RIETI Discussion Paper Series 18-E-040

Bankruptcy Propagation on a Customer-supplier Network: An empirical analysis in Japan

Yoshiyuki ARATA
RIETI

Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
https://www.rieti.go.jp/en/

RIETI Discussion Paper Series 18-E-040
First Draft June 2018
Revised February 2019

Bankruptcy propagation on a customer-supplier network: An empirical analysis in Japan

Yoshiyuki Arata*

Research Institute of Economy, Trade and Industry

Abstract

Since firms are interrelated via customer-supplier relationships, the bankruptcy of a firm may lead to the bankruptcy of its suppliers. Due to this contagion effect, one bankruptcy may trigger many subsequent bankruptcies of direct and indirect and have a nonnegligible impact on an aggregate economy. This paper empirically analyzes this bankruptcy propagation on a customer-supplier network by using a comprehensive dataset consisting of more than one million firms and their customer-supplier relationships, and bankruptcy records over April 2013 to February 2017 in Japan. We find that the contagion effect is significant at the firm-level; for example, if 50% customers of a firm go bankrupt, the firm's bankruptcy probability approximately triples. However, it does not immediately imply that there is a substantial risk at the aggregate level that a nonnegligible fraction of firms are forced into bankruptcies by the contagion effect. In fact, by simulating our model, we find that the reach of bankruptcy propagation is very limited in most cases and it is highly unlikely that bankruptcy spread extensively on the network. This is because of the structure of the customer-supplier network. The network structure contributes to absorbing bankruptcy shocks by an aggregate economy rather than spreads bankruptcy on the network.

Keywords: Bankruptcy propagation; Production network; Survival analysis.
JEL Classification Numbers: G33; G20.

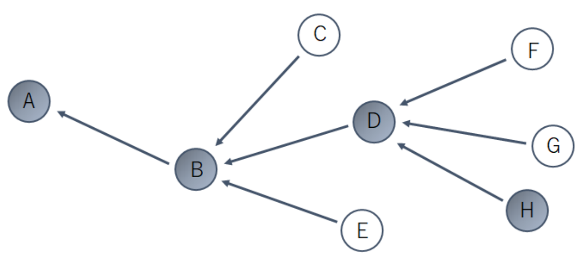
*Address: 1-3-1 Kasumigaseki Chiyoda-ku, Tokyo, 100-8901 Japan. E-mail: arata-yoshiyuki@rieti.go.jp. This research was conducted as part of the Project "Large-scale Simulation and Analysis of Economic Network for Macro Prudential Policy" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). This research was supported by MEXT as Exploratory Challenges on Post-K computer (Studies of Multi-level Spatiotemporal Simulation of Socioeconomic Phenomena, Macroeconomic Simulations). This research used computational resources of the K computer provided by the RIKEN Advanced Institute for Computational Science through the HPCI System Research project (Project ID:hp170242, hp180177). This study is financially supported by JSPS KAKENHI Grant Numbers 17H02041, 18K12788.

1 Introduction

One of the characteristic features of our modern economy is the interdependence of firms through various relationships. For example, firms purchase and sell goods simultaneously and these customer-supplier relationships generate a huge complex network. Since transactions between customers and suppliers are frequently performed on trade credit, the relationships imply that firms are exposed to counterparty risk, that is, a payment default of a firm's results in significant losses on its supplier. One bankruptcy of a firm may leads to another subsequent bankruptcy and cause bankruptcy propagation on the network.

In recent years, the importance of network structure has received increasing attentions, especially in theoretical macroeconomic literature. As theoretically shown by e.g., Acemoglu et al. (2012, 2015, 2017) and Baqaee (forthcoming), if a network has hub firms, idiosyncratic shocks to hub firms do not die out but spread on the network, causing substantial aggregate fluctuations. In particular, regarding to systemic risk and network structure, the non-monotonicity of relationship between the network density and bankruptcy propagation has been pointed out by e.g., Gai and Kapadia (2010), Gai et al. (2011), and Elliott et al. (2014).[1] Imagine as an extreme case that a network has no network ties and firms are completely independent of each other. In this case, since there is no shock propagation channel, bankruptcy does not spread. As network ties are added to this network, firms becomes more susceptible to other firms, and therefore, the risk of bankruptcy propagation increases. However, as the underlying network approaches to the other extreme, i.e., a complete network in which firms are connected to all other firms, bankruptcy is unlikely to spread because bankruptcy shocks are not concentrated but immediately diluted in the economy. Put differently, adding network ties contributes to bankruptcy propagation when the network is very sparse whereas it prevents bankruptcy propagation when the network is highly connected. Even if the contagion effect is present at the firm level, it does not immediately imply that a non-negligible fraction of firms in the economy are forced into bankruptcy because its likelihood depends on the network structure. Therefore, to assess the risk of bankruptcy propagation on a network, both the magnitude of the contagion effect at the firm level and an empirical network structure must be considered simultaneously. This paper tackles this problem.

Figure 1: Bankruptcy propagation on a customer-supplier network.



Note: Arrows represent the flow of money, e.g., firm A is a supplier of firm B, which is a supplier of firms C, D, and E.

This paper uses two comprehensive data in Japan. The first one is about customer-supplier relationships of more than one million firms in Japan, which enables us to identify the customers and suppliers of a firm. The second one is bankruptcy records over April 2013 and February 2017, which includes the bankruptcy date and identity of bankrupt firms. By merging these two dataset, we can trace how bankruptcy spreads on the customer-supplier network. First, by applying the survival analysis, we quantify the contagion effect at the firm level, that is, we measure an increase in the bankruptcy probability of a surviving firm facing bankruptcy of customers and/or suppliers. We find that the contagion effect is economically and statistically

[1] Related to this literature, the structure of financial networks, e.g., interbank networks, has been discussed for evaluating systemic risk. See Allen and Gale (2000); Dasgupta (2004); Leitner (2005); Brunnermeier and Pedersen (2008); Allen et al. (2012); Billio et al. (2012); Bigio and La'O (2016); Glasserman and Young (2015); Cabrales et al. (2017); Gofman (2017); for a review, Glasserman and Young (2016) and Benoit et al. (2017).

significant; for example, if the half of the customers of a firm go bankrupt, the bankruptcy probability of the firm approximately triples. Next, based on the estimates and the empirical network, we measure the impact of the contagion effect at the aggregate level, that is, we analyze the size of bankruptcy propagation on the network. We find that in most cases, the simulated size of bankruptcy propagation is very limited, in spite of a significant contagion effect at the firm level.

This contrast between at the firm and aggregate level is due to network structure. The empirical customer-supplier network shows high connectivity and most of firms are directly or indirectly. In particular, they are connected with short path length (the average path length is less than 5). This means that, regarding the systemic risk and network structure, the empirical network is close to the complete network, and therefore, bankruptcy shocks are immediately diluted before causing another bankruptcy. In other words, the customer-supplier network prevents—rather than contributes to—large-scale bankruptcy propagation.

Our paper is closely related to Boissay and Gropp (2013) and Jacobson and Schedvin (2015), in which detail information of bankrupt firms are examined. Focusing on the amount of trade credit extended to bankrupt firms, they show that debtor bankruptcies significantly increase the bankruptcy risk of creditors. Our paper complements their analysis in that our dataset contains not only customer-supplier relationships of bankrupt firms but surviving firms, which enables us to analyze the underlying network including bankrupt and surviving firms. In particular, our finding that network structure plays a role of preventing bankruptcy propagation contrasts with the argument in Boissay and Gropp (2013) that the existence of *deep pocket* (i.e., firms providing additional trade credit to financially constrained customers) is a key factor in preventing bankruptcy propagation. Our finding gives another explanation of why large-scale bankruptcy propagation does not occur in reality.

The rest of this paper is organized as follows.. Section 2 describes our data and show the features of the empirical customer-supplier network. Section 3 shows the results of survival analysis. Section 4 examines the aggregate impact of the contagion effect. Section 5 concludes.

2 Data

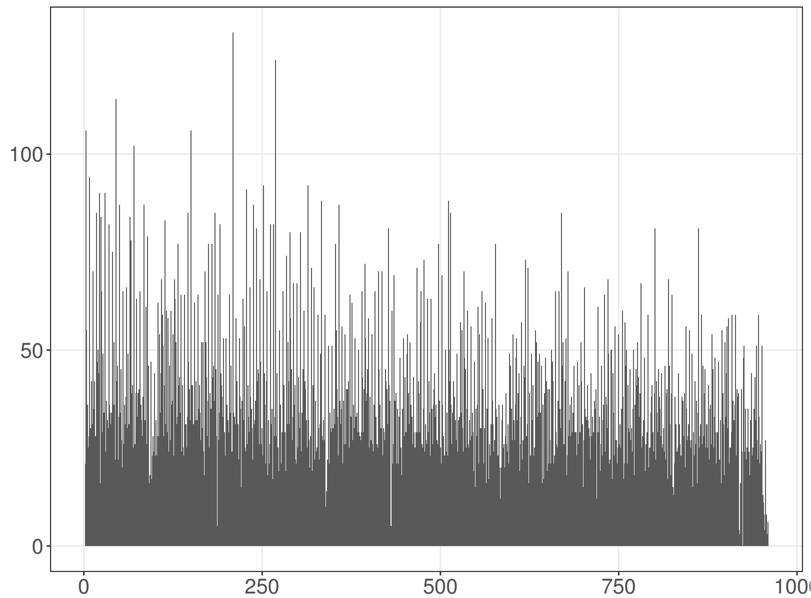
Our analysis uses two proprietary datasets complied by Tokyo Shoko Research Ltd (TSR): bankruptcy records from April 1st, 2013 to February 28th, 2017, and customer-supplier relationships of approximately one million firms in Japan. Bankruptcy records includes bankrupt firms with total debt *geq* ten millions yen, which contains the bankruptcy date, firm's attributes and financial information such as total debt.[2] The number of bankruptcies during the sample period is $35,945$. Figure 2 shows the time series of the number of bankruptcies on a daily basis.

TSR network data contains the identity of firms' important customers and suppliers (up to 24 firms in each category) as well as a set of firm characteristics such as sales, location and industry.[3] In addition, financial information from financial statements is available for about 25% of the firms. We exclude firms (with two-digit Japan Standard Industrial Classification (JSIC) codes in parentheses) operating in the divisions agriculture and forestry (01-04), finance and insurance (62-67), education and learning support (81-82), medical, healthcare and welfare services (83-85), and all subsequent sectors (86-99). Moreover, we exclude very small firms with sales less than ten millions yen. Our analysis is based on network data in 2014, but because firms that went bankrupt prior to 2014 are excluded from the 2014 data, we complement the network by adding bankrupt firms and their network ties in the 2012 data. Namely, we study firms that survive in 2014 (and may go bankrupt in subsequent years) and firms that go bankrupt prior to 2014, and consider a customer-supplier network generated by these firms. Our final data consists of $1,080,977$ firms and $14,670$ bankruptcies.

[2]Bankruptcy in our data is defined as either (1) suspension of bank transactions (2) bankruptcy under the corporate rehabilitation law (3) bankruptcy under the bankruptcy act law, (4) voluntary reduction of debts (Nai-Seiri) (5) special liquidations (6) bankruptcy under the civil rehabilitation law. In Japan, if a firm defaults on its payment twice within six months, its transaction with banks are legally prohibited (Case (1)). In Case (4), a firm privately negotiates with its debtors for debt reduction to rebuild its business. In Cases (2), (3), (5), and (6), bankruptcy date is the one on which a firm files a bankruptcy petition in the court.

[3]We assume that there is a customer-supplier relationship between firms i and j if at least either firm reports the relationship.

Figure 2: Time series of bankruptcy frequency in Japan.



Note: The horizontal axis is the number of working days since April 1st, 2013. There are 960 working days by February 28th, 2017. The sample average of bankruptcy frequency over our sample period is 37.44.

2.1 Descriptive statistics

Table 1 reports descriptive statistics of firm characteristics for (A) all firms (B) firms whose financial information is available (C) bankrupt firms (D) bankrupt firms whose financial information is available. An important empirical feature of the firm size distribution is that it is very rightly skewed. Figure 3 shows the complementary cumulative distribution function (CCDF) of sales and employees in the log-log scale, that is, the fraction of firms with firm size $\geq s$ for some s. It suggests that the firm size distribution has a fatter tail, which is roughly approximated by a straight line. This empirical feature is called Zipf's law in the literature (cf. Axtell (2001) and Gabaix (2009)). To be precise, Zipf's law means that firm size S approximately follows a power law tail:

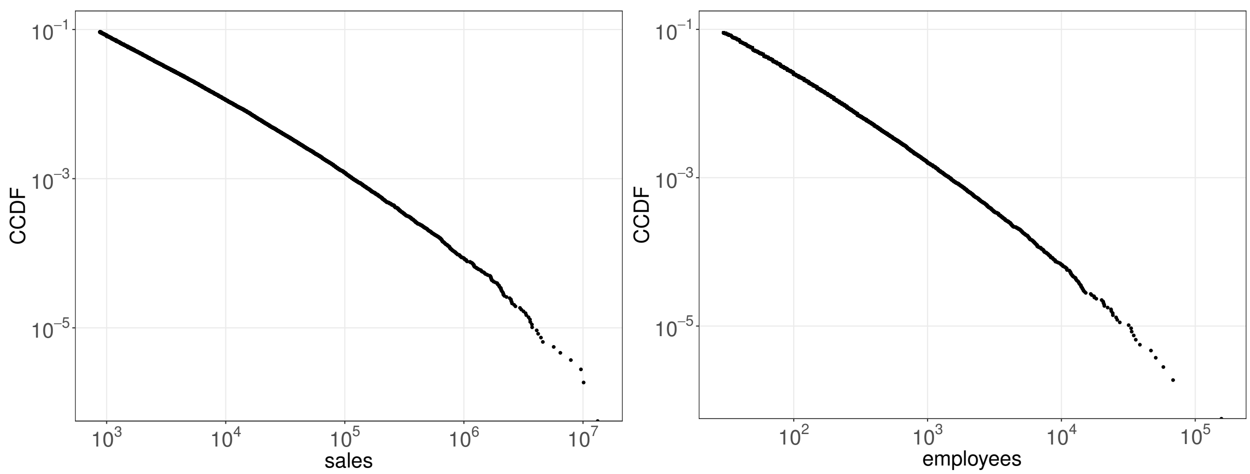
$$\Pr(S > s) \sim s^{-\beta}, \quad \text{for large s}, \tag{1}$$

where exponent β is close to 1. By using Hill's method, we estimate exponents β (s.e. in the parentheses) and find that $\hat{\beta} = 1.03$ (0.015) for sales and $\hat{\beta} = 1.28$ (0.018) for employees, respectively. Consistent with the previous literature, the exponent is close to 1, especially when firm size is measured by sales. This means that firm size is very heterogeneous and there exists extremely large firms.

(C) of Table 1 shows descriptive statistics for bankrupt firms. The sample means of firm size measured by employees and sales are 12.58, and 401.31 millions yen, respectively, and smaller compared to (A). However, this does not imply that all of the bankrupt firms are small. In fact, as shown in Figure 4 and 5, the CCDF of sales, employees, and total debts for bankrupt firms are fat-tailed and approximated by a power law. This means that, similar to the case for all firms, firm size for bankrupt firms are very heterogeneous and some extremely large firms go bankrupt.

In (B) and (D) of Table 1, we report total assets and ratios of profits, cash holdings, account payables, account receivables, current liabilities, and total liabilities to total assets as well as basic firm characteristics. (B) shows that the sample means of the ratios of account receivables and payables are 22.6% and 15.2%, suggesting that firms extend and receive a nonnegligible amount of trade credit. In other words, trade credit can be an important channel through which suppliers are affected by bankruptcy of their customer. As expected, comparison between (B) and (D) shows that bankrupt firms are lower-ranked by TSR (lower credit score), less profitable, and more heavily encumbered with short-term and total debt. For account receivables, the sample means for (B) and (D) are 22.6% and 24.5%, which means that bankrupt firms extend more trade credit to their customers. Although this is consistent with the trade credit channel hypothesis, the difference of the ratio of account receivables between all firms and bankrupt firms does not seem large.

Figure 3: CCDF of firm size for all firms.



Note: The left (right) panel shows the CCDF of sales (employees). Hill's estimates (s.e. in the parenthesis) for exponent β are $\hat{\beta} = 1.025$ (0.0145) for sales and $\hat{\beta} = 1.277$ (0.0181) for employees, respectively.

2.2 Network structure

This subsection describes the structure of a customer-supplier network of the $1,080,977$ firms in Japan. This network consists of $3,946,446$ customer-supplier relationships, and each relationship constitutes a directed network tie (from customer i to supplier j) in the network. One of the important features of the customer-supplier network is its sparsity. The empirical network shows that the network density, which is defined by the number of actual network ties divided by the maximum of network ties (i.e., $N(N-1)$ for N firms), is very low; the network density is 0.000338%, that is, only 0.000338% of all possible network ties actually exist. Consistent with the sparsity, Table 2 reports descriptive statistics of in- and out-degree of the network for each firm, that is, the number of customers and suppliers for each firm. It shows that most of firms have a limited number of customers and/or suppliers. For example, more than 90% of the firms have less than 10 customers. However, similar to the firm size distributions, the in- and out-distributions are very rightly skewed and the right tails follow a power law as shown in Figure 6. It suggests that both in- and out-degrees are very heterogeneously distributed across firms and there exist hub firms having ties to many other firms.

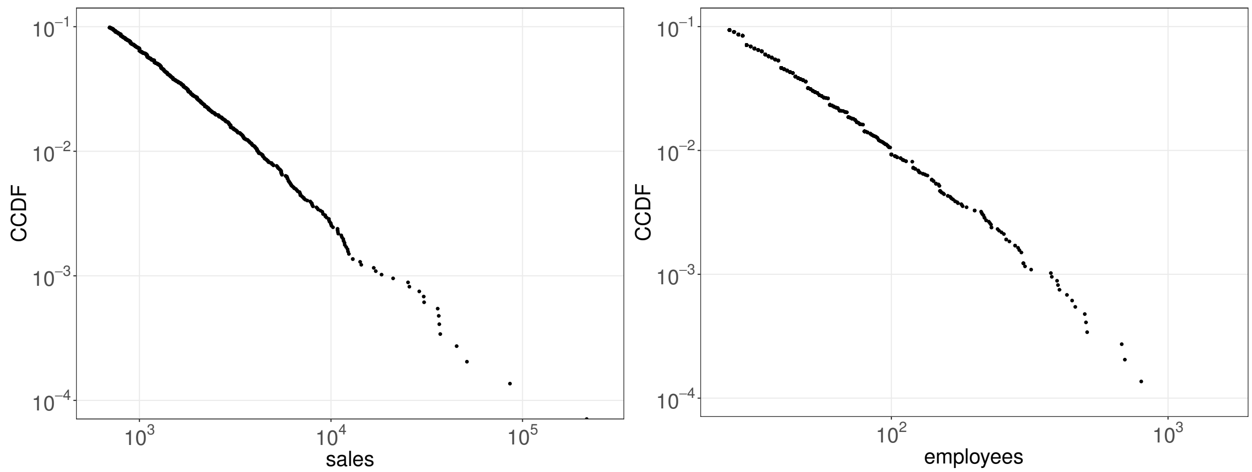
Another important feature of the network is its connectivity, which is particularly important in analysis of bankruptcy propagation because a high connectivity means that there are paths through which bankruptcy shocks potentially spread. We consider two concepts of network connectivity: weak and strong connectivity. A subnetwork is called a weakly connected component (WCC) if any pair of firms in the subnetwork is directly or indirectly connected. Similarly, a subnetwork is called a strongly connected component (SCC) if any pair of firms in the subnetwork has a directed path between the two firms. Note that direction of network ties are explicitly considered in the SCC but not in the WCC. In particular, firms i and j in the

Table 1: Descriptive statistics of firm characteristics.

	Mean	Median	S.D.	10th	90th	NA's
(A) All firms ($1,080,977$ firms)						
Age	34.373	32	17.633	12.000	58	1646
Employees	20.247	5	277.083	2.000	29	10270
Sales (in a million yen)	1100.627	90	32634.330	19.853	800	0
Credit-score	46.496	46	5.510	40.000	53	883
(B) Firms with accounting data ($263,455$ firms)						
Age	34.949	33.000	17.417	13.000	59.000	66
Employees	43.011	8.000	408.160	2.000	58.000	1560
Sales	2992.649	169.286	58896.016	27.665	2294.990	0
Credit-score	49.048	49.000	6.627	41.000	58.000	129
Total assets (a million yen)	3491.373	111.552	84102.955	12.517	1900.175	0
Ratios in percentage of total assets						
Profits	0.049	0.028	1.244	−0.077	0.194	1
Cash holdings	0.269	0.218	0.212	0.039	0.579	1
Account receivables	0.226	0.188	0.190	0.006	0.495	1
Account payables	0.152	0.095	0.343	0.000	0.369	1
Current liabilities	0.489	0.361	1.782	0.094	0.814	1
Total liabilities	0.910	0.742	2.412	0.230	1.399	1
(C) Bankrupt firms ($14,670$ firms)						
Age	33.678	31.000	16.792	13	57	3
Employees	12.575	6.000	40.017	2	25	8
Sales (a million yen)	401.309	125.483	2635.560	30	700	0
Credit-score	44.116	45.000	4.802	38	49	291
Total debt (a million yen)	376.536	94.000	6151.496	21	550	352
(D) Bankrupt firms with accounting data ($1,587$ firms)						
Age	34.096	32.000	17.598	12.000	58.400	0
Employees	17.843	8.000	64.198	3.000	35.000	1
Sales	844.803	199.780	5086.017	41.002	1353.588	0
Credit-score	43.940	45.000	5.460	38.000	50.000	47
Total assets (a million yen)	636.617	142.758	3384.478	22.291	1098.178	0
Total debt (a million yen)	641.959	168.000	4111.922	39.000	1043.700	27
Ratios in percentage of total assets						
Profits	−0.050	0.008	0.366	−0.213	0.075	0
Cash holdings	0.117	0.068	0.138	0.008	0.270	0
Account receivables	0.245	0.193	0.210	0.010	0.562	0
Account payables	0.227	0.141	0.379	0.000	0.498	0
Current liabilities	0.695	0.502	1.003	0.149	1.232	0
Total liabilities	1.575	1.042	2.154	0.712	2.665	0

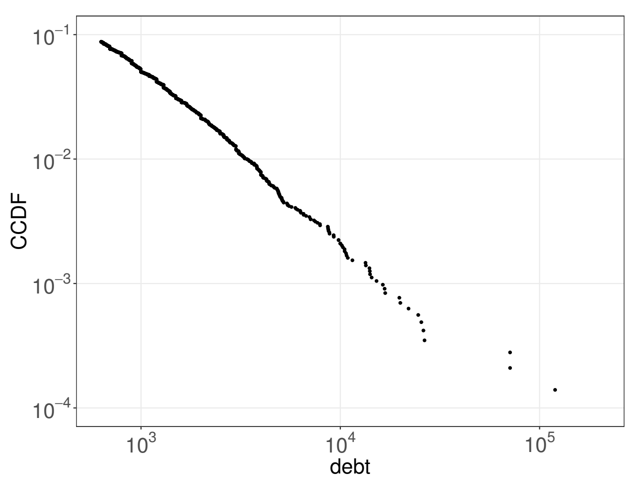
Note: Credit score given by TSR is within the range of $0 - 99$ and decreasing when the firms is poor. Account receivables is defined as the sum of three items: notes receivable-trade, accounts receivable-trade, and accounts receivable from completed construction contracts. Account payables is defined as the sum of three items: notes payable-trade, accounts payable-trade, and accounts payable for construction contracts.

Figure 4: CCDF of firm size for bankrupt firms.



Note: The left (right) panel shows the CCDF of sales (employees). Hill's estimates (s.e. in the parenthesis) for exponent $\hat{\beta} = 1.151$ (0.0515) for sales and $\hat{\beta} = 1.386$ (0.0620) for employees, respectively.

Figure 5: CCDF of total debt for bankrupt firms.



Note: Hill's estimates (s.e. in the parenthesis) for exponent β is $\hat{\beta} = 0.992$ (0.0444).

Figure 6: CCDF of in- and out-degree.

Note: The left (right) panel shows the CCDF of in-degree (out-degree). Hill's estimates (s.e. in the parenthesis) for exponent β are $\hat{\beta} = 1.317$ (0.0186) for in-degree and $\hat{\beta} = 1.326$ (0.01875) for out-degree, respectively.

Table 2: Descriptive statistics of in- and out-degree.

	Mean	Median	S.D.	10th	90th
All firms					
in degree	3.651	1	26.308	0	7
out degree	3.651	1	26.079	0	7
total degree	7.302	3	43.799	0	14
Bankrupt firms					
bank in degree	3.110	2	5.330	0	7
bank out degree	3.354	2	7.468	0	8
bank total degree	6.465	4	10.590	1	14

Note: Total degree is defined as the sum of in- and out-degree.

SCC have both directed paths from i to j and j to i, the SCC can be seen as a loop structure in the network.

We find that the largest WCC of the customer-supplier network accounts for 79.70% of all the firms. In other words, in spite of the sparsity of the network, a majority of firms are directly or indirectly connected through customer-supplier relationships and constitutes a large connected component. Figure 7 visualizes (a part of) the largest WCC of the customer-supplier network, showing that the largest components firms across different sectors. For the SCC, 39.91% of all the firms constitute the largest SCC. In light of the loop structure of the SCC, bankruptcy shocks can propagate and be further amplified by circulating in the SCC. Furthermore, Figure 8 shows the distribution of network distance between connected firms, that is, how far firms are apart from each other on the network if there is a directed path connecting the firms. Although only large firms are considered in Figure 8, it shows that firms are connected with a short path, for example, the average of path length is 3.851. In short, firms are not only connected but highly-connected with short path length.

Lastly, we report network correlation of firm characteristics in Table 3. One might be concerned that apparent bankruptcy propagation may be due to the correlation of firm characteristics between connected firms. For example, if low-performance firms are connected, they go bankrupt simultaneously without any contagion effects. To measure the correlation of firm characteristics, Table 3 reports the assortativity measure ρ proposed by Newman (2002, 2003) for sales, employees, credit-score by TSR, and network degree. It shows that on average, the customer-supplier network is disassortative mixing, that is, larger and high-performance firms are likely to be connected with smaller and low-performance firms, and vice versa. In other words, it is unlikely that a group of bad firms are concentrated locally on the network and result in a spurious bankruptcy propagation.

Table 3: Network correlation.

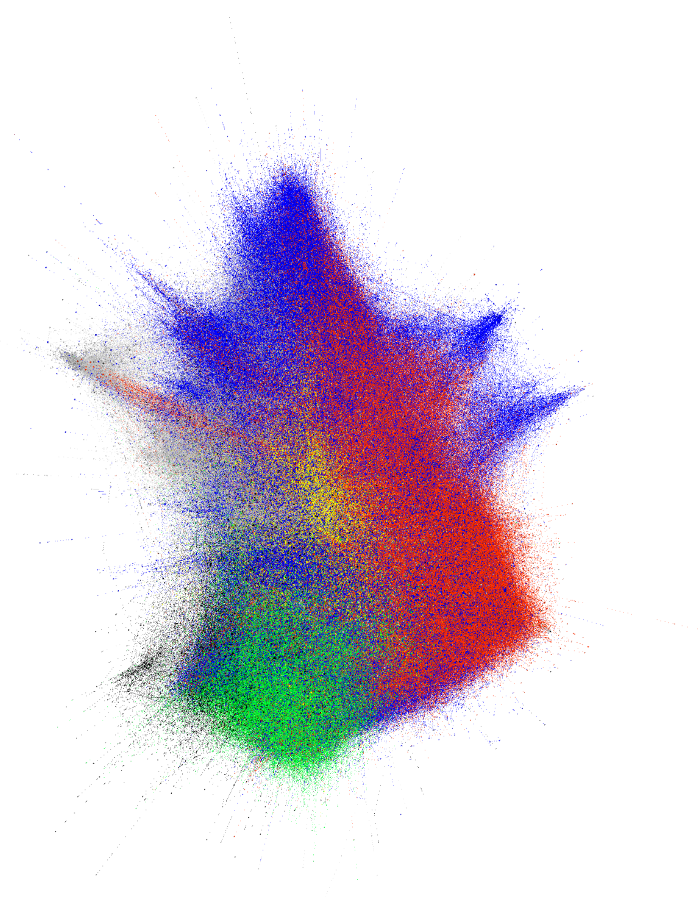
	Sales	Employees	Credit score	Degree
Assortativity measure ρ	-0.0610	-0.1415	-0.158	-0.0904

Note: Assortativity measure ρ, which is defined by Newman (2002, 2003), is the correlation coefficient between connected firms and within the range of $-1 \leq \rho \leq 1$. $\rho > 0$ (< 0) represents assortativity (disassortativity). For sales, assortativity measure ρ of $\log_{10}$(sales) is calculated. For employees, assortativity measure ρ of $\log_{10}$(employees +1) is calculated to deal with firms with zero employees.

2.3 Subnetwork of bankrupt firms

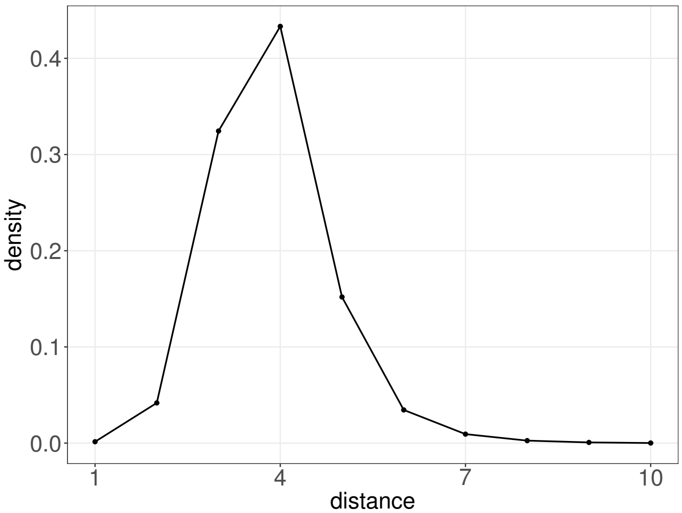
Next, we consider a subnetwork consisting of only bankrupt firms ($14,670$ bankrupt firms). Namely, we leave bankrupt firms and network ties between them from the underlying customer-supplier network. If large-scale bankruptcy propagation occurs, bankrupt firms must be connected and constitute a large connected

Figure 7: WCC of the customer-supplier network.



Note: The largest WCC consisting of firms with sales $\geq 10^6$ is shown (86,833 firms). The color of circles represents the firm's industry (1-digit). Blue, red, green, yellow and black represents wholesale & retail trade, represents manufacturing, construction, transport & postal activities, and real estate & goods rental & leasing. Other sectors are colored by grey.

Figure 8: Distribution of network distance.



Note: 6,480 firms are considered, that is, network distance between $6,480 \times 6,479 = 41,983,920$ pairs are calculated. The sample mean of path length is 3.851 and its s.d. is 0.959.

component in this subnetwork.

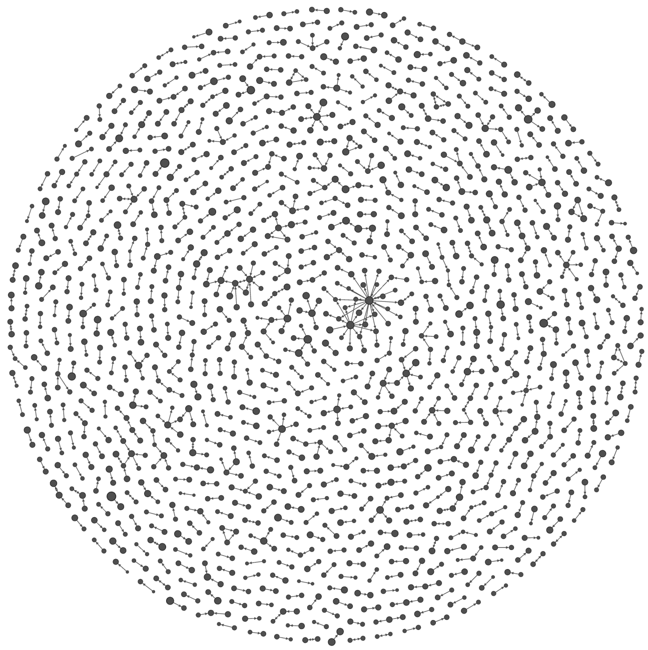
However, we find that in sharply contrast to the underlying customer-supplier network, this subnetwork of bankrupt firms shows disconnectivity. First, a majority of firms (88.71% of the bankrupt firms) in this subnetwork are isolated, that is, they have no network tie to another bankrupt firm. By definition, these isolated bankruptcies are not involved with any bankruptcy propagation. Next, by excluding these isolated bankrupt firms, we focus on remaining bankrupt firms having at least one network tie to other bankrupt firms. Figure 2 shows this subnetwork, which consists of 1,656 bankrupt firms and 1,072 network ties among them. It shows that while small-scale bankruptcy propagations are observed as shown in Figures 10 and 11, the subnetwork constitute of many small connected components. In contrast to the underlying customer-supplier network, this subnetwork has no large connected component. Indeed, the left panel of Figure 12 reports that the distribution of the size of weakly connected components, showing that for most cases, the size is 2, that is, two bankrupt firms with a network tie between them. In spite of high-connectivity of the underlying customer-supplier network, the size of empirical bankruptcy propagation is very limited.

Moreover, we consider the diameter of each connected component, which is the maximum length of shortest paths among pairs of connected bankrupt firms in the connected component. If a substantial contagion effect works and a firm's bankruptcy triggers its supplier's bankruptcy and in turn the bankruptcy of a supplier of the supplier, the connected component would be characterized by long diameter from the initial bankruptcy and the end point of bankruptcy propagation. The right panel of Figure 12 shows the distribution of the diameter across connected components, suggesting that the observed diameter are very short. Even for the largest connected components, its diameter is 3. In short, there is a sharp contrast between the underlying customer-supplier network and the subnetwork of bankrupt firms; the former is characterized by high connectivity and the latter is characterized by low connectivity.

3 Survival Analysis

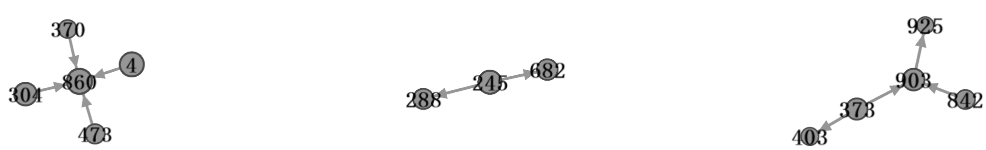
This section performs a statistical analysis to measure the contagion effect. Our model is an extension of the conventional survival analysis with interaction effects, that is, the effect of the bankruptcy of a customer/supplier.

Figure 9: Subnetwork of bankrupt firms.



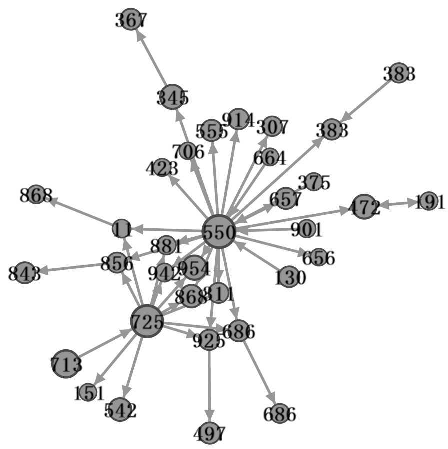
Note: Bankrupt firms and network ties between the bankrupt firms are kept from the entire customer-supplier network. Disconnected parts are gathered at the center of the figure for illustrative purposes.

Figure 10: Examples of bankruptcy propagation.



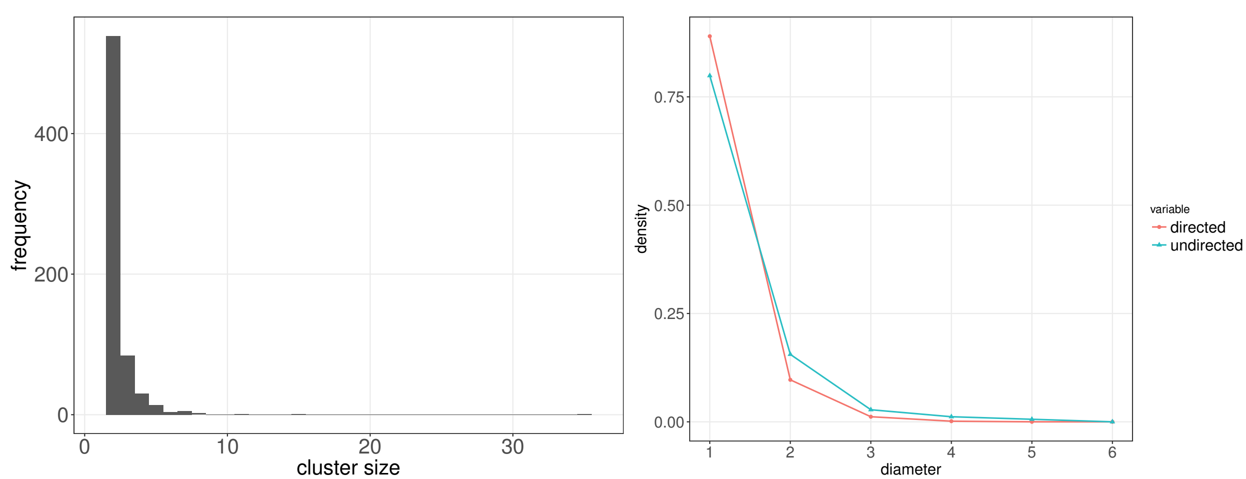
Note: These are parts of the subnetwork in Figure 9. The number in the circle represents its bankruptcy date (the number of working days since April 1st, 2013). In panel (a), three customers of a common supplier went bankrupt each on dates 456, 578, and 672. The supplier then went bankrupt on date 743. In panel (b), the common customer first went bankrupt (on date 495) and two suppliers then went bankrupt (on dates 511 and 636, respectively). In panel (c), the bankruptcy on date 373 precedes two bankruptcies on the dates 403 and 902, but the firm that went bankrupt on date 903 has two other bankrupt customers (on dates 734 and 842). Subsequently, a supplier of this firm went bankrupt on date 926.

Figure 11: Example of bankruptcy propagation.



Note: This is the largest part in Figure 9, which includes 35 bankrupt firms.

Figure 12: Cluster size and diameter.



Note: In the left panel, the histogram of cluster size of the bankrupt firms subnetwork is shown. The sample mean is 2.432, s.d. is 1.628, and the maximum cluster size is 35. In the right panel, the probability distribution of diameter of each cluster is shown. The sample mean is 1.125, s.d. is 0.376, and the maximum diameter is 4.

3.1 Econometric model

Our model consists of two components: Markov property and Cox's proportional hazard function. Let us assume discrete time $t = 0, 1, ..., T$. Markov property means that the probability that firm $i = 1, ..., n$ goes bankrupt on date t is determined by the *state* S_{t-1} on date $t-1$, which includes macroeconomic variables $\mathbf{X}_{t-1}$, firm-specific factors $\mathbf{Y}_{i,t-1}$ for i, and the firm status $\mathbf{Z_{t-1}}$ ($Z_{i,t-1} = 1$ if the firm is dead and 0 otherwise) on date $t-1$. To be precise, the probability of the stochastic process $\{\mathbf{Z}_t\}_{1 \le t \le T}$ over the entire period is written as follows:

$$P(\mathbf{Z}_{1 \le t \le T} \in \mathbf{A}_{1 \le t \le T} | S_0) = P(\mathbf{Z}_1 \in \mathbf{A}_1 | S_0) \cdot P(\mathbf{Z}_2 \in \mathbf{A}_2 | S_1) \cdots P(\mathbf{Z}_T \in \mathbf{A}_T | S_{T-1}) \tag{2}$$

Here, $P(\mathbf{Z}_t \in \mathbf{A}_t | S_{t-1})$ is the probability of bankruptcy on date $t-1$ conditional on the state S_{t-1}. By Markov property, the probability over the entire period is the product of bankruptcy probability on each date.

Second, we assume that the bankruptcy probability on each date $P(\mathbf{Z}_t \in \mathbf{A}_t | S_{t-1})$ is described by Cox's proportional hazard model. Namely, the hazard rate for firm i (i.e., the bankruptcy probability of firm i on date t), h_i, takes the following functional form:

$$h_i(S_{t-1}) := \lambda_0 \cdot \exp(\boldsymbol{\theta}_X^T \cdot \mathbf{X}_{t-1} + \boldsymbol{\theta}_Y^T \cdot \mathbf{Y}_{i,t-1} + \boldsymbol{\theta}_Z^T \cdot \mathbf{g}_i(\mathbf{Z}_{t-1})) \tag{3}$$

where parameters $\boldsymbol{\theta} := (\boldsymbol{\theta}_X, \boldsymbol{\theta}_Y, \boldsymbol{\theta}_Z)$ represents the effects of explanatory variables on the bankruptcy probability. Function $\mathbf{g}$ captures the effect of other firms on firm i's bankruptcy probability. For example, when the effect from customer side is considered, function $\mathbf{g}$ counts the number of bankrupt firms among the i's customers C_i, i.e., $\mathbf{g}_i(\mathbf{Z}_{t-1}) = \sum_{j \in C_i} Z_{j,t-1}/N_{C_i}$, where N_{C_i} is the number of customers at the initial point.

Given the hazard rate h_i, the bankruptcy probability on date t, $P(\mathbf{Z}_t \in \mathbf{A}_t | S_{t-1})$, is written as follows:

$$P(\mathbf{Z}_t \in \mathbf{A}_t | S_{t-1}) := \prod_{i \in I_t} h_i(S_{t-1})^{\delta_{i,t}} (1 - h_i(S_{t-1}))^{1-\delta_{i,t}}, \tag{4}$$

where I_t represents the set of surviving firms, and $\delta_{i,t}$ is an indicator function taking the value of 1 if firm i goes bankrupt on date t and 0 otherwise. The log-likelihood function $\log L$ for the parameters $\boldsymbol{\theta}$ is given as follows:

$$\log L(\boldsymbol{\theta} | \mathbf{z}_{0 \le t \le T}, \mathbf{x}_{0 \le t \le T}, \mathbf{y}_{i,0 \le t \le T}, \forall i) := \log P(\mathbf{z}_{0 \le t \le T} | \boldsymbol{\theta}; \mathbf{x}_{0 \le t \le T}, \mathbf{y}_{i,0 \le t \le T})$$

Our estimated parameters $\boldsymbol{\theta}$ are given by the maximum likelihood method:

$$\hat{\boldsymbol{\theta}} := \arg\max_{\boldsymbol{\theta}} \log L(\boldsymbol{\theta} | \mathbf{z}_{0 \le t \le T}, \mathbf{x}_{0 \le t \le T}, \mathbf{y}_{i,0 \le t \le T}, \forall i)$$

3.2 Contagion Effect

For estimation, we decompose our sample periods into two parts (see Figure 13). Bankrupt firms in the first part (April 1st, 2013 to September 30th, 2014) is used as part of the initial condition S_0 in Equation (4). Given this initial condition, we analyze how bankruptcy spreads on the network for the subsequent periods.

Figure 13: Sample periods.

Taking into the limitation of our data, we consider the following two specifications:

(I) All firms: Firm age, size, credit-score by TSR, industry and location dummies are used as $\mathbf{Y}_{i,t}$.

(II) Firms with financial variables: In addition to the variables used in Model (I), five financial ratios (profit rate, ratio of account receivables/payables to total assets, short-term and long-term leverage ratios) to control capital structure are included in $\mathbf{Y}_{i,t}$.

In both models, the quarterly GDP growth rate is used for $\mathbf{X}_t$ to control for the macroeconomic shocks to bankruptcy probability. For $\mathbf{g}(\mathbf{Z}_t)$, we use the ratios of bankrupt customers/suppliers on date t (denoted by g_d and g_s, respectively). The remaining firms, which are not analyzed as dependent variables, are used as exogenous variables, that is, these firms are used to calculate g_d and g_s.

Table 4 reports the estimation results. Column (I) shows that the coefficient for g_d is 1.931, which is statistically and economically significant. For example, if 50% of customers go bankrupt, the bankruptcy probability increases by a factor of $\exp(1.931\times 0.5) = 2.63$. In addition, the results suggest that the contagion effect from the customer side is larger than from the supplier side. This is consistent with the trade credit channel hypothesis in Boissay and Gropp (2013), that is, the trade credit is a source of the contagion effect. Column (II) shows the results for the model with financial variables, suggesting that even if we control for the heretogeneity of capital structure, the results are essentially the same as Model (I). Therefore, these results confirm that the contagion effect actually works and is an important factor of bankruptcy.

Table 4: Estimation results.

Model	(I) All firms	(II) Firms with financial variables
Contagion g_d	1.931***	2.250***
Contagion g_s	1.168***	1.110***
Firm attributes	Yes	Yes
Macro var.	Yes	Yes
Industry FE	Yes	Yes
Location FE	Yes	Yes
Financial ratios		Yes
pseudo-$R^2 (:= 1 - \frac{\log L}{\log L_0})$	.0177	.0446
# firms	$1,048,487$	$210,850$

Note: Coefficient of the contagion effect. Standard errors estimated by the observed Fisher information matrix are in parentheses. Firm attributes include firm age, firm size measured by the logarithm of the number of employees, and credit rating assigned by TSR. Accounting information means the four ratios: profit/total assets, account receivables/total assets, current liabilities/total assets, and total liabilities/total assets.

4 Aggregate impact

In this section, we turn our attention to the aggregate impact of the contagion effect. In subsection 4.1, by simulation method, we compare our model with the contagion effect and null model without the contagion effect. In subsection 4.2, we examine the role of network structure in bankruptcy propagation by using firm-level Leontief inverse matrix..

4.1 Simulations

To analyze the aggregate impact of the contagion effect θ_Z, we simulate our model with and without the contagion effect. The number of simulations for each model is 3,000. For comparison, we consider for each simulation the number of bankruptcies, the number of isolated bankruptcies (i.e., no ties to another bankrupt firm), the maximum size of the connected components, the maximum diameter for directed and undirected cases.

Simulation results are given in Figure 5. Compared to the null model, the contagion effect seems to contribute to bankruptcy propagation, but the increase is very small. In other words, the results imply that the significant contagion effect at the firm level does not necessary mean the occurrence of large-scale bankruptcy at the aggregate level. The network structure explains the reason.

Table 5: Simulation comparison.

	Mean	S.D.	min	max
Model with contagion				
# bankruptcies	8441.260	102.599	8251.000	8637.000
# isolated bankruptcies	0.891	0.005	0.882	0.901
Maximum size of the WCC	102.920	71.152	30.000	392.000
Maximum diameter (directed)	5.320	1.301	4.000	9.000
Maximum diameter (undirected)	10.880	3.657	7.000	24.000
Model without contagion				
# bankruptcies null	8364.580	105.271	8180.000	8590.000
# isolated null	0.898	0.006	0.883	0.908
Maximum size of the WCC	104.100	80.821	23.000	420.000
Maximum diameter (directed)	5.320	1.203	4.000	9.000
Maximum diameter (undirected)	10.200	2.955	6.000	18.000

4.2 Bankruptcy propagation and network structure

To analyze how likely and fast the network propagates shock, we consider the following equation (cf. Elliott et al. (2014)):

$$\mathbf{z} = \alpha \mathbf{W}'\mathbf{z} + \mathbf{e},$$

Here, $\mathbf{W} := \{w_{ij}\}_{i,j=1,\ldots,N}$ is the weigth matrix, in which $0 \leq w_{ij} \leq 1$ is positive if firm i is a customer of firm j and 0 otherwise. $\mathbf{e}$ is initial shocks, and decaying rate α satisfies $0 \leq \alpha < 1$. Namely. this equation describes the propagation of initial shocks $\mathbf{e}$ through $\mathbf{W}$ and can be seen as a static version of our model. By simple algebra, the equation can be written as

$$\mathbf{z} = (\mathbf{I} - \alpha \mathbf{W}')^{-1}\mathbf{e}.$$

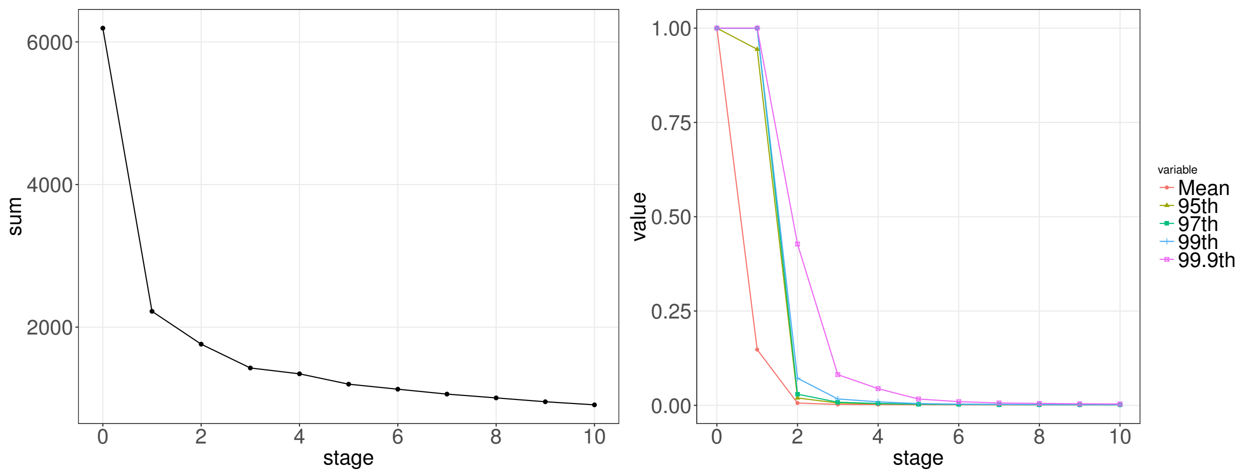
Solution $\mathbf{z}$ to this equation represents the resultant firms' states given initial shocks $\mathbf{e}$ and propagation by $\mathbf{W}$. Multiplier $\mathbf{L} := (\mathbf{I} - \alpha \mathbf{W}')^{-1}$ is Leontief's inverse matrix, which can be expanded by the sum of an infinite series:

$$\mathbf{L} = \sum_{k=0}^{\infty} \alpha^k \mathbf{W}'^k.$$

Each term in this equation represents different stage of propagation. For example, $\mathbf{W}'\mathbf{e}$ represents direct shocks from their customers (the first stage propagation) and $\mathbf{W}'^2\mathbf{e}$ represents indirect shocks from customers of their customers (the second stage propagation). By using empirical network data, we explicitly calculate the k-th stage propagation $\mathbf{W}'^k\mathbf{e}$.

The right panel of Figure 14 shows the mean, 95th, 97th, 99th and 99.9th percentile values of the cross-sectional distribution of $\mathbf{W}'^k\mathbf{e}$. It suggests that these values rapidly decrease as the stage k increases. This means that as the path length between indirectly connected firms becomes longer (3 or 4 path lengths), the effect from the indirect firms immediately die out. Because of this property, bankruptcy does not propagate because the negative shocks immediately disappear before causing another bankruptcy. This property is closely related with the high-connectivity of the network discussed in Section 2. Since the network is highly connected, there are a large number of indirect suppliers for each firm. Conversely, from the viewpoint of the indirect supplier, an indirect customer which goes bankrupt is just one of such many indirect customers, and therefore, the effect of the bankruptcy is negligible. We also consider the effect from supply side in Figure 15 and from both sides in Figure 16, and confirm that the results are similar to Figure 14. These results show that the high-connectivity reduces—rather than contributes—the risk of bankruptcy propagation.

Figure 14: Leontief inverse matrix for demand shocks.



Note: The left panel show the sum of indirect shocks across firms, i.e., $:= \sum_i (\mathbf{W}'^k \mathbf{e})_{i\times 1}$. The right panel show the mean, 95th, 97th, 99th and 99.9th percentile values of the cross-sectional distribution of $(\mathbf{W}'^k \mathbf{e})_{i\times 1}$ over i.

Figure 15: Leontief inverse matrix for supply shocks.

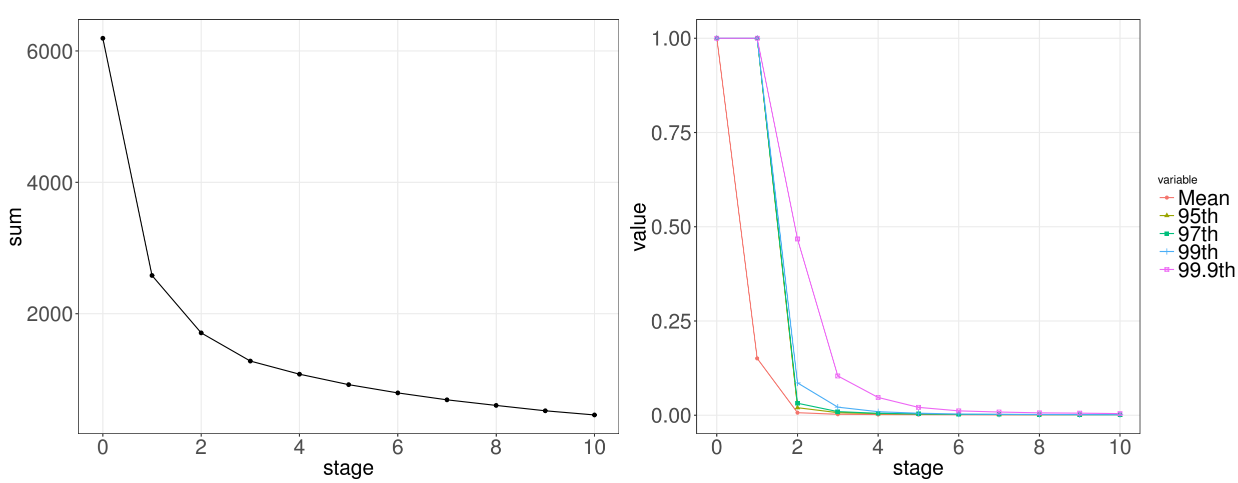
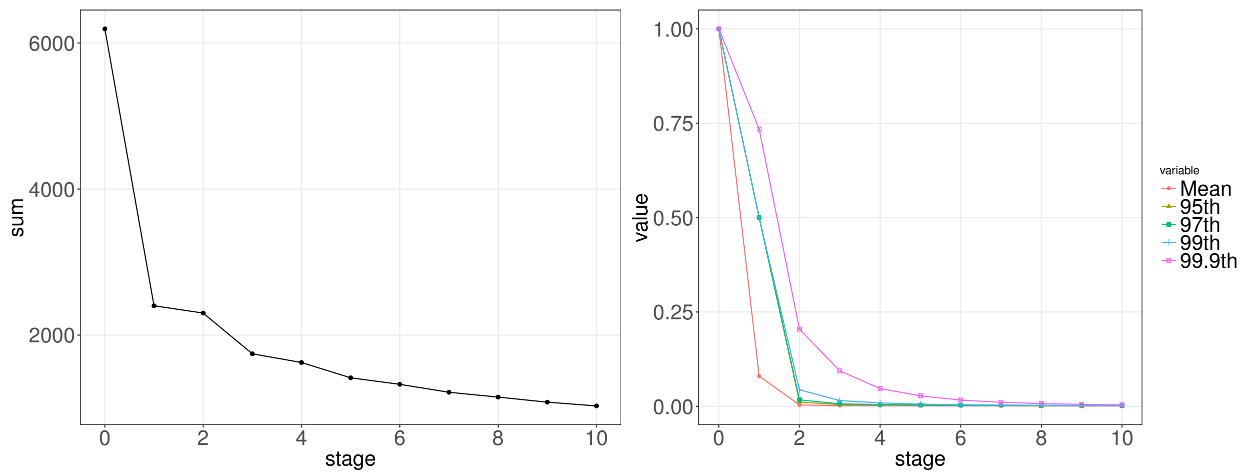


Figure 16: Leontief inverse matrix for both demand and supply shocks.



5 Concluding Remarks

The importance of customer-supplier networks has received increasing attention in recent years. One of important examples in which network plays a crucial role is the bankruptcy propagation via customer-supplier relationships. A firm's bankruptcy may lead to a considerable loss on its suppliers and force them into bankruptcy, resulting in a severe economic downturn. The assessment of such risk is important from both research and policy perspectives.

This paper tackled this issue by exploiting a comprehensive dataset of more than one million firms and five million transaction relationships as well as bankruptcy records. This dataset enables us to trace how bankruptcies spread on the network. We developed a statistical model similar to survival analysis and estimate the contagion effect. We found that the contagion effect is significant, and in particular, the effect from the demand side is larger than from supply side, consistent the trade credit channel hypothesis. At the firm level, the contagion effect is an important factor in corporate bankruptcy.

However, this does not necessarily imply that one bankruptcy trigger subsequent bankruptcy cascades and causes nonnegligible aggregate fluctuations. This is because bankruptcy propagation is determined by the combined effects of the contagion effect at the firm level and network structure. We found that by simulating our model, the aggregate impact of the contagion effect is negligible. Since the network is highly connected, bankruptcy shocks immediately diluted before causing another bankruptcy. In other words, due to the network structure, bankruptcy shocks are immediately absorbed by an aggregate economy. Our results suggest that the potential risk of severe economic downturn by bankruptcy propagation, which is often used to justify policy intervention, has been overemphasized in reality.

The network sometimes becomes a vehicle propagating shocks, and sometimes a firewall preventing shock propagation. With the increasing use of big data recently, an in-depth examination at the firm level has become possible and the detail network structure has been revealed. Further analysis based on detail firm level information has become crucial for the better understanding of the aggregate behavior of complex systems. Our finding contributes to this promising area.

Acknowledgments

This research was conducted as part of the Project "Large-scale Simulation and Analysis of Economic Network for Macro Prudential Policy" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). This research was supported by MEXT as Exploratory Challenges on Post-K computer (Studies of Multi-level Spatiotemporal Simulation of Socioeconomic Phenomena, Macroeconomic Simulations). This research used computational resources of the K computer provided by the RIKEN Advanced Institute for Computational Science through the HPCI System Research project (Project ID: hp170242, hp180177). This study is financially supported by JSPS KAKENHI Grant Numbers 17H02041, 18K12788. Conflicts of interest: none.

References

Acemoglu, D., Carvalho, V. M., Ozdaglar, A., Tahbaz-Salehi, A., 2012. The network origins of aggregate fluctuations. Econometrica 80 (5), 1977–2016.

Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A., 2015. Systemic risk and stability in financial networks. American Economic Review 105 (2), 564–608.

Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A., 2017. Microeconomic origins of macroeconomic tail risks. American Economic Review 107 (1), 54–108.

Allen, F., Babus, A., Carletti, E., 2012. Asset commonality, debt maturity and systemic risk. Journal of Financial Economics 104 (3), 519–534.

Allen, F., Gale, D., 2000. Financial contagion. Journal of Political Economy 108 (1), 1–33.

Axtell, R. L., 2001. Zipf distribution of us firm sizes. science 293 (5536), 1818–1820.

Baqaee, D. R., forthcoming. Cascading failures in production networks. Econometrica.

Benoit, S., Colliard, J.-E., Hurlin, C., Pérignon, C., 2017. Where the risks lie: A survey on systemic risk. Review of Finance 21 (1), 109–152.

Bigio, S., La'O, J., 2016. Financial frictions in production networks. Tech. rep., National Bureau of Economic Research.

Billio, M., Getmansky, M., Lo, A. W., Pelizzon, L., 2012. Econometric measures of connectedness and systemic risk in the finance and insurance sectors. Journal of financial economics 104 (3), 535–559.

Boissay, F., Gropp, R., 2013. Payment defaults and interfirm liquidity provision. Review of Finance 17 (6), 1853–1894.

Brunnermeier, M. K., Pedersen, L. H., 2008. Market liquidity and funding liquidity. The Review of Financial Studies 22 (6), 2201–2238.

Cabrales, A., Gottardi, P., Vega-Redondo, F., 2017. Risk sharing and contagion in networks. The Review of Financial Studies 30 (9), 3086–3127.

Dasgupta, A., 2004. Financial contagion through capital connections: A model of the origin and spread of bank panics. Journal of the European Economic Association 2 (6), 1049–1084.

Elliott, M., Golub, B., Jackson, M. O., 2014. Financial networks and contagion. American Economic Review 104 (10), 3115–53.

Gabaix, X., 2009. Power laws in economics and finance. Annu. Rev. Econ. 1 (1), 255–294.

Gai, P., Haldane, A., Kapadia, S., 2011. Complexity, concentration and contagion. Journal of Monetary Economics 58 (5), 453–470.

Gai, P., Kapadia, S., 2010. Contagion in financial networks. In: Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences. The Royal Society, p. rspa20090410.

Glasserman, P., Young, H. P., 2015. How likely is contagion in financial networks? Journal of Banking & Finance 50, 383–399.

Glasserman, P., Young, H. P., 2016. Contagion in financial networks. Journal of Economic Literature 54 (3), 779–831.

Gofman, M., 2017. Efficiency and stability of a financial architecture with too-interconnected-to-fail institutions. Journal of Financial Economics 124 (1), 113–146.

Jacobson, T., Schedvin, E., 2015. Trade credit and the propagation of corporate failure: an empirical analysis. Econometrica 83 (4), 1315–1371.

Leitner, Y., 2005. Financial networks: Contagion, commitment, and private sector bailouts. The Journal of Finance 60 (6), 2925–2953.

Newman, M. E., 2002. Assortative mixing in networks. Physical review letters 89 (20), 208701.

Newman, M. E., 2003. Mixing patterns in networks. Physical Review E 67 (2), 026126.