Tax Havens and Cross-border Licensing

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Abstract  
Multinational enterprises (MNEs) are incentivized to use transfer pricing in tax planning when corporate tax rates differ in different countries. The incentive is stronger when MNEs own intangible assets which can easily be shifted across countries. To mitigate such strategic avoidance of tax payments, the OECD proposed the arm's length principle (ALP). This paper investigates how the ALP affects MNEs' licensing strategies and welfare in the presence of a tax haven. We specifically deal with two methods of the ALP: the comparable uncontrolled price method and the transactional net margin method. The ALP may distort MNEs' licensing decisions, because providing a license to unrelated firms restricts the MNE's profit-shifting opportunities due to the emergence of comparable transactions. In particular, the avoidance of licensing in the presence of the ALP may worsen domestic welfare if the licensee and the MNE's subsidiary do not compete in the domestic market, but may improve welfare if they compete.

Keywords: Multinational enterprises; Licensing; Royalties; Transfer pricing; Arm's length principle  
JEL classification: D45; F23; H26; L12

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1 Introduction

Multinational enterprises (MNEs) have a strong presence in the world economy. According to Zucman (2014), the share of global corporate profits made by MNEs was about 15%. It has also been reported that MNEs often artificially shift their profits across countries to avoid taxation. MNEs take advantage of differences in corporate tax rates and preferential tax measures provided by various countries. For example, Zucman (2014) pointed out that more than 50% of the profits of U.S. firms were reported in low-tax countries in 2012.\footnote{These countries include Ireland, the Netherlands, Luxembourg, Switzerland, Bermuda and other Caribbean countries, and Singapore. This share was less than 20% in 1984 and grew over time.} According to the estimation of Torslov et al. (2018), more than 600 billion US dollars were shifted to tax havens. This kind of profit-shifting is often conducted via transfer pricing of intra-firm transactions.\footnote{In reality, profit-shifting is executed using highly complex methods. Well known methods of profit-shifting include transfer pricing of tangible assets, internal debt, and licensing payments. Hopland et al. (2018), for example, introduces multiple methods of profit-shifting in their analyses.} With respect to the prices of goods and services within a firm (i.e., transfer prices), there is no market mechanism. Thus, MNEs manipulate transfer prices for tax planning. This is called “transfer pricing.” The member countries of Organisation for Economic Co-operation and Development (OECD) have cooperatively tackled this problem by setting guidelines for transfer pricing and carrying out the Base Erosion and Profit Shifting (BEPS) project.

OECD guidelines stipulate

> When independent enterprises transact with each other, the conditions of their commercial and financial relations (e.g. the price of goods transferred or services provided and the conditions of the transfer or provision) ordinarily are determined by market forces. (Chapter I, p33)

> These market-driven conditions are codified into the “arm’s length principle (ALP).” As a method of the ALP, the comparable uncontrolled price (CUP) method is considered ideal. It suggests that tax authorities audit tax-avoidance behaviors by comparing the prices used in intra-firm transactions with those of similarly uncontrolled transactions between independent parties (i.e., arm’s length (AL) prices).\footnote{See OECD guideline Chapter 2, p.101.}

> Reality, unfortunately, is not as simple. In particular, it is very difficult to audit intra-firm transfers of intangible assets because of the ambiguous nature of intangible
First, it is easy to shift intangible assets across countries without accompanying production. Thus, MNEs tend to locate their intangible assets in tax havens to minimize payments. For instance, profits shifted to Ireland via royalties accounted for approximately 23% of its annual GDP between 2010 and 2015. Second and more importantly, finding appropriate fees or royalties is difficult, because there is often no comparable transaction for intangible assets. As pointed out by the OECD guidelines, “Tax administrations should not automatically assume that associated enterprises have sought to manipulate their profits. There may be a genuine difficulty in accurately determining a market price in the absence of market forces or when adopting a particular commercial strategy. (Chapter I, p33)”

In the case of transactions of intangible assets, therefore, it is difficult to apply the CUP method. In practice, practitioners heavily rely on a different method called the transactional net margin (TNM) method because of its ease in uses. The method examines the profit-level indicator (PLI), defined as net profits relative to an appropriate base (e.g., sales) that a taxpayer realizes from a controlled transaction. With the TNM method, the PLI of the taxpayer from the controlled transaction should be equal to the PLI obtained in a comparable transaction by an independent enterprise (i.e., a reference firm). The selection criteria of the reference firm are based upon the evaluation of the functional risks of the taxpayer and the reference firm (e.g., R&D risk and credit risk). This implies that they may not operate in the same industry. Moreover, even if a taxpayer chooses a reference firm for the TNM method, the tax authority often proposes a different reference firm.

According to the Internal Revenue Service, the most frequently used transfer pricing method for both tangible and intangible property in 2016 was the comparable profits method (CPM) or the TNM method, which accounted for 89%. Furthermore, the most frequently used PLI is operating margin (i.e., the ratio of operating profits to sales) which

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4 One of the most famous examples of profit shifting through royalties is that of the “Double Irish with a Dutch Sandwich” conducted by Google. It was reported that Google saved at least $3.7 billion in taxes in 2016 using this method. https://www.irishtimes.com/business/economy/double-irish-and-dutch-sandwich-saved-google-3-7bn-in-tax-in-2016-1.3343205

5 See Dischinger and Riedel (2011), Karkinsky and Riedel (2012), and Griffith et al. (2014), among others, for empirical evidence of location choices for intangible assets.

6 See the Financial Times. https://www.ft.com/content/d6a75b56-215b-11e8-a895-1ba1f72c2c11

7 For details, see https://www.irs.gov/irb/2017-15_IRB. The CP method is mainly used in the U.S. to calculate appropriate transfer prices. Basically, these two methods are the same, but the only difference is that the TNM method deals with investigations based on transaction units, whereas the CP method investigates firm-level transactions.
accounts for 67%.\footnote{There are several other measures of the PLI, such as belly ratio and return on assets or capital employed, which accounted for 33%. On service transactions, the CP method or the TNM method were used in 76% of the cases. The most commonly used PLI was the operating margin (43%).}

Against this background, we explore the relationship between the ALP and MNE technology transfers through licensing. Specifically, we investigate how the ALP affects MNE’s licensing strategy and welfare in the presence of a tax haven. On one hand, if the MNE decides to license its technology to unrelated firms, a comparable transaction appears, and the CUP method becomes applicable. Thus, the MNE needs to set the same royalty for both related and unrelated parties. On the other hand, if the MNE transfers its technology only internally, there is no comparable transaction, and the tax authority relies on the TNM method. When making a licensing decision, the MNE faces a trade-off between the license revenue from the unrelated parties and the greater opportunity of profit-shifting via transfer pricing. Thus, the presence of the ALP may affect the MNE’s licensing decisions and welfare.

We contribute to transfer-pricing literature by capturing this aspect of profit-shifting using intangible assets. Apart from the importance of profit-shifting, most papers have dealt with intra-firm transactions through physical products, internal debt, and interest payments. Profit-shifting via intangible assets has been analyzed in only a few studies, including those of Hopland et al. (2018) and Juranek et al. (2018a,b). They incorporated royalty payments in their analysis but they did not consider licensing to external firms. Moreover, despite the fact that licensing improves production costs, the interaction between licensing and the market has been largely overlooked, because extant literature has often only considered perfectly competitive markets.

Many studies about licensing have assumed licensing either by means of a per-unit royalty or a fixed fee alone. However, as documented by San Martin and Saracho (2010), most license contracts have adopted the ad-valorem scheme for royalty payments instead of a per-unit royalty or a fixed fee. In our analysis, therefore, we focus on ad valorem royalties as licensing payments.

We show that the ALP increases tax revenue while potentially harming consumers. As a result, the ALP can worsen economic welfare. In particular, the avoidance of licensing in the presence of the ALP is harmful to consumers if the licensee and the MNE’s subsidiary do not compete in the domestic market. However, it is beneficial to consumers if they do compete. When the licensee is a rival for the MNE’s subsidiary in the market, the MNE strategically decreases subsidiary output to save on tax payments.
As a result, consumers lose.

The rest of the paper is organized as follows. Section 2 presents a basic setup and analyzes how the ALP (i.e., CUP and TNM methods) affects MNE incentives to license in the presence of a tax haven. In the basic model, the good produced by the licensee is not substitutable with a good produced by the MNE. Section 3 explores the effects of the ALP on domestic welfare with a tax haven. Section 4 extends the basic model. We consider substitutability of goods in the extended model. The last section concludes this paper.

2 Basic Model

Consider the world composed of a domestic country, a foreign country, and another foreign country, labeled $D$, $F$, and $H$, respectively. Country $H$ is a tax haven. Its corporate tax rate is lower than that of country $D$ and is normalized to zero. We assume for simplicity that there is no source tax on royalty payments. There is a single MNE, the headquarters of which is located in country $F$. There is a single local firm (called firm $Y$) in country $D$. Firm $X$, a subsidiary of the MNE located in country $D$ and firm $Y$, respectively, produce goods $X$ and $Y$. The two goods are independent and not substitutable. Each firm is a monopolist in country $D$. Because we are primarily interested in MNE profit-shifting from the domestic country to the tax haven and the domestic welfare consequences of introducing the ALP, we assume that both goods are consumed only in country $D$.

The original marginal cost (MC) of producing good $i$ ($i = X, Y$) is $c_i$. However, the MNE owns a patent which can reduce MCs. Although the two goods are not substitutes, the patented technology is assumed to be applied to the production of both goods. With the patent, each firm can reduce its MC from $c_i$ to zero. Thus, firm $X$’s MC is always zero, whereas firm $Y$’s MC is zero only when the patent is granted to the local firm. We assume that the licensing contract is by means of ad valorem royalties. The MNE offers ad valorem royalties $r_x$ for internal licensing (i.e., licensing to firm $X$) and $r_y$ for external licensing (i.e., licensing to firm $Y$), respectively.

Let $\pi_i(\omega)$ denote the monopoly profits when MC is given as $\omega$ ($i = X, Y$). Since

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9 For example, royalty payments within the EU are exempted from the source tax.
10 In section 4, we consider the case in which the goods are substitutes.
11 Basically, the effects of licensing by means of fixed fees are equivalent to those by means of ad valorem royalties.
the internal licensing always occurs, firm X’s profits are always \( \pi_x(0)(\equiv \pi_{x0}) \). Firm Y’s profits depend on whether licensing takes place or not. The profits are \( \pi_y(c_y)(\equiv \pi_{yc}) \) without licensing and \( \pi_y(0)(\equiv \pi_{y0}) \) with licensing, respectively.

We solve the following three-stage game. In the first stage, the MNE determines its internal and external royalty rates. The MNE specifically makes a take-it-or-leave-it offer to firm Y. After observing the royalty rates, firm Y decides whether to accept the license contract. Finally, firms X and Y produce and supply their products in country \( D \).

2.1 The benchmark case: without a tax haven

To clarify the effects of a tax haven and the ALP, this subsection analyzes the case without the tax haven. We assume that the domestic corporate tax rate, \( t \), and the foreign corporate tax rate are the same.\(^{12}\)

The MNE has a patent which reduces MC from \( c_i \) to 0. Therefore, the subsidiary’s MC is always 0, whereas the local firm’s MC is either \( c_y \) or 0. If the MNE grants a license to the local firm, the local firm pays a license fee to the MNE. The royalty rate of the license is \( r_y \in [0, 1] \). Formally, the profits can be written as

\[
\Pi_M = (1-t)(\pi_{x0} + \lambda r_y \pi_{y0}), \quad (1)
\]

\[
\Pi_y = (1-t)\{\lambda(1-r_y)\pi_{y0} + (1-\lambda)\pi_{yc}\}, \quad (2)
\]

where \( \lambda \) is a binary variable which takes one if the external licensing arises and zero otherwise. It should be noted that a change in \( t \) does not affect output levels.

Given Eq.(2), the local firm accepts the licensing offer if and only if

\[
\Pi_y]\big|_{\lambda=1} \geq \Pi_y]\big|_{\lambda=0} \iff r_y \leq 1 - \frac{\pi_{yc}}{\pi_{y0}} = \frac{\Omega}{\pi_{y0}}, \quad (3)
\]

where \( \Omega \equiv \pi_{y0} - \pi_{yc} > 0 \). Since the two firms do not interact in the markets, the MNE is always willing to license its technology to the local firm. From Eq.(1), it is always optimal for the MNE to obtain license revenue by setting the highest royalty subject to Eq.(3), \( r_y = \frac{\Omega}{\pi_{y0}} \equiv r_y^*(< 1) \). In other words, the MNE will set the royalty rate such that license revenue equals \( \Omega \).

\(^{12}\)Even if the foreign tax rate is higher than the domestic one, the analysis in this subsection would not change with the assumption that the MNE establishes a shell company in the domestic country and transfers its patent to the shell company.
In equilibrium, the profits become
\[ \Pi^*_M = (1-t)(\pi_x \Omega) \quad \text{and} \quad \Pi^*_y = (1-t)(1-r^*_y)\pi_y \Omega. \quad (4) \]

As seen in the above equation, the optimal license contract makes the local firm indifferent between with and without licensing.

2.2 A tax haven without the ALP

We now introduce a tax haven into the analysis. We assume that the MNE establishes a shell company, firm S, in country H without any cost. Obviously, transferring the patent to the shell company is the optimal strategy for the MNE, because it can make more profits in the tax haven not only by profit-shifting from firm X but also by license revenue from firm Y. The profits of the MNE and firm Y are, respectively, given by

\[ \Pi^T_{TH}^M = (1-t)(1-r^T_{TH})\pi_x \Omega + r^T_{TH} \pi_x \Omega + r^T_{TH} \pi_y \Omega, \]
\[ \Pi^T_{TH}^y = (1-t)(1-r^T_{TH})\pi_y \Omega, \]

where the first term of \( \Pi^T_{TH}^M \) is the post-tax profits of firm X and the second and third terms are the license revenues from firms X and Y recorded in country H.

We first consider the case without the ALP. In this case, the MNE can set ad valorem royalties without constraint. The optimal royalty rate is one with which all profits of firm X are shifted to firm S. Thus, \( r^T_{TH} = 1 \), whereas the arm’s length royalty rate is the same as the benchmark case (i.e., \( r^T_{TH} = r^*_y \)).

As a result, we obtain the same licensing strategy as the benchmark case in equilibrium. This is because the country where the MNE reports the tax base simply changes from country D to country H. Because the corporate tax is proportionally imposed on the profits, the tax rates never affect the licensing strategy. Therefore, the post-tax profits are computed as

\[ \Pi^T_{TH}^M = \pi_x \Omega, \quad \text{and} \quad \Pi^T_{TH}^y = (1-t)\pi_y \Omega \left( = \Pi^*_y \right). \quad (5) \]

2.3 A tax haven with the ALP

Finally, we investigate the effect of the ALP in the presence of a tax haven. The ALP restricts the MNE’s profit-shifting strategy through one of two methods, the CUP method and the TNM method.
First, if the MNE licenses the technology to the local firm, the CUP method applies. The MNE is unable to price-discriminate because of the emergence of a comparable transaction and arm’s length royalty. Put differently, the MNE must set a uniform royalty rate, \( r_{\text{CUP}} \). The MNE’s problem can be stated as follows:

\[
\begin{align*}
\max_{r_{\text{CUP}}} \Pi_{\text{CUP}}^M &= (1 - t)(1 - r_{\text{CUP}})\pi_{x0} + r_{\text{CUP}}(\pi_{x0} + \pi_{y0}) \\
&= (1 - t)(1 + \frac{t}{1 - t}r_{\text{CUP}})\pi_{x0} + r_{\text{CUP}}\pi_{y0}
\end{align*}
\]

subject to

\[
\Pi_{\text{CUP}}^y|_{\lambda=1} \geq \Pi_{\text{CUP}}^y|_{\lambda=0} \iff r \leq 1 - \frac{\pi_{yc}}{\pi_{y0}} = r^* (< 1).
\]

Because \( \Pi_{\text{CUP}}^M \) is strictly increasing in \( r_{\text{CUP}} \), the optimal royalty rate is given by \( r^*_{\text{CUP}} = r_{y}^* \). This strategy generates the following post-tax profits:

\[
\begin{align*}
\Pi^*_{\text{CUP}}^M &= (1 - t) \left[ 1 + \frac{t}{1 - t} \left( 1 - \frac{\pi_{yc}}{\pi_{y0}} \right) \right] \pi_{x0} + \Omega \\
&= (1 - t) \left[ 1 + \frac{t}{1 - t} \left( \frac{\Omega}{\pi_{y0}} \right) \right] \pi_{x0} + \Omega, \\
\Pi^*_{\text{CUP}}^y &= (1 - t)\pi_{yc} \quad (= \Pi^*_y = \Pi^*_{\text{TH}}).
\end{align*}
\]

Note that the imposition of the ALP does not lead to the elimination of profit-shifting.\(^\text{13}\) As seen in Eq.(6), the MNE shifts only a part of its profits to the tax haven. As discussed in Section 2.1, \( r^*_{\text{CUP}} \) is determined only by the market condition of good \( Y \). This means that the MNE’s global post-tax profits under the CUP method become larger with \( \Omega \).

Alternatively, if the MNE does not license its technology, no comparable transaction appears. Hence, the royalty rate is regulated by the TNM method. With the TNM method, the royalty rate \( r_{\text{TNM}}^x \) is set such that the PLI of firm \( X \) equals the PLI of the reference firm which is exogenously given in this subsection. We denote such a royalty rate by \( \eta \). That is, \( r^*_{\text{TNM}}^x = \eta \) holds. Thus, we have the following post-tax profits under the TNM method:

\[
\begin{align*}
\Pi^*_M^\text{TNM} &= (1 - t)(1 - \eta)\pi_{x0} + \eta\pi_{x0} \\
&= (1 - t)\pi_{x0} + t\eta\pi_{x0}, \quad (8) \\
\Pi^*_y^\text{TNM} &= (1 - t)\pi_{yc} \quad (= \Pi^*_y = \Pi^*_{\text{TH}} = \Pi^*_{\text{CUP}}).
\end{align*}
\]

\(^\text{13}\)Some existing literature considers cases in which the ALP completely eliminates the opportunity of profit-shifting. In our model, however, ALP makes the royalties equal between related and unrelated firms. Thus, the MNE still enjoys profit-shifting.
A comparison of the two post-tax profits reveals the condition used to determine whether to license the technology. Formally, the MNE grants the license to the local firm if and only if

\[
\Delta \Pi_M \equiv \Pi_M^{\text{CUP}} - \Pi_M^{\text{TNM}}
\]

\[
= \left(1 - t\right) \left[1 + \frac{t}{1 - t} \left(\frac{\Omega}{\pi_y}\right)\right] \pi_x 0 + \Omega - \{(1 - t)\pi_x 0 + t\eta \pi_x 0\}
\]

\[
= \Omega - t \left[\eta - \frac{\Omega}{\pi_y}\right] \pi_x 0 > 0.
\]

We can easily confirm that

\[
\frac{\partial \Delta \Pi_M}{\partial \eta} < 0, \quad \frac{\partial \Delta \Pi_M}{\partial t} < 0, \quad \frac{\partial \Delta \Pi_M}{\partial \pi_x 0} < 0, \quad \frac{\partial \Delta \Pi_M}{\partial \Omega} > 0.
\]

Thus, given the other parameters, we can define a threshold of $\eta$, $\eta^L$, such that the MNE is indifferent to licensing and non-licensing. Licensing arises if and only if $\eta \leq \eta^L (\equiv \Omega \pi_y 0 \pi_x 0$. It is obvious that $\eta^L = \frac{\Omega}{\pi_y 0} = \eta^*_y$.

Thus, we arrive at the following proposition.

**Proposition 1** The introduction of the ALP in the presence of the tax haven results in non-licensing if $\eta$, $t$ or $\pi_x 0$ is sufficiently large or if $\Omega$ is sufficiently small.

The proposition is intuitive. The MNE faces a trade-off between license revenue from the local firm and the profit-shifting from its subsidiary to the tax haven. The latter is likely to dominate the former as $\eta$, $t$ and $\pi_x 0$ become larger and $\Omega$ becomes smaller.

### 3 Welfare analysis

Following the previous literature (e.g., Kind et al., 2005), we assume that the MNE is owned by residents in the foreign country. Thus, domestic welfare comprises consumer surplus, firm Y’s profits, and domestic tax revenue. Note that firm Y’s profits are always constant and equal to $(1 - t)\pi_y$ and that consumer surplus in the market of good X also remains constant. Thus, a change in domestic welfare is simply the sum of changes in consumer surplus in the market of good Y, $CS$, and in tax revenue from the MNE, $TR$. Obviously, $CS$ is larger with licensing than without.
Domestic welfare with the tax haven (without the ALP) is always less than that of
the benchmark case, because the presence of the tax haven does not affect the licensing
strategy. Instead, it leads to leakage of tax revenue from the domestic country to the tax
haven. Thus, the presence of the tax haven is always harmful for the domestic country.

We now investigate the welfare effects of the ALP in the presence of the tax haven.
To this end, we compare domestic welfare between with and without the ALP. If the
introduction of the ALP does not affect the licensing strategy of the MNE, that is, if the
MNE is still engaged in licensing with the ALP (which is the CUP method in this case),
the impact of the ALP is straightforward. Obviously, CS is not affected. Under the CUP
method, MNE’s profit-shifting is restricted, which means that TR increases. Thus, the
ALP increases domestic welfare by $t(1 - r^{*CUP})\pi_0$, implying $W^{*CUP} > W^{*TH}$ holds.
However, if the ALP changes the licensing strategies, that is, if the MNE stops licensing
under the ALP, a trade-off arises. On one hand, the ALP decreases MNE’s profit-
shifting to the tax haven and hence TR increases. On the other hand, non-licensing
lowers productivity of the local firm and hence CS decreases. Thus, $W^{*TNM} > W^{*TH}$
may or may not hold. We then obtain the following lemma.

Lemma 1 While $W^{*CUP} > W^{*TH}$ holds, $W^{*TNM} > W^{*TH}$ may not hold.

$W^{*TNM}$ is decreasing in $\eta$ while both $W^{*CUP}$ and $W^{*TH}$ are independent of $\eta$.
Thus, $W^{*TNM} > W^{*CUP}$ and $W^{*TNM} > W^{*TH}$ are likely if $\eta$ is close to 0 and vice
versa if $\eta$ is close to 1. Recall that whether licensing occurs or not depends on $\Delta \Pi_M (\equiv \Pi^{CUP}_M - \Pi^{TNM}_M)$. We let $\eta^L$ denote the level of $\eta$ with which $\Delta \Pi_M = 0$. Then, licensing
occurs if and only if $\eta \leq \eta^L$.

The following computation reflecting linear demands clarifies the above point. Assume
that the inverse demands are given by

$$p_x = A - ax \quad \text{and} \quad p_y = B - by.$$ 

First, domestic welfare without the ALP, $W^{TH}$, is compared to that with the TNM
method, $W^{TNM}$:

$$W^{TNM} - W^{TH} = CS|_{\lambda=0} - CS|_{\lambda=1} + t(1 - \eta) \frac{A^2}{4a}\quad (13)$$

$$= -\frac{c_y(2B - c_y)}{8b} + t(1 - \eta) \frac{A^2}{4a} \geq 0 \iff \eta \leq 1 - \frac{ac(2B - c_y)}{2tbA^2} \equiv \eta^W. \quad (14)$$
\( W^{*TNM} < W^{*TH} \) holds if and only if \( \eta > \eta^W \), because greater \( \eta \) results in more opportunity of profit-shifting for the MNE. The increase in tax revenue caused by the ALP (which is the TNM method in this case) is not large enough to cover the decrease in consumer surplus in the market of good \( Y \).

Thus, we have two cases. With \( \eta^L < \eta^W \), the ALP may enhance domestic welfare even if licensing does not occur in the presence of the ALP. More specifically, if \( \eta^L < \eta < \eta^W \), domestic welfare increases even without licensing. With \( \eta^L > \eta^W \), however, the ALP improves domestic welfare if and only if licensing arises.

We can thus derive the condition with respect to \( t \) under which \( \eta^L < \eta^W \) holds:

\[
\eta^L < \eta^W \iff t > \frac{AB^2(2B^2 + 2Bc_y - c_y^2)}{2bA^2(B - c_y)^2} \equiv t^*.
\]

This is illustrated in Figure 1. Therefore, if licensing does not occur in the presence of the ALP, the ALP is necessarily harmful to the domestic country with \( t < t^* \) but may be beneficial with \( t > t^* \).

The results are illustrated in the Figures 2 and 3.\(^{14}\) The figures show how \( \eta \) affects the MNE’s licensing strategy and domestic welfare. Figure 2 is drawn with \( t = 0.3 < \)

\(^{14}\)We set \( A = B = a = b = 1 \) and \( c_y = \frac{1}{10} \) in Figures 2 and 3.
meaning $\eta_L > \eta_W$. If $\eta < \eta_L$, the MNE has an incentive for licensing and domestic welfare is larger with the ALP (i.e., the CUP method) than without. If $\eta > \eta_L$, on the other hand, licensing does not occur and domestic welfare is smaller with the ALP (i.e., the TNM method) than without. Thus, if the MNE terminates licensing because of the ALP, the domestic country loses. Figure 3 is drawn with $t = 0.5 > \underline{t}$, meaning $\eta_L < \eta_W$. In this case, even if the ALP leads the MNE to stop licensing, the domestic country may not lose.

These results are summarized in the following proposition.

**Proposition 2** Suppose that the ALP is introduced with a tax haven. The ALP improves domestic welfare if $\eta < \eta_L$ (i.e., licensing occurs). With $t < \underline{t}$, the ALP improves domestic welfare if and only if $\eta < \eta_L$. With $t > \underline{t}$, the ALP improves domestic welfare if and only if $\eta < \eta_W$.

We next take the choice of $\eta$ into account. As described in the introduction, the tax authority takes the initiative when selecting the reference firm. Although the government cannot freely choose the reference firm or $\eta$, it still has some freedom of choice.

We consider an extended game where in Stage 0, prior to the MNE’s decision on royalty rates, the domestic government chooses $\eta$ from a certain range to maximize domestic welfare. We specifically assume that the government can choose $\eta \in [\underline{\eta}, \bar{\eta}]$ where $0 < \underline{\eta} < \bar{\eta} < 1$. If $\underline{\eta} < \eta_L$, the government sets $\eta$ to induce licensing. As long as licensing is induced, the size of $\eta$ does not matter. This is because domestic welfare with licensing is independent of $\eta$. If $\underline{\eta} \geq \eta_L$ on the other hand, the government chooses $\eta = \underline{\eta}$. Note that the ALP harms the domestic country if $\underline{\eta} > \max\{\eta_W, \eta_L\}$.

Thus, we have the following proposition.

**Proposition 3** Suppose that the government chooses $\eta$ from the domain $[\underline{\eta}, \bar{\eta}]$ where $0 < \underline{\eta} < \bar{\eta} < 1$. The optimal royalty rate $\eta^*$ is given by $\eta^* = \underline{\eta}$ if $\eta_L \leq \underline{\eta}$ and $\underline{\eta} \leq \eta^* \leq \min\{\eta_L, \eta_W\}$ if $\underline{\eta} < \eta_L$.

### 4 Substitutable goods

In the last section, to clarify our point, we have assumed that both goods $X$ and $Y$ are not at all substitutable. In this section, we consider the case in which the two goods are substitutable. Specifically, we assume that the MNE and the local firm produce
a homogeneous good and are engaged in Cournot competition. We also assume the following linear demand:

\[ \tilde{p} = A - a(\tilde{x} + \tilde{y}) . \]

### 4.1 Without a tax haven

We begin with the case without a tax haven. The profits \( Y \) are

\[
\tilde{\Pi}_M = (1 - t) \left( \frac{\tilde{p} \tilde{x}}{\pi_x} + \lambda \tilde{y} \frac{\tilde{p} \tilde{y}}{\pi_y(\lambda=1)} \right),
\]

\[
\tilde{\Pi}_y = (1 - t) \{ \lambda(1 - \tilde{r}_y) \frac{\tilde{p} \tilde{y}}{\pi_y(\lambda=1)} + (1 - \lambda)(\tilde{p} - c_y)\tilde{y} \}.
\]

In the following analysis, the MNE is assumed to make centralized decisions.

If the MNE does not grant a license to the local firm (or, \( \lambda = 0 \)), the equilibrium is given by

\[
\tilde{x}^*|_{\lambda=0} = \frac{A + c_y}{3a}, \quad \tilde{y}^*|_{\lambda=0} = \frac{A - 2c_y}{3a}, \quad \tilde{p}^*|_{\lambda=0} = \frac{A + c_y}{3},
\]

\[
\tilde{\Pi}_M^*|_{\lambda=0} = (1 - t) \left( \frac{A + c_y}{9a} \right)^2, \quad \tilde{\Pi}_y^*|_{\lambda=0} = (1 - t) \left( \frac{A - 2c_y}{9a} \right)^2.
\]

If the MNE licenses its technology to the local firm (or, \( \lambda = 1 \)), the first order conditions (FOCs) are

\[
\text{FOC for } x : A - 2a\tilde{x} - a\tilde{y} - \tilde{r}_y a\tilde{y} = 0, \quad (17)
\]

\[
\text{FOC for } y : (1 - \tilde{r}_y)(A - a\tilde{x} - 2a\tilde{y}) = 0, \quad (18)
\]

which yield the following output and price levels:

\[
\tilde{x}|_{\lambda=1} = \frac{(1 - \tilde{r}_y)A}{(3 - \tilde{r}_y)a}, \quad \tilde{y}|_{\lambda=1} = \frac{A}{(3 - \tilde{r}_y)a}, \quad \tilde{p}|_{\lambda=1} = \frac{A}{3 - \tilde{r}_y}.
\]

Given these equilibrium outcomes, the profits are

\[
\tilde{\Pi}_M|_{\lambda=1} = (1 - t) \left( \frac{(1 - \tilde{r}_y)A^2}{(3 - \tilde{r}_y)^2a} + \tilde{r}_y \frac{(1 - \tilde{r}_y)A^2}{(3 - \tilde{r}_y)^2a} \right), \quad \tilde{\Pi}_y|_{\lambda=1} = (1 - t) \left( \frac{(1 - \tilde{r}_y)A^2}{(3 - \tilde{r}_y)^2a} \right).
\]

The MNE sets \( r_y \) such that

\[
\tilde{\Pi}_y|_{\lambda=1} \geq \tilde{\Pi}_y|_{\lambda=0} \iff \tilde{r}_y \leq \frac{A \sqrt{A^2 + 32Ac - 32c_y^2} - (A^2 + 8Ac_y - 8c_y^2)}{2(A - 2c_y)^2} \equiv \tilde{r}_y^*. \]

With the MNE’s take-it-or-leave-it license offer, therefore, the royalty rate becomes \( \tilde{r}_y^* \).

The equilibrium profits are

\[
\tilde{\Pi}_M|_{\lambda=1} = (1 - t) \left( \frac{(1 - \tilde{r}_y^*)A^2}{(3 - \tilde{r}_y^*)^2a} + \tilde{r}_y^* \frac{(1 - \tilde{r}_y^*)A^2}{(3 - \tilde{r}_y^*)^2a} \right), \quad \tilde{\Pi}_y^*|_{\lambda=1} = (1 - t) \left( \frac{(1 - \tilde{r}_y^*)A^2}{(3 - \tilde{r}_y^*)^2a} \right).
\]
4.2 A tax haven without the ALP

In the presence of a tax haven, the profits with licensing are

\[
\begin{align*}
\widetilde{\Pi}_M^{TH} \bigg|_{\lambda=1} &= (1-t)(1-\tilde{r}_x)\tilde{p}\tilde{x} + \tilde{r}_x\tilde{p}\tilde{x} + \tilde{r}_y\tilde{p}\tilde{y} = \xi\tilde{p}\tilde{x} + \tilde{r}_y\tilde{p}\tilde{y}, \\
\widetilde{\Pi}_y^{TH} \bigg|_{\lambda=1} &= (1-t)(1-\tilde{r}_y)\tilde{p}\tilde{y},
\end{align*}
\]

where \(r_x\) represents the internal royalty rate and \(\xi \equiv 1 - t + t\tilde{r}_x\). In this case, FOC is also a function of \(\xi\). The expressions of equilibrium variables are similar to those without the tax haven:

\[
\begin{align*}
\tilde{x}^{TH} \bigg|_{\lambda=1} &= \frac{(\xi - \tilde{r}_y)A}{(3\xi - \tilde{r}_y)a}, \quad \tilde{y}^{TH} \bigg|_{\lambda=1} = \frac{\xi A}{(3\xi - \tilde{r}_y)a}, \quad \tilde{p}^{TH} \bigg|_{\lambda=1} = \frac{\xi A}{3\xi - \tilde{r}_y},
\end{align*}
\]

and MNE’s post-tax profits are

\[
\begin{align*}
\widetilde{\Pi}_M^{TH} \bigg|_{\lambda=1} &= \frac{\xi^2 A^2}{(3\xi - \tilde{r}_y)^2a} \quad \text{and} \quad \frac{\partial \widetilde{\Pi}_M^{TH}}{\partial \xi} \bigg|_{\lambda=1} = \frac{3A^2\xi^2}{a(3\xi - \tilde{r}_y)^3}(2\xi - \tilde{r}_y)t > 0.
\end{align*}
\]

Thus, the MNE sets the internal royalty rate as high as possible (i.e., \(\tilde{r}_x^{TH} = 1\)) in the absence of the ALP. With \(\tilde{r}_x^{TH} = 1\), \(\xi = 1\) also holds, and the optimal arm’s length royalty becomes \(\tilde{r}_y^{TH} = \tilde{r}_y^a\), which is the same as the case without the tax haven. Thus, we have

\[
\begin{align*}
\tilde{x}^{TH} \bigg|_{\lambda=1} &= \frac{(1-\tilde{r}_y^a)A}{(3-\tilde{r}_y^a)a}, \quad \tilde{y}^{TH} \bigg|_{\lambda=1} = \frac{A}{(3-\tilde{r}_y^a)a}.
\end{align*}
\]

The equilibrium profits are given by

\[
\begin{align*}
\widetilde{\Pi}_M^{TH} \bigg|_{\lambda=1} &= \left(\frac{(1-\tilde{r}_y^a)A^2}{(3-\tilde{r}_y^a)^2a} + \tilde{r}_y^a \frac{(1-\tilde{r}_y^a)A^2}{(3-\tilde{r}_y^a)^2a}\right), \\
\widetilde{\Pi}_y^{TH} \bigg|_{\lambda=1} &= (1-t)\frac{(1-\tilde{r}_y^a)A^2}{(3-\tilde{r}_y^a)^2a}.
\end{align*}
\]

The profits without licensing are given by

\[
\begin{align*}
\widetilde{\Pi}_M^{TH} \bigg|_{\lambda=0} &= (1-t)(1-\tilde{r}_x)\tilde{p}\tilde{x} + \tilde{r}_x\tilde{p}\tilde{x} = \xi\tilde{p}\tilde{x}, \\
\widetilde{\Pi}_y^{TH} \bigg|_{\lambda=0} &= (1-t)\tilde{p}\tilde{y}.
\end{align*}
\]

Since the MNE sets \(r_x = 1\) without the ALP, we have

\[
\begin{align*}
\tilde{x}^{TH} \bigg|_{\lambda=0} &= \frac{A + c_y}{3a}, \quad \tilde{y}^{TH} \bigg|_{\lambda=0} = \frac{A - 2c_y}{3a}, \\
\widetilde{\Pi}_M^{TH} \bigg|_{\lambda=0} &= \frac{(A + c_y)^2}{9a}, \quad \widetilde{\Pi}_y^{TH} \bigg|_{\lambda=0} = (1-t)\frac{(A - 2c_y)^2}{9a}.
\end{align*}
\]
4.3 A tax haven with the ALP

In the case of licensing with the ALP, the MNE cannot price-discriminate between its subsidiary and the local firm.

\[ \tilde{\Pi}_{M}^{\text{CUP}} = (1-t)(1-\tilde{\tau}^{\text{CUP}})\tilde{p}x + \tilde{\tau}^{\text{CUP}}\tilde{p}x + \tilde{\tau}^{\text{CUP}}\tilde{p}\gamma = (1-t + t\tilde{\tau}^{\text{CUP}})\tilde{p}x + \tilde{\tau}^{\text{CUP}}\tilde{p}\gamma, \]
\[ \tilde{\Pi}_{y}^{\text{CUP}} = (1-t)(1-\tilde{\tau}^{\text{CUP}})\tilde{p}\gamma. \]

Then the outputs are

\[ \tilde{x}^{\text{CUP}} = \frac{((1-t + t\tilde{\tau}^{\text{CUP}}) - \tilde{\tau}^{\text{CUP}})A}{3(1-t + t\tilde{\tau}^{\text{CUP}}) - \tilde{\tau}^{\text{CUP}}a}, \quad \tilde{y}^{\text{CUP}} = \frac{(1-t + t\tilde{\tau}^{\text{CUP}})A}{3(1-t + t\tilde{\tau}^{\text{CUP}}) - \tilde{\tau}^{\text{CUP}}a}. \]

Noting \( \tilde{\Pi}_{y}^{\text{TNM}} = \tilde{\Pi}_{y}^{\text{CUP}}\big|_{\lambda=0} \), the optimal royalty rate, \( \tilde{\tau}^{*}\text{CUP} \), satisfies the following condition:

\[ \tilde{\Pi}_{y}^{\text{TNM}} = (1-t)(1-\tilde{\tau}^{*}\text{CUP})(1-t + \tilde{\tau}^{*}\text{CUP})^2A^2 \]
\[ = (1-t)\frac{(A - 2c_y)^2}{9a} = \tilde{\Pi}_{y}^{\text{CUP}}\big|_{\lambda=0}. \]

Thus, the MNE’s profits with the CUP method are

\[ \tilde{\Pi}_{M}^{*}\text{CUP} = \frac{(1-t + t\tilde{\tau}^{*}\text{CUP})^3A^2}{3(1-t + t\tilde{\tau}^{*}\text{CUP}) - \tilde{\tau}^{*}\text{CUP}}. \]

In the case of non-licensing, the MNE has to set the internal royalty rate equal the comparable value \( \eta \). The profits are

\[ \tilde{\Pi}_{M}^{\text{TNM}} = (1-t)(1-\eta)\tilde{p}x + \eta\tilde{p}x = (1-t + \eta t)\tilde{p}x, \]
\[ \tilde{\Pi}_{y}^{\text{TNM}} = (1-t)\tilde{p}\gamma. \]

Since the outputs are independent of the internal royalty rate without licensing, we obtain

\[ \tilde{x}^{\text{TNM}} = \frac{A + c_y}{3a}, \quad \tilde{y}^{\text{TNM}} = \frac{A - 2c_y}{3a}. \]

Thus, the MNE’s profits with the TNM method become

\[ \tilde{\Pi}_{M}^{*}\text{TNM} = (1-t + t\eta)\frac{(A + c_y)^2}{9a}. \]

\( \tilde{\Pi}_{M}^{\text{CUP}} > \tilde{\Pi}_{M}^{*}\text{TNM} \) may or may not hold. We can confirm that \( \Delta\tilde{\Pi}_{M}(= \tilde{\Pi}_{M}^{\text{CUP}} - \tilde{\Pi}_{M}^{*}\text{TNM}) < 0 \) is possible only if \( c_y \) is small. Small \( c_y \) implies that licensing is not very attractive to the MNE, because the smaller the \( c_y \), the smaller the license revenue. Figure
Figure 4: Figure 4 shows the relationship between $\eta$ and $\Delta \tilde{\Pi}_M$ with three different levels of $t$. $\tilde{\Pi}_x^{TNM}$ is more likely to exceed $\tilde{\Pi}_x^{CUP}$ if $\eta$ is relatively large and $t$ is relatively small.\(^{15}\)

The MNE attempts to reduce tax payments through two channels. The first is by shifting the profits of its subsidiary to the tax haven. This channel is likely to be more efficient when the MNE is not engaged in licensing and $\eta$ is relatively large. The second is by increasing the license revenue from the local firm. For this channel, licensing is essential. When $t$ is relatively small, the first channel dominates the second channel. The MNE tries to take advantage of the first channel and is more unlikely to license its technology to the local firm.

4.4 Welfare comparison

We examine how domestic welfare changes when the ALP is introduced in the presence of a tax haven. As in Section 3, changes in domestic welfare are measured by changes in domestic tax revenue and consumer surplus.

First, we can prove the following lemma.\(^{16}\)

**Lemma 2** (i) $\tilde{x}^*_{CUP} + \tilde{y}^*_{CUP} < \tilde{x}^*_{TNM} + \tilde{y}^*_{TNM}$, (ii) $\tilde{x}^*_{CUP} + \tilde{y}^*_{CUP} < \tilde{x}^*_{TH} + \tilde{y}^*_{TH}$ if $t \leq \frac{1}{3}$, and (iii) $\tilde{x}^*_{TH} + \tilde{y}^*_{TH} < \tilde{x}^*_{TNM} + \tilde{y}^*_{TNM}$.

\(^{15}\)Figures 4-7 are drawn with $c_y = \frac{1}{1000}$ and $A = a = 1$.

\(^{16}\)See Appendix for the proof.
Lemma 2 (i) says that in the presence of the ALP, the total supply of the good is greater without licensing than with licensing. This seems surprising because the total output is less with licensing regardless of whether licensing leads both firms to produce the good with zero MC. The negative effect on the total output caused by the MNE’s centralized decision with licensing by means of ad valorem royalties dominates the positive effect of the cost reduction of the local firm. When the goods are substitutes, the MNE decreases the output of its subsidiary to increase the output of the local firm and the price. As a result, the MNE obtains more license revenue from the local firm.

Lemma (ii) and (iii) says that $CS$ without the ALP may be larger than $CS$ with the CUP method but is smaller than $CS$ with the TNM method. Thus, as a result of the introduction of the ALP, consumers may lose if licensing occurs but gain if it does not.

Therefore, noting that the ALP increases tax revenue, we can establish the following proposition.

**Proposition 4** Suppose that the MNE’s subsidiary and the local firm compete in the market. The ALP may harm consumers and worsen domestic welfare if the MNE keeps licensing to the local firm but benefits consumers and improves domestic welfare if the MNE stops licensing to the local firm.

Figures 5-7 illustrate whether the introduction of the ALP improves domestic welfare. Each figure is drawn with a different tax rate. In Figures 5 and 6, in the presence of the ALP, licensing occurs if and only if $\eta < \eta^L$. The ALP always improves domestic welfare in Figure 5. However, in Figures 6 and 7, the ALP worsens domestic welfare if licensing occurs.

5 Concluding remarks

This paper has dealt with the MNE transfer pricing of intangible assets licensed by means of ad valorem royalties. Specifically, we have explored the effects of the ALP on MNE licensing strategies and economic welfare in the presence of a tax haven.

Our analysis in the basic model provides two messages. First, the ALP may distort the MNE licensing strategy. In the absence of the ALP, the MNE is willing to offer a licensing contract to an unrelated firm regardless of the existence of a tax haven. In the presence of the ALP, however, the MNE may refrain from offering the contract. Thus, the comparable transaction of licensing may vanish, enabling the MNE to enjoy more opportunity for profit-shifting from its subsidiary.
Figure 5:

Figure 6:
Second and more importantly, the disincentivization of licensing may worsen the welfare of high-tax countries. One may expect that anti tax-avoidance policies such as BEPS actions prevent MNEs from profit-shifting and contribute to welfare improvement through an increase in the tax revenue. Our model, however, has shown that such a positive aspect may appear at the expense of consumers, because the MNEs may stop licensing to remove comparable transactions.

We also investigated the case in which the goods are substitutes as an extension. In this case, consumers may lose even if the licensing still occurs with the ALP. This is because the MNE decreases the output of its subsidiary to take more advantage of the license revenue from the local firm. As a result, the ALP harms consumers.

Although our model has shed new light on the link between licensing and profit-shifting, further analysis on this topic is essential. A potential extension will consider policies focusing more on patents (e.g., the patent box). Although several empirical studies have focused on these kinds policies rapidly prevailing in Europe, theoretical analyses have not been very satisfactory.
Appendix

The appendix proves Lemma 2.

Proof. First, we prove (i) \( \tilde{x}^*_{\text{CUP}} + \tilde{y}^*_{\text{CUP}} < \tilde{x}^*_{\text{TNM}} + \tilde{y}^*_{\text{TNM}} \). The total supply of the good is

\[
\tilde{x}^*_{\text{TNM}} + \tilde{y}^*_{\text{TNM}} = \frac{2A - sy}{3a}, \quad \text{with the TNM method,}
\]

\[
\tilde{x}^*_{\text{CUP}} + \tilde{y}^*_{\text{CUP}} = \frac{A(2(1 - t + \tilde{r}^*_{\text{CUP}}) - \tilde{r}^*_{\text{CUP}})}{a(3(1 - t + \tilde{r}^*_{\text{CUP}}) - \tilde{r}^*_{\text{CUP}})}, \quad \text{with the CUP method.}
\]

We can then derive the condition under which the total supply is greater under the TNM method than under the CUP method:

\[
(\tilde{x}^*_{\text{TNM}} + \tilde{y}^*_{\text{TNM}}) - (\tilde{x}^*_{\text{CUP}} + \tilde{y}^*_{\text{CUP}}) = \frac{(A + cy)(\tilde{r}^*_{\text{CUP}} - 3(1 - t + \tilde{r}^*_{\text{CUP}})A_y)}{3a(3(1 - t + \tilde{r}^*_{\text{CUP}}) - \tilde{r}^*_{\text{CUP}})} \geq 0
\]

\[
\Leftrightarrow \tilde{r}^*_{\text{CUP}} \geq \frac{3(1 - t)cy}{A + cy - 3tcy} \equiv r^*_{\text{CS}}.
\]

We can then verify that the local firm accepts the license offer if the MNE sets \( r^*_{\text{CS}} \) because the following holds:

\[
\tilde{\Pi}_y^{\text{CUP}} \bigg|_{r=r^*_{\text{CS}}} - \tilde{\Pi}_y^{\text{TNM}} = \frac{(1 - t)(1 - r^*_{\text{CS}})A^2(1 - t + tr^*_{\text{CS}})}{a(3(1 - t + tr^*_{\text{CS}}) - r^*_{\text{CS}})^2} - \frac{(1 - t)(A - 2cy)^2}{9a}
\]

\[
= \frac{(1 - t)(A - 2cy)}{9a(A + cy - 3tcy)} \left[ \frac{(A + cy)^2}{A + tc_y} A^2 - (A - 2cy)(A + cy - 3tc_y) \right]
\]

\[
= \frac{(1 - t)(A - 2cy)}{9a(A + cy - 3tcy)} \left[ \left( \frac{(A + cy)^2}{A + tc_y} - 1 \right) A^2 + cy \left( A + 2cy + 3t(A - 2cy) \right) \right]
\]

\[
> 0 \tag{A-1}
\]

Note that the MNE has an incentive to set the royalty rate as high as possible, because \( \frac{\partial \tilde{\Pi}_y^{\text{CUP}}}{\partial r_{\text{CUP}}} = (1 - t + \tilde{r}^*_{\text{CUP}}) \tilde{p}_x + \tilde{p}_y > 0 \) holds. Eq.(A-1) suggests that \( r^*_{\text{CS}} \) is acceptable for the local firm but is not optimal for the MNE, because \( \tilde{\Pi}_y^{\text{CUP}} = \tilde{\Pi}_y^{\text{TNM}} \) is not satisfied. Thus, the optimal royalty is greater than \( r^*_{\text{CS}} \). In view of Eq.(A-3), the optimal royalty results in more total supply under the TNM method than under the CUP method.

Second, we prove (ii) \( \tilde{x}^*_{\text{CUP}} + \tilde{y}^*_{\text{CUP}} < \tilde{x}^*_{\text{TH}} + \tilde{y}^*_{\text{TH}} \) if \( t \leq \frac{1}{3} \). We have

\[
\tilde{\Pi}_y^{\text{CUP}} = \left( \frac{1 - t}{a} \right) \Psi, \quad \tilde{\Pi}_y^{\text{TNM}} = \left( \frac{1 - t}{a} \right) \frac{(A - 2cy)^2}{9a},
\]

where \( \Psi \equiv \frac{(1 - r^*_{\text{CUP}})A^2(1 - t + tr^*_{\text{CUP}})^2}{3(1 - t + tr^*_{\text{CUP}}) - r^*_{\text{CUP}}} \). The optimal \( \tilde{r}^*_{\text{CUP}} \) is determined by

\[
\Delta \tilde{\Pi}_y \equiv \tilde{\Pi}_y^{\text{CUP}} - \tilde{\Pi}_y^{\text{TNM}} = \left( \frac{1 - t}{a} \right) \left( \Psi - \frac{(A - 2cy)^2}{9a} \right) = 0. \tag{A-2}
\]
Suppose that the domestic tax rate is zero under the CUP method. Then, the optimal royalty rate is the same as the one in the benchmark case $\tilde{\gamma}^{CUP}|_{t=0} = \tilde{\gamma}^*$. This is because there is no tax avoidance motive. Thus, $\tilde{x}^{CUP} + \tilde{y}^{CUP} = \tilde{x}^{TH} + \tilde{y}^{TH}$ holds at $t = 0$. We next examine how $\tilde{x}^{CUP} + \tilde{y}^{CUP}$ changes as $t$ increases. We have

$$\frac{\partial \Psi}{\partial t} = \frac{2(1 - \tilde{\gamma}^{CUP})^2 A^2 (1 - t + \tilde{\gamma}^{CUP} t) \tilde{\gamma}^{CUP}}{3(1 - t + \tilde{\gamma}^{CUP}) - \tilde{\gamma}^{CUP})^3} > 0.$$  

We can also show

$$\frac{\partial \Psi}{\partial \gamma^{CUP}} = \left[ \frac{A^2 (1 - t + \tilde{\gamma}^{CUP} t)}{3(1 - t + \tilde{\gamma}^{CUP}) - \tilde{\gamma}^{CUP})^3} \right] \times \left[ (1 - t)(1 - \tilde{\gamma}^{CUP})(1 - t + \tilde{\gamma}^{CUP} t) + 2(1 - t) \left( \tilde{\gamma}^{CUP} - (1 - \tilde{\gamma}^{CUP})^2 t \right) + 2t(\tilde{\gamma}^{CUP})^2 \right] < 0 \text{ if } 0 < t \leq \frac{1}{3}. \quad (A-3)$$

To show (A-3), we examine the inside of the second square brackets.

$$F(\tilde{\gamma}^{CUP}; t) = (1 - t)(1 - \tilde{\gamma}^{CUP})(1 - t + \tilde{\gamma}^{CUP} t) + 2(1 - t) \left( \tilde{\gamma}^{CUP} - (1 - \tilde{\gamma}^{CUP})^2 t \right) + t(\tilde{\gamma}^{CUP})^2.$$ 

$F(0; t) = -(3t - 1)(t - 1)$ and $F(1; t) = -2$. Thus, if $t < \frac{1}{3}$, $F(\tilde{\gamma}^{CUP}; t) < 0$ holds for $\tilde{\gamma}^{CUP} \in [0, 1]$. If $t = \frac{1}{3}$, $F(\tilde{\gamma}^{CUP}; \frac{1}{3}) = -2\tilde{\gamma}^{CUP} < 0$ holds for $\tilde{\gamma}^{CUP} \in (0, 1]$. Noting that the inside of the first square brackets is positive, we can confirm (A-3). Therefore, as $t$ increases in the range of $(0, \frac{1}{3}]$, the MNE increases $\tilde{\gamma}^{CUP}$ to achieve (A-2). That is, the optimal royalty rate is increasing in $t$ in the range of $(0, \frac{1}{3}]$. In addition, we can confirm

$$\frac{\partial (\tilde{x}^{CUP} + \tilde{y}^{CUP})}{\partial t} = \frac{-(1 - t) A}{a \{3(1 - t + \tilde{\gamma}^{CUP}) - \tilde{\gamma}^{CUP})^2 \}^2} < 0,$$

$$\frac{\partial (\tilde{x}^{CUP} + \tilde{y}^{CUP})}{\partial \tilde{\gamma}^{CUP}} = \frac{-(1 - \tilde{\gamma}^{CUP}) A}{a \{3(1 - t + \tilde{\gamma}^{CUP}) - \tilde{\gamma}^{CUP})^2 \}^2} < 0.$$  

Eqs.(A-1) and (A-2) imply that an increase in $t$ increases in the range of $(0, \frac{1}{3}]$ the total supply under the CUP method directly and indirectly. The indirect increase is through an increase in $\tilde{\gamma}^{CUP}$ caused by the increase in $t$. Thus, we obtain $\tilde{x}^{CUP} + \tilde{y}^{CUP} < \tilde{x}^{TH} + \tilde{y}^{TH}$ if $0 < t \leq \frac{1}{3}$.

Lastly, with respect to (iii) $\tilde{x}^{TH} + \tilde{y}^{TH} < \tilde{x}^{TNM} + \tilde{y}^{TNM}$, Proposition 2 (ii) of San Martin and Saracho (2010) actually prove that consumer surplus with licensing is lower than without licensing. Thus, the TNM method increases the total supply, i.e., $\tilde{x}^{TH} + \tilde{y}^{TH} < \tilde{x}^{TNM} + \tilde{y}^{TNM}$. ❑

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References


