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Speedy Bankruptcy Procedures and Bank Bailouts

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Abstract

To reduce the future occurrence of bank bailouts, after the global financial crisis of 2008, the financial stability policies seem to settle into stronger prudential regulations (e.g., capital requirements) and speedier bankruptcy procedures especially for big banks (e.g., living wills). Speedier bankruptcy procedures have also been adopted to help heavily indebted households and corporations in many countries. However, I argue here an opposite consequence. Because of simple and speedy bankruptcy procedures, along with prudential regulations, bank bailouts can be justified in a crisis. This result emerges as an implication of optimal contracts in general equilibrium with an endogenous, competitive banking sector.

Keywords: Bank bailout, prudential regulation, deposit insurance, insolvency, tail risk, endogenous banking sector

JEL classification: E44, G21, G28

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I. INTRODUCTION

After the global financial crisis of 2008, financial stability policies seem to settle with stronger prudential regulations (e.g., capital requirements in Basel III) and speedier bankruptcy procedures especially for big banks (e.g., living wills). These reform agenda, led by the Financial Stability Board and the Basel Committee, and then adopted by major countries, have targeted, at least partially, to reduce future occurrence of bank bailouts, especially for too-big-to-fail banks. The reasons for those policy reforms seems straightforward (see, e.g., IMF, 2013). A larger capital buffer reduces the need for public funding in crisis. A speedy bankruptcy procedure reduces the increasing costs and uncertainties associated with lengthy bankruptcy negotiations.

However, I argue here for an opposite consequence. Because of a simple and speedy bankruptcy procedure, with prudential regulations, bank bailouts are justified in crisis. This result emerges in general equilibrium financial contracts with an endogenous, competitive banking sector. Assuming that the speedy bankruptcy based on a simple rule is beneficial to lower bankruptcy costs, then a simple and speedy bankruptcy rule should be adopted from the cost-savings point of view. In the model, some are depositors, others are borrowers, and the rests are bankers. Bank bailouts ex post cannot be Pareto improving as the resource are reallocated from some to others. However, from the ex ante viewpoint, bailouts can be regarded as a risk insurance mechanism to help depositors who lose most if banks default. Such contingent policy could be also called as deposit insurance with ex post funding from the government, though typical deposit insurance scheme is ex ante set fees from banks. Under either names, such insurance scheme can be ex ante Pareto improving. Note that the state-contingent resource redistribution makes sense only when the market equilibrium is inefficient. And, the simple bankruptcy procedure can indeed result in different, and thus inefficient, allocation compared to the optimal allocation that could be reached after state verification and negotiation, which however are costly.

As a general setup to understand a banking crisis and related policy issues, especially bank bailouts, from a viewpoint of equilibrium contracts, this paper provides a micro-foundation of typical macro models with financial frictions. Since Diamond and Dybvig (1983), the literature discusses heavily on the liquidity problem but I focus on the insolvency problem of banks because bank bailouts and other restructuring efforts with fiscal money were essential in the banking crises and stemmed from insolvency of banks. Namely, I characterize equilibrium financial contracts with defaults in a simple one-period general equilibrium model with depositors, borrowers, and bankers. A general equilibrium perspective is important since systemic importance has been stressed as a major reason for bank bailouts in many countries. Also, because regulations and bailout expectations affect the banking sector size, policy implications are better to be studied recognizing endogeneity of the banking sector size.

I assume segregated households, in contrast to a "big household" that shares all the agents' income risks perfectly and is often assumed in the macroeconomic literature.[2] Note that a fictitious "big household" assumption is convenient to track macroeconomic dynamics, but it is not usual in reality

[2]In typical general equilibrium macroeconomic models with financial frictions, income risks are assumed to be shared among bankers, borrowers, and depositors, with population of each type exogenously given (e.g., Bernanke, Gertler, and Gilchrist (1999) and Gertler and Kiyotaki (2010)). Moreover, banks are often not well separated from firms in the literature.

that typical households share their income with bankers. Also, the big household assumption is not theoretically consistent with financial frictions that govern the lending activities in these models.

Instead, this paper assumes that there are ex ante identical agents but in the beginning of the period they choose different occupations, either entrepreneurs in the production sector or bankers in the banking sector. I assume other extreme, i.e., no cross shareholdings, so that they do not become a ”big household.” In this case, a banker necessarily assumes her bank’s risk and an entrepreneur assumes his firm’s risks. Still, occupational arbitrage implies that the expected utility of bankers should equate with those of entrepreneurs.

Before production, entrepreneurs draw talents or business ideas, and are sorted out to two types, either having high or low productivity. A highly productive entrepreneurs borrow capital from bankers while those with a low productivity deposits. Loan and deposit contracts determine how risks are shared among three types of agents. In equilibrium, the bankers own positive capital with which they provide a partial insurance for depositors against aggregate shocks. As an insurance premium, bankers have expected income from a spread between the deposit and loan rates. Then, both high and low productivity agents produces with capital in hand. Outputs are subject to idiosyncratic and aggregate shocks. With a bad tail shock, many borrowers may default and bankers may face bankruptcy.

To study bankruptcies, debt contracts should be chosen in equilibrium. Townsend (1979) is a seminal paper based on costly state verification. However, it implicitly assume ex ante (partially) contingent contract, without a possibility of debt restructuring negotiations. Because a simple and speedy bankruptcy procedures have been adopted to mitigate the negotiation costs and debt overhang in general, I need to introduce incomplete contract which require negotiation ex post. Following (implicitly) Chari and Kehoe (2016), I assume such state verification and negotiation ex post are costly, for example, to pay accountants and lawyers and then debt overhang costs due to delays in settlements.

Then, as the third regime but main focus of this paper is the simple and speedy bankruptcy procedure. In this case, following incomplete contract literature (e.g., Hart, 1995), borrower’s asset is seized upon default with some retention. Implicitly, the state is assumed to be revealed to the creditor when seizes the asset. The negotiation would not occur and thus negotiation cost is zero. In this way, this simple but speedy bankruptcy procedure can save bankruptcy related costs.

However, the asset allocation is no longer well aligned with the creditors’ and borrowers’ marginal valuations, which are the case in Townsend (1979) or costly negotiation regime. Instead, as is often the case with reality and macro models, all the assets are seized by creditors except for a specific amount of assets retained by borrowers. With large negative shock, such a simple asset allocation rule, though speedy, would allow borrowers and bankers to walk away easily from their debts but then make depositors to assume all the tail risks (*tail-risk dumping*).

Bank bailouts that insure deposits funded by consumption tax, together with a capital ratio requirement, can be welfare improving, even ex ante, since bailouts can make the tail-risk allocation among segregated households more equal. Also welfare improving are the bailouts with additional bankruptcy rule reforms to ask more direct burden sharing for both depositors and borrowers.

This optimality of bailouts relies on the limited liability with a simple asset seizure rule but also on the special power of a government to tax on defaulters (e.g., via consumption tax). In this sense, if the government cannot seize assets from defaulters, I also show that bank bailouts or any other policies cannot improve the market allocation. Although numerous papers implicitly or explicitly argue the needs for government intervention to bank restructuring in crisis, to the best of my knowledge, theoretical argument (i.e., better sharing of tail risks) proposed by this paper is a novel one, not articulated before in the literature.[3]

The literature so far identified a few different justifications for bank bailouts. In many theoretical models, as in this paper, distinction between bank depositors and other creditors are not well delineated. Hence, theories to support deposit insurance could also support bank bailouts. A seminal paper for the need of deposit insurance is Diamond and Divbig (1983). Because a bank borrows short-term funds (deposits) but lends to long-term projects, it cannot repay to all the depositors at once if all of them asked to do so. Knowing this, if depositors expect a bank run, they would try to withdraw their deposits as fast as possible. In this sense, a bank run becomes a self-fulfilling equilibrium. Deposit insurance can eliminate depositors' incentive to withdraw deposits even if they know many other depositors would do so. He and Xiong (2012) extended this analysis to the market based funding. Depending on parameter values, they support public protection of investors in a short-term funding market. However, such contingent protection, if provided, is never used in equilibrium. Moreover, Allen and Gale (2004) and Killenthong and Townsend (2016) argue that the Diamond-Divbig type mechanism does not necessarily require government interventions as the market can achieve the constrained Pareto optimal allocation.

More plausible reason to bail out banks seems its ex post optimality due to a (reduced form) cost of bank's bankruptcy and debt overhang.[4] Indeed, it seems apparently optimal to bail out banks ex post to avoid a large bankruptcy cost, as several empirical papers (e.g., Ashcraft, 2005; Peek and Rosengren, 2000) found sizable aggregate costs stemming from banks' bankruptcies. Theoretically, Chari and Kehoe (2016) argues that the government ends up bailing out banks ex post to save bankruptcy costs as the subgame perfect equilibrium, even though it is not ex ante optimal. In this case, expecting bailouts, banks may engage risky behaviors ex ante, for which prudential regulations plays a role, similar to Kareken and Wallace (1978).

In the real world, calls for speedier bankruptcy procedures have started at least since 2000, to mitigate bankruptcy costs and more generally debt-overhang problems. For example, Germany and Japan changed its insolvency related laws and precedents at least a few times to adopt US Chapter 11 like debt restructuring laws for private entities (e.g., 1999, 2005, and 2012 in Germany; 2000, 2003, and 2006 in Japan). Before such changes, when a firm becomes insolvent in these countries, liquidation with lengthy court process was the norm. And, to avoid it, insolvencies often ignited lengthy private negotiations, often without involving a court, between creditors and a borrower. In the aftermath of the Global Financial Crisis, even more simpler and speedier debt restructuring schemes were adopted in many countries (IMF, 2013). For example, the U.S. adopted the Home Affordable Modification Program (HAMP) in 2009 to expedite massive mortgage bankruptcy cases to settle smoothly. A similar program was adopted in the U.K. and even wider ones were in place in

[3] Green (2010) is related. In a model similar to Diamond and Dibvig (1983) but with production by firms (no banks in his model), the only available contract is subject to limited liability. A better contract in terms of incentive to induce higher production is the contract without limited liability. The difference needs to be fill in by the government's tax-subsidy system.

[4] A deeper argument of debt overhang problem is as follows. If it is close to bankruptcy, a bank may not lend to profitable projects (Myers, 1977) or it may lend to highly risky projects (Jensen and Meckling, 1976).

crisis-hit European countries (e.g., GIIPs, Iceland, eastern European countries). The International Monetary Fund has also been recommending crisis-hit countries to adopt such simple and speedy bankruptcy procedures (Claessens, et. al, 2014). Moreover, in the midst of the banking crisis, although a key tool for a bank rescue is recapitalization, other tools include subsidized purchase of bad assets, which took a form of a specialized asset management company or good-bank-bad-bank separation (see e.g., Landier and Ueda, 2009). Those schemes can be also classified as speedier bankruptcy procedures.

Not only the real world has progressed towards simpler and speedier bankruptcy procedures, but also a few strands of academic literature take them as given. The empirical law-and-finance literature show speedier bankruptcy regimes are growth and efficiency enhancing (e.g., cross-country panel regression studies by Djankov, et. al., (2008) and by Claessens, Ueda, and Yafeh (2014)). Many dynamic stochastic general equilibrium macroeconomic models assume implicitly or explicitly a simple and speedy bankruptcy procedure—typically borrowers retain some portion of their assets and creditors take the rest in bankruptcy without any additional direct costs (e.g., Gertler and Kiyotaki, 2010).

Note that macro models' simple asset allocation rule is in line with the reality and I assume such simple rule. Under the simple rule, borrowers retain some portion of their assets and creditors take the rest in bankruptcy without any additional direct costs. Indeed, Chapter 11-type debt restructuring procedure is often characterized not only by a speedy procedure but also debtor in possession, which allows debtors to keep key assets to run a firm as a going concern or, in household bankruptcy cases (Chapter 13 in the US), to keep key assets such as a house to ensure a minimum consumption level. The allowed retained asset values are sometimes large. For example, in the State of Florida, the value of the primary residence that can be retained by a defaulted borrower is almost unlimited as long as it is less than half acre.

If bank bailout is a good policy, why doesn't a bank recapitalizes itself by issuing equity? In my model, when the recapitalization is needed, borrowers and bankers are bankrupt and depositors lose their promised deposit return. In this situation, no private agent is willing to recapitalize banks. Indeed, without government power of tax on defaulters, equilibrium bankruptcies are constrained optimal. In general, the literature also show the bank recapitalization by private agents is difficult, especially in crisis, although bank recapitalization would be beneficial to banks and the economy. This is because recapitalization by a bank itself by issuing additional equity is often blocked by its shareholders on the ground that it benefits mostly debt holders and dilute shareholders' values (Landier and Ueda, 2009). In particular, when the default is imminent, public recapitalization may be worth to pursue (Philippon and Schnabl, 2013). Empirically, some argues that the Japanese lost decade is a result of reluctant government involvement on decisive recapitalization and speedy cleaning up of nonperforming loans (Hoshi and Kashyap, 2010). Recently crisis-hit European countries seems to share similar problems (IMF, 2013).

This paper also endogenizes the banking sector size. In most of the macroeconomic models with financial frictions, bank defaults are absent and the capital ratio is not determined endogenously. If the banking sector size were exogenously given in my model, then it would be difficult to identify a full scale of distortions. For example, the capital adequacy ratio requirement would create higher monopoly rents for bankers in an exogenously given banking sector, but such rents would dissipate with endogenous entry of bankers. Still, a capital ratio requirement can be effective when it counteracts the distortions caused by institutionalized bank bailouts financed by consumption tax.

Only a few papers have investigated the endogenous nature of the financial sector size. The U.S. financial sector has grown over time with increased bankers' wage that compensates increased bankers' income risk (Phillippon, 2008). In an occupational choice model, Bolton, Santos, and Scheinkman (2016) argues that the financial traders attract too many talents due to profitable opportunities in the opaque OTC market. Their papers apparently bring important arguments but have little to say about policies towards deposit-taking banks.

Because it relies on specific financial frictions, this paper obviously misses other issues relating to bank bailouts, in particular, the moral hazard related to making low efforts or diverting funds by bank managers. This does not imply that the moral hazard is not important but please note that the optimal contract under the moral hazard by hidden effort is typically output contingent, equity-type contract, not debt-type ones, so that discussing bankruptcy would be difficult. Even so, when making an actual policy decision, all the issues on bailouts, including the moral hazard, should be carefully considered. Indeed, since Kareken and Wallace (1978), the moral hazard of too much risk taking by banks due to deposit insurance and other forms of government protections have been well recognized and called for the regulations. Moreover, Calomiris and Gorton (1991) argue, based on historical evidences, that the bank runs often associated with insolvency and not characterized as random events that models with self-fulfilling equilibria would imply. Demirgüç-Kunt and Detrageache (2002) show more formally that crisis probability is not reduced by the presence of deposit insurance in their regression analysis. This paper also shows that, under the optimal bailout scheme, bankers have thinner capital, more leverage, and default thresholds of equilibrium contracts become higher, allowing more frequent defaults. However, these happens in the social optimum.

In summary, this paper identifies a novel reason why many governments ended up to bail out banks in crises. Moreover, ex ante optimality validates an institutionalized bailout scheme (e.g., a resolution fund) together with a capital ratio requirement. It is a way to protect depositors against large negative aggregate shocks when the legally allowed retained assets by borrowers and bankers are unconditional on the aggregate output levels. When a tail risk event hits, defaulted borrowers and bankers can still enjoy consumption levels protected by the limited liability but depositors are not (*tail-risk dumping*). By transferring the goods from defaulters to depositors using consumption tax, a government can mitigate the incomplete risk sharing arrangement embedded in the bankruptcy procedure that is efficient in normal times. An improvement on explicit bankruptcy rules to be (more) contingent on aggregate shocks could improve welfare and could eliminate the needs for bank bailouts and associated prudential regulations. However, questions remain how to retain the speediness due to a simple rule by changing the bankruptcy rule to a complex one and whether writing such a complex rule can be done without prohibitive costs.

II. Model Setup

A. Demography, Utility, and Technology

I analyze a simple one-period model to understand the basic characteristics of a simple default procedure in allocating factors among depositors, borrowers, and bankers. A continuum of ex ante identical agents lives in the interval of $[0, 1]$and endowed with the same initial capital $k_0 > 0$. An agent chooses to become an entrepreneur or a banker endogenously. An entrepreneur then becomes either a depositor or a borrower depending on his talent. I denote bankers' population by μ and entrepreneurs' by $1-\mu$. I assume bankers locate from 0 to μ on the unit line, as if indexed by subscript h. A half of the remaining $1-\mu$ people are depositors, as if indexed by i and the rest are borrowers indexed by j.

Bankers intermediates the capital market in this paper. In a Walrasian equilibrium, in tradition of Arrow and Debreu (1954), there would be an auctioneer who offers price and matches demand and supply. This paper departs from the Walrasian setup in a few ways.[5] First, instead of an auctioneer, there is a continuum of nonatomic bankers who intermediate capital markets. Second, a banker offers a more general form of "price" in the capital market, that is, deposit and loan repayment schedules. Given the deposit and loan contract offers that specify repayment schedules, entrepreneurs pick the best contracts and decide the amounts of deposits and loans. After the production takes place, goods are allocated among borrowers, depositors, and bankers in accordance with the repayment schedules. Note that repayment schedules may be constrained by a bankruptcy regime, which I will discuss extensively later.

There is essentially one period, which is split to seven stages as follows.

- Stage I: Each agent chooses occupation, either a banker or an entrepreneur.
- Stage II: Business ideas of entrepreneurs are chosen by "nature." They are revealed to others freely.
- Stage III: Bankers offer deposit and loan contracts.
- Stage IV: Each entrepreneurpicks the best contract and decides to make deposits or to take loans depending on his idea. In other words, entrepreneurs are sorted to depositors or borrowers. Deposit and loan markets clear.
- Stage V: Production takes place with aggregate and idiosyncratic productivity shocks, which are chosen by "nature" and private information without costly verification.
- Stage VI: Borrowers repay loans and bankers repay deposits according to agreed contracts. Defaults may occur. Defaulters' outputs and assets are divided according to the bankruptcy regime.
- Stage VII: Agents consume what they have at the end of the period. Note that, in a later section, I introduce a possible government redistribution.

Because they are ex ante identical, both entrepreneurs and bankers have the same utility function from consumption. Each agent maximizes the expected utility $E[u(c)]$. For the sake of simplicity, I assume constant relative risk aversion, that is, $u(c) = c^{1-\sigma}/(1-\sigma)$ with a positive parameter value $\sigma > 0$. Note that, the utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing $u' > 0$ and concave $u'' < 0$ and satisfies the Inada conditions.

In Stage II, once an agent becomes an entrepreneur, he observes his business idea e.[6] After observing his idea e, he makes an investment decision on endowed capital $k_0 \in \mathbb{R}_{++}$. The capital reallocation

[5] The equilibrium concept can be regarded as a variant of Prescott and Townsend (1984 a,b) or Ueda (2013).

[6] This e can be also regarded as his talent.

is assumed to be intermediated only by specialists called bankers. Looking at deposit and loan contract offers from bankers made in Stage III, in Stage IV, depositor i makes deposits $s_i \in [0, k_0]$, which is the sum of deposits in each bank s_{hi}, and borrower j take loans $l_j \in \mathbb{R}_+$, which is the sum of loans in each bank l_{hj}. An entrepreneur then invest capital and produce outputs.

In Stage V, while producing goods, entrepreneurs are hit by aggregate and idiosyncratic productivity shocks. In total, there are three types of shocks that each entrepreneur faces: In Stage II, the ex ante idiosyncratic idea shock e from the cumulative distribution $F(e) : [\underline{e}, \overline{e}] \to [0, 1]$ with mean one and $\underline{e} > 0$; In Stage V, the ex post idiosyncratic productivity shock ϵ from the cumulative distribution $H(\epsilon) : [\underline{\epsilon}, \overline{\epsilon}] \to [0, 1]$ also with mean one and $\underline{\epsilon} > 0$; and, at the same time, the ex post aggregate productivity shock A from the cumulative distribution $G(A) : [\underline{A}, \overline{A}] \to [0, 1]$ with mean greater than one and $0 < \underline{A} < 1$ to make a possibility of aggregate negative net return, e.g., as a tail risk. Without loss of generality, I assume only two levels of business ideas, good idea e^U and bad idea e^D (i.e., up or down) with equal probability $1/2$. Note that $e^U > 1 > e^D > 0$ and $(e^U + e^D)/2 = 1$.

The production function is Cobb-Douglas with capital share $0 < \alpha < 1$ as in a standard macroeconomic model. One unit of labor is assumed to be inelastically supplied by each agent to his own project. The output is then expressed as

$$
\begin{aligned}
y_i^D &= y(s_i, e^D, A, \epsilon) = \epsilon_i A e^D (k_0 - s_i)^\alpha \quad \text{for those make deposits } s_i; \\
y_j^U &= y(l_j, e^U, A, \epsilon) = \epsilon_j A e^U (k_0 + l_j)^\alpha \quad \text{for those take loans } l_j.
\end{aligned}
\tag{1}
$$

The production function exhibits diminishing marginal returns to capital. Also, note that the marginal product of capital for a depositor's own business goes infinity when $s_i \to k_0$, implying that some capital is always invested in own business.

In Stage IV, without borrowing and depositing, entrepreneurs who received a good idea e^U would have higher expected marginal returns on the endowed capital k_0 than those with a bad idea. Accordingly, such entrepreneur j would like to borrow capital l_j from bankers until the expected marginal returns equate to the expected loan repayment. From banker h's perspective, she lends l_h to firms who submit loan demands with business idea, for which the banker is assume to be able to tell the quality, good or bad, without costs. Bank h's deposit intake is denoted as s_h, which is the sum of deposits from all the depositors.

On the other hand, an entrepreneur with bad idea e^D would have a lower expected marginal return if investing only the endowed capital k_0 than the effective deposit rate. He is thus willing to deposit s_i, some portion of his endowed capital, to bankers, and operate own business more productively in a smaller scale. In the end, a depositor's expected marginal return from own business would be equated to the expected deposit return after correcting for risks.

With a risk averse utility function, deposit and loan amounts are affected by risk sharing considerations and so do the expected deposit return and loan repayment in equilibrium. With a positive spread between the deposit and loan rates, some entrepreneurs might not want to engage in transactions with banks. However, in the case with two ideas, a sufficient difference between the two are assumed so that those with a good idea always become borrowers and those with bad idea always become depositors for a reasonable range of the spread.

After he finishes producing goods, borrower j transfers outputs to banker h according to a potentially nonlinear loan repayment schedule, $R^L(l_{hj}, A, \epsilon_j) : \mathbb{R}_+ \times [\underline{A}, \overline{A}] \times [\underline{\epsilon}, \overline{\epsilon}] \rightarrow \mathbb{R}_+$, which is a gross return rate to lenders and potentially depends on the borrower's loan amount. The function space from which R^L is chosen is denoted by Λ. If there is a flat rate portion in the repayment schedule, the return rate is denoted by $(1 + \rho^L)$, where $\rho^L \in \mathbb{R}_+$ is called as a promised loan rate. For the sake of simplicity and without loss of generality, I assume that capital inputs depreciate 100 percent after they are used in the production process.[7]

Without loss of generality, a banker is assumed not to price-discriminate among borrowers and depositors so that banker h offers the same loan repayment schedule $\tilde{R}^L_h \in \Lambda$ for all borrowers.[8] Since Williamson (1987) shows a possibility of credit rationing with costly state verification, and more generally market frictions may require optimal contracts to link price and quantity (e.g., Prescott and Townsend, 1984), I assume here that bankers can specify both loan repayment schedule and amount in their contracts. This is also in line with the real world contracts, which often specify both loan rates and amounts.[9] Specifically, banker h can specify the loan amount in her offer $l_{hj} \in \mathbb{R}_+ \times \{N.S.\}$ where $\{N.S.\}$ denotes the non-specified option. Note that a banker's offer with $l_{hj} = 0$ means that banker h rejects lending to firm j. With denoting the loan amount offer to all possible borrowers $l_h = l_{h1} \times l_{h2} \times \cdots$, banker h offers a loan contract (R^L, l_h) from $\overline{\Lambda} \equiv \Lambda \times (\mathbb{R}_+ \times \{N.S.\})^{\frac{1-\mu}{2}}$, which is the strategy space covering for all possible loan amounts for every borrower with one loan repayment schedule.[10] I denote $\Omega \in \overline{\Lambda}^\mu$ as the set of loan contract offers from all the bankers.

It is taken for granted that a banker always pools idiosyncratic shocks by allocating loans equally among a sizable set of borrowers. This implies that deposit repayment from a banker depends only on the aggregate shock A. An active banker transfers outputs to a depositor according to a potentially nonlinear deposit repayment schedule, $R^D(S, A) : \mathbb{R}_+ \times [\underline{A}, \overline{A}] \rightarrow \mathbb{R}_+$, which is a gross return rate to depositors and potentially depends on the bank's deposit intake. The function space from which R^D is chosen is denoted by Δ. If there is a flat rate portion, the return rate is $(1 + \rho^D)$, where $\rho^D \in \mathbb{R}_+$ denotes a promised deposit rate. Again, without loss of generality, a banker is assumed not to price-discriminate among depositors so that the repayment function offer R^D must be the same for all depositors. Then, banker h offers a same deposit repayment schedule $R^D_h \in \Delta$ for all depositors. Similar to loan contracts, I assume that a banker can specify the deposit amount in her offer $s_{hi} \in \mathbb{R}_+ \times \{N.S.\}$. With denoting the deposit amount offer to all possible depositors $s_h = s_{h1} \times s_{h2} \times \cdots$, banker h offers a deposit contract (R^D, s_h) from $\overline{\Delta} \equiv \Delta \times (\mathbb{R}_+ \times \{N.S.\})^{\frac{1-\mu}{2}}$, which is a banker's strategy space covering for all possible deposit amounts from every depositor with one deposit repayment schedule. I denote $\Psi \in \overline{\Delta}^\mu$ as the set of all deposit contract offers.

Bankers offer loan and deposit contracts to entrepreneurs. Entrepreneurs submit their loan demands and their deposit supplies to bankers. An entrepreneur i's demand for loan contact offered by banker

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[7] More generally, any depreciation rate can be assumed in the model. Theoretical implications would still go through.

[8] Non-discrimination assumption would be redundant even in equilibrium with possible price discrimination, since one repayment schedule would be chosen optimally following essentially same lemmas and propositions shown below.

[9] This formulation of competition in both price and quantity by intermediaries follows Ueda (2013).

[10] Superscript $(1 - \mu/2)$ means $(1 - \mu/2)$-time Cartesian product of $(\mathbb{R}_+ \times \{N.S.\})$.

h is chosen from the constrained choice set $G^L_h i(\Omega)$ depending on all bankers' offers Ω, that is,

$$\begin{aligned} G^L_{hj}(\Omega) &\equiv R^L_h \times \overline{\mathbb{R}}_+ \quad \text{if banker } h \text{ specifies } R_h \text{ only}, \\ &\equiv R^L_h \times (l_{hj} \cup \{0\}) \quad \text{if banker } h \text{ specifies both } R^L_h \text{ and } l_{hj}. \end{aligned} \tag{2}$$

The borrower's choice set of the last case is either (R^L_h, l_{hj}) or $(R^L_h, 0)$; i.e., a borrower has the choice to "*accept*" or "*reject*" the offer. Note that bankers are always would like to specify price, otherwise, borrower choose zero price, which bring bankers negative profit as long as deposit rate is positive. A borrower submits his replies to bankers' offers Ω, chosing from the Cartesian product of the constrained choice sets, $G^L_j(\Omega) \equiv G^L_{h1}(\Omega) \times G^L_{h2}(\Omega) \times \cdots$. The constrained choice set for depositor i to bank h, G^D_{hi} can be similarly defined. A depositor submits his replies to bankers' offers Ψ, chosing from the Cartesian product of the constrained choice sets,
$G^D_i(\Psi) \equiv G^D_{h1}(\Psi) \times G^D_{h2}(\Psi) \times \cdots$.

At the end of the period, a borrower under loan contracts with average repayment R^L and total loan $l_j = \sum_h l_{hj}$ repays R^L for loan l_j from his own output y^U_j. In other words, consumption of a borrower is determined by the budget constraint. It is expressed as a function, if selecting the loan repayment schedule and the loan amount, $(R^L_j(l_j, A, \epsilon_j), l_j)$. That is,
$c^U : G^L_j(\Omega) \times [\underline{A}, \overline{A}] \times [\underline{\epsilon}, \overline{\epsilon}] \to \mathbb{R}_+$ such that

$$c^U(l_j, R^L_j, A, \epsilon_j) = y(l_j, e^U, A, \epsilon_j) - \tau 1_N - R^L_j(l_j, A, \epsilon_j) l_j. \tag{3}$$

Although loans are given by each bank l_{hj}, the loan repayment schedule, in particular default, is conditional on the borrower j's total loans l_j. Note that *pari passu* in bankruptcy procedure is assumed following the real world practice, that is, the creditors are treated equally when default happens.

More discussions on costs are given in the next section but, without loss of generality, a borrower ends up paying the verification cost τ when he needs to do so. In (3), the cost payment is denoted by an indicator function $1_N \in \{0, 1\}$, i.e., it is $1_N = 1$ if paid, otherwise $1_N = 0$. I also assume that cost τ is small relative to endowed capital k_0 so that it can be always paid. This assumption will be clarified later.

A depositor under deposit contracts with average repayment R^D and total deposits s_i consumes the return from his returned deposits $R^D s_i$ and his own product y^D_i. Note that a banker may not repay the promised deposit rate $1 + \rho^D$ in full and gross return R^D can be even less than one ex post (i.e., the net return can be negative). In the worst case, a depositor receives nothing from a banker, i.e., $R^D = 0$. Consumption of a depositor is expressed as $c^D : G^D_j(\Psi) \times [\underline{A}, \overline{A}] \times [\underline{\epsilon}, \overline{\epsilon}] \to \mathbb{R}_+$ such that

$$c^D(s_i, R^D_i, A, \epsilon_i) = y(s_i, e^D, A, \epsilon_i) + R^D_i(s_h, A) s_i. \tag{4}$$

The deposit repayment schedule R^D, particularly when a bank defaults, is conditional on the banker h's deposit intake, not by depositor i's deposits in an equilibrium, and assuming *pari passu* for depositors in bank bankruptcy. Note that (3) and (4) also represent the budget constraints with nonlinear repayment schedule for a borrower and a depositor, respectively.

A banker lends loan l_{hj} to a borrower funded by her own capital $k^B_0 = k_0$ and the deposits that she collected, i.e., $s_h = s_{hi}(1 - \mu)/(2\mu)$. She then earns, and consumes, the spread income. Banker's consumption is determined by the budget constraint and expressed as a function of her own loan and deposit contract offers given all offers and aggregate shock, i.e., $c^B : [\underline{A}, \overline{A}] \times \overline{\Lambda}^{\mu} \times \overline{\Delta}^{\mu} \to \mathbb{R}_+$, if entrepreneurs choose her contracts among other offers.

$$c^B(A, R_h^L, l_h, R_h^D, s_h) = \int_{\underline{\epsilon}}^{\overline{\epsilon}} R_h^L(l_j, A, \epsilon_j) l_h dH(\epsilon) - R_h^D(s_h, A)s_h. \tag{5}$$

B. Equilibrium Definition

Before specifying the institutional setup on financial intermediation, a decentralized equilibrium can be defined generally as follows. Note that I denote a generic loan from bank h to entrepreneur j by l_{hj}, the optimal lending by a banker (i.e., the supply) by $\tilde{l}_{hj}$, the optimal borrowing by an entrepreneur (i.e., the demand) by l_{hj}^*, and the equilibrium loan by L_{hj} (see below). I use similar notations for deposits.

For the sake of simplicity, I focus on a pure strategy symmetric equilibrium. In such an equilibrium, the loan and deposit repayment schedules should be symmetric $\tilde{R}_h^L = R^L$ and $\tilde{R}_h^D = R^D$ in equilibrium. Also, in a symmetric equilibrium, the equal market shares in the deposit and loan markets are achieved by those bankers who offer the best contracts. That is, borrower j borrows loans from equally from all active bankers in equilibrium, $l_j = l_{hj}/\mu = l_h(1-\mu)/2\mu$.[11] As for the deposit market, a depositor is also assumed to make his deposits to all active bankers in equilibrium and $s_i = s_{hi}/\mu = s_h(1-\mu)/2\mu$.[12] Note that, in an off-equilibrium, heterogeneous offers of loan and deposit contracts by bankers are possible and accordingly loan and deposit rates as well as the market shares could become heterogeneous.

Definition 1. *An equilibrium is the number of bankers μ, the capital allocation s_{hi} and l_{hj}, and the consumption allocations represented by deposit repayment schedule $R^D(s_h, A)$ and loan repayment schedule $R^L(l_j, A, \epsilon_j)$, that satisfy the following conditions:*

- *Given the choice set G_j^L constrained by loan offers Ω, a borrower chooses the best reply set of the loan repayment schedule and the loan amount $(R_j^{L*}, l_j^*) \in G_j^L(\Omega)$ to maximize his expected utility,*

$$V^U(k_0, \Omega) = \max_{(R_j^L, l_j) \in G^L(\Omega)} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^U(l_j, R_j^L, A, \epsilon_j)\right) dH(\epsilon) dG(A). \tag{6}$$

subject to budget constraint (3). Here, a borrower choses the loan repayment schedule R_j^{L*} from Ω, a set of $\tilde{R}_h^L$. He may pick several contracts and decides total borrowings $l_j^* = \sum_h l_{hj}^*$. Again, in symmetric equilibrium, I focus on, $l_j = l_{hj}/\mu = l_h(1-\mu)/2\mu$. By summing up best responses from all borrowers, the loan demand of all (symmetric) borrowers to banker h's specific offer $(\tilde{R}_h^L, \tilde{l}_h)$, given other bankers' offer Ω_{-h}, is expressed as $l_h^*((\tilde{R}_h^L, \tilde{l}_h), \Omega_{-h})$, or simply $l_h^*(\Omega)$. Note that constrained choice set implies $R_h^{L*} = \tilde{R}_h$.

[11] This assumption can be relaxed easily so that a borrower does not necessarily borrow from all banks.

[12] Again, this assumption can be relaxed easily so that a depositor may make his deposit in a certain number of bankers.

- *Given the choice set G_i^D constrained by deposit offers Ψ, a depositor chooses the best reply set of deposit repayment schedule and the deposit amount $(R_i^{D*}, s_i^*) \in G_i^D(\Psi)$ to maximize his expected utility,*

$$V^D(k_0, \Psi) = \max_{(R_i^D, s_i^*) \in G^D(\Psi)} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^D(s_i, R_i^D, A, \epsilon_i)\right) dH(\epsilon) dG(A). \tag{7}$$

  subject to budget constraint (4). He may pick several contracts and decides total deposits $s_i^*(\Psi) = \sum_h s_{hi}^*(\Psi)$. By summing up best responses from all depositors, the deposit supply of all (symmetric) depositors to banker h's specific offer $(\tilde{R}_h^D, \tilde{s}_h)$, given other bankers' offer Ψ_{-h}, is expressed as $s_h^*((\tilde{R}_h^D, \tilde{s}_h), \Psi_{-h})$, or simply $s_h^*(\Psi)$. Note that constrained choice set implies $R_h^{D*} = \tilde{R}_h$.

- *A banker offers her loan contract $(\tilde{R}_h^L.\tilde{l}_h) \in \overline{\Lambda}$ and deposit contract $(\tilde{R}_h^D, \tilde{s}_h) \in \overline{\Delta}$ to maximize her expected utility, given the best response functions of entrepreneurs (i.e., loan demand $l_h^*((\tilde{R}_h^L, \tilde{l}_h), \Omega_{-h})$ and deposit supply $s_h^*((\tilde{R}_h^D, \tilde{s}_h), \Psi_{-h})$ and other bankers' loan offers Ω_{-h} and deposit offers Ψ_{-h},*

$$\begin{aligned} &V^B(k_0) \\ &= \max_{(R_h^L, l_h) \in \overline{\Lambda}, (R_h^D, s_h) \in \overline{\Delta}} \int_{\underline{A}}^{\overline{A}} u\left(c^B(A, R^L, l_h^*((\tilde{R}_h^L, \tilde{l}_h), \Omega_{-h}), R^D, s_h^*((\tilde{R}_h^D, \tilde{s}_h), \Psi_{-h}))\right) dG(A), \end{aligned} \tag{8}$$

  subject to budget constraint (5).

- *Bankers offer the same deposit and loan repayment schedules to entrepreneurs in a symmetric equilibrium (i.e., fixed point conditions)*

$$R^L = \tilde{R}_h^L = R_j^{L*}, \quad R^D = \tilde{R}_i^D = R_i^{D*}. \tag{9}$$

  As for the loan and deposit amounts, including non-specified option, the best responses becomes the equilibrium allocations

$$L_h = l_h^*(\Omega), \quad S_h = s_h^*(\Psi), \tag{10}$$

  and

$$\Omega = (R^L, L_h)^\mu, \quad \Psi = (R^D, S_h)^\mu. \tag{11}$$

  Note that if the equilibrium loan offer $\tilde{l}_{hi}$ is equal to borrower's willing to borrow at offered repayment schedule $\tilde{R}_h$, then the equilibrium loan offer would not be specified, i.e., $\{N.S.\}$.

- *Aggregate capital market clears. In a symmetric equilibrium, a representative banker h takes deposits S_h and make loans L_h from depositors and borrowers, respectively. She also invests her own capital $k_0^B = k_0$ as a part of loans to borrowers. Adjusting the relative size, the resource constraint in the capital market for the representative bank is expressed as,*

$$\frac{1-\mu}{2\mu} L_j = L_h = S_h + k_0^B = \frac{1-\mu}{2\mu} S_i + k_0^B. \tag{12}$$

- *After the production takes place, consumption goods market clears for any realization of* *aggregate shock $A \in [\underline{A}, \overline{A}]$,*

$$\begin{aligned}&\frac{1-\mu}{2}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\left(c^U(L_j, R^L, A, \epsilon_j) + c^D(S_i, R^D, A, \epsilon_i)\right)dH(\epsilon) + \mu c^B(A, R^L, L_h, R^D, S_h)\\ =&\frac{1-\mu}{2}\int_{\underline{\epsilon}}^{\overline{\epsilon}}\left(y(e^U, A, \epsilon_j) + y(e^D, A, \epsilon_i)\right)dH(\epsilon_i).\end{aligned} \tag{13}$$

- *In an intermediated equilibrium, banker population is sizable $\mu > 0$ or measure zero $\mu = 0$, and the ex ante arbitrage condition for occupational choice to become a banker or an entrepreneur before observing talent $e = e^U$ or e^L holds. The equilibrium bank population μ must be consistent with this occupational arbitrage condition.*

$$V^B(k_0) = V^E(k_0) \equiv \frac{1}{2}V^U(k_0) + \frac{1}{2}V^D(k_0). \tag{14}$$

- *In a unintermediated equilibrium, no banker exists ($\mu = 0$) and no capital is exchanved. In this case, $V^B(k_0) < V^E(k_0)$ for a potential measure-zero banker as well as for a positive measure banker.*

Note that, in accounting, the bank balance sheet is reported differently from the banker's budget constraint (5) as well as the resource constraint (12). In equilibrium, a deposit takes a form of debt contract, which will be proved in Section III. B below. In other words, the deposit contract has a flat payment portion (i.e., a promised payment) and a default region. A typical accounting standard uses the market values for assets but the face values for liabilities. Then, the ex ante accounting balance sheet of the representative bank in a symmetric equilibrium is expressed as

$$\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} R^L(L_j, A, \epsilon_i)L_h dH(\epsilon)dG(A) = (1+\rho^D)S_h + w(k_0^B, S_h), \tag{15}$$

where $w(k_0^B, S_h)$ is the accounting valuation of the net worth. Ex post, depending on the realization of the aggregate shock, the net worth can become tiny or even negative as a bank needs to repay deposits in full if not default.

The equilibrium notion follows the perfect Bayesian Nash equilibrium with resource constraints. In particular, I require sequential rationality, that is, at any stage, when a banker, a borrower, or a depositor takes an action, she or he should maximize her or his expected utility from that stage. For simultaneous moves within a stage, beliefs on other agents' strategies (e.g., other bankers' offers) have to be consistent in actual strategies taken in equilibrium. Importantly, with this equilibrium notion, I can find the equilibrium by solving backwards.

III. Financial Intermediation

A. Institutional Assumptions

Because this paper discusses bankruptcy-related issues, the model's institutional setup on financial intermediation needs to allow debt-type contracts with bankruptcy in equilibrium.[13] This paper essentially follows the costly state verification setup introduced by Townsend (1979), who shows that costly state verification gives rise to a debt contract, which has a flat payment (i.e., honoring the face value) for good realizations of shocks and state-contingent payments (i.e., default and partial recovery) for bad realizations of shocks.[14] In essence, Townsend (1979) introduces one friction in the complete state contingent securities market. That is, information regarding which state is realized can be verified only with a cost. Hence, the state contingent return of a security includes additional endogenous contingency, i.e., its return when state is verified and its return otherwise. Apparently, the return becomes insensitive to state realizations when state is not verified. However, in Townsend (1979), the state contingent return in case of verification is supposed to be prescribed in the security at its issuance as in the Arrow-Debreu market. Townsend (1979) also assumes implicitly free enforcement of contracts, again as in the Arrow-Debreu market. In summary, both explicit and implicit assumptions of Townsend (1979) on the financial market can be summarized as follows.

Assumption 1. ***[CSV - Contingent Contract Regime]***
(a-0) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ.
(b-0) [Free Contingent Contracts] It is free to write ex ante contingent loan contracts for all possible states supposing the realized state is verified.
(c-0) [Free Enforcement] Repayment by a borrower is enforced freely according to the contract, including default cases.
(d-0) [Small Cost] $\tau \leq \underline{\epsilon A} e^U k_0^\alpha$.

Note that, in the literature, the verification cost is sometimes assumed to be paid by borrowers (e.g. hiring an accounting firm) or by a bank (e.g., examining documents), who would however charge the cost to the borrower in equilibrium. In either case, the cost ends up to be deducted from the borrower's income and assets in equilibrium. Assumption (d-0) assumes that the cost is small enough to be paid even by the autarkic production with the worst realization of the productivity shocks.

While Townsend (1979) and many followers assume such a partially state-contingent contract can be written ex ante and enforced freely ex post, the financial crisis literature has been concerned about costs associated with bankruptcy as by Chari and Kehoe (2016). They recognize that the government

[13] To focus on symmetric equilibrium and to avoid any other complexities, bankers' claims are treated equally (i.e., *pari passu*) following the real world convention, throughout this paper.

[14] On the contrary, if completely hidden information is assumed, the optimal contracts are usually equity type, having payoffs contingent on outputs, to contain moral hazard in case of hidden efforts and to mitigate adverse selection in case of hidden types.

has an incentive to bail out banks to save private agents' state verification costs (and other costs) associated with bankruptcy ex post.[15]

Here, I simply add an ex post cost associated with bankruptcy as costly negotiation. To do so, I also need to assume ex ante impossibility of writing (or enforcing) a state-contingent contract.[16]

Assumption 2. ***[Costly Negotiation Regime]***
(a-1) [Costly State Verification] A specific realization of combined productivity shock ϵA is private information to a borrower but can be verified by the borrower with cost τ to his lender only.
(b-1) [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent loan contracts for any possible states.
(c-I) [Costly Negotiation] Once the shock realization is verified, loan repayment by a borrower is decided by Nash bargaining after the negotiation cost T is subtracted from the output. More specifically, for any realization of triple (l, A, ϵ), a borrower-bank pair maximizes its joint surplus, given a bargaining power parameter ξ^L,[17]
(d-1) [Small Cost] $\tau + T \leq \underline{\epsilon A} e^U k_0^\alpha$.

$$\max u(c^U)^{\xi^L} u(c^B)^{(1-\xi^L)}. \tag{16}$$

Assumption (c-1), costly negotiation, is the major assumption I introduce here. Assumption (b-1), incomplete contract, is naturally associated with ex post negotiations because writing contingent contracts ex ante for states that will be verified would make ex post negotiation redundant.[18] Assumption (d-1) is a slight modification of Assumption (d-0) to make both verification and negotiation costs to be small enough to be paid even in the worst case scenario.

Assumption (a-1), costly state verification, is almost the same as Assumption (a-0) in the CSV-Contingent Contract Regime, as it needs to make debt-type contracts exist in equilibrium. A slight clarification is added that the verified information remains private to the borrower and the lender who obtained the information. Assumption (a-1) makes a loan contract to be possibly contingent on each borrower's combined shock if verified, but not on the aggregate shock alone. It implies that the aggregate shock is not public information if no borrower declares default. Moreover, that the verified information remains private to a borrower and his lender implies that a borrower cannot recognize for sure the other borrowers' combined shocks ϵA and thus the aggregate shock A, either. Note that, in reality, aggregate GDP growth rate is revealed to public but a quarter to a year later, i.e., with a substantial time lag. This is consistent with private information assumption regarding the aggregate shocks when a borrower makes his repayment decision in my model.

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[15]The bankruptcy cost could be considered as a part of debt overhang cost. The bankruptcy cost could have a spillover effect (e.g., Igan, Mishra, and Tressel, 2011). However, this paper assumes away the spillover effect for the sake of simplicity.

[16]This regime is somewhat a hybrid of CSV (Townsend, 1979) and incomplete contract literature (e.g., Hart, 1995), which I discuss more later.

[17]The cost of negotiation can be defined as a part of c^L, borrower's consumption by the same way as the cost of verification.

[18]Even if any promise is not honored and requires negotiation ex post, without assumption (b-1), a contingent contract for verified states would be written as a negotiation-proof contract. Hence, if contingent contracts can be written freely, it avoids costly negotiation. But, then, the model misses the important friction that focuses on.

In the original Townsend (1979) model, it does not matter much if the verified information remains to be private for these two parties or becomes public. This is because contingent contracts can be written ex ante for the disclosed information and enforced freely. However, if the information on the realized state becomes public, a third party could involve ex post in the distressed asset market freely. It is not only inconsistent to Assumptions (b-1) and (c-1) but also to the costs paid (and profits enjoyed) by the real world specialists such as loan servicers and vulture funds.[19]

Moreover, as noted in the introduction, bankruptcy procedures in major countries (e.g., Germany, Japan, and Italy) have been expanding bankruptcy procedures from just liquidation to include reorganization procedure to help entities as going concerns. The reorganization procedures allow defaulters to keep substantial assets and to restart their lives and businesses quickly. Often involved is agreements on viable business plans after debt restructuring. These negotiations requires the state of the firm to be verified, which is costly in the real world. Theoretically, optimal contingent contracts can be written with renegotiation proof and hence the assumption of incomplete contracting (b-1) is required. Incompletely prescribed contingent returns on verified state naturally calls for ex post negotiation.

Firm owners would engage actively in negotiation to restructure debt, or to split assets and outputs, and to articulate the business plan. Empirical studies (e.g., Djankov, et. al., 2008) show that disputes and lengthy negotiations for a bankrupt entity between creditors and borrowers often occurs. Such costs associated with disputes and lengthy negotiations, including any real and opportunity losses (possibly also by debt overhang), should be included in bankruptcy costs and are represented simply as the negotiation cost assumption (c-1) as in this paper.

Regarding the equilibrium allocations, the CSV-Costly Negotiation Regime is *not* different from the CSV-Contingent Contract Regime, except for loss of negotiation cost T. When he needs to do so, a borrower verifies the realized state of the world. Then, he negotiates with the banker to split the borrower's assets and outputs by Nash bargaining, which is by definition Pareto optimal, as it would be written in the contingent contract in the CSV-Contingent Contract Regime. And, this whole process can be done with cost $\tau + T$. Therefore, the goods allocation in the CSV-Costly Negotiation Regime must be the same as in the CSV-Contingent Contract Regime, except for a cost difference by T in case of default. And, it is obvious that the bank bailouts (or any policy interventions), which introduce a different goods allocation than the market, would not improve the welfare because Townsend (1979) already shows that the general equilibrium allocation in his model, i.e., what I call the CSV-Contingent Contract Regime, is Pareto optimal.

The CSV-Costly Negotiation Regime, however, seems to miss another important change that the real world experiences. Simple and speedy bankruptcy procedures have been increasingly adopted in many countries since around 2000 and especially after the global financial crisis of 2008 (see the discussions in introduction). A specific, often simple, asset allocation rule upon default can effectively eliminate negotiation process (e.g., HAMP in the U.S.).

Theoretically, those simple and speedy rule is consistent with the incomplete contract literature (e.g., Hart (1995)). This literature stresses prohibitive costs of specifying all the states and writing contracts for all the contingencies, and emergence of shareholders (i.e., firm owners) as residual

[19] I do not model the distressed asset specialists in this paper. However, as long as they have to pay the costs of verification (and negotiation), they can be considered as one function of bankers in my model. Indeed, Diamond and Rajan (2001) argues that the loan collection skill is a key feature of a bank.

claimants after paying to flat promised returns to debt holders. In this case, borrowers do not need to verify states to creditors as long as they can repay promised returns. In case that borrowers cannot repay in full, creditors get collateral (i.e., seize assets) and shareholders are left with remaining assets. In this case, verification of states is not required to be costly, rather, it seems implicitly assumed that the state (i.e., the value of seized assets) is revealed freely to the creditor, the new owner of the assets. Such a bankruptcy regime is often assumed in the literature of macroeconomics with financial frictions following Kiyotaki and Moore (1997).

The literature often assume that the defaulter can retain a fixed portion of his assets and the rest is given to the creditors. This assumption is not so much different from the relatively new bankruptcy schemes adopted in many countries. Therefore, I assume such a bankruptcy regime in this paper as follows.

Assumption 3. ***[Speedy Bankruptcy Regime]***
(a-2) [State Revelation to the Owner] A specific realization of combined productivity shock ϵA is free private information to a business owner, who is a borrower but also is a creditor if seizes the assets.
(b-1) [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent loan contracts for any possible states.
(c-2) [Simple Asset Seizure Rule] When a borrower declares default, his business is seized by bankers except for the outputs that he is allowed to retain. It is simply assumed worth $\lambda > 0$ portion of his invested capital, i.e., λk_0.[20]
(d-2) [Small Asset Retainment] $\lambda k_0 \leq \underline{\epsilon A} e^U k_0^\alpha$. (Note that the cost of default is $y_j^U - \lambda k_0$ for a borrower.)

Assumption (c-2), a simple asset seizure rule is the key in this paper. This rule creates a potentially non-optimal allocations, but it allows the creditors and the borrowers to save the costs associated with lengthy negotiations, represented by T, as well as the verification cost τ.[21] It is a simplest way to represent the *raison d'etre* of a speedy bankruptcy scheme. In other words, this explains why many countries adopt speedy bankruptcy rules and also why those rules bring efficient outcomes.

Note that Assumption (b-1), incomplete contract, is the same as in the CSV-Costly Negotiation Regime. If writing contingent contracts were freely done and enforced, it would easily eliminate the simple rule and cannot explain the efforts done to create the speedy rules in many countries. Under Assumption (b-1), costly negotiations could occur, and thus it lays the basis for a cost advantage of adopting a speedy bankruptcy rule. For the sake of simplicity, Assumption (d-2) is a modification to Assumptions (d-0) and (d-1), meaning that retained outputs can be paid by the borrower's outputs even in the worst case.

In reality, both renegotiation procedure (the CSV-Constly Negotiation Regime) and the Speedy Bankruptcy Regime coexist and are chosen by the related parties based on their costs and benefit. However, below, in this paper, I focus on the Speedy Bankruptcy Regime, which seems to describe relatively small but many debts, e.g., mortgages and SME loans. Note that the regime is so far

[20]If the model were to incorporate a dynamic setup, it would make sense to assume that a defaulted borrower retains his capital to continue production as a going concern or keeps his own house.

[21]Without the simple rule, the optimal contract with CSV implies a potentially non-linear risk sharing between two parities when a default occurs. The literature has so far shown that it becomes a straight line when at least one party is risk neutral (see a review by Fulghieri and Goldman, 2008).

defined for the loan market but, in a later section, essentially the same setup will be also assumed for the deposit market, between depositors and bankers. The key issue is that, as borrowers and bankers are guaranteed to have the minimum consumption level of λk_0, depositors assume all the tail risks (*tail-risk dumping*).

B. Set of Loan Contracts Restricted by Informational Assumption and Bankruptcy Procedure

A debt contract is chosen optimally when state is revealed with cost (i.e., asset seizure) in the Speedy Bankruptcy Regime. This state revelation cost upon default makes a debt contract as an optimal contract (Townsend, 1979). In this regard, mathematically both CSV and incomplet contracting based optimal contract is similar, and hence proof for the following lemma is omitted. Essentially, a borrower would not want to pay the cost to reveal for all realizations. To minimize the cost payment, he would do so only when necessary. But, not revealing states implies that the repayment schedule is non-contingent on state, i.e., there is a flat portion on repayment schedule, which can be considered as the promised fixed interest rate. A borrower reveals states only when doing so is beneficial (i.e., strategic default).

Lemma 1. *Assumption 3 restricts the set of equilibrium loan repayment schedules $R^L(L_j, A, \epsilon_j)$ as follows.*

- *Above a unique default threshold θ^L, a borrower pays the flat loan repayment in full, $(1+\rho^L)$.*
- *Below the threshold θ^L, a borrower defaults with retaining assets λk_0, and thus a defaulter's repayment schedule (per loan) has an intercept term and a linearly increased portion with respect to the realized (combined) productivity shock $\epsilon_j A$:*

$$\frac{\epsilon_j A e^U (k_0 + L_j)^\alpha - \lambda k_0}{L_j}. \tag{17}$$

Accordingly, a borrower consumes,

$$c^U(L_j, A, \epsilon_j) = \lambda k_0, \quad \text{if default, i.e., for} \quad \epsilon_j A \in [\underline{\epsilon A}, \theta^L), \tag{18}$$

otherwise

$$c^U(L_j, A, \epsilon_j) = \epsilon_j A e^U (k_0 + L_j)^\alpha - (1+\rho^L) L_j, \quad \text{for} \quad \epsilon_j A \in [\theta^L, \overline{\epsilon A}]. \tag{19}$$

At the threshold, a rational borrower should equate consumption from defaulting and that from not-defaulting. This decision is made after the production takes place given loan size L_j.

$$\theta^L e^U (k_0 + L_j)^\alpha - (1+\rho^L) L_j = \lambda k_0. \tag{20}$$

I assume small enough $\underline{\epsilon A}$ to avoid a corner solution $\theta^L = \underline{\epsilon A}$ because a borrower never defaults in this case, which is not interesting. I also assume large enough $\overline{\epsilon A}$ to avoid another corner solution $\theta^L = \overline{\epsilon A}$, with which the loan contract becomes an equity contract.

C. Set of Deposit Contracts Restricted by Informational Assumption and Bankruptcy Procedure

Regarding defaults of bankers in the deposit market, institutional assumptions on bankruptcy for bankers is made in a manner similar to the loan market.

Assumption 4. *[Speedy Bankruptcy Regime for Bankers]*
(a-2') [State Revelation to the Owner] A specific realization of aggregate productivity shock A is free private information to a banker, but also is revealed freely if depositors seizes the bank assets.
(b-1) [Incomplete Contract] There is a prohibitive cost for agents to write ex ante contingent deposit contracts for all possible states.
(c-2') [Simple Debt Restructuring Rule] When a banker declares default (i.e., does not pay the flat deposit rate as promised), her business is immediately seized by depositors except for the assets that she is allowed to retain. It is assumed worth $\lambda > 0$ portion of her invested capital, i.e., λk_0.
(d-2') [Small Cost] $\lambda k_0 + \lambda k_0^B \leq \underline{\epsilon A} e^U k_0^\alpha$.
(e) [Bank Population] $\mu \leq 1/3$.

Assumption (e) is a natural restriction on banker population to be at most the same number as borrowers or depositors. In case $\mu = 1/3$, all three types agents have measure $1/3$, i.e., one banker intermediate one depositor and one borrower. For the sake of simplicity, Assumption (d-2') is made as an extension to Assumption (d-2) to allow the retained assets of both borrowers and bankers to be paid by the borrowers even in the worst case scenario with highest number of bankers at $\mu = 1/3$. Note that the recovery rate upon default by creditors is quite high, more than 90 percent in U.K. and 60 percent in France (Tirole, 2006), implying that the retained asset ratio λ of a borrower is likely a way below $1/2$, and thus Assumption (d-2') also seems likely to be satisfied in the real world.

The deposit repayment schedule is simply but optimally chosen by bankers under the specific bankruptcy regime described in Assumption 4.

Lemma 2. *The optimal deposit repayment schedule takes a form of a standard debt-type contract. The deposit repayment schedule $R^D(S_h, A)$ has a flat portion with full-pay deposit rate ρ^D above the threshold θ^D defined for aggregate shock A. Below θ^D is the bank default region, and the repayment is contingent on the realization of aggregate shock A.*

Proof. In a region where a borrower does not default (i.e., $\epsilon A \geq \theta^L$), repayment from the borrower is constant $(1 + \rho^L)$. On the other hand, in a region where a borrower defaults (i.e., $\epsilon_j A < \theta^L$), repayment from the borrower is increasing with aggregate shock A with possible zero return.

A banker's gross income from all borrowers, denoted by B, is expressed as a function of the aggregate shocks given the loan market outcomes (i.e., the loan repayment schedule, loans per bank, and loans per firm):[22]

$$
\begin{aligned}
B(A|R^L, L_h, L_j) &= \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^L(A, \epsilon_j) L_h dH(\epsilon) \\
&= \left(1 - H\left(\frac{\theta^L}{A}\right)\right)(1+\rho^L)L_h \\
&\quad + \int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} \left(\epsilon_j A e^U (k_0 + L_j)^\alpha - \lambda k_0\right) \frac{L_h}{L_j} dH(\epsilon).
\end{aligned} \tag{21}
$$

Note that $B(A|R^L, L_h, L_j) \geq 0$ for all A because of Assumption 4 (d-2'). Also note that $\partial B/\partial A > 0$, that is, the banker's gross income is increasing in the aggregate shock.

$$
\begin{aligned}
&\frac{\partial B}{\partial A} \\
=&(1+\rho^L)L_h \frac{\theta^L}{A^2} h - \left(\frac{\theta^L}{A} A e^U (k_0 + L_j)^\alpha - \lambda k_0\right) \frac{L_h}{L_j} \frac{\theta^L}{A^2} h + \int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} \epsilon_j e^U (k_0 + L_j)^\alpha \frac{L_h}{L_j} dH(\epsilon) \\
=&\left\{(1+\rho^L)L_h - \left(\theta^L e^U (k_0 + L_j)^\alpha - \lambda k_0\right) \frac{L_h}{L_j}\right\} \frac{\theta^L}{A^2} h + \int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} \epsilon_j e^U (k_0 + L_j)^\alpha \frac{L_h}{L_j} dH(\epsilon) \\
=&\int_{\underline{\epsilon}}^{\frac{\theta^L}{A}} \epsilon_j e^U (k_0 + L_j)^\alpha \frac{L_h}{L_j} dH(\epsilon) \\
>&0,
\end{aligned} \tag{22}
$$

where probability density function h in the second and third lines is evaluated at θ^L/A. Note that the brace term in the third line is zero because the full loan repayment $1+\rho^L$ is equal to the all the outputs plus net-of-retained portion of capital of a borrower at the default threshold of a loan (i.e., ϵA is at θ^L) as shown in (20).

Because banker's gross income B is monotonically increasing in aggregate shock A, Assumption 4, in particular, (c-2'), implies that a banker optimally choses a unique threshold θ^D above which a banker does not default and below which she defaults. If not default, the banker enjoys consumption from the income after repaying the full obligation to depositors. The deposit repayment schedule is

$$
R^D(S_h, A) = 1 + \rho^D, \quad \text{if } A \in [\theta^D, \overline{A}]. \tag{23}
$$

Below θ^D, the banker needs to settle a low consumption level. A defaulted banker retains λ portion

[22] In a symmetric equilibrium, recall that $(1-\mu)/(2L_j) = \mu L_h$ and $L_h = S_h + k_0^B$. Hence,

$$
\text{sign}\left(\frac{\partial B}{\partial \mu}\right) = \text{sign}\left(\frac{\partial \frac{1-\mu}{2\mu}}{\partial \mu}\right)
$$

and

$$
\frac{\partial \frac{1-\mu}{2\mu}}{\partial \mu} = -\frac{1}{2\mu} - \frac{1-\mu}{2\mu^2} = -\frac{1}{2\mu^2} < 0,
$$

i.e., the banker's revenue is decreasing with the banker population.

of their book capital. The deposit repayment function reflects revenues from borrowers net of retainment assets, correcting for the relative size of the banking sector:

$$R^D(S_h, A) = \frac{B(A|R^L, L_h, L_j) - \lambda k_0^B}{S_h}, \quad \text{if } A \in [\underline{A}, \theta^D). \tag{24}$$

where θ^B is the minimum aggregate shock level with which a banker has enough revenue to retain λk_0^B. Note that, if Assumption (d-2') is violated, then a defaulted banker might not obtain sufficient revenue to cover allowed retained assets. In this case, the return to depositors inevitably becomes zero. This case does not occur under Assumption (d-2'), but for a very bad shock, the return to depositor may still become almost zero.

Q.E.D.

D. Bankers' Consumption under Restricted Sets of Loan and Deposit Contracts

A banker's net income is the gross income net of (size-corrected) repayments. If every borrower repays in full and a banker can do so, a banker enjoys the maximum income from the spread between loan and deposit rates,

$$\begin{aligned} & c^B(A, R^L, L_h, R^D, S_h) \\ =& (1+\rho^L)L_h - (1+\rho^D)S_h = (\rho^L - \rho^D)S_h + (1+\rho^L)k_0^B, \quad \text{if } A \in [\frac{\theta^L}{\underline{\epsilon}}, \overline{A}]. \end{aligned} \tag{25}$$

Even if some borrowers cannot repay back the promised loan returns to a banker, a banker can still repay deposits in full to depositors using her own capital buffer,

$$c^B(A, R^L, L_h, R^D, S_h) = B(A|R^L, L_h, L_j) - (1+\rho^D)S_h, \quad \text{if } A \in [\theta^D, \frac{\theta^L}{\underline{\epsilon}}]. \tag{26}$$

In case a banker defaults and retain allowed amounts at hand,

$$c^B(A, R^L, L_h, R^D, S_h) = \lambda k_0^B, \quad \text{if } A \in [\underline{A}, \theta^D). \tag{27}$$

IV. Overview of Equilibrium

A. Illustrative Explanation of Equilibrium

I explain here what the restricted shapes of loan and deposit contracts mean in equilibrium, step by step, illustratively.

First, consider hypothetical contracts that are perfectly contingent on idiosyncratic shocks ϵ and aggregate shocks A in an economy with borrowers and depositors but without bankers. And, keep assuming that the idea shock e is not insured. Then, the contracts are written essentially to exchange capital to arbitrage the returns. Figure 1 shows the consumption schedules for a representative borrower and depositor with and without capital exchange. For the illustrative purpose, $\epsilon = 1$ case is shown.

The capital exchange means that a bad-idea entrepreneur invests part of his capital to a good type until equating the marginal product of capital. It allows a higher output for a good-idea entrepreneur and a lower output for a bad type. Hence, these two output levels (the dotted lines) diverge from the autarkic levels (the dashed lines). However, the bad type receives the returns from the good type, the consumption levels improves from the autarkic level (the solid line). Also, accepting this contract means that the good type consumption after repaying dividends is still better than the autarkic level (the solid line).

Next, still assuming only borrowers and depositors exist, but suppose available contracts are those characterized as a simple debt type that does *not* allow default. Figure 2 shows the consumption schedules for a representative borrower and depositor in this case. From the output levels after the capital exchange (the dotted line), consumption of the borrower shifts down by the flat loan repayments regardless of states as default is assumed away here, while that of the depositor shits up as much as the flat deposit return (the solid lines). Compared to the equity contract in Figure 1, the borrower (the good type) gains upside potentials but suffers from downside risks. The capital exchange itself are not likely the same as under the equity contracts.

To limit the downside risks for borrowers, assume now that the debt contract allows default with retaining some assets. The borrower's consumption is bounded by below at the allowed retained assets (see Figure 3, the solid line that kinks at θ^L). Then, when a large negative aggregate shock (i.e., a tail risk) is realized, the consequences are mostly assumed by depositors (*tail risk dumping*).

The situation for depositors improves by introducing bankers into this economy. Figure 4 adds banker's consumption (the dotted line). Still, if a tail risk materializes, both borrowers and bankers would keep minimum guaranteed consumption from their assets, but depositors cannot do so. However, the bankers provide some insurance using their capital buffers for depositors to mitigate the tail-risk dumping problem. Depositors can enjoy full repayment now as long as aggregate shock $A \geq \theta^D$ at which bankers would default after depleting most of her capital. This threshold θ^D is apparently lower than the borrowers' θ^L, thanks to bankers' capital buffer. In other words, as the insurance provider of the aggreagte tail risks, banks emerges in equilibrium in this paper.

Below, I explain more formally what are the consumption allocations, or equivalently the loan repayment and deposit repayment functions, in equilibrium. Although the model assumes standard utility and production functions, which are often used macro models with financial frictions, it allows the deposit and loan repayment schedules to have kinks (Figure 4). This may evoke a question on existence and uniqueness of equilibrium with kinked repayment functions. One way to analyze is to allow lotteries (i.e., correlated or mixed strategies) to convexify the kinks *à la* Prescott and Townsend (1984 a, b). Their approach could make sure the existence of equilibrium, but with complex contracts to be issued by bankers and then securitized and traded in a Walrasian competitive market.

In this paper, I would like to analyze banking sector policies, especially bailouts. So, I write a model with bankers who strategically designing contracts, intermediate capital, and possibly default by themselves. The loan and deposit contracts can be securitized and traded in markets, but there would be no difference with or without such markets in my model. I also focus on pure strategies with identifying some restrictions on parameter values under which a unique equilibrium is supported, and provides analytical characterizations of the equilibrium and associated policy implications. Still, whenever possible, I explain similarities and differences from a general contract approach by Prescott and Townsend (1984 a, b). I show the analysis below by a constructive manner on (i) the partial equilibrium in the loan market, (ii) the partial equilibrium in the deposit market, (iii) the general equilibrium with fixed banker popluation, and (iv) the general equilibrium with endogenous banker population.

Note that rights to consume a portion of the banker's consumption allocation c^B would be called "equity shares" of a bank. In the perfect world, everyone has incentive to sell such equities and hold other people's share (i.e., perfect cross-share holdings) so that "big household" assumption would prevail. However, as stressed at the beginning, the purpose of this paper is to investigate how consumption and investment allocations would be characterized and whether any policy interventions can improve welfare in the segregated household economy without perfect risk sharing. For this focus, the model assumes away the equity issuance or cross-share holdings by bankers or entrepreneurs.[23] Technically, the non-availability of such equity-type contracts is consistent with the informational assumptions that make debt-type contracts to be chosen optimally. In other words, just assuming equity ownership of banks and firms in an ad hoc manner would not be consistent with the model with a specific information structure that makes financial contracts with possible defaults to emerge in equilibrium.

B. Overview of Proofs for Equilibrium Existence and Uniqueness

In this paper, a repayment function with default acts like a price in typical Walrasian equilibrium, but existence and uniqueness of equilibrium are not instantly guaranteed. However, a repayment function in equilibrium become similar to a simple price. Given the output split rule upon default, the repayment functions are represented by the flat pay and the default threshold, (ρ^L, θ^L) for loans and (ρ^D, θ^D) for deposits. Below, I explain that the equilibrium is analyzed similarly to an economy with a simple bond that has a fixed price without default. Note that a measure zero bank would pay depositors the whole loan repayments of borrowers without any alteration, and I define this specific deposit offer as a *hypothetical corporate bond*.

In the next several sections, I will explain the general equilibrium existence and uniqueness as follows.

- **Loan market partial equilibrium** is represented by the inverse loan demand function, $\rho^L = g^L(l_j^*|\mu)$, which is an decreasing function given banker population μ.
- **Deposit market partial equilibrium** is represented by the equilibrium deposit supply function $S_i = f(\rho^D|\mu)$, which is an increasing function given banker population μ. This deposit supply function can be translated to the inverse loan supply function $\rho^L = \rho^D + \pi = g^D(\tilde{l}_j|\mu)$ where π denotes the spread.
- **General equilibrium with fixed banker population** exists and is unique.
- **General equilibrium with endogenous banker population** is determined by occupational arbitrage, ex ante. It exists and is unique.

[23] If bankers' income were shared among people perfectly, no demonstrations would have occurred against the bank failure and bailouts in the aftermath of the global financial crisis.

V. Loan Market Partial Equilibrium

Given banker population and deposits, the loan market partial equilibrium is uniquely determined. It is represented by the inverse loan demand function, $\rho^L = g^L(l_j^*|\mu)$, which is decreasing in the l_j-ρ^L plane given banker population μ. I explain these results in detail in the following order.

- Given loan demand l_j^*, loan repayment schedule (ρ^L, θ^L) is uniquely determined by ex post strategic default decision of a borrower.
- Larger loan demand l_j^* is associated with lower loan rate ρ^L together with higher default threshold θ^L. That is, $\partial\rho^L/\partial l_j^* < 0$, $\partial\theta^L/\partial\rho^L < 0$, and $\partial\theta^L/\partial l_j^* > 0$.
- Given banker population μ and deposits S_i, loan supply $\tilde{l}_j$ is fixed, utilizing all the deposits and bank capital, i.e., $\tilde{l}_j = S_i + \frac{2\mu}{1-\mu}k_0^B$.
- Loan supply is passively met with loan demand, $\tilde{l}_j = l_j^* = L_j$, at equilibrium loan repayment schedule R^L, without credit rationing. Hence, the inverse loan demand function becomes dependent only on the equilibrium loan demand and banker population, i.e., $\rho^L = g^L(l_j^*|\mu)$.

A. Loan Demand by Borrowers

A borrower makes three decisions. Before the production, he chooses the best loan contract from all offers and simultaneously decides how much he should borrow. After the produciton, he considers whether he should repay the loan in full or default. I analyze these decisions backwards.

At the repayment stage, borrower's default decision is determined as shown by Lemma 1. For any given amount of loans l_j, a borrower may borrow at a high rate with high chance of defaulting or at a low rate with low chance of defaulting. This relation can be drawn as a *iso-loan default curve* on the θ^L-ρ^L plane (e.g., the dotted line in Figure 5) based on the default condition (20):

$$1 + \rho^L = \theta^L e^U \frac{(k_0 + l_j)^\alpha}{l_j} - \frac{\lambda k_0}{l_j}. \tag{28}$$

Lemma 3. *For any given amount of loans l_j, the iso-loan default curve on the θ^L-ρ^L plane is monotonically increasing in default threshold θ^L. It shifts down-right and becomes flatter with a larger loan l_j on the θ^L-ρ^L plane.*

The proof is provided in Appendix.

Knowing his own behavior at the repayment stage, a borrower choses his demand for a loan at the borrowing stage for a given repayment schedule R^L. Let $\eta_j \equiv \epsilon_j A$ denote the combined shock with cumulative distribution function $M \equiv G \circ H$ and associated probability density funciton m. The first order condition for the borrower's problem to ask for loans (6) is

$$\int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha\eta_j e^U (k_0 + l_j)^{\alpha-1} - (1 + \rho^L)\right) u'(c^U) dM(\eta) = 0. \tag{29}$$

This is essentially the optimal leverage problem for a limited-liability entrepreneur. An entrepreneur borrows capital until the expected marginal product of capital become equal to the loan rate but only for the non-default region because, if defaulted, the borrower would consume only the retained initial wealth regardless of the loan amount.

Here, the *iso-loan demand curve* of the loan rate ρ^L with respect to default threshold θ^L given loan amount l_j is expressed as an implicit function of the borrower's first order condition (29),

$$\chi(\theta^L, \rho^L) \equiv \int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha\eta_j e^U (k_0 + l_j)^{\alpha-1} - (1+\rho^L)\right) u'(c^U) dM(\eta) = 0. \tag{30}$$

Lemma 4. *For any given amount of optimally chosen loans l_j^*, the iso-loan demand curve on the θ^L-ρ^L plane is monotonically increasing in default threshold θ^L. It moves down-right with a larger loan l_j^*.*

The proof is provided in Appendix. On the θ^L-ρ^L plane, the *iso-loan demand function* can be drawn like the solid line in Figure 5.

Lemma 3 implies that ex post default decision by a borrower pins down the relationship between the default threshold θ^L and the loan rate ρ^L as the iso-loan default curve. For a Perfect Baysian Nash equilibrium, this relation must be consistent with the pair (θ^L, ρ^L) in the loan contract when a borrower decides to take loans before the production. The latter relation is represented by the iso-loan demand curve. The cross points of those two curves are credible choices by borrowers. Call this (ρ^{L*}, θ^{L*}) pair as an *admissible loan contract* given l_j^*.

A question may arise if a pair of loan rate and default threshold of the admissible contract is determined uniquely by a borrower given a loan amount. Proposition 1 below assures the uniqueness, given a loan amount, under reasonable parameter values with a not-so-tight restriction on banker population.

Assumption 5. *[Curvature Restriction]*

Maximum curvatures of utility and production functions (i.e., σ and α) relative to banker population μ,

$$\sigma + \alpha \leq \frac{2}{(1+\frac{1}{Z})\mu + 1 - \frac{1}{Z}}, \tag{31}$$

where Z denotes essentially the relative difference in talents,

$$Z = \left(\frac{e^U}{e^D}\right)^{\frac{1}{1-\alpha}}. \tag{32}$$

In the extreme case, where I assume any talent difference is possible, i.e, $Z \to \infty$, then the assumption becomes[24]

$$\sigma + \alpha \leq \frac{2}{\mu+1}. \tag{33}$$

[24] The same condition appears if I assume possibility of the maximum loan equaling to the whole capital owned by depositors (i.e., there is no production by depositors) plus banker's capital. In this case, $l = k_0 + 2\mu k_0/(1-\mu)$.

Even this tighter condition allows most of reasonable parameter values assumed in the macroeconomics literature. For example, the relative risk aversion parameter $\sigma = 1.2$ and the capital share $\alpha = 0.3$ can allow the equilibrium banker population up to $1/3$, at which banker population equals to the population of borrowers (or depositors) in a symmetric equilibrium, conforming Assumption 4 (e). If I can focus on more reasonable range of the banker population, say up to 10 percent, then, $\sigma = 1.5$ and $\alpha = 0.3$ satisfy this tighter assumption.

For more general cases for parameter values of (σ, α, μ), it is sufficient to restrict the maximum talent difference as (32) in Assumption 5.[25] It is not so restrictive in the real world application. Take, for example, $Z = 2$ and $\alpha = 0.5$, then $e^U \leq \sqrt{2}e^D$, that is, at most about 40 percent difference in productivity. In this case, $\sigma = 2$ can allow the equilibrium banker population up to 20 percent, which covers almost all the countries, even financial centers like Switzerland.[26]

Proposition 1. *Under Assumption 5, given a borrower's credible and optimal loan amount l_j^*, there exists unique loan admissible contract (ρ^{L*}, θ^{L*}) demanded by a borrower.*

See the proof in Appendix.

Corollary 1. *If borrower's optimally chosen loan amount l_j^* is larger, then the associated loan rate ρ^{L*} of admissible loan contracts is lower but the default threshold θ^{L*} is higher. In other words, the loan demand l_j^* is one-to-one mapping to admissible loan rate ρ^{L*} and strictly decreasing in ρ^{L*}, $\partial l_j^* / \partial \rho^{L*} < 0$. The associated default threshold θ^{L*} becomes strictly higher, $\partial \theta^{L*} / \partial l_j^* > 0$ and thus $\partial \theta^{L*}\ \partial \rho^{L*} < 0$ by the chain rule. Hence, banker population μ does not influence the loan demand directly.*

Proof. With a larger loan amount, the iso-default curve shifts down-right (Lemma 3) and also the iso-loan demand curve shifts down-right (Lemma 4). In proofs of both lemmas, the loan rate declines but the default threshold increases with the higher loan amount. Hence, the cross point must shift down-right as the loans increase. Note that the borrower's utility (6) is not directly affected by banker population μ. *Q.E.D.*

B. Loan Supply by Bankers

A banker determines the loan supply to maximize her utility (8) subject to borrower's default decision (20) or equivalently the iso-loan default curve (28). Given deposits per banker s_h, a banker should naturally supply loans up to its balance sheet condition, i.e., $\tilde{l}_h = S_h + k_0^B$. This is because deposits and banker's capital would be wasted, if they are not used in production.[27] This leads to Lemma 5 below.

[25] If we allow higher banker population, $\mu \leq 1/3$ as in Assumption 4 (e) and also assume potentially any α up to 0.5, then the tighter restriction (33) becomes $\sigma \leq 1$. This is quite restrictive compared to usual assumptions of σ between one and two. Hence, in general, we also need to focus a reasonable range of the talent difference.

[26] Just before the Global Financial Crisis, in a height of a credit boom, financial sector GDP to total GDP ratio of Switzerland is around 12 percent and the employment share of the financial sector is around 6 percent (OECD Statistics).

[27] As noted previously, I am assuming 100 percent depreciation of capital. However, a case with less than 100 percent depreciation can be analyzed similarly.

Lemma 5. *Given deposits and a profitable pair of positive spread and default thresholds, a banker supplies loans as much as possible, up to her balance sheet constraint, to maximize her utility.*

Importantly, bakers do not hoard deposits. This means that the loan supply is essentially determined by the deposit market.

Often we take it granted that, given deposit rate and loan amounts that utilizes all the deposits and bank capital, higher loan rate and lower loan default threshold must be good for a banker. However, it is not so straightforward. Following (reasonable) restriction on tail-risks are needed to ensure that a banker prefers the higher loan rate. Still, a banker would not like lower loan default threshold, rather prefers a specific loan threshold associated with the loan rate.

Assumption 6. *[Tail Risk Restriction]*
A banker's retained income upon default, relative to her normal-period income, correcting for her risk aversion, is larger than the probability of banker's default multiplied by firm's survival probability in a tail-risk scenario,

$$\frac{\lambda k_0^B}{(\rho^L - \rho^D)S_h + (1+\rho^L)k_0^B} \geq \left\{ G(\theta^D) \left(1 - H\left(\frac{\theta^L}{\theta^D}\right)\right)\right\}^{\frac{1}{\sigma}}. \tag{34}$$

Assumption 6 on banker's income is unlikely a binding restriction in a real world when the tail risk is materialized. Since $G(\theta^D)$ is the probability of deposit default by all bankers in the model, it is quite a rare event. Moreover, $H(\theta^L/\theta^D)$ is the idiosyncratic probability of firm's default given the tail risk (of bankers' default) is materialized, and hence $1 - H(\theta^L/\theta^D)$ is a less likely event in which a borrower survives with a lucky idiosyncratic productivity shock even when many other borrowers and bankers go bankrupt. Often said is that a banking crisis occur in almost once in thirty years in an advanced country—if so, then $G(\theta^D) = 1/30 \approx 3\%$. Also, in such a case, many firms go bankrupt, and $H(\theta^L/\theta^D) = 1/3$ (i.e., 1/3 of borrowers defaults in the tail-risk event) is not an extreme situation. Then, with $\sigma = 1$, the right-hand-side is just 1% of the promised income and, with $\sigma = 2$, it becomes 10%. As for the left-hand-side, a stronger condition can be obtained at $s_h = k_0 = k_0^B$ assuming banker population μ at most $1/3$ under Assumption 4 (e). Suppose also quite a large loan rate of 25% and a spread of 5%. Then, the left-hand-side becomes $\lambda/1.3$. Overall, a strong condition can be expressed as $\lambda \geq 1.3\left(G(\theta^D)\right)^{1/\sigma}$, i.e., 1.3% for $\sigma = 1$ case and 13% for $\sigma = 2$ case.

Lemma 6. *Under Assumption 6, banker's utility V^B is increasing in loan rate ρ^L. Banker's indifference curve on θ^L-ρ^L plane is U-shaped with the unique flat bottom crossed at the borrower's iso-loan default curve. In other words, to achieve the highest utility for a given loan rate, a banker chooses a specific default threshold on the borrower's iso-loan default curve.*

Please see Appendix for the proof.

C. Loan Market Equilibrium

In general, with a debt-type contract, credit rationing could occur, i.e., loan supply could be lower than loan demand in equilibrium. Williamson (1987) shows that a higher loan rate is not necessarily preferred by bankers in his simpler model of the loan market with costly state verification, because too high loan rate would induce a higher chance of default. However, Williamson's result seems to stem from the exogenous leverage (i.e., 100 percent in his simple model) and fixed output seizure when default. In contrast, in this paper, higher default threshold is not necessarily bad for creditors. This is because the leverage is chosen endogenously and because more outputs are seized by creditors if a borrower were to default with higher outputs than his optimal default threshold.

It is necessary and sufficient to show impossibility of credit rationing with a price-taker assumption in a typical Walrasian-like market setup that the loan supply is always increasing in loan rate after considering effects on default. However, in my model, bankers are strategically intermediating the market and thus require additional consideration for a possibility of credit rationing.

Lemma 7. *Credit rationing does not occur and an equilibrium contract must be an admissible contract.*

Proof. Lemma 5 says that a banker utilizes all the deposits and own capital. Still, a possible credit rationing could occur if, at an equilibrium loan repayment schedule, the loan demand were strictly larger than the loan supply. In this case, the prevailing loan contract must be different from, i.e., more favorable to, borrowers (i.e., less favorable to bankers) than the admissible contract associate with the same loan amounts of the prevailing loan contract. However, recall that the admissible contract is the one reflects borrower's willingness to pay. Hence, a deviant banker can offer the admissible contract, which is more favorable to bankers than the prevailing contract. Then, because the loan were rationed, the deviant banker could attract the same residual loan demand and thus obtain a higher utility. Therefore, the prevailing loan repayment schedule with credit rationing must not be an equilibrium contract. Following this argument, only admissible contracts survive in equilibrium.[28] *Q.E.D.*

Proposition 2. *Given banker population μ and deposits s_i, there exists a unique partial equilibrium of loan rate ρ^L and default threshold θ^L in the loan market.*

Proof. The loan demand l_j^*, including the effect on default threshold θ^L, is strictly decreasing function of loan rate ρ^L on l_j-ρ^L plane (Corollary 1). The loan supply $\tilde{l}_j$, on the other hand, is determined by deposits and bankers' capital in a partial equilibrium, regardless of loan rate and default threshold. Therefore, there exists a unique pair (ρ^L, l_j) as a partial equilibrium given deposits S_h and banker population μ.

The default threshold θ^L is then determined by the demand side as the unique admissible contract (Proposition 1) given the equilibrium loan amounts and the associated loan rate (ρ^L, l_j). *Q.E.D.*

Lemma 6 states that, on θ^L-ρ^L plane, given deposits and banker's capital, the banker's indifference curve can be drawn as U-shaped with flat portion near the iso-loan default curve. The banker's indifference curve and the borrower's iso-loan default curve has a unique cross point, at which $\tilde{\theta}^L$ is determined given $(\tilde{\rho}^L, \tilde{l}_j)$. Please see Figure 5. Also, on θ^L-ρ^L plane, the ex ante loan demand by borrowers can be drawn. It also has a unique cross point with borrower's iso-loan default curve,

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[28] Over-investment by a borrower is not induced by bankers, either. This would be a case if all bankers offer loan repayment schedule that are less favorable to borrowers than the admissible contract associated with the prevailed loan amount. This is not an equilibrium because a deviant banker can specify a smaller but borrower's optimal amount of loans with a bit higher loan rate with a bit higher deposit rate to become a monopolist and enjoy a better payoff.

which represents ex post strategic default decision by a firm (Proposition 1). This is an admissible contract. The two cross points must be the same in equilibrium since bankers offer the admissible contract in equilibrium (Lemma 7).

Overall, given banker population and deposits, the supply of loan amount is passively chosen as $\tilde{l}_h = S_h + k_0^B$. (Lemma 5). Then, however, the loan contract (ρ^L, θ^L) in the loan market partial equilibrium is uniquely determined, essentially by the borrowers' decision, which relies on the marginal product of capital. When equilibrium loans $l_j = \tilde{l}_j$ become larger either by bankers' capital or deposits, loan rate ρ^L decreases and default threshold θ^L increases (Corollary 1). This decreasing relation between loans l_j and the loan rate ρ^L is essentially the same as typical loan demand function without default, which can be represented as the inverse loan demand function $\rho^L = g^L(l_j^*|\mu)$.

VI. DEPOSIT MARKET PARTIAL EQUILIBRIUM

Given banker population and loan market outcome, the deposit market partial equilibrium is uniquely determined. It is represented by deposit supply function $S_i = f(\rho^D|\mu)$ and equilibrium spread $\pi = \rho^L - \rho^D = h(\rho^L, S_i|\mu)$. I explain these results in the following order.

- Given deposit demand $\tilde{s}_h$ by a banker, deposit repayment schedule offer (ρ^D, θ^D) is credible if consistent with ex post strategic default decision of a banker.
- Larger deposit demand $\tilde{s}_h$ by a banker makes the credible deposit offer curve to shift lower right on the θ^D-ρ^D plane.
- Using own capital, bankers earn spread $\pi = \rho^L - \rho^D$ per her deposits s_h as insurance premium for repaying deposits in full for a lower part of productivity, which would be in the default region of a hypothetical corporate bond intermediated by a measure-zero bank. Given μ, the premium per deposits obviously declines with safer (lower) loan rate and larger deposits (i.e., thinner capital per deposit), $\pi = h(\rho^L, S_i|\mu)$ in equilibrium.
- By paying the insurance premium as spread, depositors become indifferent between bank deposits and a hypothetical corporate bond. Hence, deposit supply s_i^* is determined by the arbitrage between the return on own business and the return on a hypothetical corporate bond, which is equal to the loan repayments by borrowers. In other words, deposit supply s_i^* is essentially the same as demand for a hypothetical corporate bond.
- Deposit supply and deposit demand are met, $\tilde{s}_i = s_i^* = S_i$, at equilibrium deposit repayment schedule R^D. Larger S_i is supported by higher ρ^D associated with higher θ^D in equilibrium. Overall, the equilibrium deposits is an increasing function of deposit rate (with associated default threshold), given banker population, $S_i = f(\rho^D|\mu)$.

A. Credible Deposit Demand by Bankers

When a person decides to be a banker, he offers a deposit contract, which describes a deposit repayment schedule and also possibly specifies deposit amounts. However, as long as the spread income is positive, a banker is happy to take as much deposits as possible. Indeed, $\partial V^B/\partial s_h > 0$ since an increase in s_h has an apparent positive effect on banker's consumption C^B defined in (25)-(27). In other words, the unconstrained deposit demand by a bank is inelastic at ∞ to any given profitable pair of positive spread and default thresholds.

Importantly, a banker's offer faces a constraint. Since a banker may default ex post strategically, not all pairs of deposit rate and default threshold (ρ^D, θ^D) are credible to depositors. A banker is supposed to repay deposits in full, $(1+\rho^D)$, under a deposit contract $R^D(S_h, A)$ as long as the aggregate shock A is above the default threshold θ^D. However, this repayment schedule is credible only if a banker chooses the default threshold in his own behalf. Default means for a banker to give up his revenue to depositors. Hence, a higher default threshold θ^D for a given spread does not necessarily provide a banker more profits. Indeed, given loan rate ρ^L and deposit rate ρ^D (or spread $\pi = \rho^L - \rho^D$), a banker can maximize his utility by choosing threshold θ^D such that consumption under default (27) is equal to consumption under full deposit repayment (26) for any level of deposits demand s_h for banker h ex post,[29]

$$\lambda k_0^B = B(\theta^D | R^L, L_h, L_j) - (1+\rho^D)s_h. \tag{35}$$

Given initial capital k_0^B, banker population μ, and loan market variables (R^L, L_h, L_j), this constraint (35) should hold and appears as *credible deposit contract offer curve* $s^d(\theta^D, \rho^D)$ on the θ^D-ρ^D plane for each deposit demand s_h as (see Figure 6),

$$(1+\rho^D) = \frac{B(\theta^D | R^L, L_h, L_j) - \lambda k_0^B}{s_h}. \tag{36}$$

The banker's balance sheet condition, $s_h = L_h + k_0^B$, pins down a specific credible deposit contract offer, given per banker loans L_h, i.e., per borrower loans L_j and banker population μ.

Lemma 8. *Given banker population μ and loan per borrower L_j, the credible deposit contract offer curve is strictly increasing in θ^D on the θ^D-ρ^D plane.*

Proof. Multiply both sides of (36) by s_h and take a derivative of the right hand side with respect to θ^D, that is, (22) at specific aggregate shock, $A = \theta^D$:

$$\int_{\underline{\epsilon}}^{\frac{\theta^L}{\theta^D}} \epsilon_j e^U (k_0 + L_j)^\alpha \frac{L_h}{L_j} dH(\epsilon) > 0. \tag{37}$$

Q.E.D.

[29] A banker decides if defaults or not, after she observes loan repayments by borrowers. The first order condition for (8) with respect to θ^D is

$$u(\lambda k_0^B) = u(B(\theta^D | R^L, L_h, L_j) - (1+\rho^D)s_h),$$

which is simplified to (35).

Corollary 2. *With larger deposits s_h, given default threshold θ^D, the credible deposit contract offer curve shifts down (i.e., lower ρ^D) on the θ^D-ρ^D plane. On the other hand, given deposit rate ρ^D, it shifts right (i.e., higher θ^D). Overall, the larger deposits make the curve to shift down right.*

This result directly follows the signs of derivatives of (36), that is, $\partial\rho^D/\partial s_h < 0$ and $\partial\theta^D/\partial s_h > 0$ from (22) when $A = \theta^D$.

Note that banker's demand for deposits never saturates since the all terms in the following expression is positive,[30]

$$\frac{\partial V^B}{\partial s_h} + \frac{\partial V^B}{\partial \rho^D}\frac{\partial \rho^D}{\partial s_h} + \frac{\partial V^B}{\partial \theta^D}\frac{\partial \theta^D}{\partial s_h} > 0. \tag{38}$$

This implies that the credible deposit contract offer curve represents the banker's (constrained) deposit demand. However, bankers might offer something different as they compete strategically for deposits, but they would not do so. I will explain this banker's behavior after looking into the deposit supply.

B. Deposit Supply by Depositors

Proposition 3 (Optimal Deposit Size). *Given a deposit contract $R^D(S_h, A)$, a depositor determines deposit amount $s_i^* \in (0, 1)$ uniquely to maximize his utility (7) in an intermediated equilibrium.*

The first order condition for depositor's problem (7) with respect to deposits s_i is, assuming an internal solution $s_i \in (0, k_0)$,

$$\begin{aligned} 0 = &- \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \alpha\epsilon_i A e^D (k_0 - s_i)^{\alpha-1} u'(c^D(s_i, A, \epsilon_i)) dH(\epsilon) dG(A) \\ &+ \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(S_h, A) u'(c^D(s_i, A, \epsilon_i)) dH(\epsilon) dG(A) \\ &\equiv \Phi(\theta^D, \rho^D). \end{aligned} \tag{39}$$

Note that a corner solution $s_i^* = 0$ if and only if $\Phi < 0$, and another corner solution $s_i^* = k_0$ if and only if $\Phi > 0$. For the uniqueness of the internal solution $s_i^* \in (0, k_0)$, see the following validity of the second order condition in the proof in Appendix,

$$\frac{\partial \Phi}{\partial s_i} < 0. \tag{40}$$

For given initial capital k_0 and parameter values of the production and utility functions as well as the deposit repayment schedule R^D, the utility level is determined in equilibrium by optimally chosen deposit s_i^*. On the other hand, equation (39) shows the relation between the deposit rate ρ^D and the default threshold θ^D given a specific level of deposits s_i^* and loan market variables. This relation can be drawn on θ^D-ρ^D plane as the *iso-deposit supply curve*.

[30] $\partial V^B/\partial\theta^D = 0$ because of strategic default. In other words, banker's income are the same either slightly above the threshold (26) or below it (27).

Given a FOC-satisfying deposit level s_i^*, the slope of the iso-deposit supply curve is determined by the implict function theorem applied to the FOC (39):[31]

$$\frac{d\rho^D}{d\theta^D} = -\frac{\partial\Phi(\theta^D, \rho^D)/\partial\theta^D}{\partial\Phi(\theta^D, \rho^D)/\partial\rho^D}. \tag{41}$$

Lemma 9. *For deposits $s_i^* \in (0, k_0)$, the iso-deposit supply curve on the θ^D-ρ^D plane has zero slope in the neighborhood of the credible deposit contract offer curve. Its slope is negative on the right of the default threshold θ^D and positive on the left side, creating an inverted-U shape.*

See the proof in Appendix. Intuitively, this result is interpreted as the following. If a banker were to default at a higher default threshold than the credible one, $\theta^D > \tilde{\theta}^D$, then by construction the banker's income would be seized by depositors more than the promised repayment $(1 + \tilde{\rho}^D)\tilde{s}_h$ for aggregate shock $\theta \in (\tilde{\theta}^D, \theta^D)$. Hence, in the right of the credible deposit contract offer, the expected deposit return including the default region is higher, inducing a higher deposit supply. In other words, the iso-deposit supply curve should decline, $\partial\rho^D/\partial\theta^D < 0$, for $\theta^D > \tilde{\theta}^D$. On the other hand, if a banker were to default at a lower default threshold than the credible one, $\theta^D < \tilde{\theta}^D$, then by construction the banker's income would be lower than λk_0^B for aggregate shock $\theta \in (\theta^D, \tilde{\theta}^D)$. Hence, in the left of the credible deposit contract offer, the expected deposit return including the default region is also higher, inducing a higher deposit supply. So, the iso-deposit supply curve should increase, $\partial\rho^D/\partial\theta^D > 0$, for $\theta^D < \tilde{\theta}^D$.

Corollary 3. *With larger deposits $s_i^* \in (0, k_0)$, the iso-deposit supply curve shifts up (i.e., higher ρ^D) given θ^D. On the other hand, given ρ^D, it does not change in terms of default threshold θ^D in the neighborhood of the credit deposit contract offer curve. However, the slope $\partial\rho^D/\partial\theta^D$ of the iso-deposit supply curve becomes flatter in the right and left of the credible deposit contract offer curve.*

See the proof in Appendix.

C. Deposit Market Equilibrium

Given a deposit repayment schedule R^D by a banker, depositors decides how much to supply deposits. The depositors choose the best contracts among offers from many bankers for themselves. Bankers strategically compete for deposits.

Lemma 10. *[Lower Bound of Deposit Returns] Banker's deposit contract offer R^D needs to bring at least the same utility for a depositor as the equilibrium loan contract R^L.*

$$\max_{s_i} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^D(s_i, R^D, A, \epsilon_i)\right) dH(\epsilon)dG(A) \geq \max_{s_i} \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} u\left(c^D(s_i, R^L, A, \epsilon_i)\right) dH(\epsilon)dG(A). \tag{42}$$

[31]The derivative changes at default threshold θ^D depending on whether depositors expect default or not by bankers at the threshold. The derivative is taken from right for the non-default neighborhood and from left for the default neighborhood.

This is because a deviant measure-zero banker can always offer the loan contract R^L as her deposit contract with effectively no capital buffer per loan.

The following proposition shows that the equilibrium outcome is similar to a typical non-strategic competitive equilibrium. That is, an equilibrium is determined at the cross point of deposit demand and supply functions on the "price-quantity" plane.

Proposition 4. *A partial equilibrium exists in the deposit market, given total deposits, banker population, and loan market outcome. In equilibrium, bankers receive all the depositor's willingness to pay for the bank capital buffer as insurance against tail risks.*

The proof is provided in the Appendix. Although they strategically behave, bankers' behavior can be analyzed as a representative banker in the equilibrium due to the tight connection between the deposits amount and the default threshold given deposit rate. Here, I can further show that the deposit market partial equilibrium is unique. The proof is in Appendix.

Proposition 5. *Given loan market variables and total deposits S_i, the deposit market equilibrium—deposits per depositor s_i and per banker s_h and deposit repayment schedule R^D represented by deposit rate ρ^D and bank default threshold θ^D—is uniquely determined. Moreover, the equilibrium relation is expressed as $S_i = f(\rho^D|\mu)$, strictly increasing in deposit rate ρ^D given banker population μ with associated higher default threshold θ^D.*

Corollary 4. *A race to zero profit would not occur among bankers ex post, unlike the Bertrand competition.*

Proof. Proposition 4 ensures the equilibrium spread to be strictly positive, i.e., $\pi = \rho^L - \rho^D > 0$ and Proposition 5 states that the spread is determined uniquely. *Q.E.D.*

The positive banker profits after entry assures that the Nash equilibrium can be refined to a sequentially rational one, i.e., the perfect Baysian equilibrium. Intuitively, positive banker profits after entry is possible because the deposit repayment schedule, especially the default threshold, is tightly linked to deposits per capital, which is like endogenous capacity constraints. And, the Bertrand competition with capacity constraints ends up like the Cournot competition with positive profits—this is proven by Kreps and Scheinkman (1983) in which the price and the quantity are determined independently. However, this paper is different from their paper in that the price (repayment schedule) and the capacity (deposits) cannot be determined independently. As is clear in the proof of Proposition 4, if a banker attract more deposits, the banker must worsen its insurance service based on per deposit banker's capital.

VII. General Equilibrium

A. General Equilibrium given Banker Population

Proposition 6. *Given banker population μ, there exists a unique general equilibrium.*

Proof. By Proposition 4, the equilibrium deposit amount is one-to-one mapping with deposit rate, i.e., $S_i = f(\rho^D|\mu)$. Given banker population, the loan supply per borrower is the sum of deposits and banker capital (Lemma 5), that is, $\tilde{l}_j = f(\rho^D|\mu) + 2\mu k_0^b/(1-\mu)$. Lemma 10 and Proposition 4 imply that the (42) is satisfied with equality. That is, the deposit supply is essentially the same as the demand for underlying corporate loans, $S_i = f(\rho^D|\mu) = \hat{f}(\rho^L|\mu)$. It is strictly increasing in ρ^L (Proposition 5). Then, the inverse loan supply function can be represented by one-to-one function, $\rho^l = \rho^D + \pi = g^D(\tilde{l}_j|\mu)$.

Apparently, this inverse loan supply is an increasing function on l_j-ρ^L plane. But, the loan demand $\rho^L = g^L(l_j^*|\mu)$ is also one-to-one mapping and a decreasing function (Corollary 1). Therefore, the general equilibrium exists and unique, given μ. *Q.E.D.*

B. Positive Banker Population in General Equilibrium

A sizable banking sector ($\mu > 0$) can exist in equilibrium to provide an insurance to mitigate the tail-risk dumping to depositors. A depositor faces the shock-contingent deposit return up to threshold θ^D and then flat income ρ^D. With large enough variations in the aggregate shocks, there are some chances that deposits are not repaid in full. Then, the depositors would prefer a more insured contract that provides a same expected return with a lower deposit rate but also with a lower default risk. A banker also prefers to provide this insurance contract, implying that he has strictly positive capital per depositor. In other words, the banking sector is not measure zero in equilibrium.

Proposition 7. *In a general equilibrium, if exists, the banking sector is sizable $\mu > 0$ if deposits are positive $s_i > 0$. The capital ratio k_0^B/L_h and the spread $\pi = \rho^L - \rho^D$ are strictly positive for all bankers.*

Proof. Suppose that deposits are positive $\hat{s}_i > 0$ but that the banking sector size $\hat{\mu}$ is measure zero. Then, as tiny bankers serve all depositors, the capital ratio becomes (almost) zero, and the expected loan and deposit repayment schedules becomes (almost) the same by arbitrage. By Lemma 10, there is one threshold $\hat{\theta}^D = \hat{\theta}^L/\underline{\epsilon}$ below which both borrowers and bankers default. The full-pay deposit and loan rate becomes (almost) equal and the spread is zero, $\hat{\pi} = \hat{\rho}^L - \hat{\rho}^D = 0$.

While keeping the same loan contract, consider a slightly different deposit contract with positive capital buffer, which gives the same expected return to depositors with less volatile repayment. Here, less volatile means a lower deposit rate $\underline{\rho}^D$ but with lower threshold $\underline{\theta}^D$ and higher repayment in case of banker's default (due to capital buffer). This contract (partially) insures depositors' income for the combined idiosyncratic and aggregate productivity shocks and thus it is preferred by a risk averse depositor. For illustrative purpose, I draw the original and new deposit repayment schedules with the aggregate shock realization on the x-axis and the repayment on the y-axis in Figure 7.

Think about a banker's default case. Repayment to a depositor is $R^D s_i$, where R^D is defined as the seized banker's income (24). Because of symmetric equilibrium, deposit repayment per depositor is backed by loan repayment by a firm, but the bank capital per depositor needs to be adjusted by relative banker size per depositor, i.e., $\hat{w} = 2\mu/(1-\mu)$.

Under the measure zero banking sector, the seized bank capital per depositor is (almost) zero. Thus, a depositor's gain from the new contract with the sizable bank capital buffer is positive as the new repayment schedule is above the original one (see Figure 7). When bankers default, the gain is represented by the increase in the recovery rate of deposits just by the capital buffer, net of banker's retained portion, i.e., $\hat{w}(1-\lambda)k_0^B$.

The expected gain is obtained by the integral of that gain over the default region weighted by the probability of bank default. A lower bound of the depositor's expected gain is the parallelogram area consisting of the height of the capital buffer and the range between the new default threshold $\underline{\theta}^D$ and the lowest aggregate shock $\underline{A}$ (see Figure 7):

$$Gain > \underline{Gain} = \int_{\underline{A}}^{\underline{\theta}^D} \hat{w}(1-\lambda)k_0^B dH = \hat{w}(1-\lambda)k_0^B H(\underline{\theta}^D). \tag{43}$$

Recall that this new contract is assumed to give the same expected repayments to depositors, but here the banker is offering a lower deposit rate $\underline{\rho}^D$ for the same loan rate $\hat{\rho}^L$. This change of the deposit rate is denoted by the increase in the spread $\triangle\pi$ from zero. This would create a loss to depositor in the non-default region.

The depositor's loss is strictly lower than the upperbound of the loss, which is measured by the rectangle made by the change in the spread and the cumulative probability above the new threshold in Figure 7. That is,

$$Loss < \overline{Loss} = \triangle\pi(1-H(\underline{\theta}^D)). \tag{44}$$

Because the new contract is assumed to have the same expected returns for a depositor, the gain and the loss must be the same. Therefore, the lowerbound of the gain must be strictly lower than the upperbound of the loss: That is,

$$\triangle\pi > \hat{w}(1-\lambda)k_0^B \frac{H(\underline{\theta}^D)}{1-H(\underline{\theta}^D)}. \tag{45}$$

Because the original banking sector size is almost zero, I evaluate this at the limit $\hat{\mu} \to 0$ (i.e., $\hat{w} \to 0$) to see the profit gain stemming from a new contract. But, it is strictly positive in the limit, too:

$$\lim_{\hat{\mu}\to 0} \triangle\pi > 0. \tag{46}$$

That is, an increase in the spread by offering a insuring contract is strictly positive near $\mu = 0$. The spread is literary an insurance premium for the strictly positive value of the capital buffer as an insurance for depositors.

For a deviant banker to create sizable bank capital per depositor, she may need to ration loans and deposits. By doing so, her income could become less because the total income is affected by the spread times deposits. But, recall that the spread under the original contract is zero. Thus, the deviant banker's income increases from zero to positive. Therefore, the new offer $(\underline{\rho}^D, \underline{\theta}^D)$ with sizable bank capital per depositor can make profits for the deviant banker.

This also works for the same deviation by a sizable set of bankers. Indeed, both depositors and bankers prefer the new contract, given the same loan contract (i.e., the same utility for borrowers).

Therefore, in equilibrium, the banker population is sizable $\mu > 0$, associated with sizable bank capital per depositor $w > 0$ and positive spread $\pi > 0$ in a symmetric equilibrium. *Q.E.D.*

C. General Equilibrium with Endogenous Banker Population

Ex ante, occupation is chosen by arbitrage. Proposition 7 shows that banker population is sizable $\mu > 0$ in a general equilibrium, if exists. Here, I show indeed there exists a general equilibrium with endogenous banker population.

The spread income of a banker is the spread π times the deposits S_h.

$$\pi S_h = \begin{cases} \pi \frac{1-\mu}{2\mu} S_i & \text{if } \mu > 0, \quad \text{when } \pi > 0, \\ 0 & \text{if } \mu = 0 \text{ (and hence } \pi = 0) \end{cases} \tag{47}$$

Note that the second case covers the case with measure zero bankers. If there is no banker, the spread income for them is not defined. The spread income is obviously discontinuous at $\mu = 0$, because it approaches to ∞ with $\mu \to 0$ from right while it is zero at $\mu = 0$. On the other hand, apparently, spread income becomes zero at $\mu = 1$.

I focus on realistic cases regarding default probability of borrowers, in particular, less than 50 percent on average.

Assumption 7. $H(\theta^L) < 1/2$

Lemma 11. *In the case of positive banker population $\mu > 0$, banker's spread income πS_h and hence banker's utility V^B are strictly decreasing in μ under Assumption 7.*

See the proof in Appendix.

Lemma 12. *Ex ante entrepreneur utility V^E increases strictly with μ.*

Proof. Lemma 10 and Proposition 4 imply that depositor's utility (and deposits) do not increase with μ as it is kept at the participation constraint (42). On the other hand, because bankers invest all their capital to borrowers as loans, total capital used by a borrower increases strictly with larger banker population.[32] Hence, borrower's production capital increases strictly with banker population (i.e., capital buffer) and so does borrower's income $y^U - R^L L_j$ (see borrower's first order condition (29)). *Q.E.D.*

Think about banker's capital as foreign investment to a country with two entrepreneurs. Although only one of them receives investments from bankers, overall funds available to the set of two entrepreneurs are larger than before. This implies higher outputs for two entrepreneurs, though with a lower MPK and less entrepreneur population.

Proposition 8. *A general equilibrium exists and is unique even with endogenous occupation choice.*

[32] Even without deposit intake, a banker could invest her own capital, like a money lender.

Proof. Entrepreneur's utility is strictly increasing with μ by Lemma 12 with bounded positive income. On the other hand, banker's income is strictly decreasing by Lemma 11 from very large income level near $\mu = 0$ and is approaching zero income as $\mu \to 1$. Apparently, bank utility follows this path. Therefore, a unique equilibrium exist in endogenous occupation choice (see Figure 8). *Q.E.D.*

VIII. POLICY IMPLICATIONS

A. Welfare Theorem

The institutional assumptions restrict the shape of the loan and deposit repayment schedules R^L and R^D, which agents in this economy as well as the social planner obey. Because R^L and R^D also dictates the consumption allocation, the constrained social planner can determine the consumption allocation indirectly by choosing the loan and deposit repayment schedules.

The social planner's problem can be expressed as maximizing a representative banker's utility, similar to (8), but also with controls μ, l, and s in addition to R^L and R^D:

$$\max_{\mu,l,s,R^L,R^D} \int_{\underline{A}}^{\overline{A}} u\left(c^B(A, \tilde{R}^L, \Omega, \tilde{R}^D, \Psi)\right) dG(A), \tag{48}$$

subject to the representative borrower's utility maximization, i.e., an incentive constraint, (6), the representative depositor's utility maximization, i.e., another incentive constraint, (7), the occupational choice, i.e., yet another incentive constraint, (14), and resource constraints (12) and (13). Note that the social planner can set the all the loan and deposit contracts to be the same, $\Omega = \tilde{R}^{L\mu}$ and $\Psi = \tilde{R}^{D\mu}$, i.e., μ-Cartesian products of $\tilde{R}^L$ and $\tilde{R}^D$, respectively.

Definition 2. *The constrained social optimal allocation is the solution to the social planner's problem in which the social planner faces the same restrictions as the private agents.*

Note that the social planner's problem can be set up as maximizing the social weight weighted sum of utilities $\mu V^B + (1-\mu)V^E$ subject to the occupation arbitrage condition $V^B = V^E$. However, by substituting the occupation arbitrage condition into the weighted sum of utilities, the objective becomes simply to maximize V^E (or V^B) with the condition $V^B = V^E$. This is the same the formulation in (48). In other words, the occupational arbitrage condition implies that the weighted sum version of the social welfare function has a corner solution of $V^B = V^E$. With this condition, the unconstrained marginal condition ($V^B + \mu\partial V^B/\partial\mu - V^E + (1-\mu)\partial V^E/\partial\mu$) is not equal to zero. In a sense, a potential externality arising from population allocation exists but it cannot be fixed by the constrained social planner due to the occupational arbitrage condition.

Proposition 9. *The decentralized equilibrium achieves the constrained social optimum.*

The proof is straightforward and sketched here. Inside the social planner's problem, the incentive constraints for borrowers (6), for depositors (7), and for occupational choice (14) are exactly the same problems that are solved in the decentralized equilibrium under the same resource constraints and Assumptions. Only difference is the problem for bankers. In decentralized equilibrium, a banker maximizes his utility by choosing loan and deposit repayment schedules given the reaction functions (i.e., loan demand and deposit supply functions) from borrowers and depositors. The social planner achieves the social optimum by maximizing the representative banker's utility by choosing loan and deposit repayment schedules and also by selecting loans and deposits. However, those are chosen from the sets restricted by incentive constraints for borrowers and depositors. The restrictions are the same as reaction functions in the decentralized equilibrium.

An immediate result is that any policy intervention is not necessary for the decentralized equilibrium. These include the capital adequacy ratio requirement, deposit insurance, and so on. However, next, I explore possible fiscal intervention in the form of bank bailout, which may have a different outcome.

B. Deposit Insurance and Bank Bailout

The depositors assume tail risks in the Speedy Bankruptcy Regime. The allocation of goods when hit by a tail risk is quite unequal. While the borrowers and bankers can retain their consumption at λk_0, depositors face less consumption depending on the shock. In such tail risk region, the CSV-Costly Negotiation Regime can achieve Pareto optimal, more symmetric allocation, given equal social weights. Indeed, if the negotiation cost is low, the CSV-Costly Negotiation Regime is the one, a country should adopt under the incomplete contract assumption. As already discussed, the CSV-Costly Negotiation Regime, if negotiation cost is low, achieves essentially the same allocation as in the CSV Regime (Townsend, 1979), which I call Townsend equilibrium.

In the Speedy Bankruptcy Regime, a question arises whether a government intervention can achieve better allocation than the decentralized equilibrium. In such cases, given the intrinsic costs, the goal would be to achieve or get to close to the Townsend equilibrium. In particular, to ease the tail-risk burdens for depositors in the Speedy Bankruptcy Regime, a question might arises if a deposit insurance would work. A typical deposit insurance can be defined as a protection for depositors' income in case that a banker would default. It is usually financed by taxing the bankers, ex ante. This "taxing bankers ex ante" is the difference from the bailout, which is "taxing everyone ex post."

Consider a case that the depositors will not lose the face value of the deposit, that is, a full coverage deposit insurance. Because of the welfare theorem (Proposition 9), the following claim is obvious.

Claim. *The full coverage deposit insurance with ex ante fees does not improve welfare.*

Here is the sketch of the proof. The full coverage deposit insurance with ex ante fee is essentially the same as restricting the bankers' offer of deposit contracts to be very safe, close to zero default $\theta^D \approx 0$, associated with high spread π to pay the insurance fee. This restriction on the bankers' offers of deposit contracts is an obvious distortion to the economy and the associated social planner's problem. Therefore, such a scheme cannot improve the social welfare.

Note that a partial, but substantial, coverage deposit insurance—which covers the full amount down to the government-set threshold θ_G^D—with ex ante fees would create the similar distortion as the full coverage version. Essentially, the bankers are constrained to choose the deposit repayment schedule and thus the welfare decreases.

A better insurance scheme for deposits is to set fees ex post, depending the shock realizations, and share the funding among all the agents. This scheme, I would say, is essentially called as the bank bailouts financed by consumption tax.[33] A bailout policy is defined here as guaranteeing a banker's income in case that a banker would default without the bailout policy. It enables a banker to repay deposits in full. This description represents actual bailouts (see e.g., Landier and Ueda, 2009).

So far, the social planner or the benevolent government is assumed to face the same constraints as the private agents. This assumption supports the welfare theorem, Proposition 9. However, here, I now assume that the government has a taxing power, which is an extra power compared to the private agents. This violates the assumption beneath the welfare theorem (Proposition 9). This is not a great assumption from a viewpoint of pure theoretical arguments, but it seems realistic assumption.

In particular, the government can tax on consumption of those who defaulted. By doing so, the government can make defaulters to contribute to the expenditure needed to share tail risks more equally among agents. The debt contract with limited liability implies that the borrowers and bankers are well insured for a very low realization of aggregate shock while the depositors absorb the whole tail risk. If there is a way to redistribute the borrowers' retained assets to the depositors, the allocation would become closer to the Townsend equilibrium and raise the ex ante welfare. Given the limited liability laws, for private agents, it is legally difficult to collect funds from those who defaulted. One way to do so is to use the taxing power of the government.

I do not assume that the government has an informational advantage over private agents. However, it seems consistent to assume that the government can know the state of bankers (i.e., the aggregate shock realizations) by becoming a (parital) owner through capital injection. Even for more general form of bailouts, other than capital injection, it seems natural to assume that the government can know at least the aggregate shocks when it bails out bankers by examining their balance sheets. Still, it is likely that only bankers know idiosyncratic shocks of client firms through costly verification.

Assumption 8. ***[Speedy Bankrupty Regime with Bank Bailout]***
In addition to Assumptions 3 and 4,
(a-3) [State Revelation to the Government] A specific realization of aggregate productivity shock A is revealed freely when bailing out bankers.[34]
(f) [Government Power to Tax] The government can collect consumption tax even from those who defaulted.

The bailout can be designed so that the bankers would not benefit from them directly.

Definition 3. *A "transparent" bailout transfers funds to depositors via bankers without benefiting bankers directly, while an "untransparent" bailout benefits bankers directly.*

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[33]In the real world, bank bailouts are often financed by government bonds, which the government repays over time, for example, by consumption tax. Other real-world examples include inflation tax on monetary assets or (future) income tax on human capital, though they are not modeled in this paper.

[34]To include more realistic timing is left for a further research, but in reality, the speedy recognition of economic condition is important for a government, especially in crisis. In normal times in the real world, after a quarter or two, the government can figure out the realized aggregate shock. But, in crisis, by bailing out banks, the government seems to recognize the magnitude of the aggregate shock at first hand.

Note that, under a transparent bailout, the bankers may benefit indirectly through the general equilibrium effect, but not directly. Hence, if looking at the direct effects, a transparent bailout serves as an insurance for depositors at the cost of borrowers. Depositors can have the perfectly constant deposit repayment under this bailout policy for any realizations of the aggregate shocks A. Everyone needs to pay contingent consumption tax $\kappa(A)$ per person ex post to finance the bailouts. The consumption changes to C^D_{BO}, C^U_{BO}, and C^B_{BO} for depositors, borrowers, and bankers, respectively, and they are defined as the original consumption C^D, C^U, and C^B, respectively, minus consumption tax κ plus bank bailout transfers.

Under a transparent bailout, when a banker faces default, the government transfers funds to a banker just to repay the deposit in full so that the banker's consumption schedule C^B remains unchanged. The tax-transfer system based on idiosyncratic shocks is ruled out under (a-3) in Assumption 8. Hence, I focus on a conventional tax policy, which is not discriminatory among people and can be conditional only on the aggregate shocks. Transfers then become conditional on the aggregate shocks only but can be targeted to depositors.

The bailout fund transfer occurs only when the aggregate shock is much lower than the banker's default threshold, $A < \bar{\theta}^D_{BO} < \theta^D_{BO}$, where the new threshold $\bar{\theta}^D_{BO}$ is the shock level at which depositor's total income becomes smaller than the retained assets by borrowers and bankers, i.e., $R^D s_i + y^D < \lambda k_0$. This happens in the tail risk region. In this case, per depositor transfer $\nu(A)$ is the difference between the borrower's and banker's retained asset λk_0 and the depositor's average income $R^D s_i + E[y^D|A] < \lambda k_0$, to make every occupation's average pre-tax income to be equal at λk_0. More precisely,

$$\nu(A) = \lambda k_0 - \left(\frac{B(A|R^L, L_h, L_j) - \lambda k_0^B}{S_h} \right) s_i - \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon_i A e^D (k_0 - s_i)^\alpha dH(\epsilon), \quad \text{if} \quad A \in [\underline{A}, \bar{\theta}^D_{BO}];$$
$$\nu(A) = 0, \quad \text{otherwise}. \tag{49}$$

Note that this bailout does not guarantee the full repayment $(1 + \rho^D)$ of deposits for depositors. In this sense, it is a partial bailout.

Bank bailout, when formulated ex post, it is often the case that bank shareholders need to approve any capital injections (Landier and Ueda, 2009) for bank pre-tax profits. Hence, in the model, I require the following bank's participation constraint to the bailout scheme ex post:

$$c^B_{Bailout} \geq c^B_{NoBailout}. \tag{50}$$

Obviously, this is also a condition for Pareto superiority of the bank bailout compared with the no-bailout equilibrium.

Lemma 13. *A sudden, unexpected transparent bank bailout can improve the social welfare from the ex ante point of view.*

Proof. The repayment function R^D and R^L were not reoptimized with this unexpectd transparent bailout. Then, this unexpected bailout essentially relax the additional constraints of the simple debt restructuring rules (c-2 in Assumption 3 and c-2' in Assumption 4) in the economy. Because the ex post bank participation constraint (50) is met with equality for after-tax terms, bankers do not lose nor gain.[35] The depositors and borrowers would share the cost of bank defaults.

A particular example is to tax $\kappa(A) = \nu(A)/2$ on everyone. Then, transfer $(1-\mu)\nu(A)/2 = (1-\mu)\kappa(A)$ to depositors; transfer $\mu\nu(A)/2 = \mu\kappa(A)$ to bankers (i.e., $\nu(A)/2 = \kappa(A)$ back to a banker).

From the ex ante viewpoint, with the bailout policy, the entrepreneurial risk is reduced. Reduces is the consumption volatility stemming from uncertainty for talent shocks that makes an entrepreneur either a borrower or a depositor. In particular, this risk sharing occurs at the low consumption level with high marginal utility. Hence, better sharing the risks between a depositor and a borrower for a large negative aggregate shock is welfare improving for an entrepreneur, when assessed ex ante based on a symmetrically weighted utilitarian social welfare function. *Q.E.D.*

A problem is that an unexpected bailout, announced ex post suddenly, is not supported by everyone. In case bank participation constraint (50) is met, the bankers remains at the same utility lavel. But, the borrowers are worse off at the time of bailout ex post, although they would agree from the ex ante point of view.

Then, it may be better to institutionalize (and commit) bank bailouts ex ante, for example, by establishing a resolution fund.[36] On the other hand, many (casually) argue that institutionalized expectation of bank bailouts is bad from ex ante point of view because it induces distorted behaviors of agents and increase the probability of bank defaults. Indeed, if the bailout is ex ante designed and expected, it may be the case in the new equilibrium as shown in Lemma 14 below.

Lemma 14. *Institutionalizing transparent bank bailouts financed by consumption tax improves tail-risk sharing but distorts efficiency in capital allocation and occupational choice. Both deposits and loans increases. Deposit and loan repayments becomes lower in returns and higher in risks. Overall effects on the social welfare is lower than the Townsend equilibrium but uncertain compared to the original Speedy Bankruptcy Regime without bank bailout.*

Proof. The government can adopt the tax-transfer system to potentially mimic the shape of consumption allocation in the Townsend equilibrium of the CSV-contingent contract regime, except for idiosyncratic shock adjustment for the borrowers. As shown in Lemma 13, this policy enables agents to share tail-risks better. Thus, it could bring a Pareto superior equilibrium compared to the no-bailout case if there were no distortions in the decisions on deposits and loans as well as banker leverage and population.

However, in the deposit market equilibrium, it is obvious that the credible deposit contract offer perceived by depositors become flat in θ^D-ρ^D plane of Figure 6 because deposits are perfectly safe for depositors and bankers essentially claim so in their offers. Then, to have an equilibrium in the deposit market, deposits need to become much larger, apparently settling with lower promised return

[35] If the condition is met only in pre-tax, which is likely more realistic, then the bankers would lose just for tax payments. Still, overall social welfare from ex ante point of view may improve.

[36] Too-big-to-fail problem often assumes people expects bank bailouts even if it is institutionalized. However, because it is not institutionalized ex ante, some oppositions are based solely on ex post point of views, for example, arguing that it is “unfair” to transfer funds to banks from a general tax base.

ρ^D with higher (hidden) deposit default threshold θ^D. Larger deposits translates into larger loans in the loan market. This makes equilibrium promised loan rate ρ^L lower and loan default threshold θ^L higher (see Lemma 3 and Figure 5).

Although (almost) the same allocation in tail-risk region as in the Townsend equilibrium is achieved, funding for the transfer system is not directly linked to deposit and borrowing decisions in this regime, creating classical fiscal externality (tragedy of commons). That is, this scheme creates too large deposits and loans (and thus lower banker capital) with higher risks and lower returns to bankers and depositors. Hence, the value of entrepreneurs and bankers are both lower than in the Townsend equilibrium.

Q.E.D.

The bank bailout financed by consumption tax by everyone is essentially a simple but formal "big household" creation in my model, at least for tail-risk sharing. Because the private contracts alone cannot achieve the tail-risk sharing, the model require the government intervention. However, Lemma 14 suggests that this simple "big household" insurance creates additional distortion because the costs of risk insurance is not contingent on the (potential) use of risk insurance if it is financed by a (simple) consumption tax system. Here is a potential role for a regulation.

Proposition 10. *With appropriate bank capital ratio requirement, institutionalizing transparent bank bailouts financed by consumption tax can (almost) mimic the Townsend equilibrium and thus improves the social welfare ex ante.*

Proof. This is rather a conventional argument and thus I just sketch the proof. The distortion in Lemma 14 is too large deposits (and loans) because the perceived risk of deposit contract offer is too safe. Introducing a capital ratio requirement can limit the deposits and loans in the balance sheet. If it is set to mimic the deposit to capital ratio (or loans to capital ratio) of the Townsend equilibrium, with (almost) the same contingent allocation of goods, the new equilibrium can (almost) mimic the Townsend equilibrium .

Q.E.D.

It may be obvious but substantial loan guarantees by the government would create similar distortion as the institutionalized bank bailouts. The loans become safe, making deposits safe. Then, the deposit market distortion is the same as above. However, now, the loan market is also distorted, i.e., the perceived iso-loan default curve become flat in Figure 5. This would create even larger loan demands (and larger deposits in general equilibrium). The similar arguments on the capital ratio requirement as in the deposit market goes through for the loan market distortion.

C. Bankruptcy Rule Reforms

Another possible way to improve welfare, without using a capital regulation, is somewhat obvious but to change the bankruptcy rule to be more in line with the contingent contracts that would be written in the Townsend equilibrium. Of course, the key issue of this paper is that the ex ante contingent contracts may not be written and the ex post negotiation costs are large, so that simple and speedy bankruptcy rule are required.

Hence, here, I would like to present a rule-based bankruptcy, as simple as possible. In essence, each agent should faces the same repayment schedules as in the CSV-contingent contract regime. However, the reformed procedure may become complex. And, I discuss below how it may be difficult to be implemented.

For depositors, the problem is that the financial burden of bailouts as a safety net is not charged per each depositor's deposits if consumption tax is used. This can be fixed by changing the financing scheme to depositor (creditor) bail-in. That is, whenever bankers are bailed out, depositors (creditors) should also give up some portion $\xi(A)$ of deposit claims S_i. By doing this, unlike consumption tax $\kappa(A)$, depositors now face the financing burden of guaranteed deposits to be dependent on his own deposits. A problem of implementing depositor bail-in is that insuring deposits is an important policy in widely accepted paper. Without securing deposits institutionally, depositor bail-in may invoke inefficient self-fulfilling bank-run in the Diamond-Dybvig model (1983).

For borrowers, if a government can tax each defaulted borrower directly at a different rate conditional on realized aggregate and also idiosyncratic shocks as well as loan amounts, then the government can relax a constraint, the simple debt restructuring rule, even more. Here, the government tax-transfer can mimic the loan repayment schedule in the Townsend equilibrium. Then, the loan demand become the same as the Townsend equilibrium. A problem of this scheme is that the government has to know the idiosyncratic shock realizations of each borrower. This violates Assumption 3 (f), and it is hard to assume that the government has perfect information while bankers have to gather them with costs.

Yet another way may be possible. For borrowers (and bankers), a change in limited liability by seizing a portion of income of borrowers (and bankers), even from λk_0 which is protected previously by the limited liability. This scheme can be considered as *bail-in of defaulters*, which is similar in spirit to CEO compensation clawbacks in the real world when bailed out or found misconducts. Such penalties have been adopted in several major advanced countries, especially for banks, after the global financial crisis. However, though not rigorously analyzed here, this scheme may degrade the benefits of speedy bankruptcy with some retained wealth, which is supposed to facilitate a fresh start as a going concern of distressed firms, and thereby reduce the debt-overhang problem.

Note that the unlimited liability or "double liability" of bankers as in the pre-Great Depression in the U.S. would not work well.[37] Under the double liability regime, in essence, bankers always had to pay deposits in full, otherwise they were jailed (i.e., their consumption level is almost zero). In this regime, bankers were the ones that assumed all the tail risks. This would not be the optimal risk sharing among different types of agents, and thus is not socially optimal. Note that in this paper, the

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[37] See a history and theory paper by Kane and Wilson (1998) and an empirical work by Grossman (2001).

key friction is not the limited liability itself, but rather the limited liability being non-contingent to the aggregate shocks.

Overall, repayment schedules with reformed bankruptcy rules could make the economy closer to the Townsend equilibrium. The institutionalized bailout policy with reformed bankruptcy rule, then, allows more optimal (larger) risk taking by entrepreneurs and bankers due to a better risk sharing, compared to the case with speedy bankruptcy without bailouts. This is reminiscent to Obstfeld (1994) that shows in a different model setup (i.e., perfect information with Epstein-Zin preference) that a better risk sharing makes people to invest higher-risk-higher-return projects optimally and improves welfare.

Several questions, however, remains for the bankruptcy rule reforms. First, it is difficult to know for the government what is the best bankruptcy rule. in particular if people are heterogeneous in wealth and utility functions. Second, politically, if reforms were hastily set, the asset allocation upon default may favor a specific group of people. Third, if seriously debated, the time and costs to negotiate to pin down the best bankruptcy rule would be huge. Fourth, the reforms should be also robust to other distortions assumed in related literature. Finally, if people are heterogeneous in wealth and utility functions, the best asset allocation upon default is not the common rule but specific to each borrower-creditor pair as implicitly predicted in the CSV-Costly Negotiation Regime.[38] But, then the negotiation costs would be large given heterogeneity. In this regard, the Speedy Bankruptcy Regime with Bank Bailout with a capital ratio requirement may work relatively well.

D. Untransparent Bailouts and Income Shifting

So far, I have focused on the optimal transparent bailout scheme in a realistic institutional setup and find that a bank bailout, if designed well, can be welfare improving. However, in the real world, there can be a bad bailout. In particular, the literature (and newspaper articles) often discuss about corruption and other problems like moral hazard.[39] In other words, ex post "looting" opportunity may also be available for banks if banks can seize a part of bailout funds (similar to Akerlof and Romer, 1993). In this case, bailouts are not transparent as defined in Definition 3, but include some hidden subsidies to banks. I call this *untransparent bank bailouts*. Some of them may be necessary to persuade bank owners to agree on bailouts (e.g., Landier and Ueda, 2009) but others may well be a result of political influence by bank lobby (e.g., Igan, Mishra, and Tressel, 2011). The latter case is the problem. Here, I do not attempt to theorize the underlying mechanism of such practices in this paper but characterize the implications of this bad bailout scheme.

[38]Such a heterogeneity is a reason of lengthy negotiations.

[39]Distortions in the presence of the government protection in the financial system has been discussed mostly in a partial equilibrium framework. For example, the risk shifting problem induced by deposit insurance requires prudential regulations such as a capital adequacy ratio requirement in Kareken and Wallace (1978), Keeley (1990), and Allen and Gale (2007). The moral hazard problem from expected bailouts requires prudential regulations in Chari and Kehoe (2009) or tax in Kocherlakota (2010) although Chari and Kehoe (2016) admit the bailout of firms via banks is ex post efficient to avoid assumed fixed costs associated with bankruptcy. In a general equilibrium framework, Van den Heuvel (2008) argues that the capital adequacy ratio requirement is costly as it limits the liquidity available in the general equilibrium. Related issue is the effect of competition policy as regulations such as capital adequacy ratio requirement reduces competition. Some argue that risk taking becomes too excessive under freer competition (Allen and Gale, 2000) because monopolistic rents limit the banks' risk taking behavior. The others argue the opposite (Boyd and De Nicolo, 2005) because bank's higher monopolistic rents implies firms' lower rents that lead to higher risk taking at the firm level.

Under an *untransparent* bailout policy, the banker's consumption increases by excessive transfer $\underline{\kappa}$ for low realizations of aggregate shock, $A < \bar{\theta}^D_{BO}$, where the bank default threshold $\bar{\theta}^D_{BO}$ could be higher than that under a transparent bailout scheme, because a bank has a little more incentive to declare default to receive extra transfer $\underline{\nu}$. The overall transfer $\bar{\nu}(A)$ to a depositor is the same $\nu(A)$ as before but with the addition of this extra transfer:

$$\begin{aligned}&\bar{\nu}(A) = \nu(A) + \underline{\nu} \quad \text{if} \quad A \in [\underline{A}, \bar{\theta}^D_{BO}];\\&\bar{\nu}(A) = 0, \quad \text{otherwise.}\end{aligned} \tag{51}$$

An example is to tax $\kappa(A) = \nu(A)/2 + \mu\underline{\nu}/2$ on everyone. Then, transfer $(1-\mu)\nu(A)/2$ to depositors; transfer $\mu\nu(A)/2 + \mu\underline{\nu}/2$ to bankers, i.e., $\nu(A)/2 + \underline{\nu}$ back to a banker. By this bailout process, a banker gains $(1-\mu)\underline{\nu}/2$.

When institutioned, the distortions to loan and deposit markets are similar as in the transparent bailout. Now, on top of them, another distortion is added to the banking sector, because bankers are enriched by bailouts. The bailout expectations will create distorted incentives for people to become bankers rather than productive entrepreneurs. As a result, there will be too many bankers and too little production. Lower production implies lower entrepreneurs' utility, and so is bankers' utility through occupational arbitrage in a general equilibrium. I call this *income shifting* problem.

Almost tautologically, this problem requires a policy to limit bank profits so as not to attract too many people to become bankers. Introducing an additional capital adequacy ratio can mitigate a unnecessarily high incentive to become a banker by lowering bankers' utility. Introducing a bank levy to lower the (present value of) transfer works as well. With these regulations to countervail bad transfers $\underline{\nu}$, an untransparent bank bailout, if not eliminated due to e.g. bank lobby's power, might still become a reasonable response to tail-risk events in the Speedy Bankruptcy Regime.

IX. CONCLUSION

During the global financial crisis, the world witnessed bank bailouts everywhere, together with protests. In the aftermath of the crisis, the prudential regulations have been strengthened and bank resolution (i.e., bankruptcy) procedures have been reformed. More general corporate and household bankruptcy procedures have been evolved towards speedier rule-based ones at least since early 2000, and further facilitated since 2008.

In this paper, I try to understand in what conditions these policy reactions can be justified in a general equilibrium. Since Diamond and Dybvig (1983), the literature seems to focus on the liquidity problem but obviously the banking crises stems also, if not primarily, from the insolvency problem. So, I focus on the insolvency issues.

The model needs to allow defaults of debt contracts in equilibrium and thus uses the costly state verification framework (Townsend, 1979). However, the original costly state verification model assumes (implicitly) ex ante contingent contract for the default region of shock realizations, and therefore bankruptcy procedure is almost frictionless after paying state verification costs. As this is not in line with sometimes lengthy bankruptcy procedures in the world (Djankov, et. al., 2008), I assume to prohibit writing ex ante contingent contracts but allow ex post Nash negotiation. As long as negotiation is efficient, the allocation can mimic the contingent contracts.

However, in reality, lengthy negotiation is costly. Once it is admitted so, a way to lower the negotiation cost is to rely on a simple rule based bankruptcy procedure, which however may lose efficiency in allocating outputs and wealth upon default. My analysis starts from here and, to see welfare implications, it uses a general equilibrium setup.

Essentially, I develop a simple macro model with realistic financial frictions in bank lending, namely, costly state verification and limited liability with simple and speedy bankruptcy procedure. These frictions prohibit perfect income risk sharing among bankers, borrowers, and depositors. Moreover, I assume endogenous bank size—a banker is an occupation under a strict assumption of not sharing income (i.e., ownership). Ex ante, banker's income is equated in expectation with entrepreneur's in expectation. Entrepreneurs are further sorted to either borrowers or depositors depending on the idea shocks they draw. Overall outputs are affected by financial frictions and possible policy distortions. Inefficiency can appear in the labor allocation (i.e., the occupational choice) and the capital allocation (i.e., deposits and loans), which are affected by the spreads, set endogenously by bankers.

I show that the optimal loan and deposit contracts take a form of a standard debt contract, following costly state verification and incomplete contract literature. The optimal bank capital is shown to be positive to provide a buffer to depositors and bankers themselves. And, the banking sector is proven to be sizable, challenging a typical assumption of measure-zero banks.

However, when a large negative shock hits, both borrowers and bankers would walk away with retained assets because of limited liability protection. The depositors would assume all the tail risk. This *tail-risk dumping* problem creates too risky consumption profile ex post and thus make the occupational choice too risky ex ante. This is the new perspective to the existing literature on macro models with financial frictions.

A government could play a role in the economy. Once the government is allowed to tax on consumption and can acquire information of banks upon bailouts, it can de facto relax the limited liability constraint and the simple asset split rule. The government can then make transfers to be contingent on the aggregate shocks. This transparent bank bailout can mitigate the *tail-risk dumping* problem. An optimal deposit insurance, if funded ex post by tax, can mimic such a transparent bailout.

However, the bailouts or deposit insurance induces a free-riding problem of depositors relying on general tax revenue for their insurance premium, reducing their need for bank capital buffer and resulting too much deposits. Loan guarantees can be considered similarly. Here, a typical "big household" way of simply insuring tail risks each other is distortionary. But, these distortions can be remedied by a capital ratio requirement. Then, the overall welfare can be improved ex ante.

The bank bailout essentially provides insurance against tail risks faced by depositors. These additional regulations associated with bailouts would not be necessary, if the insurance are financed directly by each agent. This calls for the reforms in bankrupty rules to implement the Townsend equilibrium (in the original CSV contingent contracts or in Nash negotiation ex post). Specifically, it should not allow defaulters to walk away completely. That is, *bail-in* of defaulters is called for. Also, it is likely the case that depositors should not ask for the full repayment in the tail-risk event (i.e., depositor bail-in).

Bankruptcy procedures can be reformed further to incorporate aggregate risk contingencies. However, I conjecture that the complete implementation would be difficult. After the global financial crisis, creditor bail-in is already proposed and implemented but not depositor bail-in, not to mention contingencies on aggregate macroeconomic variables, Although deposit insurance has been somewhat restricted and harmonized across countries, a complete depositor bail-in (i.e., abolishing any bailouts and deposit insurance) may not be robustly a good policy considering justifications by other theories (e.g., Diamond and Dybvig, 1983). As for the defaulters' bail-in, more restrictions on limited liability in general and CEO compensation clawbacks for banks may indeed a way forward. But, again, a complete defaulter bail-in (i.e., abolishing limited liablity), contingent on aggregate shocks, may not be robustly a good policy either, considering justifications by other concerns (e.g., mitigating debt overhang). After all, complex contingencies embedded in contracts would create ex post lengthy negotiations.

In summary, this paper provides a more solid micro-foundation for the macroeconomic models with financial frictions by looking at the incomplete consumption sharing, especially for tail events, among different types of households as the logical consequence of a typical set of financial frictions. With this framework, the financial sector policies—bank bailouts, deposit insurance, and regulations—can be examined as redistribution policies, in addition to efficiency perspective. Notably, bank bailouts are shown to be welfare improving, in an environment with a simple and speedy bankruptcy procedures, together with prudential regulations. The market incompleteness for tail-risk events, which supports government interventions, would be likely to remain at least for a while since the speedy bankruptcy regime can minimize costs of negotiations and debt overhang in general. Hopefully, a breakthrough happens, perhaps through further evolution in information technology.

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Figure 1. Capital Exchange with Complete Contracts

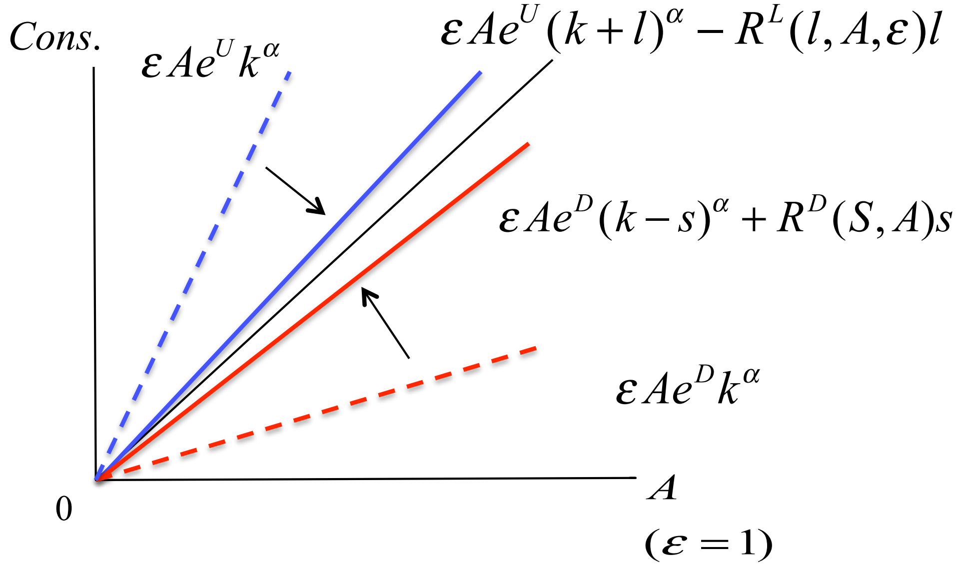


Figure 2. Capital Exchange with Simple Debt Contracts

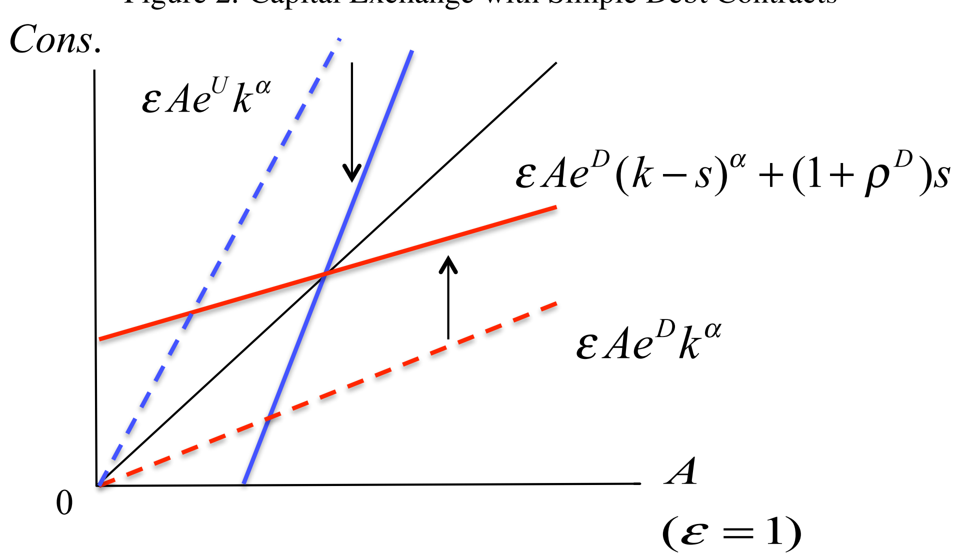


Figure 3. Debt Contracts with Limited Liability

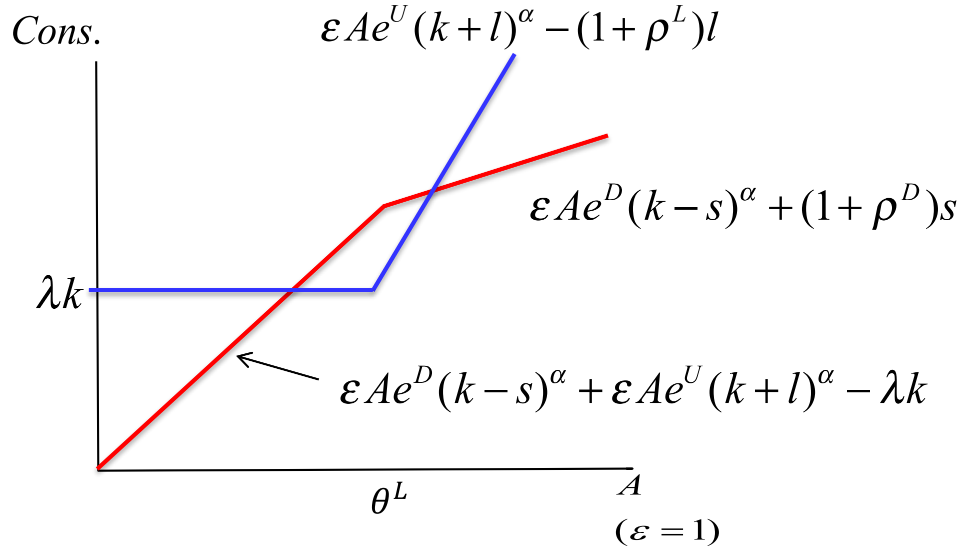


Figure 4. Debt Contracts and Banks

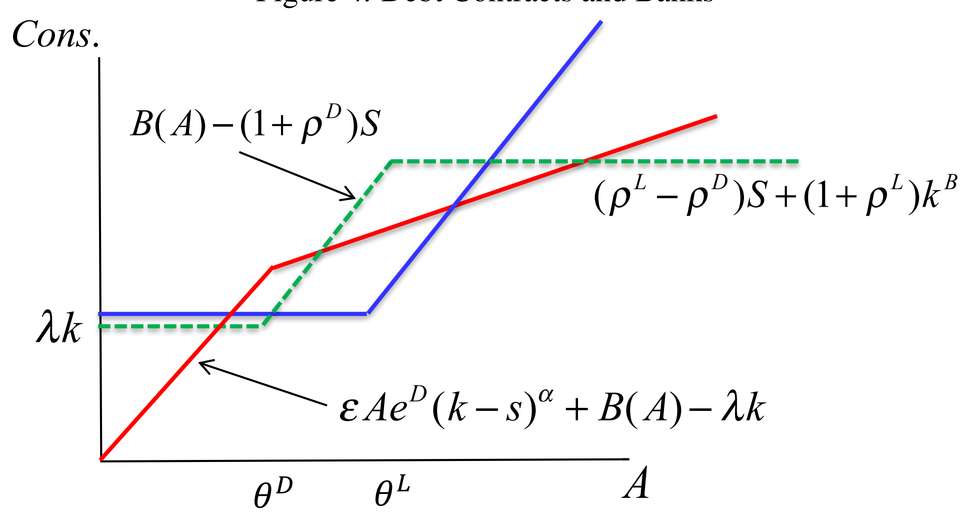


Figure 5. Loan Market Partial Equilibrium

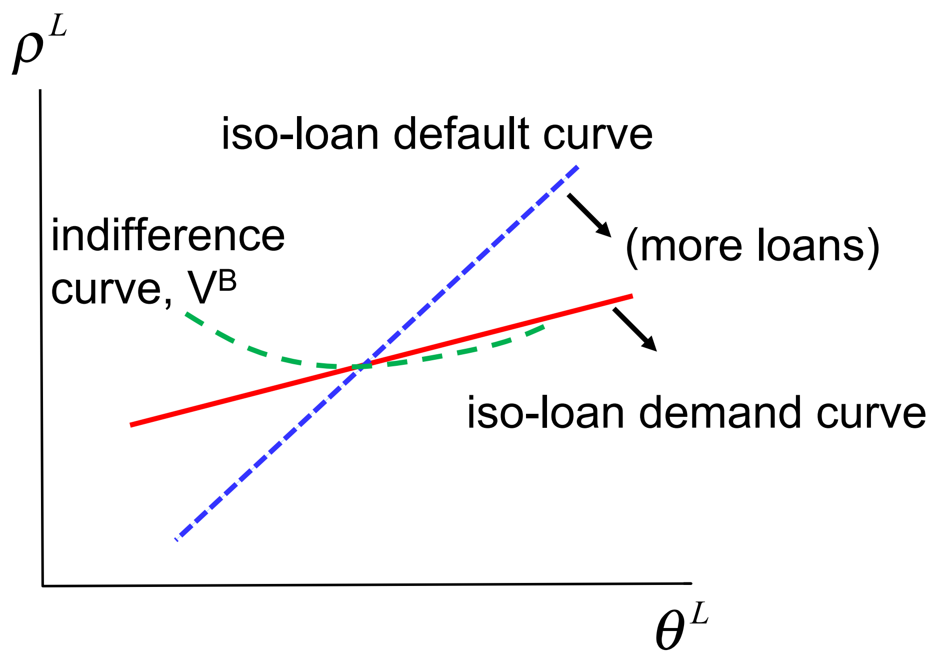


Figure 6. Deposit Market Partial Equilibrium

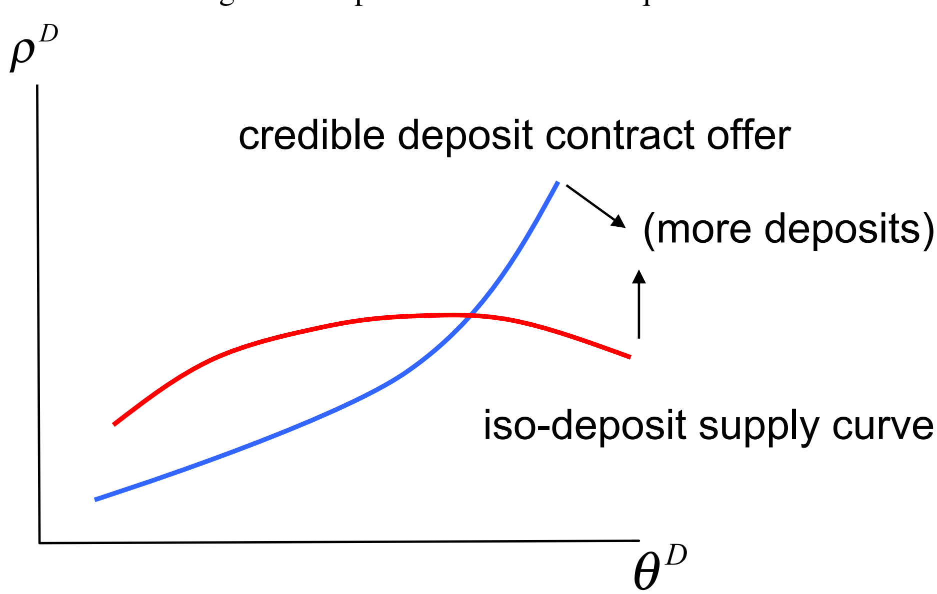


Figure 7. Deposit Return per Depositor

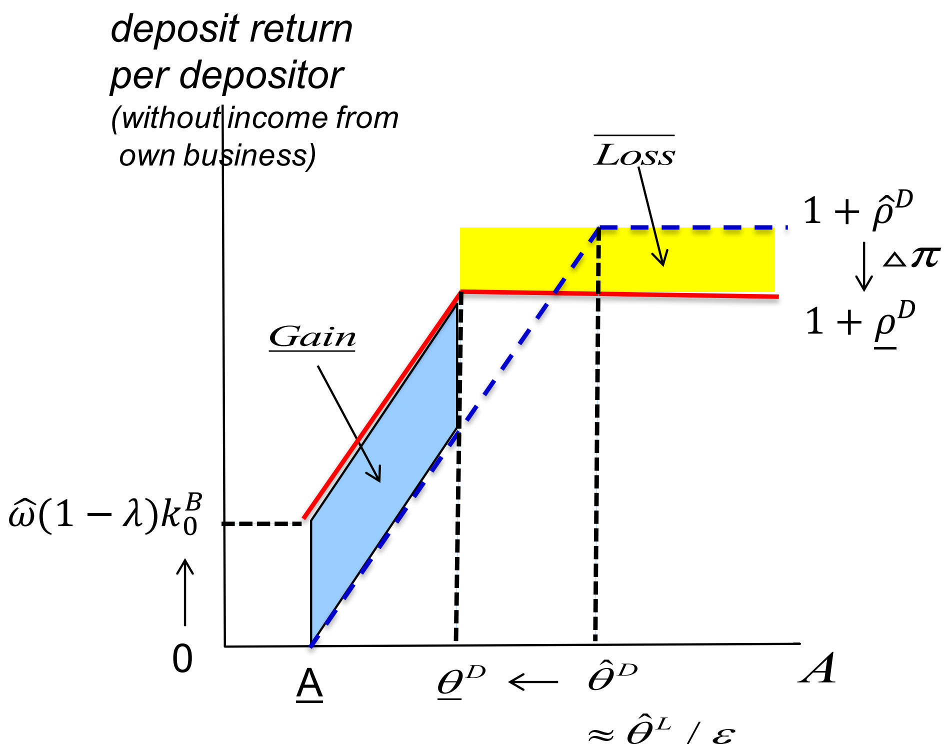


Figure 8. Banker Population and General Equilibrium

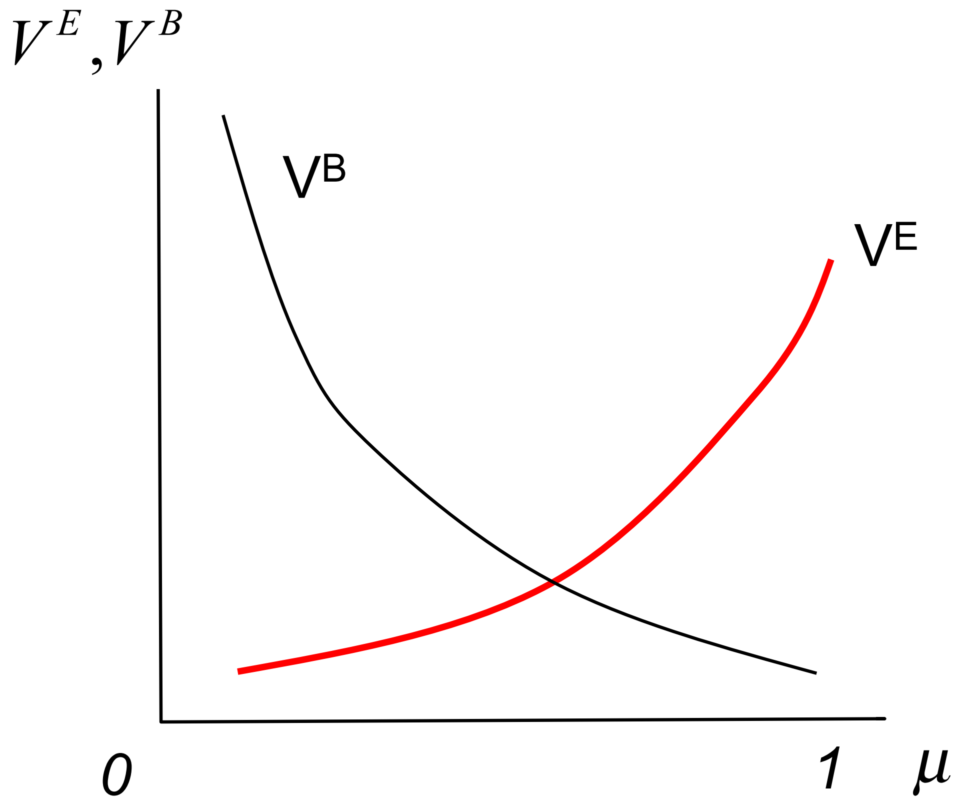
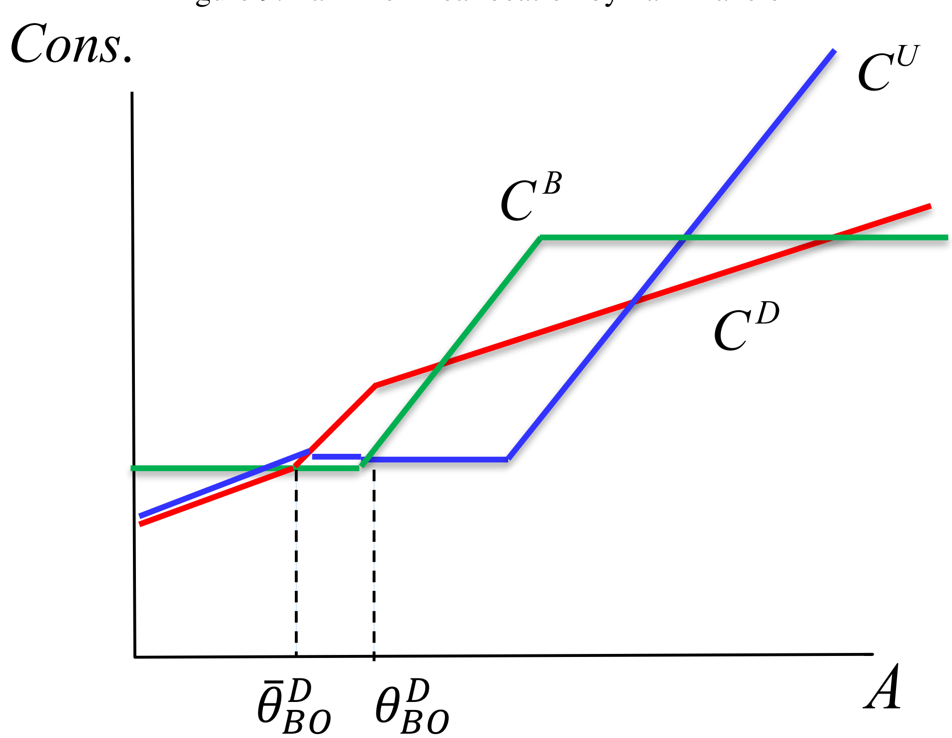


Figure 9. Tail-Risk Reallocation by Tax-Transfer



APPENDIX I. PROOFS

A. Proof for Lemma 3

Proof. Increasing in θ^L on the θ^L-ρ^L plane is easy to see because the slope of the iso-loan default function (28) with respect to θ^L is positive:

$$e^U \frac{(k_0 + l_j)^\alpha}{l_j} > 0. \tag{A1}$$

The slope of (28) becomes flatter with a larger loan l_j since its derivative with respect to l_j is negative

$$\alpha e^U \frac{(k_0 + l_j)^{\alpha-1}}{l_j} - e^U \frac{(k_0 + l_j)^\alpha}{l_j^2} = -\frac{1}{l_j^2} e^U (k_0 + l_j)^{\alpha-1}(k_0 + (1-\alpha)l_j) < 0. \tag{A2}$$

The intercept k_0/l_j of (28) on the θ^L-ρ^L plane is affected an increase in l_j as its derivative with respect to l_j is positive,

$$\frac{\lambda k_0}{l_j^2} > 0. \tag{A3}$$

The overall effect of *iso-loan default curve* by larger loans l_j is the higher intercept and flatter slope, though always increasing in θ^L. Moreover, under Assumption 4 (e), the iso-loan default curve becomes lower for all relevant domain of θ^L. It is because the whole change of the right-hand-side of (28) by a larger loan is the sum of its effect on the slope and the intercept, which is negative under Assumption 4 (e),

$$-\frac{1}{l_j^2}\left\{\theta^L e^U (k_0 + l_j)^{\alpha-1}(k_0 + (1-\alpha)l_j) - \lambda k_0\right\} \leq -\frac{1}{l_j^2}\left\{\theta^L e^U (k_0 + (1-\alpha)l_j)^\alpha - \lambda k_0\right\} \leq 0. \tag{A4}$$

Hence, *iso-loan default curve* moves lower down with flatter slope for the relevant range of θ^L on θ^L-ρ^L plane.

Q.E.D.

B. Proof for Lemma 4

Proof. The derivative of the iso-loan demand function with respect to θ^L is expressed as

$$\frac{\partial \chi(\theta^L, \rho^L)}{\partial \theta^L} = \left((1+\rho^L) - \alpha\theta^L e^U (k_0 + l_j)^{\alpha-1}\right) u'(\underline{c}^U) m(\theta^L) > 0, \tag{A5}$$

where $\underline{c}^U$ is evaluated at $\eta = \theta^L$. This is positive because the first order condition (29) implies that the loan rate $(1+\rho^L)$ is on average equal to the marginal product of capital conditional on not defaulting. Hence, the loan rate $(1+\rho^L)$ must be higher than the marginal product of capital with the lowest non-default shock θ^L, which is the second term in the parenthesis.

The derivative with respect to ρ^L is

$$\begin{aligned}\frac{\partial\chi(\theta^L,\rho^L)}{\partial\rho^L} &= \int_{\theta^L}^{\overline{\epsilon A}} \left(-u'(c^U) - (\alpha\eta_j e^U(k_0+l_j)^{\alpha-1} - (1+\rho^L))lu''(c^U)\right) dM \\ &= -\int_{\theta^L}^{\overline{\epsilon A}} u'(c^U)dM + \sigma l_j^* \int_{\theta^L}^{\overline{\epsilon A}} (\alpha\eta_j e^U(k_0+l_j)^{\alpha-1} - (1+\rho^L))\frac{u'(c^U)}{c^U}dM \\ &< 0.\end{aligned} \tag{A6}$$

Note that inside the second integral in the penultimate line has a "weight" of u'/c^U, which has higher weights for lower realizations of shocks and lower weights for higher realizations of shocks compared to the weight u' in the first order condition (30). Because the second integral is different only in this "weight" from the borrower's first order condition (30), the second integral must be smaller than $\chi(\theta^L,\rho^L) = 0$ of (30), and thus negative.

In summary,

$$\frac{d\rho^L}{d\theta^L} = -\frac{\partial\chi/\partial\theta^L}{\partial\chi/\partial\rho^L} > 0. \tag{A7}$$

Moreover,

$$\begin{aligned}\frac{\partial\chi(\theta^L,\rho^L)}{\partial l_j^*} &= \int_{\theta^L}^{\overline{\epsilon A}} (\alpha-1)\alpha\eta e^U(k_0+l_j^*)^{\alpha-2}u'(c^L)dM(\eta) \\ &+ \int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha\eta e^U(k_0+l_j^*)^{\alpha-1} - (1+\rho^L)\right)^2 u''(c^L)dM(\eta) \\ &< 0.\end{aligned} \tag{A8}$$

Hence by the implicit function theorem, with a larger loan, the loan rate decreases,

$$\frac{d\rho^L}{dl_j^*} < 0, \tag{A9}$$

that is, the iso-loan demand curve shifts down on the θ^L-ρ^L plane. At the same time, with a larger loan, the default threshold increases,

$$\frac{d\theta^L}{dl_j^*} > 0, \tag{A10}$$

that is the iso-loan demand curve shifts right on the θ^L-ρ^L plane. *Q.E.D.*

C. Proof for Proposition 1

Proof. On the θ^L-ρ^L plane, the iso-loan default curve is increasing (Lemma 3) and the iso-loan demand curve is also increasing (Lemma 4).

As $\theta^L \to 0$, the iso-loan default curve converges to the intercept of (28), i.e., $-1 - \lambda k_0/l_j$, which is lower than -1.

As for the iso-loan demand curve, the first order condition (29) can be expressed as

$$1 + \rho^L = \frac{\int_{\theta^L}^{\overline{\epsilon A}} \left(\alpha \eta_j e^U (k_0 + l_j)^{\alpha-1}\right) u'(c^L) dM(\eta)}{\int_{\theta^L}^{\overline{\epsilon A}} u'(c^L) dM(\eta)}. \tag{A11}$$

This implies$(1 + \rho^L) > 0$ no matter what the level of θ^L. Hence, the intercept of the iso-loan demand curve at $\theta^L = 0$ is bigger than -1 on the θ^L-ρ^L plane, and hence bigger than the intercept of the iso-loan default curve.

Next, I show below that the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve. This means that the iso-loan default curve crosses the iso-loan demand curve from below only once on the θ^L-ρ^L plane. Therefore, for each loan amount l_j, the pair (θ^L, ρ^L) is determined uniquely in an equilibrium of the loan market.

First, for the iso-loan demand curve, I simplify the term inside the second integral of (A6), as follows

$$\begin{aligned}
&\alpha \eta_j e^U (k_0 + l)^{\alpha-1} - (1 + \rho^L) \\
<&\eta_j e^U (k_0 + l_j)^{\alpha-1} - (1 + \rho^L) \\
<&\eta_j e^U (k_0 + l_j)^{\alpha-1} - (1 + \rho^L)\frac{l_j}{k_0 + l_j} \\
=&\frac{c^U}{k_0 + l_j} \quad \text{for} \quad \eta_j = \epsilon_j A \geq \theta^L.
\end{aligned} \tag{A12}$$

This implies

$$\begin{aligned}
\frac{\partial \chi(\theta^L, \rho^L)}{\partial \rho^L} &< \int_{\theta^L}^{\overline{\epsilon A}} u'(c^U) dM + \sigma l_j \int_{\theta^L}^{\overline{\epsilon A}} \frac{c^L}{k_0 + l_j} \frac{u'(c^U)}{c^U} dM \\
&= -\left(1 - \frac{\sigma l_j}{k_0 + l_j}\right) \int_{\theta^L}^{\overline{\epsilon A}} u'(c^U) dM.
\end{aligned} \tag{A13}$$

Hence, the slope of iso-loan demand curve is

$$\begin{aligned}\frac{d\rho^L}{d\theta^L} &= -\frac{\partial\chi/\partial\theta^L}{\partial\chi/\partial\rho^L}\\ &< \frac{\left((1+\rho^L)-\alpha\theta^L e^U(k_0+l_j)^{\alpha-1}\right)u'(\underline{c}^U)m(\theta^L)}{\left(1-\frac{\sigma l_j}{k_0+l_j}\right)\int_{\theta^L}^{\overline{\epsilon A}}u'(c^U)dM}\\ &< \frac{(1+\rho^L)-\alpha\theta^L e^U(k_0+l_j)^{\alpha-1}}{1-\frac{\sigma l_j}{k_0+l_j}}.\end{aligned} \tag{A14}$$

Note that the last line uses the apparent relation

$$u'(\underline{c}^U)m(\theta^L) < \int_{\theta^L}^{\overline{\epsilon A}} u'(c^U)dM. \tag{A15}$$

Hence, the slope of the iso-loan demand curve is always flatter than that of the iso-loan default curve if

$$\frac{(1+\rho^L)-\alpha\theta^L e^U(k_0+l_j)^{\alpha-1}}{1-\frac{\sigma l_j}{k_0+l_j}} < \frac{e^U(k_0+l_j)^{\alpha}}{l_j}, \tag{A16}$$

or equivalently,

$$\frac{(1+\rho^L)-\alpha\theta^L e^U(k_0+l_j)^{\alpha-1}}{k_0+l_j-\sigma l_j} < \frac{e^U(k_0+l_j)^{\alpha-1}}{l_j}. \tag{A17}$$

Here, I first show that the numerator of the left-hand side is smaller than that of the right-hand side. Because the marginal product of capital is increasing in productivity shock η while the marginal utility is decreasing in the same shock, covariance of these two terms are negative.[40]. Then, based on the first order condition (A11),

$$\begin{aligned}1+\rho^L &< \frac{\left\{\int_{\theta^L}^{\overline{\epsilon A}}\alpha\eta_j e^U(k_0+l_j)^{\alpha-1}dM(\eta)\right\}\left\{\int_{\theta^L}^{\overline{\epsilon A}}u'(c^L)dM(\eta)\right\}}{\int_{\theta^L}^{\overline{\epsilon A}}u'(c^L)dM(\eta)}\\ &= \int_{\theta^L}^{\overline{\epsilon A}}\alpha\eta_j e^U(k_0+l_j)^{\alpha-1}dM(\eta).\end{aligned} \tag{A18}$$

Hence, to prove the numerator of the left hand side of (A17) is smaller than that of the right hand side, it suffices to show

$$\int_{\theta^L}^{\overline{\epsilon A}}\alpha\eta_j e^U(k_0+l_j)^{\alpha-1}dM(\eta) - \alpha\theta^L e^U(k_0+l_j)^{\alpha-1} \le e^U(k_0+l_j)^{\alpha-1}, \tag{A19}$$

[40]Recall that, for any two random variables ξ and ν, $E[\xi,\nu] = E[\xi]E[\nu] + cov(\xi,\nu)$.

or equivalently,

$$\alpha\left(\int_{\theta^L}^{\overline{\epsilon A}} \eta_j dM(\eta) - \theta^L\right) \leq 1, \tag{A20}$$

which can be simplified to

$$\alpha\left(1 - M(\theta^L) - \theta^L\right) \leq 1. \tag{A21}$$

But, because the inside of the parenthesis is less than 1, this is always satisfied for any $\alpha \in [0, 1]$.

Next, consider the whole condition (A17). Let $\zeta \equiv \alpha(1 - M(\theta^L) - \theta^L)$. Then, the condition (A17) is satisfied if

$$\begin{aligned} k_0 + l_j - \sigma l_j &\geq \zeta l_j \\ k_0 &\geq (\sigma - (1-\zeta))l_j \\ \frac{k_0}{l_j} &\geq \sigma - (1-\zeta) \equiv \tilde{\sigma} \end{aligned} \tag{A22}$$

or

$$\frac{l_j}{k_0} \leq \frac{1}{\tilde{\sigma}}. \tag{A23}$$

This l_j/k_0 becomes largest under the largest capital exchange (l^{FB}, s^{FB}), which occurs in the first best allocation. In this case, the marginal product of capital equates in each state, i.e.,

$$\alpha\eta_j e^U (k_0 + l_j^{FB})^{\alpha-1} = \alpha\eta_j e^D (k_0 - s_i^{FB})^{\alpha-1}. \tag{A24}$$

This can be simplified to

$$\frac{k_0 + l_j^{FB}}{k_0 - s_i^{FB}} = \left(\frac{e^U}{e^D}\right)^{\frac{1}{1-\alpha}} = Z, \tag{A25}$$

where Z is appeared in Assumption 5. Using $l_j^{FB} = s_i^{FB} + 2\mu k_0^B/(1-\mu)$, this can be expressed as

$$(Z+1)l_j^{FB} = (Z-1)k_0 + \frac{2\mu}{1-\mu} Z k_0^B, \tag{A26}$$

and then, because $k_0^B = k_0$ by assumption,

$$\frac{k_0}{l_j^{FB}} = \frac{Z+1}{\frac{1+\mu}{1-\mu}Z - 1}. \tag{A27}$$

By construction $l_j \leq l_j^{FB}$, and thus, to prove (A22), it is sufficient to show $k_0/l_j^{FB} \geq \tilde{\sigma}$, that is,

$$\frac{Z+1}{\frac{1+\mu}{1-\mu}Z - 1} \geq \tilde{\sigma} = \sigma - (1-\zeta). \tag{A28}$$

This is simplified to

$$\sigma + \zeta \leq \frac{2Z}{(Z+1)\mu + Z - 1}. \tag{A29}$$

Because $\zeta < \alpha$, it suffices to show

$$\sigma + \alpha \leq \frac{2Z}{(Z+1)\mu + Z - 1} = \frac{2}{\left(1 + \frac{1}{Z}\right)\mu + 1 - \frac{1}{Z}}. \tag{A30}$$

This is true under Assumption 5. In other words, the slope of the iso-loan default curve is always steeper than that of the iso-loan demand curve under Assumption 5.

Q.E.D.

D. Proof for Lemma 6

Proof. The partial derivative of banker's expected utility with respect to the promised loan rate consists of the direct effect and the indirect effect through its influence on default threshold.

Consider the indirect effect of higher loan rate through possibly higher default threshold. The partial derivative of banker's expected utility with respect to the default threshold is

$$\begin{aligned}\frac{\partial V^B}{\partial \theta^L} =& -\left\{(1+\rho^L)l_h - (1+\rho^D)S_h\right\} u'g\left(\frac{\theta^L}{\underline{\epsilon}}\right)\frac{1}{\underline{\epsilon}} \\ &+ \left\{B\left(\frac{\theta^L}{\underline{\epsilon}}|R^L, l_h, L_j\right) - (1+\rho^D)S_h\right\} u'g\left(\frac{\theta^L}{\underline{\epsilon}}\right)\frac{1}{\underline{\epsilon}} \\ &+ \int_{\theta^D}^{\frac{\theta^L}{\underline{\epsilon}}} \left\{-(1+\rho^L)l_h u'h\left(\frac{\theta^L}{A}\right)\frac{1}{A} + (\theta^L e^U(k_0+L_j)^\alpha - \lambda k_0 - \tau)\frac{l_h}{L_j}u'h\left(\frac{\theta^L}{A}\right)\frac{1}{A}\right\} dG(A).\end{aligned} \tag{A31}$$

Note that each u' is evaluated at the specific aggregate shock level appearing in immediately following probability density g or h.

The first line is the loss of promised loan repayments due to a higher default threshold. The second line is the extra seizure of outputs upon default, which is however $B\left(\frac{\theta^L}{\underline{\epsilon}}|R^L, l_h, L_j\right) = (1+\rho^L)l_h$ (see (21)). This means that the loss in the first line and the gain in the second line sum up to zero. Similarly, in the third line, the first term is the loss of promised loan repayments and the second term is the extra seizure.

The default condition (20) implies that the loss in the first term and the gain in the second term sum up to zero. Hence, to be consistent with the ex post optimal behavior (i.e., sequentially rational), it must be the case that

$$\frac{\partial V^B}{\partial \theta^L} = 0. \tag{A32}$$

That is, on θ^L-ρ^L plane, V^B is flat on the iso-loan default curve.

Since the indirect effect through θ^D is zero, now I look at only the direct effect.

$$\begin{aligned}\frac{\partial V^B}{\partial \rho^L} &= \int_{\frac{\theta^L}{\epsilon}}^{\overline{A}} u' l_h dG(A) + \int_{\theta^D}^{\frac{\theta^L}{\epsilon}} u' \left(1 - H\left(\frac{\theta^L}{A}\right)\right) l_h dG(A) \\ &= l_h \left\{ u'|_{\overline{A}} G(\overline{A}) - \left(1 - H\left(\frac{\theta^L}{\theta^D}\right)\right) G(\theta^D) u'|_{\theta^D} \right\},\end{aligned} \tag{A33}$$

where $u'|_A$ denote the marginal utility value evaluated at a specific aggregate shock A.

I show below that (A33) is positive. Because $G(\overline{A}) = 1$, it is equivalent to show

$$u'|_{\overline{A}} - \left(1 - H\left(\frac{\theta^L}{\theta^D}\right)\right) G(\theta^D) u'|_{\theta^D} > 0. \tag{A34}$$

With the assumed CRRA utility, this condition becomes,

$$\frac{c^B|_{\theta^D}}{c^B|_{\overline{A}}} \geq \left\{ G(\theta^D) \left(1 - H\left(\frac{\theta^L}{\theta^D}\right)\right) \right\}^{\frac{1}{\sigma}}. \tag{A35}$$

And,

$$\begin{aligned}\frac{c^B|_{\theta^D}}{c^B|_{\overline{A}}} =& \frac{\lambda k_0^B}{B(\overline{A}) - (1+\rho^D) S_h} \\ =& \frac{\lambda k_0^B}{(1+\rho^L) l_h - (1+\rho^D) S_h} = \frac{\lambda k_0^B}{(\rho^L - \rho^D) S_h + (1+\rho^L) k_0^B}.\end{aligned} \tag{A36}$$

Hence, with Assumption 6, condition (A35) is satisfied, and thus condition (A33) is positive.

Q.E.D.

E. Proof for Proposition 3

Proof. The first order condition (39) essentially is the optimal portfolio problem of allocating capital so as to equate the internal marginal product from own business (*MPK*) to the outside opportunity, which is the deposit to banks, weighted by the marginal utility, *u'*, that is,

$$0 = -\int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} (MPK) u' dH(\epsilon) dG(A) + \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(S_h, A) u' dH(\epsilon) dG(A) \tag{A37}$$

To secure the uniqueness, I show below that the second order condition with respect to deposit s_i is negative.

$$\begin{aligned}\frac{\partial \Phi}{\partial s_i} =& -\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \frac{\partial MPK}{\partial s_i} u' dH(\epsilon)dG(A) - \int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} MPK \frac{\partial u'}{\partial s_i} dH(\epsilon)dG(A) \\ &+ \int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(S_h, A)\frac{\partial u'}{\partial s_i} dH(\epsilon)dG(A).\end{aligned} \tag{A38}$$

Note that

$$\frac{\partial MPK}{\partial s_i} = -(\alpha-1)\alpha\epsilon_i A e^D (k_0 - s_i)^{\alpha-2} = \frac{1-\alpha}{k_0 - s_i} MPK, \tag{A39}$$

and

$$\frac{\partial u'}{\partial s_i} = \frac{\partial c^D}{\partial s_i} u'' = \left(-MPK + R^D(S_h, A)\right) u''. \tag{A40}$$

Then, the second order condition (A38) becomes

$$\begin{aligned} &-\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \frac{1-\alpha}{k_0 - s_i}(MPK)u' dH(\epsilon)dG(A) - \int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} MPK(-MPK + R^D)u'' dH(\epsilon)dG(A) \\ &+ \int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} R^D(-MPK + R^D)u'' dH(\epsilon)dG(A) \\ =& -\int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \frac{1-\alpha}{k_0 - s_i}(MPK)u' dH(\epsilon)dG(A) + \int_{\underline{A}}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} (R^D - MPK)^2 u'' dH(\epsilon)dG(A) \\ <& 0. \end{aligned} \tag{A41}$$

Q.E.D.

F. Proof for Lemma 9

Proof. For deposits $s_i^* \in (0, k_0)$, noting that $\partial c^D / \partial \rho^D = s_i$ for $A \in [\theta^D, \overline{A}]$, the denominator of (41) becomes

$$\frac{\partial \Phi}{\partial \rho^D} = \int_{\theta^D}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} u' dH(\epsilon)dG(A) + \int_{\theta^D}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - MPK\right\} s_i u'' dH(\epsilon)dG(A) \tag{A42}$$

where MPK denotes the marginal product of capital from depositor's own business. I will show this is positive.

Using the definition of relative risk aversion, $u'' = -\sigma u'/c$, the second term becomes

$$-\sigma s_i \int_{\theta^D}^{\overline{A}}\int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - MPK\right\} u' \frac{1}{c_i} dH(\epsilon)dG(A). \tag{A43}$$

I will show this is positive, or equivalently,

$$0 > \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - MPK\right\} u' \frac{1}{c_i} dH(\epsilon) dG(A). \tag{A44}$$

Note that $u'/c = c^{-(\sigma+1)}$.

First, I would like to show the following without the term $1/c$ is negative,

$$0 > \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - MPK\right\} u' dH(\epsilon) dG(A). \tag{A45}$$

Because of the first order condition (39), it is equivalent to show the integral for the tail-risk region is positive,

$$0 < \int_{\underline{A}}^{\theta^D} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{R^D - MPK\right\} u' dH(\epsilon) dG(A). \tag{A46}$$

Note that, in the tail-risk region, $A \in [\underline{A}, \theta^D)$,

$$R^D s_i = y^u - 2\lambda k_0 = MPK^U \frac{k_0 + l_j}{\alpha} - 2\lambda k_0, \tag{A47}$$

where MPK^U denotes the marginal product of capital for a borrower. I claim that $R^D > MPK^U$, or equivalently, (A47) $> MPK^U s_i$, i.e.,

$$MPK^U \left(\frac{k_0 + l_j}{\alpha} - s_i \right) > 2\lambda k_0. \tag{A48}$$

Because $l_j \geq s_i$ in a symmetric equilibrium, it suffices to show

$$2\lambda k_0 < MPK^U \frac{k_0 + (1-\alpha) l_j}{\alpha} = \epsilon A e^U (k_0 + l_j)^{\alpha - 1} (k_0 + (1-\alpha) l_j). \tag{A49}$$

But, Assumption 4 (d-2') says that

$$2\lambda k_0 < \underline{\epsilon A} e^U k_0^\alpha \leq \underline{\epsilon A} e^U (k_0 + l_j)^{\alpha-1} (k_0 + (1-\alpha) l_j), \tag{A50}$$

which is the right-hand-side of (A49) at the worst case $\underline{\epsilon A}$, and thus (A49) is true.

Then, for (A46) to be satisfied, it suffices to show

$$0 \leq \int_{\underline{A}}^{\theta^D} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{MPK^U - MPK\right\} u' dH(\epsilon) dG(A). \tag{A51}$$

This is the case indeed. With the hypothetical Arrow-Debreu market, capital exchange would make $E[MPK^U] = E[MPK]$. But, here, the return from the debt contract in an equilibrium is a flat pay, without any up-side like an Arrow-Debreu security, for the non-default region of shocks, while it is a state-contingent pay like an Arrow-Debreu security in the default region. Apparently, such an debt contract attracts investment less than the Arrow-Debreu security. Hence, the equilibrium deposit is less than (or at most equal to) the first best amount, making more capital to be remained at a depositor's business and lower its marginal product of capital, $E[MPK^U] \geq E[MPK]$. Because the productivity shocks ϵA enter linearly in the production function, for any same level of the productivity shocks, $MPK^U \geq MPK$.

Therefore, (A45) is established.

Now suppose that the risk aversion is higher $\hat{\sigma} = \sigma + 1$. Under this higher risk aversion, I denote utility, consumption, deposits, and MPK with hat-bearing variables, i.e., $\hat{u}$, $\hat{c}$, $\hat{s}$, and $\widehat{MPK}$. Note that $\hat{u}' = \hat{c}^{-\sigma-1}$. Obviously, the FOC (39) and its non-default portion (A45) are satisfied under higher risk aversion $\hat{\sigma}$ with a different level of deposit $\hat{s}$ and the marginal product of capital of own business $\widehat{MPK}$, which are determined by a depositor facing the same deposit contract offer R^D. That is,

$$0 > \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{ (1 + \rho^D) - \widehat{MPK} \right\} \hat{u}' dH(\epsilon) dG(A). \tag{A52}$$

Here, I claim $\hat{s} < s$, the deposit under the higher risk aversion is lower than the original amount. To see this, take the derivative of the FOC (39) with respect to the risk aversion σ,

$$\frac{\partial \Phi(\theta^D, \rho^D)}{\partial \sigma} = \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} (R^D - MPK) u' \log c_i^D dH(\epsilon) dG(A). \tag{A53}$$

Compared with the FOC (39), the above expression has an additional term $\log c$. While $(R^D - MPK)u'$ is strictly decreasing in productivity shock realizations, the additional "weight" $\log c$ puts a large weight for a low realization of shocks and a small weight for a high realization of shocks. Therefore, this expression (A53) should have a lower value than the FOC (39), i.e.,

$$(A53) < \int_{\underline{A}}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} (R^D - MPK) u' dH(\epsilon) dG(A) = 0. \tag{A54}$$

This means that Φ, the right-hand-side of the FOC (39), with the same level of deposit but with higher risk aversion would be lower than zero.[41] However, under the new $\hat{\sigma}$, the new Φ must be zero, i.e., the new FOC must be also satisfied. To restore zero for the new Φ, with facing the same R^D (i.e., the same ρ^D and θ^D), it must be the case that MPK needs to be lower, i.e., $\widehat{MPK} < MPK$.

[41] The deposit dictates consumption level, given deposit contract offer R^D and the technology of own business.

This implies that more capital is retained to be invested in the own business and hence the deposit is lower, $\hat{s} < s$.

Now consider the consumption under the higher σ for the non-default region.

$$\hat{c}_i^D = (1+\rho^D)\hat{s}_i + \epsilon_i A e^D (k_0 - \hat{s}_i)^\alpha. \tag{A55}$$

In the ϵA-c graph, this has an intersection $(1+\rho^D)\hat{s}$, which is lower than the original $(1+\rho^D)s$, and has a slope of $e^D(k_0-\hat{s})^\alpha$, which is higher than the original $e^D(k_0-\hat{s})^\alpha$. In sum, $\hat{c} < c$ for a low realization of shocks and $\hat{c} > c$ for a high realization of shocks. This is due to more allocation of capital to own business that is riskier than flat-pay deposits.

Below I substitute $\hat{c}$ by c in (A52) in the second line,

$$\begin{aligned} 0 &> \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - \widehat{MPK}\right\} \hat{c}^{-\hat{\sigma}} dH(\epsilon) dG(A) \\ &> \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - \widehat{MPK}\right\} c^{-\hat{\sigma}} dH(\epsilon) dG(A). \end{aligned} \tag{A56}$$

This inequality is due to the fact that $1/c$ puts smaller "weight" than $1/\hat{c}$ for lower realizations of return difference $(1+\rho^D - \widehat{MPK})$, which is strictly decreasing in shock realizations.

Now, substitute $\widehat{MPK}$ by MPK. Because $MPK > \widehat{MPK}$ for any shock realizations,

$$\begin{aligned} 0 &> \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - \widehat{MPK}\right\} c^{-\hat{\sigma}} dH(\epsilon) dG(A) \\ &> \int_{\theta^D}^{\overline{A}} \int_{\underline{\epsilon}}^{\overline{\epsilon}} \left\{(1+\rho^D) - MPK\right\} c^{-\hat{\sigma}} dH(\epsilon) dG(A). \end{aligned} \tag{A57}$$

That is, (A44) is just shown because $\hat{\sigma} = \sigma + 1$.

Therefore,

$$\frac{\partial \Phi}{\partial \rho^D} > 0. \tag{A58}$$

Next, look at the numerator of (41), which can be expressed as

$$\frac{\partial \Phi(\theta^D, \rho^D)}{\partial \theta^D} = \left\{\left(B(\theta^D | R^L, L_i, L_j) - \lambda k_0^B\right) - (1+\rho^D)s_i^*\right\} g(\theta^D) \int_{\underline{\epsilon}}^{\overline{\epsilon}} u'(c^D(s_i^*, \theta^D, \epsilon_i)) dH(\epsilon), \tag{A59}$$

where $g(A)$ is pdf for cdf $G(A)$.

Note that the term inside the brace of (A59) is zero if θ^D is also satisfying the credible deposit contract offer (36), that is, $s_i^* = \sum_h \tilde{s}_{hi}$. For this specific case, let $(\tilde{\rho}^D, \tilde{\theta}^D)$ to denote the pair of deposit rate and default threshold. For a larger $\theta^D > \tilde{\theta}^D$ given $\tilde{\rho}^D$ under the same deposits, the brace term is positive, i.e., $\partial\Phi/\partial\theta^D > 0$, and vice versa.[42]

Therefore, together with (A58), in the right of the credible deposit contract offer, $\theta^D > \tilde{\theta}^D$, the iso-deposit supply curve is declining, $\partial\rho^D < \partial\theta^D$, and vice versa.

Q.E.D.

G. Proof for Corollary 3

Proof. Regarding the relation between the deposits and the deposit rate, the implicit function theorem, together with (40) and (A44), implies

$$\frac{\partial s_i}{\partial \rho^D} = -\frac{\partial\Phi/\partial\rho^D}{\partial\Phi/\partial s_i} > 0. \tag{A60}$$

As for the relation between the deposits and the deposit default threshold, the implicit function theorem with (40) and (A59) shows that

$$\frac{\partial s_i}{\partial \theta^D}\Big|_{\theta^D=\tilde{\theta}^D} = -\frac{\partial\Phi/\partial\theta^D}{\partial\Phi/\partial s_i} = 0. \tag{A61}$$

as in the proof of Lemma 9 at $\theta^D = \tilde{\theta}^D$.

On the right of the credible deposit contract offer curve, the numerator is positive (i.e., $\partial s/\partial\theta^D > 0$) and thus the larger deposits is associated with higher default threshold. This means less decline in the right of the credible deposit contract offer curve. The opposite is true for the left side. Hence, overall, the iso-deposit supply curve on θ^D-ρ^D plane becomes flatter with larger s_i though the slope remains at zero in the neighborhood of the credit deposit contract offer curve. *Q.E.D.*

H. Proof for Proposition 4

Proof. A banker offers one contract from a set of credible deposit contracts. Lemma 8 states that a higher deposit rate ρ^D is associated with a higher default threshold θ^D given banker's deposit intake s_h. Corollary 2 states that a larger deposit intake shifts the credible deposit contract offer to down right, i.e., lower deposit rate and higher default threshold.

[42] Recall that $\partial B > \partial A$ as shown in (22).

On the deposit supply side, on θ^D-ρ^D plane, given deposits s_i, the equilibrium deposit repayments cannot be strictly lower than the graph of the iso-deposit supply curve. This is obvious. However, the equilibrium deposit repayment offer by bankers cannot also be strictly higher than the iso-deposit supply curve. I prove this by contradiction as below.

Suppose that the equilibrium deposit repayments were to be strictly higher than the depositor's willingness to accept, which is depicted as the iso-deposit supply curve. Then, given such an offer, a depositor wants to deposit more than the current s_i. This means that the deposits per depositor s_i is rationed in equilibrium. Note that this equilibrium set of s_i and R^D should lie on the credible deposit contract offer curve, but now it is supposed to lie above the iso-deposit supply curve. Then, a banker could deviate to offer a deposit contract with a bit lower rate ρ^D and lower default threshold θ^D on the same credible deposit contract offer curve, but a bit down-left towards the iso-deposit supply curve. Because this deviant's offer would still lie above the iso-deposit supply curve and other bankers would keep rationing their deposit intakes, the deviant banker could keep the same deposit amounts. This strategy would be obviously profitable for a deviant banker. Hence, the equilibrium deposit rate cannot be strictly higher than the iso-loan supply curve, i.e., the depositor's willingness to accept.

Therefore, only possible equilibrium deposit repayments must be equal to the depositor's willingness to accept, that is, on the iso-deposit supply curve, and deposits are not rationed. Still, a question remains whether there is any profitable deviation by a banker to upset such possible equilibrium contracts.

Now consider a deviation by a banker from the equilibrium deposit repayments, which is equal to the depositor's willingness to accept. One possible deviation to attract more deposits $\triangle s_h$ could be to offer a deposit repayment schedule that is more beneficial for depositors, $R^D + \triangle R^D$, which is a higher deposit rate but a higher default threshold, because the offer still needs to be credible (i.e., on a credible deposit contract offer curve). Note that Corollary 2 states that, with larger deposits, the credible contract offer curve shifts down right.

Here, there is an obvious loss from the thinner spread from existing deposits, $-\triangle R^D s_h$ but there could be a gain from more deposits by stealing customers from other bankers. That is, the deviant banker could have more deposits, so that the deviant strategy is a set of $(R^D + \triangle R^D, s_h + \triangle s_h)$. Indeed, the deviant banker can attract $\triangle s_h > 0$ as long as it is tiny relative to the whole banking sector. But, the deviant banker has to ration the deposit intake so that the pair of deposit rate and default threshold is credible, on the creible deposit contract offer curve with larger deposits. The new deposit level $s_h + \triangle s_h$ is at most the same as the depositor's willingness to accept under a new term $R^D + \triangle R^D$. Then, in essence, the deviant banker could not steal deposits from rival bankers but would face the same per banker deposit supply function (i.e., the representative deposit supply function).

For this deviation to be not profitable, the loss should be larger than or equal to the gain, $-\triangle\rho^D s_h \geq \rho^D \triangle s_h$, on the credible deposit contract curve (36). I prove this below.

By switching s_h in the right-hand-side and $(1+\rho^D)$ in the right-hand-side of (36), s_h can be viewed as a function of ρ^D. Indeed, by denoting the numerator of the right-hand-side of (36) as X, the credible deposit contract offer (36) can be written as

$$s_h = \frac{X}{1+\rho^D}. \tag{A62}$$

Take the derivative on this,

$$\frac{\partial s_h}{\partial \rho^D} = -\frac{X}{(1+\rho^D)^2}. \tag{A63}$$

Then,

$$-\frac{\partial s_h}{\partial \rho^D}\frac{\rho^D}{s_h} = \frac{X}{(1+\rho^D)^2}\frac{(1+\rho^D)\rho^D}{X} = \frac{\rho^D}{(1+\rho^D)} < 1. \tag{A64}$$

Therefore,

$$-\triangle\rho^D s_h \geq \rho^D \triangle s_h. \tag{A65}$$

Another possible deviation could be for a banker to offer a deposit repayment schedule that is a bit less beneficial for depositors (i.e., lower deposit rate and associated default threshold on the credible contract offer curve). There could be a gain for the deviant banker from a higher spread income per deposits. However, unlike the previous case, the deviant banker loses almost all the deposits by offering a repayment schedule inferior to all other bankers' offers. This is because the deviant banker's measure is tiny compared with other bankers and thus other bankers can absorb deviant banker's deposits without much increase in their own deposit intake under a symmetric equilibrium. *Q.E.D.*

I. Proof for Proposition 5

Proof. Given total deposits, on the θ^D-ρ^D plane, the iso-deposit supply curve has a zero slope only in the neighborhood of the credible deposit contract offer curve with inversed U shape (Lemma 9). The credible deposit contract offer curve is strictly increasing in θ^D (Lemma 8) and thus crosses the inversed U iso-deposit supply curve at the peak ρ^D, where the slope is zero (see Figure 6).

Given a crossing point at the peak of the iso-deposit supply curve, two curves never crosses again. To see this, suppose on the contrary that the iso-deposit supply curve once again crosses the deposit contract offer curve in the lower left region of the original crossing point, because the credible deposit contract offer curve is strictly increasing in that region. This contradicts to the characteristics of the iso-deposit supply curve in Lemma 9. Recall that the iso-deposit supply curve is inversed U shape, having a zero slope only at the peak crossing point and has a positive slope in the lower left region of that crossing point.

Corollary 3 implies that the iso-deposit supply curve shifts up with larger deposits per depositor $s'_i > s_i$. Apparently, for any deposit level, the crossing point, i.e., the equilibrium, must be on this path. On the other hand, Corollary 2 states that the credible deposit contract offer curve shifts down right with larger deposits per banker $s'_h > s_h$. With the equilibrium condition $s'_h = \sum_i s'_{hi}$ and the restriction on the crossing point, the fact that one curve goes up and the other goes down with larger deposits implies that these two curves meet only under one pair of deposit amounts (s_i, s_h) given the total deposits $S = \sum_{h,i} s_{h,i}$. And, obviously, only one deposit repayment schedule (ρ^D, θ^D) lies at the crossing point of two curves.[43]

When total deposits S_i increases, the credible deposit contract curve shifts down right. However, the iso-deposit supply curve shifts up and so does its peak, at which two curves crosses. To be consistent with both curves' movements, the crossing point must be shifts up right. In other words, larger equilibrium deposits S_i is associated with higher equilibrium deposit rate ρ^D and higher deposit default threshold θ^D. The relation of the deposit market can thus been represented by $S_i = f(\rho^D|\mu)$, which is increasing in ρ^D given μ, associated with equilibrium default threshold θ^D. *Q.E.D.*

J. Proof for Lemma 11

Proof.

$$\begin{aligned}\frac{\partial(\pi S_h)}{\partial \mu} &= \frac{\partial \pi}{\partial \mu} S_h + \pi\left(\frac{1-\mu}{2\mu}\frac{\partial S_i}{\partial \mu} + \frac{\frac{1-\mu}{2\mu}}{\partial \mu} S_i\right) \\ &= \frac{1-\mu}{2\mu} S_i \frac{\partial \pi}{\partial \mu} - \pi S_i \frac{1}{2\mu^2}.\end{aligned} \tag{A66}$$

Banker population μ does not matter for deposits per depositor S_i, $\partial S_i/\partial \mu = 0$, in the second line

[43] Here, implicitly I focus on symmetric equilibrium. However, given the Cobb-Douglas production function, depositor's s_i, which adjusts marginal product of capital of own production, becomes symmetric in equilibrium even it is not assumed. Also, banker's s_h becomes symmetric in equilibrium since ex ante utility level is arbitraged (see sections below).

above.[44] Here, (A66) is less than zero if

$$\begin{aligned} \frac{1-\mu}{2\mu}\frac{\partial\pi}{\partial\mu} &< \pi\frac{1}{2\mu^2} \\ \frac{\mu\pi'}{\pi} &< \frac{1}{1-\mu} \\ \frac{\log\pi}{\log\mu} &< \frac{1}{1-\mu} \end{aligned} \tag{A67}$$

Banker's spread π increases with banker capital per depositor, which becomes larger with banker population μ, i.e., $\partial\pi/\partial\mu > 0$. Diminishing marginal utility for insurance implies concavity, $\partial\pi^2/\partial^2\mu < 0$. Because $\pi'(\mu) > 0$ and $\pi''(\mu) < 0$, $\pi'(\mu)$ is positive and largest near $\mu = 0$ but still $\pi'(\mu) < \infty$ as insurance premium. On the other hand, $1/(1-\mu)$ is smallest near $\mu = 0$. Hence, it is sufficient to show the elasticity of the spread with respect to banker population is less than one near $\mu = 0$, i.e.,

$$\lim_{\mu\to 0}\frac{\log\pi}{\log\mu} < 1. \tag{A68}$$

But, an increase in banker population by one percent near $\mu = 0$ raises banker capital per depositor by 2 percent because

$$\begin{aligned} \lim_{\mu\to 0}\frac{\partial\log\frac{2\mu k_0^B}{1-\mu}}{\partial\log\mu} &= 2 - \lim_{\mu\to 0}\frac{\partial\log(1-\mu)}{\partial\log\mu} \\ &= 2 - \lim_{\mu\to 0}\frac{\partial\log(1-\mu)}{\partial\mu}\frac{\partial\mu}{\partial\log\mu} \\ &= 2 + \lim_{\mu\to 0}\frac{\mu}{1-\mu} \\ &= 2. \end{aligned} \tag{A69}$$

But, banker capital is given to depositors only when a banker defaults. As long as the default probability of corporate bonds are less than 50 percent under Assumption 7, the expected income gain for depositor is at most one percent. Overall, expected income gain by one percent more bank capital buffer is at most one percent. Then, the depositor would pay at most one percent additional premium. Therefore, the above condition (A68) is satisfied. *Q.E.D.*

[44]Recall Lemma 10, that is, depositors arbitrage with potential measure zero banker's intermediation. Hence, demand for deposits is determined in equilibrium by corporate loan returns, which is not affected by actual bank size μ.