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# **Complementarity between Firm Exporting and Firm Importing on Industry Productivity and Welfare**

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# Complementarity between Firm Exporting and Firm Importing on Industry Productivity and Welfare<sup>1</sup>

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## Abstract

How different are the impacts of trade barriers on trade flows between intermediate inputs and final goods? How large are the welfare gains from trade for intermediate inputs relative to final goods? To address these questions, we develop a heterogeneous-firm model in which firm exporting and firm importing play a key role in industry productivity and welfare. We derive a gravity equation in intermediate-input trade to show that reductions in intermediate-input trade costs increase aggregate trade flows more than those in final-good trade costs, due to an extra adjustment operating through the extensive margin. We also find the general condition under which the welfare gains from trade are greater in intermediate-input trade than those in final-good trade.

Keywords: Firm exporting, firm importing, heterogeneous firms

JEL classification: F12, F14

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# 1 Introduction

How different are the impacts of trade barriers on trade flows between intermediate inputs and final goods? How large are the welfare gains from trade for intermediate inputs relative to final goods? Though the recent theoretical trade literature has devoted enormous effort to developing new models that match with empirical evidence, there is few theoretical work that investigates a distinctive mechanism through which intermediate-input trade can impact on trade flows and welfare that is absent from final-good trade. The present paper tries to fill this important gap in the literature by deriving a gravity equation in intermediate-input trade and relating the trade elasticity obtained from the gravity equation with the welfare gains from trade.

Intermediate-input trade has been growing faster and its share in total trade is larger than final-good trade, due to outsourcing or offshoring that leads to fragmentation of production across the globe (Hummels et al., 2001; Hanson et al., 2005; Johnson and Noguera, 2012). Some recent empirical work has revealed that when firms import intermediate inputs from foreign countries, firm importing displays a number of the same performance differences as firm exporting. A series of work by Bernard et al. (2007, 2012, 2018a) unveil empirical regularity on firm importing that, just as in exporters, importers are larger and more productive than non-importers within same industries, and only a small fraction of firms can import. Other empirical work also finds that the impact of trade liberalization is quite different between final-good trade and intermediate-input trade. For example, Amiti and Konings (2007) find that input tariff reductions increase industry productivity twice greater than output tariff reductions in Indonesia; Topalova and Khandelwal (2011) similarly find that firms' gains from input tariff reductions can be ten times greater than those from output tariff reductions in India. Further, input tariff reductions give rise to different productivity gains from output tariff reductions by expanding technological possibilities of firms, as shown by Goldberg et al. (2010).

This paper develops a heterogeneous-firm model in which firm exporting and firm importing play a crucial role in industry productivity and welfare. In our model, an industry is composed of two production sectors (i.e., upstream and downstream sectors), where the former (latter) sector exports (imports) intermediate inputs. Whereas intermediate-input firms in the upstream sector are modeled in a similar way to Melitz (2003), one of main departures from Melitz (2003) is that final-good firms in the downstream sector have to incur a fixed cost when procuring intermediate inputs from domestic/foreign markets. As a result, selection occurs not only into the upstream sector but also into the downstream sector, which allows us to capture the empirical pattern that only more productive firms can source foreign intermediate inputs. This framework also allows us to show that input tariff reductions lead to resource reallocations in both sectors, while output tariff reductions lead to reallocations only in the downstream sector, which may help explain why welfare gains from trade can be greater for input tariff reductions than output tariff reductions. For the sake of parsimony, we focus on two symmetric countries; nonetheless, our model delivers new welfare gains from selection into exporting and importing in the two production sectors.

Our first contribution is in showing that the impact of trade costs, both variable and fixed, on aggregate trade flows are in general greater for intermediate-input trade than final-good trade. This is most clearly seen in a special case of a Pareto distribution with free entry. In this case, aggregate intermediate-input exports are expressed as

$$R_t^I = \psi^I L (B^I)^{\frac{k}{\sigma-1}} \tau^{-\frac{k(\sigma-1)}{2(\sigma-1)-k}} f_t^{1-\frac{k}{2(\sigma-1)-k}},$$

whereas aggregate final-good exports are expressed as

$$R_t^F = \psi^F L (B^F)^{\frac{k}{\sigma-1}} (\tau\Gamma)^{-k} f_t^{1-\frac{k}{\sigma-1}},$$

where  $\psi^i$  is some constant term,  $L$  is exporting country size,  $B^i$  is importing country size, and  $\Gamma$  is unit cost of final-good production. It follows immediately from the expressions that the partial trade elasticities with respect to a variable trade cost  $\tau$  and a fixed trade cost  $f_t$  are respectively given by

$$\begin{aligned} \zeta_o^I &\equiv -\frac{\partial \ln R_t^I}{\partial \ln \tau} = \frac{k(\sigma-1)}{2(\sigma-1)-k}, & \xi_o^I &\equiv -\frac{\partial \ln R_t^I}{\partial \ln f_t} = \frac{k}{2(\sigma-1)-k} - 1, \\ \zeta_o^F &\equiv -\frac{\partial \ln R_t^F}{\partial \ln \tau} = k, & \xi_o^F &\equiv -\frac{\partial \ln R_t^F}{\partial \ln f_t} = \frac{k}{\sigma-1} - 1. \end{aligned}$$

Therefore,  $\zeta_o^I > \zeta_o^F$  and  $\xi_o^I > \xi_o^F$  if and only if  $k > \sigma - 1$ . (Notice that  $\zeta_o^F$  and  $\xi_o^F$  are exactly the same as those in Chaney (2008).) This theoretical finding is consistent with the empirical findings by Amiti and Konings (2007) and Topalova and Khandelwal (2011) in that input tariff reductions have a greater impact than output tariff reductions. Although these papers focus on the impact of tariff reductions on industry productivity, we find that the partial trade elasticities are greater for intermediate-input trade than final-good trade in the gravity equation (see Section 4).

Our second contribution is in showing the general condition under which the welfare gains from trade are greater in intermediate-input trade than those in final-good trade. In particular, we find that whether changes in welfare are greater for intermediate-input trade than final-good trade depends critically on changes in the mass of entrants. This suggests that in the special case of a Pareto distribution in which entry does not respond to changes in trade costs, changes in welfare are the same between intermediate-input trade and final-good trade. Put it differently, only when we apply a (untruncated) Pareto distribution, does the welfare formula by Arkolakis et al. (2012) hold for the welfare comparison between the two types of trade, which is the similar caveat raised by Melitz and Redding (2015). More formally, changes in welfare can be expressed in terms of the domestic trade share  $\lambda$  and the trade elasticity for intermediate-input trade  $\zeta_o^I$ :

$$\begin{aligned} \widehat{W}^I &= \widehat{\lambda}^{-\frac{2(\sigma-1)-k}{k(\sigma-1)}} \\ &= \widehat{\lambda}^{-\frac{1}{\zeta_o^I}}, \end{aligned}$$

where  $\widehat{\lambda} \equiv \lambda'/\lambda$  is the change in the share of domestic expenditure. For final-good trade, changes in welfare can be also expressed in terms of  $\lambda$  and the trade elasticity for final-good trade  $\zeta_o^F$ :

$$\begin{aligned}\widehat{W}^F &= \widehat{\lambda}^{-\frac{1}{k}} \\ &= \widehat{\lambda}^{-\frac{1}{\zeta_o^F}}.\end{aligned}$$

Though useful, the welfare results hold only for a (untruncated) Pareto distribution. From this perspective, our contribution relative to the existing literature is to derive the general condition under which the welfare gains from trade are greater in intermediate-input trade than those in final-good trade, which includes the above expressions as a special case of our general result.

This paper is related to the recent theoretical literature that explores the impact of imported intermediate inputs on industry productivity and welfare. Antràs et al. (2017) develop a multi-country sourcing model in which more productive final-good firms profitably import intermediate inputs from a larger number of countries. Under the condition that final goods are non-tradable, they find, like ours, that the aggregate trade elasticity with respect to a variable trade cost tend to be higher than the firm-level trade elasticity in the gravity equation of intermediate-input trade. Since they employ the Eaton-Kortum framework for sourcing inputs, the upstream sector is characterized by perfect competition and hence selection into export markets does not work for intermediate-input firms. Further, they do not address a difference in the welfare gains between intermediate-input trade and final-good trade, which is one of the main focuses in our paper.

Bernard et al. (2018b) develop a model of two-sided heterogeneity in terms of the productivity levels in both intermediate-input firms and final-good firms. Under the condition that final goods are non-tradable, they find that there is negative degree assortivity between these two types of firms, in that more productive intermediate-input firms match with less productive final-good firms. While intermediate-input firms self-select into exporting by incurring a fixed export cost, selection of final-good firms is made by matching without paying a fixed import cost. In practice, however, such a fixed import cost is empirically relevant for why a small fraction of firms import (see, e.g., Kasahara and Lapham 2013; Halpern et al. 2015). Abstracting from matching between the two types of firms, we show that trade-induced changes in the domestic productivity cutoffs arise in both production sectors as a result of self-selection into exporting and importing.

Our finding of the welfare gains from intermediate-input trade is also related to Melitz and Redding (2014b). They show that when non-traded final goods are produced by using a sequence of traded intermediate inputs, the welfare gains from intermediate-input trade are magnified by increasing domestic productivity. They consider, however, perfectly competitive markets in all production sectors in which any firm-level variables do not play a significant role in resource reallocations by input tariff reductions. The current paper, while keeping non-traded final goods produced by a sequence of production stages, focuses on two-sided heterogeneity and show that such reductions raise the domestic productivity cutoff not only in the upstream sector but also in the downstream sector, thereby creating new welfare gains from trade.

## 2 Model

### 2.1 Setup

Consider a model in which two symmetric countries produce both differentiated final goods and differentiated intermediate inputs. An economy is composed of one industry with monopolistic competition and heterogeneous firms (for the production of final goods and intermediate inputs). For simplicity, we assume that final goods are non-tradable but intermediate inputs are tradable. Labor is only a factor of production and each country is endowed with  $L$  units of labor which serves as a numeraire of the model.

Firm behavior is similar to that in Melitz (2003). Upon paying an entry cost  $f_e$ , intermediate-input firms draw productivity  $\varphi$  from a distribution  $G(\varphi)$ ; and final-good firms draw productivity  $\phi$  from a distribution  $G(\phi)$ . The functional forms of the distributions are potentially different. We first consider general distribution functions without any parameterizations and later impose specific distribution functions to obtain closed-form solutions. Intermediate-input firms incur a fixed production cost  $f$ , and if they also choose to export, they incur a variable trade cost  $\tau$  and a fixed trade cost  $f_t$  where  $\tau^{\sigma-1} f_t > f$ . On the other hand, final-good firms incur a fixed production cost  $f$ , and if they also choose to import, they incur a fixed trade cost  $f_t$  where  $\tau^{\sigma-1} f_t > f$ . That is, domestic-sourcing firms who use only domestic input incur only  $f$ , while foreign-sourcing firms who use both domestic input and foreign input incur both  $f$  and  $f_t$ . These costs are the same between final-good firms and intermediate-input firms.

Following Helpman et al. (2008) and Melitz and Redding (2014a), it is useful to define

$$J(a) = \int_a^\infty \left[ \left( \frac{\phi}{a} \right)^{\sigma-1} - 1 \right] dG(\phi),$$

$$V(a) = \int_a^\infty \phi^{\sigma-1} dG(\phi),$$

where  $J(a)$  and  $V(a)$  are strictly decreasing in  $a$ , with  $J(a) + 1 - G(a) = a^{1-\sigma} V(a)$ .

### 2.2 Consumers

Consumers' preferences are given by a C.E.S. utility function with elasticity  $\sigma (> 1)$ :

$$U = \left[ \int_{v \in V} q(v)^{\frac{\sigma-1}{\sigma}} dv + \int_{v \in \tilde{V}} \tilde{q}(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}},$$

where  $q(v)$  and  $\tilde{q}(v)$  are final goods produced by domestic-sourcing firms and foreign-sourcing firms respectively. Utility maximization yields the demand function for final goods:

$$q(v) = R^F P^{\sigma-1} p(v)^{-\sigma},$$

$$\tilde{q}(v) = R^F P^{\sigma-1} \tilde{p}(v)^{-\sigma},$$

where  $R^F = \int_{v \in V} p(v)q(v)dv + \int_{v \in \tilde{V}} \tilde{p}(v)\tilde{q}(v)dv$  is consumers' expenditure of final goods and

$$P = \left[ \int_{v \in V} p(v)^{1-\sigma} dv + \int_{v \in \tilde{V}} \tilde{p}(v)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}$$

is the price index of final goods. Defining an aggregate final good  $Q \equiv U$ , we have  $PQ = R^F$ .

### 2.3 Final-good Firms

Final-good firms' technologies are also given by a C.E.S. production function with elasticity  $\sigma$ :

$$\begin{aligned} q(\phi) &= \phi \left[ \int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \\ \tilde{q}(\phi) &= \phi \left[ \int_{\omega \in \tilde{\Omega}} \tilde{x}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega \in \tilde{\Omega}} \tilde{x}_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \end{aligned} \quad (1)$$

where  $x(\omega)$  is domestic inputs used by domestic-sourcing firms, while  $\tilde{x}(\omega)$  and  $\tilde{x}_t(\omega)$  are domestic and foreign inputs used by foreign-sourcing firms. To facilitate the analysis, we follow Bernard et al. (2018b) in assuming that the elasticity of substitution among intermediate inputs is identical to the elasticity of substitution among final goods. The expenditures of these final-good firms are

$$\begin{aligned} e &= \int_{\omega \in \Omega} \gamma(\omega)x(\omega)d\omega, \\ \tilde{e} &= \int_{\omega \in \tilde{\Omega}} \tilde{\gamma}(\omega)\tilde{x}(\omega)d\omega + \int_{\omega \in \tilde{\Omega}} \tilde{\gamma}_t(\omega)\tilde{x}_t(\omega)d\omega, \end{aligned}$$

where  $\gamma(\omega)$  is domestic input prices faced by domestic-sourcing firms, while  $\tilde{\gamma}(\omega)$  and  $\tilde{\gamma}_t(\omega)$  are domestic and foreign input prices faced by foreign-sourcing firms. It follows from pricing rules of intermediate-input firms that these input prices satisfy

$$\tilde{\gamma}_t(\omega) = \tau\tilde{\gamma}(\omega) = \tau\gamma(\omega).$$

Cost minimization yields the input demand function for domestic-sourcing firms:

$$\begin{aligned} x(\omega) &= e\Gamma^{\sigma-1}\gamma(\omega)^{-\sigma}, \\ \Gamma &= \left[ \int_{\omega \in \Omega} \gamma(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \end{aligned} \quad (2)$$

and the input demand function for foreign-sourcing firms:

$$\begin{aligned} \tilde{x}(\omega) &= \tilde{e}\tilde{\Gamma}^{\sigma-1}\tilde{\gamma}(\omega)^{-\sigma}, \quad \tilde{x}_t(\omega) = \tau^{-\sigma}\tilde{x}(\omega), \\ \tilde{\Gamma} &= \left[ \int_{\omega \in \tilde{\Omega}} \tilde{\gamma}(\omega)^{1-\sigma} d\omega + \int_{\omega \in \tilde{\Omega}} \tilde{\gamma}_t(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \end{aligned} \quad (3)$$

$\Gamma$  and  $\tilde{\Gamma}$  are the dual unit cost functions for final-good production by domestic-sourcing firms and foreign-sourcing firms respectively. These dual unit cost functions satisfy

$$\begin{aligned}\tilde{\Gamma}^{1-\sigma} &= \int_{\omega \in \tilde{\Omega}} \tilde{\gamma}(\omega)^{1-\sigma} d\omega + \int_{\omega \in \tilde{\Omega}} \tilde{\gamma}_t(\omega)^{1-\sigma} d\omega \\ &= (1 + \tau^{1-\sigma} \Lambda^I) \int_{\omega \in \tilde{\Omega}} \tilde{\gamma}(\omega)^{1-\sigma} d\omega \\ &= (1 + \tau^{1-\sigma} \Lambda^I) \Gamma^{1-\sigma},\end{aligned}\tag{4}$$

where  $\Lambda^I (< 1)$  is the (endogenous) market share of input exporters in the domestic market. To see this, suppose that  $\tau$  is sufficiently large that no intermediate-input firms profitably export. Then,  $\Lambda^I = 0$  and the unit cost is the same between domestic-sourcing firms and foreign-sourcing firms. In the analysis below, we will focus on the range of  $\tau$  such that only more productive firms profitably export intermediate inputs. Then,  $\Lambda^I < 1$  and together with  $\tau^{1-\sigma} < 1$ , (4) implies that the unit cost is lower for foreign-sourcing firms than domestic-sourcing firms. Intuitively, using both domestic and foreign inputs, foreign-sourcing firms are able to exploit a love-of-variety effect for production, which allows them to produce relatively more efficiently.

Substituting (2) and (3) into (1) yields

$$\begin{aligned}q(\phi) &= \phi \frac{e}{\Gamma} \iff e(\phi) = \frac{\Gamma}{\phi} q(\phi), \\ \tilde{q}(\phi) &= \phi \frac{\tilde{e}}{\tilde{\Gamma}} \iff \tilde{e}(\phi) = \frac{\tilde{\Gamma}}{\phi} \tilde{q}(\phi).\end{aligned}$$

The profits of the two types of firms are then

$$\begin{aligned}\pi^F(\phi) &= p(\phi)q(\phi) - \frac{\Gamma}{\phi} q(\phi) - f, \\ \tilde{\pi}^F(\phi) &= \tilde{p}(\phi)\tilde{q}(\phi) - \frac{\tilde{\Gamma}}{\phi} \tilde{q}(\phi) - f - f_t.\end{aligned}$$

The pricing rules are given by

$$\begin{aligned}p(\phi) &= \frac{\sigma}{\sigma - 1} \frac{\Gamma}{\phi}, \\ \tilde{p}(\phi) &= \frac{\sigma}{\sigma - 1} \frac{\tilde{\Gamma}}{\phi}.\end{aligned}$$

As usual, the pricing rules are a constant markup over marginal cost. Since the unit cost differs between the two types of firms, however, the equilibrium price is lower for foreign-sourcing firms than domestic-sourcing firms for a given productivity level:

$$\tilde{\Gamma} < \Gamma \implies \tilde{p}(\phi) < p(\phi).$$



Using (4), the equilibrium revenues of the two types of firms are

$$\begin{aligned} r^F(\phi) &= \sigma \Gamma^{1-\sigma} B^F \phi^{\sigma-1}, \\ \tilde{r}^F(\phi) &= \sigma(1 + \tau^{1-\sigma} \Lambda^I) \Gamma^{1-\sigma} B^F \phi^{\sigma-1}, \end{aligned}$$

where

$$B^F = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} R^F P^{\sigma-1}$$

is the index of final-good market demand. The equilibrium profits are

$$\begin{aligned} \pi^F(\phi) &= \frac{r^F(\phi)}{\sigma} - f = \Gamma^{1-\sigma} B^F \phi^{\sigma-1} - f, \\ \tilde{\pi}^F(\phi) &= \frac{\tilde{r}^F(\phi)}{\sigma} - f - f_t = (1 + \tau^{1-\sigma} \Lambda^I) \Gamma^{1-\sigma} B^F \phi^{\sigma-1} - f - f_t. \end{aligned}$$

Figure 1 draws  $\pi^F(\phi)$  and  $\tilde{\pi}^F(\phi)$  in the  $(\phi^{\sigma-1}, \pi^F)$  space, where the slope is measured by the unit cost and the intercept is measured by the fixed cost. Simple inspection of the two profit functions reveal that the slope of  $\tilde{\pi}^F(\phi)$  is greater than that of  $\pi^F(\phi)$  but the intercept is also greater, which is a key tradeoff that foreign-sourcing firms face: while they reduce the unit cost by exploiting the love-of-variety effect for production, they have to incur the additional fixed cost  $f_t$  to source foreign inputs. Thus only more productive firms that profitably incur this fixed cost are able to enjoy the benefit of lowering the unit cost by importing foreign inputs.

To characterize the equilibrium of the downstream sector, we identify the productivity cutoffs that satisfy  $\pi^F(\phi^*) = 0$  and  $\pi^F(\phi_t^*) = \tilde{\pi}^F(\phi_t^*)$ . This condition, which we hereafter refer to as the zero cutoff profit (ZCP) condition, yields

$$\Gamma^{1-\sigma} B^F (\phi^*)^{\sigma-1} = f, \tag{5}$$

$$\tau^{1-\sigma} \Lambda^I \Gamma^{1-\sigma} B^F (\phi_t^*)^{\sigma-1} = f_t, \tag{6}$$

which imply that

$$\left( \frac{\phi_t^*}{\phi^*} \right)^{\sigma-1} = \frac{1}{\Lambda^I} \frac{\tau^{\sigma-1} f_t}{f} > 1. \tag{7}$$

From Figure 1, there arises the following sorting pattern in the downstream sector: firms with lowest productivity below  $\phi^*$  exit immediately; firms with intermediate productivity between  $\phi^*$  and  $\phi_t^*$  source only domestic input; and firms with highest productivity above  $\phi_t^*$  source domestic input and foreign input. This prediction of the model is consistent with the recent evidence that selection is crucial not only for firm exporting but also for firm importing (see, e.g., Bernard et al., 2007; 2012; 2018a).

In addition to (5) and (6), we need to impose the free entry (FE) condition. Free entry implies that the expected profits must be equal to the fixed entry cost ex ante. Since the productivity levels for domestic-sourcing firms are between  $\phi^*$  and  $\phi_t^*$  and those for foreign-sourcing firms are

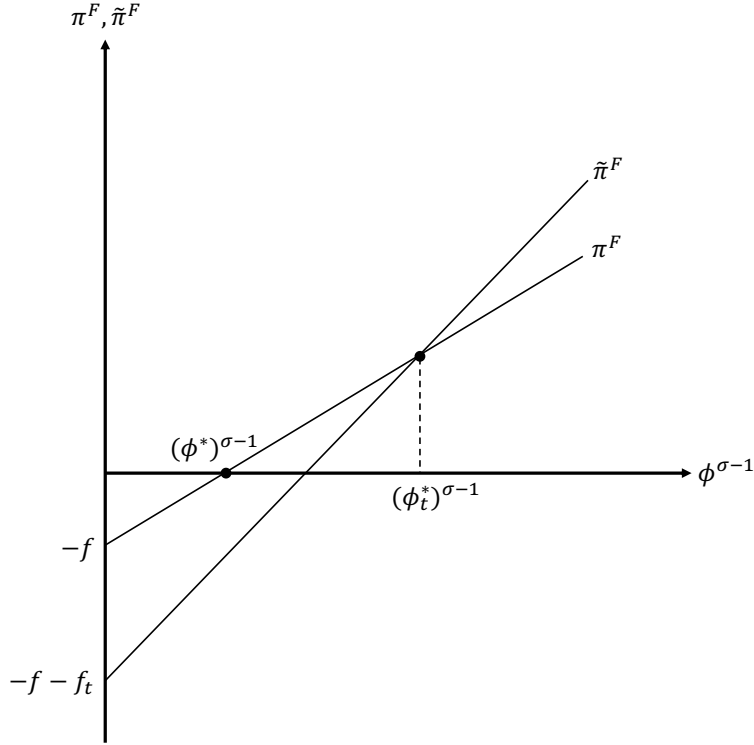


FIGURE 1 – Profits by domestic-sourcing firms and foreign-sourcing firms

above  $\phi_t^*$ , the free entry condition for final-good firms is defined as

$$\begin{aligned}
 & \int_{\phi^*}^{\phi_t^*} \pi^F(\phi) dG(\phi) + \int_{\phi_t^*}^{\infty} \tilde{\pi}^F(\phi) dG(\phi) = f_e \\
 \Leftrightarrow & \int_{\phi^*}^{\infty} \pi^F(\phi) dG(\phi) + \int_{\phi_t^*}^{\infty} (\tilde{\pi}^F(\phi) - \pi^F(\phi)) dG(\phi) = f_e \\
 \Leftrightarrow & fJ(\phi^*) + f_t J(\phi_t^*) = f_e.
 \end{aligned} \tag{8}$$

It is important to note that the ZCP and FE conditions cannot characterize the equilibrium of the downstream sector in this model. As shown by (7), the productivity cutoffs are affected by the market share of input exporters  $\Lambda^I$ . As will be demonstrated later, input trade liberalization induces endogenous changes in  $\Lambda^I$  in the upstream sector, which in turn has a critical impact on selection in the downstream sector.

We also need to define the labor market clearing (LMC) condition of the economy. Since labor is used in the two production sectors in the model, let us first consider aggregate amount of labor used for final-good production:

$$L^F = N_e f_e + N_e \int_{\phi^*}^{\phi_t^*} f dG(\phi) + N_e \int_{\phi_t^*}^{\infty} (f + f_t) dG(\phi).$$

The first, second and third terms in the right-hand side are respectively labor used by entrants, domestic-sourcing firms, and foreign-sourcing firms, where  $N_e$  is the mass of entrants in the downstream sector. Labor is used only for the fixed costs in the final-good production, because final-good firms purchase intermediate inputs from the markets and hence labor is not used for the variable costs (i.e., for transforming inputs into outputs). Using (8),  $L^F$  is expressed as

$$\begin{aligned} L^F &= N_e \int_{\phi^*}^{\phi_t^*} (r^F(\phi) - e(\phi)) dG(\phi) + N_e \int_{\phi_t^*}^{\infty} (\tilde{r}^F(\phi) - \tilde{e}(\phi)) dG(\phi) \\ &= R^F - E, \end{aligned} \quad (9)$$

where  $R^F$  and  $E$  are aggregate revenue and expenditure of final-good firms respectively.

Aggregate revenue of final-good firms is written as

$$\begin{aligned} R^F &= N_e \int_{\phi^*}^{\phi_t^*} r^F(\phi) dG(\phi) + N_e \int_{\phi_t^*}^{\infty} \tilde{r}^F(\phi) dG(\phi) \\ &= N_e \sigma \Gamma^{1-\sigma} B^F V(\phi^*) (1 + \tau^{1-\sigma} \Lambda^F \Lambda^I), \end{aligned} \quad (10)$$

where

$$\Lambda^F = \frac{V(\phi_t^*)}{V(\phi^*)} < 1$$

is the (endogenous) market share of foreign-sourcing firms in the domestic market. This  $\Lambda^F$  is a counterpart to  $\Lambda^I$  in (4), which appears in the ZCP condition of intermediate-input firms later. Noting that aggregate revenue of final-good firms equals aggregate expenditure of consumers, the share of domestic expenditure in aggregate expenditure (from a viewpoint of consumers) is

$$\lambda = \frac{N_e \int_{\phi^*}^{\infty} r^F(\phi) dG(\phi)}{R^F} = \frac{1}{1 + \tau^{1-\sigma} \Lambda^F \Lambda^I}.$$

The domestic share  $\lambda$  proves to be one of important statistics for estimating the welfare gains from trade in our model.

As for the mass of entrants, using (5), (6), (8) and (9), we have

$$N_e = \frac{L^F + E}{\sigma \{f[1 - G(\phi^*)] + f_t[1 - G(\phi_t^*)] + f_e\}}.$$

The masses of domestic-sourcing firms and foreign-sourcing firms, in turn, are  $N = [1 - G(\phi^*)]N_e$  and  $N_t = [1 - G(\phi_t^*)]N_e$  respectively.

Using the productivity cutoffs, the price index is defined as

$$\begin{aligned} P^{1-\sigma} &= N_e \int_{\phi^*}^{\phi_t^*} p(\phi)^{1-\sigma} dG(\phi) + N_e \int_{\phi_t^*}^{\infty} \tilde{p}(\phi)^{1-\sigma} dG(\phi) \\ &= N_e \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \Gamma^{1-\sigma} V(\phi^*) (1 + \tau^{1-\sigma} \Lambda^F \Lambda^I). \end{aligned}$$

From (10), the price index is related to market demand  $B^F$ :

$$P^{1-\sigma} = \frac{(\sigma - 1)^{\sigma-1} R^F}{\sigma^\sigma B^F}.$$

Finally, we derive final-good demands and input demands in terms of market demand  $B^F$  and productivity levels  $\phi, \varphi$ . Substituting the pricing rules of final goods, we get final-good demands by domestic-sourcing firms and foreign-sourcing firms:

$$\begin{aligned} q(\phi) &= (\sigma - 1)\Gamma^{-\sigma} B^F \phi^\sigma, \\ \tilde{q}(\phi) &= (\sigma - 1)\tilde{\Gamma}^{-\sigma} B^F \phi^\sigma, \end{aligned}$$

which gives us individual expenditures of these firms:

$$\begin{aligned} e(\phi) &= (\sigma - 1)\Gamma^{1-\sigma} B^F \phi^{\sigma-1}, \\ \tilde{e}(\phi) &= (\sigma - 1)\tilde{\Gamma}^{1-\sigma} B^F \phi^{\sigma-1}. \end{aligned}$$

Using these, the aggregate expenditure of final-good firms is given by

$$\begin{aligned} E &= N_e \int_{\phi^*}^{\phi_t^*} e(\phi) dG(\phi) + N_e \int_{\phi_t^*}^{\infty} \tilde{e}(\phi) dG(\phi), \\ &= N_e (\sigma - 1) \Gamma^{1-\sigma} B^F V(\phi^*) (1 + \tau^{1-\sigma} \Lambda^F \Lambda^I), \end{aligned} \tag{11}$$

which is given in (9). It follows from (10) that this expenditure turns out to be a fraction of the aggregate expenditure of consumers:

$$E = \left( \frac{\sigma - 1}{\sigma} \right) R^F.$$

Similarly, substituting the pricing rules of intermediate input derived later into (2) and (3), we also get intermediate-input demands by domestic-sourcing firms and foreign-sourcing firms:

$$\begin{aligned} x(\phi, \varphi) &= \tilde{x}(\phi, \varphi) = (\sigma - 1) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma B^F \phi^{\sigma-1} \varphi^\sigma, \\ \tilde{x}_t(\phi, \varphi) &= \tau^{-\sigma} x(\phi, \varphi). \end{aligned} \tag{12}$$

Observe that while final-good demands depend only on the productivity level of final-good firms, intermediate-input demands depend on the productivity levels of both intermediate-input firms and final-good firms.

This is the characterization of the downstream sector. Since the interaction between the two production sectors is key to examining the impact of trade in the present model, let us next turn to characterizing the upstream sector and explore the channel through which the two production sectors intersect with each other.

## 2.4 Intermediate-input Firms

Intermediate-input firms' technologies are represented by a linear cost function of input. The cost function needs to reflect the fact that only a fraction of final-good firms source domestic and foreign inputs among entrants: the productivity levels for final-good firms who source domestic (foreign) inputs must be greater than  $\phi^*$  ( $\phi_t^*$ ) among  $N_e$  entrants. From  $x(\phi, \varphi) = \tilde{x}(\phi, \varphi)$ , labor used for domestic production and exporting by intermediate-input firms is

$$l = f + N_e \int_{\phi^*}^{\infty} \frac{x(\phi, \varphi)}{\varphi} dG(\phi),$$

$$l_t = f_t + N_e \int_{\phi_t^*}^{\infty} \frac{\tau \tilde{x}_t(\phi, \varphi)}{\varphi} dG(\phi),$$

where  $x(\phi, \varphi)$  and  $\tilde{x}_t(\phi, \varphi)$  are input demands by final-good firms in (12).

The revenues of domestic firms and exporting firms also depend on the productivity levels of final-good firms: domestic (exporting) firms sell their input to final-good firms whose productivity is greater than  $\phi^*$  ( $\phi_t^*$ ) among  $N_e$  entrants. The profits of the two types of firms are then

$$\pi^I = N_e \int_{\phi^*}^{\infty} \gamma(\varphi) x(\phi, \varphi) dG(\phi) - N_e \int_{\phi^*}^{\infty} \frac{x(\phi, \varphi)}{\varphi} dG(\phi) - f,$$

$$\pi_t^I = N_e \int_{\phi_t^*}^{\infty} \tilde{\gamma}_t(\varphi) \tilde{x}_t(\phi, \varphi) dG(\phi) - N_e \int_{\phi_t^*}^{\infty} \frac{\tau \tilde{x}_t(\phi, \varphi)}{\varphi} dG(\phi) - f_t.$$

The pricing rules are given by

$$\gamma(\varphi) = \tilde{\gamma}(\varphi) = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi},$$

$$\tilde{\gamma}_t(\varphi) = \frac{\sigma}{\sigma - 1} \frac{\tau}{\varphi},$$

and thus the pricing rules of intermediate-input firms satisfy  $\tilde{\gamma}_t(\varphi) = \tau \gamma(\varphi) = \tau \tilde{\gamma}(\varphi)$  for a given productivity level as shown in Section 2.3.

Using (12) and the above pricing rules, the equilibrium revenues of the two types of firms are

$$r^I(\varphi) = N_e \sigma \left( \frac{\sigma - 1}{\sigma} \right)^\sigma B^F V(\phi^*) \varphi^{\sigma-1},$$

$$r_t^I(\varphi) = N_e \tau^{1-\sigma} \sigma \left( \frac{\sigma - 1}{\sigma} \right)^\sigma B^F V(\phi_t^*) \varphi^{\sigma-1}.$$

These revenues can be expressed in terms of intermediate-input market demand. To show this, note that aggregate revenue of intermediate-input firms  $R^I$  must equal aggregate expenditure of final-good firms  $E$  in (11) in equilibrium:

$$R^I = E \iff R^I = \left( \frac{\sigma - 1}{\sigma} \right) R^F,$$

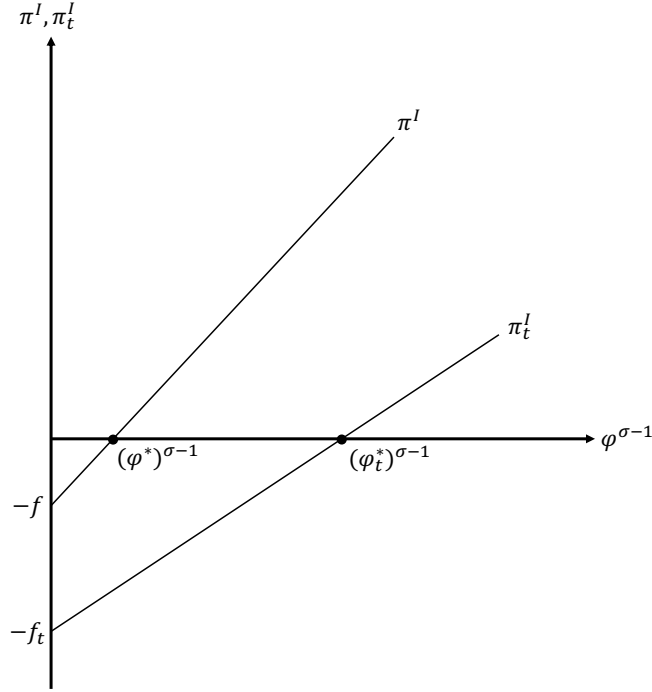


FIGURE 2 – Profits by domestic firms and exporting firms

where aggregate revenue of final-good firms  $R^F$  is given in (10). Substituting (10) into the above equality and rearranging, the equilibrium revenues are expressed as

$$\begin{aligned} r^I(\varphi) &= \sigma B^I \varphi^{\sigma-1}, \\ r_t^I(\varphi) &= \sigma \tau^{1-\sigma} \Lambda^F B^I \varphi^{\sigma-1}, \end{aligned}$$

where

$$B^I = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} \lambda R^I \Gamma^{\sigma-1}$$

is the index of intermediate-input market demand. The equilibrium profits are then given by

$$\begin{aligned} \pi^I(\varphi) &= \frac{r^I(\varphi)}{\sigma} - f = B^I \varphi^{\sigma-1} - f, \\ \pi_t^I(\varphi) &= \frac{r_t^I(\varphi)}{\sigma} - f_t = \tau^{1-\sigma} \Lambda^F B^I \varphi^{\sigma-1} - f_t. \end{aligned}$$

Figure 2 draws  $\pi^I(\varphi)$  and  $\pi_t^I(\varphi)$  in the  $(\varphi^{\sigma-1}, \pi^I)$  space. Under the condition that  $\tau^{1-\sigma} \Lambda^F < 1$ , the slope of exporting firms is smaller than the slope of domestic firms. Suppose that  $\tau$  is sufficiently large that no intermediate-input firm exports, and hence no final-good firm imports. Then, the share of foreign-sourcing firms is zero ( $\Lambda^F = 0$ ) and exporting firms cannot earn revenue from the foreign market. In the analysis below, we will focus on the range of  $\tau$  such that only more productive firms profitably import ( $\Lambda^F < 1$ ), in which case  $\pi_t^I(\varphi)$  is flatter than  $\pi^I(\varphi)$ .

The ZCP condition identifies the productivity cutoffs that satisfy  $\pi^I(\varphi^*) = 0$  and  $\pi_t^I(\varphi_t^*) = 0$ :

$$B^I(\varphi^*)^{\sigma-1} = f, \quad (13)$$

$$\tau^{1-\sigma} \Lambda^F B^I(\varphi_t^*)^{\sigma-1} = f_t, \quad (14)$$

which imply that

$$\left(\frac{\varphi_t^*}{\varphi^*}\right)^{\sigma-1} = \frac{1}{\Lambda^F} \frac{\tau^{\sigma-1} f_t}{f} > 1. \quad (15)$$

As in Figure 1, Figure 2 shows the following sorting pattern in the upstream sector: firms with lowest productivity below  $\varphi^*$  exit immediately; firms with intermediate productivity between  $\varphi^*$  and  $\varphi_t^*$  produce only for the domestic market; and firms with highest productivity above  $\varphi_t^*$  also export to the foreign market. Despite the similarity between firm exporting and firm importing, there exists a key difference between them:  $\pi_t^I(\varphi)$  is the *additional* profits from exporting, while  $\tilde{\pi}^F(\phi)$  is the *total* profits from foreign sourcing. Because of this difference, the profit function of exporting (importing) firms is flatter (steeper) than that of domestic firms, even with the similar selection in both production sectors.

In addition, we need to impose the FE condition:

$$\begin{aligned} & \int_{\varphi^*}^{\infty} \pi^I(\varphi) dG(\varphi) + \int_{\varphi_t^*}^{\infty} \pi_t^I(\varphi) dG(\varphi) = f_e \\ \Leftrightarrow & fJ(\varphi^*) + f_t J(\varphi_t^*) = f_e. \end{aligned} \quad (16)$$

As in the downstream sector, the ZCP and FE conditions cannot characterize the equilibrium in the upstream sector. The productivity cutoffs are affected by  $\Lambda^F$ , which captures selection in the downstream sector (see (15)). Thus, when examining the impact of trade, it is necessary to take into account the interaction between the upstream sector and downstream sector.

To characterize the LMC condition of the economy, we need to take into account not only the aggregate amount of labor used for final-good production but also that used for intermediate-input production:

$$L^I = M_e f_e + M_e \int_{\varphi^*}^{\infty} \left( f + \int_{\phi^*}^{\infty} \frac{x(\varphi, \phi)}{\varphi} dG(\phi) \right) dG(\varphi) + M_e \int_{\varphi_t^*}^{\infty} \left( f_t + \int_{\phi_t^*}^{\infty} \frac{\tau \tilde{x}_t(\varphi, \phi)}{\varphi} dG(\phi) \right) dG(\varphi).$$

The first, second, and third terms in the right-hand side are respectively labor used by entrants, domestic firms, and exporting firms, where  $M_e$  is the mass of entrants in the upstream sector. Using (16),  $L^I$  is expressed as

$$\begin{aligned} L^I &= M_e \int_{\varphi^*}^{\infty} r^I(\varphi) dG(\varphi) + M_e \int_{\varphi_t^*}^{\infty} r_t^I(\varphi) dG(\varphi) \\ &= R^I. \end{aligned}$$

Aggregate revenue of intermediate-input firms is written as

$$\begin{aligned} R^I &= M_e \int_{\varphi^*}^{\infty} r^I(\varphi) dG(\varphi) + M_e \int_{\varphi_t^*}^{\infty} r_t^I(\varphi) dG(\varphi) \\ &= M_e \sigma B^I V(\varphi^*) (1 + \tau^{1-\sigma} \Lambda^F \Lambda^I). \end{aligned} \quad (17)$$

Noting that aggregate revenue of intermediate-input firms equals aggregate expenditure of final-good firms, the domestic share (from a viewpoint of final-good firms) is given by

$$\lambda = \frac{M_e \int_{\varphi^*}^{\infty} r^I(\varphi) dG(\varphi)}{R^I} = \frac{1}{1 + \tau^{1-\sigma} \Lambda^F \Lambda^I},$$

which is exactly the same as the domestic share from a viewpoint of consumers.

As for the mass of entrants, using (13), (14), (16) and (17),

$$M_e = \frac{L^I}{\sigma \{f[1 - G(\varphi^*)] + f_t[1 - G(\varphi_t^*)] + f_e\}}.$$

The masses of domestic firms and exporting firms, in turn, are given by  $M = [1 - G(\varphi^*)]M_e$  and  $M_t = [1 - G(\varphi_t^*)]M_e$  respectively.

Using the productivity cutoffs, the unit cost of domestic-sourcing firms is

$$\begin{aligned} \Gamma^{1-\sigma} &= M_e \int_{\varphi^*}^{\infty} \gamma(\varphi)^{1-\sigma} dG(\varphi) \\ &= M_e \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} V(\varphi^*). \end{aligned}$$

Similarly, the unit cost of foreign-sourcing firms is

$$\begin{aligned} \tilde{\Gamma}^{1-\sigma} &= M_e \int_{\varphi^*}^{\infty} \tilde{\gamma}(\varphi)^{1-\sigma} dG(\varphi) + M_e \int_{\varphi_t^*}^{\infty} \tilde{\gamma}_t(\varphi)^{1-\sigma} dG(\varphi) \\ &= M_e \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} V(\varphi^*) \left( 1 + \frac{\tau^{1-\sigma} V(\varphi_t^*)}{V(\varphi^*)} \right) \\ &= \Gamma^{1-\sigma} (1 + \tau^{1-\sigma} \Lambda^I), \end{aligned}$$

where

$$\Lambda^I = \frac{V(\varphi_t^*)}{V(\varphi^*)} < 1$$

is the (endogenous) market share of exporters in the domestic market (see (4)). From (17), these unit costs are related to market demand  $B^I$ :

$$\begin{aligned} \Gamma^{1-\sigma} &= \frac{(\sigma - 1)^{\sigma-1} \lambda R^I}{\sigma^\sigma B^I}, \\ \tilde{\Gamma}^{1-\sigma} &= \frac{(\sigma - 1)^{\sigma-1} \lambda R^I}{\sigma^\sigma B^I} (1 + \tau^{1-\sigma} \Lambda^I). \end{aligned}$$



Finally, we define the LMC condition of the economy. The LMC condition implies that the amount of labor used in the production sectors equals the fixed labor endowment:  $L^F + L^I = L$ . From  $L^F = R^F - R^I$  (as  $R^I = E$ ) and  $L^I = R^I$ , final-good expenditure equals aggregate payments to labor:  $R^F = L$ . Further, from  $R^I = \left(\frac{\sigma-1}{\sigma}\right) R^F$ , labor allocations are exogenously fixed:

$$L^F = \left(\frac{1}{\sigma}\right) L, \quad L^I = \left(\frac{\sigma-1}{\sigma}\right) L.$$

This completes the characterization of the model. The next section solves for the equilibrium for the general distribution function first, and for the untruncated Pareto distribution next.

### 3 Equilibrium

#### 3.1 General Properties

##### 3.1.1 Equilibrium Variables

Having described the equilibrium conditions, we characterize the important variables in general equilibrium. Since there are the four ZCP conditions and two FE conditions ((5), (6), (8), (13), (14) and (16)), these conditions jointly provide the implicit solutions for the following six unknowns:

$$\phi^*, \phi_t^*, B^F, \varphi^*, \varphi_t^*, B^I.$$

The LMC condition can be omitted by Walras's law by setting labor as a numeraire of the model. The other equilibrium variables can be written as a function of these endogenous variables.

In what follows, we will consider trade liberalization of intermediate inputs because we have assumed that final goods are non-tradable. As shown in the Appendix, reductions in the variable trade cost  $\tau$  or the fixed trade cost  $f_t$  give rise to the following effects:

$$\begin{aligned} \frac{d\phi^*}{d\tau} < 0, \quad \frac{d\phi_t^*}{d\tau} > 0, \quad \frac{dB^F}{d\tau} > 0, \quad \frac{d\varphi^*}{d\tau} < 0, \quad \frac{d\varphi_t^*}{d\tau} > 0, \quad \frac{dB^I}{d\tau} > 0, \\ \frac{d\phi^*}{df_t} < 0, \quad \frac{d\phi_t^*}{df_t} > 0, \quad \frac{dB^F}{df_t} > 0, \quad \frac{d\varphi^*}{df_t} < 0, \quad \frac{d\varphi_t^*}{df_t} > 0, \quad \frac{dB^I}{df_t} > 0. \end{aligned} \tag{18}$$

These hold if

$$(\sigma - 1)^2 - \theta^F \theta^I > 0, \tag{19}$$

where

$$\theta^F \equiv -\frac{d \ln V(\phi^*)}{d \ln \phi^*} > 0, \quad \theta^I \equiv -\frac{d \ln V(\varphi^*)}{d \ln \varphi^*} > 0$$

are the extensive margin elasticities in the respective sectors. Hence (19) requires the extensive margin elasticity not too large relative to the intensive margin elasticity  $(\sigma - 1)$ , which is assumed hereafter.

Reductions in the trade costs increase the domestic productivity cutoff; however, this occurs not only in the upstream sector but in the downstream sector. Such reductions also decrease the export/import productivity cutoffs. This suggests that trade liberalization of intermediate inputs gives rise to the Melitz-type reallocations in both production sectors. Perhaps surprisingly, trade liberalization of final goods does *not* generate these two-sided reallocations in the current setting, in which case only the downstream sector experiences the reallocations (see Appendix).

Given the impact of the trade costs on the six unknowns, it is straightforward to analyze the impact on the other equilibrium variables. Regarding the mass of firms, we have

$$\begin{aligned} \frac{dM_e}{d\tau} \geq 0, \quad \frac{dM}{d\tau} > 0, \quad \frac{dM_t}{d\tau} < 0, \quad \frac{dN_e}{d\tau} \geq 0, \quad \frac{dN}{d\tau} > 0, \quad \frac{dN_t}{d\tau} < 0, \\ \frac{dM_e}{df_t} \geq 0, \quad \frac{dM}{df_t} > 0, \quad \frac{dM_t}{df_t} < 0, \quad \frac{dN_e}{df_t} \geq 0, \quad \frac{dN}{df_t} > 0, \quad \frac{dN_t}{df_t} < 0. \end{aligned}$$

Reductions in the trade costs decrease the mass of domestic firms in the two production sectors. Such reductions also increase the mass of importing firms as well as the mass of exporting firms. This co-movement in vertical linkages is consistent with empirical evidence in Ara et al. (2018), who find that import tariff reductions after China's WTO accession induce not only entry of other countries' input exporters into China but also entry of Chinese input importers.

As for the impact on the shares of foreign firms and domestic expenditure, we have

$$\begin{aligned} \frac{d\Lambda^F}{d\tau} < 0, \quad \frac{d\Lambda^I}{d\tau} < 0, \quad \frac{d\lambda}{d\tau} > 0, \\ \frac{d\Lambda^F}{df_t} < 0, \quad \frac{d\Lambda^I}{df_t} < 0, \quad \frac{d\lambda}{df_t} > 0. \end{aligned}$$

Reductions in the trade costs increase the share of exporters/importers in the domestic market, which indirectly narrow the productivity gaps between domestic firms and exporting/importing firms (see (7) and (15)). Simultaneously, such reductions lower the share of domestic expenditure, which has a direct impact on the welfare gains from trade, as will be shown later.

### 3.1.2 Aggregate Trade Flows

Let us next turn to the impact of the trade costs on aggregate input exports (as only intermediate inputs are tradable in the current setting), which are the last term in the right-hand side of (17). To consider this, it is useful to decompose aggregate input exports into the extensive margin and the intensive margin:

$$\begin{aligned} R_t^I &= M_e \int_{\varphi_t^*}^{\infty} r_t^I(\varphi) dG(\varphi) \\ &= [1 - G(\varphi_t^*)] M_e \times \frac{1}{1 - G(\varphi_t^*)} \int_{\varphi_t^*}^{\infty} r_t^I(\varphi) dG(\varphi) \\ &= M_t \times \bar{r}_t^I, \end{aligned}$$

where  $M_t$  denotes the mass of exporting firms (extensive margin) and  $\bar{r}_t^I$  denotes average exports per exporting firm (intensive margin). It follows immediately from (18) that

$$\begin{aligned} \frac{dR_t^I}{d\tau} < 0, & \quad \frac{dM_t}{d\tau} < 0, & \quad \frac{d\bar{r}_t^I}{d\tau} \geq 0, \\ \frac{dR_t^I}{df_t} < 0, & \quad \frac{dM_t}{df_t} < 0, & \quad \frac{d\bar{r}_t^I}{df_t} \geq 0. \end{aligned}$$

Aggregate input exports naturally increase with reductions in the trade costs. As shown above, the extensive margin also increases with such reductions. Since  $\bar{r}_t^I = R_t^I/M_t$ , net changes in the intensive margin are generally ambiguous, depending on the extent to which  $R_t^I$  and  $M_t$  rise.

To see the sensitivity of aggregate input exports to changes in the trade costs in more detail, let us derive the full trade elasticity with respect to the variable trade cost. Following Melitz and Redding (2015), this trade elasticity is given by

$$\begin{aligned} \zeta^I &= -\frac{d \ln \left( \frac{1-\lambda}{\lambda} \right)}{d \ln \tau} \\ &= (\sigma - 1) - \frac{d \ln \Lambda^F}{d \ln \tau} - \frac{d \ln \Lambda^I}{d \ln \tau}, \end{aligned} \tag{20}$$

where the first term is the intensive margin elasticity, and the second and third terms are the extensive margin elasticity in final goods and intermediate inputs respectively. The fact that the extensive margin elasticity stems from both production sectors indicates that there is an extra adjustment operating through the extensive margin in intermediate-input trade. Intuitively, reductions in the trade costs induce entry of importing firms as well as entry of exporting firms, where entry of importing firms raises foreign input demand. The claim also applies to the fixed trade cost and the full trade elasticity with respect to fixed trade cost is given by

$$\begin{aligned} \xi^I &= -\frac{d \ln \left( \frac{1-\lambda}{\lambda} \right)}{d \ln f_t} \\ &= 0 - \frac{d \ln \Lambda^F}{d \ln f_t} - \frac{d \ln \Lambda^I}{d \ln f_t}. \end{aligned} \tag{21}$$

It is interesting to compare (20) and (21) with the full trade elasticities for final-good trade in the current setting. As shown in the Appendix, the extensive margin elasticity stems only from the downstream sector and hence the full trade elasticities are represented by

$$\begin{aligned} \zeta^F &= (\sigma - 1) - \frac{d \ln \Lambda}{d \ln \tau}, \\ \xi^F &= 0 - \frac{d \ln \Lambda}{d \ln f_t}, \end{aligned} \tag{22}$$

where  $\Lambda$  is the market share of *final-good* exporters in the domestic market. (Note that  $\Lambda^I$  is the market share of *intermediate-input* exporters and  $\Lambda^F$  is the market share of *intermediate-input*

importers in the current model.) The comparison between (20)–(22) immediately reveals that, due to an extra adjustment operating through the extensive margin, the trade elasticities are in general greater for intermediate-input trade than final-good trade:

$$\begin{aligned}\zeta^I &> \zeta^F, \\ \xi^I &> \xi^F.\end{aligned}$$

We will confirm the result for a Pareto distribution in the next subsection.

### 3.1.3 Welfare

We next explore welfare. From (5) and (10),

$$\begin{aligned}\frac{1}{P} &= \left(\frac{\sigma-1}{\sigma}\right) \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \frac{\phi^*}{\Gamma}, \\ \frac{1}{\Gamma} &= \left(\frac{\sigma-1}{\sigma}\right) \left(\frac{L^I}{\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi^* \lambda^{\frac{1}{\sigma-1}}.\end{aligned}$$

Further, from  $PQ = R^F = L$ , welfare per worker is expressed as

$$W^I = \frac{1}{P} = \left(\frac{\sigma-1}{\sigma}\right)^2 \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \left(\frac{L^I}{\sigma f}\right)^{\frac{1}{\sigma-1}} \phi^* \varphi^* \lambda^{\frac{1}{\sigma-1}}. \quad (23)$$

Thus the domestic productivity cutoffs  $\phi^*$ ,  $\varphi^*$  and the domestic share  $\lambda$  are sufficient statistics for welfare in intermediate-input trade.

To explore the welfare gains from trade, suppose that input trade is not possible in autarky. Since  $\lambda = 1$  in autarky and labor allocations between the two production sectors are fixed,

$$W_A^I = \frac{1}{P_A} = \left(\frac{\sigma-1}{\sigma}\right)^2 \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \left(\frac{L^I}{\sigma f}\right)^{\frac{1}{\sigma-1}} \phi_A^* \varphi_A^*,$$

where the subscript  $A$  denotes autarky. The welfare gains from trade are given by

$$\frac{W^I}{W_A^I} = \left(\frac{\phi^*}{\phi_A^*}\right) \left(\frac{\varphi^*}{\varphi_A^*}\right) \lambda^{\frac{1}{\sigma-1}}.$$

We are interested in comparing the welfare gains between final-good trade and intermediate-input trade. As shown in the Appendix, welfare in final-good trade in the current model is

$$W^F = \frac{1}{P} = \left(\frac{\sigma-1}{\sigma}\right)^2 \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \left(\frac{L^I}{\sigma f}\right)^{\frac{1}{\sigma-1}} \phi^* \varphi^*,$$

where only the domestic productivity cutoffs  $\phi^*$ ,  $\varphi^*$  are sufficient statistics for welfare in final-good trade. In addition, we find that only the domestic productivity cutoff of final-good firms  $\phi^*$

is variant to the trade costs for final-good trade, and the welfare gains from trade are given by

$$\frac{W^F}{W_A^F} = \frac{\phi^*}{\phi_A^*}.$$

Compared with final-good trade, intermediate-input trade gives rise to trade-induced changes in the domestic productivity cutoffs in both production sectors ( $\phi^* > \phi_A^*$ ,  $\varphi^* > \varphi_A^*$ ), suggesting that when the non-traded final goods are produced by vertical disintegration, the welfare gains from trade are magnified by raising domestic productivity (Melitz and Redding, 2014b). At the same time, intermediate-input trade also reduces the domestic trade share ( $\lambda < \lambda_A = 1$ ) and the above welfare comparison does not tell us which welfare gains from trade are greater.

To answer the question, we consider changes in welfare in the two types of trade. Changes in welfare in intermediate-input trade are

$$\widehat{W}^I = \widehat{\phi}^* + \widehat{\varphi}^* + \frac{1}{\sigma - 1} \widehat{\lambda},$$

where  $\widehat{v} \equiv \frac{dv}{v} = d \ln v$ . Moreover, changes in the domestic productivity cutoffs are (see Appendix)

$$\begin{aligned} \widehat{\phi}^* &= \frac{1}{(\sigma - 1) + \theta^F} (\widehat{N}_e - \widehat{\lambda}), \\ \widehat{\varphi}^* &= \frac{1}{(\sigma - 1) + \theta^I} (\widehat{M}_e - \widehat{\lambda}). \end{aligned} \quad (24)$$

Substituting (24), changes in welfare are expressed as

$$\widehat{W}^I = \frac{1}{(\sigma - 1) + \theta^F} \widehat{N}_e + \frac{1}{(\sigma - 1) + \theta^I} \widehat{M}_e + \frac{(\sigma - 1)^2 - \theta^F \theta^I}{(\sigma - 1)[(\sigma - 1) + \theta^F][(\sigma - 1) + \theta^I]} (-\widehat{\lambda}). \quad (25)$$

Note that not only are the coefficients of the first and second terms, but also the coefficient of the third term is positive under (19). Thus, welfare rises with the masses of entrants in each sector but falls with the domestic share. For final-good trade, only the domestic productivity cutoff of final-good firms  $\phi^*$  is variant to the trade costs (i.e.,  $\widehat{\varphi}^* = 0$ ). As a result, changes in welfare are

$$\widehat{W}^F = \frac{1}{(\sigma - 1) + \theta} \widehat{N}_e + \frac{1}{(\sigma - 1) + \theta} (-\widehat{\lambda}), \quad (26)$$

where  $\theta$  is the extensive margin elasticity in final-good trade.

To compare the welfare changes in (25) and (26), we need to use the *partial* trade elasticity that is only empirically observable, since it is estimated from a gravity equation with origin and destination fixed effects, whereby incomes and price indices are held constant (see Arkolakis et al., 2012; Melitz and Redding, 2015). While Melitz and Redding (2015) take the partial derivative with respect to  $\tau$  holding the domestic cutoff  $\phi^*$  constant, we instead take this partial derivative holding the export/import cutoffs  $\phi_t^*$ ,  $\varphi_t^*$  constant, as in Head et al. (2014).

Under the circumstance, the partial trade elasticity with respect to the variable trade cost in intermediate-input trade is given by

$$\begin{aligned}
\zeta_o^I &= -\frac{\partial \ln \left( \frac{1-\lambda}{\lambda} \right)}{\partial \ln \tau} \Big|_{\phi_t^*, \varphi_t^*} \\
&= (\sigma - 1) - \frac{\partial \ln \Lambda^F}{\partial \ln \phi^*} \frac{\partial \ln \phi^*}{\partial \ln \tau} \Big|_{\phi_t^*, \varphi_t^*} - \frac{\partial \ln \Lambda^I}{\partial \ln \varphi^*} \frac{\partial \ln \varphi^*}{\partial \ln \tau} \Big|_{\phi_t^*, \varphi_t^*} \\
&= \frac{(\sigma - 1)[(\sigma - 1) + \theta^F][(\sigma - 1) + \theta^I]}{(\sigma - 1)^2 - \theta^F \theta^I},
\end{aligned} \tag{27}$$

which is the inverse of the coefficient of the third term in (25). The corresponding trade elasticity in final-good trade is given by

$$\begin{aligned}
\zeta_o^F &= -\frac{\partial \ln \left( \frac{1-\lambda}{\lambda} \right)}{\partial \ln \tau} \Big|_{\phi_t^*} \\
&= (\sigma - 1) - \frac{\partial \ln \Lambda}{\partial \ln \phi^*} \frac{\partial \ln \phi^*}{\partial \ln \tau} \Big|_{\phi_t^*} \\
&= (\sigma - 1) + \theta,
\end{aligned} \tag{28}$$

which is the inverse of the coefficient of the first and second terms in (26). These trade elasticities can be estimated from the gravity equation for each case.

We are ready for comparing the changes in welfare across the different trade liberalizations. From (25) and (27), the changes in welfare in intermediate-input trade are given by

$$\widehat{W}^I = \frac{\mu^F}{\zeta_o^I} \widehat{N}_e + \frac{\mu^I}{\zeta_o^I} \widehat{M}_e + \frac{1}{\zeta_o^I} (-\widehat{\lambda}), \tag{29}$$

where

$$\mu^F \equiv \frac{(\sigma - 1)[(\sigma - 1) + \theta^I]}{(\sigma - 1)^2 - \theta^F \theta^I} > 1, \quad \mu^I \equiv \frac{(\sigma - 1)[(\sigma - 1) + \theta^F]}{(\sigma - 1)^2 - \theta^F \theta^I} > 1.$$

On the other hand, from (26) and (28), the changes in welfare in final-good trade are

$$\widehat{W}^F = \frac{1}{\zeta_o^F} \widehat{N}_e + \frac{1}{\zeta_o^F} (-\widehat{\lambda}). \tag{30}$$

Thus, conditional on the estimated trade elasticities with respect to the variable trade cost, the changes in welfare are greater in intermediate-input trade than final-good trade ( $\widehat{W}^I > \widehat{W}^F$ ) if and only if the masses of entrants increase by reductions of the trade costs ( $\widehat{M}_e > 0$  and  $\widehat{N}_e > 0$ ). However, whether the masses of entrants increase by such reductions is in general ambiguous, which depends critically on the productivity distribution  $G(\cdot)$ . In the next subsection, thus, we apply a specific distribution, namely a (untruncated) Pareto distribution, to obtain closed-form solutions, which also helps us to derive quantitative predictions for aggregate trade flows and welfare gains from trade.

## 3.2 Specific Properties under Pareto

### 3.2.1 Equilibrium Variables

To get closed-form solutions, we restrict our attention to a Pareto distribution in the following:

$$G(\phi) = 1 - \left( \frac{\phi_{\min}}{\phi} \right)^k, \quad G(\varphi) = 1 - \left( \frac{\varphi_{\min}}{\varphi} \right)^k,$$

where  $k - (\sigma - 1) > 0$  with support  $\phi \in [\phi_{\min}, \infty)$  and  $\varphi \in [\varphi_{\min}, \infty)$ . For simplicity, we assume a common shape parameter  $k$  but different lower bounds  $\phi_{\min}, \varphi_{\min}$ . With this specific distribution,  $J(a)$  and  $V(a)$  are simple power functions of the productivity cutoffs. In case of  $G(\phi)$ , we have

$$J(a) = \frac{(\sigma - 1)\phi_{\min}^k}{k - (\sigma - 1)} \frac{1}{a^k},$$

$$V(a) = \frac{k\phi_{\min}^k}{k - (\sigma - 1)} \frac{1}{a^{k-(\sigma-1)}},$$

with  $J(a) = \frac{\sigma-1}{k} a^{1-\sigma} V(a)$ .

Applying Pareto, the extensive margin elasticity is simply written as  $\theta^F = \theta^I = k - (\sigma - 1)$ . Then the parameter restriction in (19) holds if

$$\sigma - 1 < k < 2(\sigma - 1). \quad (31)$$

From (31), under the Pareto distribution, (19) implies that the shape parameter  $k$  is not too large, i.e., productivity dispersion is not too small.

The equilibrium property in (18) continues to hold under Pareto. One of important departures from the previous analysis lies in the masses of entrants in the two production sectors. Though these masses depend in general on the productivity cutoffs, the dependence is eliminated under Pareto and they depend only on labor supply  $L, L^I$ :

$$N_e = \frac{\sigma - 1}{k\sigma} \frac{L}{f_e},$$

$$M_e = \frac{\sigma - 1}{k\sigma} \frac{L^I}{f_e}. \quad (32)$$

As a result, the masses of entrants do not respond to changes in the trade costs:

$$\frac{dM_e}{d\tau} = 0, \quad \frac{dN_e}{d\tau} = 0,$$

$$\frac{dM_e}{df_t} = 0, \quad \frac{dN_e}{df_t} = 0.$$

As before, reductions in the trade costs induce exit of domestic firms, but such reductions also induces entry of exporting/importing firms in each production sector.

### 3.2.2 Aggregate Trade Flows

We next turn to analyzing aggregate input exports under the Pareto distribution. Using (32), the extensive and intensive margins are expressed as

$$M_t = \left( \frac{\varphi_{\min}}{\varphi_t^*} \right)^k \frac{\sigma - 1}{k\sigma} \frac{L^I}{f_e},$$

$$\bar{r}_t^I = \frac{k\sigma}{k - (\sigma - 1)} f_t.$$

This expression shows that while the trade costs have an impact on the extensive margin through the export productivity cutoff  $\varphi_t^*$ , the variable cost has no impact on the intensive margin. (Any reduction in the fixed trade cost decreases the intensive margin.)

$$\frac{dM_t}{d\tau} < 0, \quad \frac{d\bar{r}_t^I}{d\tau} = 0,$$

$$\frac{dM_t}{df_t} < 0, \quad \frac{d\bar{r}_t^I}{df_t} > 0.$$

The property of intermediate-input trade under the Pareto distribution is the same as that of final-good trade (see Melitz and Redding (2014a) for a recent literature review).

The difference between the two types of trade arises when considering the trade elasticities. The decomposition into the extensive and intensive margins under Pareto allows us to express aggregate input exports as a gravity equation. Substituting  $\varphi_t^*$  from (14) into  $M_t$  in (32), we get<sup>1</sup>

$$R_t^I = \psi^I L (B^I)^{\frac{k}{\sigma-1}} \tau^{-\frac{k(\sigma-1)}{2(\sigma-1)-k}} f_t^{1-\frac{k}{2(\sigma-1)-k}}. \quad (33)$$

As in the usual gravity equation, aggregate input exports  $R_t^I$  are a function of exporting country size  $L$ , importing country demand  $B^I$ , and the bilateral trade costs, both variable  $\tau$  and fixed  $f_t$ . If we derive the gravity equation for final-good trade in the current setting, aggregate final-good exports are expressed as (see Appendix)<sup>2</sup>

$$R_t^F = \psi^F L (B^F)^{\frac{k}{\sigma-1}} (\tau\Gamma)^{-k} f_t^{1-\frac{k}{\sigma-1}}. \quad (34)$$

While the functional forms in (33) and (34) are similar to that in Chaney (2008), they show that the trade elasticities with respect to the variable and fixed trade costs are different between intermediate-input trade and final-good trade. Recall that only the partial trade elasticities can be empirically estimated in the gravity equation with origin and destination fixed effects. Thus, holding market demand  $B^I$  constant in (33), the partial trade elasticities in intermediate-input

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<sup>1</sup> $\psi^I \equiv \frac{(\sigma-1)^2}{\sigma(k-(\sigma-1))} \frac{(\varphi_{\min})^k}{f_e} f^{\frac{k}{\sigma-1} \frac{k-(\sigma-1)}{2(\sigma-1)-k}}$ .

<sup>2</sup> $\psi^F \equiv \frac{\sigma-1}{k-(\sigma-1)} \frac{(\varphi_{\min})^k}{f_e}$ .



trade are given by

$$\zeta_o^I \equiv -\frac{\partial \ln R_t^I}{\partial \ln \tau} = \frac{k(\sigma - 1)}{2(\sigma - 1) - k},$$

$$\xi_o^I \equiv -\frac{\partial \ln R_t^I}{\partial \ln f_t} = \frac{k}{2(\sigma - 1) - k} - 1.$$

On the other hand, holding market demand  $B^F$  and the unit cost  $\Gamma$  constant in (34), the partial trade elasticities in final-good trade are given by

$$\zeta_o^F \equiv -\frac{\partial \ln R_t^F}{\partial \ln \tau} = k,$$

$$\xi_o^F \equiv -\frac{\partial \ln R_t^F}{\partial \ln f_t} = \frac{k}{\sigma - 1} - 1,$$

which are the exactly the same as Chaney (2008). Comparing these under condition (31), we get

$$\zeta_o^I > \zeta_o^F,$$

$$\xi_o^I > \xi_o^F. \quad (35)$$

The first inequality in (35) is also confirmed by substituting  $\theta^F = \theta^I = k - (\sigma - 1)$  into (27) and (28). This finding confirms the previous assertion that, due to an extra adjustment operating through the extensive margin, the trade elasticities are in general greater for intermediate-input trade than final-good trade.

Further, the partial trade elasticities are equal to the full trade elasticities given by (20)–(22). From (33) and (34), the domestic share is expressed as

$$\frac{1 - \lambda}{\lambda} = \begin{cases} \tau^{-\frac{k(\sigma-1)}{2(\sigma-1)-k}} \left(\frac{f_t}{f}\right)^{1-\frac{k}{2(\sigma-1)-k}} & \text{if intermediate-input trade,} \\ \tau^{-k} \left(\frac{f_t}{f}\right)^{1-\frac{k}{\sigma-1}} & \text{if final-good trade.} \end{cases}$$

The shares of exporters/importers in the domestic market are also expressed as

$$\Lambda^F = \Lambda^I = \left(\frac{\tau^{\sigma-1} f_t}{f}\right)^{-\frac{k-(\sigma-1)}{2(\sigma-1)-k}},$$

$$\Lambda = \left(\frac{\tau^{\sigma-1} f_t}{f}\right)^{-\frac{k-(\sigma-1)}{\sigma-1}}.$$

Applying these expressions to (20), (21) and (22) and comparing with the partial trade elasticities establishes the result. It is important to note that the property is very specific to the untruncated Pareto distribution, and even a slight departure from this distribution inevitably makes the two trade elasticities different. As stressed by Melitz and Redding (2015), this is particularly crucial for evaluating the welfare gains from trade.

### 3.2.3 Welfare

We conclude this subsection by deriving a specific welfare property under the Pareto distribution. Since  $\theta^F = \theta^I = k - (\sigma - 1)$  and  $\widehat{M}_e = \widehat{N}_e = 0$  under this distribution, (25) is simplified as

$$\widehat{W}^I = - \left( \frac{2(\sigma - 1) - k}{k(\sigma - 1)} \right) \widehat{\lambda},$$

and the domestic share provides the single sufficient statistic for the welfare gains from trade. We can alternatively see this by rewriting the domestic productivity cutoffs as

$$\begin{aligned} (\phi^*)^k &= \frac{\phi_{\min}^k}{\lambda} \frac{\sigma - 1}{k - (\sigma - 1)} \frac{f}{f_e}, \\ (\varphi^*)^k &= \frac{\varphi_{\min}^k}{\lambda} \frac{\sigma - 1}{k - (\sigma - 1)} \frac{f}{f_e}. \end{aligned} \tag{36}$$

Substituting (36) into (23) gives the following welfare expression:

$$\begin{aligned} d \ln W^I &= d \ln \phi^* + d \ln \varphi^* + \frac{1}{\sigma - 1} d \ln \lambda \\ &= - \left( \frac{2(\sigma - 1) - k}{k(\sigma - 1)} \right) d \ln \lambda, \end{aligned}$$

which is the same as the above expression because  $d \ln v = \widehat{v}$ . As a consequence, the ACR formula applies to intermediate-input trade under the Pareto distribution:

$$\begin{aligned} \widehat{W}^I &= \widehat{\lambda}^{-\frac{2(\sigma-1)-k}{k(\sigma-1)}} \\ &= \widehat{\lambda}^{-\frac{1}{\zeta_o^I}}. \end{aligned}$$

As for final-good trade, it follows immediately from (26) that changes in welfare in this case are given by

$$\begin{aligned} \widehat{W}^F &= \widehat{\lambda}^{-\frac{1}{k}} \\ &= \widehat{\lambda}^{-\frac{1}{\zeta_o^F}}. \end{aligned}$$

These observations suggest that, conditional on the estimated trade elasticities, the changes in welfare depend only on the two sufficient statistics  $\lambda, \zeta_o$  and the welfare gains from trade are the same between the two types of trade. This result, however, holds only for the untruncated Pareto distribution, and the welfare gains are in general different between the two types of trade (see Melitz and Redding (2015) for the similar argument of the welfare comparison between the homogeneous and heterogeneous firm models). In the current model, (29) and (30) highlight how the changes in the masses of entrants in general generate the different changes in welfare, even conditional on the domestic share and trade elasticity.

To demonstrate this caveat, we follow Helpman et al. (2008) and Melitz and Redding (2015) in assuming a slight departure from an untruncated Pareto to a truncated Pareto. Recall that the changes in welfare are greater in intermediate-input trade than final-good trade if and only if the mass of entrants increases (which does not change under the untruncated Pareto distribution). For simplicity, we will focus on the impact of the variable trade cost on the mass of entrants.

Differentiating  $N_e$  and  $M_e$  with respect to  $\tau$  yields

$$\begin{aligned}\frac{dN_e}{d\tau} &= fg(\phi^*) \frac{d\phi^*}{d\tau} \left(1 - \frac{\theta_t^F}{\theta^F}\right), \\ \frac{dM_e}{d\tau} &= fg(\varphi^*) \frac{d\varphi^*}{d\tau} \left(1 - \frac{\theta_t^I}{\theta^I}\right),\end{aligned}\tag{37}$$

where  $g(\cdot)$  is a probability distribution of  $G(\cdot)$  and

$$\theta_t^F \equiv -\frac{d \ln V(\phi_t^*)}{d \ln \phi_t^*} > 0, \quad \theta_t^I \equiv -\frac{d \ln V(\varphi_t^*)}{d \ln \varphi_t^*} > 0$$

are the extensive margin elasticities of foreign firms. Under an untruncated Pareto distribution,  $\theta^F = \theta_t^F = \theta^I = \theta_t^I = k - (\sigma - 1)$  and hence  $M_e$  and  $N_e$  do not respond to the variable trade cost. In contrast, under a truncated Pareto distribution with support  $[\phi_{\min}, \phi_{\max}]$  for  $G(\phi)$ ,  $\theta^F$  and  $\theta_t^F$  become (see Melitz and Redding, 2015)

$$\begin{aligned}\theta^F &= (k - (\sigma - 1)) \frac{\left(\frac{\phi_{\min}}{\phi^*}\right)^{k-(\sigma-1)}}{\left(\frac{\phi_{\min}}{\phi^*}\right)^{k-(\sigma-1)} - \left(\frac{\phi_{\min}}{\phi_{\max}}\right)^{k-(\sigma-1)}}, \\ \theta_t^F &= (k - (\sigma - 1)) \frac{\left(\frac{\phi_{\min}}{\phi_t^*}\right)^{k-(\sigma-1)}}{\left(\frac{\phi_{\min}}{\phi_t^*}\right)^{k-(\sigma-1)} - \left(\frac{\phi_{\min}}{\phi_{\max}}\right)^{k-(\sigma-1)}},\end{aligned}$$

and hence  $\theta_t^F > \theta^F$  as long as  $\phi_t^* > \phi^*$  under (7). Similarly,  $\theta_t^I > \theta^I$  as long as  $\varphi_t^* > \varphi^*$  under (15). It then follows from (18) and (37) that reductions in the variable trade cost reduces the mass of entrants in the two production sectors. Further, from  $\widehat{M}_e < 0$  and  $\widehat{N}_e < 0$ , (29) and (30) suggest that the change in welfare should be *smaller* in intermediate-input trade than final-good trade ( $\widehat{W}^I < \widehat{W}^F$ ).

This theoretical finding seems contrary to the empirical evidence that input tariff reductions increase industry productivity more than output tariff reductions (e.g., Amiti and Konings, 2007; Topalova and Khandelwal, 2011). One of possible reasons is that the (truncated or untruncated) Pareto distribution might fail to quantify the effect of the variable trade cost on intermediate-input trade, relative to final-good trade. We will elaborate on this point by estimating the gravity equation in the next section.

## 4 Evidence

This section assesses empirically the relevance of one of our main theoretical predictions: the trade elasticity with respect to the variable trade cost is greater for intermediate-input trade than final-good trade. Section 4.1 discusses the data source, Section 4.2 presents the regression specifications and Section 4.3 reports the estimation results.

### 4.1 Data

The dataset used in the estimation is the census of annual firm-level export transactions in China for the period from 2000 to 2009, collected by China Customs. The dataset contains the information on the trade value, quantity, and destination at the 8-digit Harmonized System (HS) product classification. We use the publicly available concordance tables for 1997, 2002 and 2007 HS codes to make the product code consistent over time. For brevity, we will focus attention on 2005 data in the below, but it should be noted that results for other years are similar before the 2008 global financial crisis.

To implement our regression, we aggregate the original China Customs dataset at the 8-digit HS product level into the 6-digit HS product level. This allows us to decompose aggregate trade values into the number of exporters with positive trade (extensive margin) and average export values conditional on trade being positive (intensive margin) in terms of thousand U.S. dollar for each destination at the product level.

We focus on differentiated goods in the empirical analysis for being consistent with theory. For this purpose, we divide our dataset into differentiated goods and homogeneous goods by the Rauch (1999) classification where the Standard International Trade Classification (SITC) codes are converted into the HS codes, and thereby commodities and reference-priced goods are treated as homogeneous goods. To test the trade elasticity difference in the two types of trade, we also divide our dataset into intermediate inputs and final goods by the Broad Economic Categories (BEC) classification. Thus, China's aggregate exports, and the extensive and intensive margins, are classified along with the two dimensions. In the case of China's exports to Japan, for example, the numbers of homogeneous intermediate inputs and differentiated intermediate inputs are 1,073 and 1,331, while the numbers of homogeneous final goods and differentiated final goods are 183 and 1,310. After excluding homogeneous intermediate inputs and homogeneous final goods, our dataset covers 193 destination countries and roughly 1,500 products, and in total roughly 100,000 observations for differentiated intermediate inputs and differentiated final goods at the 6-digit level from manufacturing industries in 2005.

### 4.2 Specifications

We examine the trade elasticities with respect to the variable trade cost for intermediate-input trade and final-good trade by estimating the gravity equations derived for the Pareto distribution

in (33) and (34). While controlling for source-country characteristics with a constant, we conduct the following regressions:

$$\ln Z_{pc}^i = \alpha_0^i + \alpha_1^i \ln dist_c + \alpha_2^i \ln GDP_c + \alpha_3^i border_c + \alpha_4^i Chinese_c + \alpha_5^i FTA_c + \theta_p^i + \epsilon_{pc}^i. \quad (38)$$

$Z_{pc}^i$  is either aggregate export values  $R_{pc}^i$ , the number of exporting firms  $M_{pc}^i$  (extensive margin), or average export values per exporting firm  $\bar{r}_{pc}^i$  (intensive margin) from China to destination country  $c$  at product  $p$ . Conditional on trade being positive, these variables satisfy

$$R_{pc}^i = M_{pc}^i \times \bar{r}_{pc}^i.$$

The superscript  $i$  attached to each of these variables indicates that a product is either final goods ( $i = F$ ) or intermediate inputs ( $i = I$ ).

We employ distance and GDP as proxies for variable trade costs and market size, respectively. In (37),  $dist_c$  is population-weighted average of the great-circle distances between the 20 largest cities in China and destination country  $c$  and  $GDP_c$  is GDP in destination country  $c$ . In addition,  $border_c$  is a dummy variable which is equal to one if destination country  $c$  shares the national border with China, and zero otherwise;  $Chinese_c$  is a dummy variable which is equal to one if destination country  $c$  shares the common language (i.e., Chinese), and zero otherwise;  $FTA_c$  is a dummy variable which is equal to one if destination country  $c$  has an FTA with China, and zero otherwise; and  $\theta_p^i$  is the product fixed effect.  $\epsilon_{pc}^i$  is the error term.

Our interest in this gravity equation lies in the coefficient  $\alpha_1^i$ . In particular, in light of (35), we hypothesize that  $|\alpha_1^I| > |\alpha_1^F|$  for aggregate export values  $R_{pc}^i$ . The theory also predicts that the major part of this difference comes from the extensive margin, leaving the intensive margin independent of distance between the two types of trade. Although the prediction on the intensive margin is very specific to the Pareto distribution, we hypothesize that  $|\alpha_1^I|$  is significantly greater than  $|\alpha_1^F|$  for the extensive margin  $M_{pc}^i$ , whereas  $|\alpha_1^I|$  and its difference between the two types of trade are insignificant for the intensive margin  $\bar{r}_{pc}^i$ .

In order to check whether there is a statistically significant difference in the trade elasticity between final-good trade and intermediate-input trade, we also conduct the following regression with an interaction term:

$$\ln Z_{pc}^F = \beta_0 + \beta_1 \ln dist_c + \beta_2 dist_c * inter_c + \beta_3 \ln GDP_c + \beta_4 border_c + \beta_5 Chinese_c + \beta_6 FTA_c + \theta_p + \epsilon_{pc}. \quad (39)$$

where  $inter_c$  is a dummy variable which is equal to one for intermediate-input trade and zero for final-good trade. In regressing (38), we pool our dataset on final-good trade and intermediate-input trade together. Note that, for final-good trade where  $inter = 0$ , the regression results are basically the same as those in (37). Since the theory predicts that intermediate-input trade is more responsible to the variable trade cost than final-good trade, we hypothesize that  $\beta_2 < 0$ , especially for aggregate export values  $R_{pc}^i$  and the extensive margin  $M_{pc}^i$ .

### 4.3 Estimation Results

Let us first report the estimation results of (37). Since we divide our dataset into final-good trade and intermediate-input trade for (37), the results for each type of trade are separately given in Panels (a) and (b) of Table 1. The first column of Table 1 shows the well-known regularity that aggregate trade flows decline with distance and increase with market size. While this regularity holds for both types of trade, note importantly that the estimated coefficient of  $\ln dist_c$  is greater for intermediate-input trade than final-good trade. In the second and third columns of Table 1, the relationship is decomposed into the extensive and intensive margins. For final-good trade, the negative relationship between aggregate export values and distance is mostly explained by the extensive margin, leaving the role of the intensive margin minor in the gravity equation, which accords well with the existing literature (e.g., Bernard et al. 2007, 2011; Eaton et al. 2004, 2011). For intermediate-input trade, in contrast, the negative relationship comes from both the extensive and intensive margins, although the extensive margin plays a relatively bigger role. Overall, we find theory-consistent evidence that the trade elasticity with respect to the variable trade cost is greater for intermediate-input trade than final-good trade, particularly through the extensive margin.

Let us next report the estimation results of (38) in Table 2. Note first that, in contrast to (37), we make use of the pooled dataset to estimate (38) and, except for the second row of  $dist_c * inter_c$ , the estimated coefficients are almost the same as those in Table 1(a). The coefficients of  $dist_c * inter_c$  are all negative and statistically significant at the 1% level, and therefore, the variable trade cost has a greater impact on intermediate-input trade than final-good trade, through both the extensive and intensive margins. (Adding the coefficients of  $dist_c$  and  $dist_c * inter_c$  are almost the same as that of  $dist_c$  in Table 1(b).) This is an unsatisfactory result. While the theory shows that only the extensive margin can explain the difference in the trade elasticities as long as the Pareto distribution is imposed, the evidence shows that not only does the extensive margin but the intensive margin also plays a key role in explaining this difference.

The regression results in Tables 1 and 2 indicate that the Pareto distribution might fail to quantify the effect of the variable trade cost on intermediate-input trade, relative to final-good trade, through the intensive margin. As stressed in Section 3, reductions in the variable trade cost can in general increase the intensive margin, and the property that the intensive margin is unaffected by the variable trade cost is very specific to the Pareto distribution. Our empirical finding requires us to move from the Pareto distribution to other distributions, which seems to be in particular relevant to intermediate-input trade. To capture the role of the intensive margin, the recent literature tries to develop new trade models without Pareto (e.g., Fernandes et al. 2018), but, to the best of our knowledge, most of existing work does not distinguish intermediate-input trade and final-good trade. We believe this distinction is worth investigating because the trade elasticities are in general different between the two types of trade, which in turn gives rise to different welfare effects, as demonstrated by our theory.

Table 1 — Estimates of the gravity equation

	$\ln R_{pc}^F$	$\ln M_{pc}^F$	$\ln \bar{r}_{pc}^F$
$\ln dist_c$	-0.249** (0.104)	-0.184*** (0.065)	-0.065 (0.053)
$\ln GDP_c$	0.837*** (0.028)	0.449*** (0.016)	0.388*** (0.015)
$border_c$	-0.228 (0.239)	-0.182 (0.132)	-0.046 (0.133)
$Chinese_c$	0.918*** (0.252)	0.652*** (0.221)	0.266*** (0.061)
$FTA_c$	0.986 (0.982)	1.142*** (0.436)	-0.156 (0.585)
<b>Product FE</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>
<b>No. of products</b>	<b>1,550</b>	<b>1,550</b>	<b>1,550</b>
<b>No. of destinations</b>	<b>193</b>	<b>193</b>	<b>193</b>
<b>No. of observations</b>	<b>110,458</b>	<b>110,458</b>	<b>110,458</b>
<b>Adj. <math>R^2</math></b>	<b>0.516</b>	<b>0.614</b>	<b>0.408</b>

(a) Differentiated final goods, 2005

	$\ln R_{pc}^I$	$\ln M_{pc}^I$	$\ln \bar{r}_{pc}^I$
$\ln dist_c$	-0.617*** (0.100)	-0.343*** (0.062)	-0.274*** (0.049)
$\ln GDP_c$	0.705*** (0.029)	0.375*** (0.017)	0.331*** (0.015)
$border_c$	0.029 (0.261)	-0.007 (0.179)	0.035 (0.101)
$Chinese_c$	1.008*** (0.232)	0.668*** (0.179)	0.340*** (0.068)
$FTA_c$	0.406 (0.956)	0.688* (0.394)	-0.283 (0.598)
<b>Product FE</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>
<b>No. of products</b>	<b>1,603</b>	<b>1,603</b>	<b>1,603</b>
<b>No. of destinations</b>	<b>193</b>	<b>193</b>	<b>193</b>
<b>No. of observations</b>	<b>96,295</b>	<b>96,295</b>	<b>96,295</b>
<b>Adj. <math>R^2</math></b>	<b>0.451</b>	<b>0.596</b>	<b>0.330</b>

(b) Differentiated intermediate inputs, 2005

Note: Standard errors clustered at country-level are in brackets

\* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$

Table 2 — Robustness checks of the gravity equation

	$\ln R_{pc}^F$	$\ln M_{pc}^F$	$\ln \bar{r}_{pc}^F$
$\ln dist_c$	-0.267** (0.105)	-0.199*** (0.065)	-0.068 (0.053)
$dist_c * inter_c$	-0.333*** (0.078)	-0.130*** (0.044)	-0.203*** (0.038)
$\ln GDP_c$	0.775*** (0.026)	0.414*** (0.015)	0.361*** (0.014)
$border_c$	-0.095 (0.219)	-0.092 (0.146)	-0.003 (0.098)
$Chinese_c$	0.966*** (0.237)	0.662*** (0.198)	0.304*** (0.058)
$FTA_c$	0.693 (0.960)	0.915** (0.408)	-0.222 (0.589)
<b>Product FE</b>	<b>Yes</b>	<b>Yes</b>	<b>Yes</b>
<b>No. of products</b>	<b>3,153</b>	<b>3,153</b>	<b>3,153</b>
<b>No. of destinations</b>	<b>193</b>	<b>193</b>	<b>193</b>
<b>No. of observations</b>	<b>206,753</b>	<b>206,753</b>	<b>206,753</b>
<b>Adj. <math>R^2</math></b>	<b>0.485</b>	<b>0.605</b>	<b>0.369</b>

Differentiated final goods, 2005

Note: Standard errors clustered at country-level are in brackets

\* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$



## 5 Conclusion

This paper presented a heterogeneous-firm model in which firm exporting and firm importing play a crucial role in industry productivity and welfare. We found that the impact of trade costs, both variable and fixed, on aggregate trade flows are in general greater for intermediate-input trade than final-good trade. This theoretical finding is consistent with the empirical findings in that input tariff reductions have a greater impact than output tariff reductions. We also showed the general condition under which the welfare gains are greater in intermediate-input trade than those in final-good trade. In particular, whether changes in welfare are greater for intermediate-input trade than final-good trade depends crucially on changes in the mass of entrants.

To highlight an extra adjustment operating through the extensive margin that impacts on the gravity equation and welfare, we have restricted attention to a simple setting in which only intermediate inputs are tradable between two symmetric countries. Our next question must be the extent to which the results generalize. Among others, the impact of country asymmetry on the pattern of trade is of particular interest for us. Will a larger country host disproportionately more final-good firms compared to a smaller country and hence is it a net exporter of final goods? Will asymmetric trade liberalization in either kind of trade induce agglomeration of final-good firms in a larger country and hence give a higher welfare gain from trade there by allowing for increased final-good variety? We shall address such questions in future research.

## Appendix A: Proofs

### A.1 Proof of (18)

Noting the definition of  $\Lambda^I$  and  $\Lambda^F$ , differentiating (7) and (15) with respect to  $\tau$  gives

$$(\sigma - 1) \frac{\dot{\phi}_t^*}{\phi_t^*} - (\sigma - 1) \frac{\dot{\phi}^*}{\phi^*} = -\theta^I \frac{\dot{\varphi}^*}{\varphi^*} + \theta_t^I \frac{\dot{\varphi}_t^*}{\varphi_t^*} + (\sigma - 1) \frac{1}{\tau}, \quad (\text{A.1})$$

$$(\sigma - 1) \frac{\dot{\varphi}_t^*}{\varphi_t^*} - (\sigma - 1) \frac{\dot{\varphi}^*}{\varphi^*} = -\theta^F \frac{\dot{\phi}^*}{\phi^*} + \theta_t^F \frac{\dot{\phi}_t^*}{\phi_t^*} + (\sigma - 1) \frac{1}{\tau}, \quad (\text{A.2})$$

where a dot represents the derivative with respect to  $\tau$  (e.g.,  $\dot{\phi}^* \equiv \frac{d\phi^*}{d\tau}$ ). Further, differentiating (8) and (16) with respect to  $\tau$  gives

$$\dot{\phi}_t^* = -C\dot{\phi}^*, \quad (\text{A.3})$$

$$\dot{\varphi}_t^* = -D\dot{\varphi}^*, \quad (\text{A.4})$$

where  $C \equiv \frac{fJ'(\phi^*)}{f_t J'(\phi_t^*)} > 0$  and  $D \equiv \frac{fJ'(\varphi^*)}{f_t J'(\varphi_t^*)} > 0$ . Note that (A.1) – (A.4) are four equations which have four unknowns ( $\dot{\phi}^*$ ,  $\dot{\phi}_t^*$ ,  $\dot{\varphi}^*$ ,  $\dot{\varphi}_t^*$ ). Substituting (A.3) and (A.4) into (A.1) and (A.2) yields

$$\begin{aligned} -(\sigma - 1) \left( \frac{1}{\phi^*} + \frac{C}{\phi_t^*} \right) \dot{\phi}^* &= - \left( \frac{\theta^I}{\varphi^*} + \frac{D\theta_t^I}{\varphi_t^*} \right) \dot{\varphi}^* + (\sigma - 1) \frac{1}{\tau}, \\ -(\sigma - 1) \left( \frac{1}{\varphi^*} + \frac{D}{\varphi_t^*} \right) \dot{\varphi}^* &= - \left( \frac{\theta^F}{\phi^*} + \frac{C\theta_t^F}{\phi_t^*} \right) \dot{\phi}^* + (\sigma - 1) \frac{1}{\tau}. \end{aligned}$$

These are two equations with two unknowns ( $\dot{\phi}^*$ ,  $\dot{\varphi}^*$ ), which are solved for

$$\begin{aligned} \frac{d\phi^*}{d\tau} &= - \frac{(\sigma - 1) \frac{1}{\tau} \left[ (\sigma - 1) \left( \frac{1}{\varphi^*} + \frac{D}{\varphi_t^*} \right) + \left( \frac{\theta^I}{\varphi^*} + \frac{D\theta_t^I}{\varphi_t^*} \right) \right]}{\Delta}, \\ \frac{d\varphi^*}{d\tau} &= - \frac{(\sigma - 1) \frac{1}{\tau} \left[ (\sigma - 1) \left( \frac{1}{\phi^*} + \frac{C}{\phi_t^*} \right) + \left( \frac{\theta^F}{\phi^*} + \frac{C\theta_t^F}{\phi_t^*} \right) \right]}{\Delta}, \end{aligned}$$

where

$$\Delta \equiv (\sigma - 1)^2 \left( \frac{1}{\phi^*} + \frac{C}{\phi_t^*} \right) \left( \frac{1}{\varphi^*} + \frac{D}{\varphi_t^*} \right) - \left( \frac{\theta^F}{\phi^*} + \frac{C\theta_t^F}{\phi_t^*} \right) \left( \frac{\theta^I}{\varphi^*} + \frac{D\theta_t^I}{\varphi_t^*} \right).$$

Simple inspection reveals that  $\Delta > 0$  under (19). Then we have  $\frac{d\phi^*}{d\tau} < 0$ ,  $\frac{d\varphi^*}{d\tau} < 0$  and from (A.3) and (A.4),  $\frac{d\phi_t^*}{d\tau} > 0$ ,  $\frac{d\varphi_t^*}{d\tau} > 0$ , and from (5) and (13),  $\frac{dB^F}{d\tau} > 0$ ,  $\frac{dB^I}{d\tau} > 0$ .

Under the Pareto distribution,  $\theta^F = \theta_t^F = \theta^I = \theta_t^I = k - (\sigma - 1)$ , and hence

$$\Delta = [(\sigma - 1)^2 - (k - (\sigma - 1))^2] \left( \frac{1}{\phi^*} + \frac{C}{\phi_t^*} \right) \left( \frac{1}{\varphi^*} + \frac{D}{\varphi_t^*} \right).$$

Together with  $k > \sigma - 1$ , the condition that  $\Delta > 0$  is satisfied under (31).

Similarly, we have

$$\begin{aligned}\frac{d\phi^*}{df_t} &= -\frac{\frac{1}{f_t} \left[ (\sigma - 1) \left( \frac{1}{\varphi^*} + \frac{D}{\varphi_t^*} \right) F + \left( \frac{\theta^I}{\varphi^*} + \frac{D\theta_t^I}{\varphi_t^*} \right) G \right]}{\Delta}, \\ \frac{d\varphi^*}{df_t} &= -\frac{\frac{1}{f_t} \left[ (\sigma - 1) \left( \frac{1}{\phi^*} + \frac{C}{\phi_t^*} \right) G + \left( \frac{\theta^F}{\phi^*} + \frac{C\theta_t^F}{\phi_t^*} \right) F \right]}{\Delta},\end{aligned}$$

where  $F \equiv 1 - (\sigma - 1)\rho(\phi_t^*) - \frac{\theta_t^I}{\varphi_t^*} \frac{J(\varphi_t^*)}{J'(\varphi_t^*)}$ ,  $G \equiv 1 - (\sigma - 1)\rho(\varphi_t^*) - \frac{\theta_t^F}{\phi_t^*} \frac{J(\phi_t^*)}{J'(\phi_t^*)}$ , and  $\rho(a) \equiv -\frac{J(a)}{aJ'(a)}$ . Since  $J'(a) = -(\sigma - 1)a^{-\sigma}V(a)$ , we have  $1 - (\sigma - 1)\rho(a) > 0$  and hence  $F > 0, G > 0$ . Then, under condition that  $\Delta > 0$ , we have  $\frac{d\phi_t^*}{df} < 0, \frac{d\varphi_t^*}{df} < 0$ , which implies that  $\frac{d\phi_t^*}{df_t} > 0, \frac{d\varphi_t^*}{df_t} > 0, \frac{dB^F}{df_t} > 0, \frac{dB^I}{df_t} > 0$ .

## A.2 Proof of (24)

Let us first show the first equation of (24). Using the domestic trade share  $\lambda$ , let us rewrite the price index as

$$P^{1-\sigma} = N_e \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \Gamma^{1-\sigma} V(\phi^*) \frac{1}{\lambda}.$$

Taking the log and totally differentiating this equation gives

$$-(\sigma - 1) \frac{dP}{P} = \frac{dN_e}{N_e} - (\sigma - 1) \frac{d\Gamma}{\Gamma} - \theta^F \frac{d\phi^*}{\phi^*} - \frac{d\lambda}{\lambda}.$$

Further, taking the log and totally differentiating (5) gives

$$\frac{d\phi^*}{\phi^*} = - \left( \frac{dP}{P} - \frac{d\Gamma}{\Gamma} \right).$$

Combining the above two equations and rearranging,

$$\frac{dN_e}{N_e} - \frac{d\lambda}{\lambda} = \frac{d\phi^*}{\phi^*} ((\sigma - 1) + \theta^F).$$

The result follows directly from noting that  $\hat{v} \equiv \frac{dv}{v}$ .

Next we show the second equation of (24). Taking the log and totally differentiating the unit cost of domestic-sourcing firms  $\Gamma^{1-\sigma}$  gives

$$-(\sigma - 1) \frac{d\Gamma}{\Gamma} = \frac{dM_e}{M_e} - \theta^I \frac{d\varphi^*}{\varphi^*}.$$

Further, taking the log and totally differentiating (13) gives

$$\frac{d\lambda}{\lambda} - \left( \frac{dM_e}{M_e} - \theta^I \frac{d\varphi^*}{\varphi^*} \right) + (\sigma - 1) \frac{d\varphi^*}{\varphi^*} = 0.$$

Combining the above two equations and rearranging,

$$\frac{dM_e}{M_e} - \frac{d\lambda}{\lambda} = \frac{d\varphi^*}{\varphi^*} ((\sigma - 1) + \theta^I).$$

The result follows directly from noting that  $\hat{v} \equiv \frac{dv}{v}$ .

### A.3 Proof of (27)

Taking the log and partially differentiating (7) with respect to  $\ln \tau$  gives

$$-\frac{\partial \ln \phi^*}{\partial \ln \tau} = -\frac{\theta^I}{\sigma - 1} \frac{\partial \ln \varphi^*}{\partial \ln \tau} + 1.$$

Similarly, taking the log and partially differentiating (15) with respect to  $\ln \tau$  gives

$$-\frac{\partial \ln \varphi^*}{\partial \ln \tau} = -\frac{\theta^F}{\sigma - 1} \frac{\partial \ln \phi^*}{\partial \ln \tau} + 1.$$

Substituting the second equation into the first equation and rearranging,

$$\begin{aligned} \frac{\partial \ln \phi^*}{\partial \ln \tau} &= -\frac{(\sigma - 1)[(\sigma - 1) + \theta^I]}{(\sigma - 1)^2 - \theta^F \theta^I}, \\ \frac{\partial \ln \varphi^*}{\partial \ln \tau} &= -\frac{(\sigma - 1)[(\sigma - 1) + \theta^F]}{(\sigma - 1)^2 - \theta^F \theta^I}. \end{aligned}$$

Noting that  $\frac{\partial \ln \Lambda^F}{\partial \ln \phi^*} = \theta^F$  and  $\frac{\partial \ln \Lambda^I}{\partial \ln \varphi^*} = \theta^I$ , and substituting the above two equations into the second equality of (27) establishes the desired result.

### A.4 Proof of (36)

Let us first show the first equation of (36). Noting that aggregate domestic expenditure  $\lambda R^F$  is decomposed into the extensive margin and intensive margin, we have

$$\lambda R^F = N \frac{k\sigma}{k - (\sigma - 1)} f \iff N = \frac{\lambda L}{\frac{k\sigma}{k - (\sigma - 1)} f},$$

where  $R^F = L$ . Under the Pareto distribution, this mass is also expressed as

$$N = \left( \frac{\phi_{\min}}{\phi^*} \right)^k \frac{\sigma - 1}{k\sigma} \frac{L}{f_e}.$$

Combining the above two expressions and rearranging gives the result. Following the similar steps, it is straightforward to show the second equation of (36).

## A.5 Proof of (37)

Let us first show the first equation of (37). Differentiating  $N_e$  with respect to  $\tau$ , we have

$$\begin{aligned}
 \frac{dN_e}{d\tau} &= fg(\phi^*) \frac{d\phi^*}{d\tau} + f_t g(\phi_t^*) \frac{d\phi_t^*}{d\tau} \\
 &= fg(\phi^*) \frac{d\phi^*}{d\tau} + f_t g(\phi_t^*) \left( -\frac{f}{f_t} \frac{J'(\phi^*)}{J'(\phi_t^*)} \frac{d\phi^*}{d\tau} \right) \quad (\text{using (A.3)}) \\
 &= fg(\phi^*) \frac{d\phi^*}{d\tau} \left( 1 - \frac{g(\phi_t^*)}{g(\phi^*)} \frac{J'(\phi^*)}{J'(\phi_t^*)} \right).
 \end{aligned}$$

Next, differentiating  $J(a)$  with respect to  $a$ ,

$$J'(a) = -(\sigma - 1)a^{-\sigma}V(a).$$

Further, differentiating  $J(a) + 1 - G(a) = a^{1-\sigma}V(a)$  with respect to  $a$  and rearranging,

$$g(a) = a^{-\sigma}V(a) \left( -\frac{d \ln V(a)}{d \ln a} \right).$$

Substituting these, we have

$$\begin{aligned}
 \frac{dN_e}{d\tau} &= fg(\phi^*) \frac{d\phi^*}{d\tau} \left[ 1 - \left( \frac{\phi_t^*}{\phi^*} \right)^{-\sigma} \frac{V(\phi_t^*)}{V(\phi^*)} \frac{\theta_t^F}{\theta^F} \left( \frac{\phi^*}{\phi_t^*} \right)^{-\sigma} \frac{V(\phi^*)}{V(\phi_t^*)} \right] \\
 &= fg(\phi^*) \frac{d\phi^*}{d\tau} \left( 1 - \frac{\theta_t^F}{\theta^F} \right),
 \end{aligned}$$

which establishes the desired result. Following the similar steps, it is straightforward to show the second equation of (37).

## Appendix B: Final-good Trade

In this appendix, we develop a model in which only final goods are tradable in the baseline setup, which is used to compare aggregate trade flows and welfare between the two types of trade. To make it easier to compare the result, the appendix is organized in a similar way to the main text.

### B.1 Model

#### B.1.1 Consumers

Consumers' preferences are given by a C.E.S. utility function with elasticity  $\sigma$ :

$$U = \left[ \int_{v \in V} q(v)^{\frac{\sigma-1}{\sigma}} dv + \int_{v \in V_t} q_t(v)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}},$$

where  $q(v)$  and  $q_t(v)$  are final goods produced by domestic firms and foreign exporters. Utility maximization yields the demand function for final goods:

$$\begin{aligned} q(v) &= R^F P^{\sigma-1} p(v)^{-\sigma}, \\ q_t(v) &= R^F P^{\sigma-1} p_t(v)^{-\sigma}, \end{aligned}$$

where  $R^F = \int_{v \in V} p(v)q(v)dv + \int_{v \in V_t} p_t(v)q_t(v)dv$  is final-good expenditure and

$$P = \left[ \int_{v \in V} p(v)^{1-\sigma} dv + \int_{v \in V_t} p_t(v)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}$$

is the price index of final goods.

#### B.1.2 Final-good Firms

Final-good firms technologies are also given by a C.E.S. function with elasticity  $\sigma$ :

$$\begin{aligned} q(\phi) &= \phi \left[ \int_{\omega \in \Omega} x(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \\ q_t(\phi) &= \phi \left[ \int_{\omega \in \Omega_t} x_t(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \end{aligned} \tag{B.1}$$

where  $x(\omega)$  and  $x_t(\omega)$  are domestic inputs used by domestic firms and foreign exporters. The corresponding expenditures of these final-good firms are given by

$$\begin{aligned} e &= \int_{\omega \in \Omega} \gamma(\omega)x(\omega)d\omega, \\ e_t &= \int_{\omega \in \Omega_t} \gamma_t(\omega)x_t(\omega)d\omega, \end{aligned}$$

where  $\gamma(\omega)$  and  $\gamma_t(\omega)$  are domestic input prices faced by domestic firms and foreign exporters. It follows from the pricing rules of intermediate-input firms that these input prices satisfy

$$\gamma(\omega) = \gamma_t(\omega).$$

Cost minimization yields the demand function for domestic firms:

$$\begin{aligned} x(\omega) &= e\Gamma^{\sigma-1}\gamma(\omega)^{-\sigma}, \\ \Gamma &= \left[ \int_{\omega \in \Omega} \gamma(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \end{aligned} \tag{B.2}$$

and the input demand function for foreign exporters:

$$\begin{aligned} x_t(\omega) &= e_t\Gamma_t^{\sigma-1}(\tau\gamma_t(\omega))^{-\sigma}, \\ \Gamma_t &= \left[ \int_{\omega \in \Omega_t} (\tau\gamma_t(\omega))^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}, \end{aligned} \tag{B.3}$$

where  $\Gamma$  and  $\Gamma_t$  are the dual unit cost functions for final-good production by domestic firms and foreign exporters respectively. The dual unit cost functions satisfy

$$\Gamma_t = \tau\Gamma.$$

Since intermediate inputs are non-tradable, there is no love-of-variety effect and the unit cost of foreign exporters is higher than that of domestic firms simply due to the variable trade cost.

Substituting (B.2) and (B.3) into (B.1) yields

$$\begin{aligned} q(\phi) &= \frac{\phi e}{\Gamma} \iff e(\phi) = \frac{\Gamma}{\phi} q(\phi), \\ q_t(\phi) &= \frac{\phi e_t}{\Gamma_t} \iff e_t(\phi) = \frac{\tau\Gamma}{\phi} q_t(\phi). \end{aligned}$$

The profits of the two types of firms are then

$$\begin{aligned} \pi(\phi) &= p(\phi)q(\phi) - \frac{\Gamma}{\phi} q(\phi) - f, \\ \pi_t(\phi) &= p_t(\phi)q_t(\phi) - \frac{\tau\Gamma}{\phi} q_t(\phi) - f_t. \end{aligned}$$

The pricing rules are given by

$$\begin{aligned} p(\phi) &= \frac{\sigma}{\sigma-1} \frac{\Gamma}{\phi}, \\ p_t(\phi) &= \frac{\sigma}{\sigma-1} \frac{\tau\Gamma}{\phi}. \end{aligned}$$

The equilibrium revenues of the two types of firms are

$$\begin{aligned} r^F(\phi) &= \Gamma^{1-\sigma} \sigma B^F \phi^{\sigma-1}, \\ r_t^F(\phi) &= (\tau\Gamma)^{1-\sigma} \sigma B^F \phi^{\sigma-1}, \end{aligned}$$

where

$$B^F = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} R^F P^{\sigma-1}$$

is the index of final-good market demand. The equilibrium profits are

$$\begin{aligned} \pi^F(\phi) &= \frac{r^F(\phi)}{\sigma} - f = \Gamma^{1-\sigma} B^F \phi^{\sigma-1} - f, \\ \pi_t^F(\phi) &= \frac{r_t^F(\phi)}{\sigma} - f_t = (\tau\Gamma)^{1-\sigma} B^F \phi^{\sigma-1} - f_t. \end{aligned}$$

The ZCP condition identifies the productivity cutoffs that satisfy  $\pi^F(\phi^*) = 0$  and  $\pi_t^F(\phi_t^*) = 0$ :

$$\Gamma^{1-\sigma} B^F (\phi^*)^{\sigma-1} = f, \quad (\text{B.4})$$

$$(\tau\Gamma)^{1-\sigma} B^F (\phi_t^*)^{\sigma-1} = f_t, \quad (\text{B.5})$$

and

$$\frac{\phi_t^*}{\phi^*} = \tau \left( \frac{f_t}{f} \right)^{\frac{1}{\sigma-1}} > 1.$$

In addition to (B.4) and (B.5), we need to impose the FE condition:

$$\begin{aligned} \int_{\phi^*}^{\infty} \pi^F(\phi) dG(\phi) + \int_{\phi_t^*}^{\infty} \pi_t^F(\phi) dG(\phi) &= f_e \\ \iff fJ(\phi^*) + f_t J(\phi_t^*) &= f_e. \end{aligned} \quad (\text{B.6})$$

It is important to note that (B.5) does not contain  $\Lambda^I$ . Thus, (B.4)–(B.6) solely provide implicit solutions for three unknowns  $\phi^*, \phi_t^*, B^F$  without referring to the upstream sector.

The aggregate amount of labor for final-good production is

$$L^F = N_e f_e + N_e \int_{\phi^*}^{\infty} f dG(\phi) + N_e \int_{\phi_t^*}^{\infty} f_t dG(\phi),$$

which is rewritten as

$$\begin{aligned} L^F &= N_e \int_{\phi^*}^{\infty} (r^F(\phi) - e(\phi)) dG(\phi) + N_e \int_{\phi_t^*}^{\infty} (r_t^F(\phi) - e_t(\phi)) dG(\phi) = L^F \\ &= R^F - E. \end{aligned} \quad (\text{B.7})$$

As in the main text, the aggregate amount of labor equals aggregate revenue minus aggregate expenditure in the downstream sector.



Aggregate revenue of final-good firms is

$$\begin{aligned} R^F &= N_e \int_{\phi^*}^{\infty} r^F(\phi) dG(\phi) + N_e \int_{\phi_t^*}^{\infty} r_t^F(\phi) dG(\phi) \\ &= N_e \sigma \Gamma^{1-\sigma} B^F V(\phi^*) (1 + \tau^{1-\sigma} \Lambda), \end{aligned} \tag{B.8}$$

where

$$\Lambda = \frac{V(\phi_t^*)}{V(\phi^*)}$$

is the (endogenous) market share of *final-good* exporters in the domestic market. (Note that  $\Lambda^I$  is the market share of *intermediate-input* exporters and  $\Lambda^F$  is the market share of *intermediate-input* importers in the current model.)

It is useful to define the share of domestic expenditure in total final-good expenditure (from a viewpoint of consumers):

$$\lambda = \frac{N_e \int_{\phi^*}^{\infty} r^F(\phi) dG(\phi)}{R^F} = \frac{1}{1 + \tau^{1-\sigma} \Lambda}.$$

In contrast to intermediate-input trade, the domestic share contains only the market share of foreign exporters  $\Lambda$ .

As for the mass of entrants, using (B.4), (B.5), (B.6) and (B.7), we have

$$N_e = \frac{L^F + E}{\sigma \{f[1 - G(\phi^*)] + f_t[1 - G(\phi_t^*)] + f_e\}}.$$

The masses of domestic firms and foreign exporters are  $N = [1 - G(\phi^*)]N_e$  and  $N_t = [1 - G(\phi_t^*)]N_e$  respectively.

The price index is defined as

$$\begin{aligned} P^{1-\sigma} &= N_e \int_{\phi^*}^{\infty} \left( \frac{\sigma}{\sigma-1} \frac{\Gamma}{\phi} \right)^{1-\sigma} dG(\phi) + N_e \int_{\phi_t^*}^{\infty} \left( \frac{\sigma}{\sigma-1} \frac{\tau\Gamma}{\phi} \right)^{1-\sigma} dG(\phi) \\ &= N_e \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} \Gamma^{1-\sigma} V(\phi^*) (1 + \tau^{1-\sigma} \Lambda). \end{aligned}$$

From (B.8), this is related to market demand  $B^F$ :

$$P^{1-\sigma} = \frac{(\sigma-1)^{\sigma-1} R^F}{\sigma^\sigma B^F}.$$

Finally, we derive final-good demands of domestic firms and foreign exporters:

$$\begin{aligned} q(\phi) &= (\sigma-1) \Gamma^{-\sigma} B^F \phi^\sigma, \\ q_t(\phi) &= \tau^{-\sigma} (\sigma-1) \Gamma^{-\sigma} B^F \phi^\sigma. \end{aligned}$$

These in turn give us the expenditure of these firms:

$$\begin{aligned} e &= (\sigma - 1)\Gamma^{1-\sigma} B^F \phi^{\sigma-1}, \\ e_t &= \tau^{1-\sigma} (\sigma - 1)\Gamma^{1-\sigma} B^F \phi^{\sigma-1}. \end{aligned}$$

Using these, aggregate expenditure of final-good firms is

$$\begin{aligned} E &= N_e \int_{\phi^*}^{\infty} e(\phi) dG(\phi) + N_e \int_{\phi_t^*}^{\infty} e_t(\phi) dG(\phi) \\ &= N_e (\sigma - 1)\Gamma^{1-\sigma} B^F V(\phi^*) (1 + \tau^{1-\sigma} \Lambda), \end{aligned} \tag{B.9}$$

which is a fraction of aggregate expenditure of consumers (see (B.8)):

$$E = \left( \frac{\sigma - 1}{\sigma} \right) R^F.$$

On the other hand, intermediate-input demands of domestic firms and foreign exporters are

$$\begin{aligned} x(\phi, \varphi) &= (\sigma - 1) \left( \frac{\sigma - 1}{\sigma} \right)^\sigma B^F \phi^{\sigma-1} \varphi^\sigma, \\ x_t(\phi, \varphi) &= \tau^{-\sigma} x(\phi, \varphi). \end{aligned}$$

This is the characterization of the downstream sector. The crucial difference from intermediate-input trade is that there is no interaction with the upstream sector in final-good trade.

### B.1.3 Intermediate-input Firms

Intermediate-input firms' technologies are represented by a linear cost function of input. Since domestic firms (foreign exporters) above the cutoff  $\phi^*$  ( $\phi_t^*$ ) source intermediate input among  $N_e$  entrants, labor used for domestic production by intermediate-input firms is

$$l(\varphi) = f + N_e \int_{\phi^*}^{\infty} \frac{x(\phi, \varphi)}{\varphi} dG(\phi) + N_e \int_{\phi_t^*}^{\infty} \frac{x_t(\phi, \varphi)}{\varphi} dG(\phi),$$

where  $x(\phi, \varphi)$  and  $x_t(\phi, \varphi)$  are input demands of domestic firms and foreign exporters

The revenue of intermediate-input firms also depends on the productivity cutoffs of final-good firms. Recalling that intermediate inputs are non-tradable, the profit of these firms is

$$\pi^I(\varphi) = N_e \int_{\phi^*}^{\infty} \left[ \gamma(\varphi) x(\phi, \varphi) - \frac{x(\phi, \varphi)}{\varphi} \right] dG(\phi) + N_e \int_{\phi_t^*}^{\infty} \left[ \gamma_t(\varphi) x_t(\phi, \varphi) - \frac{x_t(\phi, \varphi)}{\varphi} \right] dG(\phi) - f.$$

The pricing rules are given by

$$\gamma(\varphi) = \gamma_t(\varphi) = \frac{\sigma}{\sigma - 1} \frac{1}{\varphi},$$

and intermediate-input firms set the same price for domestic firms and foreign exporters.

The equilibrium revenue of intermediate-input firms is

$$\begin{aligned} r^I(\varphi) &= N_e \int_{\phi^*}^{\infty} \gamma(\varphi) x(\phi, \varphi) dG(\phi) + N_e \int_{\phi_t^*}^{\infty} \gamma_t(\varphi) x_t(\phi, \varphi) dG(\phi) \\ &= N_e \sigma \left( \frac{\sigma - 1}{\sigma} \right)^\sigma B^F V(\phi^*) (1 + \tau^{1-\sigma} \Lambda) \varphi^{\sigma-1}. \end{aligned}$$

This revenue can be expressed in terms of intermediate-input market demand. To show this, note that aggregate revenue of intermediate-input firms  $R^I$  must equal aggregate expenditure of final-good firms  $E$  in (B.9) in equilibrium:

$$R^I = E \iff R^I = \left( \frac{\sigma - 1}{\sigma} \right) R^F,$$

where aggregate revenue of final-good firms  $R^F$  is given in (B.8). Substituting (B.8) into the above equality and rearranging, the equilibrium revenue is expressed as

$$r^I(\varphi) = \sigma B^I \varphi^{\sigma-1},$$

where

$$B^I = \frac{(\sigma - 1)^{\sigma-1}}{\sigma^\sigma} R^I \Gamma^{\sigma-1}$$

is the index of intermediate-input market demand. The equilibrium profit is then given by

$$\pi^I(\varphi) = \frac{r^I(\varphi)}{\sigma} - f = B^I \varphi^{\sigma-1} - f.$$

The ZCP condition identifies the productivity cutoff that satisfies  $\pi^I(\varphi^*) = 0$ :

$$B^I (\varphi^*)^{\sigma-1} = f. \tag{B.10}$$

In addition, we need to impose the FE condition:

$$\begin{aligned} \int_{\varphi^*}^{\infty} \pi^I(\varphi) dG(\varphi) &= f_e \\ \iff f J(\varphi^*) &= f_e. \end{aligned} \tag{B.11}$$

As in (B.4)–(B.6), (B.10) and (B.11) solely provide implicit solutions for two unknowns  $\varphi^*, B^I$  without referring to the interaction with the downstream sector.

The aggregate amount of labor for intermediate-input production is

$$L^I = M_e f_e + M_e \int_{\varphi^*}^{\infty} \left( f + \int_{\phi^*}^{\infty} \frac{x(\phi, \varphi)}{\varphi} dG(\phi) \right) dG(\varphi).$$

Using (B.11), this condition is rewritten as

$$\begin{aligned} L^I &= M_e \int_{\varphi^*}^{\infty} r^I(\varphi) dG(\varphi) \\ &= R^I. \end{aligned} \tag{B.12}$$

Condition (B.12), together with (B.7), is used to characterize the LMC condition of the economy, as will be shown later.

Aggregate revenue of intermediate-input firms is

$$\begin{aligned} R^I &= M_e \int_{\varphi^*}^{\infty} r^I(\varphi) dG(\varphi) \\ &= M_e \sigma B^I V(\varphi^*). \end{aligned} \tag{B.13}$$

As for the mass of entrants, using (B.10), (B.11) and (B.12),

$$M_e = \frac{L^I}{\sigma \{f[1 - G(\varphi^*)] + f_e\}}.$$

The mass of domestic firms is given by  $M = [1 - G(\varphi^*)]M_e$ .

The unit cost of domestic firms is

$$\begin{aligned} \Gamma^{1-\sigma} &= M_e \int_{\varphi^*}^{\infty} \left( \frac{\sigma}{\sigma-1} \frac{1}{\varphi} \right)^{1-\sigma} dG(\varphi) \\ &= \left( \frac{\sigma}{\sigma-1} \right)^{1-\sigma} M_e V(\varphi^*). \end{aligned}$$

From (B.13), this is related to market demand  $B^I$ :

$$\Gamma^{1-\sigma} = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \frac{R^I}{B^I}.$$

The unit cost of foreign firms is simply given by  $\Gamma_t = \tau\Gamma$  (see (B.3)).

Finally, we define the LMC condition of the economy. The LMC condition implies that the amount of labor used in production equals the fixed labor endowment, i.e.,  $L^F + L^I = L$ . From  $L^F = R^F - R^I$  (since  $R^I = E$ ) and  $L^I = R^I$ , final-good expenditure equals aggregate payments to labor  $R^F = L$ . Further, from  $R^I = \left(\frac{\sigma-1}{\sigma}\right) R^F$ , labor allocations are exogenously fixed:

$$L^F = \left(\frac{1}{\sigma}\right) L, \quad L^I = \left(\frac{\sigma-1}{\sigma}\right) L.$$

This completes the characterization of the model. Note importantly that the upstream sector also has no interaction with the downstream sector, which stands in contrast to intermediate-input trade. This difference is key to the impact of final-good trade liberalization.

## B.2 Equilibrium

### B.2.1 Equilibrium Variables

In the model of final-good trade, there are five unknowns which are jointly characterized by (B.4), (B.5), (B.6), (B.10) and (B.11):

$$\phi^*, \phi_t^*, B^F, \varphi^*, B^I.$$

In what follows, we will consider trade liberalization of final goods because we have assumed that intermediate inputs are non-tradable.

As stressed in the previous section, the equilibrium can be separately analyzed between the upstream sector and the downstream sector. In the upstream sector, the FE condition in (B.11) uniquely determines the productivity cutoff of intermediate-input firms  $\varphi^*$ . The ZCP condition in (B.10) in turn uniquely determines the input market demand  $B^I$ :

$$B^I = f(\varphi^*)^{1-\sigma}.$$

Then, reductions in the trade costs, either the variable trade cost  $\tau$  or the fixed trade cost  $f_t$ , have no impact on  $\varphi^*$  or  $B^I$ , because the trade costs do not appear in (B.10) and (B.11):

$$\begin{aligned} \frac{d\varphi^*}{d\tau} &= 0, & \frac{dB^I}{d\tau} &= 0, \\ \frac{d\varphi^*}{df_t} &= 0, & \frac{dB^I}{df_t} &= 0. \end{aligned} \tag{B.14}$$

In the downstream sector, on the other hand, the ZCP and FE conditions jointly determine the productivity cutoffs of final-good firms  $\phi^*, \phi_t^*$  and the final-good market demand  $B^F$ . Solving (B.4), (B.5) and (B.6) simultaneously, we find that reductions in the trade costs gives rise to the following impacts on the three unknowns:

$$\begin{aligned} \frac{d\phi^*}{d\tau} &< 0, & \frac{d\phi_t^*}{d\tau} &> 0, & \frac{dB^F}{d\tau} &> 0, \\ \frac{d\phi^*}{df_t} &< 0, & \frac{d\phi_t^*}{df_t} &> 0, & \frac{dB^F}{df_t} &> 0. \end{aligned} \tag{B.15}$$

It follows immediately from (14) and (15) that final-good trade liberalization induces resource reallocations *only* in the downstream sector. This stands in contrast to input trade liberalization where resource allocations occur not only in the upstream sector, but also in the downstream sector. Intuitively, intermediate-input firms do not import final goods for their production and hence selection for firm importing does not arise in the upstream sector.

Given this impact of the trade costs in (B.14) and (B.15), final-good trade liberalization has no impact on the mass of upstream firms, while leading to the similar impact on the mass of downstream firms as in the main text.

## B.2.2 Aggregate Trade Flows

Let us next turn to the impact of the trade costs on aggregate final-good trade exports, which are the second term in the right-hand side of (B.8). It is useful to decompose the aggregate final-good exports into the extensive margin and the intensive margin:

$$\begin{aligned}
 R_t^F &= N_e \int_{\phi_t^*}^{\infty} r_t^F(\varphi) dG(\varphi) \\
 &= [1 - G(\phi_t^*)] N_e \times \frac{1}{1 - G(\phi_t^*)} \int_{\phi_t^*}^{\infty} r_t^F(\varphi) dG(\varphi) \\
 &= N_t \times \bar{r}_t^F.
 \end{aligned}$$

From (B.15), we have that

$$\begin{aligned}
 \frac{dR_t^F}{d\tau} < 0, & \quad \frac{dN_t}{d\tau} < 0, & \quad \frac{d\bar{r}_t^F}{d\tau} \geq 0, \\
 \frac{dR_t^F}{df_t} < 0, & \quad \frac{dN_t}{df_t} < 0, & \quad \frac{d\bar{r}_t^F}{df_t} \leq 0,
 \end{aligned}$$

which implies that the impact of trade costs on the extensive and intensive margins is similar between final-good trade and intermediate-input trade.

The full trade elasticity with respect to the variable trade cost is given by

$$\begin{aligned}
 \zeta^F &= -\frac{d \ln \left( \frac{1-\lambda}{\lambda} \right)}{d \ln \tau} \\
 &= (\sigma - 1) - \frac{d \ln \Lambda}{d \ln \tau}.
 \end{aligned} \tag{B.16}$$

Note that (B.16) is exactly the same as that shown by Melitz and Redding (2015), even in the presence of vertical linkages between the upstream and downstream sectors. On the other hand, the full trade elasticity with respect to the fixed trade cost is given by

$$\begin{aligned}
 \xi^F &= -\frac{d \ln \left( \frac{1-\lambda}{\lambda} \right)}{d \ln f_t} \\
 &= 0 - \frac{d \ln \Lambda}{d \ln f_t}.
 \end{aligned} \tag{B.17}$$

These suggests that, due to (B.14), there is no extra adjustment operating through the extensive margin in final-good trade.

To obtain closed-form solutions, let us apply a Pareto distribution. In this case, we have

$$\Lambda = \left( \frac{\tau^{\sigma-1} f_t}{f} \right)^{-\frac{k-(\sigma-1)}{\sigma-1}},$$

and (B.16) and (B.17) are expressed as

$$\begin{aligned}\zeta^F &= (\sigma - 1) + (k - (\sigma - 1)) = k, \\ \xi^F &= 0 + \frac{k - (\sigma - 1)}{\sigma - 1} = \frac{k}{\sigma - 1} - 1,\end{aligned}$$

which are exactly the same as those in Chaney (2008). Further, under this specific distribution, the extensive and intensive margins are expressed as

$$\begin{aligned}N_t &= \left(\frac{\phi_{\min}}{\phi_t^*}\right)^k \frac{\sigma - 1}{k\sigma} \frac{L}{f_e}, \\ \bar{r}_t^F &= \frac{k\sigma}{k - (\sigma - 1)} f_t.\end{aligned}$$

Substituting  $\phi_t^*$  from (B.5) into  $N_t$  and noting  $R_t^F = N_t^F \times \bar{r}_t^F$ , aggregate final-good exports are expressed as the gravity equation form:

$$R_t^F = \psi^F L (B^F)^{\frac{k}{\sigma-1}} (\tau\Gamma)^{-k} f_t^{1-\frac{k}{\sigma-1}}. \quad (\text{B.18})$$

where  $\psi^F = \frac{\sigma-1}{k-(\sigma-1)} \frac{(\phi_{\min})^k}{f_e}$ . From this expression, the partial trade elasticities with respect to the variable and fixed trade costs are given by

$$\begin{aligned}\zeta_o^F &\equiv -\frac{\partial \ln R_t^F}{\partial \ln \tau} = k, \\ \xi_o^F &\equiv -\frac{\partial \ln R_t^F}{\partial \ln f_t} = \frac{k}{\sigma - 1} - 1,\end{aligned}$$

which are the same as the full trade elasticities above. From (B.18), it is also straightforward to express the domestic trade share as

$$\frac{1 - \lambda}{\lambda} = \tau^{-k} \left(\frac{f_t}{f}\right)^{1-\frac{k}{\sigma-1}}.$$

Applying this expression to (B.16) and (B.17) also establishes the identity between the full trade elasticity and partial trade elasticity.

### B.2.3 Welfare

We next explore welfare. From (B.4) and (B.10),

$$\begin{aligned}\frac{1}{P} &= \left(\frac{\sigma - 1}{\sigma}\right) \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \frac{\phi^*}{\Gamma}, \\ \frac{1}{\Gamma} &= \left(\frac{\sigma - 1}{\sigma}\right) \left(\frac{L^I}{\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi^*,\end{aligned}$$

and welfare is written as

$$W^F = \frac{1}{P} = \left(\frac{\sigma-1}{\sigma}\right)^2 \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \left(\frac{L^I}{\sigma f}\right)^{\frac{1}{\sigma-1}} \phi^* \varphi^*. \quad (\text{B.19})$$

Thus, the productivity cutoffs of domestic production,  $\phi^*, \varphi^*$  are sufficient statistics for welfare. Note importantly that the domestic share  $\lambda$  does not appear in (B.19), which also constitutes sufficient statistics for welfare in intermediate-input trade. Recalling that  $\varphi^*$  is not affected by final-good trade liberalization (see (B.14)), the welfare gains from trade are given by

$$\frac{W}{W_A} = \frac{\phi^*}{\phi_A^*}.$$

We finally consider changes in welfare in (B.19). It follows from (B.14) that  $\widehat{\varphi}^* = 0$  and hence changes in welfare are expressed as

$$\begin{aligned} \widehat{W}^F &= \widehat{\phi}^* \\ &= \frac{1}{(\sigma-1) + \theta} \widehat{N}_e + \frac{1}{(\sigma-1) + \theta} (-\widehat{\lambda}), \end{aligned} \quad (\text{B.20})$$

where

$$\theta \equiv -\frac{d \ln V(\phi^*)}{d \ln \phi^*} > 0.$$

(A.20) often appears in the literature when analyzing the changes in welfare for the case of the single-stage production (see, e.g., Head et al., 2014). Under the Pareto distribution,  $\theta = k - (\sigma - 1)$  and  $\widehat{N}_e = 0$ , and hence (B.20) is simplified as

$$\begin{aligned} \widehat{W}^F &= \widehat{\lambda}^{-\frac{1}{k}} \\ &= \widehat{\lambda}^{-\frac{1}{\zeta_o^F}}. \end{aligned}$$

Thus, in this special case, the changes in welfare is captured only by the two sufficient statistics  $\lambda, \zeta_o$  and the welfare gains from trade are the same between intermediate-input trade and final-good trade.



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