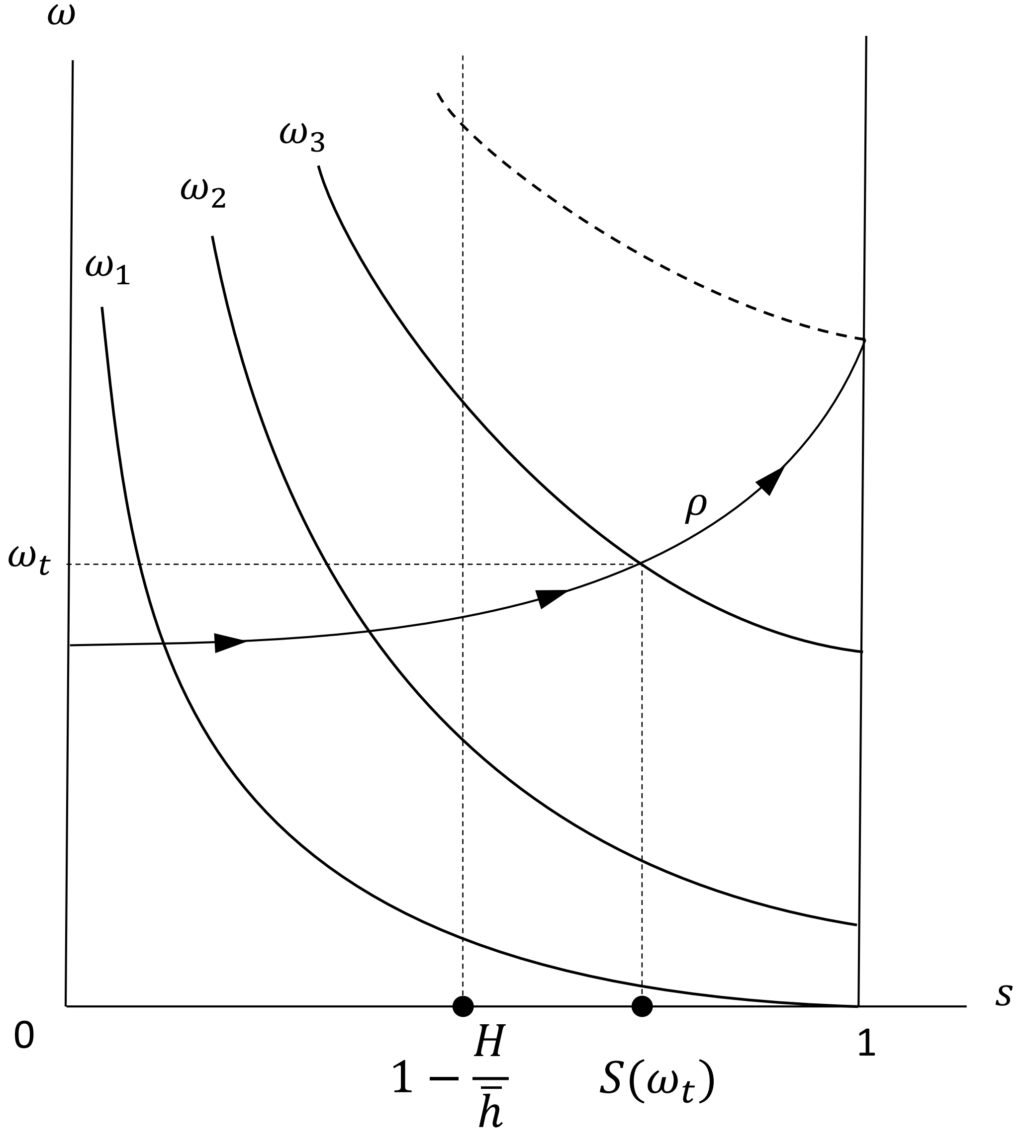

Figure 1: Job Destruction