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Stagnation Traps in an Open Economy

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Abstract

In this paper, we construct a continuous time endogenous growth model in which innovation is the engine of growth, and study the effect of trade restriction on the employment and the growth rate. In our model, the nominal wage rate has downward rigidity, and therefore the model may have involuntary unemployment. Monetary policy is determined by the Taylor rule. We first show that there are two balanced growth paths. In one path, the workers are fully employed and the nominal interest rate is positive. In another path, some workers are unemployed and the nominal interest rate is zero. We next show that, in the unemployment equilibrium, trade restriction by raising tariffs may lower the unemployment rate without reducing the economic growth rate. However, increasing the tariff rate excessively reduces the growth rate without improving the labor market.

Keywords: Endogenous growth, Secular stagnation, Liquidity trap, Trade restriction, Tariff

JEL classification: E5

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1 Introduction

Many developed countries are experiencing low economic growth rates especially after the Great Recession. Summers (2014) argues that such countries suffer from secular stagnation. The secular stagnation was originally formulated by Hansen (1939) just after the Great Depression. Secular stagnation shows the long periods of low economic growth in the developed economies. As the economy grows up, the level of savings begins to go beyond the investment which is needed to keep the high economic growth rate. If investment continues to be below the needed amount, the economy begins to stagnate. The concept of secular stagnation, which was originally used for describing the Great Depression, but it again became popular after the speech of Summers (2014). [1]

Recently, some papers analyze the secular stagnation in an economy with unemployment. Schmitt-Grohe and Uribe (2017) studies the model in which equilibria are characterized by liquidity traps and unemployment. [2]They showed that raising nominal interest rates can escape the liquidity traps and increase the unemployment. Benigno and Fornaro (2018, henceforth BF) consider the discrete time endogenous growth model. In Benigno and Fornaro, there are two equilibria, one with full employment and the other with unemployment and zero nominal interest rate. They find that subsidizing innovation may increase the employment and also welfare in the unemployment equilibrium. Eggertson et al. (2018) studies the quantitative overlapping generations models and find that low real interest rates are closely connected to the output reduction. In their models, the nominal wage rate is downwardly rigid and both individuals and firms take the wage rate as given. Therefore labor supply may exceed the labor demand. The authors call the difference as involuntary unemployment.

In this paper, we study the continuous time endogenous growth model in which innovation is the engine of growth, and study the effect of monetary policy, and trade restriction on the employment and growth rate. In our model, nominal wage rate has downward

[1] See his website at http://larrysummers.com/2017/06/07/secular-stagnation/

[2] For the literature on the liquidity trap, see Krugman (1998), Benhabib and Farmer (1994), Benhabib et al. (2001) and Mertens and Ravn(2014).

rigidity and therefore the model has involuntary unemployment. Monetary policy is determined by the Taylor rule. We first show that there are two balanced growth paths. In one path, the workers are fully employed and the nominal interest rate is positive. However, in another path, some workers are unemployed and the nominal interest rate is zero. We next show that in the unemployment equilibrium, trade restriction by tariff may reduce the unemployment rate and raises welfare, but if we increase the tariff too much, then it reduces the growth rate and welfare without improving the labor market.

Recent literature investigates the open economy models of secular stagnation. Schmitt-Grohe and Uribe (2016) studies the small open economy model with nominal rigidity and capital mobility, and find that when the capital is fully mobile, then the fixed exchange rate may generate unemployment. Eggertson et al. (2017) study two-country overlapping generations models with secular stagnations. However, these models do not investigate the effect of trade restriction on employment.

Some empirical literature investigates the effect of free trade on unemployment. For example, Autor et al. (2014) and Autor et al. (2016) argue that imports from China reduced the employment and innovation in the United States. These facts support the idea of restricting the free trade to some extent. Textbook style argument which propose the free trade does not apply to the economy with low employment. Here we argue that if the economy suffers from both the liquidity trap and the high unemployment, restricting free trade may solve the problem.

This paper is organized as follows. Section 2 describes the basic structure of the model. Section 3 investigates secular stagnation. Section 4 considers several extensions. Section 5 concludes the paper.

2 The Model

In this section, we provide an overview of our model. The model is very close to Romer (1990) and Rivera-Batiz and Romer (1991) but here we consider the monetary policy. Time is continuous. There are two countries, home and foreign. In each country, there is a continuum of individuals who lives for ever. The total population is set to unity. An individual supplies labor in the labor market, receives wage income in each period, consumes and accumulates asset. Each individual has one units of time each period, but the individual supplies labor less than one units due to the nominal wage rigidity.

There are three goods, final good, differentiated output and labor. The final good and the intermediate goods are freely traded, but the international trade of the differentiated output is subject to tariff. The labor is immobile. The production of the final good needs labor supply and intermediated good. The one unit of intermediate good need one unit of final good. The variety of the intermediate good increases by the innovation. Each innovation needs labor supply. Although labor is immobile across countries, but it is freely mobile across sectors.

2.1 Consumer's problem

The individual i in the Home country maximizes the following expected intertemporal utility:

$$\max \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt \tag{1}$$

subject to the following budget constraint

$$\dot{X}_t = W_t L_t + i_t X_t - P_t C_t + P_t TR_t,$$

where ρ is the discount factor, σ is the inverse of the intertemporal elasticity of substitution, C_t is the consumption level of individual i at time t, P_t is the price of the final consumption good, X_t is the asset level in period t, i_t is the nominal interest rate in period t, W_t is the nominal wage income in period t, TR_t is the government subsidy in period t, and L_t is a time spent on working. The individual supplies labor both in the final goods sector and the innovation sector.

Following Ogaki and Reinhart (1989) and BF, we assume that the parameter σ is strictly greater than one. The time spent on working, L_t must be less than 1. Tariff revenue is redistributed to the individuals by a lump-sum. Let $\pi_t = \dot{P}_t / P_t$ denote the inflation rate. The budget constraint in real terms is written as

$$\dot{x} = wL + rx - C + TR,$$

where x_t is the asset level in real terms in period t, $r_t = i_t - \pi_t$ is the real interest rate in period t, $w_t = W/P$ is the nominal wage income in period t. Similarly, the individual i in the Foreign country maximizes the following expected intertemporal utility:

$$\max \int_0^{\infty} e^{-\rho t} \frac{C_t^{*1-\sigma}}{1-\sigma} dt \tag{2}$$

subject to the following budget constraint

$$\dot{X}_t^* = W_t^* L_t + i_t^* X_t^* - P_t^* C_t^* + P_t^* TR_t^*,$$

where C_t^* is the consumption level of Foreign individual, P_t is the price of the Foreign, X_t^* is the asset level of the Foreign individual, i_t^* is the nominal interest rate, W_t^* is the nominal wage income in period t, T_t^* is the Foreign government subsidy in period t, and L_t^* is a time spent on working.

The consumption Euler equation in each country is

$$\begin{aligned} \sigma \hat{C}_t &= r_t - \rho, \\ \sigma \hat{C}_t^* &= r_t^* - \rho, \end{aligned}$$

where $r_t^* = i^* - \pi^*$ is the real interest rate in the Foreign country.

2.2 The final good firm's problem

The model is a textbook style of endogenous growth model that is found in Barro and Sala-i Martin (2004). In each country, there are many identical firms that produce final output which is tradable. The firms uses labor and differentiated intermediate output. They import the intermediate good that is produced in the foreign country.

The production function F has constant returns to scale with respect to labor and intermediate output. The production function in the Home country is

$$Y = L_Y^{\alpha} \left[\int_0^{A} d(\theta)^{1-\alpha} d\theta + \int_0^{A^*} m(\theta^*)^{1-\alpha} d\theta^* \right], \tag{3}$$

where L_Y is the amount of labor devoted to the final goods production in the Home country, A is the number of variety which the intermediate goods firms in the Home country produces, A^* is the number of variety which the intermediate goods firms in the Foreign country produces, $d(\theta)$ is the level of intermediate good θ which is produced in the Home country and is used for the final goods production in the Home country, and $m(\theta^*)$ is the level of intermediate good θ^* produced in the Foreign country used for the final goods production in the Home country. Similarly, the production function in the foreign country is

$$Y^* = L_Y^{*\alpha} \left[\int_0^A d^*(\theta)^{1-\alpha} d\theta + \int_0^{A^*} m^*(\theta^*)^{1-\alpha} d\theta^* \right], \tag{4}$$

where $d^*(\theta)$ is the level of intermediate good θ which is produced in the Home country and is used for the final goods production in the Foreign country, and $m^*(\theta^*)$ is the level of intermediate good θ^* produced in the Foreign country used for the final goods production in the Foreign country.

The final good firm in Home maximizes the profit (in the real terms)

$$\pi = Y - w_t L_Y - \int_0^A p_d(\theta) d(\theta) d\theta - (1+\tau) \int_0^{A^*} p_m(\theta^*) m(\theta^*) d\theta^*,$$

where $p_d(\theta)$ is the price (in terms of the final good) of the Home intermediate good, $p_m(\theta)$ is the price (in terms of the final good) of the foreign intermediate good, τ is the tariff. The intermediate good firm in the foreign country maximizes the profit (in the real terms)

$$\pi^* = Y^* - w_t^* L_Y^* - (1+\tau) \int_0^A p_d^*(\theta) d(\theta) d\theta - \int_0^{A^*} p_m^*(\theta^*) m(\theta^*) d\theta^*, \tag{5}$$

where $p_d^*(\theta)$ is the price of the Home intermediate good which is sold to the Foreign country, $p_m^*(\theta)$ is the price (in terms of the final good) of the foreign intermediate good which is sold to the Foreign country.

The first order conditions (FOCs) on labor are

$$w_t = \alpha L_Y^{\alpha-1} \left[\int_0^A d(\theta)^{1-\alpha} d\theta + \int_0^{A^*} m(\theta^*)^{1-\alpha} d\theta^* \right] \tag{6}$$

$$w_t^* = \alpha L_Y^{*\alpha-1} \left[\int_0^A d^*(\theta)^{1-\alpha} d\theta + \int_0^{A^*} m^*(\theta^*)^{1-\alpha} d\theta^* \right] \tag{7}$$

The FOCs on the intermediate goods for the home country are

$$(1-\alpha)L_Y^{\alpha}d(\theta)^{-\alpha} = p_d(\theta) \tag{8}$$

$$T^{-1}(1-\alpha)L_Y^{\alpha}m(\theta^*)^{-\alpha} = p_m(\theta) \tag{9}$$

where $T = \tau + 1$. Similarly, the FOCs on the intermediate goods for the foreign country are

$$(1-\alpha)L_Y^{*\alpha}m^*(\theta^*)^{-\alpha} = p_m^*(\theta^*), \tag{10}$$

$$T^{-1}(1-\alpha)L_Y^{*\alpha}d^*(\theta)^{-\alpha} = p_d^*(\theta), \tag{11}$$

In the competitive equilibrium, prices and quantities are independent of the variety parameters θ and θ^*.

2.3 The intermediate goods firm

The differentiated intermediate good firms first buy the blueprint of the variety, and produces one unit of differentiated intermediate good from the one unit of the final good. Therefore the intermediate good firm in the Home country earns profit from the Home country by $\Pi_d = p_d d - d$ unit, and $\Pi_d^* = p_d^* d^* - d^*$ units from the foreign country. (All profits are measured in the final good). Using the FOCs above, these profits are written as

$$\Pi_d = p_d d - d = (1-\alpha)L_Y^{\alpha}d^{1-\alpha} - d \tag{12}$$

$$\Pi_d^* = p_d^* d^* - d^* = T^{*-1}(1-\alpha)L_Y^{*\alpha}(d^*)^{1-\alpha} - d^* \tag{13}$$

The firm maximizes total profit $\Pi_d + \Pi_d^*$ by choosing d and d^*. Similarly, the intermediate good firm in the foreign country earns profit from the Home country by $\Pi_m = p_m m - m$ unit, and $\Pi_m^* = p_m^* m^* - m^*$ units from the foreign country. These profits are written as

$$\Pi_m = T^{-1}(1-\alpha)L_Y^{\alpha}m^{-\alpha} - m \tag{14}$$

$$\Pi_m^* = (1-\alpha)L_Y^{*\alpha}m^{*1-\alpha} - m^* \tag{15}$$

The FOCs are

$$1 = (1-\alpha)^2 L_Y^{\alpha} d^{-\alpha} = T^{*-1}(1-\alpha)^2 L_Y^{*\alpha}(d^*)^{-\alpha} \tag{16}$$

$$1 = (1-\alpha)^2 L_Y^{*\alpha} m^{*-\alpha} = T^{-1}(1-\alpha)^2 L_Y^{\alpha}(m)^{-\alpha} \tag{17}$$

Therefore the prices of these goods in the symmetric equilibrium are given by

$$p_d = p_d^* = p_m = p_m^* = \frac{1}{1-\alpha} \tag{18}$$

The optimal quantities for the home firms satisfy

$$d = (1-\alpha)^{2/\alpha} L_Y, \tag{19}$$

$$d^* = T^{*-1/\alpha}(1-\alpha)^{2/\alpha} L_Y^* \tag{20}$$

The optimal quantities for the foreign firms satisfy

$$m^* = (1-\alpha)^{2/\alpha} L_Y^*, \tag{21}$$

$$m = T^{-1/\alpha}(1-\alpha)^{2/\alpha} L_Y \tag{22}$$

These equations implies that the quantities are proportional to the labor supply devoted to the final good production.

The profit of the intermediate good in the home country is

$$\Pi_d^{tot} = \frac{\alpha}{1-\alpha}(d+d^*) \tag{23}$$

$$\Pi_m^{tot} = \frac{\alpha}{1-\alpha}(m+m^*) \tag{24}$$

In the following, we focus on the constant growth path along which the capital rental rate r and the wage rate w is time independent constant. We first characterize the path by assuming that w and r are constant, and later we show that the capital rental rate and the wage rate are in fact constant along the path.

2.4 Innovation

The increase in the variety of the intermediate goods is proportional to the number of workers devoted to the innovation activities. The variety thus evolves according to

$$\dot{A} = \delta L_A A^T, \tag{25}$$

$$\dot{A}^* = \delta L_A^* A^T, \tag{26}$$

where $\delta > 0$ is a parameter, $A^T = A + A^*$ is the total level of variety, L_A is the labor supply devoted to innovation in the Home country, and L_A^* is the labor supply devoted to innovation in the Foreign country. Each entrepreneur takes A^T as given.

If we let $a^* = A^*/A$ denote the relative level of foreign variety, we let

$$\hat{A} = \delta(L - L_Y)(1 + a^*), \tag{27}$$

$$\hat{A}^* = \delta(L^* - L_Y^*)(1 + 1/a^*), \tag{28}$$

where $\hat{A}$ is the growth rate of A. Now let p_A and p_{A^*} denote the relative price of the blue print in Home and Foreign countries. The nominal prices are $P \cdot p_A$ and $P^* \cdot p_{A^*}$, respectively.

Because of the non-arbitrage condition, the price of the variety of the intermediate good must be equal to the present discounted value of the profit of the each intermediate good. Thus $p_A = \int \exp(-it)\frac{P_t}{P_0}\Pi_d^{tot} dt$. Since $\hat{P} = \pi$, we get

$$p_A = \frac{\Pi_d^{tot}}{r} = \frac{1}{r}\frac{\alpha}{1-\alpha}(d + d^*) \tag{29}$$

$$p_{A^*} = \frac{\Pi_m^{tot}}{r} = \frac{1}{r}\frac{\alpha}{1-\alpha}(m + m^*) \tag{30}$$

The marginal productivity of labor in the innovation sector in the home country and the one in the foreign country are the same and is equal to $p_A \delta A^{tot}$ and $p_A \delta A^{tot}$, respectively. This must be wage income in the final good sector. Therefore, we get

$$\delta p_A(A + A^*) = \alpha L_Y^{\alpha-1}(A d^{1-\alpha} + A^* m^{1-\alpha})$$

$$\delta p_{A^*}(A + A^*) = \alpha L_Y^{*\alpha-1}(A d^{*1-\alpha} + A^* m^{*1-\alpha})$$

Dividing each side of the equation by the technology level A, we get

$$\delta p_A(1+a^*) = \alpha L_Y^{\alpha-1}(d^{1-\alpha} + a^* m^{1-\alpha}), \tag{31}$$

$$\delta p_{A^*}(1+a^*) = \alpha L_Y^{*\alpha-1}(d^{*1-\alpha} + a^* m^{*1-\alpha}). \tag{32}$$

2.5 Labor market imperfections

We now describe the nominal wage rigidity in the labor market. Following Benigno and Fornaro (2018), we assume that nominal wage is downwardly rigid, and the degree of rigidity depends on the slackness of the labor market:

$$\hat{W} \geq \omega(L), \text{ and } \hat{W} = \omega(L) \text{ if } L < 1 \tag{33}$$

$$\hat{W}^* \geq \omega(L^*), \text{and } \hat{W}^* = \omega(L^*) \text{ if } L^* < 1 \tag{34}$$

where $\omega(L)$ is the weakly increasing function of L. If the nominal wage is upwardly rigid, then the inequality is written as $\hat{W} \leq \omega(L)$. In that case, when the inequality binds, the full employment is always achieved.

The measure of the unemployed in the Home country is

$$U = \bar{L} - L = \bar{L} - L_Y - L_A.$$

The measure of the unemployed in the foreign country is similarly defined.

Along the balanced growth path, the growth rate of nominal wage payment must be the same as the growth rate of the nominal consumption expenditure, Therefore we get

$$\hat{W} = \pi + \hat{C}, \tag{35}$$

$$\hat{W}^* = \pi^* + \hat{C}^*. \tag{36}$$

We finally describe the monetary policy rule of each country.

2.6 Monetary and fiscal policies

Monetary policy rules in the two countries follow the Taylor rule

$$1 + i(L) = \max((1+\underline{i})(L)^{\phi}, 1) \tag{37}$$

$$1 + i^*(L^*) = \max((1+\underline{i}^*)(L^*)^{\phi}, 1) \tag{38}$$

where $\phi > 1$ is the constant. $\underline{i} > 0$ and $\underline{i}^* > 0$ are the levels of targeted nominal interest rate. If labor is fully employed, $i = \underline{i}$.

The government collects tax revenues only from taxing imports, and the government redistribute the revenue to the consumer by a lump-sum manner. Therefore

$$T^* = \tau \int_0^A p_d^*(\theta)d(\theta)d\theta, T = \tau \int_0^{A^*} p_m(\theta^*)m(\theta^*)d\theta^* \tag{39}$$

In the symmetric equilibrium, $T^* = \tau A p_d^* d$ and $T = \tau A^* p_m m$.

3 Balanced growth paths

In this section, we characterize the balanced growth path in which consumption and output all grow at the same rate.

3.1 Asymmetric equilibrium

Along the balanced growth path, the growth rate of the output, say g, must be the same as the growth rate of the consumption and the one of the innovation. Therefore

$$g = \hat{C} = \hat{A} = \hat{C}^* = \hat{A}^*. \tag{40}$$

The next proposition characterizes the balanced growth path.

Proposition 1 *The balanced growth allocation* $\{r, g, a^*, L, L^*, L_Y, L_Y^*, d, d^*, m, m^*, p_A, p_A^*\}$ *is determined by the non-arbitrage conditions in the labor market, (31) and (32), the equilibrium quantities in the intermediate goods sector, (19), (20), (21) and (22), the price equation on the blueprint of the new varieties (29) and (30), and the following conditions*

$$g = \delta(L - L_Y)(1 + a^*) \tag{41}$$

$$g = \delta(L^* - L_Y^*)(1 + 1/a^*), \tag{42}$$

$$\sigma g = r - \rho, \tag{43}$$

$$\omega(L) \leq i(L) - r + g (= \hat{W}) \tag{44}$$

$$\omega(L^*) \leq i^*(L^*) - r + g (= \hat{W}^*) \tag{45}$$

In the last two equations, when the strictly inequality holds, then $L = L^* = 1$, and when the equality holds, then $L \leq 1$ and $L^* \leq 1$

Proof. See the Appendix. ■

We first investigate the full employment equilibrium. The conditions are written as

$$g = \delta(1 - L_Y)(1 + a^*), \tag{46}$$

$$g = \delta(1 - L_Y^*)(1 + 1/a^*), \tag{47}$$

$$\sigma g = r - \rho, \tag{48}$$

$$i = \underline{i}, i^* = \underline{i}^* \tag{49}$$

We now investigate the equilibrium under liquidity trap. In that case, the conditions are written as

$$g = \delta(L - L_Y)(1 + a^*) \tag{50}$$

$$g = \delta(L^* - L_Y^*)(1 + 1/a^*), \tag{51}$$

$$\sigma g = r - \rho, \tag{52}$$

$$\omega(L) = \omega(L^*) = -r + g. \tag{53}$$

In that case, level of employment is endogenously determined.

3.2 Symmetric equilibrium

To simplify the calculation, we focus on the symmetric equilibrium path with involuntary unemployment. To simplify the model, we suppose that nominal wage stickiness is independent of the labor market conditions. In the symmetric equilibrium, $d = m^*$, $d^* = m$, $L = L^*$, $L_Y = L_Y^*$ and $a^* = 1$. Therefore $m = T^{-1/\alpha}d$. Symmetric balanced growth allocation with involuntary unemployment, $\{r, g, L, L_Y, d, p_A\}$ is determined by the Euler equation $\sigma g = r - \rho$, and

$$2\delta p_A = \alpha L_Y^{\alpha-1}(1+T^{-(1-\alpha)/\alpha})d^{1-\alpha}, \tag{54}$$

$$g = 2\delta(L - L_Y) \tag{55}$$

$$\bar{\omega} = i(L) - r + g \tag{56}$$

$$d = (1-\alpha)^{2/\alpha} L_Y \tag{57}$$

$$rp_A = (1/\alpha - 1)^{-1}(1+T^{-1/\alpha})d \tag{58}$$

Suppose that the economy suffers from the liquidity trap. In this case, the economic growth rate satisfies the Euler equation and

$$\bar{\omega} = -r + g = -(\sigma - 1)g - \rho \tag{59}$$

Thus the balanced growth rate is independent of the tax, and is equal to

$$g = \bar{g} = \frac{-\bar{\omega} - \rho}{\sigma - 1} \tag{60}$$

Now we get the following proposition.

Proposition 2 *In the balanced growth equilibrium with involuntary unemployment and liquidity trap, the effect of trade restriction by tariff is nonlinear. When the tariff is small, it reduces the unemployment rate and raises welfare. However, if we increase the tax rate too much, then it reduces the growth rate and welfare without improving the labor market.*

Proof. See the appendix. ■

Recent empirical literature investigates the effect of free trade on unemployment. For example, Autor et al. (2014) and Autor et al. (2016) use the microdata in the United States and find that imports from China reduced the domestic employment and also the domestic innovation. These facts support the idea of setting moderate restrictions on the free trade. Traditional arguments which support the free trade do not apply to the economy with zero interest rate and low employment. Here we argue that if the economy suffers from both the liquidity trap and the high unemployment, restricting free trade by setting a tariff in a cooperative way may solve the problem.

4 Conclusion

In this paper, we study the discrete time endogenous growth model in which innovation is the engine of growth, and study the effect of monetary policy, and trade restriction on the growth rate and the welfare. In our model, nominal wage rate has downward rigidity and therefore the model has involuntary unemployment. Monetary policy is determined by the Taylor rule. We first show that there are two balanced growth paths. In one path, the workers are fully employed and the nominal interest rate is positive. However, in another path, some workers are unemployed and the nominal interest rate is zero. We next show that in the unemployment equilibrium, trade restriction by tariff may reduce the unemployment rate and raises welfare, but if we increase the tariff too much, then it reduces the growth rate and welfare without improving the labor market. Our proposition is consistent with the recent empirical literature on trade, growth and unemployment.

As a future study, we would like to investigate the liquidity trap equilibrium with other types of endogenous growth models such as Aghion et al. (1992, 2013, 2014). Especially, we want to investigate the human capital growth models such as Caballe and Santos (1993), Josten (2000), and Benhabib and Perli(1994). For example, Caballe and Santos (1993) analyze the dynamics of the optimal path. However, they do not study unemployment. [3]Some economists propose the concept of hysteresis on stagnation. By using the endogenous growth models with both physical and human capital, we would like to study how secular stagnation interacts with human capital accumulation.

[3]Benhabib and Perli (1994) find that human capital externality generates equilibrium indeterminacy. Jones et al. (1993) studies the optimal tax policy. Josten (2000) studies OLG model of endogenous growth.

Appendix

The Appendix provides proofs for propositions.

A Proof of Proposition 1

With respect to the first and the second equations, along the balanced growth path, the growth rate of the varieties must be equal to the balanced growth rate g. The consumption growth rate is equal to g, we get the third equation. The inflation rate is by definition the difference between the nominal interest rate and the real interest rate $i(L) - r$. The nominal wage growth rate must be equal to the sum of the real consumption growth and inflation rate. Therefore we get the fourth and the fifth equation. ■

B Proof of Proposition 2

In the liquidity trap equilibrium, the real interest rate $\bar{r}$, the economic growth rate g are both independent of T. Since $d = (1-\alpha)^{2/\alpha} L_Y$, we have $rp_A = (1/\alpha - 1)^{-1}(1+T^{-1/\alpha})(1-\alpha)^{2/\alpha} L_Y$, Substitution of $2\delta p_A = \alpha L_Y^{\alpha-1}(1 + T^{-(1-\alpha)/\alpha})d^{1-\alpha}$ into this equation yields

$$L_Y = \kappa \frac{1 + T^{-(1-\alpha)/\alpha}}{1 + T^{-1/\alpha}} = \kappa \bar{r} T \frac{T^{1/\alpha - 1} + 1}{1 + T^{1/\alpha}},$$

where $\kappa = \frac{\alpha(1-\alpha)^{2(1-\alpha)/\alpha}}{(1/\alpha-1)^{-1}} \frac{1}{2\delta}$ and $\bar{r} = \sigma \bar{g} + \rho$. Therefore the total labor supply is

$$L = \frac{\bar{g}}{2\delta} + L_Y.$$

If we let $x = T^{-1/\alpha}$, then $L_Y = \kappa \frac{1+x^{1-\alpha}}{1+x}$. One can easily show that L_Y has a U-shaped curve on x. Therefore L_Y, and also L is inverted U shaped on the tariff T. ■

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