

RIETI Discussion Paper Series 18-E-055

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The Research Institute of Economy, Trade and Industry https://www.rieti.go.jp/en/

August 2018

Analysis of Inflation/Deflation: Clusters of micro prices matter!*

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Abstract

Micro price data show that individual price settings are not time-invariant. Furthermore, the analysis of autocorrelations shows that interactions of micro prices with leads and lags play a significant role in explaining the behavior of aggregate price index. We present a new method of extracting information on the nature of such interactions of micro prices. For Japan's data, we identify two macro shocks—one external and the other domestic—to drive dynamics of prices, but find that irrespective of the sources of shocks, there exists a robust flow of changes of domestic prices from upstream to downstream. Prices change in clusters. We identify such clusters. Our analysis suggests that inertia arising from input/output relationships in production explains the behavior of aggregate prices.

Keywords: CPI, Sticky prices, Cluster, PCA JEL classification: E31, D12, C40

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^{*}The authors would like to thank Yuichi Ikeda, Makoto Nirei, Wataru Souma, Kenichi Ueda, Tsutomu Watanabe, and Yoshihiro Yajima for their encouraging and helpful comments and discussions. We are also grateful to the participants of seminars at the Bank of Japan, University of Tokyo, and RIETI for their useful comments. This study was supported in part by the project "Large-scale Simulation and Analysis of Economic Network for Macro Prudential Policy" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). Also it was supported in part by MEXT as Exploratory Challenges on Post-K computer (Studies of Multi-level Spatiotemporal Simulation of Socioeconomic Phenomena, Macroeconomic Simulations), JSPS KAKENHI Grant Numbers 25400393, 17H02041, and the Kyoto University Supporting Program for Interaction-based Initiative Team Studies: SPIRITS, as part of the Program for Promoting the Enhancement of Research Universities, MEXT, JAPAN.

How prices behave is of primary importance in economics. In macroeconomics, inflation, together with unemployment, is one of the most important policy issues. More recently, deflation is regarded as threat to the macroeconomy. In the late 1990's, Japan was first of all the advanced countries trapped into deflation, and amid deflation faced the zero interest bound. Krugman (1998), in place of the traditional interest policy, advanced the quantitative easing (QE) coupled with the firmly committed inflation target as a possible remedy. Many others have elaborated on the idea. Today, most central banks are indeed committed to explicit inflation target such as the annual two percent increase of consumer price index (CPI). Facing the zero interest bound, they struggle against deflation by resorting to unconventional monetary policy such as quantitative easing, forward guidance, and negative interest rate. The efficacy of such policy with respect to control of price depends, of course, on how prices are determined.

In macroeconomic theory, prices are said to be "sticky". In fact, in the modern DSGE (dynamic stochastic general equilibrium) models, monetary policy is effective to the extent that prices are sticky. There are a number of theories which attempt to explain sticky prices: the Taylor–Calvo model of desynchronized staggered wage/price changes (Calvo, 1983) and menu cost models (Mankiw, 1985), just to name a few. Based on such micro-foundations, the standard framework for understanding the role of monetary policy is the New Keynesian Phillips curve (NKPC).

The key property of the NKPC is that inflation is primarily a forward-looking process. That is, expectations on future inflation largely determine current inflation. This justifies recent emphasis on expectations management and communications as tools of monetary policy. There is a great amount of literature on the NKPC. However, after a long survey of the literature, Mavroeidis et al. (2014) reached quite disappointing conclusion. Namely, their major finding is that estimation of the NKPC using macro data is subject to a severe weak instruments problem. Indeed, they find that "the evidence is consistent both with the view that expectations matter a lot, as well as with the opposite view that they matter very little". They thus conclude that identification of the NKPC is too weak to warrant research on conceptually minor extensions. The traditional analysis based on macro data has its clear limitations.

Meanwhile, recent empirical works on micro price-setting as surveyed by Klenow and Malin (2011) have uncovered hitherto little known dynamics of micro prices. Bils and Klenow (2004), for example, by examining the frequency of price changes for 350 categories of goods and services demonstrate that half of prices last 5.5 months or less. Their findings seem to suggest that individual prices are actually not rigid. There are substantial differences across goods, however; prices of raw materials and foodstuff are flexible while those of services less flexible. The fact is well known. Long time ago, Kalecki (1954) proposed a two-sector model of prices: Prices of raw materials and foodstuff are determined flexibly by market forces of supply and demand whereas prices of most manufactured products and

services are determined by suppliers based on their production costs. Today, central banks are committed to inflation targeting with respect to the "core" CPI which excludes prices of foodstuff and energy.

Studies of micro prices provide useful information. However, changes of aggregate price index are entirely different matter from changes of individual prices. For changes of aggregate price index such as CPI, the frequency of individual price changes and synchronization on which many empirical works focus provides only partial information. The reason is that deflation and inflation are nothing but changes in the aggregate price *over time* while the existing theoretical literature on micro prices focuses mostly on *cross-sectional* differences among micro prices. There still remains much to be done.

The purpose of this paper is to fill this gap. First, the existing literature on micro price dynamics either explicitly or implicitly assumes that frequency and synchronization of micro price changes are independent of each other and are time-invariant. More generally, probability distribution of micro price changes is often assumed to be given and time-invariant. Alternative theories are proposed to account for such given distribution (Golosov and Lucas (2007), Midrigan (2011)). However, distribution of micro price changes is actually not time-invariant. Using Norwegian micro price data (1975–2004), Wulfsberg (2016) demonstrates that prices change more frequently in high inflation period than in low inflation period. Our study of Japanese data for the period of the 2008 financial crisis shows that the pattern can drastically change even within much shorter period, that is, only a few months. Note that it is precisely this change of probability distribution of micro price index.

It is also important to recognize that prices of individual goods and services affect each other *with leads and lags*. In Section II, we formally demonstrate this fact by way of the analysis of autocorrelations of prices. Therefore, it is essential to analyze dynamics of micro prices taking explicitly account of these lead and lag relationships. The present paper precisely does it.

Secondly, individual prices occasionally change simultaneously responding to certain macro shocks. Despite of our primary interest in macroeconomics and monetary policy, the existing literature does not *empirically* link the findings on micro price behavior to changes in macroeconomic variables. In some papers such as Golosov and Lucas (2007) and Midrigan (2011), money is explicitly introduced, but it is simply assumed that money stock must directly affect all the micro prices. Here, theory is ahead of hard empirical evidence. The analysis of micro price dynamics should be able to provide useful *empirical* information.

The most important point we demonstrate is that *micro prices change in clusters.* Thus, for exploring dynamics of micro prices, it is not enough to talk about just micro prices, but we must look at *clusters* of micro prices. The analysis sheds light on the central question for macroeconomics and monetary policy, namely the relative importance of expectations and inertia as determinant of aggregate price. In section I, studying prices of 940 goods and services for Japan, we demonstrate that the frequency of individual price changes and synchronization are not constant but time-varying. As stated above, the existing theoretical literature routinely assumes that distribution of micro price changes is constant. This assumption is simply not borne out by data. Frequency, synchronization, and size of price changes are all time-varying.

In section II, we study the autocorrelations of individual (intermediate) CPI and the aggregate CPI. This analysis demonstrates the importance of interdependence of prices with leads and lags. To uncover such dynamics of micro prices with leads and lags and explore what macro-variables are major driving forces, Section III resorts to a new method called the Complex Hilbert Principal Component Analysis (CHPCA). Our analysis identifies two driving forces, one external and the other domestic, but at the same time shows that irrespective of the nature of shock, there exists a robust propagation mechanism of domestic prices. Section IV then demonstrates that domestic prices change in clusters. Section V offers concluding remarks.

I. The Behavior of Prices

Beginning the late 1990's, Japan experienced more than a decade long notorious deflation. Amid deflation, the Bank of Japan (BoJ) faced the zero interest bound. The U.S. Federal Reserve and the European Central Bank eventually followed the suit. The world paid much attention to this phenomenon; Even the word "Japanization" was coined.

At the early stage, most macroeconomists believed that sizable quantitative easing should turn deflation to mild deflation (Krugman (1998)). However, things turned out to be not so easy. For example, the BoJ has pursued unprecedentedly sizable QE by increasing monetary base from 40 trillion yen as of December, 2012 to 400 trillion yen by the end of 2017. And yet, the rate of the change of consumer price remains only 0.4 percent, falling short of the target rate of 2%. In the U.S., the Federal Reserve chairman Janet Yellem regards "low price" as a kind of puzzle. Clearly, further analysis of price is needed.

Fig. 1 shows three aggregate prices, namely, import price, producer price, and consumer price of Japan for three decades. Plainly, import price is much more volatile than both producer and consumer prices; Note that changes of import price are measured on the right axis with different scale. In fact, import price often declined substantially. For example, in the fall of 1985, the Plaza Agreement was reached, and the yen started appreciating from 240 yen per dollar to 120 yen per dollar. As a consequence, the import price in terms of yen fell to a half. In parallel with it, the producer price also declined by 7%. The decline of oil price also affected Japan's import price substantially as observed for 2009 and 2016. However, we are interested mainly in consumer price simply because it is the target for central bank's monetary policy.

We begin by examining the Japanese monthly data of the three categories



FIGURE 1. OVERVIEW OF THE AGGREGATE PRICES

Note: Time-series of monthly price indices (PI) for Import PI (blue and right-axis), Producer PI (red), and Consumer PI (green) from 1980 up to present. Dashed vertical lines correspond to the three months in which VAT was raised, namely, April of 1989, 1997 and 2014 (VAT 3%, 5%, 8% respectively), and September 2008 in which the Lehman Brothers went into bankruptcy.

of individual prices for the period from January 1985 to December 2016 (384 months). The data we have collected are made of the following three categories:

- IPI: Import Price Index, compiled by the Bank of Japan (BoJ) consists of "prices of \cdots imports at the stage of entry into Japan." It covers 84 goods (Bank of Japan, 2017).
- PPI: Producer Price Index, compiled by the BoJ, "surveys the prices of goods traded among companies, specifically domestically produced goods for domestic markets, mainly at the stage of shipment from producers and partly from wholesalers." It covers 490 goods (Bank of Japan, 2017).
- CPI: Consumer Price Index, compiled by the Statistics Bureau of the Ministry of Internal Affairs and Communications covers 366 consumption goods and services (Statistics Bureau, 2017).¹

All together, this data set of 940 time series of length 384 (months) covers a wide range of goods and services from raw materials such as crude oil to consumables.²

¹The number of the data in each categories are for individual prices only and does not include any aggregated prices.

 $^{^{2}}$ The number of goods and services of all three indices have been gradually increasing since 1980, reflecting the appearance of new products or services in the market. We use only those that are listed every month during the whole period from January 1980 to June 2013 for consistency of our analysis.

For 940 individual prices, we now denote them by $p_{\alpha}(t)$ where $\alpha = 1, 2, \dots, 940$ (:= N) is the kind of goods and services, and $t = 1, 2, \dots, 384$ stands for the month during the period from January, 1985 to December, 2016. We examine monthly changes of the individual price³ defined by

(1)
$$r_{\alpha}(t) := \log_{10} \left[\frac{p_{\alpha}(t+1)}{p_{\alpha}(t)} \right].$$

Heterogeneity of micro prices found in the existing literature can be easily confirmed for the Japanese data we analyze. Table 1 shows the mean duration d (in months) of the period during which individual prices remains unchanged for 39 groups of goods and services. The table also shows λ , the monthly frequency or the probability that the price changes within a month (not directly observed). If one assumes that the prices can change at any instance of time with the constant probability, a simple Poisson process leads to the result that d is equal to $-1/\ln(1-\lambda)$. Given d, the values for λ in the table are estimated by this formula⁴.

The mean duration varies from 10 months for business machinery and transportation equipment to one month for food, cloths and most imported goods and materials. In between is 6 months for chemicals in PPI and services in CPI. On the whole, prices of imported goods and materials are very flexible. They are broadly consistent with the results obtained in previous works on prices of other countries (Klenow and Malin, 2011).

³The individual prices we use are not seasonally adjusted, because only a limited number of them such as clothing and vegetable shows seasonality (Statistics Japan, 2014). Applying seasonal adjustment on some selected individual prices while not doing so for others necessarily brings in some ad-hoc assumptions. They are not desirable for our analysis. Using the year-to-year rate of change is an alternative to seasonal adjustment and has an advantage of being free from ad-hoc assumptions. It, however, has a severe disadvantage of having a year-long aftereffect from a big change, such as the introduction and raise of the consumption tax, and therefore are not adopted in our analysis.

⁴Assume that the price changes according to a homogeneous Poisson process with parameter θ , namely a constant probability of change at any instance of time. For a realization of *n* changes of the price at times $0 \equiv t_0 < t_1 < t_2 < \cdots < t_n \equiv T$, the likelihood function is given by $L = \theta^n \exp(-\theta T)$, because the inter-occurrence times $T_k = t_k - t_{k-1}$ ($k = 1, 2, \ldots, n$) are independent and identically distributed by an exponential distribution with parameter θ . The maximum likelihood estimate is then obtained by $\theta = n/T = 1/d$. On the other hand, the probability that the price changes in a month, λ , is related to the parameter θ by $\lambda = 1 - e^{-\theta}$ as easily shown. It therefore follows that $d = -1/\ln(1 - \lambda)$. See Basawa and Prakasa Rao (1980, Chap.6.2) for example.

ID	CLASSIFICATION OF SECTOR	#Goods	Months	Freq
	IPI — Import PI			
01	Foodstuffs & feedstuffs	20	1.02	62.34
02	Textiles	7	1.17	57.56
03	Metals & related products	20	1.06	61.08
04	Wood, lumber & related products	3	1.01	62.73
05	Petroleum, coal & natural gas	8	1.03	62.16
06	Chemicals & related products	10	1.41	50.69
07	General purpose, production & business oriented machinery	2	1.11	59.50
08	Electric & electronic products	3	1.09	59.90
09	Other primary products & manufactured goods	11	1.11	59.32
	ALL	84	1.11	59.48
	PPI — Producer PI			
01	Food, beverages, tobacco & feedstuffs	83	2.64	31.56
02	Textile products	20	7.79	12.05
03	Lumber & wood products	11	3.37	25.67
04	Pulp, paper & related products	25	3.13	27.34
05	Chemicals & related products	<u>-</u> 0 64	5.82	15.78
06	Petroleum & coal products	11	1.92	40.56
07	Plastic products	14	3.61	24.19
08	Ceramic stone & clay products	30	4 21	21.10
09	Iron & steel	27	4 17	21.10
10	Nonferrous metals	21	1.17	49.38
11	Metal products	21	4.07	21 78
12	General nurnose machinery	21	4.07	18.38
12	Production machinery	21 91	3.16	27.12
14	Business oriented machinery	14	6.21	1/ 88
15	Electronic components & devices	8	2 30	35.31
16	Electrical machinery & equipment	25	3.37	25.68
17	Information & communications equipment	20	2.56	20.00
18	Transportation equipment	11	9.20	10.21
10	Other manufacturing industry products	25	7.25	12.85
20	Agriculture forestry & fishery products	18	4.76	18.07
20	Minorals	2	4.70	0.56
21	Floetric power, gas le water	3	8.03	10.60
22	Seran ly waste	5	1 11	50.50
		490	4 31	20.70
	CPI — Consumer PI	450	4.01	20.10
01	End End	151	1 99	52.08
01	Food	101	1.55	02.90 44.19
02	Fuel Light & Water Charges	15	2.20	25.40
03	Fueri, Light & Water Onarges	0 99	2.29	56 10
04	Clothes & Footware	50 20	1.21	46.04
00	Medical Care	14	1.58	40.94 22.15
00	Transportation & Communication	14	2.48 19 74	5 20
01	Figure 1 and a Communication	23	18.14	0.20
08	Culture & Decreation	11	9.90	9.00
10	Vulture & Recreation	33 04	0.22	14.80
10	Miscellaneous	24	3.84	22.94
	ALL	300	3.39	$_{20.00}$

TABLE 1—PROPERTIES OF MICRO PRICES

Note: List of IDs, classification of sectors, the numbers of goods, the durations and frequencies of price changes for the commodities of IPI, PPI and CPI. The sectors for IPI and PPI correspond to major groups based on the BoJ datasets. Those for CPI are classified by the authors partially based on the original classification and identities. Months is the mean duration between price changes, denoted by d. Freq is the constant monthly frequency of price changes or probability (in percent) that the price changes in a month, λ , estimated from d based on a simple assumption of Poisson process, i.e., by $d = -1/\ln(1-\lambda)$.

Micro prices of individual goods and services have different volatilities. They must reflect differences in industrial organization and the nature of goods and services. Prices of imported oil and other materials are globally determined in well-organized auction markets, where speculation plays important role. Import prices are also affected by changes of the exchange rate. In contrast, many prices of manufactured goods and services are determined by price makers. To take into account these differences in volatility, in what follows, we consider the *normalized* price change. Denoting by $\langle r_{\alpha} \rangle_t$ and σ_{α} the sample average and standard deviation of the time-series $r_{\alpha}(t)$, respectively, we define the normalized time-series by

(2)
$$w_{\alpha}(t) := \frac{r_{\alpha}(t) - \langle r_{\alpha} \rangle_{t}}{\sigma_{\alpha}}$$

Seeing is believing. Fig. 2 shows the normalized changes $w_{\alpha}(t)$ defined by Eq.(2) for 940 individual prices of the goods and services during the period from January 1985 to December 2016. In Fig. 2 $|w_{\alpha}(t)| < 1$, namely changes smaller than one standard deviation, are shown as blank space. Blue and red colors of each point indicate significant positive and negative changes, namely $w_{\alpha}(t) > 1$ and $w_{\alpha}(t) < -1$, respectively. The portions of "IPI", "PPI" and "CPI" indicated on the vertical axis in Fig.2 correspond, respectively, to 366 goods and services for CPI 490 goods for PPI, and 84 goods comprising IPI. These individual points are grouped into the sectors that they belong to. The lists of sectors for IPI, PPI, CPI are summarized in Table 1.

Fig. 2 demonstrates that the simultaneous changes of individual prices or the synchronization occasionally occur without any clear periodicity. The April 1989, the April 1997 and the April 2014 are three examples of extreme synchronization as indicated by thin vertical lines with labels "VAT1", "VAT2" and "VAT3" in Fig. 2. In Japan, the three percent value added tax (VAT) called the consumption tax was introduced in April 1989, and the tax rate was raised from three to five percent in April 1997, and it was further raised to eight percent in April 2014. Almost all the prices were raised then. Note, however, that individual prices were not mechanically raised by three, two and three percent, respectively.

It is important to recognize that synchronization is observed not only for these VAT hikes but for other occasions as well. The most notable are the post Plaza agreement period of 1985-86, and the Great Recession following the bankruptcy of the Lehman Brothers in 2008. We will take up these two periods as major episodes in subsequent analysis.

Fig. 2 demonstrates the several important points. First, the *average* frequency of price change of individual goods or service provides only a very limited information on the behavior of aggregate price because changes of micro prices are not time homogeneous. And it is precisely this change of probability distribution of micro prices that determines dynamics of *aggregate* price. The year 2008 is a good example. In the first half of the year, many prices were rising, but the



FIGURE 2. NORMALIZED PRICE CHANGES $w_{\alpha}(t)$

Note: Blue circles and red triangles denote $w_{\alpha}(t) > 0.5$ and $w_{\alpha}(t) < -0.5$, respectively, with their linear size proportional to $w_{\alpha}(t)$. Blank areas correspond to no "significant" change in this aspect. Five thin vertical lines show months when Plaza Agreement was reached by "Plaza", three VAT changes to x% by "VAT (x%)" and the Bankruptcy of Lehman Brothers (sub-prime mortgage crisis) by "Lehman". Subcategories listed in Table 1 with more than or equal to 10 items are shown by numbers.

bankruptcy of the Lehman Brothers turned the tide, and afterwords, many prices started declining, some significantly. In other times, most prices simply remained unchanged for a long time period.

Secondly, in order to fully understand the behavior of aggregate price, we must explicitly consider subsets or clusters of prices, not just a single macro-group of prices, and also interactions of clusters of prices. For example, look at Fig.2 for the period during 1995-2000 vertically. Except for April,1997 when VAT was raised, prices of some goods went up or down in clusters while others remained unchanged.

In most theoretical models, an individual firm is assumed to strategically set or reset its price considering the behaviors of all the other firms. It is commonly assumed that firm j is interested in P_j/P where P_j is the firm j's own price and P is the aggregate price index. In other words, it is routinely assumed that the universe in which each firm optimizes is the economy as a whole. However, the behavior of individual prices shown in Fig. 2 does not support this presumption; it shows that there is a significant tendency that a cluster of prices change together while at the same time prices which belong to other clusters do not. In Section IV, we will explicitly identify such clusters. The standard theoretical model takes the macroeconomy as if it were a single industry or a group of retailers in a region. Such a model may serve for the purpose of industrial organization, but does not fit the purpose of macroeconomics and monetary policy.

II. Inertia and Interactions of Individual Prices

How dynamics of individual prices and the behavior of the aggregate price index relate to each other? In macroeconomics and monetary policy, we are interested mainly in changes of *the aggregate price index* such as CPI. The aggregate price is nothing but weighted average of micro prices. In what follows, we compare autocorrelations of 47 consumer prices comprising CPI with that of the aggregate price index, namely CPI.

For this purpose, we study autocorrelations of the rate of change of price relative to the twelve months earlier. That is, we examine $\pi_{\alpha}(t)$, the rate of change of monthly price p_{α} at time t with 0 mean:

(3)
$$\pi_{\alpha}(t) = \log \frac{p_{\alpha}(t)}{p_{\alpha}(t-12)} - \left\langle \log \frac{p_{\alpha}(t)}{p_{\alpha}(t-12)} \right\rangle_{t},$$

where $\langle \cdot \rangle_t$ means the average over the time t. And we define the following index $\pi(t)$ by taking a weighted average of $\pi_{\alpha}(t)$:

(4)
$$\pi(t) = \sum_{\alpha=1}^{n} g_{\alpha} \pi_{\alpha}(t) \; .$$

where g_{α} is the statistical weight of item α as used in the aggregation for CPI

and $n = 47.^{5}$

Stationarity of the year-to-year change of $\pi(t)$ was confirmed by conducting the Phillips-Perron test with no drift term; its *p*-value takes 0.00219, much smaller than the standard significance level $\alpha = 0.05$. Also the year-to-year changes of 41 prices out of the 47 CPI-constituting prices pass the unit-root test at the same significance level.⁶

The autocorrelations of $\pi(t)$ and individual prices are related by

(5)
$$\langle \pi(t_0)\pi(t_0+t)\rangle_{t_0} = \sum_{\alpha} g_{\alpha}^2 \langle \pi_{\alpha}(t_0)\pi_{\alpha}(t_0+t)\rangle_{t_0}$$
$$+ \sum_{\alpha\neq\beta} g_{\alpha}g_{\beta} \langle \pi_{\alpha}(t_0)\pi_{\beta}(t_0+t)\rangle_{t_0} .$$

The first term in the right-hand side of Eq. (5) represents the autocorrelations of individual prices whereas the second term represents the interactions among individual prices. The autocorrelation functions, $\phi(t)$ and $\phi_{\alpha}(t)$, of $\pi(t)$ and $\pi_{\alpha}(t)$ are then defined by

(6)
$$\phi(t) = \frac{\langle \pi(t_0)\pi(t_0+t)\rangle_{t_0}}{\langle \pi(t_0)^2 \rangle_{t_0}},$$

and

(7)
$$\phi_{\alpha}(t) = \frac{\langle \pi_{\alpha}(t_0)\pi_{\alpha}(t_0+t)\rangle_{t_0}}{\langle \pi_{\alpha}(t_0)^2\rangle_{t_0}},$$

respectively.

Figure 3 shows the autocorrelations of prices of 47 goods and services which comprise CPI. Although the autocorrelations of individual prices considerably differ across goods and services, they share a clearly observed common pattern. Namely, the autocorrelations almost linearly decline up to 12 months, and then flatten afterwords. This pattern is explained by the following random walk model for monthly log $p_{\alpha}(t)$:

(8)
$$\log p_{\alpha}(t) - \log p_{\alpha}(t-1) = \epsilon_t ,$$

where ϵ_t is white noise with 0 mean. In this case, we observe

(9)
$$\pi_{\alpha}(t) = \log p_{\alpha}(t) - \log p_{\alpha}(t-12) - \dots = \epsilon_t + \epsilon_{t-1} + \dots + \epsilon_{t-11} .$$

⁵The rate of change of aggregate price index $\pi(t)$ defined by (3) is different from that of the official CPI because the latter is obtained by aggregating individual prices themselves, not their year-to-year changes. However, the difference between the two is quite small.

⁶The 6 exceptional price data are those designated by ID as 13, 14, 22, 27, 29, and 43 in Table 3.



Figure 3. Autocorrelation functions for individual prices of 47 goods and services comprising CPI.

Note: The mean (dot) of 47 autocorrelation functions $\phi_{\alpha}(t)$'s with the standard deviation of their distribution (error bar) is plotted at every time difference.

The autocorrelation function $\phi_{\alpha}(t)$ for $\pi_{\alpha}(t)$ in the random walk model is then

(10)
$$\phi_{\alpha}(t) = 1 - \frac{t}{12} \ (0 \le t \le 12) \ ,$$

with $\phi_{\alpha}(t) = 0$ beyond t = 12. Figure 3 suggests that monthly individual prices more or less follow random walk, and, therefore, that monthly micro shocks to the *level* of individual price are not transitory but basically permanent.

Figure 4 (a) shows the autocorrelation function of CPI, $\phi(t)$. The autocorrelation of CPI has a very different pattern from those of individual prices. Specifically, it follows an exponential decay.

(11)
$$\phi(t) = \exp(-t/\tau) \; .$$

It means that in contrast to the autocorrelations of individual prices which have comparatively short memory, the aggregate price index contains substantial long memory.

Why has the aggregate price such inertia? For exploring this problem, Fig. 4 (b) shows the weighted average $\phi_{self}(t)$ of autocorrelations of 47 prices defined as

(12)
$$\phi_{\text{self}}(t) = \sum_{\alpha=1}^{47} d_{\alpha} \phi_{\alpha}(t)$$



FIGURE 4. AUTOCORRELATION FUNCTION FOR $\pi(t)$ (A), COMPARED WITH ITS SELF-COMPONENT (B).

Note: The dashed curve in panel A shows an exponential decay form fitted to the numerical results (dots) for $t \leq 6$; its characteristic decay time τ is 15.7 months. The dotted curve in panel A depicts the autocorrelation of CPI exclusive of imputed rent. The dashed line in panel B shows Eq. (10).

Here, the weight d_{α} is given by

(13)
$$d_{\alpha} = \frac{g_{\alpha}^2 \sigma_{\alpha}^2}{\sum_{\alpha=1}^{47} g_{\alpha}^2 \sigma_{\alpha}^2},$$

with its variance σ_{α}^2 .

Figure 4(a) corresponds to the left-hand side of Eq. (5) whereas Fig. 4(b) to the first term on the right-hand side of Eq. (5). In other words, the weighted average of autocorrelations of individual prices shown in Fig. 4(b) excludes the effects arising from interactions of individual prices with leads and lags, the second term in the right-hand side of Eq. (5). To the extent that Figs. 4(a) and 4(b) are significantly different, we must take into account interactions of individual prices with leads and lags for our fully understanding behavior of the aggregate price index.

To test the statistical significance of interdependence of individual prices, we prepare a null model by randomly rotating time-series of individual prices in the time direction with a periodic boundary condition imposed. This randomization procedure destroys only the cross-autocorrelations involved in the original data, leaving the autocorrelations as they are. That is, it is mathematically equivalent to omitting the second term on the right-hand side of Eq. (5) for time-series data. Details of the data shuffling method, referred to as rotational random shuffling (RRS), are given in the supplemental material.



FIGURE 5. TEST OF STATISTICAL SIGNIFICANCE FOR INTERDEPENDENCY OF INDIVIDUAL PRICES.

Note: The autocorrelation function of $\pi(t)$ as shown in Fig. 4a is compared with statistical variations of the corresponding correlation function calculated instead with individual prices which are randomly rotated in the time direction; their median is shown by solid curve; their lower 5%, upper 5%, and upper 1% significance levels, by dotted curves; the number of samples is 100,000. This shuffling provides us with a null hypothesis by destroying cross-correlations among prices with their autocorrelations preserved. The degree of the autocorrelation of $\pi(t)$ for $t \leq 15$ is out of the statistical fluctuations even at the 99% level of confidence.

By repeating the RRS, we generated 100,000 samples to evaluate statistical variations of the autocorrelation function of $\pi(t)$ based on individual prices thus randomized; the statistical fluctuations arise from finiteness of the time-series data. In Fig. 5, their median, lower 5% level, upper 5% level, and upper 1% level are compared with the autocorrelations of $\pi(t)$. We can first confirm that the median agrees well with $\phi_{\text{self}}(t)$ in Fig. 4(b) as it is expected. We then see that the autocorrelations of $\pi(t)$ level in the null model for $t \leq 15$. We thus conclude that interdependency among individual prices is statistically significant even at the 99% level of confidence.

The fact that the autocorrelations of individual prices (the first term of Eq. (5)) are not significant after 12 months means that the menu costs which are to generate autocorrelations of individual prices are not really significant for the purpose of macroeconomics and monetary policy. The present analyses demonstrate that the long-ranged autocorrelations of the *aggregate* price index results mainly from *cross*-autocorrelations of individual prices. Namely, interdependence of individual prices with leads and lags plays an important role in determining the rate of change of aggregate price, namely deflation/inflation. In the next section, we analyze such interactions of prices and their relation to macro variables by new method.

III. Dynamics of Prices and Macroeconomic Variables: Complex Hilbert Principal Component Analysis

In the previous section, we demonstrated that the persistence of the aggregate price index significantly arises from interactions of individual prices. Obviously, for understanding behavior of aggregate price, it is extremely important to explore the nature of (1) interactions of individual prices and (2) their relationships to changes of macro variables. In this section, we resort to new analytical method called Complex Hilbert Principal Component Analysis.

A. Data

We use the Japanese monthly data of the following 80 prices for the period of January 1985 through December 2016:⁷

- Import Price Index (IPI), 10 prices, ID=1–10 in Table 2.
- Producer Price Index (PPI) 2015 base Intermediate classification, excluding consumption tax, Bank of Japan, 23 prices, ID=11–33 in Table 2.
- Consumer Price Index (CPI) 2015 base Intermediate classification, Statistics Bureau of Japan, 47 prices, ID=34–80 in Table 3.

We also use the following seven monthly macroeconomic variables, ID=81-87:

⁷Because the CPI data show jumps when sales tax was imposed (3% in April, 1989) and its rate was raised (from 3% to 5% in April, 1997 and from 5% to 8% in April, 2014), we removed the sales tax effects simply by taking average of the values just before and after the sales tax shocks.

TABLE 2—LIST OF PRICES IN THE PPI AND IPI CATEGORIES

ID	IPI	
1	Foodstuffs & feedstuffs	
2	Textiles	
3	Metals & related products	
4	Wood, lumber & related products	
5	Petroleum, coal & natural gas	
6	Chemicals & related products	
7	General purpose, production & business oriented machinery	
8	Electric & electronic products	
9	Transportation equipment	
10	Other primary products & manufactured goods	
ID	PPI	
11	Food, beverages, tobacco & feedstuffs	
12	Textile products	
13	Lumber & wood products	
14	Pulp, paper & related products	
15	Chemicals & related products	
16	Petroleum & coal products	
17	Plastic products	
18	Ceramic, stone & clay products	
19	Iron & steel	
20	Nonferrous metals	
21	Metal products	
22	General purpose machinery	
23	Production machinery	
24	Business oriented machinery	
25	Electronic components & devices	
26	Electrical machinery & equipment	
27	Information & communications equipment	
28	Transportation equipment	
29	Other manufacturing industry products	
30	Agriculture, forestry & fishery products	
31	Minerals	
32	Electric power, gas & water	

33 Scrap & waste

TABLE 3—LIST OF PRICES IN THE CPI CATEGORY

ID	CPI
34	Cereals
35	Fish & seafood
36	Meats
37	Dairy products & eggs
38	Vegetables & seaweeds
39	Fruits
40	Oils, fats & seasonings
41	Cakes & candies
42	Cooked food
43	Beverages
44	Alcoholic beverages
45	Meals outside the home
46	Rent
47	Repairs & maintenance
48	Electricity
49	Gas
50	Other fuel & light
51	Water & sewerage charges
52	Household durable goods
53	Interior furnishings
54	Bedding
55	Domestic utensils
56	Domestic non-durable goods
57	Domestic services
58	Clothes
59	Shirts, sweaters & underwear
60	Footwear
61	Other clothing
62	Services related to clothing
63	Medicines & health fortification
64	Medical supplies & appliances
65	Medical services
66	Public transportation

- 66 67Private transportation
- 68Communication
- 69School fees
- School textbooks & reference books for study 70
- 71Tutorial fees
- 72Recreational durable goods
- Recreational goods 73
- Books & other reading materials 74
- Recreational services 75
- 76Personal care services
- 77Toilet articles
- 78Personal effects
- 79Tobacco
- 80 Other miscellaneous

- 81. Japanese Yen to US Dollar Exchange Rate (JPY/USD) Tokyo market, monthly average, Bank of Japan.
- 82–84. Index of Business Condition (Leading, Coincident, Lagging) Composite Index 2015 base, outlier processed, Cabinet Office, Government of Japan.
 - 85. Money Stock (M2) Bank of Japan.
 - 86. Monetary Base. Bank of Japan
 - 87. Nominal wage (Contractual cash earnings (Manufacturing)), Health, Labour and Welfare Ministry, Japan.

All together, we have time-series of 87 variables consisting of 80 micro prices and 7 macro variables with the length of 384 months.

B. Complex Hilbert Principal Component Analysis

The interactions and comovements among individual prices and macroeconomic variables can be studied by their correlations. One may think that the ordinary principal component analysis (PCA) or factor analysis, widely used in economics as well as in other disciplines, to uncover "hidden" factors which generate comovements of multi-variables may be appropriate. There is, however, a serious problem because there exist significant *leads and lags* among the variables we analyze. For example, some prices are affected by changes of some other prices, with significant lags. One might still think that PCA with time-shifts can take care of this problem. It can not. For a given pair of time-series, it may work. One can simply shift one of them by *m*-months and calculate the correlation coefficients for several values of *m* to find the value of *m* for which the absolute value of the correlation coefficient is maximized. However, in our case, we have 87*86/2 = 3,741 pairs in our data set. It makes necessary calculations practically impossible.

Complex Hilbert Principal Component Analysis (CHPCA) solves the problem in unified manner. It allows us to do just one calculation for the whole set to extract significant comovements with leads and lags which often span the whole set. This method has been successfully applied to various subjects from meteorology/climatology to signal processing to finance and economics (Gabor (1946); Granger and Hatanaka (1964); Rasmusson et al. (1981); Barnett (1983); Aoyama et al. (2017)). However, the method is still little known in economics. In the following, we shall briefly explain the CHPCA method. CHPCA is made of the following steps.

1. First, we complexify each time series. For this purpose, we decompose it to Fourier components, and then replace $\sin(\omega t)$ by $ie^{-i\omega t}$ and $\cos(\omega t)$ by $e^{-i\omega t}$ in each component. Note that by this operation the original time-series remains as the real part of the complexified time-series. The resulting complex components rotates in the clock-wise direction on its complex plane.

2. Next, we calculate the complex correlation coefficient

(14)
$$\widetilde{C}_{\alpha\beta} := \langle \widetilde{w}_{\alpha} \widetilde{w}_{\beta}^* \rangle_t,$$

where \tilde{w}_{α} is a normalized complex time-series with average equal to zero and standard deviation equal to 1. The complex correlation coefficient $\tilde{C}_{\alpha\beta}$ gives the strength of the correlation between time-series α and β by its absolute value, and the time-delay between them by its phase.

We then obtain the eigenvalues $\lambda^{(n)}$ and the eigenvectors $V^{(n)}$ for $\widetilde{C}_{\alpha\beta}$:

(15)
$$\widetilde{C} V^{(n)} = \lambda^{(n)} V^{(n)}$$

We note that $V^{(n)\dagger} \cdot V^{(m)} = \delta_{nm}$ and $\sum_{n=1}^{N} \lambda^{(n)} = N$ hold where N is the number of the time-series.

3. In order to find significant eigenmodes that represent statistically significant co-movements (signals), we carry out the significance test by Rotational Random Shuffling (RRS) simulation. Even for the ordinary principal component (factor) analysis, one faces difficulty in carrying out significance test for eigenvectors. RRS is a well-established significance test (Iyetomi et al. (2011); Aoyama et al. (2017)). In this test, the null is eigenvalues which are obtained for randomized data. Thus, in this simulation, in order to destroy correlations between time series, each time-series are rotated (with its head and the end joined) randomly and independently, and then the eigenvalues are calculated. By carrying out this for many times, we obtain the distribution of each eigenvalues. Any eigenvalue above the RRS corresponding distribution is identified to be associated with significant comovements.

In what follows, we show our results obtained for data comprising 80 prices and 7 macro variables.

C. Significance test of principal components

First, we compute eigenvalues of the complex correlation matrix C constructed from the price data,⁸ and then carry out significance test for the principal components based on the RRS as null model.

Figure 6 compares the actual eigenvalues of the CHPCA with the results of the RRS simulation with 1000 samples. Here we take the upper limit of the largest eigenvalue predicted by the RRS at 3σ confidence level as a criterion for the significance test. In Figure 6, we observe that the two largest eigenvalues are significant. The eigenvectors associated with those eigenvalues are hence regarded as statistically significant correlations among individual prices.

In what follows, we focus on the first and second eigenmodes. We note that

 $^{^{8}}$ We removed seasonal components involved in the time series of individual prices using the X13-ARIMA-SEATS with the default setting for its options. According to the seasonal adjustment tool, 12 time series among them have no significant seasonal components.



FIGURE 6. EIGENVALUES OBTAINED BY THE CHPCA WITH THE RRS CRITERION.

Note: The points are the actual CHPCA eigenvalues. The solid horizontal line shows the upper limit of the largest eigenvalue predicted by the RRS simulation with 1000 samples at 3σ confidence level while the dashed line shows its average value; the largest and second largest eigenvalues are statistically meaningful.

the basic properties of the eigenvectors do not depend on whether the seasonal adjustment is applied to the time series data or not.

D. Interpretation of the first and second eigenmodes

In Figure 7, the complex components of the first and second eigenvectors are shown. Note that the significant eigenvectors generate dynamics of the group of micro prices as a whole, and thereby aggregate price index. By looking at the components of eigenvectors, we can understand the missing link between micro prices and aggregate price, and also the nature of driving macro-variables.

In Figure 7, the vertical axis measures the absolute value while the horizontal axis measures the phases. The absolute value of each component in significant eigenvector measures to what extent the corresponding price (#1-80) or macro-variable (#81-87) contributes to the eigenmode, namely significant movements of prices as a whole. On the other hand, the phase difference between a pair of components in a significant eigenvector represents lead-lag relationships between the corresponding pair of prices and macro-variables in the eigenmode. Number shown in the figure indicates identification of each price (#1-80) and macro-variables (#81-87).

Prices whose components have large magnitude in the eigenvectors play an important role in their correlation structures. To determine whether prices and macroeconomic variables are statistically significant components in the eigenvectors, we reiterated the CHPCA for the price data to which an auxiliary random



FIGURE 7. THE EIGENVECTORS ASSOCIATED WITH THE LARGEST AND SECOND LARGEST EIGENVALUES

Note: The upper panel plots the complex components of the first eigenvector in a phase-magnitude plane with dotted lines which are the criteria of the auxiliary random variable method to detect significant components at 5% (lower line) and 1% (upper line) significance levels. The lower panel is the same plot as the upper one, but for those of the second eigenvector. According to the present definition of the Fourier transformation, a price changes ahead of (behind) prices on its right-hand (left-hand) side.

time series was added as the 88th component⁹. We then determined the 5% and 1% significance levels as regards magnitude of the eigenvector components by collecting 10,000 samples with different random time series. Although the basic structures of the two eigenvectors are robust against addition of such a random time series, not all of the components are statistically significant. In Fig. 7, two horizontal dotted lines indicate the significance levels, 1% (upper line) and 5% (lower line). We can dismiss components having the absolute value below the 1% significance level. Tables 4 and 5 list the components whose absolute values are above the 1% significance level for the 1st and 2nd eigenvectors, respectively.

The phase difference between prices does not straightforwardly translate into lead-lag relations in real time, because the phase of the complex correlation coefficient is a nonlinear average over the phase (and thus time) difference of Fourier components of the prices. In our particular case, however, we have the business cycle indicators which have well-established lead-lag relations in real time. That is, the leading index leads the coincident index by four months on average, and the coincident index, in turn, leads the lagging index by six months. The difference between the leading and lagging indices is roughly $\pi/3$ to $\pi/2$ in phase while it is ten months in real time. Given this information, we may estimate that phase difference of about π which the components in the eigenvectors span roughly corresponds to 2 years in real time. This estimate of the time scale for the lead-lag relationship among prices is comparable to the characteristic decay time of the autocorrelation of CPI as determined in Section II.

In the first eigenmode, the exchange rate (#81) is by far the most dominant macro-variable leading prices (Fig. 7(a)). The business cycle indicators, the leading (#82), the coincident (#83), and the lagging (#84) indices accompany the exchange rate. Also prices of raw materials and energy sources such as scrap & waste (#33), nonferrous metals (#20), petroleum & coal (#16) and other fuel & light (#50) synchronize with the exchange rate. The remaining PPI prices change with delay, and then finally the CPI prices follow. In short, the exchange rate first affects import prices (#1–10), then producer prices (#11–33), and finally consumer prices (#34–80). It is noted that the absolute values tend to get lower from upstream (IPI) to downstream, (PPI and CPI). In fact, the absolute values of many consumer prices are insignificant. External macroeconomic shocks such as changes of exchange rate and the oil price gradually attenuate in the course of their propagation from upstream to downstream across domestic prices.

 $^9\mathrm{We}$ refer to this significance test as the auxiliary random variable method.

TABLE 4—THE FIRST EIGENVECTOR.

Abs	θ [rad]	Items
1.43	0	82 Index of Business Condition Leading Index
1.41	0.15	33 PPI Scrap & waste
2.07	0.19	3 IPI Metals & related products
2.08	0.25	1 IPI Foodstuffs & feedstuffs
1.80	0.25	81 US Dollar to Japanese Yen Exchange Rate
1.89	0.26	20 PPI Nonferrous metals
2.04	0.29	10 IPI Other primary products & manufactured goods
1.74	0.29	4 IPI Wood, humber & related products
1.81	0.30	7 IPI General purpose, production & business oriented machinery
1.73	0.3	8 IPI Electric & electronic products
1.85	0.32	5 IPI Petroleum, coal & natural gas
1.17	0.36	83 Index of Business Condition Coincident Index
1.88	0.37	6 IPI Chemicals & related products
1.75	0.38	2 IPI Textiles
1.58	0.39	9 IPI Transportation equipment
1.60	0.51	16 PPI Petroleum & coal products
1.17	0.54	67 CPI Private transportation
1.48	0.60	15 PPI Chemicals & related products
1.40	0.73	50 CPI Other fuel & light
1.34	0.76	12 PPI Textile products
1.40	0.87	84 Index of Business Condition Lagging Index
0.83	0.90	13 PPI Lumber & wood products
0.65	0.99	87 Contractual cash earnings (Manufacturing)
1.36	1.14	19 PPI Iron & steel
0.78	1.30	49 CPI Gas
0.84	1.37	78 CPI Personal effects
1.31	1.39	21 PPI Metal products
0.62	1.43	37 CPI Dairy products & eggs
0.97	1.46	36 CPI Meats
1.15	1.50	17 PPI Plastic products
1.04	1.64	11 PPI Food, beverages, tobacco & feedstuffs
0.52	1.76	23 PPI Production machinery
0.74	1.95	58 CPI Clothes
1.11	2.00	56 CPI Domestic non-durable goods
0.70	2.01	26 PPI Electrical machinery & equipment
0.64	2.06	54 CPI Bedding
1.18	2.12	40 CP1 Ohs, fats & seasonings
0.87	2.12	10 UP1 Recreational services
0.03	2.13	22 FF1 General purpose machinery 20 PDI Other menufacturing inductor products
0.94	2.15	29 r r i Other manufacturing industry products
0.01	2.17	41 CPI Calcas & condias
0.79	2.19	41 OF I Cakes & Califies 14 PPI Pulp, paper & related products
0.70	2.20	55 CPI Domostic utonsils
0.79	2.01	45 CPI Mosle outside the home
0.00	2.01	60 CPI Footwar
0.95	2.07	42 CPI Cooked food
1.00	2.44	62 CPI Services related to clothing
0.55	2.40	27 PPI Information & communications equipment
0.82	2.53	43 CPI Beverages
0.93	2.59	28 PPI Transportation equipment
0.61	2.64	53 CPI Interior furnishings
1.08	2.77	47 CPI Repairs & maintenance
0.83	3.15	76 CPI Personal care services
5.00	0.10	

Note: The components of the 1st eigenvector with absolute value greater than 0.502 (1% significance level) are listed in the ascending order of θ which is the phase measured in reference to that of Leading Index.

TABLE 5—THE SECOND EIGENVECTOR.

Abs	θ [rad]	Items
1.07	0	82 Index of Business Condition Leading Index
0.75	0.31	16 PPI Petroleum & coal products
0.55	0.57	67 CPI Private transportation
1.37	0.61	83 Index of Business Condition Coincident Index
0.79	0.69	87 Contractual cash earnings (Manufacturing)
0.82	1.27	50 CPI Other fuel & light
1.17	1.42	84 Index of Business Condition Lagging Index
0.85	2.05	19 PPI Iron & steel
1.29	2.11	21 PPI Metal products
0.94	2.32	40 CPI Oils, fats & seasonings
1.25	2.35	17 PPI Plastic products
1.17	2.37	14 PPI Pulp, paper & related products
0.82	2.63	56 CPI Domestic non-durable goods
0.77	2.71	35 CPI Fish & seafood
0.34	2.73	39 CPI Fruits
0.91	2.73	58 CPI Clothes
1.40	2.74	18 PPI Ceramic, stone & clay products
0.91	2.79	36 CPI Meats
0.79	2.81	12 PPI Textile products
1.10	2.81	49 CPI Gas
0.90	2.84	31 PPI Minerals
1.07	2.84	42 CPI Cooked food
1.02	2.91	22 PPI General purpose machinery
0.73	2.92	48 CPI Electricity
1.29	2.93	26 PPI Electrical machinery & equipment
0.85	2.98	25 PPI Electronic components & devices
1.19	3.08	28 PPI Transportation equipment
1.22	3.09	29 PPI Other manufacturing industry products
0.91	3.11	41 CPI Cakes & candies
1.62	3.19	62 CPI Services related to clothing
1.26	3.31	55 CPI Domestic utensils
1.08	3.41	27 PPI Information & communications equipment
0.81	3.41	61 CPI Other clothing
1.35	3.51	47 CPI Repairs & maintenance
0.76	3.55	46 CPI Rent
1.34	3.69	76 CPI Personal care services
1.37	4.04	6 IPI Chemicals & related products
2.30	4.06	8 IPI Electric & electronic products
1.87	4.06	4 IPI Wood, lumber & related products
2.47	4.09	81 US Dollar to Japanese Yen Exchange Rate
1.88	4.10	9 IPI Transportation equipment
1.66	4.11	1 IPI Foodstuffs & feedstuffs
2.29	4.12	7 IPI General purpose, production & business oriented machinery
1.02	4.13	3 IPI Metals & related products
2.16	4.15	10 IPI Other primary products & manufactured goods
0.72	4.19	43 CPI Beverages
		~

0.72 4.19 43 CP Beverages 2.03 4.21 2 IPI Textiles Note: All the components of the 2nd eigenvector with absolute value greater than 0.702 (1% significance level) are listed in the ascending order of θ which is the phase measured in reference to that of Leading Index.

The second eigenmode, on the other hand, represents the domestic business condition (Fig. 7(b)). It drives domestic prices. Propagation of shocks across prices does not have such damping behavior as observed in the first eigenmode.

The exchange rate and import prices except for price of petroleum, coal & natural gas (#5) also have large absolute values in the second eigenvector, but curiously, they lag behind other variables. In fact, they lie outside of 2π . This apparent lag of the exchange rate behind domestic prices is, in fact, nothing but a mathematical necessity, and therefore, we can disregard it as such. It is shown in Appendix A.

In both eigenmodes, nominal wage (#87) plays a notable role in determining the dynamics of domestic prices. It leads changes of most prices. In contrast, neither money stock (#85) nor monetary base (#86) is significant in two eigenmodes.

E. A robust comovement of domestic prices



FIGURE 8. LEAD-LAG RELATIONSHIP AMONG DOMESTIC PRICES IN THE FIRST AND SECOND EIGENMODES.

Note: The lead-lag relationship among the significant domestic prices of the first eigenmode is compared with that of the second eigenmode. The phase θ_1 of each price in the first eigenvector is plotted on the horizontal axis and the phase θ_2 of the same price in the second eigenvector, on the vertical axis.

Closer look at the first and the second eigenvectors reveals that the lead-lag re-

lationships among domestic prices, namely PPI and CPI, in the two eigenmodes are, in fact, quite similar to each other. Figure 8 compares phases of the significant domestic prices in the first eigenvector with the corresponding phases in the second eigenvector. The prices are well aligned on the correlation plot. We can also confirm the strong resemblance between the lead-lag relations of domestic prices in the two eigenmodes by taking inner product of the corresponding complex vectors, a generalization of the cosine similarity. The result is 0.73, which is highly significant in reference to the similarity between two random complex vectors. The associated p value takes an extremely small value, 1.5×10^{-23} .

This means that there exists robust internal dynamics of domestic prices irrespective of their driving forces, that is, the exchange rate accompanied by import prices in the first eigenmode or domestic business condition in the second eigenmode. Domestic prices are thereby interconnected by their mutual interactions to form a chain-like correlation structure. The important point is that this chain-like correlation arises by way of clusters of individual prices as schematically depicted in Figure 9. In the next section, we explicitly demonstrate the presence of such clusters.



FIGURE 9. DYNAMICS OF INDIVIDUAL PRICES IN CLUSTERS

Note: Schematic diagram of comovement of domestic prices originating from their mutual interactions with its driving forces, the dollar-yen exchange rate in the first eigenmode and domestic demand in the second eigenmode. The filled triangles and circles represent individual prices belonging to the PPI and CPI categories, respectively.

IV. Formation of Price Clusters

Figure 10 shows propagations of macroeconomic shocks across individual prices from upstream to downstream. It is the macroscopic picture of the comovements of domestic prices shown in the previous section. The data shown in Fig. 10 are basically the same as Fig. 2, but here, prices are ordered vertically not by their original identification numbers (1-87) but their phases in the first eigenvector of CHPCA (Fig. 7(a)) from top (leading or upstream) to bottom (lagging or downstream). By so doing, we are able to identify clusters: For both price increase and decrease, prices change in clusters. In what follows, we identify such clusters.



FIGURE 10. TEMPORAL CHANGES OF INDIVIDUAL PRICES

Note: Standardized monthly log differences of individual prices and macroeconomic variables are visualized basically in the same way as Fig. 2. But all the data are plotted here, and the positive and negative changes are depicted by circles in the panels (a) and (b), respectively. The variables are ordered by their phases in the first eigenvector of the CHPCA from top (upstream) to bottom (downstream) in the vertical direction.

A. Methodology

We can resort to percolation model (Kirkpatrick (1973)) for identifying price clusters seen in Fig. 10. We take it that cluster is a set of price changes which are

linked because they are "similar". The degree of similarity is defined as follows. First, the data is placed on a square lattice (see Fig. 11). The first and second neighbors of each site are candidates for linkage. We measure the strength $g_{\alpha\beta}$ of coupling between price α at time t and price β at time t' (t' = t or t' = t ± 1) by geometric mean of their monthly changes $w_{\alpha}(t)$ and $w_{\beta}(t')$:

(16)
$$g_{\alpha\beta}(t,t') = \sqrt{w_{\alpha}(t)w_{\beta}(t')} \; .$$

The two neighboring prices are regarded as being *linked* if their coupling constant is larger than a certain threshold g_c :

(17)
$$g_{\alpha\beta}(t,t') > g_c \; .$$

Obviously, identification of price clusters depends crucially on the choice of g_c . If we adopt a too small value of g_c , prices would fragment to a number of tiny pieces. For a too large value of g_c , on the other hand, most prices would be connected to form a single group. With increased g_c , the clustering characteristics of prices show a crossover from the one extreme case (all prices are isolated) to the other extreme case (all prices are connected), and at a certain point of g_c , called the percolation threshold, the size distribution of clusters has scale-free nature with a power law tail. This is an example of the percolation transitions. If we carefully adjust g_c close to the percolation threshold in the price lattice system, clusters of various scales are illuminated. At the percolation threshold, we can thereby extract information on the clustering properties of prices in the most effective way. This algorithm for detecting price clusters is illustrated in Figure 11.

By reiterating the percolation calculations with varied g_c , we have found a percolation transition takes place around $g_c = 0.5$ in the model system for both positive and negative changes of prices. Incidentally, the total numbers of clusters obtained at $g_c = 0.5$ are 857 and 896 for positive and negative changes of prices, respectively.

B. Results

Figure 12 shows major price clusters thus identified. We focus on two periods in which the formation of clusters drastically changed. First is the post Plaza Agreement period (1985-87), and second, the Great Recession (2008-09).

After the Plaza Agreement in the fall of 1985, the yen started appreciating from 250 yen per dollar to 120 yen within 2 years (see Figure 12(d)). This sharp appreciation of the yen prompted declines of import prices and related producer prices, but not consumer prices (Fig. 12(b)). It corresponds to the first eigenmode in the analysis of the previous section. We note that there is no cluster of price increase observed during the period.

Beginning 1987, Japan experienced the major boom sustained by substantial in-



FIGURE 11. ILLUSTRATION OF THE CLUSTER DETECTION

Note: This diagram illustrates the algorithmic way explained in the text for detecting price clusters in Figure 10. Prices are arranged on a square lattice and the thick line between a pair of prices indicates that the two prices are connected according to the criterion (17). Here we find three price clusters, which are encompassed with dotted lines.

creases of asset prices which in retrospect, turned out the be bubbles (Fig. 12(c)). Prices rose, first in upstream sectors, and subsequently in downstream sectors. Thus, clusters move from northwest to southeast over time in Fig. 12(a). During 1987-90, no cluster of price decline was observed. After the price bubble burst and the Japanese economy entered severer recession in 1991, upstream prices started declining while downstream prices still kept rising.

The year 2008 also provides us with an excellent opportunity for case study. The long-lasted boom which started in 2002 eventually generated price increases again from upstream to downstream sectors for the period of 2006 to the fall of 2008 (Fig. 12(a)). During the period, no cluster of price decline is identified (Fig. 12(b)). In the fall of 2008, Japan's notorious deflation finally appeared to end. However, the bankruptcy of the Lehman Brothers turned the tide. Prices abruptly started declining in clusters.

It is noted that unlike the U.S. economy, Japan's "Great Recession" was *not* caused by financial crisis; the Japanese financial system was basically stable. Japan's sharp recession in 2009 as shown in Fig. 12(c) was caused by unprecedented fall of exports due to the global recession: see Fig. 13 (Iyetomi et al. (2011)).

The fall of aggregated demand, exports in particular, prompted the Japanese economy into recession causing decreases of prices (Fig. 12(b)). Rise of price abruptly turned into fall of price. This change due to weakened domestic economy is captured by the second eigenmode in CHPCA in the previous section.

Beginning 2013, the yen started depreciating (Fig. 12(c)), while at the same time the oil price rose. Import prices rose, and their increases were propagated from upstream prices to downstream prices; We observe in Fig. 12(a) that once again, clusters move from northwest to southwest during the period.

The present analysis demonstrates that prices change neither all at once nor



FIGURE 12. PRICE CLUSTERS WITH MACROECONOMIC VARIABLES

Note: The panels (a) and (b) show the 20 largest clusters as detected by the percolation analysis with $g_c = 0.5$ for positive and negative changes of prices in Figure 10 (a) and (b). The panels (c) and (d) show temporal variations of the coincident index and the dollar-yen exchange rate for comparison of their behavior with formation of the price clusters.





FIGURE 13. INDICES OF EXPORTS (SOLID LINE) AND INDUSTRIAL PRODUCTION (DASHED LINE) NORMALIZED TO 100 AT 2005 (FROM IYETOMI ET AL. (2011)).

in isolation. They change in clusters. We note that identification of clusters of price change is made possible by CHPCA which provides us with information on phase, namely leads and lags.

V. Concluding Remarks

The world today is in an age of low inflation. Low inflation may be welcome, but is next only to deflation which can be a threat to the macroeconomy. Deflation, if it is not collapse of price Irvin Fisher faced during the 1930's still causes serious problem to monetary policy particularly when nominal interest rate is extremely low, say zero. The Bank of Japan, the U.S. Federal Reserve and the European Central Bank have all faced the zero interest bound amid low inflation or even deflation. Central banks have resorted to sizable quantitative easing (QE), but they have difficulty in achieving the target rate of inflation, namely 2%. Why prices do not rise remains a puzzle. Obviously we need to know more about prices change. The standard framework for understanding the behavior of aggregate price is the New Keynesian Phillips curve (NKPC). The key property of the NKPC is that inflation is primarily a forward-looking process. That is, expectations on future inflation largely determine current inflation. This justifies recent emphasis on expectations management and communications as tools of monetary policy under the extraordinary zero-interest situations. There is a great amount of literature on the NKPC. However, after a long survey of the literature, Mavroeidis et al. (2014) concluded that identification of the NKPC is too weak to warrant research on conceptually minor extensions. The traditional analysis based on macro data has its clear limitations.

Meanwhile, recent empirical works on micro price-setting as surveyed by Klenow

and Malin (2011) have uncovered hitherto little known dynamics of micro prices. Bils and Klenow (2004), for example, by examining the frequency of price changes for 350 categories of goods and services demonstrate that half of prices last 5.5 months or less. Many others also found that there is a considerable *cross-sectional* heterogeneity in the frequency and/or the hazard rate of price change across goods and services (Carvalho (2006), Klenow and Malin (2011)). The information is useful for understanding industrial organization of particular market. However, it provides only a limited information on deflation/inflation precisely because understanding deflation/inflation amounts, after all, to understanding *changes in the behavior of the aggregate price over time*. The existing literature focuses on *cross-sectional* distribution of micro price changes, and assumes that the distribution is given and time-invariant. We note that micro-optimization exercise results in a particular pattern of price setting which is time-invariant. However, Section I of this paper demonstrated that distribution of changes of micro prices which ultimately produces deflation/inflation is *time-varying*.

The analysis of autocorrelations in Section II demonstrated the significance of cross-autocorrelations of micro prices in aggregate price dynamics. This means the limitation of analyses of price setting behavior on the assumption of representative firm because sluggishness of aggregate price arises mainly from interactions of micro prices.

Section III analyses such interactions of micro prices by a new method called the complex Hilbert principal component analysis (CHPCA). Though it is little known in economics, it has been successfully applied in many fields of natural science. This method takes care of lead-lag relationships present in micro price dynamics. We note that the ordinary (real) principal component (factor) analysis fails to uncover hidden common factors when there are significant leads and lags in variables under investigation. CHPCA enables us to capture systematic behavior of the group of micro prices as a while and thereby the aggregate price, and the nature of driving macro shocks. We have applied CHPCA to 80 micro prices and 7 macro variables and found that there exist two dominant factors (eigenmodes) which generate systematic dynamics of micro prices, as a whole, and thereby aggregate price.

The first significant eigenmode generating the systemic fluctuations of individual prices is significantly correlated with the exchange rate and crude oil price. In open economy like the Japanese economy, changes in the exchange rate and oil price affect the import prices without lags, and they, in turn, change the costs of energy and materials used in the production of a wide range of goods and services. With lags, many prices follow suit. Gopinath et al. (2010) find that exchange rates systematically affect import prices for the U.S. as well, but that the elasticity of import prices with respect to changes in exchange rates is rather small, namely that firms adjust prices by only 0.25% for each 1% change in the exchange rate. The case study of the Post Plaza Agreement period when the yen sharply appreciated from 240 per dollar to 120 amply demonstrates the

present of this mechanism. In fact, Brown and Ozga (1955) studying the longterm data (1870–1950) for the U.K. found that the most important determinant of the British price was terms of trade which was in turn basically determined by prices of raw materials. It is easy to dismiss this finding by saying that price is nominal whereas terms of trade are real. But that is what data tells us. For the Japanese economy, real price of energy and the real exchange rate affect the nominal aggregate price.

The second significant eigenmode represents domestic business condition. In both eigenmodes, nominal wage plays a role in determining dynamics of domestic prices. It leads change of most prices. In contrast, neither money stock nor monetary base is significant.

We have found that irrespective of the nature of driving forces, there exists robust internal dynamics of domestic prices. Most important is the find that micro prices change *in clusters*. Clusters of price changes shift from upstream (raw materials) to downstream (final consumer goods) over time.

Most likely, this cluster dynamics arises from input/output relationships in production of goods and services. The propagation of price change in clusters suggests that prices of final gods and services are made based on costs rather than expectations. We can recall that the cost-based mark-up pricing was once said to be prevalent (Hall and Hitch (1939), Nordhaus and Godley (1972)). Eichenbaum et al. (2011) using scanner data from a US supermarket chain, also shows that retail prices, reference prices excluding temporary sales in particular, tend to change so as to keep the product's mark up over marginal cost at its average level. This conclusion is in accordance with the following remark made by Gordon (2011).

"(Recent research on inflationary expectations is) flawed because it placed the information barriers in the wrong place, in an inability to perceive costless macro information, instead of where the information barriers really exist, at the micro level of costs and supplierproducer relationships. Producers of final goods are unable to perceive cost increases of crude and intermediate materials that may be in the pipeline, and they have no choice but to wait until they receive notification of actual cost changes (with the exception of crude materials like oil where prices are determined in public auction markets). \cdots A fundamental source of persistence is not just explicit wage contracts as analyzed by Taylor, but also explicit or implicit price contracts between suppliers and producers of final goods. Even without contracts, persistence and inertia are introduced by lags between price changes of crude materials, intermediate goods and final goods. For some goods, e.g. cars or aircraft, there are literally thousands of separate intermediate goods, and most of these are made up of further layers of intermediate goods." (Gordon, 2011, pp.32–33)

Deflation and inflation are macroeconomic phenomena. However, we cannot

fully understand them by only exploring macro data because the behavior of aggregate price such as CPI depends crucially on interactions of micro prices. The results we obtained strongly suggest that for our better understanding the behavior of aggregate price, namely, inflation/deflation, we should redirect our research from the analysis of price setting on the assumption of representative firm to interactions of firms/sectors based on input/output linkages or intersectoral production networks. Recent theoretical and empirical investigations on business cycles resurrect the importance of input-output linkages in production (Acemoglu et al., 2012). Out analysis suggests that it holds true not only for quantity but also for price.

34

REFERENCES

- Acemoglu, Daron, Vasco M Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The network origins of aggregate fluctuations," *Econometrica*, 2012, 80 (5), 1977–2016.
- Aoyama, Hideaki, Yoshi Fujiwara, Yuichi Ikeda, Hiroshi Iyetomi, Wataru Souma, and Hiroshi Yoshikawa, Macro-Econophysics –New Studies on Economic Networks and Synchronization, Cambridge University Press, 2017.
- Bank of Japan, "Outline of Statistics and Statistical Release Schedule," 2017. https://www.boj.or.jp/en/statistics/outline/exp/pi/excgpi02.htm/.
- Barnett, TP, "Interaction of the monsoon and Pacific trade wind system at interannual time scales Part I: the equatorial zone," *Monthly Weather Review*, 1983, 111 (4), 756–773.
- Basawa, I. V. and B. L. S. Prakasa Rao, Statistical Inference for Stochastic Processes, Academic Press, 1980.
- Bils, Mark and Peter J Klenow, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, October 2004, *112*, 947–85.
- Brown, E. H. Phelps and S. Andrew Ozga, "Economic growth and the price level," *The Economic Journal*, 1955, pp. 1–18.
- Calvo, Guillermo A, "Staggered prices in a utility-maximizing framework," Journal of monetary Economics, 1983, 12 (3), 383–398.
- Carvalho, Carlos, "Heterogeneity in price stickiness and the real effects of monetary shocks," *Frontiers in Macroeconomics*, 2006, 2 (1).
- Eichenbaum, M., N. Jaimovich, and S. Rebelo, "Reference Prices, Costs, and Nominal Rigidities," *American Economic Review*, 2011, 101, 234–262.
- Gabor, D., "Theory of communication," J. Inst. Electr. Eng.-Part III, Radio Commun. Eng., 1946, 93, 429–457.
- Golosov, Mikhail and Robert E Lucas, "Menu costs and Phillips curves," Journal of Political Economy, April 2007, 115, 171–199.
- Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon, "Currency choice and exchange rate pass-through," *American Economic Review*, 2010, 100 (1), 304–336.
- Gordon, Robert J, "The history of the Phillips curve: consensus and bifurcation," *Economica*, 2011, 78 (309), 10–50.

- Granger, Clive William John and Michio Hatanaka, Spectral analysis of economic time series., Princeton Univ. Press., 1964.
- Hall, Robert L and Charles J Hitch, "Price theory and business behaviour," Oxford economic papers, 1939, 2, 12–45.
- Iyetomi, Hiroshi, Yasuhiro Nakayama, Hiroshi Yoshikawa, Hideaki Aoyama, Yoshi Fujiwara, Yuichi Ikeda, and Wataru Souma, "What causes business cycles? Analysis of the Japanese industrial production data," *Journal of the Japanese and International Economies*, 2011, 25 (3), 246–272.
- Kalecki, Michał, Theory of economic dynamics, George Allen & Unwin, London, 1954.
- Kirkpatrick, Scott, "Percolation and Conduction," Rev. Mod. Phys., Oct 1973, 45, 574–588.
- Klenow, Peter J and Benjamin A Malin, "Microeconomic evidence on pricesetting," in Benjamin H. Friedman and Michael Woodford, eds., Handbook of Monetary Economics, Vol. 3A, North-Holland, 2011, chapter 6.
- Krugman, Paul R, "It's baaack: Japan's slump and the return of the liquidity trap," Brookings Papers on Economic Activity, 1998, 1998 (2), 137–205.
- Mankiw, N Gregory, "Small menu costs and large business cycles: A macroeconomic model of monopoly," *The Quarterly Journal of Economics*, 1985, pp. 529–537.
- Mavroeidis, Sophocles, Mikkel Plagborg-Møller, and James H Stock, "Empirical evidence on inflation expectations in the New Keynesian Phillips Curve," *Journal of Economic Literature*, 2014, 52 (1), 124–188.
- Midrigan, Virgiliu, "Menu costs, multiproduct firms, and aggregate fluctuations," *Econometrica*, 2011, 79 (4), 1139–1180.
- Nordhaus, William D and Wynne Godley, "Pricing in the trade cycle," The Economic Journal, 1972, pp. 853–882.
- Rasmusson, E. M., P. A. Arkin, W.-Y. Chen, and J. B. Jalickee, "Biennial variations in surface temperature over the United States as revealed by singular decomposition," *Mon. Wea. Rev.*, 1981, 109, 587–598.
- Statistics Bureau, "Consumer Price Index," 2017. http://www.stat.go.jp/ english/data/cpi/index.htm.
- Statistics Japan, "Q&A on the Consumer Price Index (in Japanese)," 2014. http://www.stat.go.jp/data/cpi/4-1.htm.
- Wulfsberg, Fredrik, "Inflation and price adjustments: micro evidence from Norwegian consumer prices 1975–2004," American Economic Journal: Macroeconomics, 2016, 8 (3), 175–194.

Appendix A. Mathematical Structure of Eigenvectors of Two-variable Model

In Section III, the exchange rate and import prices, though they have large absolute values, lay behind domestic prices in the second mode (Figure 7(b)). In this appendix, we show that it is nothing but a mathematical necessity in two-variable model. To understand the correlation structures observed in the first and second eigenmodes, we introduce a simple two-variable model. For this purpose, we first replace the group motion of domestic prices by a single collective coordinate, that is, the mode signal of the first eigenmode of the CHPCA applied to the reduced data set in which only domestic prices are retained. Also we replace the dollar-yen exchange rate and import prices by another collective coordinate. Adopting the two collective coordinates reduces the economic system under study to a two-variable model.

In this two-variable model, the complex correlation matrix \tilde{C} has such a reduced form as

(A1)
$$\tilde{\boldsymbol{C}} = \begin{pmatrix} \sigma_1 & \sigma_{12} \\ \sigma_{12}^* & \sigma_2 \end{pmatrix} ,$$

where σ_1 and σ_2 are the variances of the collective coordinates for domestic prices and the exchange rate accompanied by import prices, respectively, and σ_{12} is a complex correlation coefficient between the two coordinates.

If σ_1 and σ_2 take an identical value σ , the two eigenvalues λ_{\pm} are calculated as

(A2)
$$\lambda_{\pm} = \sigma \pm |\sigma_{12}| ,$$

with their eigenvectors V_{\pm} given by

(A3)
$$V_{+} = \begin{pmatrix} 1 \\ \exp(-i\theta) \end{pmatrix}, V_{-} = \begin{pmatrix} 1 \\ -\exp(-i\theta) \end{pmatrix},$$

where θ is the phase angle of σ_{12} . We see that the relationship between the comovement of domestic prices and the exchange rate in V_+ is reversed in V_- . When $0 < \theta < \pi/2$, for example, the exchange rate leads the comovement of domestic prices with phase difference θ in V_+ , while the exchange rate follows the comovement of domestic prices with phase difference $\pi - \theta$.

In the actual data, we obtain $\sigma_1 = 7.81$ and $\sigma_2 = 7.42$. The former is the largest eigenvalue of the submatrix of \tilde{C} for domestic prices, and the latter, that for the exchange rate and import prices. Thus, the condition $\sigma_1 = \sigma_2$ is approximately satisfied. The model with (A3) of V_+ and V_- therefore well explains the correlation structures in the two dominant eigenmodes. The exchange rate drives the comovement of domestic prices in the first eigenmode to fix the phase difference θ between the two collective coordinates. On the other hand, in the second eigenmode, the lead-lag relationship of the exchange rate with the comovement of domestic prices is automatically determined by $\pi - \theta$. This is basically what we observe in Figure 7(b). It is simply a mathematical necessity in a two-variable model.

Given this mathematical fact, it is not the end of the story. Because replacement of the exchange rate by a completely random time series would result in the same mathematical relation between V_+ and V_- as long as the condition $\sigma_1 \simeq \sigma_2$ is satisfied. The random time series is fixed to the comovement of domestic prices at any phase angle. The remaining issue to be addressed is thereby whether the fixed phase difference θ between the two collective coordinates in the first eigenmode is statistically significant or not. To test statistical significance of the phase angle between the comovement of domestic prices and the exchange rate. we reiterated the CHPCA calculation for the data set in which the exchange rate and import prices are substituted by a random time series with the variance kept the same; the new results serve as a null model. The strength of coupling between the two collective coordinates is represented by the magnitude of σ_{12} and hence by difference of the two dominant eigenvalues as shown in Eq. (A2). Figure A1 demonstrates distribution of $\lambda_1 - \lambda_2$ in the null model. On the other hand, the actual result for $\lambda_1 - \lambda_2$ is 2.401 and its *p*-value is given as 0.006 according to the null hypothesis. This comparison allows us to infer that the fixed phase angle between the comovement of domestic prices and the exchange rate is statistically meaningful.

In conclusion, the correlation structures in the two dominant eigenmodes are fully understandable with a two-variable model. And also we confirm that the exchange rate is certainly a driving factor for the first eigenmode.



Figure A1. Distribution of the eigenvalue separation, $\lambda_1 - \lambda_2$, in the CHPCA with a random time series in place of the exchange rate and import prices (sampled 10,000 times), where the variance of random time series is kept the same as $\sigma_2 = 7.42$.