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Heterogeneous Labor and Agglomeration over Generations*

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Abstract

The productivity in cities is enhanced by the interaction between heterogeneous workers who are born and raised in various regions and countries. However, such benefit does not last forever because the composition of workers in cities becomes homogenized over generations. To evaluate the agglomeration economies and diseconomies of labor heterogeneity, this paper constructs a two-region non-overlapping generations model. Workers are assumed to be differentiated in terms of their birthplaces. Although they may migrate from their home regions to other regions to work as foreigners, they should incur an adjustment cost due to cultural differences. Assuming that the distribution of workers' births depends on their previous generation's residency choices, this study obtained the following results: (i) In the short run, residency choice leads workers to disperse across regions in each period. In the long run, however, the accumulation of residency choices over time makes birth distributions concentrated in a single region. Consequently, the composition of the workers becomes homogenized and they continue to reside in one region in a steady-state equilibrium. (ii) Social welfare is maximized by an even distribution of births involving a persistent circulation of heterogeneous labor. A comparison between the social optimum and the steady-state equilibrium indicates a dynamic inefficiency due to generational transition. (iii) When housing consumption is introduced as a dispersion force, social welfare can be maximized in a steady-state equilibrium with an equal distribution. (iv) Contrarily, even when another agglomeration economy is introduced on account of the quantity of labor, distribution of births in a steady-state equilibrium is still concentrated in comparison to the social optimum.

Keywords: Heterogeneous labor, Non-overlapping generations model, Migration, Agglomeration economies

JEL classification: R12, R23, J61

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1 Introduction

Along with the progress of agglomeration and urbanization in the world economy, recent decades have witnessed an increasing interest in labor diversity. World Urbanization Prospects (2014 Revision) reports that "the urban population of the world has grown rapidly since 1950, from 746 million to 3.9 billion in 2014." Running parallel with the such progressively concentrated population, the number of immigrants has also been increasing in the world. For instance, International Migration Outlook (2017) reports that the foreigners as a share of population in the United States increased from 10.7 % in 2000 to 13.5 % in 2015. As Jacobs (1961) pointed out, labor diversity a source of productivity in urban areas. Cities attract heterogeneous workers who have different values, cultures and ways of thinking, which enhances urban productivity. In fact, as many have noted, the prosperity of Silicon Valley is reinforced by foreign workers from various countries. [1]

Numerous researchers have studied the effect of labor diversity on productivity in recent years. For instance, Florida (2002) emphasizes that regional creativity is explained by cultural heterogeneity and tolerance to it, which are measured by various indices such as the "Gay index," the "Bohemian index," and so on. The studies conducted by Ottaviano and Peri (2005, 2006) and Bellini et al. (2013) respectively show the effects of cultural heterogeneity on urban productivity in the United States and Europe. Sparber (2009) also argues that racial diversity has a positive effect on wages in the United States, especially in legal services, computer manufacturing, and computer software. Furthermore, Iranzo et al. (2008) and Navon (2010) explain that diversity in skills and knowledge is beneficial for firms and plants alike. In a theoretical study, Berliant and Fujita (2008; 2012) explained the micro-foundation of knowledge creation and pointed out the role of culture and communication costs in it.

Turning to the relation between agglomeration and labor diversity, the new economic geography (NEG) presents a useful framework. Originating with Krugman (1991) and Fujita et al. (1999), numerous studies have attempted to explain the cumulative agglomeration of economic activities in ongoing globalization processes. Focusing on labor diversity, for instance, Amiti and Pissarides (2005) explained that trade liberalization causes industrial agglomeration and interregional trade when labor is heterogeneous, and Ottaviano and Prarolo (2009) showed that multicultural cities emerged when communication between them was easy.

Adding to the stream of work on agglomeration and labor heterogeneity, this paper introduces a negative aspect of agglomeration: labor heterogeneity does not last forever. As Berliant and Fujita (2008; 2012) pinpoint, the heterogeneity of workers will decline if they communicate and collaborate together for a long time, because their knowledge and way of thinking are gradually homogenized. Similar tendencies may be also observed in the transition of generations. Some of the significant parts of labor characteristics are formed under the influence of one's regional environment, e.g., culture, lifestyle, habits, language, etc. As a consequence, the original characteristics that migrant workers have, which contribute to productivity, can change over generations. In other words, even though heterogeneous workers from various regions and countries migrate to a city, their children (i.e., the next generation workers) will become homogenous.

To consider negative effects related to the agglomeration process, this paper constructs a two-region, non-overlapping generations model. The remainder of this paper is organized as follows. Section 2 presents the framework of the model, and Section 3 considers the essential case that will provide the main results of the paper. Sections 4 and 5 expand the analysis by introducing housing consumption and another agglomeration economies due to labor amount and, finally, Section 6 concludes this paper.

[1] As will be stated later, labor heterogeneity could be of little consequence (or sometimes even harmful) to some traditional production sectors and industries, while it is still quite important and positively impacts creative industries such as R&D and high-tech industries. This paper largely focuses on the latter aspect.

2 The model

This section presents a two-region non-overlapping generations model, whose structure is summarized as follows. In each period, workers are born, then choose a region in which to work, and finally leave the job market. The interregional distribution of births is decided through the residency choices of the prior generation (i.e., one's parents). The workers are assumed to be differentiated in terms of birthplace. If they migrate from their home region to the other region for work, they incur an adjustment cost imposed by cultural differences.

2.1 Production sector

Following Ottaviano et al. (2005), homogenous consumption goods are produced using labor and capital. Assuming labor is differentiated in terms of its origin, the production function is expressed as

$$Q_{it} = A_{it} K_{it}^{1-\alpha} \sum_{j=1}^{2} l_{jit}^{\alpha}, \tag{1}$$

where A_{it} means the level of technology in region i in period t, K_{it} is the amount of capital locating in region i, and l_{jit} is the number of workers who were born in region j and reside and work in region i. [2]

Letting r_{it}^{K} and w_{jit} be respectively the reward to capital and the wage for labor, and letting p be the price of consumption goods, the profit function is expressed as

$$\pi_{it} = pQ_{it} - r_{it}^{K} K_{it} - \sum_{j=1}^{2} w_{jit} l_{jit}, \tag{2}$$

and the profit maximization yields the following factor prices:

$$r_{it}^{K} = (1-\alpha) p A_{it} K_{it}^{-\alpha} \sum_{j=1}^{2} l_{jit}^{\alpha}, \tag{3}$$

$$w_{jit} = \alpha p A_{it} K_{it}^{1-\alpha} l_{jit}^{\alpha-1}. \tag{4}$$

Capital is assumed to be freely mobile between the regions, thus $r_{1t}^{K} = r_{2t}^{K}$ yields the capital distribution as

$$\frac{K_{1t}}{K_{1t}+K_{2t}} = \frac{A_{1t}^{1/\alpha}(l_{11}^{\alpha}+l_{21}^{\alpha})^{1/\alpha}}{A_{1t}^{1/\alpha}(l_{11}^{\alpha}+l_{21}^{\alpha})^{1/\alpha} + A_{2t}^{1/\alpha}(l_{22}^{\alpha}+l_{12}^{\alpha})^{1/\alpha}}. \tag{5}$$

Workers can also choose their residential regions. However, the workers migrating from their home region to the other (foreign) one, must incur adjustment costs for cultural differences in, e.g., language, customs, lifestyle, habits. On the one hand, supposing the adjustment costs take in an iceberg form, foreigners' *effective* labor amount available for production is rewritten as

$$l_{jit} = L_{jit}\tau, \qquad \text{for} \quad i \neq j, \tag{6}$$

where L_{jit} be the *actual* number of foreign workers, and $\tau \in (0, 1)$ be the easiness to adjust. Consequently the effective wage which each worker receives is written as

$$W_{jit} = w_{jit}\tau, \qquad \text{for} \quad i \neq j. \tag{7}$$

[2]In this production function, labor input is in a CES form, where the elasticity of substitution of labor is assumed to equal to $1/(1-\alpha)$ to make the model tractable. Such a restriction does not lose the generality of the results in this paper.

On the other hand, local workers remaining in their home region do not incur any adjustment costs, thus $l_{iit} = L_{iit}$ and $W_{iit} = w_{iit}$. As a result, effective wages are expressed as

$$W_{11t} = \alpha p A_{1t}^{1/\alpha} K^{1-\alpha} \left(\frac{(L_{11}^{\alpha} + (L_{21}\tau)^{\alpha})^{1/\alpha}}{A_{1t}^{1/\alpha}(L_{11}^{\alpha} + (L_{21}\tau)^{\alpha})^{1/\alpha} + A_{2t}^{1/\alpha}(L_{22}^{\alpha} + (L_{12}\tau)^{\alpha})^{1/\alpha}} \right)^{1-\alpha} L_{11}^{\alpha-1}, \tag{8}$$

$$W_{12t} = \alpha p A_{2t}^{1/\alpha} K^{1-\alpha} \left(\frac{(L_{22}^{\alpha} + (L_{12}\tau)^{\alpha})^{1/\alpha}}{A_{1t}^{1/\alpha}(L_{11}^{\alpha} + (L_{21}\tau)^{\alpha})^{1/\alpha} + A_{2t}^{1/\alpha}(L_{22}^{\alpha} + (L_{12}\tau)^{\alpha})^{1/\alpha}} \right)^{1-\alpha} L_{12}^{\alpha-1}\tau^{\alpha}, \tag{9}$$

$$W_{22t} = \alpha p A_{2t}^{1/\alpha} K^{1-\alpha} \left(\frac{(L_{22}^{\alpha} + (L_{12}\tau)^{\alpha})^{1/\alpha}}{A_{1t}^{1/\alpha}(L_{11}^{\alpha} + (L_{21}\tau)^{\alpha})^{1/\alpha} + A_{2t}^{1/\alpha}(L_{22}^{\alpha} + (L_{12}\tau)^{\alpha})^{1/\alpha}} \right)^{1-\alpha} L_{22}^{\alpha-1}, \tag{10}$$

$$W_{21t} = \alpha p A_{1t}^{1/\alpha} K^{1-\alpha} \left(\frac{(L_{11}^{\alpha} + (L_{21}\tau)^{\alpha})^{1/\alpha}}{A_{1t}^{1/\alpha}(L_{11}^{\alpha} + (L_{21}\tau)^{\alpha})^{1/\alpha} + A_{2t}^{1/\alpha}(L_{22}^{\alpha} + (L_{12}\tau)^{\alpha})^{1/\alpha}} \right)^{1-\alpha} L_{21}^{\alpha-1}\tau^{\alpha}. \tag{11}$$

In the final section of this paper, we will examine the effect of introducing another kind of agglomeration economy that depends on the amount of labor, in addition to the agglomeration economy due to labor heterogeneity. For this purpose, we have the following equation:

$$A_{it} = [1 + \lambda_{it}L_{it} + (1-\lambda_{jt})L_{jt}\tau]^{\gamma}. \tag{12}$$

This equation means the technology level of each region is increasing in the total amount of the effective labor, where γ captures the degree of the agglomeration economy according to the amount of labor.

2.2 Consumption behavior

Consumption behavior is described as follows. Utility function of each worker born in region j and resides in region i is

$$U_{jit} = c_{jit}^{1-\mu} h_{jit}^{\mu}, \tag{13}$$

where c_{jit} and h_{jit} are respectively the amounts of consumption goods and housing land, and μ is the consumption share for housing. The budget constraint is given by $y_{jit} = pc_{jit} + r_{jit}^{H} h_{jit}$, where y_{jit} means personal income and r_{it}^{H} means housing rent. Utility maximization yields the following optimum consumptions:

$$c_{jit} = (1-\mu)y_{jit}/p, \tag{14}$$

$$h_{jit} = \mu y_{jit}/r_{it}^{H}. \tag{15}$$

Considering the housing market, housing rent is given by

$$r_{it}^{H} = \mu \frac{L_{iit}y_{iit} + L_{jit}y_{jit}}{H_i}, \tag{16}$$

where H_i is the total amount of land for housing. As a result, the indirect utility is given by

$$V_{jit} = \left(\frac{1-\mu}{p} \right)^{1-\mu} \frac{y_{jit}H_i^{\mu}}{(L_{jj}y_{jj} + L_{ij}y_{ij})^{\mu}}. \tag{17}$$

Assuming both capital and land for housing are equally owned by all the workers, the income of each worker is expressed as

$$y_{jit} = W_{jit} + RK_t + RH_t, \quad RK_t = \frac{\sum_{j=1}^{2} r_{jt}^{K} K_{jt}}{\sum_{i=1}^{2}\sum_{j=1}^{2} L_{ji}}, \quad RH_t = \frac{\sum_{j=1}^{2} r_{jt}^{H} H_{jt}}{\sum_{i=1}^{2}\sum_{j=1}^{2} L_{ji}}. \tag{18}$$

2.3 Short-run and Long-run analyses

The *short-run* analysis is of the residential choice of workers in each period. Since workers migrate to a region where they can enjoy higher indirect utility, the short-run equilibrium is given by the residency distribution equalizing indirect utilities found in the two regions. Now let L_{it} be the number of births (i.e., the number of workers who were born) in region i, and λ_{it} be the share of workers who choose to remain in their home region (and work as local workers). Then we can rewrite the number of workers as $L_{iit} = \lambda_{it} L_{it}$ and $L_{ijt} = (1 - \lambda_{it}) L_{it}$. Consequently the share of workers in the short-run equilibrium, λ^*_{it}, can be derived from $V_{iit} = V_{ijt}$, taking L_{it} as given.

In addition, let us organize the notation for analytical simplicity. The total amounts of labor and capital are respectively set to unity: $L_{1t} + L_{2t} = 1$ and $K_{1t} + K_{2t} = 1$. Similarly, the total amount of housing land in each region equals unity: $H_1 = H_2 = 1$. In addition, we choose a unit such that $p = 1/\alpha$.

Finally, let us describe the *long-run* analysis of generational transition. The transition of the number of births is given by the following difference equation:

$$L_{1t+1} = \lambda^*_{1t}(L_{1t}) \cdot L_{1t} + (1 - \lambda^*_{2t}(L_{1t})) \cdot (1 - L_{1t}) \equiv f(L_{1t}). \tag{19}$$

That is, the number of workers residing in region 1 in period t, $f(L_{1t})$, equals to the number of workers who will be born in the next period, L_{1t+1}.[3] Following this equation, the interregional distribution of workers will converge to a steady state.

3 The essential analysis

We first consider an essential case which eliminates both the housing consumption and agglomeration economies due to the amount of labor by setting $\mu = 0$ and $\gamma = 0$. Although we will expand the analysis in the next sections, this section derives the main results of this paper.

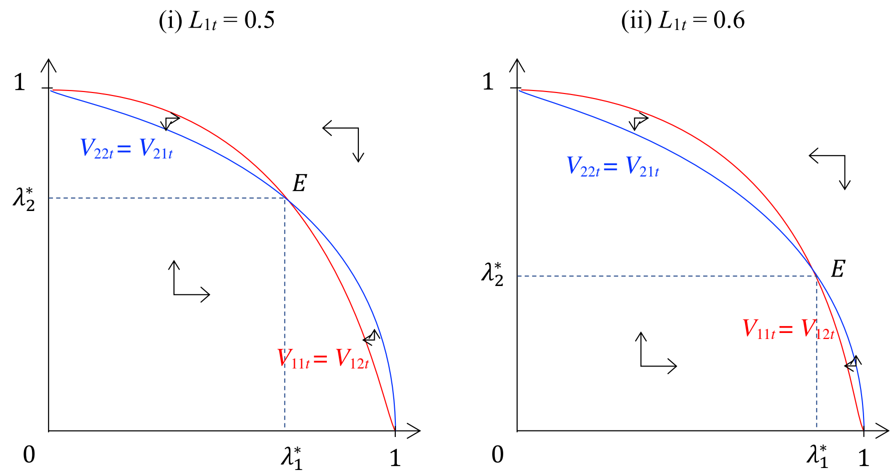
3.1 The short-run Analysis of residency choice

The first analysis considers the short run. Workers choose their residency region in each period, taking their distribution of births as given. At equilibrium, the share of workers staying in their home region, λ^*_{it}, is derived from $V_{iit} = V_{ijt}$. Appendix 1 shows the precise form of the indirect utilities. To grasp the basic feature of this model, Figure 1 numerically shows the implicit functions, $V_{11t} = V_{12t}$ and $V_{22t} = V_{21t}$, in the $(\lambda_{1t}, \lambda_{2t})$ plane. The left panel is for the case of $L_{it} = 0.5$, and the right panel is for the case of $L_{it} = 0.6$. Point E at which two lines intersect indicates equilibrium, and the horizontal and vertical arrows are respectively the migration choices of workers born in regions 1 and 2. Considering the directions of the arrows, we can see that the equilibrium point is stable in Figure 1.

Fig. 1. The short-run equilibrium of residency choice

[3] The difference equations of the two regions are expressed below. Substituting $L_{2t} = 1 - L_{1t}$ into these equations we can confirm that the equations are the same.

$$L_{1t+1} = \lambda^*_{1t}(L_{1t}, L_{2t}) \cdot L_{1t} + (1 - \lambda^*_{2t}(L_{1t}, L_{2t})) \cdot L_{2t},$$
$$L_{2t+1} = \lambda^*_{2t}(L_{1t}, L_{2t}) \cdot L_{2t} + (1 - \lambda^*_{1t}(L_{1t}, L_{2t})) \cdot L_{1t}.$$



3.1.1 Under the equal division of births

As a beginning of the analysis, we will examine the case of symmetric distribution of births, $L_{1t} = 1/2$. Substituting $L_{1t} = 1/2$ into V_{ijt} and solving $V_{11t} = V_{12t}$ and $V_{22t} = V_{21t}$ for λ_{it}, the equilibrium share of workers choosing to remain in their home region is given by

$$\lambda_{it}^* = \frac{1}{1+\tau^{\alpha/(1-\alpha)}} > \frac{1}{2}. \tag{20}$$

Note that λ_{it}^* is decreasing in τ, which means workers less hesitate to migrate to foreign region, as the adjustment cost falls. The stability of the equilibrium is calculated by the total derivative derived from $V_{11t} = V_{12t}$:

$$\left.\frac{d\lambda_{2t}}{d\lambda_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} = -1 - \frac{\left(1-\tau^{\alpha/1-\alpha}\right)^2}{2\tau^{\alpha/1-\alpha}} < -1. \tag{21}$$

This equation means the slope of the line $V_{11t} = V_{12t}$ is less than -1 at the equilibrium point E in Figure 1. Contrarily, the slope of the line $V_{22t} = V_{21t}$ is larger than -1, which is derived from the inverse of the RHS of equation (21). Since the slope of $V_{11t} = V_{12t}$ is steeper than that of $V_{22t} = V_{21t}$, we can see that the equilibrium is stable. Besides, Appendix 2 investigates the corner point $(\lambda_{1t}, \lambda_{2t}) = (1, 0)$, where the slope of $V_{22t} = V_{21t}$ is always steeper than that of $V_{11t} = V_{12t}$. Consequently, we have

Lemma 1. *In the case of symmetric distribution of births, workers are dispersed among the regions at short-run equilibrium.*

Next, if the births distribution deviates from this symmetry, how will the equilibrium share change? The answer is given by $d\lambda_{it}^*/dL_{it}$, which is derived from the total derivatives of $V_{iit} = V_{ijt}$. Appendix 1 shows details of the calculation.

$$\left.\frac{d\lambda_{1t}^*}{dL_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} = -\left.\frac{d\lambda_{2t}^*}{dL_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} = \frac{4\tau^{\alpha/1-\alpha}}{1-\tau^{2\alpha/1-\alpha}} > 0. \tag{22}$$

As shown in the right panel of Figure 1, the larger share is of births distribution, the more workers will tend to stay in their home region.

3.1.2 Under the core-periphery division of births

Let us consider a core-periphery structure in which all workers are born in one of the two regions. For instance, if we consider a case where region 1 is the core, the following equation explains that all the workers choose to remain in their home region.

$$\left.\frac{V_{11t}}{V_{12t}}\right|_{L_{1t}=1,\lambda_{1t}=1} = \frac{1}{\tau} > 1. \tag{23}$$

Therefore we have

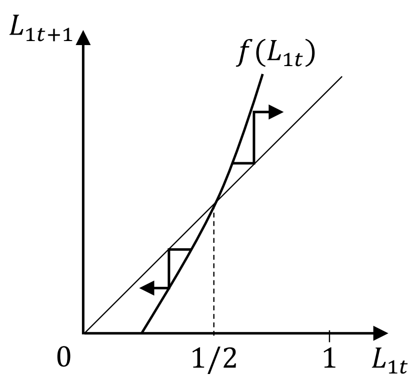
Lemma 2. *In the core-periphery structure of the births distribution, the concentration equilibrium of the residency choice is always sustainable.*

It is clear that the existence of the adjustment cost for foreign workers keeps the workers in their home region.

3.2 The long-run analysis of generational transition

Now we consider the transitional dynamics of births distribution over periods. As mentioned above, equation (19) relates the births distributions of current and next generations. A numerical example of the transition dynamics (for the case of $\alpha = 0.7$ and $\tau = 0.6$) is shown in Figure 2. This figure reveals that the symmetric structure is not stable in the long run, and the number of births converges to the core-periphery structure across generations.

Fig 2. Transitional dynamics of births distribution



Let us investigate the stability of the symmetric structure in detail. First, we can easily confirm that the symmetric structure is in a steady state, by substituting $L_{1t}^* = 1/2$ and equation (20) into equation (19). Then, the total derivative of equation (19) yields

$$\frac{df}{dL_{1t}} = L_{1t}\frac{d\lambda_{1t}^*}{dL_{1t}} - (1 - L_{1t})\frac{d\lambda_{2t}^*}{dL_{1t}} + \lambda_{1t}^* + \lambda_{2t}^* - 1, \tag{24}$$

and substituting equations (20) and (22) into equation (24) yields

$$\left.\frac{df}{dL_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} = \frac{1+\tau^{\alpha/(1-\alpha)}}{1-\tau^{\alpha/(1-\alpha)}} > 1. \tag{25}$$

This equation means the slope of the line $f(L_{1t})$ is larger than 1 at $L_{1t} = 1/2$. Hence we obtain the following proposition.

Proposition 1. *The equal division of the births is always unstable in the long run.*

That is, if the births distribution deviates from the symmetry in a period, the residency distribution of workers becomes more concentrated than the births distribution. Since the residency distribution is turned over to the next generation's births distribution, the accumulation of generational change leads workers to concentrate in one of the two regions step-by-step. Consequently, the distribution of workers converges to the steady-state equilibrium of $L_1^* = 0$ or $L_1^* = 1$. Besides, it is noteworthy that the cumulative agglomeration process in the long run is highly contrasting to the dispersion of residency choice in the short run.

3.3 Social optimum

Now we consider a government which chooses λ_1 and λ_2 to maximize the social welfare SW in the steady state. The social welfare in this paper is given as the aggregation of the indirect utility of workers. Thus the behavior of the government is described as

$$\max_{\lambda_1,\lambda_2} SW = V_{11t}\lambda_1 L_{1t} + V_{12t}(1-\lambda_1)L_{1t} + V_{22t}\lambda_2 L_{2t} + V_{21t}(1-\lambda_2)L_{2t}. \tag{26}$$

Similarly to equation (19), the transitional dynamics of the births distribution is expressed as follows.

$$L_{1t+1} = \lambda_1 L_{1t} + (1-\lambda_2)(1-L_{1t}). \tag{27}$$

Note that λ_i is not depending on L_{1t}, unlike equation (19). Then, the steady state is given by omitting t from equation (27) and solving it for L_1:

$$\tilde{L}_1 = \frac{1-\lambda_2}{2-\lambda_1-\lambda_2}. \tag{28}$$

Though the social welfare function SW is somewhat complicated, we can see some of the features by

$$SW|_{L_1=\tilde{L}_1,\lambda_1=1} = SW|_{L_1=\tilde{L}_1,\lambda_2=1} = 1, \tag{29}$$

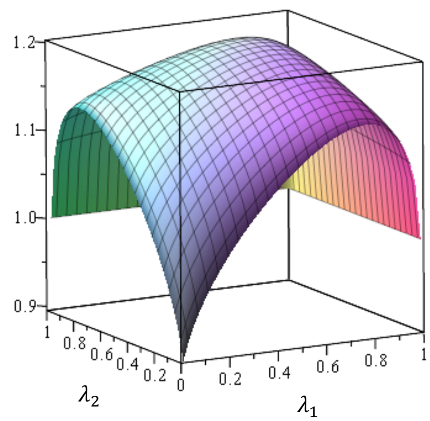
$$SW|_{L_1=\tilde{L}_1,\lambda_2=\lambda_1} = (\lambda_1)^\alpha + ((1-\lambda_1)\tau)^\alpha. \tag{30}$$

From these equations, we can see the social welfare is maximized at $\lambda_i = 1/(1+\tau^{\alpha/(1-\alpha)})$, which implies $\tilde{L}_1 = 1/2$. As shown in Appendix 3, it must be noted that the case of ($\lambda_1 = 1$, $\lambda_2 = 1$) is a special one, where there is no migration between the regions. Consequently, we have

Proposition 2. *The equal division of the births maximizes the social welfare.*

In other words, this result implies that the interregional circulation of workers maintains labor heterogeneity. Figure 3 shows a numerical example for the case of $\alpha = 0.6$ and $\tau = 0.7$. A comparison between Propositions 1 and 2 concludes that the social optimum cannot be achieved by the steady-state equilibrium. It is noteworthy that such a dynamic inefficiency conducted by the transition of generations is not observed in the ordinary NEG models so far.

Fig 3. Social welfare



4 Housing consumption

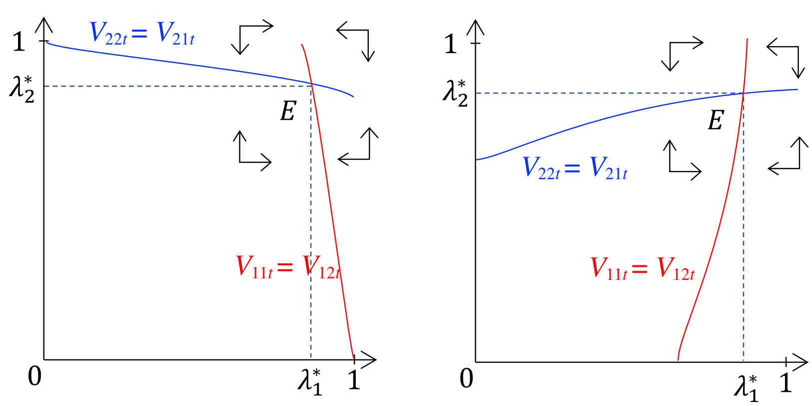
Sections 4 and 5 extend the above analysis to explore the relation between the steady-state equilibrium and the social optimum. First, we consider a case of $\mu > 0$ and $\gamma = 0$, where housing consumption is positive, which will raise incentives to disperse within residency choices.

4.1 The short-run analysis of residency choice

In the same way as the previous section, we first consider the residency choice in a symmetric births distribution, $L_{1t} = 1/2$.[4] Before the detailed analysis, numerical examples of the implicit functions $V_{11t} = V_{12t}$ and $V_{11t} = V_{12t}$ are shown in Figure 4, for the case of $\alpha = 0.6$ and $\tau = 0.3$. Depending on the value of μ, the slope of the lines changes from negative to positive. However, as shown in the two panels, the direction of the arrows reveals the equilibrium points are stable in both cases.

Fig 4. The residency choice

(i) $\mu = 0.1$ (ii) $\mu = 0.3$



[4]This section focuses only on the symmetric structure of births distribution and does not consider the core-periphery structure, since the housing consumption interrupts the full-agglomeration of population.

Let us approach to the equilibrium more precisely. First, the equilibrium share of the workers remaining in the home region is given by equation (20), by solving $V_{iit} = V_{ijt}$ for the case of $L_{1t} = 1/2$. Next, the stability of the equilibrium point E is given by the total derivative of $V_{11t} = V_{12t}$:

$$\left.\frac{d\lambda_{2t}}{d\lambda_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} = -1 - \frac{\alpha(1-\alpha)\left(1-\tau^{\alpha/(1-\alpha)}\right)^2 + 4\mu\tau^{\alpha/(1-\alpha)}}{2[\alpha(1-\alpha)(1-\mu)-\mu]\tau^{\alpha/(1-\alpha)}}. \tag{31}$$

Hence, the slope of the line $V_{11t} = V_{12t}$ at point E is negative and less than -1 when $\mu < \mu^* \equiv \alpha(1-\alpha)/(1+\alpha(1-\alpha))$; it is positive and larger than 1 when $\mu > \mu^*$; and it becomes ∞ as $\mu \to \mu^*$. As a consequence, the equilibrium point E is stable for all the cases of μ.

Next, the effect on the equilibrium share of a slight change in the births distribution from symmetry is expressed as

$$\left.\frac{d\lambda_{1t}^*}{dL_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} = -\left.\frac{d\lambda_{2t}^*}{dL_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} = \frac{4(\alpha(1-\alpha)(1-\mu)-\mu)\tau^{\alpha/(1-\alpha)}\left(1-\tau^{\alpha/(1-\alpha)}\right)}{\left(\alpha(1-\alpha)(1-\mu)\left(1-\tau^{\alpha/(1-\alpha)}\right)^2 + 4\mu\tau^{\alpha/(1-\alpha)}\right)\left(1+\tau^{\alpha/(1-\alpha)}\right)}, \tag{32}$$

which becomes negative when $\mu > \mu^*$. Therefore, when the share of housing consumption is sufficiently high, an increase in the number of births pushes out the workers to foreign regions because of a rise in housing rent. Contrarily, if μ is sufficiently low, such an increase in the number of births results in a rise of the number of workers remaining in their home regions, which is similar to what was said in Section 3.

4.2 The long-run analysis of generational transition

Secondly, we analyze the steady state of the generational transition. The symmetric structure, $L_{1t}^* = 1/2$, is in a steady state, and the stability is examined by equation (32) and the total differential of equation (19):

$$\left.\frac{df}{dL_{it}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} \equiv g(\tau) = \frac{\alpha(1-\alpha)(1-\mu)\left(1-\tau^{2\alpha/(1-\alpha)}\right)}{\alpha(1-\alpha)(1-\mu)\left(1-\tau^{\alpha/(1-\alpha)}\right)^2 + 4\mu\tau^{\alpha/(1-\alpha)}}. \tag{33}$$

Recalling the argument on Figure 2 in Section 3, the symmetric structure is unstable when the value of $f(L_{it})$ is larger than 1. The value is investigated in the following manner. First, we can easily see that $g(0) = 1$ and $g(1) = 0$. Then, the slope of $g(\tau)$ at $\tau = 0$ is given by

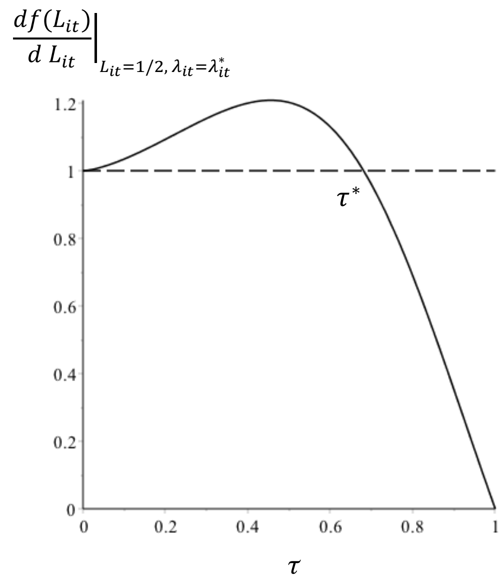
$$\lim_{\tau\to 0}\frac{dg}{d\tau} = \frac{2\alpha(\alpha(1-\alpha)(1-\mu)-2\mu)}{\alpha(1-\alpha)^2(1-\mu)} \cdot \lim_{\tau\to 0}\tau^{(2\alpha-1)/(1-\alpha)}. \tag{34}$$

Therefore, if $\mu < \mu^{**} \equiv \alpha(1-\alpha)/(2+\alpha(1-\alpha))$, the symmetric structure becomes stable when adjustment cost is sufficiently low such that

$$\tau > \tau^* = \left(\frac{\alpha(1-\alpha)(1-\mu)-2\mu}{\alpha(1-\alpha)(1-\mu)}\right)^{(1-\alpha)/\alpha}. \tag{35}$$

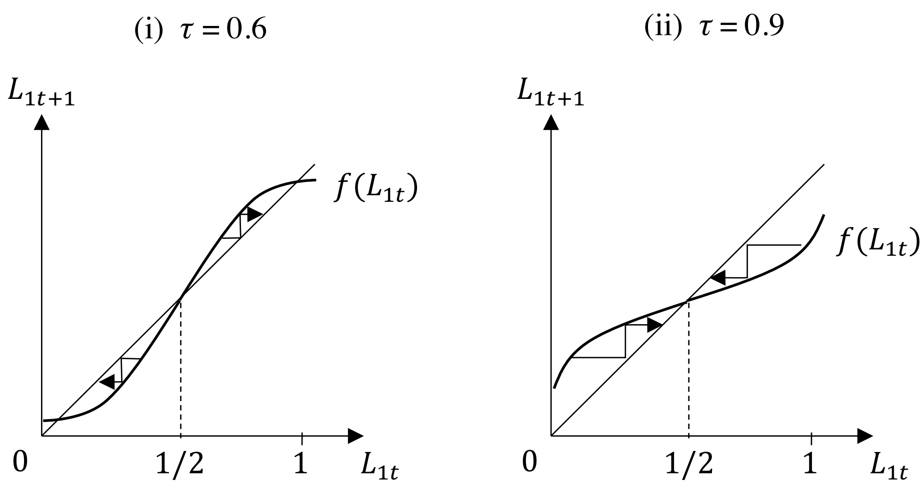
Figure 5 shows equation (33) as a function of τ for this case (specifically, $\alpha = 0.6$ and $\mu = 0.05$). However, if $\mu > \mu^{**}$, the steady state is always stable since the sign of equation (34) is negative. The relation between the stability and τ is intuitively explained as follows. A fall in the adjustment cost (a raise in τ) means the decline of the incentive for remaining to home region. If the dispersion forces emanating from housing consumption is sufficiently high, workers will want to disperse among regions when adjustment costs are sufficiently low.

Fig 5. The value of $df(L_{it})/dL_{it}$



Consequently, there exist two different patterns of transitional dynamics, as shown in Figure 6. The left panel is for the case of $\tau = 0.6$, $\alpha = 0.6$ and $\mu = 0.05$, where the number of births converges to the concentration structure, which is numerically obtained by $\lambda_1^* \simeq 0.983$, $\lambda_2^* \simeq 0.052$ and $L_1^* \simeq 0.982$, or its reverse. The right panel is for larger levels of adjustment costs, $\tau = 0.9$. In this case, the dispersion structure is stable, and equation (20) yields $\lambda_1^* = \lambda_2^* \simeq 0.539$ thus $L_1^* = 0.5$.

Fig 6. The transitional dynamics of births distribution



4.3 Social optimum

When we consider the social welfare maximization of the government, the residency distribution is chosen such that $\lambda_i = 1/(1 + \tau^{\alpha/(1-\alpha)})$, because

$$SW|_{L_1=\tilde{L}_1,\lambda_1=1} = SW|_{L_1=\tilde{L}_1,\lambda_2=1} = 1, \tag{36}$$

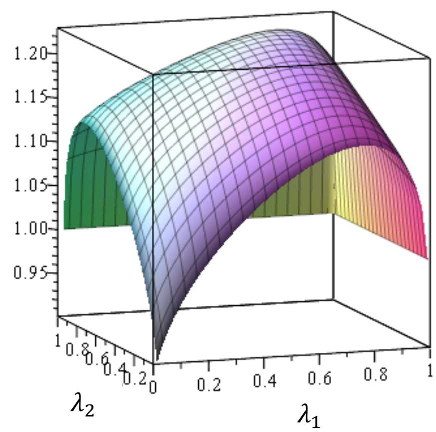
$$SW|_{L_1=\tilde{L}_1,\lambda_2=\lambda_1} = 2^{\mu}\left((\lambda_1)^{\alpha} + ((1-\lambda_1)\tau)^{\alpha}\right)^{1-\mu}. \tag{37}$$

Comparing this with the steady-state equilibrium above, we arrive at the following proposition.

Proposition 3. *When the housing consumption is sufficiently large or when adjustment costs are sufficiently small, the social welfare is maximized in a steady-state equilibrium.*

In this case, the number of births is equally divided between the regions. Figure 7 depicts a numerical example of SW as a function of λ_1 and λ_2, for the case of $\alpha = 0.6$, $\mu = 0.1$, and $\tau = 0.6$.

Fig 7. Social welfare



5 Another agglomeration economy

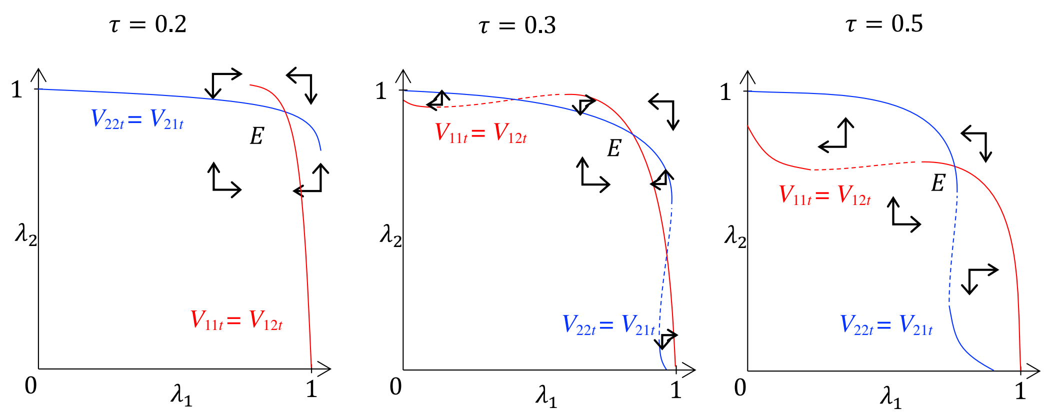
Above Sections 3 and 4 explained that a symmetric births distribution maximized social welfare. However, may the social optimum not be achieved by a concentrated structure of births distribution in any cases? To investigate the possibility that the core-periphery structure of births distribution becomes the social optimum, Section 5 introduces another kind of agglomeration economy due to labor amount by considering equation (12) with $\gamma > 0$. We eliminate the housing consumption by setting $\mu = 0$ for the tractability of the model.

5.1 The short-run analysis of residency choice

5.1.1 Under the equal division of births

In the structure of a symmetric births distribution ($L_{1t} = 1/2$), the residency choice in a symmetric equilibrium, λ_{it}^*, is given by equation (20) in this section, too. However, it is no longer always stable. To understand this, Figure 8 shows the residency choice for three different levels of τ. Other parameters are set as $\alpha = 0.6$, $\gamma = 1$. The solid line means a stable residency choice of the workers born in each region, and the broken line means an unstable one. The unstable choice means that a small group of workers moving from the distribution on the line would, in turn, stimulate more workers' migrating. Then, focusing on the equilibrium point E, a careful look at the slopes of $V_{11t} = V_{12t}$ and $V_{22t} = V_{21t}$ reveals that point E is stable in the left and middle panels, while it is unstable in the right panel and, consequently, all the workers are going to reside in one of the two regions.

Figure 8. The residency choice



More precisely, the slope of the line $V_{11t} = V_{12t}$ at point E is expressed as

$$\left.\frac{d\lambda_{2t}}{d\lambda_{1t}}\right|_{L_{1t}=1/2,\lambda_{it}=\lambda_{it}^*} \equiv h(\tau)$$
$$= \frac{\gamma(1+\tau)(1+\tau^{\alpha/(1-\alpha)}) - \alpha(1-\alpha)\left(3+\tau^{\alpha/(1-\alpha)}+(1+\tau)\tau^{\alpha/(1-\alpha)}\right)\left(\tau^{\alpha/(1-\alpha)}+\tau^{-\alpha/(1-\alpha)}\right)}{\gamma(1+\tau)(1+\tau^{\alpha/(1-\alpha)}) + \alpha(1-\alpha)\left(3+\tau^{\alpha/(1-\alpha)}+(1+\tau)\tau^{\alpha/(1-\alpha)}\right)}. \quad (38)$$

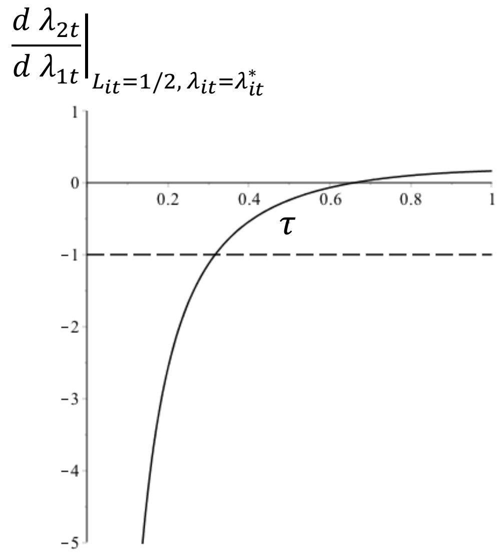
The following calculation explains the value of equation (38):

$$\lim_{\tau\to 0} h(\tau) = -\infty, \quad (39)$$

$$h(1) = \frac{\gamma - 3\alpha(1-\alpha)}{\gamma + 3\alpha(1-\alpha)} \begin{cases} = (-1,0) & \text{for} \quad \gamma < 3\alpha(1-\alpha), \\ > 0 & \text{for} \quad \gamma > 3\alpha(1-\alpha). \end{cases} \quad (40)$$

Figure 9 shows equation (38) as a function of τ, for the case of $\alpha = 0.6$ and $\gamma = 1$ (satisfying $\gamma > 3\alpha(1-\alpha)$). Thus we can see that the symmetric equilibrium point E becomes unstable when τ is sufficiently large. This is because, the raise in τ increases agglomeration economies due to the amount of labor, as we assumed in equation (12).

Fig 9. Slope of the line $V_{11t} = V_{12t}$



5.1.2 Under the core-periphery division of births

If all the workers are born in one region, it is obvious that the concentration of residents in their home region is always sustainable because

$$\left.\frac{V_{11t}}{V_{12t}}\right|_{L_{1t}=1,\lambda_{1t}=1} = 2^{\gamma/\alpha}/\tau > 1. \tag{41}$$

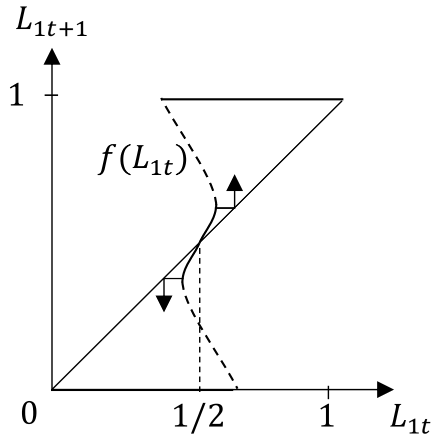
5.2 The long-run analysis of generational transition

Even if the dispersion equilibrium of the residency choice is stable in the short run, the accumulation of the residency choice across periods leads the births distribution to a concentrated structure in the long run. Similar to the previous sections, the total derivative of equation (19) at the symmetric structure, $L_{it} = 1/2, \lambda_{it} = \lambda_{it}^*$, is derived as

$$\left.\frac{df}{dL_{it}}\right|_{L_{it}=1/2,\lambda_{it}=\lambda_{it}^*} = \frac{\alpha(1-\alpha)\left(3+2\tau^{\alpha/(1-\alpha)}+\tau^{1/(1-\alpha)}\right)\left(1-\tau^{\alpha/(1-\alpha)}\right)\left(1+\tau^{\alpha/(1-\alpha)}\right)-2\gamma\left(1+\tau^{\alpha/(1-\alpha)}\right)\left(\tau^{1/(1-\alpha)}-\tau^{\alpha/(1-\alpha)}\right)}{\alpha(1-\alpha)\left(3+2\tau^{\alpha/(1-\alpha)}+\tau^{1/(1-\alpha)}\right)\left(1-\tau^{\alpha/(1-\alpha)}\right)^2-2\gamma\left(1+\tau^{\alpha/(1-\alpha)}\right)\left(\tau^{1/(1-\alpha)}+\tau^{\alpha/(1-\alpha)}\right)}. \tag{42}$$

The value of this equation is larger than 1 even when γ is sufficiently small, which means the symmetric structure is always unstable. Figure 10 shows numerical examples of the transitional dynamics of births distribution for the case of $\alpha = 0.6$, $\tau = 0.3$ and $\gamma = 1$. The solid and broken lines of $f(L_{1t})$, respectively, mean the stable and unstable distribution based on short-run residency choice. Thus, the distribution of births concentrates in one of the two regions in the long run.

Fig 10. The transitional dynamics of births distribution



5.3 Social optimum

Finally, we consider the maximization of the social welfare. Although the equation is complicated, the equations below might explain some of the features of the social welfare function:

$$SW|_{L_1=\tilde{L}_1,\lambda_1=1} = SW|_{L_1=\tilde{L}_1,\lambda_2=1} = 2^{\gamma}, \tag{43}$$

$$SW|_{L_1=\tilde{L}_1,\lambda_2=\lambda_1} = ((\lambda_1)^{\alpha} + ((1-\lambda_1)\tau)^{\alpha})\left(1 + \frac{1}{2}\lambda_1 + \frac{1}{2}(1-\lambda_1)\tau\right)^{\gamma}, \tag{44}$$

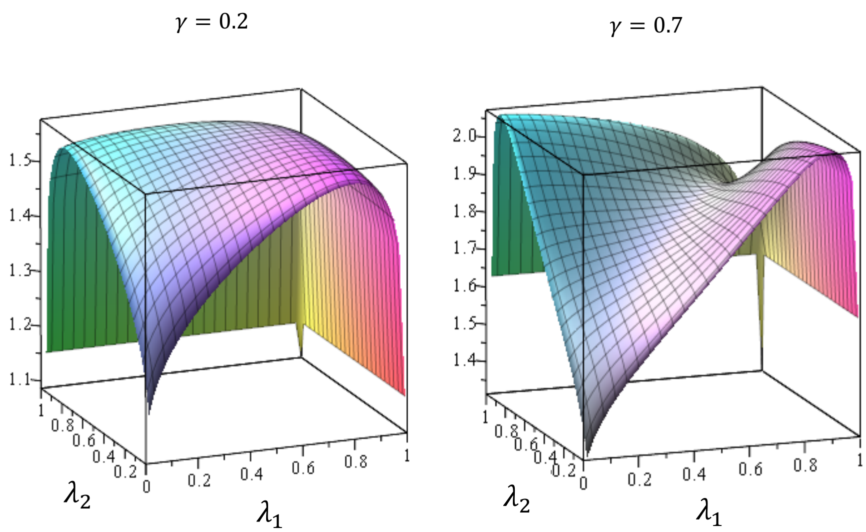
$$\begin{aligned}\lim_{\lambda_1\to 1} SW|_{L_1=\tilde{L}_1} &= \lim_{\lambda_1\to 1}\left(-\frac{2^{\gamma}\tau^{\alpha}\alpha}{(1-\lambda_1)^{1-\alpha}} - \frac{(\lambda_2^{\alpha} + ((1-\lambda_2)(1-\tau))^{\alpha})^{1/\alpha}}{2^{\gamma(1-\alpha)/\alpha}(1-\lambda_2)}\right)\\ &+ \frac{2^{\gamma}\left(\gamma((1-\lambda_2)(1-\tau)+1) + 2\left(\alpha + (1-\alpha)2^{-\gamma/\alpha}(\lambda_2^{\alpha} + ((1-\lambda_2)\tau)^{\alpha})^{1/\alpha}\right)\right)}{2(1-\lambda_2)} = -\infty.\end{aligned} \tag{45}$$

The important point is that the social welfare certainly gets worse as $\lambda_i \to 1$. If the government concentrates workers completely in one of the two regions, labor diversity will be lost in the steady state. To avoid this, a *moderate* concentration is desirable. Here we obtain the final proposition:

Proposition 4. *Even if there exists an agglomeration economy due to the amount of labor, the births distribution in the steady-state equilibrium was still concentrated compared to the social optimum.*

Figure 11 numerically illustrates SW with $\alpha = 0.3$ and $\tau = 0.5$. The left panel shows the case of $\gamma = 0.2$, whose feature is similar to that in Section 3. In this case, the social welfare is maximized at the symmetric distribution, $\lambda_1 = \lambda_2 \simeq 0.614$ (implying $\tilde{L}_1 = 0.5$). As the degree of agglomeration economy increases, the verge of SW on $\lambda_i = 1$ rises. When the agglomeration economy gets sufficiently large, the peak of SW changes from the symmetric distribution to the concentration in one of the two regions. However, as equation (45) shows, the concentration in this situation is imperfect. The right panel of Figure 11 shows the case of $\gamma = 0.7$ in which SW is maximized when $\lambda_1 \simeq 0.909$ and $\lambda_2 \simeq 0.018$ (implying $\tilde{L} \simeq 0.915$) or vice versa. Recalling that workers completely concentrate in one of the two regions in the steady-state equilibrium, we can conclude that the steady-state equilibrium does not maximize the social welfare even if we introduce another kind of agglomeration economy. To maintain labor heterogeneity, it is necessary to keep the interregional circulation of workers in every period.

Fig 11. Social welfare



6 Concluding remarks

Labor heterogeneity is a source of agglomeration. On the other hand, agglomeration reduces labor heterogeneity in the long run. Focusing on agglomeration economies and diseconomies over generations, this paper investigated the relation between labor heterogeneity and interregional labor distribution. In our two-region, non-overlapping generations model, workers were differentiated in terms of their origins, and foreign workers incurred an adjustment cost due to having to cope with cultural differences. Workers were born in each region at the beginning of each period, then chose their residency regions to work in, and finally left the job market at the end of the period. The distribution of births in each period was decided by the previous generation's residency choices.

We had the following results. (i) The residency distribution in each period resulted in an even distribution of workers, while the accumulation of the residency choices over various periods led the birth population to concentrate in each region. Consequently, all the workers would be homogenized and thus continue to reside in one region under a steady-state equilibrium. (ii) A dynamic inefficiency due to the generational transition is observed: The social optimum was given by an equal division of births among the regions, which was not achieved by a steady-state equilibrium. (iii) When we introduced housing consumption, social welfare could be maximized in a steady-state equilibrium wherein the number of births was evenly divided among the regions. (iv) Contrary, even when we introduced another agglomeration economy due to the amount of labor, the births distribution in the steady-state equilibrium was still concentrated compared to the social optimum.

Result (iv) explains well the problem of the present situation in Japan. The population share of the Tokyo Metropolitan Area (composed of Tokyo, Kanagawa, Chiba, and Saitama prefectures) gradually increased from 13.7% in 1920 to 28.4% in 2015. The expansion of the Tokyo metropolitan area may be interpreted as the path to a steady-state equilibrium of concentration, while the social optimum requires a more moderate concentration. Then how can we achieve the socially optimum distribution? Our results can suggest couple of measures be enacted to resolve this problem. The first one is to manipulate dispersion forces. As Section 3 considered housing consumption, a certain dispersion forces can handle over-concentration of population. For instance, it might be effective to reinforce incentives for housing consumption through a sort of subsidy. Besides, fostering a taste heterogeneity might also be another candidate to control the distribution of workers. As stressed in Tabuchi and Thisse (2002) and Murata (2003), heterogeneity of tastes for one's residency location is a strong dispersion force. Contrarily speaking, if people have a similar taste for location, it might be one of the reasons for the excessive flow of population into Tokyo. In such a case, it is desirable to better promote attachment to home regions. The second measure is to regulate interregional migration directly. The government of Japan has recently started trying to reduce inter-prefectural migration by not permitting raising enrollment capacities at universities in inner Tokyo. However, when we consider such a migration policy, it is quite important to note that the social optimum needs a *circulation of population* between regions and countries, thus migration itself should not be obstructed.

Appendix 1: Fundamental equations and total differentials

The fundamental equations of this model is expressed as follows.

$$V_{11t} = \frac{(\alpha(1-\mu))^{1-\mu}\left\{\Gamma A_{1t}^{1/\alpha}[(\lambda_{1t}L_{1t})^{\alpha} + ((1-\lambda_{2t})(1-L_{1t})\tau)^{\alpha}]^{(1-\alpha)/\alpha}(\lambda_{1t}L_{1t})^{\alpha-1} + R_t\right\}}{\{\Gamma[(\lambda_{1t}L_{1t})^{\alpha} + ((1-\lambda_{2t})(1-L_{1t})\tau)^{\alpha}]^{1/\alpha} + [\lambda_{1t}L_{1t} + (1-\lambda_{2t})(1-L_{1t})](R_t)\}^{\mu}}, \tag{46}$$

$$V_{12t} = \frac{(\alpha(1-\mu))^{1-\mu}\left\{\Gamma A_{2t}^{1/\alpha}[(\lambda_{2t}(1-L_{1t}))^{\alpha} + ((1-\lambda_{1t})L_{1t}\tau)^{\alpha}]^{(1-\alpha)/\alpha}((1-\lambda_{1t})L_{1t})^{\alpha-1}\tau^{\alpha} + R_t\right\}}{\{\Gamma[(\lambda_{2t}(1-L_{1t}))^{\alpha} + ((1-\lambda_{1t})L_{1t}\tau)^{\alpha}]^{1/\alpha} + [\lambda_{2t}(1-L_{1t}) + (1-\lambda_{1t})L_{1t}](R_t)\}^{\mu}}, \tag{47}$$

$$V_{22t} = \frac{(\alpha(1-\mu))^{1-\mu}\left\{\Gamma A_{2t}^{1/\alpha}[(\lambda_{2t}(1-L_{1t}))^{\alpha} + ((1-\lambda_{1t})L_{1t}\tau)^{\alpha}]^{(1-\alpha)/\alpha}(\lambda_{2t}(1-L_{1t}))^{\alpha-1} + R_t\right\}}{\{\Gamma[(\lambda_{2t}(1-L_{1t}))^{\alpha} + ((1-\lambda_{1t})L_{1t}\tau)^{\alpha}]^{1/\alpha} + [\lambda_{2t}(1-L_{1t}) + (1-\lambda_{1t})L_{1t}](R_t)\}^{\mu}}, \tag{48}$$

$$V_{21t} = \frac{(\alpha(1-\mu))^{1-\mu}\left\{\Gamma A_{1t}^{1/\alpha}[(\lambda_{1t}L_{1t})^{\alpha} + ((1-\lambda_{2t})(1-L_{1t})\tau)^{\alpha}]^{(1-\alpha)/\alpha}((1-\lambda_{2t})(1-L_{1t}))^{\alpha-1}\tau^{\alpha} + R_t\right\}}{\{\Gamma[(\lambda_{1t}L_{1t})^{\alpha} + ((1-\lambda_{2t})(1-L_{1t})\tau)^{\alpha}]^{1/\alpha} + [\lambda_{1t}L_{1t} + (1-\lambda_{2t})(1-L_{1t})](R_t)\}^{\mu}}, \tag{49}$$

$$\Gamma = \left\{A_{1t}^{1/\alpha}[(\lambda_{1t}L_{1t})^{\alpha} + ((1-\lambda_{2t})(1-L_{1t})\tau)^{\alpha}]^{1/\alpha} + A_{2t}^{1/\alpha}[(\lambda_{2t}(1-L_{1t}))^{\alpha} + ((1-\lambda_{1t})L_{1t}\tau)^{\alpha}]^{1/\alpha}\right\}^{\alpha-1}, \tag{50}$$

$$\begin{aligned} R_t \equiv RK_t + RH_t = &\frac{1-\alpha(1-\mu)}{\alpha(1-\mu)} \\ &\times \left\{A_{1t}^{1/\alpha}[(\lambda_{1t}L_{1t})^{\alpha} + ((1-\lambda_{2t})(1-L_{1t})\tau)^{\alpha}]^{1/\alpha} + A_{2t}^{1/\alpha}[(\lambda_{2t}(1-L_{1t}))^{\alpha} + ((1-\lambda_{1t})L_{1t}\tau)^{\alpha}]^{1/\alpha}\right\}^{\alpha}, \end{aligned} \tag{51}$$

$$A_{it} = [1 + \lambda_{it}L_{it} + (1-\lambda_{jt})L_{jt}\tau]^{\gamma}, \quad i, j = 1, 2, \quad i \neq j. \tag{52}$$

To avoid complex calculation, this paper divides the analysis into three sections. That is, we set $\mu = 0$ and $\gamma = 0$ in Section 3; $\mu > 0$ and $\gamma = 0$ in Section 4; and $\mu = 0$ and $\gamma > 0$ in Section 5. At the symmetric equilibrium of $L_{it} = 1/2$ and $\lambda_{it} = \lambda_{it}^*$, the total differential of $V_{11t} = V_{12t}$ is give by

$$\begin{aligned}
&\left(\frac{\gamma(1-\mu)(1+\tau^{\alpha/1-\alpha})}{\alpha(3+2\tau^{\alpha/1-\alpha}+\tau^{1/1-\alpha})} - \frac{\mu}{\alpha(1-\mu)} - (1-\alpha)\tau^{\alpha/1-\alpha}\right)d\lambda_{1t} \\
&-\left(\frac{\gamma(1-\mu)(1+\tau^{\alpha/1-\alpha})}{\alpha(3+2\tau^{\alpha/1-\alpha}+\tau^{1/1-\alpha})} - \frac{\mu}{\alpha(1-\mu)} + 1-\alpha\right)d\lambda_{2t} \\
&+2\left(\frac{\gamma(1-\mu)(1-\tau^{\alpha/1-\alpha})}{\alpha(3+2\tau^{\alpha/1-\alpha}+\tau^{1/1-\alpha})} - \frac{\mu}{\alpha(1-\mu)}\frac{1-\tau^{\alpha/1-\alpha}}{1+\tau^{\alpha/1-\alpha}} - 2(1-\alpha)\frac{\tau^{\alpha/1-\alpha}}{1+\tau^{\alpha/1-\alpha}}\right)dL_{1t} \\
&= -\left(\frac{\gamma(1-\mu)(1+\tau^{\alpha/1-\alpha})}{\alpha(3+2\tau^{\alpha/1-\alpha}+\tau^{1/1-\alpha})} - \frac{\mu}{\alpha(1-\mu)} - \frac{1-\alpha}{\tau^{\alpha/1-\alpha}}\right)d\lambda_{1t} \\
&+\left(\frac{\gamma(1-\mu)(1+\tau^{\alpha/1-\alpha})}{\alpha(3+2\tau^{\alpha/1-\alpha}+\tau^{1/1-\alpha})} - \frac{\mu}{\alpha(1-\mu)} + 1-\alpha\right)d\lambda_{2t} \\
&-2\left(\frac{\gamma(1-\mu)(1-\tau^{\alpha/1-\alpha})}{\alpha(3+2\tau^{\alpha/1-\alpha}+\tau^{1/1-\alpha})} + \frac{\alpha(1-\alpha)(1-\mu)-\mu}{\alpha(1-\mu)}\frac{1-\tau^{\alpha/1-\alpha}}{1+\tau^{\alpha/1-\alpha}} + 1-\alpha\right)dL_{1t}.
\end{aligned} \tag{53}$$

If we eliminate the terms of dL_{1t} from equation (53), we can derive $d\lambda_{2t}/d\lambda_{1t}$ at point E in Figures 1, 4, and 8. On the other hand, substituting $d\lambda_{2t} = -d\lambda_{1t}$ into equation (53) yields $d\lambda_{1t}/dL_{1t}$, which explains the effect of a small deviation from the symmetry.

Appendix 2: The slope of line $V_{iit} = V_{ijt}$ at the corner point

Appendix 2 examines the relation between the slopes of $V_{11t} = V_{12t}$ and $V_{22t} = V_{21t}$ at the corner point ($\lambda_{1t} = 1, \lambda_{2t} = 0$) in Figures 1 and 8. Since Figure 1 is derived as a specific case of $\gamma = 0$ for Figure 8, we mainly consider Figure 8. The relation between the two lines is calculated by the following steps. First, substituting $L_{1t} = 1/2$ into $V_{11t} = V_{12t}$ in Section 5, we have

$$\begin{aligned}
&\left(1 + \frac{\lambda_{1t} + (1-\lambda_{2t})\tau}{2}\right)^{\gamma/1-\alpha}\left(1 + \left(\frac{(1-\lambda_{2t})\tau}{\lambda_{1t}}\right)^{\alpha}\right) \\
&= \left(1 + \frac{\lambda_{2t} + (1-\lambda_{1t})\tau}{2}\right)^{\gamma/1-\alpha}\left(1 + \left(\frac{\lambda_{2t}}{(1-\lambda_{1t})\tau}\right)^{\alpha}\right)\tau^{\alpha/1-\alpha}.
\end{aligned} \tag{54}$$

Secondly, manipulating the total derivative of equation (54) and the limit of equation (54) as $\lambda_{1t} \to 1$ and $\lambda_{2t} \to 0$, we obtain the slope of $V_{11t} = V_{12t}$ at the end point expressed as

$$\begin{aligned}
&\lim_{\lambda_{1t}\to 1, \lambda_{2t}\to 0} \frac{d\lambda_{2t}}{d\lambda_{1t}} = \lim_{\lambda_{1t}\to 1, \lambda_{2t}\to 0} -\frac{\lambda_{2t}}{(1-\lambda_{1t})} \\
&= -\left(\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}(1+\tau^{\alpha}) - \tau^{\alpha/1-\alpha}\right)^{1/\alpha}\tau^{-\alpha/1-\alpha} \equiv \xi_1(\tau).
\end{aligned} \tag{55}$$

Thirdly, rewriting $V_{22t} = V_{21t}$ as

$$\begin{aligned}
&\left(1 + \frac{\lambda_{2t} + (1-\lambda_{1t})\tau}{2}\right)^{\gamma/1-\alpha}\left(1 + \left(\frac{(1-\lambda_{1t})\tau}{\lambda_{2t}}\right)^{\alpha}\right) \\
&= \left(1 + \frac{\lambda_{1t} + (1-\lambda_{2t})\tau}{2}\right)^{\gamma/1-\alpha}\left(1 + \left(\frac{\lambda_{1t}}{(1-\lambda_{2t})\tau}\right)^{\alpha}\right)\tau^{\alpha/1-\alpha},
\end{aligned} \tag{56}$$

the same manipulation of equation (56) yields

$$\lim_{\lambda_{1t}\to 1,\lambda_{2t}\to 0}\frac{d\lambda_{2t}}{d\lambda_{1t}}=-\frac{1}{\left([(3+\tau)/2]^{\gamma/1-\alpha}\left(1+\tau^{-\alpha}\right)-\tau^{-\alpha/1-\alpha}\right)^{1/\alpha}\tau^{\alpha/1-\alpha}}\equiv\xi_2(\tau). \tag{57}$$

Note that the RHS of equation (56) should be larger than 1, whose condition is given by

$$\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}\left(1+\frac{1}{\tau^{\alpha}}\right)-\frac{1}{\tau^{\alpha/1-\alpha}}>0. \tag{58}$$

Otherwise, the line of $V_{22t} = V_{21t}$ does not pass the corner point. Finally, comparing the values of equations (55) and (57), we have

$$\begin{aligned}\frac{\xi_1(\tau)}{\xi_2(\tau)}=&\left\{\left(\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}\left(1+\frac{1}{\tau^{\alpha}}\right)-\frac{1}{\tau^{\alpha/1-\alpha}}\right)\left(\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}\left(1+\tau^{\alpha}\right)-\tau^{\alpha/1-\alpha}\right)\right\}^{1/\alpha}\\=&\left\{\left(\frac{3+\tau}{2}\right)^{2\gamma/1-\alpha}-\left[1-\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}\tau^{\alpha/1-\alpha}\right]\left[\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}\frac{1}{\tau^{\alpha^2/1-\alpha}}-1\right]\right.\\&\left.-\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}\left[\frac{1}{\tau^{\alpha/1-\alpha}}\left(1-\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}\tau^{\alpha^2/1-\alpha}\right)-\left(\left(\frac{3+\tau}{2}\right)^{\gamma/1-\alpha}-\tau^{\alpha^2/1-\alpha}\right)\right]\right\}^{1/\alpha}.\end{aligned} \tag{59}$$

If $\gamma = 0$, the value of equation (59) is less than 1. That is, as shown in Figure 1, the absolute value of the slope of $V_{11t} = V_{12t}$ is smaller than $V_{22t} = V_{21t}$. On the other hand, as shown in Figure 8, the rise in γ decreases the value of equation (59), thus the slope of $V_{11t} = V_{12t}$ becomes larger than that of $V_{22t} = V_{21t}$.

By the way, we can easily see that the corner point itself is sustainable since

$$\left.\frac{V_{11t}}{V_{12t}}\right|_{L_{1t}=1/2,\lambda_{1t}=1,\lambda_{2t}=0}=\frac{((3+\tau)/2)^{\gamma/\alpha}\left(1+\tau^{\alpha}\right)^{(1-\alpha)/\alpha}}{\tau}>1. \tag{60}$$

Appendix 3: Social welfare at the corner point

Considering the government's choice of λ_i, we should draw attention to the particularity of the case where $\lambda_1 = 1$ and $\lambda_2 = 1$. In this case, there is no labor circulation between the regions, thus the births distribution is fixed at the initial distribution, $L_1 = L_{1o}$. (Thus $L_1 \neq \tilde{L}_1$.) In Figure 3 (in Section 3), the value of SW at the corner point ($\lambda_1 = 1, \lambda_2 = 1$) equals unity. In Figure 7 (in Section 4), it becomes

$$SW = L_{1o}^{1-\mu} + (1 - L_{1o})^{1-\mu}, \tag{61}$$

and in Figure 11 (in Section 5), it becomes

$$SW = \left((1 + L_{1o})^{\gamma/\alpha} L_{1o} + (2 - L_{1o})^{\gamma/\alpha}(1 - L_{1o})\right)^{\alpha}. \tag{62}$$

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