Trade, Location and Multiproduct Firms

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Abstract

In this paper we study how trade liberalization affects the location and the product scope of firms. We find that the largest and most productive multiproduct firms concentrate to the larger market as a result of trade liberalization. Given some relocation costs, we also find that these firms will expand their product range in the larger market while firms in the smaller market will contract their product scope. These effects are magnified with firm-level productivity. The findings are consistent with Japanese manufacturing firm data.

Keywords: Multi-product firms, Heterogeneous firms, Spatial sorting, Trade

JEL classification: F12, F15

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1 Introduction

Multiproduct firms are important. For example, in U.S. data 1987-1997 they stand for 87 percent of total output (Bernard et al. 2010). Multiproduct firms are also important for the dynamics of the economy. Using Universal Product Codes for U.S. data, Broda and Weinstein (2010) show that product creation and product destruction are four times more important than plant creation and destruction, and Goldberg et al. (2010) estimate that an increasing product scope among existing firms explained about 25 percent of the increase in India’s manufacturing output between 1989 and 2003.\footnote{Using Japanese plant and product level data, Bernard and Okubo (2016) show how the product churning is most intensive during recessions.} Several recent real business cycle (RBC) papers point to a pro-cyclical product scope as an important factor magnifying fluctuations due to technology shocks, see e.g. Minniti and Turino (2013).

In this paper, we study how trade liberalization affects the location and the product scope of firms. We find that the largest and most productive multiproduct firms concentrate to the larger market as a result of trade liberalization. Given some relocation costs, we also find that these firms will expand their product range in the larger market while firms in the smaller market will contract their product scope. The effects are magnified for more productive firms, which is consistent with Japanese manufacturing firm data.

The novelty of this paper is that we allow firms to choose location and product scope as trade is liberalized. The trade literature on multiproduct firms has focused on how trade liberalization affects the product scope, for a given location of firms. One set of papers finds that trade liberalization reduces firms’ product scope as firms concentrate on their core products as trade is liberalized. This effect tends to occur in oligopolistic frameworks where the firm has a core product and new products with a higher marginal cost compete with those already produced by the firm (the so-called "cannibalization effect"). Examples of such models are Blanchard et al. (2012), Eckel and Neary (2010) and Ju (2003). A similar result is found by Mayer et al. (2014) who use a monopolistically competitive model with heterogeneous firms and linear demand. The model by Bernard et al. (2011) has heterogeneous firms that match its “ability" to various product attributes (or consumer preferences). Here trade liberalization can lead to a larger or smaller product scope depending on whether consumer preferences are product or product-market specific. Finally, the paper by Feenstra and Ma (2007) uses standard CES-preferences but relaxes the large group assumption. By having firms accounting for their own effect on the aggregate price index, they get a cannibalization effect from new products. This paper produces larger firms with a higher product scope as a result of trade liberalization.

Another set of recent papers points to different effects of trade liberalization for high and low productive firms, where high productive firms increase their product scope as they gain better access to the foreign market whereas low productive firms contract the product scope due to the increased competition in the home market (Dhingra 2013, Nocke and Yeaple 2014,
and Qiu and Zhou 2013). Eckel et al. (2015) allow for vertical (quality) upgrading as well as horizontal (scope) upgrading. Trade liberalization (tariff reductions) here leads to a lower scope for all firms, but the firms enjoying a more effective investment upgrade quality.

The notion that trade liberalization has a differential effect on high and low productive firms or on non-exporters and exporters has some empirical support. For example, Baldwin and Gu (2009) find that non-exporting Canadian manufacturing firms decrease their product scope as a result of stronger competition due to CUSFTA, whereas no such effect is seen for exporters. Using the same experiment, Lopresti (2016) finds that firms with less than 10-20% foreign sales reduce their product scope, whereas firms above this threshold instead increase their product scope.

While our paper is most closely related to the trade literature, there is also a literature that analyzes multiproduct firms in an essentially closed economy. Chisholm and Norman (2004) analyze the location choices of multiproduct firms in a Hotelling framework with heterogeneous consumers. Flach and Irlacher (2018) use a framework with non-homothetic preferences and linear demand à la Eckel et al. (2015) to analyze multiproduct firms that can invest in both product and process innovation. They show that a larger market size implies higher R&D investments of both types. However, in highly differentiated industries, the cannibalization effect is lower and firms invest more in product innovation.

Our paper analyzes agglomeration and spatial sorting due to trade liberalization in a model with heterogeneous firms à la Melitz (2003) and Baldwin and Okubo (2006) that is extended with an endogenous product scope. We assume that the fixed cost of a new product increases with the number of products and that firms are heterogeneous in marginal costs à la Melitz (2003). The fact that per product fixed costs increase with a firm's product scope may be viewed either as increasing coordination costs or as within-firm diminishing returns to product development. A firm determines its optimal product scope by trading off fixed coordination costs that increase with the number of products against higher operating profits. Trade liberalization implies increased agglomeration to the core. The most productive firms in the periphery, which are also the firms with the largest product scope in this market, relocate from the periphery to the core. Given some relocation costs, the relocating firms will expand their product range in the larger market, whereas the firms remaining in the smaller market contract their product range.

That the largest multiproduct firms relocate away from the periphery, while the remaining firms decrease their product scope, may seem a worrying outcome for the periphery. However, our analysis shows that the introduction of an endogenous product scope also dampens

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2 This assumption is also employed by e.g. Qui and Zou (2013) who use a linear-quadratic preference structure à la Melitz and Ottaviano (2008).

3 This is consistent with a higher level of product churning due to trade liberalization as documented by Iacovone and Javorcik (2010).

4 This is consistent with the findings by Manova and Yu (2017).
agglomeration, and that the periphery always gains from trade liberalization.

Our model is consistent with stylized evidence from Japanese firm-level data, which shows that the scope of firms increases in productivity and that, controlling for productivity, the scope of firms increases when located in the large core market. Similar patterns have been documented by other studies in the international trade literature, such as multi-product firms being larger than single-product firms (Bernard et al., 2011, Goldberg et al. 2010) and that they are more productive (Schoar, 2002).

The following section presents some stylized evidence, and section 3 presents the model. Finally, section 4 concludes the paper.

2 Stylized facts

We here document several stylized facts concerning the product scope of Japanese firms. We use the 47 prefectures with varying market size as proxies for different markets, and use firm-level manufacturing data for Japan from the METI Basic Survey of Japanese Business Structure and Activities, which is an annual firm-level survey conducted by METI (Ministry of Economy, Trade, and Industry, Japan). The data set covers the years 1994-2013 (census year 1995-2014). The firm-level data is matched with product data from the Census of Manufacture at the six-digit level by METI and contains around 11,000 manufacturing firms each year. We use time-consistent product codes à la Pierce and Schott (2012) at the six-digit level, which implies that we have 2060 time-consistent product codes. Our firm-level data includes all manufacturing firms with more than 50 regular employees and with at least 30 million Yen (approximately US$275,000) of capital assets.

A majority of the firms in our data set produce several products. Overall, the average number of products among manufacturing firms is 2.85 with a standard deviation of 3.15. The location of a firm is associated with the location of its headquarter. The product scope of Japanese firms is first illustrated in two box plots.

Figure 1 shows a box plot with firms’ product scope by prefecture in 2012. It is seen that the firms with a very large product scope (the outliers) are highly concentrated to the core regions, here defined as Greater Tokyo (Tokyo, Kanagawa, Chiba, and Saitama), Greater Osaka (Osaka, Kyoto, and Hyogo) and Aichi prefecture. The variance in the product scope is considerably more narrow in the small regions. The box plot also shows the median, which is two in almost all cases due to the discreteness of the product scope.

The sectorial variation in product scope is illustrated in Figure 2 that shows a box plot with the number of products per firm by two-digit manufacturing sectors in the core and in

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5 The METI Basic Survey of Japanese Business Structure and Activities contains around 12,000-13,000 manufacturing firms per year. The matched sample with Census of Manufacture is around 11,000 firms.

6 The core regions Tokyo and Osaka do additionally have a couple of extreme outliers not shown in the figure with close to almost 60 products.
the periphery for the year 2012.\textsuperscript{7} The figure shows a substantial variation in the product scope among firms in different sectors, but this variation comes in terms of a different prevalence of large multiproduct firms. The median product scope is generally two for both the core and the periphery, while a smaller number of firms, primarily located in the core areas, have a very large product scope.

Turning to the average product scope among 47 prefectures, Figure 3 first shows a plot of the average number of products per firm in each prefecture against the regional GDP.\textsuperscript{8} The figure shows a clear positive association between the average product scope and the regional market size. Figure 4 compares the average product scope in the core and the periphery by sector. Here, it is seen how the average product scope is larger in the core in all cases except two. The fact that firms in the core have a higher product scope is highly consistent with our model below.

We next turn to firm-level regressions that allow us to add more control variables. We regress the number of firm $i$’s products in sector $j$ at time $t$, $N_{ijt}$, against firm productivity, a dummy for being located in the core, and other firm characteristics. We also allow for an interaction term between productivity and the core dummy to pick up non-linear effects, which

\textsuperscript{7}For presentational purposes, we have truncated the product scope at 30, but there are a few even larger outliers with close to 60 products in the Tokyo and Osaka areas.

\textsuperscript{8}The regression line has the slope 0.18, with a 1 percent significance level. $R^2=0.208$. 

Figure 1: Product scope by prefecture
Figure 2: Product scope by two-digit sector

Figure 3: Regional GDP and the average product scope in the region.
Figure 4: Mean product scope by sector
could be indicated by the outliers in the box plots above.

\[ N_{ijt} = \alpha + \beta \log TFP_{it} + \gamma CORE_{it} + \delta \log TFP_{it} \cdot CORE_{it} + X_{it} + \eta_j + \epsilon_{ijt}, \]

where \( i \) is the firm index, \( j \) is a two-digit level sector index and \( t \) is year. \( \eta_j \) is a two-digit sector dummy and \( CORE_{it} \) is a core dummy. If firm \( i \) locates in the core region at year \( t \), the dummy takes the value one, otherwise it is zero. The core region is again defined as Greater Tokyo, Greater Osaka and Aichi, and TFP is calculated using the Levinsohn-Petrin method (Levinsohn and Petrin, 2003). \( X_{it} \) is firm characteristics such as firm age, R&D to sales ratio and KL ratio. Since the data is at the firm level, the product scope is a count variable that is larger or equal to one, and highly skewed towards one, as seen in figures 1 and 2. Therefore, we use negative binomial regressions with robust standard errors.

Table 1 reports the regression results. The numbers reported are incident rate ratios (IRR), which means that a coefficient above one implies a positive effect. Column (1) of the table shows that high productive firms produce a larger number of varieties, and that firms in the core area of Japan produce more products compared to other more peripheral regions in Japan. The incident rate ratio for the core dummy, 1.03, implies that if we compare firms in the core with those in the periphery, the number of products will increase by a factor of 1.03. Thus, location in the core has a relatively small effect on the product scope when we control for productivity. However, the relationship between the product scope in the core and productivity is highly non-linear. When interacting the core dummy with TFP in Column (2), we see that in particular the high productive firms have a high product scope in the core. Column (3) adds some firm characteristics. Age, capital/labor ratio (KL) and R&D/sales ratios are all positive significant. Older firms and capital intensive firms are likely to have a larger product scope. The non-linear relationship between the product scope in the core and productivity is illustrated by the bin scatter-plot in Figure 5, which shows the product scope in the core and the periphery when firms have been divided into 35 productivity bins. Here, it is seen that essentially the entire difference between the core and the periphery comes from the seven highest productivity bins. As shown below, these patterns are consistent with our model in the presence of relocation costs.

3 Model

Here, we introduce multiproduct firms in the Baldwin and Okubo (2006) model. This model is based on the Melitz (2003) heterogeneous firms trade model combined with the 'footloose capital' new economic geography model by Martin and Rogers (1995).

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9To calculate TFP, we use sectoral capital book values from Hosono et al. (2017). We thank them for providing data.
Table 1: Stylized evidence

<table>
<thead>
<tr>
<th>Dependent var: # of prod. per firm</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lnTFP</td>
<td>1.35***</td>
<td>1.18***</td>
<td>1.15***</td>
</tr>
<tr>
<td></td>
<td>(0.0078)</td>
<td>(0.0072)</td>
<td>(0.0072)</td>
</tr>
<tr>
<td>Core dummy</td>
<td>1.03***</td>
<td>0.58***</td>
<td>0.59***</td>
</tr>
<tr>
<td></td>
<td>(0.0049)</td>
<td>(0.013)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>Core dummy*TFP</td>
<td>1.26***</td>
<td>1.22***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>1.13***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>KL</td>
<td>1.03***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R&amp;D/sales</td>
<td>1.03***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Sector fixed effects</th>
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<tr>
<td>Year fixed effects</td>
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</tr>
<tr>
<td>number of obs</td>
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<td>184830</td>
<td>184830</td>
</tr>
</tbody>
</table>

***significant at the 1% level, **significant at the 5% level, *
*significant at the 10% level. Robust standard errors.

Figure 5: Estimation of product scope against TFP in bins.
3.1 Basics

There are two markets with an asymmetric population (market size). Market $j$ is the larger core market and market $k$ is a smaller market also denoted by ‘*‘. There are two types of production factors, capital and labor, and each country has the same proportion of capital to labor. That is, markets are identical except for size. Market $j$ has a share $s_j$ of both factors of production. There is a fixed global stock of capital and labor $K^W, L^W$. Capital can move between markets but capital owners do not. Labor can move freely between sectors but is immobile between markets. A homogeneous good is produced with a constant-returns technology using only labor. Differentiated manufactures are produced with increasing-returns technologies using both capital and labor.

All individuals in a market $j$ have the utility function

$$U_j = C_{M,j}^{\mu} C_{A,j}^{1-\mu}, \quad C_{M,j} = \left[ \int_{\Psi} c_l^{(\sigma-1)/\sigma} dl \right]^{\sigma/(\sigma-1)},$$

(1)

where $\mu \in (0, 1)$, and $\sigma > 1$ are constants. $\Psi$ is the set of consumed variety. $C_{M,j}$ is a consumption index of manufacturing goods and $C_{A,j}$ is consumption of the homogeneous good. $c_l$ is the amount consumed of variety $l$.

Each consumer spends a share $\mu$ of his income on manufactures. The total demand for a variety $i$ in market $j$ is

$$x_{i,j} = \frac{p_{i,j}}{P_j^{1-\sigma}} \cdot \mu Y_j,$$

(2)

where $p_{i,j}$ is the consumer price of variety $i$, $P_j$ is the CES price index, and $Y_j$ is income in market $j$.

Ownership of capital is assumed to be fully regionally diversified; that is, if one market owns $X$–percent of the world capital stock, it will own $X$–percent of the capital in each market. Therefore, the income of each market is independent of the location of capital. Total expenditure equals total factor income. Firms’ fixed factor of production is capital and the variable factor is labor. The return to capital therefore equals firms’ operating profit in equilibrium. Thus, the total equilibrium expenditures can be written $E^w = wL^w + rK^w = wL^w + \mu E^w/\sigma$. Without loss of generality, we choose units so that $L^w = 1$, which gives $E^w = \frac{\sigma}{\sigma-\mu}$. This also means that $r = \frac{\mu}{\sigma-\mu} \frac{1}{K^w}$ is a constant. We assume trade balance so that income equals expenditures in each market. The income of market $j$ is therefore equal to its share of total expenditures:

$$Y_j = s_j E^w = s_j \frac{\sigma}{\sigma-\mu}.$$

(3)

$Y_j$ is thus constant irrespective of the location of capital; i.e. also out of long-run equilibrium. For ease of notation, we suppress the market subscript when possible in the following.

Turning to the supply side, the homogeneous good sector has constant returns and perfect competition. The unit factor requirement of the homogeneous good is one unit of labor. The good is freely traded and since it is also chosen as the numeraire, we have
\[ p_A = w = 1, \]  

where \( w \) is the wage of workers in all markets.

\textit{Manufacturing Firms}

Each firm needs one unit of capital to start production, which implies that \( n + n^* = K^W \). Without loss of generality, we normalize so that \( K^W = 1 \). The firm chooses the range of varieties to produce (the product scope), \( m_i \in [0, \infty) \), after having drawn its marginal cost \( a_i \) from a cumulative distribution function \( G(a) \). It is assumed that the fixed cost of a firm increases in the range of varieties it chooses to produce, and that producing many varieties implies an increased per variety fixed cost. This may be due to higher overhead/coordination costs or diminishing returns of developing blueprints for more varieties at the firm level. The fixed cost of adding a new product therefore increases in the number of products the firm produces, and the fixed cost of the \( m \):th variety is given by:

\[ f(m) = w \cdot \lambda \cdot m^\theta_i \]  

where \( \theta > 0 \), and where \( \lambda \) is a scaling parameter that we normalize to one. The total cost function of a firm with the product scope \( m \) is

\[ TC_i = r + w \int_0^{m_i} z^\theta_i dz + m_i w a_i x_i. \]  

The total profit of a firm is given by

\[ \pi_i = \frac{m_i p_i x_i}{\sigma} - \frac{m_i^{\theta+1}}{1 + \theta} - r. \]  

Geographical distance is represented by trade costs. Shipping the manufactured good involves a frictional trade cost of the “iceberg” form: for one unit of good from firm \( i \) in market \( j \) to arrive in market \( k \), \( \tau_{ijk} > 1 \) units must be shipped. The trade costs are symmetric between markets \( \tau_{ijk} = \tau_{i \forall j, k} \).

Profit maximization by manufacturing firms leads to a constant mark-up over the marginal cost

\[ p_i = \frac{\sigma}{\sigma - 1} a_i, \]  

and the export price is \( p_i \tau_i \).

Using this and (2) we can write the profit as

\[ \pi_i = m_i a_i^{1-\sigma} B + m_i a_i^{1-\sigma} \phi B^* - \frac{m_i^{\theta+1}}{1 + \theta} - r, \]  

where \( r \) is the cost of one unit of capital, \( B = \frac{1}{\frac{\sigma}{\sigma - \mu} \Delta} \), and \( B^* = \frac{1}{\frac{\mu(1-s)(s\sigma)}{\Delta^2}} \), and where
\[ \Delta = P^{1-\sigma} = s \int_0^1 m_i p_i^{1-\sigma} dG + (1 - s) \int_0^1 m_i^* p_i^{1-\sigma} dG + (1 - s) \int_{a_R}^a m_i p_i^{1-\sigma} dG, \quad (10) \]

\[ \Delta^* = P^{*(1-\sigma)} = s \phi \int_0^1 m_i p_i^{1-\sigma} dG + (1 - s) \int_0^1 m_i^* p_i^{1-\sigma} dG + (1 - s) \phi \int_{a_R}^a m_i p_i^{1-\sigma} dG. \quad (11) \]

We use \( s \) for the core’s endowment (expenditure) share, and \( 1 - s \) for the periphery’s share. \( a_R \) is the input coefficient of the marginal firm that is indifferent between locating in the two markets. \( B \) is a measure of the market potential and it is exogenous from the point of view of an individual firm.

We can now from (9) calculate the profit maximizing product scope of a firm

\[ \tilde{m}_i = \left( B + \phi B^* \right)^{\frac{1}{\sigma}} a_i^{\frac{1-\sigma}{\sigma}}. \quad (12) \]

The expression leads to the following propositions:

**Proposition 1** The product scope of a firm increases in the market size.

**Proposition 2** More productive firms have a larger product scope.

Firms trade off the increase in fixed cost to the additional operating profit of an extra variety when determining the product scope. More productive firms have lower marginal costs and higher operating profits. The break-even fixed cost is consequently higher, which means that these firms will opt for a larger product range. This result is consistent with the stylized evidence above.

It is also seen from (12) that the product scope of a firm of a given productivity decreases in \( \theta \), and that \( \tilde{m}_i \) goes to one as \( \theta \) goes to infinity.

### 3.2 Deviation tendencies

We now turn to the question of which firm that would have the strongest incentive to move to the core market. Following Baldwin and Okubo (2006), we assume that firms move in turn and that the firm with the strongest incentive to move at each point in time will be the one that moves.\(^{10}\) For now, we apply their assumption that the cost of migrating falls to zero as the migrating pressure disappears. This means that the migrating cost disappears in equilibrium, which greatly facilitates the analytical solution of the model. In section 3.5, we will relax this assumption. Substituting (12) into (9) gives the profit differential in the two markets:

\[ \pi_i - \pi_i^* = \left( (B + \phi B^*)^{1+\frac{1}{\sigma}} - (\phi B + B^*)^{1+\frac{1}{\sigma}} \right) a_i^{\frac{(\sigma-1)(1+\theta)}{\sigma}} \frac{\theta}{1+\theta}. \quad (13) \]

This expression leads to the following proposition:

\(^{10}\)This could be justified by the introduction of a transport sector with limited capacity. The highest bidder for transport services will be the firm with the most to gain from moving.
Figure 6: Agglomeration to the core.

**Proposition 3** The most productive firm has the highest incentive to deviate to the core.

Relocation will continue until $\pi_i - \pi_i^* = 0$, which defines the marginal cost, $a_R$, of the firm that is indifferent between locations. Figure 6 illustrates how firms with a marginal cost below $a_R$ sort to the core market. The spatial sorting pattern is similar to Baldwin and Okubo (2006), but here the most productive firms are also the firms with the widest scope of variety.

### 3.2.1 Equilibrium

In order to solve for the equilibrium, we now impose a Pareto distribution for firm productivities:

$$G(a) = \left( \frac{a}{a_0} \right)^k,$$

where $a_0$ is a scale parameter, which we normalize to one, and $k$ is a shape parameter.

Free mobility of firms implies that $\pi_i = \pi_i^*$ in equilibrium which, in turn, from (13) implies that $B = B^*$ in equilibrium. The marginal cost of the firm that is indifferent between the two locations is $a_R$, and this marginal cost can be calculated from $\pi_i(a_R) = \pi_i^*(a_R)$:
where $s$ is the core’s share of labor and capital, and $1 - s$ the periphery’s share. Solving this using (12) gives

$$a_R = \left( \frac{(2s - 1) \phi}{(1 - s)(1 - \phi)} \right)^{k - \left( \frac{1}{\sigma + 1} \right)(\sigma - 1)}.$$  

(15)

Convergence of the integrals requires that $\frac{1 - \sigma}{\theta(1+\theta)} + k > 0$ or that $\frac{k}{\sigma - 1} > 1 + \frac{1}{\theta}$, which we assume to hold.

**Assumption 1** $\frac{k}{\sigma - 1} > 1 + \frac{1}{\theta}$.

The expression (15) leads to the following proposition:

**Proposition 4** The lower the cost of introducing new varieties, $\theta$, the less relocation to the core.

Thus, the existence of multiproduct firms dampens the agglomeration to the core. When a multiproduct firm relocates to the larger market, it increases the competition more in the new market than a relocating single product firm would do. A given change in trade costs consequently leads to the relocation of fewer firms when firms produce several varieties.

It is also seen from (15) that:

**Proposition 5** Trade liberalization leads to agglomeration to the core.

It is notable that the sustainpoint point where full agglomeration is reached, $\phi^* = \frac{1 - s}{s}$, is independent of $\theta$. This sustainpoint is identical to the one in Martin and Rogers (1995) and in Baldwin and Okubo (2006). Figure 7 compares the relocation pattern in the different models. The multiproduct model has a slower adjustment of firms towards the core as compared to the other models. Intuitively, this is because firms’ larger product scope substitutes for more firms moving there.

### 3.3 Trade liberalization and the product scope without relocation costs

The profit maximizing product scope is given by (12), and for a closed form solution, we need to solve for $B \equiv \frac{\mu s}{\sigma - \Delta} \phi - \sigma \Delta$, and $B^* = \frac{1}{\sigma} \mu \Delta^* \phi - \sigma \Delta^*$. From (13), $B = B^*$ in equilibrium, which implies that $\Delta^* = \frac{1 - \sigma}{s} \Delta$. Solving for $\Delta$ using (8), (12), and substituting $a_R$ from (15) gives

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11 This condition reduces to the standard condition in the Melitz model, $\frac{k}{\sigma - 1} > 1$, when $\theta$ goes to infinity.
Heterogeneous case (Baldwin and Okubo, 2006)

Homogeneous case (Martin and Rogers, 1995)

Multiproduct firms (this paper)

Figure 7: The relocation pattern in different models \((s = 0.6, \sigma = 3, k = 6, \theta = 2)\)

\[
\Delta = \left( \frac{\mu}{\sigma - \mu} \right)^{\frac{1}{1+\theta}} \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\theta(1-\sigma)}{1+\theta}} \left( \frac{\theta k}{(1-\sigma)(1+\theta) + \theta k} \right)^{\frac{\theta}{1+\theta}} s (1 + \phi),
\]

and using this relation gives

\[
\tilde{m}_i = \left( \frac{\mu \sigma^{\sigma-1} ((k - \sigma + 1) \theta - \sigma + 1)}{k(\sigma - \mu) (\sigma - 1)^{\sigma-1} \theta} \right)^{\frac{1}{1+\theta}} a_i^{\frac{1-\sigma}{\sigma}}.
\]

It may be noted that \(\tilde{m}_i\) goes to one as in the standard model when \(\theta\) goes to infinity. Equation (17) leads to the following proposition:

**Proposition 6** The optimal equilibrium number of products (the product scope) for a firm of a given productivity is independent of trade costs.

This result differs from the literature where trade liberalization leads firms to reduce the scope of variety as harder competition forces them to shed their marginal products (see e.g. Eckel and Neary 2010 and Mayer et al. 2011). It also differs from e.g. Qiu and Zou (2013) where the product scope increases when trade is liberalized. However, when we introduce relocation costs in equilibrium, in section 3.5, this changes. Then, we have a differential response to trade liberalization of firms in the core and in the periphery.
3.4 Welfare

The indirect utility of a representative individual is given by

\[ V = k \frac{w + rK}{P} = vP^{-\mu}, \]  

(18)

where \( v \equiv k(w + rK) \) is a constant. Welfare changes can therefore be represented by changes in \( \Delta \equiv P^{(1-\sigma)} \), which from (16) leads to the following proposition:

**Proposition 7** Welfare increases monotonically in trade liberalization for both regions

**Proof.** This follows from (16) and that \( \Delta^* = \frac{1-s}{s} \Delta \) in equilibrium. ■

The welfare of the core and the periphery will always change in proportion. This means that the introduction of multiproduct firms does not affect the periphery in any particular way. The concern that the periphery would be particularly hurt by high productive multiproduct firms leaving is not warranted in this model.

How then does the introduction of multiproduct firms affect welfare? Multiproduct firms imply more variety, which ceteris paribus is positive for welfare, but the expansion of a firm’s product scope requires labor (because of the higher fixed cost), which means that the total quantity produced falls. Thus, from a welfare perspective, there is a trade off between quantity and variety. Figure 8 illustrates how multiproduct firms affect welfare for different values of \( \theta \), by plotting \( \Delta \) for single-product and multiproduct firms. \( \Delta \) in the single product case is given by

\[
\lim_{\theta \to \infty} \Delta = \frac{k}{(1-\sigma)+k} \left( \frac{\sigma}{\sigma-1} \right)^{(1-\sigma)} s \left( 1 + \phi \right),
\]

while \( \Delta \) in the multiproduct case is given by (16). The curve for multiproduct firms is bounded by Assumption 1, which is shown by the "theta limit" line. The u-shaped curve illustrates how the introduction of multiproduct firms for most values of \( \theta \) decrease welfare, and that welfare increases for a higher \( \theta \) when we approach the single product case. However, there is a small range of low thetas, that lie between the theta limit value and the point where the single- and multiproduct curves intersect, for which multiproduct firms contribute positively to welfare.
Figure 8: Welfare (Δ) with single product and multiproduct firms (k = 8, σ = 5, μ = 0.3, s = 0.6, φ = 0.4).

3.5 Relocation costs

The previous analysis has been based on zero relocation costs in equilibrium, which makes it possible to obtain analytical solutions. We now turn to the, perhaps more realistic case, where there are costs associated with relocation. We assume a fixed migration cost, χ, which all firms have to pay when migrating even when the migrating pressure is nil. This implies that the equilibrium condition becomes

\[
\pi_i - \pi_i^* - \chi = \left( (B + \phi B^*)^{1+\frac{1}{\sigma}} - (\phi B + B^*)^{1+\frac{1}{\sigma}} \right) a_i^{\frac{(\sigma-1)(1+\theta)}{\theta}} \frac{\theta}{1+\theta} - \chi = 0. \tag{19}
\]

The model must now be solved by numerical simulation and we use the following generic parameter values in all simulations: σ = 5, θ = 2, s = 0.6, χ = 2.5, k = 8, μ = 0.3. The qualitative results are highly robust to changes in these.

The first observation is that the non-monotone home market effect now comes into play. Agglomeration forces are weak close to autarky and close to free trade, and they will therefore tend to be dominated by the migration costs for such levels of trade freeness. This is seen in
Figure 9 where the hump-shaped $a_R$-curve shows how relocation to the core is maximal for intermediate trade costs.\textsuperscript{12}

The relocation cost prevents relocation from fully evening out the market potentials, $B$ and $B^*$, in the two markets. Agglomeration forces make the larger market more profitable, which leads firms to expand their product scope in this market. The same forces work against the periphery where firms contract their product scope. This effect is shown in Figures 10a and 10b which plot the product scope of two firms of different productivity as the trade freeness increases ($a = 0.4$ and $0.6$). The firms are comparable over the entire range of trade costs since we illustrate two firms that are not productive enough to ever relocate to the core (c.f. Figure 9). The difference in product scope between a firm in the core and a firm in the periphery is maximal at an intermediate level of trade costs where the home market effect peaks. This is the same level of trade costs where $a_R$ is maximal in Figure 9. Comparing the two firms in Figure 10a and Figure 10b, and noting that the vertical distance is the same in both figures, reveals that the high productive firm can exploit the advantages of a larger market to a higher degree, and it consequently adds more products to the product range when being in the core. When in the periphery it also drops more products. This effect is consistent with our data, as illustrated

\textsuperscript{12}One implication of the hump-shaped curve is that some firms may want to relocate back to the periphery once the trade freeness has moved past the top of the curve, and they may face another migration cost in doing so. We here abstract from this problem. The need for relocating back would not arise if the movement of $\phi$ was very slow and firms had a Poisson probability of exiting.
in Figure 5, where the difference in product scope between the core and the periphery increases with firm productivity.

Figure 10a: The product scope for a medium-low productive firm (a=0.4) in the core and in the periphery.

Finally, we plot $\Delta$ as a measure of welfare in the two markets in Figure 11. Welfare in the two markets converge as we approach free trade. The curves are plotted for different values of $\theta$, which is the parameter determining the ease of increasing the product scope. A higher theta increases welfare. Thus, the presence of multiproduct firms is here negative for welfare for all trade cost.
Figure 11: Trade liberalization and welfare in the two regions for different values of $\theta$.

4 Conclusions

This paper analyzes the effects of trade liberalization on multiproduct firms in a framework where heterogeneous multiproduct firms can relocate between markets. Trade liberalization leads to a relocation of the most productive firms to the larger market, where they expand their product scope (given the relocation costs). The less productive firms with a smaller product range in the smaller market instead contract their product range because of increased competition due to trade liberalization. These effects are magnified with the productivity of the migrating firm. The expansion of the product scope when moving to the large market is stronger for high productive firms. We illustrate how these patterns are consistent with Japanese firm-level data.

Interestingly, the introduction of multiproduct firms dampens the agglomeration of firms to the large market. The reason for this is that the firms that agglomerate to the large market are the high productive firms with a large product scope. Thus, many varieties move with a firm and few firms therefore have to move in order to equalize the market potential.
Finally, we show that the introduction of multiproduct firms decreases welfare for most parameter values, but that trade liberalization leads to welfare gains for all markets.
References


