Appendix for "Reallocation Effects of Monetary Policy"

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A Data Summary and Robustness Checks

A.1 Summary Statistics

Table A.1 summarizes the basic statistics of the data, while Table A.2 shows the input inflation and firm-size dispersion measures for 14 manufacturing industries that are examined in Section 2.

A.2 Robustness Checks

In this subsection, we check the robustness of the main results reported in Section 2. In the following regressions, we use the sales growth regression in Column (2) of Table 2 as the benchmark specification and investigate its robustness when we take additional factors into account.

Firm Age

In the benchmark regression, we do not consider age effects to see the relation between firm performance and inflation. However, as Jovanovic (1982) argues that older firms tend to show better performance, pooling ages might lead to the case that they work as a compounding factor for the relation between inflation and growth. Thus, we split the sample of firms into four firm-age groups by quartiles in the pooled sample and rerun the sales growth regression for each group. We calculate the ages using the years since establishment of the firm.

Table A.3 shows the regression results that indicate the signs and significance levels of the coefficients are highly consistent across age groups.

High vs Low Leverage Ratio

To investigate the effect of credit constraints further, we split firms into high and low-leveraged groups. Table A.4 shows the regression results, in which the left two columns use the median of leverage ratios in the pooled sample as the threshold, and the right two columns use the 75 percentile point as the threshold. All results are consistent with our benchmark result. Notably, firms in the highest leveraged class are relatively more sensitive to inflation as seen in Column (4).

Other Year-windows to Define Trend Inflation

In the benchmark case, we use the 3-year average of input price inflation to obtain trend inflation. Here, we check the robustness of our results using different year-windows: 1-year and 5-year average. Year-windows for the real sales growth and the instrument variable (international primary commodity price) and the number of lags for firm size and leverage are adjusted consistently. As seen in Table A.5, the results for the 5-year average are similar with both the OLS and 2SLS. On the other hand, the effects of inflation are relatively weak when we take the 1-year lagged inflation, for which the coefficient of the interaction term is significantly positive with the 2SLS, while the coefficient of inflation is negative but insignificant. This result indicates that it is important to consider trend inflation, not a temporary inflation volatility, as a trigger of the reallocation effect.

Wage Inflation

Our theoretical model considers labor as the only input to produce final goods, while ignoring intermediate goods. Thus, the use of wage inflation rather than input price inflation is more consistent for the model. Here, we confirm that the main empirical result is robust when we use wage inflation as the inflation variable. Using data on the total salary and the number of employees in each firm, we calculate the average wage and its change (3-year average) in each industry and size (10 size-groups by deciles), and rerun the regression by replacing the input price inflation with wage inflation. Because whether the inflation in international primary commodity prices is a good instrumental variable is unclear, we also try two other variables separately as instruments. The first alternative is the country-level unemployment rate obtained from Labor Force Survey by the Ministry of Internal Affairs and Communications, and the second alternative is the change in the number of new job applications from General Employment Placement Situation by the Ministry of Health, Labor and Welfare (we use the IVs corresponding to the same year-window to capture wage trend inflation). Both instruments are exogenous at the firm level but it affects industry-level wage inflation. Because both variables are not sectoral variables, we multiply them by industry dummies to define the instruments.

Table A.6 shows that the cross-term positively and significantly affects firm-level sales growth with each IV setting, implying that the negative effect of wage inflation is partly offset when lagged sales are large, which is consistent with our benchmark result.[1]

Non-manufacturing Sector

Although the limited availability of input price data prevents us from extending our sample to non-manufacturing, we can do so if we use wage inflation as the inflation variable. Now, the sample includes all non-manufacturing industries in our dataset as long as the Tankan financing position D.I. are reported by the Bank of Japan. The list of industries are construction, real estate, lease, retail, wholesale, transportation and postal service, information and communication, and electricity and gas. We use wage inflation and the instruments described above.

Table A.7 shows that a similar result shows up for the relation between firm performance and inflation although the coefficients of wage inflation and the cross term are smaller in absolute terms than those in the manufacturing sector (Table A.6). This quantitative difference between the sectors is consistent with our story because non-manufacturing firms have relatively lower intensity in R&D activity than manufacturing firms. Hence, the implication derived from the main text is applicable to the overall economy while the quantitative effect depends on the share of manufacturing and non-manufacturing sectors.

Exit Measures under the Cut-off Problem

In the main text, we set an exit flag of one for each firm if it disappears from the data and never reappears. Given such disappearance occurs when a new company appears through consolidation (in the Basic Survey of Japanese Business Structure and Activities, the Ministry of Economy, Trade and Industry assigns a new ID to the new company), we exclude the firms that experience an M&A in the same years when they disappear from the dataset.[2] Behind our definition of exit is the premise that going under the cut-off (i.e., 50 employees) indicates a significant risk of true exit so that our exit flag is informative.

[1] The signs of wage inflation and the interaction term in the OLS is the opposite to those in the 2SLS. Because wages are an endogenous variable, a regression without an IV may be subject to a bias.

[2] We identify M&A by the RECOF data. We exclude a firm from the sample if it is listed as involved in any type of M&A in its last year in our main dataset. About 5% of the "exits" are related to M&A.

Here we check the robustness of the regression result on exit that is reported in Table 3 by using stricter exit measures so that we can double-check the abovementioned premise. The cut-off problem matters if a firm disappears at around the final year of the dataset, 2015. For example, a firm disappears in 2014 might just downsize temporarily and come back in the data in 2016. To exclude such a possible reappearance after the final year of the data, we set fixed years of allowance to measure exit, and we exclude the sample during those years of allowance. If we extend the years of allowance, the definition of exit becomes stricter at a cost of narrowed sample periods.

Table A.8 shows the regression results when the allowance years for exit definition are three, six, and nine years. The length of allowance is reasonable because the average years of the interval between disappearance and reappearance among firms that have temporary absence are about 3.5, and about 95% of those firms reappear within nine years. The regression results are robust to the choice of allowance years.

Although the fact that some firms reappear in the data is a pain in the neck, it also gives us an interesting picture on the heterogeneity of the exiting firms. To compare the final exit with the temporary exit, we redefine exit such that a firm exits in year t if it exists in t but disappears in $t+1$, following Nishimura et al. (2005). Then, we label each exit with a temporary exit if an exiting firm reappears later or with final exit if it never reappears. Figure A.1 shows distributions of the log of the number of employees at exit. The mean log employment at final exit and temporary exit are 4.7 and 5.0, respectively. We reject the null hypothesis that there is no gap in the mean employment sizes between the two groups, with a t-test with equal variances (t-statistics is 31.9). Hence, temporary-exit firms frequently have larger employment than final-exit firms. We observe that sales at temporary exit are also greater than sales at final exit. These facts imply size-dependent heterogeneity in exiting behavior: larger firms are less likely to truly exit from the market even if it goes under the cut-off of the survey.

Entry and Re-entry on the Dataset

We find that inflation has a robust relation with neither industry-level nor economy-wide entry rates. We have two options to define entry. The first option is to use the data on establishment years. However, the entry rate based on this measure is incredibly low at 0.04% on average annually, most probably because of the cut-off constraint. The second option is to use the timing of the first appearance on the data. However, as naturally expected, there is a spike around 50 employees when we draw a histogram of the number of employees over firms at their first appearance. It is unusual that firms have 50 employees or more at entry.[3]

To see if entering firms hire more than 50 employees, we compare the size distributions of new entrants and re-entering firms. We define entry such that a firm enters in year t if it appears in t while it is not on the data in $t-1$, and separate samples into new entry and re-entry in our data. As shown in Figure A.2, re-entry tends to occur with a greater number of employees that rejects the null hypothesis of no gap in the t-test. It holds true when we take sales as an alternative measure of firm size.

This fact reinforces our conjecture that entrants tend to have fewer employees than the cut-off level. If the majority share of new entrants hire more than 50 at entry (and appear on the data from the beginning), there should be a much smaller or no gap between new entries and re-entries in our data for employment and sales.

Unlike exit, we do not have any information before entry. Because firm growth is heterogeneous and fluctuates especially when young, our dataset is not appropriate to estimate the timing of entry of each firm and the aggregate entry rate.

Sample Size Harmonization at the First-Stage Estimation

In the main text, we use firm-level explanatory variables such as lagged sales and lagged leverage at the first-stage regression to follow the standard IV method even though the dependent variable, input inflation, is measured at the industry level. Therefore, the estimates are obtained in a way that an industry×year with more observations has a larger weight. Basically, we do not consider that this weight causes a problem because our dataset is a complete survey of the target industries with a clear cut-off condition. Thus, there is no arbitrary sample selection. However, one concern might be about a possible bias on this instrumented inflation.

To check the robustness of the main results from this viewpoint, we make the numbers of observations within each industry×year all the same in the first-stage regression. Within each industry×year cell, we divide firms into 10 size-groups by

[3] According to Economic Census for Business Activity in 2016 by the Ministry of Internal Affairs and Communications and the Ministry of Economy, Trade and Industry, the average number of employees per establishment among all industries (including both incumbents and entrants) is 10.6 in Japan.

deciles of real sales and define $\overline{\log(\text{Sales})}_{ist}$ as the mean of log of real sales among firm in industry i, size-group s and year t. Other explanatory variables $\mathbf{Z}_{ist}$ are similarly defined. The first-stage equation regressed is

$$\bar{\pi}_{it}^{\text{input}} = \alpha_0 + \alpha_{1i}\bar{\pi}_t^{\text{ipc}} + \alpha_2\overline{\log(\text{Sales})}_{ist-3} + \bar{\mathbf{Z}}_{ist}'\alpha + \theta_{is} + \theta_t + u_{ist}, \tag{A.1}$$

where θ_{is} is the industry$\times$size-group fixed effect. With this modification, each bin of industry$\times$year has 10 observations. Table A.9 shows the results of the second-stage regression that uses the instrumented inflation from the first-stage regression with (A.1). The estimates are consistent with Tables 2 and 3 in the main text in terms of signs, sizes, and significance levels.

B Model Details

B.1 Derivation of Equation (16) in the Main Text

The firm value comprises profit flows from each product plus the return from R&D that depends on k. Let the present-discount value of the sum of profit flows for a product with τ_i be $\nu_{\tau_i}(q|\delta, n)$. Rewrite the bellman equation:

$$\begin{aligned}
\rho v_k(T_k, q|\delta, w, n) = \max_{\gamma} \sum_{j \notin \Omega} & \left[\Pi^0(\xi_0 e^{-n\tau_j}|q) + \frac{\partial v_k(T_k', q|\delta, w, n)}{\partial \tau_j}\right] \\
& + \sum_{j \in \Omega} \left[\Pi^0(\xi_0 e^{-n\tau_j}|q) - \kappa + \frac{\partial v_k(T_k', q|\delta, w, n)}{\partial \tau_j}\right] \\
& - kwc(\gamma) \\
& + k\gamma \left[v_{k+1}(\{T_k', 0\}, q|\delta, w, n) - v_k(T_k', q|\delta, w, n)\right] \\
& + k\delta \left[\frac{1}{k}\sum_{j=1}^{k} v_{k-1}(T_{k-1,\langle j \rangle}', q|\delta, w, n) - v_k(T_k', q|\delta, w, n)\right].
\end{aligned} \tag{A.2}$$

Thus, our guess for the value function is

$$v_k(T_k, q|\delta, w, n) = \sum_{i=1}^{k} \nu_{\tau_i}(q|\delta, n) + k\psi(q|\delta, w, n). \tag{A.3}$$

The guess of equation (A.3) yields

$$\frac{1}{k}\sum_{i=1}^{k} v_{k-1}(T'_{k-1,<i>}, q|\delta, w, n) = \left(1-\frac{1}{k}\right)\sum_{i=1}^{k}\nu_{\tau'_i}(q|\delta, n) + (k-1)\psi(q|\delta, w, n), \quad \text{(A.4)}$$

and thus,

$$\frac{1}{k}\sum_{i=1}^{k} v_{k-1}(T'_{k-1,<i>}, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n) = -\frac{1}{k}\sum_{i=1}^{k}\nu_{\tau'_i}(q|\delta, n) - \psi(q|\delta, w, n). \tag{A.5}$$

Equation (A.2) is then written as

$$\begin{aligned}
&\rho\sum_{i=1}^{k}\nu_{\tau_i}(q|\delta, n) + \rho k\psi(q|\delta, w, n) \\
&\quad = \sum_{i=1}^{k}\left[\Pi^0(\xi_0 e^{-n\tau_i}) - I\{\tau_i = \Delta(q|\delta, n)\}\kappa + \frac{\partial\nu_{\tau_i}(q|\delta, n)}{\partial\tau}\right] \\
&\qquad - \delta\sum_{i=1}^{k}\nu_{\tau'_i}(q|\delta, n) - k\delta\psi(q|\delta, w, n) \\
&\qquad + k\max_{\gamma}\left\{\gamma\left[v_{k+1}(\{T'_k, 0\}, q|\delta, w, n) - v_k(T'_k, q|\delta, w, n)\right] - wc(\gamma)\right\}.
\end{aligned} \tag{A.6}$$

We observe that

$$\rho\nu_{\tau_i}(q|\delta, n) = \begin{cases} \Pi^0(\xi_0 e^{-n\tau_i}) + \frac{\partial\nu_{\tau_i}(q|\delta,n)}{\partial\tau} - \delta\nu_{\tau_i}(q|\delta, n) & \text{for } \tau_i \in [0, \Delta(q|\delta, n)), \\ \Pi^0(\xi_0 e^{-n\tau_i}) - \kappa + \frac{\partial\nu_0(q|\delta,n)}{\partial\tau} - \delta\nu_0(q|\delta, n) & \text{for } \tau_i = \Delta(q|\delta, n). \end{cases} \tag{A.7}$$

Therefore, we have

$$(\rho+\delta)\psi(q|\delta, w, n) = \max_{\gamma}\left\{\gamma[\nu_0(q|\delta, n) + \psi(q|\delta, w, n)] - wc(\gamma)\right\}. \tag{A.8}$$

The R&D intensity is determined as

$$\nu_0(q|\delta, n) + \psi(q|\delta, w, n) = wc'(\gamma). \tag{A.9}$$

Using equation (A.8) to eliminate ψ, this first-order condition can be rewritten as

$$wc'(\gamma) = \max_{\gamma\in[0,\rho+\delta)} \frac{(\rho+\delta)\nu_0(q|\delta, n) - wc(\gamma)}{\rho+\delta-\gamma}, \tag{A.10}$$

where the constraint $\gamma < \rho + \delta$ should hold because ψ is not well defined otherwise.

B.2 Firm Size and Quality Distribution

The stationary distribution satisfies

$$\begin{aligned}\gamma(q|\delta, w, n)(k-1)M_{k-1}(q|\delta, w, n) + \delta(k+1)M_{k+1}(q|\delta, w, n) \\ = (\gamma(q|\delta, w, n) + \delta)kM_k(q|\delta, w, n) \qquad \text{for } k \geq 2,\end{aligned} \tag{A.11}$$

with

$$\phi(q|\delta, n)\eta = \delta M_1(q|\delta, w, n), \tag{A.12}$$

$$\phi(q|\delta, n)\eta + 2\delta M_2(q|\delta, w, n) = (\gamma(q|\delta, w, n) + \delta)M_1(q|\delta, w, n). \tag{A.13}$$

Equation (A.11) leads to

$$M_k(q|\delta, w, n) = \frac{\phi(q|\delta, n)\eta}{\delta k}\left(\frac{\gamma(q|\delta, w, n)}{\delta}\right)^{k-1}. \tag{A.14}$$

The mass of type-q firms in the stationary state, $M(q|\delta, w, n)$, is $\sum_{k=1}^{\infty} M_k(q|\delta, w, n)$. If $\gamma(q|\delta, w, n) < \delta$ for almost all q, then $M(q|\delta, w, n)$ is well defined as

$$M(q|\delta, w, n) = \frac{\eta}{\delta}\left[\log\left(\frac{\delta}{\delta - \gamma(q|\delta, w, n)}\right)\right]\frac{\delta\phi(q|\delta, n)}{\gamma(q|\delta, w, n)}. \tag{A.15}$$

The condition $\sup_q \gamma(q|\delta, w, n) < \delta$ is supported when w is sufficiently large. The threshold level of w is determined by the first-order condition of the maximization in equation (A.10). Let $\underline{w}(q|\delta, n)$ be the individual threshold wage level such that $\gamma(q|\delta, w, n) < \delta$ for $w > \underline{w}(q|\delta, n)$. The threshold wage in the whole economy is $\underline{w}(\delta, n) \equiv \sup_q \underline{w}(q|\delta, n)$. Since $\nu(q|\delta, n)$ is monotonically increasing in q and $\lim_{q\to\infty} \nu(q|\delta, n) = \frac{1}{\rho+\delta} - \kappa$, we have

$$\underline{w}(\delta, n) = \begin{cases} \frac{1-(\rho+\delta)\kappa}{c(\delta)+\rho c'(\delta)} & \text{if } \bar{q} \to \infty, \\ \frac{(\rho+\delta)\nu(\bar{q}|\delta, n)}{c(\delta)+\rho c'(\delta)} & \text{if } \bar{q} \text{ is finite.} \end{cases} \tag{A.16}$$

Two remarks are in order. First, $\underline{w}$ is decreasing in the creative destruction rate δ.

This is because a decline in the creative destruction rate and a decline in the wage both stimulate incumbents' R&D. Second, $\underline{w}$ is independent of n if the support of $\bar{\phi}$ is not bounded, while greater nominal growth enlarges the admissible set otherwise.

Any equilibrium has $w > \underline{w}(\delta, n)$. Because the total mass of products is one, the creative destruction rate must equal the sum of the entry rate and the creation rates of all the incumbents. As long as $w > \underline{w}(\delta, n)$, we have

$$\begin{aligned} \delta &= \eta + \int_{\underline{q}(\delta,n)}^{\infty} dq \sum_{k=1}^{\infty} k M_k(q|\delta, w, n)\gamma(q|\delta, w, n) \\ &= \eta \int_{\underline{q}(\delta,n)}^{\infty} \frac{\delta\phi(q|\delta, n)}{\delta - \gamma(q|\delta, w, n)} dq, \end{aligned}$$

which leads to the following equation:

$$1 = \eta \int_{\underline{q}(\delta,n)}^{\infty} \frac{\phi(q|\delta, n)}{\delta - \gamma(q|\delta, w, n)} dq. \tag{A.17}$$

B.3 Proofs

Lemma A.1 *Suppose $n > 0$. $\Delta(q|\delta, n)$ is increasing in q and decreasing in $|n|$. $|n|\Delta$ is increasing in $|n|$. Moreover, $\Delta(q|\delta, n)$ is increasing in δ.*

Proof of Lemma A.1 From equations (11) and (12) in the main text, we obtain

$$q\kappa = \frac{e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}}{\rho + \delta} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{\rho + \delta - n}. \tag{A.18}$$

The total differentiation of equation (A.18) is

$$
\begin{aligned}
\kappa dq - & \frac{n e^{n\Delta}(1 - e^{-(\rho+\delta)\Delta})}{\rho+\delta} d\Delta \\
&= \left[e^{n\Delta} \left(\frac{\Delta e^{-(\rho+\delta)\Delta}}{\rho+\delta} - \frac{1 - e^{-(\rho+\delta)\Delta}}{(\rho+\delta)^2} \right) - \left(\frac{\Delta e^{-(\rho+\delta-n)\Delta}}{\rho+\delta-n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho+\delta-n)^2} \right) \right] d\delta \\
&\quad + \left[\frac{\Delta e^{n\Delta}(1 - e^{-(\rho+\delta)\Delta})}{\rho+\delta} + \frac{\Delta e^{-(\rho+\delta-n)\Delta}}{\rho+\delta-n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho+\delta-n)^2} \right] dn \\
&= e^{-(\rho+\delta-n)\Delta} \left[\left(\frac{\Delta}{\rho+\delta} - \frac{e^{(\rho+\delta)\Delta} - 1}{(\rho+\delta)^2} \right) - \left(\frac{\Delta}{\rho+\delta-n} - \frac{e^{(\rho+\delta-n)\Delta} - 1}{(\rho+\delta-n)^2} \right) \right] d\delta \\
&\quad + \left[q\kappa\Delta + \left(\frac{\Delta}{\rho+\delta-n} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{(\rho+\delta-n)^2} \right) \right] dn \qquad \text{(A.19)} \\
&= e^{-(\rho+\delta-n)\Delta} \left[h_2(\rho+\delta, \Delta) - h_2(\rho+\delta-n, \Delta) \right] d\delta + \left[q\kappa\Delta + h_1(\rho+\delta-n, \Delta) \right] dn,
\end{aligned}
$$

where we substitute equation (A.18) in the second equality. Functions h_1 and h_2 are defined in Lemma A.2 below. Owing to Lemma A.2, the signs of the coefficients for $d\delta$ and dn are determined uniquely; and we have $d\Delta/dq > 0$, $d\Delta/d\delta > 0$, and $d\Delta/dn < 0$. ■

Lemma A.2 *Define the following functions over $x \neq 0$ with $y \geq 0$ by*

$$
\begin{aligned}
h_1(x, y) &= \frac{y}{x} - \frac{1 - e^{-xy}}{x^2}, \\
h_2(x, y) &= \frac{y}{x} - \frac{e^{xy} - 1}{x^2}.
\end{aligned}
$$

Then, for any $x \neq 0$ and $y \geq 0$, the following relationships hold:

$$h_1(x, y) \geq 0, \quad h_2(x, y) \leq 0,$$

$$\frac{\partial h_1(x, y)}{\partial x} \leq 0, \quad \frac{\partial h_2(x, y)}{\partial x} \leq 0,$$

with equalities only when $y = 0$.

Proving the signs of the functions is straightforward:

$$h_1(x,y) = \frac{xy - (1 - e^{-xy})}{x^2} \geq 0,$$
$$h_2(x,y) = \frac{xy - (e^{xy} - 1)}{x^2} \leq 0.$$

For the derivative of h_1, we have

$$\frac{\partial h_1(x,y)}{\partial x} = -\frac{2(1+e^{-xy})}{x^3}\left[\frac{xy}{2} - \frac{1-e^{-xy}}{1+e^{-xy}}\right].$$

Since $y \geq 0$ and

$$\frac{xy}{2} \geq \frac{1-e^{-xy}}{1+e^{-xy}} \quad \text{if and only if } xy \geq 0,$$

we have $\frac{\partial h_1(x,y)}{\partial x} \leq 0$.

Last, the partial derivative of h_2 is expressed as

$$\frac{\partial h_2(x,y)}{\partial x} = -\frac{2(1+e^{xy})}{x^3}\left[\frac{xy}{2} - \frac{1-e^{-xy}}{1+e^{-xy}}\right].$$

Thus, the sign of $\partial h_2(x,y)/\partial x$ is equivalent to that of $\partial h_1(x,y)/\partial x$. ■

Lemma A.3 *$\tilde{\nu}_0(q|\delta,n)$ is strictly increasing in q and strictly decreasing in δ. For $n > 0$,*

$$\frac{\partial \tilde{\nu}_0(q|\delta,n)}{\partial n} < 0, \qquad \frac{\partial^2 \tilde{\nu}_0(q|\delta,n)}{\partial q \partial n} > 0.$$

Proof of Lemma A.3 The former part of the proposition is evident: $\partial\tilde{\nu}_0(q|\delta,n)/\partial q > 0$ and $\partial\tilde{\nu}_0(q|\delta,n)/\partial\delta < 0$. As for the latter part of the proposition,

$$\begin{aligned}
\frac{\partial^2 \tilde{\nu}_0(q|\delta,n)}{\partial q \partial n} &= \frac{\partial(n\Delta)}{\partial(n)}\frac{\partial^2 \tilde{\nu}_0(q|\delta,n)}{\partial q \partial(n\Delta)} = -\frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{n\Delta}}{q}\left(n\frac{\partial \Delta}{\partial q} - \frac{1}{q}\right) \\
&= -\frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{n\Delta}}{q}\frac{(\rho+\delta)q\kappa - \left(e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}\right)}{q\left(e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}\right)} \\
&= \frac{1}{\rho+\delta}\frac{\partial(n\Delta)}{\partial(n)}\frac{e^{n\Delta}}{q}\frac{1}{q\left(e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}\right)}\frac{\rho+\delta}{\rho+\delta-n}\left(1 - e^{-(\rho+\delta-n)\Delta}\right) \\
&> 0.
\end{aligned}$$

For the derivation of the last line, we use Lemma A.1. ■

Proof of Proposition 1 From equation (12) in the main text, $\kappa \geq \frac{1}{\rho+\delta}$ implies that $\underline{q}(\delta, n) \to \infty$ and no firm can yield profits. Thus, we assume $\kappa < \frac{1}{\rho+\delta}$ below.

When $n = 0$, then $\underline{q}(\delta, 0) = \frac{1}{1-(\rho+\delta)\kappa}$.

When $n > 0$, $\tilde{\nu}_0(q|\delta, n) < 0$ implies that $\Delta(q|\delta, n) > \frac{\log q}{n} \equiv \bar{\Delta}$. From the first-order condition when choosing Δ:

$$q\kappa = \frac{e^{n\Delta} - e^{-(\rho+\delta-n)\Delta}}{\rho+\delta} - \frac{1 - e^{-(\rho+\delta-n)\Delta}}{\rho+\delta-n},$$

$\underline{q}(\delta, n)$ is q that satisfies

$$\kappa = \frac{1 - e^{-(\rho+\delta)\bar{\Delta}}}{\rho+\delta} - \frac{1/q - e^{-(\rho+\delta)\bar{\Delta}}}{\rho+\delta-n} = \frac{1}{\rho+\delta} - \frac{q^{-1} - \frac{n}{\rho+\delta} q^{-\frac{\rho+\delta}{n}}}{\rho+\delta-n}. \tag{A.20}$$

Let $A_1(q, n)$ be the right-hand side of equation (A.20). Note that $A_1(1, n) = 0$ and $\lim_{q\to\infty} A_1(q, n) = \frac{1}{\rho+\delta}$.

Suppose $n \neq \rho + \delta$. Because of

$$\frac{\partial A_1(q, n)}{\partial q} = \frac{\rho+\delta}{q^2} \frac{1 - q^{1-\frac{\rho+\delta}{n}}}{\rho+\delta-n} > 0,$$

$\underline{q}(\delta, n)$ uniquely exists. Next, $\partial \underline{q}/\partial n < 0$ comes from

$$\frac{\partial A_1(q, n)}{\partial n} = \frac{q^{-\frac{\rho+\delta}{n}}}{(\rho+\delta-n)^2} \left[1 + \log q^{\frac{\rho+\delta}{n}-1} - q^{\frac{\rho+\delta}{n}-1}\right] < 0 \quad \text{for } q > 1.$$

Moreover, since $A_1(q, n)$ is continuous as $n \downarrow 0$, $\underline{q}(\delta, n) > 1/[1 - (\rho + \delta)\kappa]$ for $n > 0$. Thus, $\partial \underline{q}(\delta, n)/\partial n > 0$.

For the case of $n = \rho + \delta$, $A_1(q, n) = \frac{1-1/q}{\rho+\delta}$, which implies $A_1(q, n)$ is strictly increasing in q. Further, we have

$$\lim_{n\to\rho+\delta} \frac{\partial A_1(q, n)}{\partial n} = \lim_{n\to\rho+\delta} -\frac{1}{2}\frac{\rho+\delta}{n^2} q^{-\frac{\rho+\delta}{n}} (\log q)^2 = -\frac{(\log q)^2}{2q(\rho+\delta)} < 0.$$

Thus, $\underline{q}(\delta, n)$ uniquely exists and $\partial \underline{q}(\delta, n)/\partial n > 0$.

■

Proof of Proposition 2 From equation (A.10), the optimal γ satisfies

$$\frac{(\rho+\delta)\nu_0(q|\delta,n)}{w} = c(\gamma) + (\rho+\delta-\gamma)c'(\gamma), \qquad \gamma \in [0, \rho+\delta).$$

From the strict convexity of $c(\gamma)$ and the assumption of $c(0) = 0$, γ has a unique interior solution if w is sufficiently large. $\partial\gamma/\partial|n| < 0$ and $\partial\gamma/\partial w < 0$ result from the first-order condition. For the effect of δ, $(\rho+\delta)\nu_0(q|\delta,n)$ is strictly decreasing in δ for $n \geq 0$ from Lemma A.1. Thus, $\partial\gamma/\partial\delta < 0$. In addition,

$$\frac{\partial\gamma}{\partial q} = \frac{1}{wc''(\gamma)}\frac{\rho+\delta}{\rho+\delta-\gamma}\frac{\partial\nu_0(q|\delta,n)}{\partial q} > 0, \tag{A.21}$$

$$\frac{\partial^2\gamma}{\partial q\partial n} = \frac{1}{wc''(\gamma)}\frac{\rho+\delta}{\rho+\delta-\gamma}\frac{\partial^2\nu_0(q|\delta,n)}{\partial q\partial n}\begin{cases} > 0 & \text{for } n > 0 \\ < 0 & \text{for } n < 0, \end{cases} \tag{A.22}$$

$$\frac{\partial^2\gamma}{\partial q\partial w} = -\frac{1}{w^2c''(\gamma)}\frac{\rho+\delta}{\rho+\delta-\gamma}\frac{\partial\nu_0(q|\delta,n)}{\partial q} < 0. \tag{A.23}$$

■

Proof of Proposition 3 The free-entry (FE) condition is given by

$$\int_1^\infty \bar{\phi}(q)v_1(\{0\}, q|\delta,w,n)dq = wc'\left(\gamma_\eta(\delta,w,n)\right), \tag{A.24}$$

$$\gamma_\eta(\delta,w,n) \equiv \frac{\eta(\delta,w,n)}{1-\bar{\Phi}(\underline{q}(\delta,n))}\frac{1}{h}, \tag{A.25}$$

This equation can be rewritten as

$$\int_{\underline{q}(\delta,n)}^\infty \bar{\phi}(q)c'(\gamma(q|\delta,w,n))dq = c'(\gamma_\eta(\delta,w,n)), \tag{A.26}$$

by substituting ψ from equation (A.10).

Fix $\delta > 0$ arbitrarily. The right-hand side of equation (A.26) is increasing in w because $\eta(\delta,w,n)$ is increasing in w and $c'(\gamma)$ is positive. On the other hand, the left-hand side of the equation is decreasing in w because $\gamma(q|\delta,w,n)$ is decreasing in w for any $q > \underline{q}(\delta,n)$ in the admissible set. Hence, if w satisfies the free-entry condition for a given δ, then w is unique. Its existence is guaranteed if we can prove that the right-hand side is zero at $\underline{w}(\delta, n)$, because the left-hand side is strictly positive at $\underline{w}(\delta, n)$ and both the left- and right-hand sides are continuous for $w \geq \underline{w}(\delta, n)$. The right-hand side is zero at $\underline{w}(\delta, n)$ because equation (A.14) implies that in the limit of $\delta = \gamma$, η converges to zero to make the total measure of firms finite.

As long as η is positively correlated with δ in condition (A.17), an increase in δ drives up $\gamma_\eta(\delta, w, n)$ for given w and n, while it reduces the left-hand side of equation (A.26) through an increase in $\underline{q}(\delta, n)$ and declines in $\gamma(q|\delta, w, n)$ for any $q > \underline{q}(\delta, n)$. Thus, when δ increases, it is necessary to have a smaller w to satisfy the FE, because γ is decreasing in w and γ_η is increasing in w. ■

Proof of Proposition 4 Define $\delta_{FE}(w, n)$ as δ that satisfies equation (A.17) and the FE condition, (A.26). Consider $\eta(\delta_{FE}(w, n), w, n)$ by substituting $\delta_{FE}(w, n)$ into equation (A.17). As $w \to \infty$, $\underline{w}^{-1}(w, n)$ and the left-hand side of equation (A.26) go to zero, so that η (and δ) must be zero. Then, $\lim_{w\to\infty} \eta(\delta_{FE}(w, n), w, n) = 0$. On the other hand, when $w \to 0$, $\delta_{FE}(w, n) \to \infty$ because $\underline{w}^{-1}(w, n) \to \infty$. Thus, $\lim_{w\to 0} \eta(\delta_{FE}(w, n), w, n) = \infty$. In terms of employment, when $w \to \infty$, we observe

$$\lim_{w\to\infty} L_{R,ent}(\delta_{FE}(w, n), w, n) = 0, \qquad \lim_{w\to\infty} L_{R,inc} = \lim_{w\to\infty} L_X = 0,$$

which leads to $\lim_{w\to\infty} L_D(\delta_{FE}(w, n), w, n) = 0$ for any $n \geq 0$. When $w \to 0$, $\lim_{w\to 0} L_{R,ent}(\delta_{FE}(w, n), w, n) = \infty$ for any $n \geq 0$, which implies $\lim_{w\to 0} L_D(\delta_{FE}(w, n), w, n) = \infty$.

Because L_D is continuous in w, at least one w that satisfies $L_D(\delta_{FE}(w, n), w, n) = L$ exists for any $n \geq 0$.

Since the FE curve lies above the curve of $\underline{w}$, any stationary state has a nondegenerate firm-size distribution. Such a distribution cannot be compatible with no entry, because $\eta = 0$ implies that $\underline{q}$ should be the upper bound of the support of q (or infinity) to satisfy the FE condition, (A.26). ■

B.4 Labor Demand

Lemma A.4 *Fix $\delta > 0$ and $n \geq 0$. The cumulative distribution of $K(q|\delta, w_1, n)$ stochastically dominates that of $K(q|\delta, w_2, n)$ if $w_1 < w_2$.*

Proof of Lemma A.4 From equation (A.17) and the definition of K, $K(q|\delta, w, n)$ is increasing in w if and only if

$$\int_{\underline{q}(\delta,n)}^{\infty} -\frac{\partial \gamma(q'|\delta, w, n)}{\partial w} \frac{K(q'|\delta, w, n)}{\delta - \gamma(q'|\delta, w, n)} dq' > -\frac{\partial \gamma(q|\delta, w, n)}{\partial w} \frac{1}{\delta - \gamma(q|\delta, w, n)}.$$

The left-hand side is a positive constant for any q, while the right-hand side is zero at $q = \underline{q}(\delta, n)$ and monotonically increasing in q from Proposition 2. To keep $\int_{\underline{q}}^{\infty} K(q|\delta, w, n)dq = 1$, $\hat{q} > \underline{q}$ exists such that $\partial K/\partial w > 0$ if and only if $q < \hat{q}$.

Since $\frac{\partial}{\partial w} \int_{\underline{q}}^{\infty} K(q'|\delta, w, n)dq'$ must be zero by definition,

$$0 = \frac{\partial}{\partial w} \int_{\underline{q}}^{\hat{q}} K(q'|\delta, w, n)dq' + \frac{\partial}{\partial w} \int_{\hat{q}}^{\infty} K(q'|\delta, w, n)dq'.$$

Hence, for any $q \in (\hat{q}, \infty)$,

$$\frac{\partial}{\partial w} \int_{\underline{q}}^{\hat{q}} K(q'|\delta, w, n)dq' > -\frac{\partial}{\partial w} \int_{\hat{q}}^{q} K(q'|\delta, w, n)dq'.$$

Therefore, $\frac{\partial}{\partial w} \int_{\underline{q}}^{q} K(q'|\delta, w, n)dq' > 0$ for any finite q, which implies the stated stochastic dominance. ■

We assume that this effect does not dominate.

Proposition A.1 *$L_{R,ent}$ is increasing in w. $L_{R,inc}$ is decreasing in w. L_X is decreasing in w if the distribution effect is sufficiently weak.*

Proof of Proposition A.1 Fix $\delta > 0$ and n arbitrarily. $L_{R,ent}$ is increasing in w because

$$\frac{\partial \gamma_\eta(\delta, w, n)}{\partial w} \propto \frac{\partial \eta(\delta, w, n)}{\partial w} > 0.$$

Next, the response of $L_{R,inc}$ to an increase in w is

$$\begin{aligned} \frac{\partial L_{R,inc}}{\partial w} = &\int_{\underline{q}(\delta,n)}^{\infty} \frac{\partial K(q|\delta, w, n)}{\partial w} c(\gamma(q|\delta, w, n))dq \\ &+ \int_{\underline{q}(\delta,n)}^{\infty} K(q|\delta, w, n)c'(\gamma(q|\delta, w, n)) \frac{\partial \gamma(q|\delta, w, n)}{\partial w} dq \end{aligned} \tag{A.27}$$

The last term is negative according to Proposition 2. The first term on the right-hand side depends on $\partial K/\partial w$. Because $c(\gamma(q))$ is an increasing function of q, Lemma

A.4 implies that the first term on the right-hand side of equation (A.27) is negative. Hence, R&D labor demand from incumbents is decreasing in w.

The effect of an increase in w on L_X is

$$\frac{\partial L_X}{\partial w} = -\frac{L_X}{w} + \int_{\underline{q}(\delta,n)}^{\infty} \frac{\partial K(q|\delta,w,n)}{\partial w} L_{X,q}(q|\delta,w,n)dq \tag{A.28}$$

The last term is positive because $L_{X,q}(q|\delta,w,n)$ is a decreasing function of q and K is stochastically dominated when w increases, as shown in Lemma A.4.[4] Thus, to have L_X decreasing in w, the distribution effect is sufficiently small. ■

The LMC condition does not necessarily imply a monotonic relation between δ and w. Suppose that $w > \underline{w}(\delta,n)$ and $\eta(\delta,w,n)$ is increasing in δ in condition (A.17). Then, we have

$$\frac{\partial L_D}{\partial \delta} = \underbrace{\frac{\partial L_X}{\partial \delta}}_{(+)} + \underbrace{\frac{\partial L_{R,inc}}{\partial \delta}}_{-} + \underbrace{\frac{\partial L_{R,ent}}{\partial \delta}}_{+} \tag{A.29a}$$

$$\frac{\partial L_D}{\partial w} = \underbrace{\frac{\partial L_X}{\partial w}}_{(-)} + \underbrace{\frac{\partial L_{R,inc}}{\partial w}}_{-} + \underbrace{\frac{\partial L_{R,ent}}{\partial w}}_{+} \tag{A.29b}$$

The signs of responses of L_X in equations (A.29) are in parentheses because the signs do not hold in general. In equation (A.29a), with a rise in the creative destruction rate, firms extend the interval between price resets, leading to lower markups under positive nominal growth. This is the primary factor that increases production employment. However, a rise in δ also brings more takeover of product lines with the highest markups. Furthermore, there is a reallocation effect: $K(q|\delta,w,n)$ becomes less concentrated with higher q, because firm growth tends to be low under a high frequency of creative destruction[5] but we have higher $\underline{q}(\delta,n)$.

In equation (A.29b), $\partial L_X/\partial w$ could be positive when the reallocation effect is too strong. Lemma A.4 implies that higher w leads to a reduction in average markup and thereby an increase in production.

[4] $L_{X,q}(q|\delta,w,n)$ is decreasing in q because the benefit from a price revision increases with q, and thus, a firm with greater q never waits until the declining real price reaches the lower bound that is optimal for firms with smaller q. Hence, the lower bound of the real price, $e^{n\Delta(q|\delta,n)}$, is increasing in q. The average real price is definitely higher for high-quality firms.

[5] If δ is extremely high, $K(q|\delta,w,n)$ is close to $\phi(q|\delta,n)$.

B.5 Firm Size and Quality

Relation to Market Concentration Firm size and firm quality are positively correlated. The expected number of product lines supplied by a type-q firm is

$$E\left[k|q\right] = \frac{K(q,\delta,w,n)}{M(q,\delta,w,n)} = \frac{\gamma(q|\delta,w,n)}{\delta-\gamma(q|\delta,w,n)} \frac{1}{\log\left(\frac{\delta}{\delta-\gamma(q|\delta,w,n)}\right)}, \tag{A.30}$$

which is strictly increasing in q from Proposition 2. This relationship also implies that firm size and average markup are positively correlated, because q determines the maximum markup rate.

Another important implication from equation (A.30) concerns the concentration of the market. If there are more high-quality firms, more product lines are produced by those firms and the degree of concentration increases.

Nominal Sales Distribution Because of the log utility, the nominal sales in each product line equal the nominal income, E_t, independent of prices. Hence, the sales distribution across firms is the same as the distribution of the number of products, k. This is why we examined the relationship between sales distribution and inflation in Section 2.

Let $R(k|\delta,w,n)$ be the density around the sales of kE_t,

$$\begin{aligned} R(k|\delta,w,n) &\equiv \int_{\underline{q}(\delta,n)}^{\infty} \phi(q|\delta,n)\frac{M_k(q|\delta,w,n)}{M(q|\delta,w,n)}dq \\ &= \frac{1}{k}\int_{\underline{q}(\delta,n)}^{\infty} \phi(q|\delta,n)\left(\frac{\gamma(q|\delta,w,n)}{\delta}\right)^k \left[\log\frac{\delta}{\delta-\gamma(q|\delta,w,n)}\right]^{-1} dq. \end{aligned} \tag{A.31}$$

C Calibration

C.1 Weighted Distance between Data Moments and Simulated Moments

Define the vector of 12 moments by $\Gamma(\psi)$ that consist of the real growth rate g, nominal interest rate R, interval of price revision Δ, the ratio of the median of wage bill to the mean of wage bill (Med[WL]/E[WL]), the ratio of the standard deviation of wage bill to the mean of wage bill (Std[WL]/E[WL]), the ratio of the median of real sales to the mean of real sales (Med[Y]/E[Y]), the ratio of the standard deviation of real sales to the mean of real sales (Std[Y]/E[Y]), the correlation coefficient between labor productivity and employment (Cor[Y/L, L]), the correlation coefficient between real sales and wage bill (Cor[Y, WL]), the mean of real sales growth (E[dY/Y]), the standard deviation of real sales growth (Std[dY/Y]), and the correlation coefficient between real sales growth and real sales (Cor[dY/Y, Y]).

For the above 12 moments, we calculate model-based moments for a given parameter set θ. Defining model-based moments by $\Gamma^s(\theta)$, we search for $\hat{\theta}$ to minimize the weighted distance between data moments and simulated moments:

$$\theta = \arg\min_{\theta} \left(\Gamma^s(\theta) - \Gamma(\psi)\right)' A^{-1} \left(\Gamma^s(\theta) - \Gamma(\psi)\right).$$

Although this approach is based on Lentz and Mortensen (2008), we do not claim that we estimate the model. We do not provide standard errors for the estimates like their paper, and admittedly better estimates to minimize the weighted distance may exist because we did not calculate it for a sufficiently global parameter set. We calibrated the parameters as follows: searching for a good parameter set manually by using a grid search, and then using the FSOLVE function of the Matlab software with the parameter set as an initial value.

The variance–covariance matrix A is calculated from that of the data moments. We estimate the distributional variables by bootstrapping the data as in Lentz and Mortensen (2008). As for g and R, we assume that their variance–covariance is equal to their time-series variances from 1986 to 2006 and they are independent from other variables. For the variance of Δ, we calculate it from the category-level Δ's (e.g., unprocessed food and energy) reported in Chart 11 in Higo and Saita (2007). Furthermore, we find that these three variances are too large as compared with the variances of the distributional variables, so we multiply 10,000 for the latter to better fit with the moments of g, R, and Δ and to obtain quantitatively plausible implications for aggregate variables.

C.2 Simulated Moments

In this subsection, we explain how we calculate the model-based moments.

Real sales Y:[6] Given quality step q and firm size k, the real sales of the firm equal

[6] While Lentz and Mortensen (2008) call this the value added, it is essentially the same.

k. The corresponding frequency is given by $F(q,k) = \phi(q|n)\frac{M_k(q)}{M(q)}$ for $q \in [\underline{q}(n), \infty)$ and $k = 1, 2, \cdots$. Using this, we can calculate the moments. For example, the mean of real sales equals

$$E[Y] = \int_{\underline{q}(n)}^{\infty} \phi(q|n) \sum_{k=1}^{\infty} \frac{M_k(q)}{M(q)} k. \tag{A.32}$$

Wage bill WL: Note that the density function of real prices among type-q firms is

$$f(\xi(\tau)|q) = \frac{\delta e^{-\delta\tau}}{1 - e^{-\delta\Delta(q)}} \qquad \text{for } \tau \in [0, \Delta(q)].$$

We approximate the price dispersion by dividing it into N_p grids so that τ takes the value of $\tau(n_p) = n_p\Delta(q)/(N_p + 1)$ for $n_p = 1, 2, \cdots, N_p$ with the frequency of

$$\Gamma(n_p) \equiv \int_{(n_p-1)\Delta(q)/N_p}^{n_p\Delta(q)/N_p} f(\xi(\tau)|q)d\tau = \frac{e^{-\delta(n_p-1)\Delta(q)/N_p} - e^{-\delta n_p\Delta(q)/N_p}}{1 - e^{-\delta\Delta(q)}}. \tag{A.33}$$

For a firm of size k, we further approximate the frequency for the average price (i.e., $\tau = (\tau(n_p^1) + \tau(n_p^2) + \cdots + \tau(n_p^k))/k$) by $\Gamma(n_p)^k / \Sigma_{n_p=1}^{N_p} \Gamma(n_p)^k$. Given q, k, and τ, the real wage bill of the firm equals

$$wk(L_{X,q} + L_{R,inc,q}) = wk\left(\frac{1}{wqe^{-n\tau}} + c(\gamma(q))\right). \tag{A.34}$$

Labor productivity Y/L : Given q and τ, the labor productivity is given by

$$\Theta(q) = \left(\frac{1}{wqe^{-n\tau}} + c(\gamma(q))\right)^{-1}. \tag{A.35}$$

Future size evolution: Given q and k, the expected real sales in the following year are expressed as

$$\mathrm{E}\left[Y(k', q'|k, q)\right] = \sum_{k'} \Pr(k'|k, q) Y(k', q)(1 + n), \tag{A.36}$$

where $\Pr(k'|k,q) = \Pi_{n'=1}^{n} \Pr(k_{n'+1}|k_{n'},q)$ with $k_1 = k$, and

$$\Pr(k_2|k_1,q) = \begin{cases} 1 - e^{-k_1\gamma(q)/n} & \text{if } k_1 = k_2 - 1 \\ 1 - (1 - e^{-k_1\delta/n}) - (1 - e^{-k_1\gamma(q)/n}) & \text{if } k_1 = k_2 \\ 1 - e^{-k_1\delta/n} & \text{if } k_1 = k_2 + 1, \end{cases} \tag{A.37}$$

provided $k_1 > 0$. If $k_1 = 0$, $\Pi(k_2|k_1,q) = 0$. We assume $n = 26$ as in Lentz and Mortensen (2008).

The interval of price revisions Δ: Given q, the interval of price revisions is given by $\Delta(q)$. We calculate its average using the firm-size weight (i.e. each product is assigned an equal weight):

$$\int_{\underline{q}(\delta,n)}^{\infty} K(q|\delta,w,n)\Delta(q)\,dq. \tag{A.38}$$

C.3 Cut-off

"The Basic Survey of Japanese Business Structure and Activities" covers all firms with no less than 50 employees and no less than 30 million yen in capital. Considering this cut-off, we calculate model-based moments for the firms whose product size is no less than two ($k \geq 2$). This strict cut-off may cause the over-representation of exits and under-representation of entries. On the other hand, the strict criterion may be suitable for our study because there are many dormant small firms in Japan that exist mainly for non-business purposes, as Nishimura et al. (2005) and Murao and Nirei (2011) argue.

D Generalization: Endogenous Labor Supply and Non-unit Elasticity of Substitution

We generalize the model discussed in the main text in two directions. First, consumption is determined by the constant elasticity of substitution (CES) production function. Second, the labor supply is endogenous.

D.1 Generalized Model

A representative household has the following preferences over all versions of $a \in \{0, 1, \cdots, A_t(j)\}$ for each product line $j \in [0, 1]$:

$$U_t = \int_t^{\infty} e^{-\rho(t'-t)} \left(\log C_{t'} - \chi \frac{L_{t'}^{1+\omega}}{1+\omega} \right) dt', \tag{A.39}$$

$$C_{t'} = \left[\int_0^1 \left\{ \sum_{a=0}^{A_{t'}(j)} Q(j,a) x_{t'}(j,a) \right\}^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)}, \tag{A.40}$$

Parameter $\sigma \geq 1$ represents the elasticity of substitution, where we assumed $\sigma = 1$ in the main text. Parameter ω represents the inverse of the elasticity of work, while χ is a scale factor. The budget constraint is given by $P_t C_t = W_t L_t + \Pi_t$, where the nominal household expenditure is again growing at the exogenous rate of n, and Π_t represents a lump-sum transfer from firms.

This specification leads to the following changes in equilibrium conditions:

$$L = \left(\frac{W}{\chi PC} \right)^{1/\omega} = \left\{ \frac{w}{\chi} \left(1 - \kappa\delta \int \frac{K(q|\delta, w, n)}{1 - e^{-\delta\Delta(q|\delta,n)}} dq \right)^{-1} \right\}^{1/\omega}, \tag{A.41}$$

$$\tilde{\nu}_0(q|\delta, n) = \max_{\Delta, \xi_0} \sum_{i=0}^{\infty} e^{-(\rho+\delta)i\Delta} \left(\int_0^{\Delta} \Pi(\xi_0 e^{-n\tau}) e^{-(\rho+\delta)\tau} d\tau - \kappa \right), \tag{A.42}$$

where $\Pi(\xi) = (1 - W/\xi)\,(P/\xi)^{\sigma-1}$ if $\xi < qW$, and zero otherwise. The optimal (re)set real price satisfies $\xi_0(q|\delta, n) = qW$ if $\sigma/(\sigma-1) \geq q$. If $\sigma/(\sigma-1) < q$, the price is not explicitly clear, so we need to numerically solve not only Δ but also ξ_0. The optimal choice of price revision is expressed by

$$\frac{d}{d\xi_0}\tilde{\nu}_0(q|\delta, n) = 0 = \int_0^{\Delta} \Pi'(\xi_0 e^{-n\tau}) e^{-(\rho+\delta+n)\tau} d\tau \qquad \text{if } \xi_0 < qW$$
$$\xi_0 = \min\left[\frac{\sigma}{\sigma-1} W \frac{e^{(n\sigma-\rho-\delta)\Delta} - 1}{n\sigma - \rho - \delta} \frac{n\sigma - \rho - \delta - n}{e^{(n\sigma-\rho-\delta-n)\Delta} - 1}, qW \right], \tag{A.43}$$

as well as

$$\frac{d}{d\Delta}\tilde{\nu}_0(q|\delta, n) = 0 = -(\rho+\delta)\tilde{\nu}_0 + \Pi(\xi_0 e^{-n\Delta}). \tag{A.44}$$

Moreover, from equation (A.42), we have

$$\tilde{\nu}_0(q|\delta,n) = \frac{1}{1-e^{-(\rho+\delta)\Delta}}\left[\left(\frac{\xi_0}{P}\right)^{1-\sigma}\frac{e^{(n\sigma-\rho-\delta-n)\Delta}-1}{n\sigma-\rho-\delta-n} - \frac{W}{P}\left(\frac{\xi_0}{P}\right)^{-\sigma}\frac{e^{(n\sigma-\rho-\delta)\Delta}-1}{n\sigma-\rho-\delta} - \kappa\right]. \tag{A.45}$$

Labor demand for production is

$$L_{X,q}(q|\delta,w,n) = \int_0^{\Delta(q|\delta,n)} f(\xi(\tau))P^{\sigma-1}\xi(\tau)^{-\sigma}d\tau = \frac{\delta}{\delta-n\sigma}P^{\sigma-1}\xi_0^{-\sigma}\frac{1-e^{-(\delta-n\sigma)\Delta(q|\delta,n)}}{1-e^{-\delta\Delta(q|\delta,n)}}. \tag{A.46}$$

The intertemporal utility at $t=0$ is given by

$$\begin{aligned} U = \int_0^\infty e^{-\rho t}\left(\log C_t - \chi\frac{L_t^{1+\omega}}{1+\omega}\right)dt &= \frac{g}{\rho^2} + \frac{\log C}{\rho} - \frac{\chi}{\rho}\frac{L^{1+\omega}}{1+\omega} \\ &= \frac{g}{\rho^2} + \frac{\log C}{\rho} - \frac{\chi}{\rho}\frac{1}{1+\omega}\left(\frac{w}{\chi PC/E}\right)^{(1+\omega)/\omega}. \end{aligned} \tag{A.47}$$

The price level is given by

$$P_t = \left[\int_0^1 P_t(j)^{1-\sigma}dj\right]^{\frac{1}{1-\sigma}}, \tag{A.48}$$

where $P_t(j)$ represents the quality unit price. The price level at $t=0$ is

$$\begin{aligned} P &= \int K(q)\left[\int_0^\Delta f(\xi(\tau))\xi(\tau)^{1-\sigma}d\tau\right]^{\frac{1}{1-\sigma}} dq \\ &= \int K(q)\xi_0\left[\frac{\delta}{1-e^{-\delta\Delta}}\frac{e^{(n\sigma-\delta-n)\Delta}-1}{n\sigma-\delta-n}\right]^{\frac{1}{1-\sigma}} dq \qquad \text{if } \sigma>1 \qquad \text{(A.49)} \\ \log P &= \int K(q)\left\{\log\xi_0 + n\Delta\frac{e^{-\delta\Delta}}{1-e^{-\delta\Delta}} - \frac{n}{\delta}\right\}dq \qquad \text{if } \sigma=1. \qquad \text{(A.50)} \end{aligned}$$

When $\sigma > 1$, the real growth rate, g, is calculated as follows. Total production (consumption and menu costs) is given by

$$Y_t = E_t/P_t.$$

Let $P_t(j)$ and $p_t(j)$ be the quality unit price and the quality-unadjusted goods price, respectively. Then, using equation (A.48), we have

$$Y_t = E_t\left[\int_0^1 \left(\frac{Q(j)}{p_t(j)}\right)^{\sigma-1} dj\right]^{\frac{1}{\sigma-1}} = E\left[\int_0^1 \left(\frac{Q_t(j)}{\xi_t(j)}\right)^{\sigma-1} dj\right]^{\frac{1}{\sigma-1}}.$$

The change in $Q_t(j)/\xi_t(j)$ is

$$\frac{Q_{t+dt}(j)}{\xi_{t+dt}(j)} = \begin{cases} \frac{q'\xi_t(j)}{\xi_0(q')}\frac{Q_t(j)}{\xi_t(j)} & \text{with prob. } \delta dt, \\ \frac{\xi_t(j)}{\xi_{t+dt}(j)}\frac{Q_t(j)}{\xi_t(j)} & \text{with prob. } 1-\delta dt, \end{cases}$$

where q' is the step-size of new innovation. Define $\Delta\left(Q_t(j)/\xi_t(j)\right) \equiv \frac{Q_{t+dt}(j)}{\xi_{t+dt}(j)} - \frac{Q_t(j)}{\xi_t(j)}$, so that

$$\frac{Q_{t+dt}(j)}{\xi_{t+dt}(j)} = \left(1 + \frac{\Delta\left(Q_t(j)/\xi_t(j)\right)}{Q_t(j)/\xi_t(j)}\right)\frac{Q_t(j)}{\xi_t(j)}.$$

The growth rate is determined by

$$\begin{aligned}
\frac{Y_{t+dt}}{Y_t} \equiv 1 + \frac{\Delta Y_t}{Y_t} &= \left[\frac{\int_0^1 \left(\frac{Q_{t+dt}(j)}{\xi_{t+dt}(j)}\right)^{\sigma-1} dj}{\int_0^1 \left(\frac{Q_t(j)}{\xi_t(j)}\right)^{\sigma-1} dj}\right]^{\frac{1}{\sigma-1}} \\
&= \left[\int_0^1 \left(1 + \frac{\Delta\left(Q_t(j)/\xi_t(j)\right)}{Q_t(j)/\xi_t(j)}\right)^{\sigma-1} \left\{\frac{\left(\frac{Q_t(j)}{\xi_t(j)}\right)^{\sigma-1}}{\int_0^1 \left(\frac{Q_t(h)}{\xi_t(h)}\right)^{\sigma-1} dh}\right\} dj\right]^{\frac{1}{\sigma-1}} \\
&= \left(\mathbb{E}\left\{\left(1 + \frac{\Delta\left(Q_t(j)/\xi_t(j)\right)}{Q_t(j)/\xi_t(j)}\right)^{\sigma-1}\right\}\right)^{\frac{1}{\sigma-1}} \\
&= \left(\delta dt\,\mathbb{E}\left[\left(\frac{q'\xi_t(j)}{\xi_0(q')}\right)^{\sigma-1}\right] + (1-\delta dt)\,\mathbb{E}\left[\left(\frac{\xi_t(j)}{\xi_{t+dt}(j)}\right)^{\sigma-1}\right]\right)^{\frac{1}{\sigma-1}}.
\end{aligned}$$

The third equality comes because each product line has independent and identical distributions with respect to quality and price.

Conditional on survival, the real price changes as

$$\xi_{t+dt}(j) = \begin{cases} \xi_0(j) & \text{with prob. } f(\Delta)dt, \\ \xi_t(j)e^{-ndt} & \text{with prob. } 1-f(\Delta)dt. \end{cases}$$

Thus, we have

$$
\begin{aligned}
(1-\delta dt)\,\mathbb{E}\left[\left(\frac{\xi_t(j)}{\xi_{t+dt}(j)}\right)^{\sigma-1}\right] &= (1-\delta dt)\,\mathbb{E}\left[(1-f(\Delta)dt)\,e^{n(\sigma-1)dt}+f(\Delta)dt\left(\frac{\xi_\Delta(j)}{\xi_0(j)}\right)^{\sigma-1}\right] \\
&= \mathbb{E}\left[(1-\delta dt-f(\Delta)dt)\,e^{n(\sigma-1)dt}+f(\Delta)dte^{-n(\sigma-1)\Delta}\right].
\end{aligned}
$$

Substituting this back yields

$$
\frac{Y_{t+dt}}{Y_t}=\left(\delta dt\,\mathbb{E}\left[\left(\frac{q'\xi_{t+dt}(j)}{\xi_0(q')}\right)^{\sigma-1}\right]+\mathbb{E}\left[(1-\delta dt-f(\Delta)dt)\,e^{n(\sigma-1)dt}+f(\Delta)dte^{-n(\sigma-1)\Delta}\right]\right)^{\frac{1}{\sigma-1}}.
$$

The real growth rate is expressed as

$$
\begin{aligned}
g &\equiv \lim_{dt\to 0}\frac{Y_{t+dt}/Y_t-1}{dt} \\
&= \frac{1}{\sigma-1}\left[\delta\left\{\mathbb{E}\left[\left(\frac{q'\xi_t(j)}{\xi_0(q')}\right)^{\sigma-1}\right]-1\right\}+n(\sigma-1)-\mathbb{E}\left[f(\Delta)\left(1-e^{-n(\sigma-1)\Delta}\right)\right]\right],
\end{aligned}
\tag{A.51}
$$

where the expectations terms are given by

$$
\mathbb{E}\left[\left(\frac{q'\xi_t(j)}{\xi_0(q')}\right)^{\sigma-1}\right]=\int\left\{\int\left[\int_0^\Delta\left(\frac{q'\xi_\tau}{\xi_0(q')}\right)^{\sigma-1}f(\xi_\tau)d\tau\right]K(q)dq\right\}K(q')dq', \tag{A.52}
$$

and

$$
\mathbb{E}\left[f(\Delta)\left(1-e^{-n(\sigma-1)\Delta}\right)\right]=\int f(\Delta)\left(1-e^{-n(\sigma-1)\Delta}\right)K(q)dq. \tag{A.53}
$$

In the following two limiting cases, the real growth rate becomes

$$
g=\begin{cases}\frac{\delta}{\sigma-1}\left\{\mathbb{E}\left[\left(\frac{q'\xi_0(j)}{\xi_0(q')}\right)^{\sigma-1}\right]-1\right\} & \text{for } n=0,\\ \delta\,\mathbb{E}\left[\log q\right] & \text{for } \sigma\to 1.\end{cases}
$$

Note that, when $\sigma \to 1$, we have

$$\begin{aligned}
\lim_{\sigma\to 1} g &= \lim_{\sigma\to 1} \frac{d}{d\sigma}\left[\delta\left\{\mathbb{E}\left[\left(\frac{q'\xi_t(j)}{\xi_0(q')}\right)^{\sigma-1}\right]-1\right\}+n(\sigma-1)-\mathbb{E}\left[f(\Delta)\left(1-e^{-n(\sigma-1)\Delta}\right)\right]\right] \\
&= \delta\,\mathbb{E}\left[\log qe^{-n\tau}\right]+n-\mathbb{E}\left[n\Delta f(\Delta)\right] \\
&= \delta\,\mathbb{E}[\log q]+n\left\{1-(\delta\,\mathbb{E}[\tau]+\mathbb{E}\left[\Delta f(\Delta)\right])\right\}.
\end{aligned}$$

Because

$$\begin{aligned}
\mathbb{E}[\tau] &= \int K(q)dq\int_0^{\Delta}\tau\frac{\delta e^{-\delta\tau}}{1-e^{-\delta\Delta}}d\tau = \frac{1}{\delta}\int K(q)dq\left(1-\frac{\delta\Delta e^{-\delta\Delta}}{1-e^{-\delta\Delta}}\right), \\
\mathbb{E}\left[\Delta f(\Delta)\right] &= \int K(q)dq\frac{\delta\Delta e^{-\delta\Delta}}{1-e^{-\delta\Delta}},
\end{aligned}$$

we have $g \to \delta\,\mathbb{E}[\log q]$ as $\sigma \to 1$.

The other equations do not change.

D.2 Calibration of the Generalized Model

In the calibration, we set the inverse of the elasticity of work, ω, to 2 or the elasticity of substitution across different product lines, σ, to 1.5, although a number of different parameter values are empirically reported.

Figure A.3 shows the effects of nominal growth based on modified models, while Table A.10 shows the moments. As for the endogenous labor supply, a change in real wage w induced by a change in the nominal growth rate n influences the labor supply. In the calibrated model, an increase in n decreases w that decreases the labor supply. However, this effect is quantitatively small. There is almost no change observed between the benchmark model (in which ω is infinitely large) and the model with $\omega = 2$.

An increase in the elasticity of substitution from one reinforces the reallocation effect under the current parameter setting. It has three effects. First, the markup has a cap at $\sigma/(\sigma-1)$ even though firms have high q, because a leading firm competes with not only firms with secondary technology in the same product line but also firms in different product lines. Second, higher σ induces smaller firm-size dispersions if all other parameter values are equal. Since demand becomes more elastic, firms with high q gain smaller sales. Third, more elastic demand decreases an incentive to revise prices for a given menu cost κ. Indeed, Table A.10 shows that both the standard deviation and the mean decrease relative to the median (Std/Med and Mean/Med calculated from the values in the table), and the reset price interval increases, when we use the same parameter values.

Figure A.3 shows that the positive reallocation effect of nominal growth is stronger. The nominal growth causes a gradual decline in the relative prices of products whose posted prices are not revised, leading to a gradual increase in the expenditure shares of those products. Because high-q firms have constant prices with longer durations, the increase in the expenditure share toward high-q products becomes more outstanding under a greater σ, which amplifies the positive effects on real growth and welfare. This effect is also reinforced by a decline in innovation frequency caused by the nominal growth because innovation is always accompanied with the price reset that increases the price and decreases sales. We obtain no finite value for the nominal growth to maximize welfare as well as the real growth rate: welfare increases as the optimal nominal growth increases. This result holds even when we recalibrate the parameter values to match the moments based on the model with $\sigma = 1.5$.

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Table A.1: Summary Table for Variables Used for Estimation

Variable	Observation	Mean	Std. Dev.	Min	Max
$\bar{\pi}^{\text{input}}$	322	0.0100076	0.0495548	-0.1427733	0.2835882
Top/Middle (sales)	322	11.04676	23.00279	3.848761	229.983
Top/Bottom (sales)	322	43.26014	89.67692	11.27907	803.2036
Top/Middle (employment)	322	4.612411	1.499788	2.576576	15.22388
Top/Bottom (employment)	322	10.50473	4.61886	4.672131	38.53571
Industry real sales (log)	322	11.86376	0.8597284	10.01801	13.44605
Financial DI	316	0.9290612	11.05655	-29.75	28.25
Financing DI by size class	948	4.308456	14.36273	-36	46.25
Sales growth	189447	0.0567846	0.6869153	-0.9948701	120.5996
Employment growth	189447	0.0055831	0.268017	-0.9760662	20.75385
Real R&D expenditure (log)	58445	0.8828367	1.387753	0	9.080056
$\frac{\text{R\&D}}{\text{Sales}}$	58445	0.018587	0.0421069	0	4.858796
$\log \frac{\text{R\&D}}{\text{Wage}}$	58418	2.251617	2.354766	0	11.64695
$\log \frac{\text{R\&D}}{\text{Total Salary}}$	58418	0.1036888	0.1769869	0	5.332719
Exit	175463	0.0352154	0.1843244	0	1
Lagged sales (3 years, log)	189447	8.50466	1.341011	3.218876	16.307
Lagged leverage (3 years, log)	189447	3.235007	1.101904	-5.521461	8.336737

Sources: Ministry of Economy, Trade and Industry (METI), “the Basic Survey of Japanese Business Structure and Activities;” the Bank of Japan, “Producer Price Index;” the Bank of Japan, “Tankan,” etc.

Table A.2: Summary Table for Input Inflation and Firm Size Dispersions by Industries

	$\bar{\pi}^{\text{input}}$		T/M ratio				T/B ratio			
			Sales		Employment		Sales		Employment	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
Foods	0.21%	1.73%	6.48	0.32	4.43	0.15	24.45	1.72	10.57	0.65
Textile	0.28%	2.38%	4.58	0.36	2.90	0.25	17.05	1.45	5.42	0.62
Pulp, paper and wooden products	0.43%	1.98%	5.22	0.56	3.66	0.18	14.15	1.85	7.14	0.41
Chemical products	2.07%	3.72%	9.00	0.65	6.40	0.55	35.47	4.55	16.59	1.25
Petroleum and coal products	6.93%	10.70%	68.59	62.87	7.81	2.90	305.93	196.72	19.90	7.93
Ceramics, stone and clay products	0.70%	1.80%	5.04	0.28	3.51	0.21	14.98	1.19	6.98	0.43
Steel	2.92%	5.92%	5.61	0.51	4.34	0.42	23.30	3.81	9.10	0.95
Non-ferrous metal	3.13%	9.65%	7.42	1.22	4.36	0.34	28.49	6.25	9.42	0.89
Metal products	0.74%	3.07%	4.94	0.38	3.56	0.18	13.16	0.77	6.56	0.49
General machinery	-0.07%	1.29%	6.59	0.48	4.26	0.21	19.27	1.90	8.94	0.46
Electrical machinery	-2.04%	1.48%	9.30	0.63	5.14	0.27	35.17	2.14	12.81	0.59
Transportation equipments	-0.53%	1.08%	8.96	0.51	5.69	0.25	34.94	3.04	14.98	0.87
Precision instruments	-1.24%	0.91%	7.56	1.07	4.79	0.58	22.96	2.94	11.03	1.35
Miscellaneous	0.49%	1.94%	5.35	0.25	3.72	0.10	16.32	1.08	7.61	0.23

Sources: METI, “the Basic Survey of Japanese Business Structure and Activities;” the Bank of Japan, “Producer Price Index;” the Bank of Japan, “Tankan,” etc.

Table A.3: Effect of Inflation on Real Sales Growth within Firm Age Class

	(1)	(2)	(3)	(4)
	below 25%	25-50%	50-75%	above 75%
Dep. var.	Sales growth	Sales growth	Sales growth	Sales growth
$\hat{\pi}^{\text{input}}$	-2.098**	-2.452***	-5.192***	-1.128***
	(0.903)	(0.784)	(1.484)	(0.270)
Lagged sales	-1.113***	-1.060***	-1.842***	-0.694***
	(0.0104)	(0.0117)	(0.0244)	(0.00575)
Lagged sales × $\hat{\pi}^{\text{input}}$	0.313***	0.330***	0.602***	0.0793***
	(0.0987)	(0.0863)	(0.165)	(0.0275)
Financing DI	0.00798***	0.00596***	0.00475***	0.00309***
	(0.000503)	(0.000446)	(0.000826)	(0.000210)
Industry RS	-0.0555***	-0.0617***	-0.0158	-0.0134**
	(0.0111)	(0.0104)	(0.0210)	(0.00529)
Lagged leverage	0.103***	0.110***	0.224***	0.0526***
	(0.00779)	(0.00950)	(0.0216)	(0.00453)
Constant	9.863***	9.331***	15.13***	6.308***
	(0.155)	(0.155)	(0.317)	(0.0810)
Year/Firm FE	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	38864	43762	49204	57617
Num. firms	6844	7698	8931	7216
R^2				
within	0.298	0.222	0.138	0.281
overall	0.00487	0.00108	0.00107	0.000535
F	566.2	427.9	268.4	820.3
F_f	7.911	2.923	2.551	6.150

Notes: Standard errors are in the parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$). F_f represents the F test statistics with the null hypothesis that all firm fixed effects are zero.
Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" the Bank of Japan, "Producer Price Index;" the Bank of Japan, "Tankan," etc.

Table A.4: Effect of Inflation on Real Sales Growth within Leverage Ratio Class

Dep. var.	(1) $\leq$ Median Sales growth	(2) $>$ Median Sales growth	(3) $\leq$ 75%tile Sales growth	(4) $>$ 75%tile Sales growth
$\hat{\pi}^{\text{input}}$	-2.643***	-2.281***	-2.454***	-4.798***
	(0.436)	(0.768)	(0.456)	(0.844)
Lagged sales	-0.898***	-1.031***	-0.964***	-0.897***
	(0.00739)	(0.00999)	(0.00741)	(0.00974)
Lagged sales $\times$ $\hat{\pi}^{\text{input}}$	0.310***	0.245***	0.285***	0.525***
	(0.0447)	(0.0858)	(0.0480)	(0.0930)
Financing DI	0.00401***	0.00486***	0.00445***	0.00509***
	(0.000301)	(0.000398)	(0.000303)	(0.000384)
Industry RS	-0.0342***	-0.0334***	-0.0291***	-0.0267***
	(0.00715)	(0.00912)	(0.00702)	(0.00953)
Lagged leverage	0.0633***	0.143***	0.0925***	0.103***
	(0.00573)	(0.00924)	(0.00592)	(0.00910)
Constant	8.116***	8.747***	8.412***	7.856***
	(0.105)	(0.134)	(0.103)	(0.138)
Year/Firm FE	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	93932	95515	140371	49076
Num. firms	11981	11570	16275	6429
R^2				
within	0.180	0.135	0.140	0.204
overall	0.00119	0.00281	0.00154	0.00408
F	747.1	547.9	843.7	455.5
F_f	4.778	3.200	3.640	4.281

Notes: Standard errors are in the parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$). F_f represents the F test statistics with the null hypothesis that all firm fixed effects are zero.
Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" the Bank of Japan, "Producer Price Index;" the Bank of Japan, "Tankan," etc.

Table A.5: Alternative Trend Inflation

	1 year		5 years	
	(1)	(2)	(3)	(4)
	OLS	2SLS	OLS	2SLS
Dep. var.	Sales growth	Sales growth	Sales growth	Sales growth
$\hat{\bar{\pi}}^{\text{input}}$ ($\bar{\pi}^{\text{input}}$ for OLS)	-0.160	-0.308	-5.953***	-6.011***
	(0.241)	(0.318)	(0.550)	(0.599)
Lagged sales	-0.501***	-0.501***	-1.224***	-1.224***
	(0.00668)	(0.00668)	(0.00797)	(0.00798)
Lagged sales × $\hat{\bar{\pi}}^{\text{input}}$ ($\bar{\pi}^{\text{input}}$)	0.0248	0.0619*	0.532***	0.602***
	(0.0261)	(0.0335)	(0.0603)	(0.0651)
Financing DI	0.00363***	0.00359***	0.00443***	0.00431***
	(0.000283)	(0.000284)	(0.000313)	(0.000314)
Industry RS	-0.0190***	-0.0175***	-0.0369***	-0.0310***
	(0.00641)	(0.00644)	(0.00767)	(0.00772)
Lagged leverage	0.114***	0.113***	0.0833***	0.0839***
	(0.00530)	(0.00530)	(0.00635)	(0.00635)
Constant	4.206***	4.192***	10.79***	10.72***
	(0.0931)	(0.0934)	(0.112)	(0.113)
Year/Firm FE	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	248474	248474	145635	145635
Num. firms	23632	23632	16412	16412
R^2				
within	0.0283	0.0284	0.185	0.184
overall	0.000906	0.000905	0.00167	0.00164
F	252.1	252.5	1331.3	1327.4
F_f	2.973	2.973	4.401	4.389

Notes: Standard errors are in the parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$). F_f represents the F test statistics with the null hypothesis that all firm fixed effects are zero.
Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" the Bank of Japan, "Producer Price Index;" the Bank of Japan, "Tankan," etc.

Table A.6: Wage Inflation

	OLS	2SLS		
	(1)	(2)	(3)	(4)
Dep. var	Sales growth	Sales growth	Sales growth	Sales growth
IV		Int. com. price	Unemployment	ΔJob application
Wage inflation	0.216***	-5.524***	-5.167***	-5.947***
	(0.0583)	(0.236)	(0.225)	(0.216)
Lagged sales	-0.763***	-0.716***	-0.733***	-0.673***
	(0.00534)	(0.0160)	(0.0149)	(0.0142)
Lagged sales $\times$ Wage inflation	-0.0239***	0.594***	0.575***	0.582***
	(0.00653)	(0.0171)	(0.0171)	(0.0170)
Financing DI	0.00453***	0.00549***	0.00513***	0.00622***
	(0.000205)	(0.000330)	(0.000314)	(0.000305)
Industry RS	-0.0257***	-0.0206***	-0.0216***	-0.0157***
	(0.00490)	(0.00518)	(0.00514)	(0.00511)
Lagged leverage	0.0711***	0.0774***	0.0752***	0.0831***
	(0.00422)	(0.00463)	(0.00456)	(0.00453)
Constant	6.702***	6.110***	6.285***	5.621***
	(0.0727)	(0.184)	(0.172)	(0.164)
Year/Firm FE	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	158898	158898	158898	158898
Num. firms	17721	17721	17721	17721
R^2				
within	0.161	0.168	0.167	0.168
overall	0.00127	0.00181	0.00181	0.00175
F	1173.9	1235.6	1232.0	1235.4
F_f	3.339	3.353	3.345	3.361

Notes: Standard errors are in the parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$). F_f represents the F test statistics with the null hypothesis that all firm fixed effects are zero.
Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" the Bank of Japan, "Producer Price Index;" the Bank of Japan, "Tankan," etc.

Table A.7: Non-manufacturing Sector

	OLS	2SLS		
	(1)	(2)	(3)	(4)
Dep. var	Sales growth	Sales growth	Sales growth	Sales growth
IV		Int. com. price	Unemployment	ΔJob application
Wage inflation	0.130**	-1.378***	-1.311***	-1.462***
	(0.0560)	(0.168)	(0.166)	(0.168)
Lagged sales	-0.679***	-0.693***	-0.695***	-0.683***
	(0.00429)	(0.00831)	(0.00809)	(0.00834)
Lagged sales × Wage inflation	-0.0147**	0.181***	0.178***	0.175***
	(0.00618)	(0.0158)	(0.0158)	(0.0158)
Financing DI	0.00275***	0.00273***	0.00273***	0.00278***
	(0.000164)	(0.000166)	(0.000166)	(0.000166)
Industry RS	0.000743	0.00696	0.00795*	0.00338
	(0.00338)	(0.00424)	(0.00418)	(0.00425)
Lagged leverage	0.0435***	0.0429***	0.0428***	0.0435***
	(0.00291)	(0.00294)	(0.00294)	(0.00294)
Constant	6.066***	6.085***	6.098***	6.046***
	(0.0724)	(0.0778)	(0.0774)	(0.0779)
Year/Firm FE	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	129325	129325	129325	129325
Num. firms	17603	17603	17603	17603
R^2				
within	0.216	0.217	0.217	0.217
overall	0.000811	0.000894	0.000897	0.000879
F	1337.2	1344.3	1344.2	1343.7
F_f	6.522	6.533	6.532	6.532

Notes: Standard errors are in the parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$). F_f represents the F test statistics with the null hypothesis that all firm fixed effects are zero.
Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" the Bank of Japan, "Producer Price Index;" the Bank of Japan, "Tankan," etc.

Table A.8: Strict Definition of Exit

Dep. var.	(1) 3 years Exit flag	(2) 6 years Exit flag	(3) 9 years Exit flag
$\hat{\pi}^{\text{input}}$	0.501***	0.828***	0.493**
	(0.105)	(0.121)	(0.217)
Lagged sales	-0.0230***	-0.0228***	-0.0222***
	(0.00180)	(0.00222)	(0.00280)
Lagged sales × $\hat{\pi}^{\text{input}}$	-0.0532***	-0.0925***	-0.0507**
	(0.0112)	(0.0129)	(0.0236)
Financing DI	-0.0000908	-0.0000995	-0.0000618
	(0.0000686)	(0.0000756)	(0.0000872)
Industry RS	-0.00178	-0.00275	-0.00488**
	(0.00166)	(0.00195)	(0.00244)
Lagged leverage	-0.00573***	-0.00793***	-0.00888***
	(0.00145)	(0.00177)	(0.00226)
Constant	0.196***	0.218***	0.249***
	(0.0245)	(0.0291)	(0.0361)
Year/Firm FE	yes/yes	yes/yes	yes/yes
Obs.	154967	125311	95823
Num. firms	17906	16755	15403
R^2			
within	0.0278	0.0298	0.0292
overall	0.00341	0.00449	0.00482
F	186.4	184.9	161.2
F_f	3.958	3.763	3.683

Notes: Standard errors are in the parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$). F_f represents the F test statistics with the null hypothesis that all firm fixed effects are zero.
Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" the Bank of Japan, "Producer Price Index;" the Bank of Japan, "Tankan," etc.

Table A.9: Industry-level Estimation at the First Stage

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Sales growth	Emp. growth	Real R&D	$\frac{\text{R\&D}}{\text{Sales}}$	$\log\frac{\text{R\&D}}{\text{Wage}}$	$\log\frac{\text{R\&D}}{\text{Total Salary}}$	Exit
$\hat{\pi}^{\text{input}}$	-2.510***	-1.781***	-1.624***	-0.0839***	-4.767***	-1.260***	0.422***
	(0.387)	(0.155)	(0.404)	(0.0271)	(0.865)	(0.103)	(0.106)
Lagged sales	-0.884***	-0.141***	0.247***	0.000658	0.434***	0.0236***	-0.0217***
	(0.00588)	(0.00235)	(0.00645)	(0.000433)	(0.0138)	(0.00165)	(0.00167)
Lagged sales × $\hat{\pi}^{\text{input}}$	0.296***	0.192***	0.133***	0.00805***	0.440***	0.126***	-0.0468***
	(0.0412)	(0.0165)	(0.0387)	(0.00260)	(0.0830)	(0.00992)	(0.0113)
Financing DI	0.00476***	0.00281***	-0.000502**	-0.0000707***	0.00199***	-0.000517***	-0.0000695
	(0.000242)	(0.0000967)	(0.000257)	(0.0000173)	(0.000550)	(0.0000658)	(0.0000669)
Industry RS	-0.0284***	-0.00106	0.0645***	0.00126***	0.0358***	0.00503***	-0.00200
	(0.00568)	(0.00227)	(0.00622)	(0.000418)	(0.0133)	(0.00159)	(0.00158)
Lagged leverage	0.102***	0.00562***	-0.0147***	-0.000175	-0.0271**	-0.00338***	-0.00569***
	(0.00471)	(0.00188)	(0.00504)	(0.000339)	(0.0108)	(0.00129)	(0.00134)
Constant	7.720***	1.234***	-2.101***	-0.00469	-2.047***	-0.175***	0.184***
	(0.0823)	(0.0329)	(0.0911)	(0.00612)	(0.195)	(0.0233)	(0.0231)
Year/Firm FE	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes	yes/yes
Obs.	189447	189447	58445	58445	58418	58418	175463
Num. firms	19703	19703	14474	14474	14473	14473	18865
R^2							
within	0.139	0.0382	0.0637	0.00848	0.0538	0.0466	0.0284
overall	0.00179	0.000137	0.653	0.0390	0.617	0.261	0.00351
F	1142.7	280.7	124.5	15.66	104.0	89.41	199.2
F_f	3.901	3.333	24.37	12.15	16.03	11.53	3.940

Notes: Standard errors are in the parentheses (* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$). F_f represents the F test statistics with the null hypothesis that all firm fixed effects are zero.
Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" the Bank of Japan, "Producer Price Index;" the Bank of Japan, "Tankan," etc.

Table A.10: Moments: Data, Benchmark Model, and Modified Models for Japan

Target moments				
	Data	Benchmark model	$\omega = 2$	$\sigma = 1.5$
g	0.017	0.017	0.017	0.018
R	0.034	0.034	0.034	0.034
Δ	1.421	1.444	1.422	1.573
Med$[WL]$/E$[WL]$	0.260	0.322	0.322	0.381
Std$[WL]$/E$[WL]$	5.527	5.890	5.898	5.445
Med$[Y]$/E$[Y]$	0.181	0.317	0.317	0.371
Std$[Y]$/E$[Y]$	7.055	5.789	5.795	5.357
Cor$[Y/L,\, L]$	0.186	-0.100	-0.102	-0.084
Cor$[Y,\, WL]$	0.740	1.000	1.000	1.000
E$[dY/Y]$	0.071	0.061	0.061	0.059
Std$[dY/Y]$	1.246	0.182	0.182	0.180
Cor$[dY/Y,\, Y]$	-0.003	-0.100	-0.102	-0.084

Sources: METI, "the Basic Survey of Japanese Business Structure and Activities;" etc.

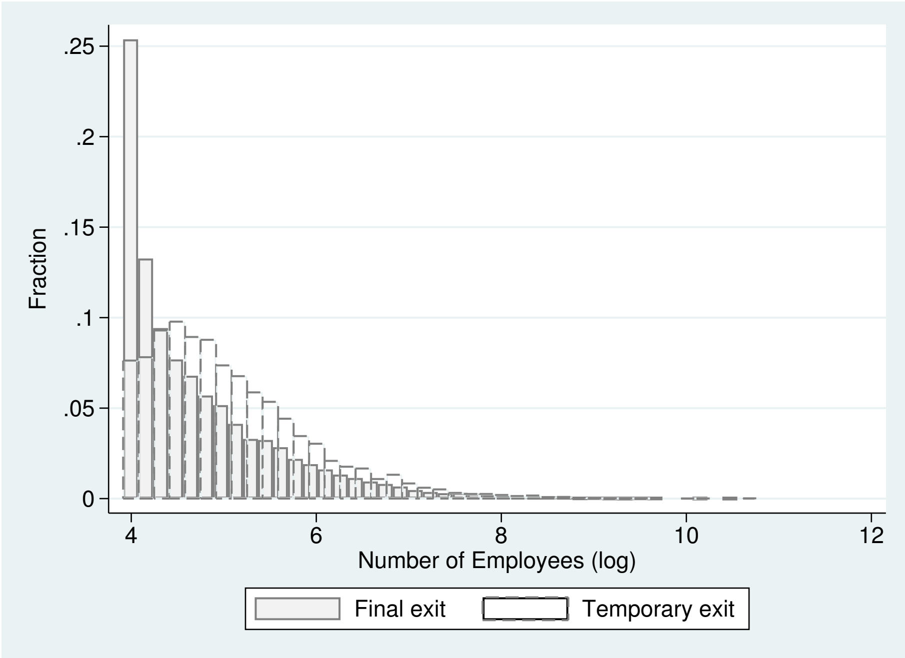

Figure A.1: Employment Distributions for Final Exit and Temporary Exit Firms
Source: METI, "the Basic Survey of Japanese Business Structure and Activities."

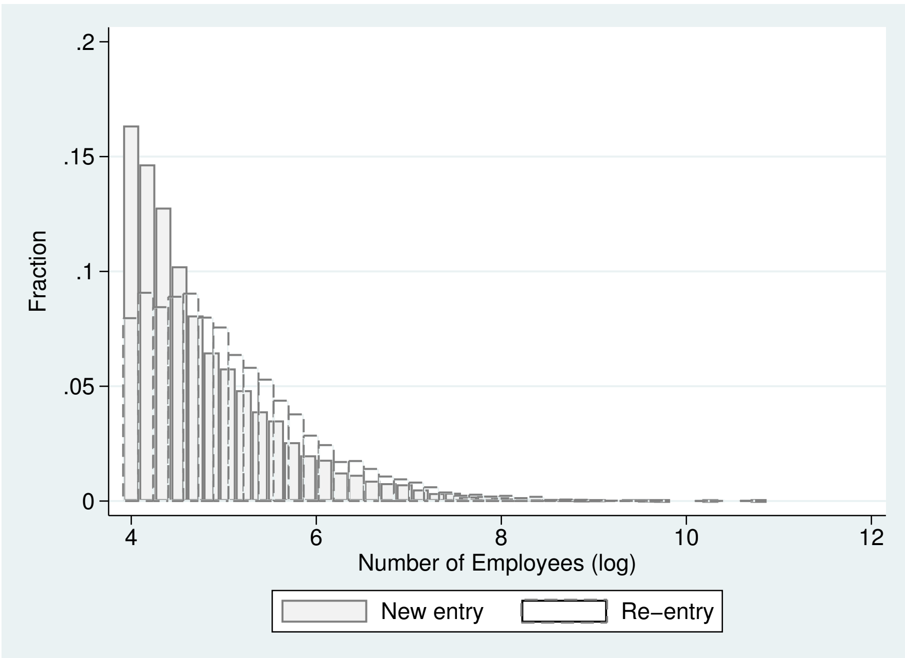

Figure A.2: Employment Distributions for New Entry and Re-entry Firms
Source: METI, "the Basic Survey of Japanese Business Structure and Activities."

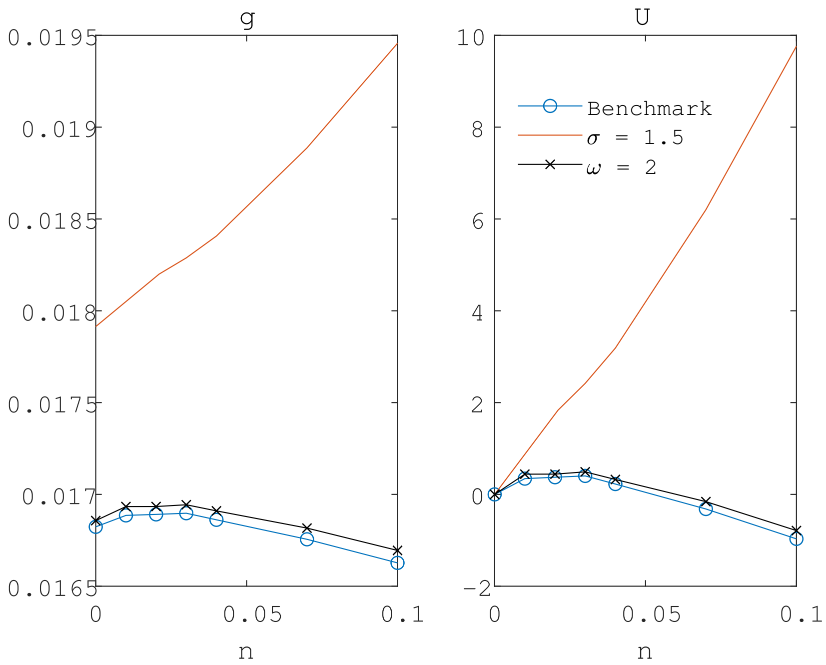

Figure A.3: Effects of Nominal Growth in Modified Models

Note: Intertemporal utility U is normalized at zero when $n = 0$.