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Agglomeration of Low-productive Entrepreneurs to Large Regions: A Simple Model[1]

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Abstract

This paper develops a simple model of spatial sorting where the least productive entrepreneurs are drawn to the large core region. This is an unusual feature. The literature on spatial sorting typically shows how the most productive individuals and firms agglomerate to the core. However, our model is consistent with data that reveals that large agglomerations also attract unproductive entrepreneurs.

Keywords: agglomeration, heterogeneous firms, spatial sorting
JEL classification: F12, F15, F21, R12

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1 Introduction

This paper develops a simple model of spatial sorting where the least productive entrepreneurs are drawn to the large core region. This is an unusual feature. The literature on spatial sorting typically shows how the most productive individuals and firms agglomerate to the core. However, empirical evidence reveals that large agglomerations also seem to attract low skilled. Glaeser et al. (2008) note that while 7.5 percent of the suburban population in the US in 2000 was poor, 19.9 percent of the population in the central cities was poor.[1] Moreover, in the developing world, where urbanization is rapid[2] the slum areas of the poor mega-cities have increased substantially over the last 40-50 years, both in absolute and relative terms, which indicates a concentration of low skilled to these cities.[3] Our model suggests one mechanism that could help explaining this pattern.

The paper introduces spatial sorting of heterogeneous entrepreneurs (firms) in the 'footloose entrepreneur' (FE) model by Forslid (1999) and Forslid and Ottaviano (2003). We here also apply quasilinear utility function à la Pfluger (2004) and Borck and Pfluger (2006).[4] A new feature of our model is that it generates spatial sorting in reverse productivity order with the least productive entrepreneur being the first to relocate. The reason for this is that the advantage of the lower price index in the larger region is the same for all entrepreneurs (because of the quasilinear utility), while the local competition effect is more important for large firms (more productive entrepreneurs). The introduction of heterogeneous firms (entrepreneurs) also implies a less drastic agglomeration pattern compared to models with homogenous firms.[5] This is the case because the gains of agglomerating to the core region are different for firms of different productivity, which implies that firms will find it optimal to relocate for different levels of trade costs. The muted agglomeration tendency agrees well with the data, since instances of abrupt or catastrophic agglomeration are rare in practice (Brakman et al. 2004, Davis and Weinstein 2002,

[1] That the poor seems to live closer to the city center than the rich has previously been noted by e.g. Margo (1992), Mieszkowski and Mills (1993), and Mills and Lubuele (1997).

[2] Only six out of 30 mega-cities are today located in high income countries. See Glaeser (2014).

[3] E.g. the slum in Ahmedabad, according to UN-Habitat (2003), has increased from 17 percent in 1971 to 21 percent in 1982 and for 1991, 40 percent of households lived in slums. For Karachi the share of shacks increased from 37 percent to 50 percent between 1978 and 2000.

[4] Our paper belongs to the trade and geography literature, with the seminal paper by Krugman (1991), that highlights how trade integration may lead to concentration or agglomeration of firms to larger countries or regions (for surveys see Fujita et al. 1999, Baldwin et al. 2003, and Combes et al. 2008). Introducing heterogeneous firms in this literature, Baldwin and Okubo (2006) show that the most productive firms are the first to agglomerate to the core when using the 'footloose capital' (FC) model by Martin and Rogers (1995).

[5] There are several other papers that generate a gradual relocation pattern. Helpman (1998) introduces a housing sector that dampens the agglomeration process. Tabuchi and Thisse (2002), Murata (2003), and Zeng (2008) introduce preference heterogeneity in different models, which generate a non-catastrophic relocation pattern, and there are also models with CES upper tier preferences may generate a gradual relocation pattern, see Pfluger and Sudekum (2008, 2011).

2008, and Redding et al. 2011).[6] Agglomeration in our model generates a higher welfare index for labour in the larger core region, which is in accordance with data.[7] However, agglomeration also generates a sharp regional income-difference among the entrepreneurs, with many poor entrepreneurs in the core.

The literature on labor (wage) and firm sorting predicts that the most productive firms and individuals agglomerate to the largest cities or regions (see e.g. Combes et al. 2008, Baum-Snow and Pavan 2011, Baum-Snow et al. 2018, Combes et al. 2012, and Duranton and Puga 2004). However, there is also evidence that the least skilled individuals agglomerate to the large cities as shown by Eckhout et al. (2014) that use data from US metropolitan areas.

Our paper is most closely related to Baldwin and Okubo (2006) that develop a trade and geography model of spatial sorting where the most productive capital agglomerates to the core as trade or transportation costs are reduced. Our paper instead finds the reverse sorting pattern, where the least productive entrepreneurs have the strongest incentives to agglomerate to the core. Our model also differs from Baldwin and Okubo (2006) by generating agglomeration from a uniform space.

Our paper is also related to the literature on systems of cities, pioneered by Henderson (1974). A more recent strand of this literature models the sorting of heterogeneous workers or entrepreneurs to markets of different size, and where the most talented entrepreneurs sort to the largest markets (see e.g. Nocke 2006, Behrens et al. 2014, Eckhout et al. 2014, Gaubert 2018, and Davis and Dingel 2019). Our model has, as noted, the opposite sorting pattern.

2 The Model

This paper uses the heterogeneous firm version of the FE-model by Forslid (1999) and Forslid and Ottaviano (2002), where we use quasilinear utility as in Pfluger (2004) and Borck and Pfluger (2006).

2.1 Basics

There are two regions, Region 1 and Region 2 (denoted by *), and two factors, human capital H and labour L. Human capital or entrepreneurs move between regions and bring with them their business. Labour can move freely between sectors but are immobile between regions. There are two sectors M (manufacturing) and A (agriculture). The A-sector produces a freely traded homogeneous good with a constant-returns technology using only labour. The M-sector produces differentiated manufactures with increasing-returns technologies using both human capital and labour. Firm productivities in the M-sector are distributed according to a cumulative density function.

[6] Using WWII data from Germany, Bosker et al. (2007) do find instances of multiple equilibria.

[7] See e.g. Combes et al. (2012).

All individuals in a region have the utility function

$$U = \mu \ln C_M + C_A, \tag{1}$$

where $\mu \in (0,1)$ is a constant and C_A is consumption of the homogenous good, and where the region subscript is suppressed. Differentiated goods enter the utility function through the index C_M, defined by

$$C_M = \left[\int_0^N c_i^{(\sigma-1)/\sigma} di \right]^{\sigma/(\sigma-1)}, \tag{2}$$

N being the mass of varieties consumed, c_i the amount of variety i consumed, and $\sigma > 1$ the elasticity of substitution.

Each consumer spends μ on manufactures, and the total demand for a domestically produced variety i is therefore

$$x_i = \frac{p_i^{-\sigma}}{P^{1-\sigma}} \cdot \mu, \tag{3}$$

where p_i is the price of variety i, and P is the price index

$$P = \left(\int_0^N p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{4}$$

The unit factor requirement of the homogeneous A-sector good is one unit of labour. This good is freely traded, and since it is also chosen as the numeraire, we have

$$p_A = w = 1, \tag{5}$$

w being the wage in all regions.

Each firm has a fixed cost in human capital. We normalize the fixed cost so that one entrepreneur is associated with one firm. Firms (entrepreneurs) are differentiated in terms of their marginal cost, and the firm-specific marginal production costs a_i are distributed according to the cumulative distribution function $G(a)$. The total cost of producing x_i units of manufactured commodity i in a region is

$$TC_i = \pi_i + a_i x_i, \tag{6}$$

where π is the return to human capital (i.e. to an entrepreneur).

Distance is represented by trading costs. Shipping the manufactured good involves a frictional trade cost of the "iceberg" type: for one unit of good from region j to arrive in region k, $\tau_{jk} > 1$ units must be shipped. Trade costs are also assumed to be equal in both directions so that $\tau_{jk} = \tau_{kj}$.

Profit maximization by manufacturing firms leads to the price

$$p_i = \frac{\sigma}{\sigma - 1} a_i. \tag{7}$$

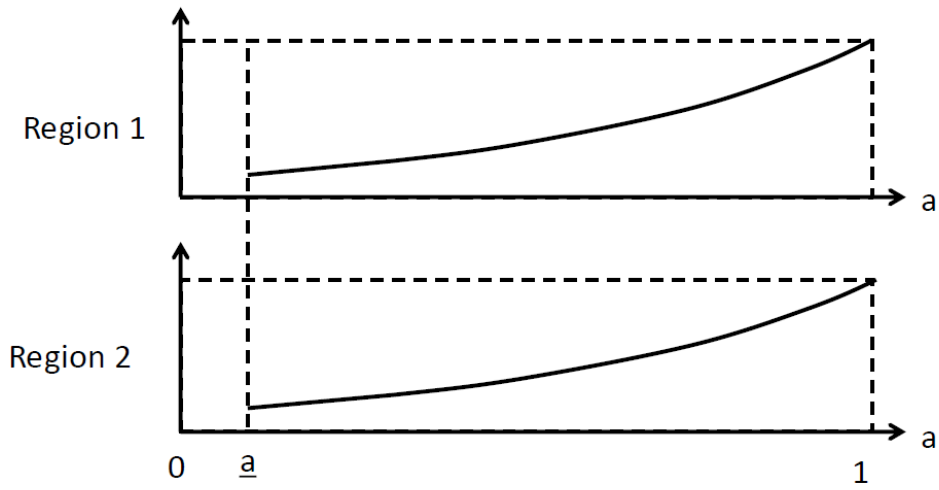

Figure 1: The productivity distribution at the initial equilibrium.

Firm heterogeneity in labour requirements, a_i, is probabilistically allocated among firms (entrepreneurs). In order to analytically solve the model, we assume the following cumulative density function of a:

$$G(a) = \frac{a^{\rho} - \underline{a}^{\rho}}{a_0^{\rho} - \underline{a}^{\rho}}, \tag{8}$$

where ρ is a shape parameter and a_0^{ρ} is a scaling factor.[8] We assume the distribution to be truncated at $\underline{a}$, where $0 < \underline{a} < a < a_0$, so that the productivity of firms is bounded, and we normalize so that $a_0 = 1$.

In the short run, the allocation of H is taken to be fixed. The model is closed by the M-sector market-clearing condition, (9), where the left-hand side is the nominal return to human capital, which equals a firm's operating profit, and the right-hand side follows from the demand functions in (3). The nominal return to entrepreneur i in Region 1 is:

$$\pi_i = \frac{\mu}{\sigma}\left(\frac{(L+H)}{\Delta} + \frac{\phi(L^* + H^*)}{\Delta^*}\right) a_i^{1-\sigma}, \tag{9}$$

where

$$\Delta \equiv \frac{P^{1-\sigma}}{\left(\frac{\sigma}{1-\sigma}\right)^{1-\sigma}} = H\int_{\underline{a}}^{1} a_k^{1-\sigma} dG(a) + \phi H^* \int_{\underline{a}}^{1} a_k^{1-\sigma} dG(a). \tag{10}$$

The object $\phi_{jk} = \tau_{jk}^{1-\sigma}$, ranging between 0 and 1, stands for "freeness" of trade between j and k (0 is autarchy and 1 is zero trade costs). These equilibrium conditions hold under the condition that the A-sector, which pins down the wage, is active in all regions.

[8] This is essentially a Pareto distribution that has been truncated (see Forslid and Okubo 2015).

2.2 Stability analysis

In the long run, entrepreneurs respond to the incentives provided by the difference in real return that can be attained in the two regions:

$$\begin{aligned} V - V^* &= (\pi_j - \mu \ln P) - (\pi_j^* - \mu \ln P^*) \\ &= \frac{\mu}{\sigma}(1-\phi)(B - B^*)\, a^{1-\sigma} - \frac{\mu}{1-\sigma}(\ln \Delta - \ln \Delta^*), \end{aligned} \tag{11}$$

where $B \equiv \frac{L+H}{\Delta}$, and $B^* \equiv \frac{L^*+H^*}{\Delta^*}$.

2.2.1 The Break Point

As is customary, a symmetric allocation of resources is always an equilibrium since everything is symmetric in the model, and we will use this equilibrium as the starting point. We will investigate whether this equilibrium is stable when perturbed a little. The experiment is to move one entrepreneur over to the other region, and then to see if the utility of the entrepreneur rises or falls as a consequence of this move. If it rises, the symmetric equilibrium is unstable, otherwise it is stable. However, since entrepreneurs are heterogeneous, it matters which entrepreneur that moves. More precisely, the effect on demand (the demand-link) is identical because of the quasi-linear preferences, whereas the effect on the price index depends on the productivity of the moving entrepreneur. We will assume that the entrepreneur with the highest gains from moving will move first. This would e.g. be consistent with entrepreneurs bidding for transportation from a transport sector with limited capacity, in which case the highest bidder will be the firm with the highest gains from moving.

The welfare effect for a marginal entrepreneur i that moves is given by

$$\frac{d(V-V^*)}{dH_i} = \frac{\mu}{\sigma}(1-\phi)\left(\frac{dB}{dH_i} - \frac{dB^*}{dH_i}\right) a_i^{1-\sigma} + \frac{\mu}{\sigma-1}\left(\frac{d\Delta}{\Delta dH_i} - \frac{d\Delta^*}{\Delta^* dH_i}\right). \tag{12}$$

This expression illustrates the agglomeration forces in action when one (infinitesimal) entrepreneur moves: The first term shows the expenditure shifting as well as the competition effect. The second term shows the price index effect or the supply link, which is positive since a larger region has a lower price index. Using that $\frac{dB}{dH} = \frac{1}{\Delta} - \frac{L+H}{\Delta^2}\frac{d\Delta}{dH}$, $\frac{dB^*}{dH} = \frac{1}{\Delta^*}\frac{dH^*}{dH} - \frac{L+H}{\Delta^{*2}}\frac{d\Delta^*}{dH^*}\frac{dH^*}{dH}$ and evaluating the expression at the symmetric equilibrium, where $\frac{d\Delta^*}{dH_i} = -\frac{d\Delta}{dH_i}$, gives

$$\left.\frac{d(V-V^*)}{dH_i}\right|_{H=H^*} = 2\frac{\mu}{\sigma}(1-\phi)\frac{1}{\Delta}a_i^{1-\sigma} - 2\frac{\mu}{\sigma}(1-\phi)\frac{L+H}{\Delta^2}\frac{d\Delta}{dH_i}a_i^{1-\sigma} + 2\frac{\mu}{\sigma-1}\frac{1}{\Delta}\frac{d\Delta}{dH_i}. \tag{13}$$

The effect on the price index of the movement of one infinitesimal entrepreneur with productivity a_i is given by

$$\frac{d\Delta}{dH_i} = \frac{(1-\phi)\lim_{\delta \longrightarrow +0} \frac{1}{\delta} H \int_{a_i}^{a_i+\delta} a^{1-\sigma} dG(a)}{g(a)H}, \tag{14}$$

where $g(a) = G'(a)$ is a probability density function.

Using l'Hopital's rule gives

$$\frac{d\Delta}{dH_i} = \frac{\frac{\rho}{1-\underline{a}^{\rho}}(1-\phi)a_i^{\rho-\sigma}}{\frac{\rho}{1-\underline{a}^{\rho}}a^{\rho-1}} = (1-\phi)a_i^{1-\sigma}, \tag{15}$$

and using this in (13) and noting that $\Delta = \frac{\rho}{1-\sigma+\rho}\frac{1-\underline{a}^{1-\sigma+\rho}}{1-\underline{a}^{\rho}}(H+\phi H^*) = \xi(1+\phi)H$, where $\xi \equiv \frac{\rho}{1-\sigma+\rho}\frac{1-\underline{a}^{1-\sigma+\rho}}{1-\underline{a}^{\rho}}$ at symmetry gives

$$\left.\frac{d(V-V^*)}{dH_i}\right|_{H=H^*} = 2\frac{\mu}{\sigma}\frac{(1-\phi)}{\xi H(1+\phi)}a_i^{1-\sigma}\left(1-\frac{L+H}{(1+\phi)H\xi}(1-\phi)a_i^{1-\sigma}+\frac{\sigma}{\sigma-1}\right). \tag{16}$$

As usual we assume that $1-\sigma+\rho > 0$, to ensure that the integrals in Δ converge.

The breakpoint, the level of trade freeness at which the symmetric equilibrium becomes unstable, is found by solving $\left.\frac{d(V-V^*)}{dH_i}\right|_{H=H^*} = 0$ for ϕ[9]:

$$\phi^B = \frac{\frac{L+H}{\xi H}a_i^{1-\sigma} - \frac{2\sigma-1}{\sigma-1}}{\frac{L+H}{\xi H}a_i^{1-\sigma} + \frac{2\sigma-1}{\sigma-1}}. \tag{17}$$

It is seen from (17) that $\phi^B < 1$, and the existence of $\phi^B > 0$ for $a \in [\underline{a}, 1]$, guaranteed by the condition that $\frac{L}{H} > \frac{2\sigma-1}{\sigma-1}\xi - 1$, which is the "no-black-hole" condition in this model, prevent full agglomeration from always being the equilibrium outcome.

Furthermore, from (17) ϕ^B decreases in a, which means that the first firm to deviate from the symmetric equilibrium, given that this equilibrium exists, will be the least productive (the firm that has the highest a). Interestingly, this sorting pattern is the opposite to that of the footloose capital (FC) model where the most productive firms are the first to move to the core (see Baldwin and Okubo 2006). Here instead the least productive entrepreneur moves first because the advantage of the lower price index in the larger region is the same for all entrepreneurs, while the local competition effect is more important for large firms (more productive entrepreneurs).

Another important difference between the models is that our model generates agglomeration starting from symmetry. In the FC model, there is no agglomeration force when markets are symmetric. Agglomeration and spatial sorting only occur when markets are different enough in size, and a larger market favours the relocation of high productive firms with high sales volumes.

The trade cost at which the symmetric equilibrium ceases to be stable, the break-point, is found by setting $a_i = 1$ in (17):

$$\phi^B = \frac{\frac{L+H}{\xi H} - \frac{2\sigma-1}{\sigma-1}}{\frac{L+H}{\xi H} + \frac{2\sigma-1}{\sigma-1}}. \tag{18}$$

The importance of firm heterogeneity can be seen by varying $\underline{a}$. Since $\frac{d\xi}{d\underline{a}} < 0$ we have from (18) that $\frac{d\phi^B}{d\underline{a}} > 0$. That is, more heterogeneity (a lower $\underline{a}$) decreases the breakpoint. Thus,

[9] Note that our breakpoint corresponds to that of Pfuger (2004) for $a_i = 1$, since $\lim_{\underline{a}\to 1}\xi = 1$.

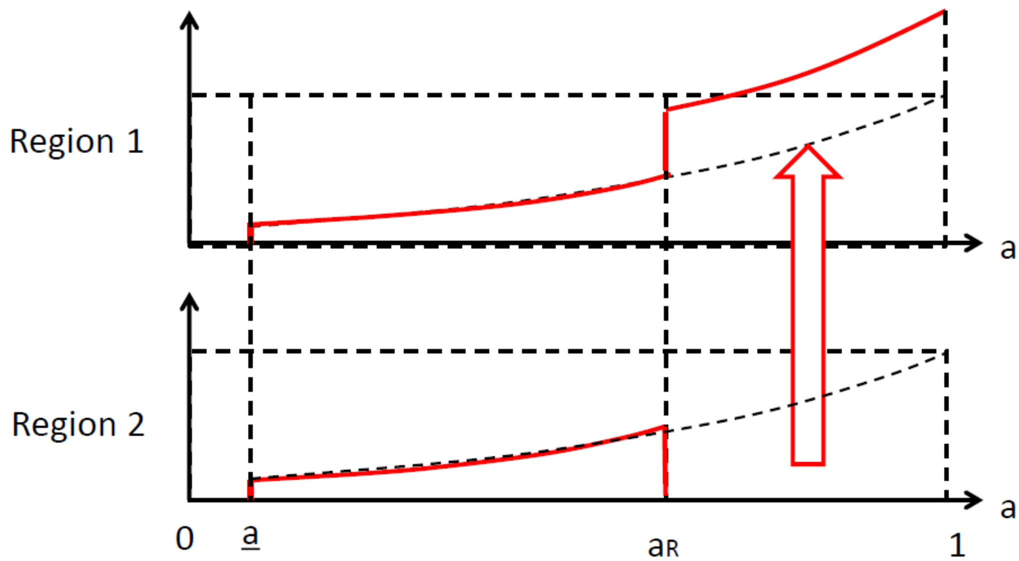

Figure 2: The relocation pattern.

heterogeneity leads to an earlier agglomeration process when trade costs fall. However, as we shall see below, it also delays full agglomeration. The agglomerations process is thus more drawn out. For $\underline{a} = 1$ the productivity distribution collapses to one point, and we have returned to the case of homogeneous firms producing the standard breakpoint with quasilinear utility (see Pluger 2004).

The sorting pattern is illustrated in Figure 2, where firms with a marginal cost above a_R sort to the larger Region 1 market.

2.2.2 The Sustain Point

Next we derive the sustain point, ϕ^S, where full agglomeration in Region 1 just becomes unstable. The condition for this is that $V - V^* = 0$ at full agglomeration in Region 1:

$$
\begin{aligned}
V - V^* &= \frac{\mu}{\sigma}(1-\phi^S)\left(\frac{L+H+H^*}{\Delta} - \frac{L^*}{\phi^S \Delta}\right) a^{1-\sigma} - \frac{\mu}{1-\sigma}(\ln \Delta - \ln \phi^S \Delta) \\
&= \frac{\mu}{\sigma}(1-\phi^S)\left(L+H+H^* - \frac{L^*}{\phi^S}\right)\frac{1}{H+H^*}\frac{a^{1-\sigma}}{\xi} - \frac{\mu}{\sigma-1}\ln \phi^S = 0. \qquad (19)
\end{aligned}
$$

Note that the last term $-\frac{\mu}{\sigma-1}\ln \phi^S$ is always positive for any $\phi^S \in (0,1)$. Thus, the first term must be negative at the sustain point, which means that $(L+H+H^* - \frac{L^*}{\phi^S}) < 0$. The most productive firm ($\underline{a}$), with the highest $a_i^{1-\sigma}$, will have the most negative first term. Therefore, it will be the first to move away from the agglomeration if trade freeness falls. Thus, the last to move into the core agglomeration are also the first to leave if the movement in trade costs is reversed. The sustain point is therefore determined by the relation:

$$\frac{\mu}{\sigma}(1-\phi^S)\left(L+H+H^*-\frac{L^*}{\phi^S}\right)\frac{1}{H+H^*}\frac{\underline{a}^{1-\sigma}}{\xi}-\frac{\mu}{\sigma-1}\ln\phi^S=0 \tag{20}$$

The effect of firm heterogeneity is determined by the term $\frac{\underline{a}^{1-\sigma}}{\xi}=\frac{(1-\sigma+\rho)(1-\underline{a}^{\rho})}{\rho(\underline{a}^{\sigma-1}-\underline{a}^{\rho})}$. As seen from this expression, $\frac{d\frac{\underline{a}^{1-\sigma}}{\xi}}{d\underline{a}}<0$ for $0<\underline{a}<1$. This means that ϕ^S increases in firm heterogeneity for $0<\underline{a}<1$. The first term in (20) vanishes when $\underline{a}=0$, in which case $\phi^S=1$. That is, full agglomeration cannot occur before free trade when we allow for infinitely productive firms. The relocation process is thus more drawn out when firms are heterogeneous. Firm heterogeneity, in this sense, delays the agglomeration process.[10]

2.3 Long-run equilibrium

Having investigated the properties of the model at the break- and sustain points, we now turn to the migration pattern as the model reaches its long equilibrium. Generally, the value of migrating depends on the productivity of the migrating entrepreneur and the entrepreneurs that have already migrated. The problem is manageable because the entrepreneurs here move in order of increasing productivity. The value of migrating for an entrepreneur with the marginal cost a_R is

$$\begin{aligned} v(a_R) &= (\pi(a_R)-\mu\ln P(a_R))-(\pi^*(a_R)-\mu\ln P^*(a_R))= \\ &\quad \frac{\mu}{\sigma}(1-\phi)\left(B(a_R)-B^*(a_R)\right)a_R^{1-\sigma}-\frac{\mu}{1-\sigma}(\ln\Delta(a_R)-\ln\Delta^*(a_R)), \end{aligned} \tag{21}$$

where a_R is the marginal cost of the entrepreneur that is next in line for migrating. $B(a_R)$ and $B^*(a_R)$ are given by:

$$B=\frac{L+H+\int_{a_R}^{1}H^*dF(a)}{\Delta(a_R)},\quad B^*\equiv\frac{L^*+H^*-\int_{a_R}^{1}H^*dF(a)}{\Delta^*(a_R)}, \tag{22}$$

and

$$\begin{aligned} \Delta(a_R) &= H\int_{\underline{a}}^{1}a^{1-\sigma}dF(a)+H^*\int_{a_R}^{1}a^{1-\sigma}dF(a)+\phi H^*\int_{\underline{a}}^{a_R}a^{1-\sigma}dF(a), \\ \Delta^*(a_R) &= H^*\int_{\underline{a}}^{a_R}a^{1-\sigma}dF(a)+\phi H^*\int_{a_R}^{1}a^{1-\sigma}dF(a)+\phi H\int_{\underline{a}}^{1}a^{1-\sigma}dF(a). \end{aligned} \tag{23}$$

[10] The sustain point is another difference as compared to the FC-model in Baldwin and Okubo (2006), which has the same sustain point as in the model by Martin and Rogers (1995).

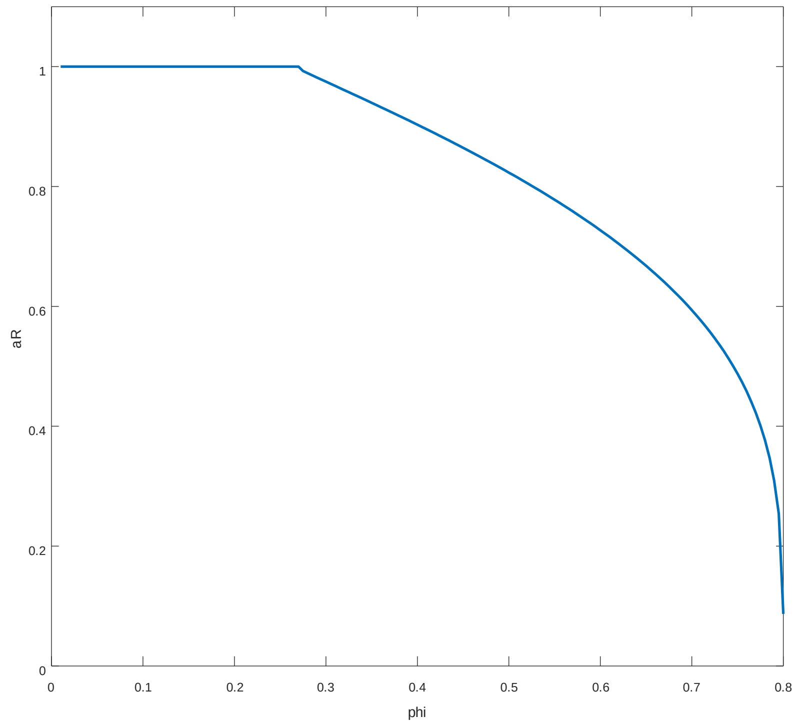

Figure 3: The effect of trade cost reduction on aR.

The long-run equilibrium is defined by $v(a_R) = 0$. This equation cannot be solved analytically and we therefore proceed by simulation. Figure 3 shows a numerical simulation of a_R for some generic parameter values: $\sigma = 5, \rho = 7, \underline{a} = 0.1, \mu = 0.2, L = 200$, and $H = H^* = 25$. The breakpoint for these parameter values $\phi^B = 0.26$ and the sustain point $\phi^S = 0.8$. The corresponding bifurcation diagram is shown in Figure 4. It is seen how the introduction of heterogeneous firms leads to a supercritical pitchfork bifurcation contrary to the subcritical (tomahawk) bifurcation in the standard FE-model. Here, there is no jump or catastrophic relocation at the breakpoint (the bifurcation point). The reason for this is simply that the heterogeneous firms here have heterogeneous gains from migrating, and this implies that they will find it optimal to relocate for different levels of trade costs. This relocation pattern may correspond well to real world data, since instances of abrupt or catastrophic agglomeration are rare in practice (see e.g. Brakman et al. 2004 and Davis and Weinstein 2002, 2008, and Redding et al. 2011).

We now turn to the welfare consequences of this migration pattern.

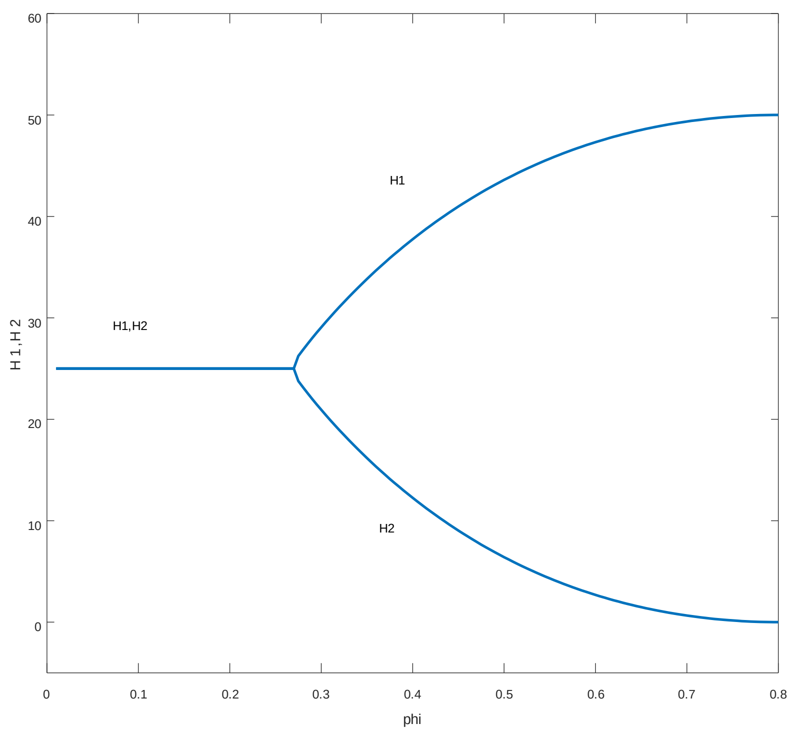

Figure 4: Bifurcation diagram

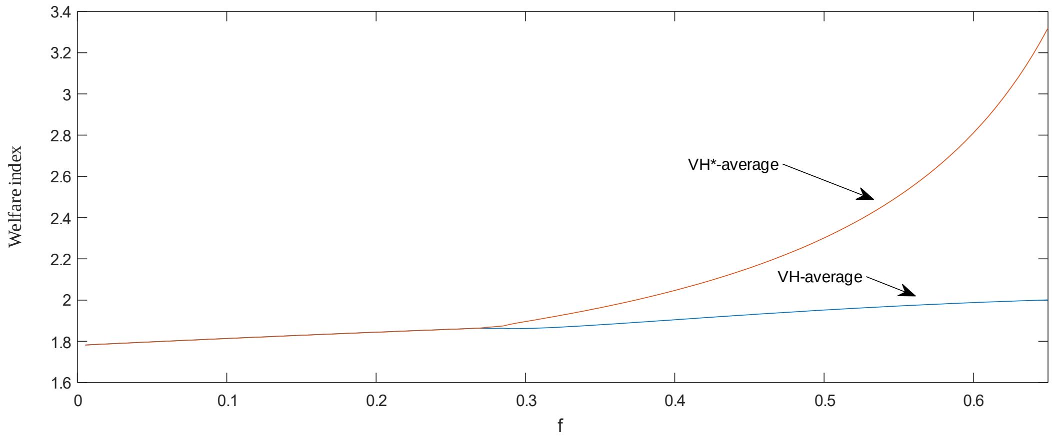

Figure 5: Average welfare of entrepreneurs in Region 1 and Region 2.

3 Welfare

Consider first the entrepreneurs. The average utility index for the entrepreneurs in the core may be expressed as:

$$\overline{V_H} = \frac{H\int\limits_{\underline{a}}^{1}\left[\frac{\mu}{\sigma}(B(a_R)+\phi B^*(a_R))a_i^{1-\sigma}-\frac{\mu}{1-\sigma}\ln\Delta(a_R)\right]dF}{H+\frac{(1-a_R^\rho)}{(1-\underline{a}^\rho)}H^*} + \frac{H^*\int\limits_{a_R}^{1}\left[\frac{\mu}{\sigma}(B(a_R)+\phi B^*(a_R))a_i^{1-\sigma}-\frac{\mu}{1-\sigma}\ln\Delta(a_R)\right]dF}{H+\frac{(1-a_R^\rho)}{(1-\underline{a}^\rho)}H^*}, \tag{24}$$

and the corresponding expression for entrepreneurs in the periphery is

$$\overline{V}_H^* = \frac{H^*\int\limits_{\underline{a}}^{a_R}\left[\frac{\mu}{\sigma}(\phi B(a_R)+B^*(a_R))a_i^{1-\sigma}-\frac{\mu}{1-\sigma}\ln\Delta^*(a_R)\right]dF}{\frac{(a_R^\rho-\underline{a}^\rho)}{(1-\underline{a}^\rho)}H^*}. \tag{25}$$

Figures 5 shows simulations of these for $\underline{a}$= 0.2, $\sigma = 5, \rho = 7, \mu = 0.2, L = 200$, and $H = H^* = 25$. Economic integration and agglomeration benefits entrepreneurs in both regions, as shown in the figure. However, there is a very stark difference between the average utility index in Region 1 and Region 2, after the breakpoint when the low productive entrepreneurs start to agglomerate in Region 1.

Figure 6, shows the corresponding relative average utility of entrepreneurs in Region 1. This ratio falls sharply as trade costs fall and low skilled agglomerate in the region.

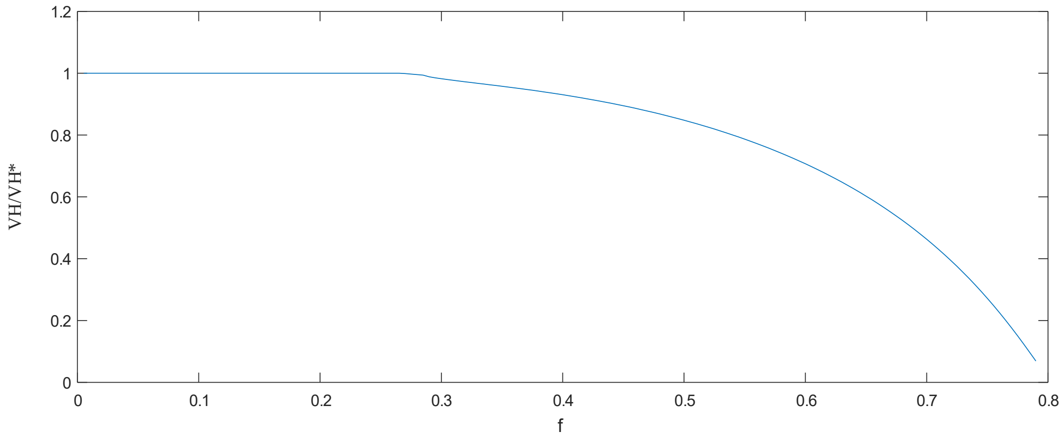

Figure 6: Relative average welfare of entrepreneurs in Region 1.

Next consider the utility of labour. The nominal wage of the immobile factor labour is fixed and determined by the productivity in the agricultural sector. The utility of this factor therefore solely depends on the price index, which falls in both regions as trade costs are reduced. The indirect utility of labour is given by

$$V = w - \mu + \mu \ln \mu + \frac{\mu}{\sigma - 1} \log \Delta. \tag{26}$$

Figure 7 ($\sigma = 5, \rho = 7, \mu = 0.2, L = 200$, and $H = H^* = 25$) shows how an index of labour welfare, as defined by (26), is developing in the two regions when trade costs fall. The first thing that stands out is that welfare is higher in the large region in spite of the reverse sorting. That is, there are advantages of agglomeration (a lower price index) even when low productive entrepreneurs move to the core. Second, all entrepreneurs are in the core once full agglomeration is reached, and the sorting pattern therefore ceases to matter at this point (where the welfare index curve for the core is flat). The figure also shows graphs for three different values of $\underline{a}$. When $\underline{a}$ approaches unity, there is less and less firm heterogeneity as the productivity distribution is compressed. It is seen in the figure how this leads to a later but more drastic agglomeration. Thus, the introduction of heterogeneous firms gives a more gradual and less abrupt localization pattern. It also means that the range of trade costs for which welfare is different in the two regions is larger. A higher $\underline{a}$ also means a lower average productivity. This explains why the welfare index curves in Figure 7 are lower when $\underline{a}$ is higher.

Welfare for the immobile factor in the periphery falls at the breakpoint but thereafter climbs as trade cost are further reduced. A sufficiently deep integration always benefits all factors of production.

Thus, agglomeration leads to higher welfare for all groups in the model when trade liberalization is deep enough. However, it also leads to sharply increasing regional income-differences among the entrepreneurs, as shown in Figure 6. The appropriate tax policy to dampen regional

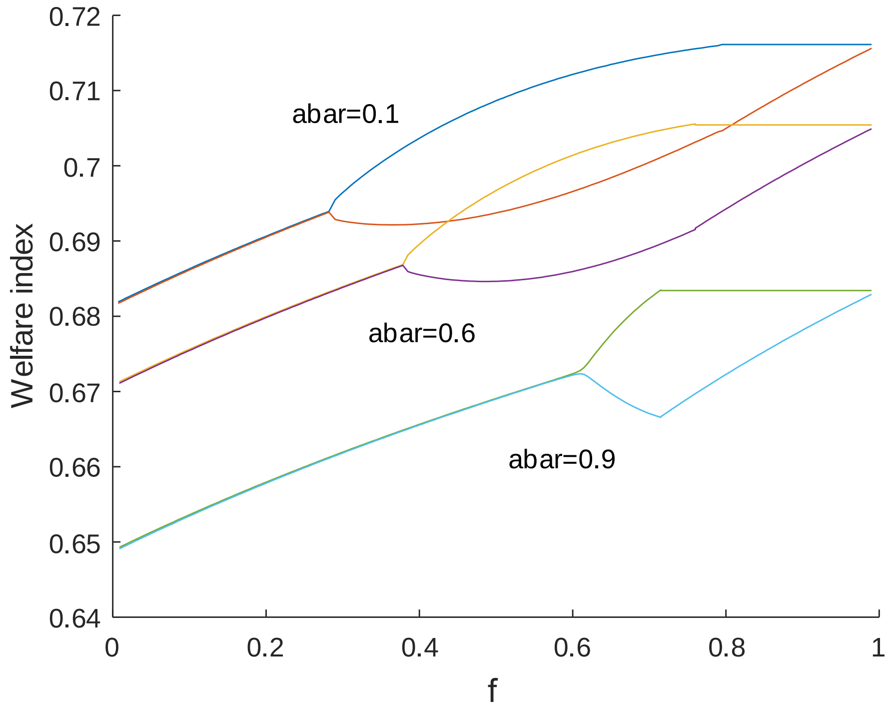

Figure 7: Trade cost reduction, bifurcations and welfare for different values of $\underline{a}$.

income differences is not obvious. Taxing entrepreneurs in the core would dampen agglomeration, but this would amount to a tax primarily on the poor entrepreneurs.

4 Conclusion

This paper develops a simple model of spatial sorting where the least productive entrepreneurs agglomerate to the large core region as trade is liberalized. This sorting pattern differs from the existing literature, and it may be one component explaining the development of poor megacities in the developing world as well as the observed concentration of poor individuals in city centres in the rich world. Agglomeration contributes to efficiency and leads to higher welfare for all groups in the model when trade liberalization is deep enough. However, it also leads to sharply increasing regional income-differences.

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