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Demand System and Liquidity Constraints: Simple Methodology for Measuring Liquidity Constraint[i]

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Abstract

This paper proposed a new methodology, which introduces a stochastic property in the field of demand analysis, for measuring the skewness of the consumption bundle distribution. We used the questionnaire data from the National Survey of Family Income and Expenditure, Japan, from 1989 to 2004. We regard two categories of variables such as the total expenditure of each household and the expenditure share of 10 commodity groups (e.g., food expenditure share, medical expenditure share, etc.) as random variables, and estimated the probability density function using the "Skew Normal Distribution." As a result, distortion in the same direction was detected in all of the 10 commodity groups. We also calculated that the measured skewness mutually relates to the major proxy of the liquidity constraint. As a proxy of households under liquidity constraint, here we employ the measurement of the fraction of Hand-to-Mouth (HtM) households. This paper was motivated by the need to provide a simple methodology for measuring the liquidity constraint even in countries with poor statistical literacy. Although several proxies have been proposed to measure the liquidity constraint, the methodology becomes complex for more rigorous measuring attempts. In contrast, our methodology only requires micro data on total consumption and a separate total for each commodity group.

Keywords: Demand System, AIDS, QUAIDS, Skew Normal Distribution.
JEL classification number: C65; D12; E21.

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1 Introduction

Analysis of consumer demand has long been recognized as a critical issue to micro and macro economics. One of the most famous work is of Engel's (Engel (1857)). Engel curves describe the change of expenditure share for commodities as a function of income. The demand analysis of Engel's was concerned in the explanation of behavioral differences between households in cross section studies. The price vector in cross section studies is constant across all households, and in such case the homogeneity of demand function plays no part. On the other hand, the adding up restriction still plays a part. Here the adding up restriction represents the relation; $\sum_i p_i g_i(x, \mathbf{p}) = x$ with total expenditure x, prices p_i, $\mathbf{p}$ and demand function $g_i(x, \mathbf{p})$. However, the traditional budget studies have set its focus on choosing functional forms of purchases or budget shares with respect to the total consumption. Such simplification enables to abstract the expression of the demand of commodity i (q_i) to be

$$q_i = f_i(x) \tag{1.1}$$

where x is the total expenditure. This relationship is commonly referred to as an Engel curve. The exploration of the functional form of Engel curves created new literature. Prais and Houthakker (1955) examined the superiority of 3 types of functional form; (i) double logarithmic, (ii) semi-logarithmic and (iii) log reciprocal, and found each models has some claim of superiority for some commodities and over part of the expenditure range. Working (1943) and Leser (1963) estimated the semi-logarithmic relation of budget shares and the logarithm of total expenditure as;

$$\omega_i = \beta_i + \gamma_i \log x + u_i \tag{1.2}$$

where β_i and γ_i are parameters to be estimated, ω_i is the budget share of a commodity i and u_i is an error term. This relation and its extension has been studied frequently, as the adding up restriction is satisfied when $\sum_i \beta_i = 1$ and $\sum_i \gamma_i = 0$. AIDS (Almost Ideal Demand System, Deaton and Muellbauer (1980)) and QUAIDS (Quadratic Almost Ideal Demand System, Banks et. al. (1997)) employs this expression as Leser (1963) found this functional form fit better than some alternatives. In addition to this expression, some employ PIGL or PIGLOG and these functional form have been favored because of their exact aggregation or representative agent properties (see, for example, Lewbel (1987)). After the development of the AIDS, several empirical studies revealed that further terms in income are required for some, but not all, expenditure share equations (see, for example, Atkinson et. al. (1990) and Hausman (1995)) and this empirical results leads to the new class of demand system, QUAIDS. AIDS and QUAIDS generally treat time-series analysis, and therefore β_i and γ_i (and also the coefficient of quadratic term in case of QUAIDS) become functions of the price vector $\mathbf{p}$.

Another frequently used functional form of Engel curves is the double logarithmic, mainly because of the empirical convenience. If we assume the double logarithmic form as

$$\log q_i = \beta_i + e_i \log x + \sum_k e_{ik} \log p_k + u_i \tag{1.3}$$

the total expenditure elasticity $(e_i = \partial \log q_i / \partial \log x)$ and price elasticity $(e_{ij} = \partial \log q_i / \partial \log p_j)$ can easily estimated directly from empirical studies. A simple calculation leads to

$$\log \omega_i = \beta_i + (e_i - 1) \log x + (e_{ii} + 1) \log p_i + \sum_{k \neq i} e_{ik} \log p_k + u_i, \tag{1.4}$$

and especially in the case of cross section studies the relation is reduced to

$$\log \omega_i = \beta_i' + \gamma_i' \log x + u_i. \tag{1.5}$$

Lastly it should be noted that the AIDS, QUAIDS and any other functional forms described above is considered as an arbitrary first (or second) order approximation to any demand system, and therefore any choices of the demand system does not directly set restrictions on the shape of the (indirect) utility function. If we assume the demand system like AIDS/QUAIDS, the cost function is required to satisfy the following 3 conditions, but no other severe restrictions is required; i) concave in the input prices, ii) homogeneous of degree 1 in the input prices and iii) second order differentiable.differentiable

In addition to the major literature on the demand system, another approach to analyze demand system by using new mathematical technology (see, for example, Velásquez-Giraldo et. al. (2018)).

Most of the empirical studies simply estimate the proposed Engel curves on time series data of expenditure, outlay, and prices with ordinary least squares or maximum likelihood method with setting the (logarithm of) demand of the commodity i or its budget share as explained variable. However, most of empirical analysis does not treat both the value of the budget share and total expenditure as stochastic variable. In this paper we focus the stochastic property of the the budget share and total expenditure, and describe its relation by analyzing its joint probability density function.

The reminder of this paper proceeds as follows. Section 2 sets up our baseline model. Section 3 presents the estimation of models and Section 4 concludes.

2 Household Survey Data and The Model

2.1 Data

We used household budget share and total consumption data from the *National Survey of Family Income and Expenditure* (hereafter, NSFIE), Japan from 1989 to 2004 in every 5 years. These data was obtained from the National Statistical Center, Japan and the survey was conducted by the Ministry of Internal Affairs and Communications. Although NSFIE is conducted for years, the micro data for each survey is not identified and the price information is not embedded in the data set. Therefore we choose to conduct cross section studies in stead of the time series analysis. The number of households of NSFIE is 44,537 in 1989, 44,687 in 1994, 44,540 in 1999 and 43,861 in 2004 (household with 2 or more persons only). The major consumption data for each household is 1) Total Expenditure, 2) Food Expenditure, 3) Dwellings Expenditure, 4) Electricity Expenditure, 5) Furniture Expenditure, 6) Wearing Expenditure, 7) Medical Expenditure, 8) Communication Expenditure, 9) Education Expenditure, 10) Entertainment Expenditure and 11) Other Expenditure. Here we made subset of households whose consumption for all expenditure (i.e., 1) to 11)) is positive. The reason to employ this restriction is that when we analyze double logarithm model, the budget share of the target commodity group have to be positive. After setting this restriction, the number of households of NSFIE reduced to 14,482 in 1989, 13,197 in 1994, 10938 in 1999 and 9,418 in 2004.

2.2 Contour and Engel curve

Before we move on to the detailed model explanation, let us introduce the contour of 2 stochastic variable, budget share and total expenditure. The Figure 1 (left side panel) represents the semi-logarithmic case; the joint probability density distribution of the logarithm of total Consumption ($\log x$, X axis) and Food expenditure share (ω_{Food}, Y axis). And the Figure 1 (right side panel) represents the double logarithmic case, which takes the logarithm of Food expenditure ($\log \omega_{Food}$, Y axis). In both cases we can observe monotone decreasing food expenditure share with respect to the total consumption.

Also, the mapping of the joint probability density distributions reveals several findings. First and foremost, the density distribution seems to be well-fitted by the Gaussian or some other related

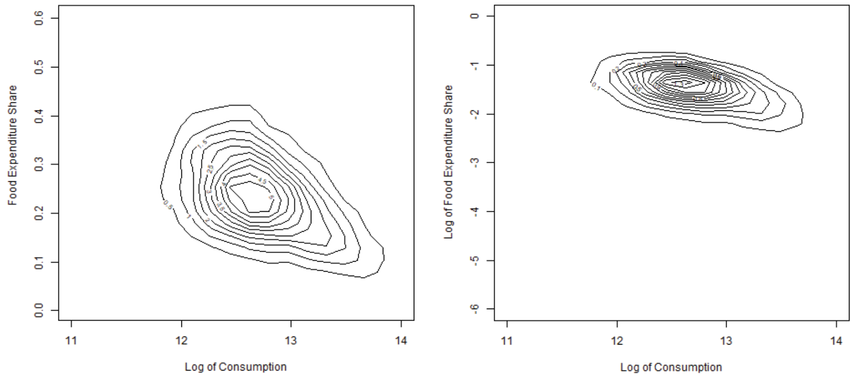

Figure 2.1: Contour of Logarithm of Consumption (X axis) and Food Expenditure Share (Y axis)

function. If we try to fit the relation of the total consumption and Food expenditure share with some function $\omega_i = f(x)$, the sample size decreases as the total income exceeds its median. So in high total expenditure region, it requires lots of samples to obtain significant estimation. However, if we fit the joint density function directly, less sample survey may work as we do not only focus on the high total expenditure region but the distribution as a whole.

The second finding is the difference of the shape of semi-logarithmic (Figure 2.1, Left) and double logarithmic (Figure 2.1, Right) distributions. In semi-logarithmic case, the tail of its distribution in low total expenditure region spreads widely compared to the high total expenditure region. In such sense, the double logarithmic case may yields better fitting estimation.

The third finding is about the top of the distribution. When we focus on the top of the distribution of double logarithmic case, the top does not locates at the exact center of the Gaussian-like ellipse contour, but slightly shifts to the upper left (i.e, low total expenditure and high Food Expenditure share) direction. The detail is discussed in the section 3.

2.3 Skew Normal Distribution

The standard normal distribution is not adequate to describe features of the joint probability density function of the total consumption and certain commodity group share, due to the skewness of the top of the density function. Azzalini et. al. (2014) provides useful measures to fit the "Skew Normal Distribution". Here the multivariate Skew Normal (SN) Distribution is defined as the followings. First denote by $f_o(\mathbf{x};\Omega)$ a probability density function of d-dimensional normal distribution with positive definite $d \times d$ correlation matrix Ω, and $\Phi\left(\alpha^{\mathrm{T}}\mathbf{x}\right)$ a probability distribution function of d-dimensional normal distribution. Then

$$f(\mathbf{x};\Omega,\alpha) = 2f_o(\mathbf{x};\Omega)\,\Phi\left(\alpha^{\mathrm{T}}\mathbf{x}\right) \tag{2.1}$$

is also a density function whose graphical appearance with different α is displayed in Figure 2.2.

Although there are many other type of multivariate SN distribution, we will introduce the easiest form of the SN distribution for the simplicity.

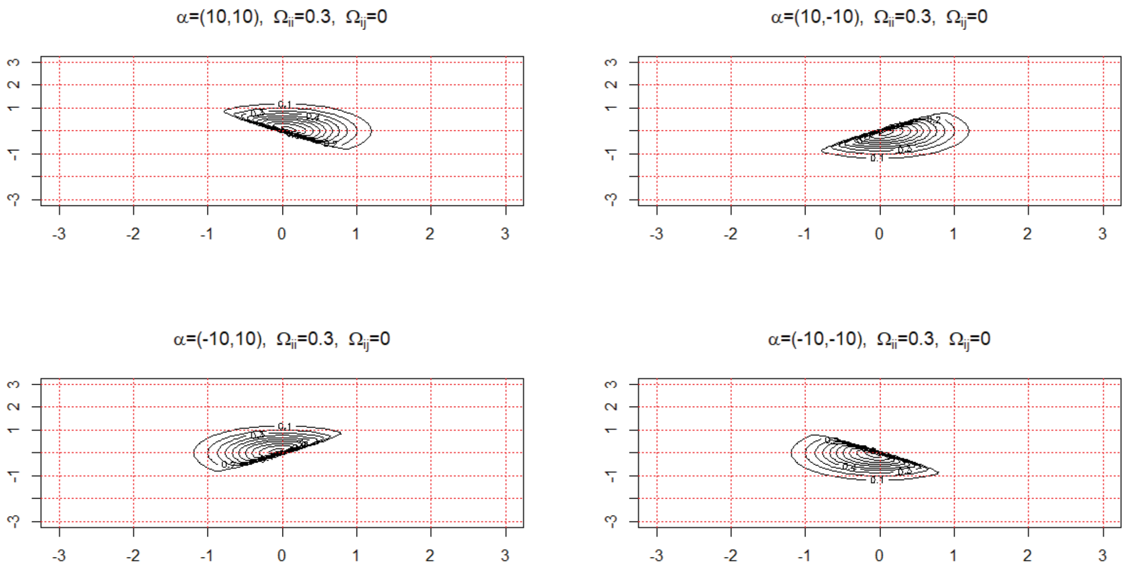

Figure 2.2: Schematic View of Contours of the Skew-Normal Distribution in Different α

2.4 Economic Interpretation: The Liquidity Constraint

Before we move onto the empirical analysis, let us spare some lines to understand the necessity and the economic background to assume the skew normal distribution. In general, the skew normal families admits several stochastic representations. As an introduction, here we provide most standard stochastic representation of the skew normal families (Azzalini et. al. (2014)); "Conditioning and Selective Sampling".

A variable $Z \sim \mathrm{SN}(0, 1, \alpha)$ can be obtained by the representation;

$$Z = \begin{cases} X_0 & \text{if } U < \alpha X_0 \\ -X_0 & \text{otherwise} \end{cases} \tag{2.2}$$

where X_0 and U are independent $\mathrm{N}(0, 1)$ variable. This representation shows that the sign of the variable X_0 becomes inverse when another independent variable U exceeds certain limit.

2.4.1 Skewness in Expenditure Share on Certain Commodity Group

When we translate this relation into economics, interesting features appears. For instance, let us assume the variable Z be the (logarithm of) the food expenditure share. If we assume this variable as SN, we have to consider 2 independent stochastic variables which 1) represents the original or potential food expenditure share (X_0), and 2) affects and sometimes changes the realized food expenditure share (U). Here let us assume U as the expenditure share of other commodities, such as furniture, medical expenditure, etc. If the household face unintended expenditure increase in certain commodity group (such as a rise in the sudden medical expenditure need), the household may reduce expenditure on other items such as food. This kind of relation can be described when $\alpha > 0$.

2.4.2 Skewness in the Total Expenditure

Also, in case of the $\alpha < 0$, opposite mechanism occurs. To understand this mechanism intuitively, it would be better to assume Z as a total consumption and U as a total income. If the household has weak consumption mind in initial ($X_0 < 0$), but faces unintended increase in the total income ($U > 0$), the household's budget constraint become loose and the household can afford wasteful expenditure ($Z = -X_0$), as long as the unintended increase in the total income exceeds the limit αX_0). On the other hand, if the household has strong consumption mind in initial ($X_0 > 0$), but faces unintended decrease in the total income ($U < 0$), the household save money considering the budget constraint and the expenditure becomes inverse ($Z = -X_0$), as long as the unintended decrease in the total income exceeds the limit same as before.

3 Estimation and Results

3.1 Contour of the Probability Density Functions

First and foremost, let us introduce the contour of the joint probability density function of each budget share and total expenditure. Figure 3.1 shows the double logarithmic plot of the contour of the joint PDF of the total consumption and each commodity group expenditure share; 1) Food Expenditure, 2) Dwellings Expenditure, 3) Electricity Expenditure, 4) Furniture Expenditure, 5) Wearing Expenditure, 6) Medical Expenditure, 7) Communication Expenditure, 8) Education Expenditure, 9) Entertainment Expenditure and 10) Other Expenditure in 2004 NSFIE. Only in the food expenditure share and Electricity Expenditure share shows clear negative trend with respect to the increase in the total consumption expenditure. Also, it should be noted that the skewness in the dwellings expenditure share becomes very large. This is mainly because the NSFIE employs "imputed rent" as the dwellings expenditure for house owners, which is not exactly equal to the market value of the house rent.

In case of the Furniture, Wearing, Medical, Communication and Entertainment Expenditure share, the distribution spread very wide and almost no correlation between each commodity group share and total consumption detected at a glance. In contrast, the contour of the Education Expenditure share and total consumption shows some interesting features. In the middle income region (Log of the total consumption ~ 12.5 or its around), some households exhibits relatively low Education Expenditure share compared to the average. On the other hand, when the total consumption increases, the Education Expenditure share notably increases. This feature may show an evidence for the interesting topic like absolute income mobility. Also, when we focus on the contour of Other Expenditure share and the total consumption, we find weak but positive correlation. The "Other Expenditure" is the only commodity group which exhibits positive correlation with respect to the total consumption. Here the "Other Expenditure" include accessories, bags, non-savings type premium and religious contribution.

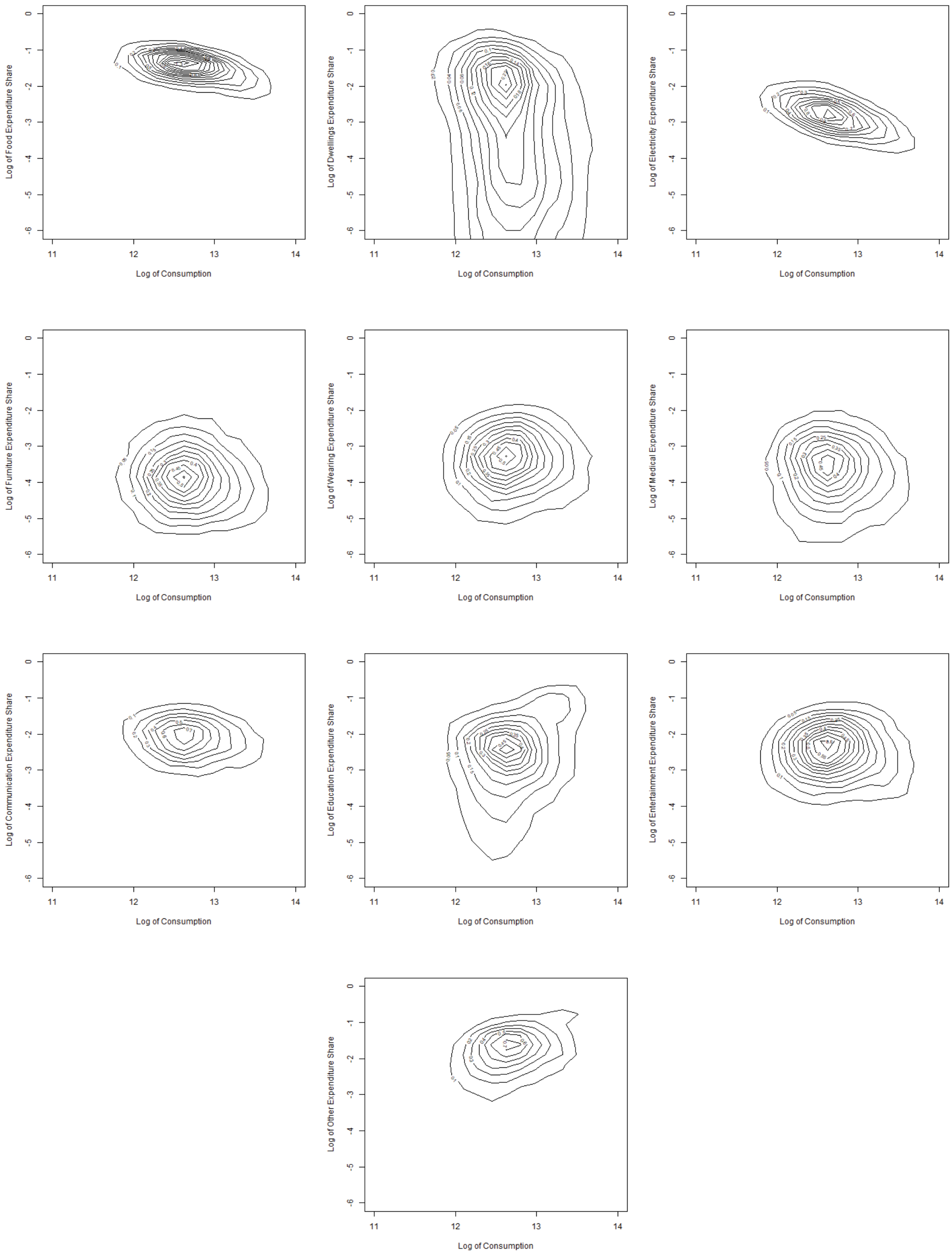

Figure 3.1: Contour of Logarithm of Consumption (X axis) and Each commodity group Expenditure Share (Y axis) of 2004 NSFIE.

3.2 Estimation Methods and Results

The estimation of the PDF itself written in Figure 3.1 can reserve many information. So we begin our analysis by providing parametric description of the density function in semi-logarithmic and double-logarithmic model. The parametric equation to be fitted is as described in (1.2) and (1.5). Given the stochastic assumptions, a sn package (v1.5-2) in R developed by Azzalini. Here let us spare several lines to explain how the estimation works. If we observe any of the figure in Figure 3.1, it might be reasonable to assume that there is some most probable point in the "Log of certain commodity group share" and the "Log of Total Consumption", and all households distributes around the most probable point in skew-normal distribution. In such sense, we do not directly assume any linear or other relationship between the "Log of certain commodity group share" and the "Log of Total Consumption", but just 1) estimate the most probable point (which equals to β_i in equation 1.2 and 1.5) and 2) how households distributes around the estimated most probable point.

The Table 1 and 2 presents parametric estimation result with using sn package in R. When we compare the estimated parameters both in case of semi-log and double-log model, we find that the skewness parameter α becomes quite unstable in the semi-log case. Especially in the estimation of the "Education Expenditure share", the estimated value of α exceeds 10^4 in x-axis and 10^7 in y-axis.

On the contrast to the result in semi-log case, the skewness parameter α behaves stable in the double-log case. The α_{Total} (total expenditure's coordinate of α) becomes positive for all commodities, while the α_{Share} (commodity group expenditure shares coordinate of α) becomes negative for all commodities. According to the Figure 2.2, this set of skewness parameters indicates that the skewness occurs especially in the region of 1) a low total consumption and 2) a high commodity group expenditure share. In addition, it should also be noted that the degree of skewness differs by commodity groups. The relatively strong skewness in commodity group expenditure share is detected in Food, Wearing, Education and Other Expenditure, while Furniture, Entertainment and Other Expenditure exhibits relatively strong skewness in total consumption.

We need a note to interpret these features, regarding the asymmetricity of the logarithm of the total consumption. In general, the income distribution of households obeys Pareto power law in high income group. This nature yields long tail in the probability density distribution in Figure 3.1, and also affects the skewness parameter. However, this nature can not adequately explain the difference of α_{Total} by commodity groups. As is written in the Table 2, the skewness in Food, Wearing, Education and Other Expenditure are strong compared to other commodity groups such as Furniture. Therefore it would be reasonable to consider there should be some underlying property for such commodity group. Here let us consider the meaning of 1) the strong skewness in commodity group expenditure share, and 2) weak skewness in total consumption. If we try to explain this feature in other words, low consumption households may wish to spend much money to the Food, Wearing, Education and Other Expenditure, however, such household can not increase these expenditure shares due to some reasons. One possible reason might be the existence of the liquidity constraint. As the sum of all commodity groups share has to be 1, there is a limitation to increase the expenditure share of basic commodities such as Food, Wearings, etc. under the liquidity constraint. The existence of the limitation create a distortion to the expenditure share allocation, and this may prevent households from fully optimizing their behavior.

3.3 Skewness and Liquidity Constraint

Then the next interest goes to how similar the skewness parameters and traditional liquidity constraint measures are. As a proxy of households under liquidity constraint, here we employ the measurement of the fraction of "Hand-to-Mouth (HtM)" households. Kaplan and Violante (2014) and Hara et. al. (2016) define a household as HtM households if its liquid wealth balance is: (1) positive and less than or equal to half of its earnings per pay-period; or (2) negative and within the sum of i) half of its per pay-period income and ii) its borrowing limit.

	α		Ω	β	
	α_{Total}	α_{Share}		β_{Total}	β_{Share}
Food Expenditure share	4.72×10^{-1}	2.47	$\begin{pmatrix} 0.225 & -0.029 \\ -0.029 & 0.014 \end{pmatrix}$	12.84	0.15
Dwellings Expenditure share	3.26×10^{4}	9.26×10^{4}	$\begin{pmatrix} 0.211 & -0.010 \\ -0.010 & 0.015 \end{pmatrix}$	12.76	0.00
Electricity Expenditure share	9.52×10^{-1}	5.84	$\begin{pmatrix} 0.255 & -0.015 \\ -0.015 & 0.002 \end{pmatrix}$	12.93	0.03
Furniture Expenditure share	2.91×10^{-1}	3.21×10^{1}	$\begin{pmatrix} 0.209 & 0.000 \\ 0.000 & 0.002 \end{pmatrix}$	12.72	0.00
Wearing Expenditure share	-7.07×10^{-2}	2.28×10^{1}	$\begin{pmatrix} 0.209 & 0.001 \\ 0.001 & 0.003 \end{pmatrix}$	12.70	0.00
Medical Expenditure share	4.40×10^{-1}	3.72×10^{2}	$\begin{pmatrix} 0.209 & -0.001 \\ -0.001 & 0.003 \end{pmatrix}$	12.72	0.00
Communication Expenditure share	2.81×10^{-1}	7.23	$\begin{pmatrix} 0.215 & 0.012 \\ 0.012 & 0.017 \end{pmatrix}$	12.63	0.04
Education Expenditure share	6.44×10^{4}	5.91×10^{7}	$\begin{pmatrix} 0.225 & 0.026 \\ 0.026 & 0.020 \end{pmatrix}$	12.58	0.00
Entertainment Expenditure share	8.56×10^{-2}	1.01×10^{1}	$\begin{pmatrix} 0.209 & 0.001 \\ 0.001 & 0.010 \end{pmatrix}$	12.70	0.02
Other Expenditure share	2.49×10^{-2}	6.57	$\begin{pmatrix} 0.238 & 0.040 \\ 0.040 & 0.033 \end{pmatrix}$	12.54	0.04

Table 1: Estimated Parameters (α, β and Ω) in 2004 NSFIE in semi-logarithmic case

	α		Ω	β	
	α_{Total}	α_{Share}		β_{Total}	β_{Share}
Food Expenditure share	0.72	-2.15	$\begin{pmatrix} 0.346 & -0.243 \\ -0.243 & 0.307 \end{pmatrix}$	12.34	-1.10
Dwellings Expenditure share	0.13	-13.87	$\begin{pmatrix} 0.221 & -0.493 \\ -0.493 & 14.301 \end{pmatrix}$	12.60	-1.06
Electricity Expenditure share	0.86	-1.12	$\begin{pmatrix} 0.364 & -0.301 \\ -0.301 & 0.417 \end{pmatrix}$	12.32	-2.41
Furniture Expenditure share	1.58	-0.62	$\begin{pmatrix} 0.378 & -0.208 \\ -0.208 & 0.874 \end{pmatrix}$	12.30	-3.50
Wearing Expenditure share	0.34	-2.46	$\begin{pmatrix} 0.218 & -0.089 \\ -0.089 & 1.590 \end{pmatrix}$	12.61	-2.58
Medical Expenditure share	0.56	-1.98	$\begin{pmatrix} 0.245 & -0.221 \\ -0.221 & 2.062 \end{pmatrix}$	12.52	-2.80
Communication Expenditure share	0.78	-1.52	$\begin{pmatrix} 0.265 & -0.120 \\ -0.120 & 0.580 \end{pmatrix}$	12.48	-1.66
Education Expenditure share	0.89	-4.93	$\begin{pmatrix} 0.213 & 0.006 \\ 0.006 & 3.543 \end{pmatrix}$	12.65	-1.41
Entertainment Expenditure share	1.49	-1.69	$\begin{pmatrix} 0.330 & -0.212 \\ -0.212 & 0.824 \end{pmatrix}$	12.36	-1.97
Other Expenditure share	1.04	-3.92	$\begin{pmatrix} 0.214 & 0.032 \\ 0.032 & 1.124 \end{pmatrix}$	12.64	-1.12

Table 2: Estimated Parameters (α, β and Ω) in 2004 NSFIE in double-logarithmic case

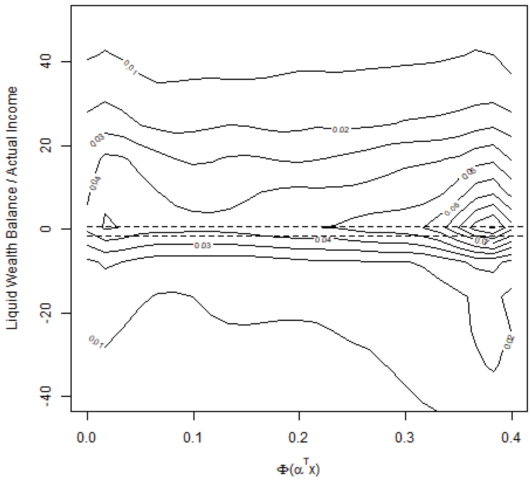

Figure 3.2: Liquidity Constraint and Skewness; Calculated from Parameters in Food Expenditure, 2004

The Figure 3.2 presents the distribution of households with respect to the skewness ($\Phi\left(\alpha^{\mathrm{T}}\mathbf{x}\right)$ in (2.1)) and liquid wealth balance divided by the pay-period income. The dashed line corresponds to the upper and lower limit for the HtM households. It is interesting that the peak of the distribution allocated along the upper limit of the HtM, i.e., its liquid wealth balance is equal to the half of its earnings per pay-period. According to the distribution in the Figure, the both of the following statements are true; 1) when we pick up HtM households, the households with high skewness becomes majority, and 2) when we pick up household with high skewness, the households with their liquid wealth balance over earnings around 0.5 becomes majority. These findings reveals that the skewness and liquidity constraint mutually related, and the skewness parameter might be a good proxy to measure the liquidity constraint.

3.4 Transition of the Skewness Parameter over Time

If we can consider the skewness parameter as an amount of distortion which prevent households from fully optimizing their behavior, it is also meaningful to analyze how these parameters have changed over time. The Table 3 presents the change in estimated parameters from 1989 to 2004. First, consider the estimated results for α_{Share}. The α_{Share} of Food, Furniture and Entertainment Expenditure share decreases slightly with time, while α_{Share} of Medical, Education and Other Expenditure shares are increasing. This result indicates that the distortion in Food, Furniture and Entertainment Expenditure is decreasing, but Medical, Education and Other Expenditure is increasing on contrast. Especially the distortion in Education and Other Expenditure should be emphasized in a senses that the distortion in these expenditure share is high and increasing.

Second, let us consider the dynamics in α_{Total}. Similar to the discussion in the dynamics of α_{Share}, the distortion in α_{Total} also increase/decrease by commodity group through the survey. However, the intuitive understanding of this distortion is much more difficult than the analysis in α_{Share}. In general, the distortion in x axis corresponds to the distortion of the top of the probability distribution function. However, the value of total consumption is same across all estimation and it is difficult to assume the distortion in x axis to differ by commodity group. One possible understanding is that the change in α_{Total} reflects the change in correlation between the total consumption and the expenditure share

	α_{Total}				α_{Share}			
	1989	1994	1999	2004	1989	1994	1999	2004
Food Expenditure share	0.46	0.50	0.54	0.72	-2.65	-2.50	-2.40	-2.15
Dwellings Expenditure share	0.14	0.13	0.26	0.13	-7.99	-9.80	-12.90	-13.87
Electricity Expenditure share	1.08	1.14	-0.10	0.86	-0.92	-0.74	-1.75	-1.12
Furniture Expenditure share	1.59	1.50	1.50	1.58	-0.69	-0.86	-0.77	-0.62
Wearing Expenditure share	0.75	0.65	0.44	0.34	-2.29	-2.52	-2.77	-2.46
Medical Expenditure share	1.21	0.85	0.69	0.56	-1.24	-1.71	-1.75	-1.98
Communication Expenditure share	0.98	0.87	0.70	0.78	-1.09	-1.51	-1.38	-1.52
Education Expenditure share	0.67	0.85	0.78	0.89	-4.00	-4.47	-4.75	-4.93
Entertainment Expenditure share	1.52	1.46	1.61	1.49	-1.58	-1.72	-1.60	-1.69
Other Expenditure share	1.00	0.96	1.00	1.04	-3.50	-3.86	-3.98	-3.92

Table 3: Estimated Parameters (α, β and Ω) in 2004 NSFIE in semi-logarithmic case

of each commodity group. As described in the Figure 3.1, the regression and correlation coefficient differs by commodity groups. The difference in regression coefficient results in the slope of the elliptic bivariate (Skewed) Gaussian, and the difference in its slope may affect the direction of the distortion.

3.5 Goodness-of-Fit

So far we only discussed the value of the estimated parameters, but did not refer any significance for each estimation. In this section we evaluate the "Goodness-of-Fit" by calculating Q-Q and P-P plot. Here Q-Q plot represents Quantile-Quantile plot, and is a graphical tool to help assessing whether a set of data plausibly came from some theoretical distribution, in this case the skew normal distribution. Q-Q plot compares two probability distributions, in most case theoretical and empirical values, by plotting their quantiles against each other. If these two distributions are similar, the Q–Q plot lies straight on the line $y = x$.

There is another methodology to assess the similarity of distribution, P-P plot. P-P plot represents Probability–Probability plot and compares the two cumulative distribution functions against each other. The difference in Q-Q and P-P plot is both known as probability plots, and the difference is the functional type of the comparison. Q-Q plot uses non-cumulative distribution function (or, density function) to calculate its quantiles, while P-P plot uses cumulative distribution function.

The Figure 3.2 and 3.3 presents Q-Q plot of Food, Wearing and Medical Expenditure share in semi-logarithmic (Figure 3.2) and double logarithmic (Figure 3.3) case. It is interesting to focus on a comparison with these results. The Food Expenditure share prefers semi-log case, whereas Wearing and Medical Expenditure share prefers double-log case. This results is same in P-P plot (Figure 3.3 and 3.4). However, most of Q-Q and P-P plots are not adequately significant and this leads we may need further modification of the model.

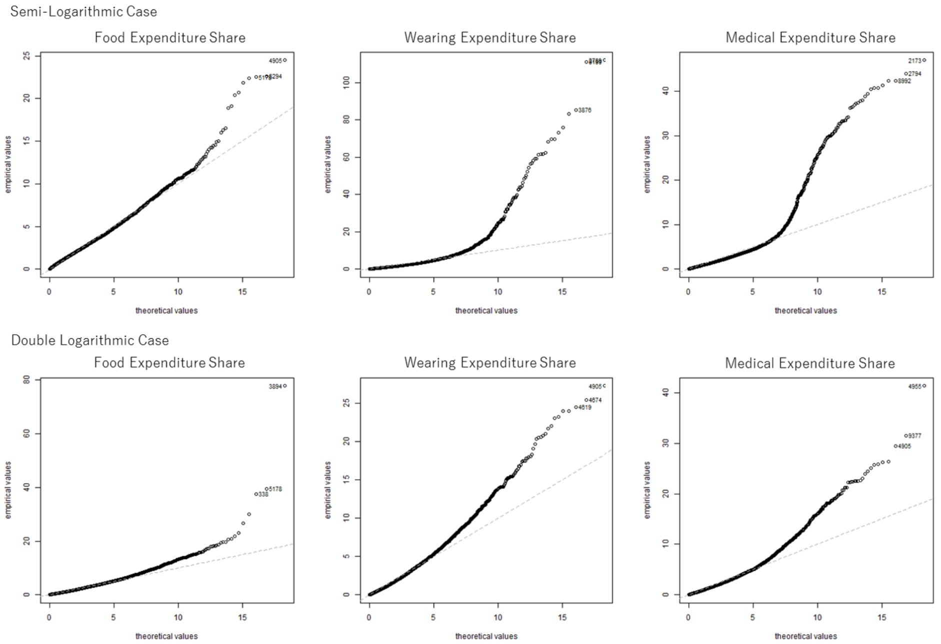

Figure 3.3: Q-Q Plot of Food (left), Wearing (middle) and Medical (right) Expenditure share of 2004 NSFIE in Semi and Double Log.

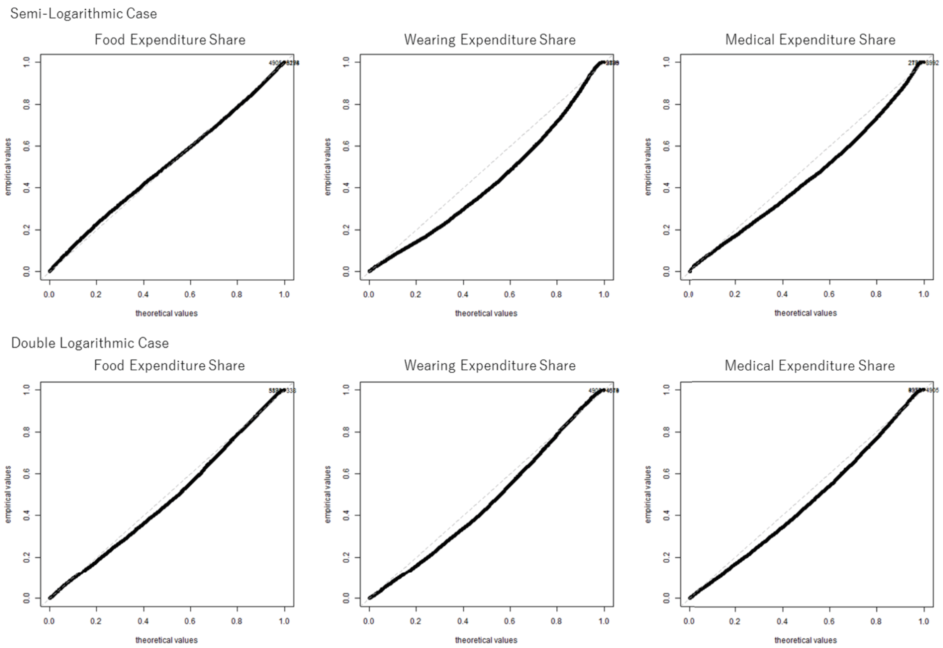

Figure 3.4: Plot of Food (left), Wearing (middle) and Medical (right) Expenditure share of 2004 NSFIE in Semi and Double Log.

3.6 Multi-Dimensional Estimation

Our new methodology also enable us to estimate all parameters in a single estimation by using multivariate skew normal distribution. First, consider 10 dimensional phase space, consist of the total expenditure and 9 commodity group's expenditure share. We eliminated 1 commodity group (Dwellings Expenditure share) from 10, to prevent multidisciplinary. The Table 4 presents the estimated parameters for this multivariate estimation in 1999 NSFIE. If we compare this result with Table 3 (such as comparing α_{Share} in Table 2 and α in Table 4), the skewness value slightly increases in most commodity groups.

	α	β	Ω									
			1)	2)	3)	4)	5)	6)	7)	8)	9)	10)
1)	0.40	12.72	0.19	-0.10	-0.13	-0.03	0.00	-0.06	0.02	0.05	-0.02	0.07
2)	-6.74	-1.28	-0.10	0.16	0.11	0.09	0.07	0.07	-0.05	-0.04	0.07	0.01
3)	-1.83	-2.77	-0.13	0.11	0.24	0.05	0.03	0.06	-0.04	-0.02	0.01	-0.04
4)	-1.61	-3.63	-0.03	0.09	0.05	0.73	0.18	0.14	-0.05	-0.10	0.12	0.06
5)	-2.71	-3.06	0.00	0.07	0.03	0.18	0.75	0.12	-0.05	-0.07	0.18	0.13
6)	-1.49	-3.75	-0.06	0.07	0.06	0.14	0.12	1.10	-0.04	-0.09	0.11	0.08
7)	-3.26	-2.43	0.02	-0.05	-0.04	-0.05	-0.05	-0.04	0.42	-0.07	-0.03	-0.04
8)	-4.04	-2.81	0.05	-0.04	-0.02	-0.10	-0.07	-0.09	-0.07	1.43	-0.12	-0.04
9)	-4.27	-2.38	-0.02	0.07	0.01	0.12	0.18	0.11	-0.03	-0.12	0.55	0.10
10)	-16.80	-1.28	0.07	0.01	-0.04	0.06	0.13	0.08	-0.04	-0.04	0.10	0.82

Table 4: Estimated Parameters (α, β and Ω) in 1999 NSFIE in multivariate, double-logarithmic case
Note: 1) Total Expenditure, 2) Food Expenditure share, 3) Electricity Expenditure share, 4) Furniture Expenditure share, 5) Wearing Expenditure share, 6) Medical Expenditure share, 7) Communication Expenditure share, 8) Education Expenditure share, 9) Entertainment Expenditure share and 10) Other Expenditure share.

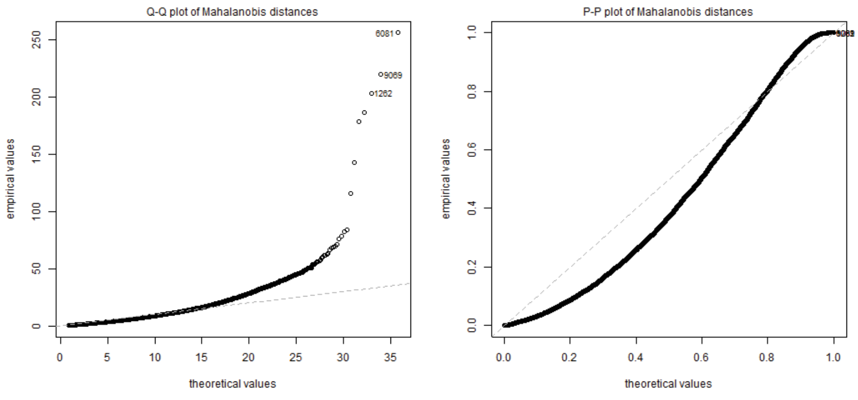

Figure 3.5: Contour of Logarithm of Consumption (X axis) and Food Expenditure Share (Y axis)

Here it should be emphasized that the estimation in multivariate skew normal is not stable compared to the bivariate case. Actually we failed to estimate parameters of 1994 and 2004 NSFIE in sn package. Although the multivariate skew normal estimation is simple and powerful, such estimation might not be appropriate in a sense of both stability and significance. As presented in Figure 3.5, the Goodness-of-Fit (Q-Q and P-P plot), provides less significance. The deviations of empirical value from the theoretical value increase at the tail in Q-Q plot and the middle in P-P plot, and these features appear in case of the less significance in general.

4 Summary and Discussion

This paper proposed new methodology to measure the skewness of the consumption bundle distribution. It is also calculated that the measured skewness mutually relates to the major proxy of the liquidity constraint; liquid wealth balance over earnings per pay-period. This paper was motivated by the needs to provide a simple methodology to measure the liquidity constraint, even in a country with poor statistical literacy. Although several proxies to measure the liquidity constraint are proposed, the methodology tend to become more complex if we try to measure it more rigorously. In contrast, our methodology only requires micro data on total consumption and its break down for each commodity group.

On the other hand, several issues are left for further discussions. First, the origin of the skewness should be considered carefully through detailed statistical analysis. As is discussed in 2.4, we assume the origin of the skewness to be a result from stochastic increase/decrease in the consumption of other commodity group. However, the effect of the skewness in the distribution of the total consumption, or income as a whole, is strong and we have to identify which effect plays significant role to estimate the skewness parameter α.

Second, the modification of the model will be required to improve the Goodness-of-Fit. The shape of Q-Q and P-P plot differs by commodity groups and estimated years, so more comprehensive or general model might be required. There are some other classes for the skewed distribution, such as skew Cauchy distributions, skew elliptical distributions, etc. The detailed understandings of each distribution and close consideration of the economic interpretation will lead to the better understandings of the hidden dynamics.

Third, the identification of the parameter for the proxy to measure the liquidity constraint is another important issue. In 3.3 we used the skewness parameter of Food Expenditure as a proxy to measure the liquidity constraint. However, the identification of the parameter as a proxy will need further discussions and validation through many statistical analysis in various countries. One of the ideal methodology to measure the skewness is to use the estimated parameters in multivariate skew distribution, but this methodology does not work in our analysis due to less stability and significance. This discussion should be continued in line with the model modification.

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