Dynamic Relation between Volatility Risk Premia of Stock and Oil Returns

NAKAMURA Nobuhiro
Hitotsubashi University

OHASHI Kazuhiko
RIETI

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NAKAMURA Nobuhiro[†]

Graduate School of Business Administration, Hitotsubashi University

and

OHASHI Kazuhiko[‡]

Graduate School of Business Administration, Hitotsubashi University

Research Institute of Economy, Trade and Industry

Abstract

Volatility risk premium (VRP), defined as the difference between implied and realized volatilities, is found to have predictive power on the returns of many different assets (e.g., stocks, exchange rates, and commodities). While most of the extant research analyzes the return predictability of VRP, in this paper, we instead investigate the relation between the VRP of different assets, specifically stocks and oil. Using daily data of VRP from May 10, 2007 to May 16, 2017, we conduct VAR analyses on the stock and oil VRP and find that the effects of the stocks VRP on the oil VRP are limited and short-lived, if any. On the other hand, the VRP of oil has significantly positive and long-lasting effects on that of stocks after the outbreak of financial crises. These results suggest that the investors' sentiments (measured by VRP) are transmitted from the oil market to the stock market over time, but not the other way around, which is rather unexpected because financialization of commodities means a massive increase in investment in commodities by investors in traditional stock and bond markets, and hence the direction of effects is thought to be from the stock market to the commodity market.

Keywords: Volatility risk premium (VRP), Implied and realized volatility, Oil and stock returns, Financialization,

JEL classification:　G11, G12, G13

[†] Professor, Graduate School of Business Administration, Hitotsubashi University E-mail: nnakamura@hub.hit-u.ac.jp,

[‡] Professor, Graduate School of Business Administration, Hitotsubashi University and faculty fellow, RIETI, E-mail: kohashi@hub.hit-u.ac.jp

1. Introduction

Volatility Risk Premium (VRP), defined as the difference between implied and realized volatilities, is found to have predictive power on returns in many different assets. For example, as a pioneering research on this topic, Bollerslev, Tauchen, and Zhou (2009) find that VRP has predictive power for the U.S. monthly aggregated stock returns, and Bollerslev, Marrone, Xu, and Zhou (2009) also find predictive power of VRP in monthly stock index returns of many other developed countries.

VRP represents the risk premium for future variation of volatility. Thus, it may be regarded as investors' sentiment (i.e., aversion of future uncertainty), and the predictability of VRP is thought due to such investors' sentiment: when the investors' sentiment gets worse (resp. better), stock prices are discounted by higher (resp. lower) premium, which results in higher (resp. lower) future returns.

Following this intuition, the scope of analysis is extended to other assets than stock. Indeed, Della Corte, Ramadorai, and Sarno (2016) and Londono and Zhou (2016) confirm predictive power of VRP in monthly exchange rates. Ornelas and Mauad (2017) investigate predictive power of VRP of different assets such as commodities currencies, stock, bond, gold, and oil on monthly returns of those assets.

Given such extant researches on VRP's return-predictability of different assets, one simple but unexplored question is how VRP of different assets are related to each other. This question is meaningful because dynamic relation of VRP between different assets is interpreted to show how the investors' sentiments on different assets transmit each other over time. Moreover, it is especially important between the VRP of stock and commodities because of the recent *financialization* of commodities (i.e., the massive increase of investment in commodities by the investors in the traditional stock and bond markets). Thus, in this paper, we investigate the dynamic relation between VRP of stock and oil using their daily returns.

There are several papers that investigate the relation among implied volatilities of stock, oil, gold, and/or exchange rate. For example, Robe and Wallen (2015), in analyzing the determinants of oil implied volatility by using a weekly data, investigates the relation between oil implied volatility and stock implied volatility. Christoffersen and Pan (2017), while investigating the effect of oil implied volatility on stock returns, analyze the relation between oil implied volatility and stock implied volatility. Liu, Ji, and Fan (2013) conduct VECM analysis on the relation among daily implied volatilities of stock, oil, gold, and exchange rate. Though the spirit of

analysis is close, this paper is different from them in that it focuses on VRP.

One paper that is closely related to this one is Hattori, Ilhoyock, and Sugihara (2018) that conduct a VAR analysis on the relation among daily stock VRP of advanced and emerging-market economies. To our knowledge, this paper is the first attempt that investigates the relation of daily VRP, not implied volatilities, of different assets i.e., stock and oil by a VAR model.

We calculate the daily VRP of stock as the difference of the VIX published by the Chicago Board of Trade (CBOE), which measures the 30-day implied volatility of S&P 500 stock index options, and the daily realized volatility of the S&P 500 stock index provided by the Oxford-Man Institute of Quantitative Finance. To obtain the daily VRP of oil, we use the OVX published by the CBOE, which measures the 30-day implied volatility of crude oil prices by applying the CBOE Volatility Index methodology to options on the United States Oil Fund (USO). Since we do not have the high frequency data of the USO prices or its daily realized volatility, we estimate the daily realized volatility of oil by applying a stochastic volatility model to its returns.

Using the daily VRP of stock and oil returns so obtained between May 10, 2007 and May 16, 2017, we conduct a VAR analysis of the VRP and obtain the following results:

In the whole period and all sub-periods, both VRP are stationary and their correlations are around 0.2 to 0.3 except the pre-crisis period (between May 10, 2007 and May 30, 2008) where the correlation is less than 0.1.

For the whole period, most of the variations of the VRP are explained by their own shocks, which may seem against what we expect from the *financialization* of commodities because *financialization* is regarded to strengthen the relation between stock and oil. Meanwhile, the shocks in both VRP of stock and oil have small but significant positive effects on each other for most of all following 20 trading days after the shock. This is in contrast to the results shown by Liu, Ji, and Fan (2013) on the relations among the implied volatilities of stock, oil, gold, and euro/dollar exchange rate in which all implied volatilities have significant but only temporary (i.e., just in the 1st trading day after the shock) effects on each other.

Such relations, however, may vary depending on the economic situation, and the economic situation surrounding stock and oil has been clearly changing. Thus, we conduct the VAR analysis on the following sub-periods: Period 1 from May 10, 2007 to May 30, 2008 (pre-crisis period), Period 2 from June 1, 2008 to June 30, 2009 (crisis outbreak period), Period 3 from July 1, 2009 to July 31, 2012 (post-crisis

recovery period 1), Period 4 from August 1, 2012 to September 30, 2014 (post-crisis recovery period 2), and Period 5 from October 1, 2014 to May 16, 2017 (plunging oil price period). Interestingly, the analysis on those sub-periods reveals a different picture on the dynamic relation between stock and oil VRP from that of the whole period.

In the pre-crisis period (period 1), we find that there are little or no relation between the VRP of stock and oil; small correlation less than 0.1, no Granger causality between the stock and oil VRP, or no significant effects on each other in impulse response functions and little effects in variance decomposition. Again, this may seem somewhat against the view of *financialization* of commodities because the effect of *financialization*, i.e., the rise of correlations among the returns of stock and commodities, is found to emerge after 2000's (Tang and Xiong, 2012; Silvennoinen and Thorp, 2013; Ohashi and Okimoto, 2016).

In the crisis outbreak period (period 2), the correlation between the VRP of stock and oil is 0.27. The stock VRP does not Granger cause the oil VRP. There are no significant effects of the stock VRP on the oil VRP in both impulse response functions and variance decomposition. On the other hand, the oil VRP Granger causes the stock VRP, has significantly positive, though small, effects on the stock VRP in impulse response, and explains 8% of the variation in the stock VRP.

In the post-crisis recovery period 1 (period 3), the correlation goes up to 0.34. Both stock and oil VRP Granger cause each other. Both have, though small, significantly positive effects on each other in impulse response functions and variance decomposition. However, their effects have quite different patterns: The VRP of oil has significantly positive and long-lasting effects (after the 2nd trading day of the shock), while the VRP of stock has significantly positive but only temporary effects (just up to the 2nd trading day) on that of oil.

In the post-crisis recovery period 2 (period 4), the correlation goes down to 0.21. Granger causality from stock to oil disappears, while the VRP of oil Granger causes that of stock. The effects of the oil VRP on the stock VRP remains significant and long-lasting similarly to those in period 3, but the effects of the stock VRP on the oil VRP disappear.

Finally, in the plunging oil price period (period 5), the correlation is 0.22. Both Granger cause each other. The effects of the VRP of stock on that of oil get back to be significantly positive but just temporary only in the 1st trading day after the shock, while the effects of the oil VRP on the stock VRP stay significantly positive up to the 8th trading day after the shock.

In sum, dynamic relation between the VRP of stock and oil depends on the economic situation, and contrary to the results for the whole period, it is revealed that the VRP of oil has significantly positive and long-lasting effects on that of stock in all sub-periods after the outbreak of financial crisis, while the effects of the stock VRP on the oil VRP are limited and, if any, much short-lived. That is, though small, the investors' sentiments are transmitted from the oil market to the stock market over time, but not the other way around. This relation between oil and stock VRP is an unexplored point by the extant literature and is rather unexpected because *financialization* of commodities i.e., the massive increase of investment in commodities by the investors in the traditional stock and bond markets, is thought to have the effects from the stock market to the commodity market.[4]

This paper is organized as follows. Section 2 explains VRP. Section 3 discusses the construction and properties of the data used in this study. Section 4 explains the selection of the model. Section 5 provides the main empirical results. Section 6 discusses robustness of the analysis. Section 7 concludes.

2. Volatility Risk Premium (VRP)

Let t be the current date. Denote by σ_{t+T} the volatility of an asset return at date $t+T$. A volatility swap that exchanges at date $t+T$ the payoff σ_{t+T} and the payment x_t, which is contracted at date t, enables its holder/investor to hedge at date t the risk of volatility variation in the future date $t+T$.[5] A simple no-arbitrage argument shows that $x_t = E_t^Q[\sigma_{t+T}]$ where Q is a risk-neutral probability. Hence, the amount that the swap investor receives at date $t+T$ is equal to $\sigma_{t+T} - E_t^Q[\sigma_{t+T}]$.

Now, if σ_{t+T} is on average less than $E_t^Q[\sigma_{t+T}]$ i.e., $E_t^P[\sigma_{t+T}] - E_t^Q[\sigma_{t+T}] < 0$ where P is the original probability, it means that the swap holder is willing to pay $E_t^P[\sigma_{t+T}]$ that is more than the expected payoff $E_t^P[\sigma_{t+T}]$ to hedge the volatility risk in the future. In this sense, $E_t^Q[\sigma_{t+T}] - E_t^P[\sigma_{t+T}]$ represents the premium that the swap

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[4] Results by extant researches about the effects of oil volatility-related variables on stock returns and volatility are mixed. For example, Ornelas and Mauad (2017) find little predictability of oil VRP on S&P 500 returns, Bams, Blanchard, Honarvar, and Lehnert (2017) find that difference of oil VRP is priced only on returns of oil-related stocks, and Christoffersen and Pan (2017) find predictability of oil implied volatility on stock returns and implied volatility.
[5] Volatility swap and variance swap, where variance is the square of volatility, are traded in over-the-counter derivative markets.

investor is willing to pay to hedge the variation risk of future volatility. Thus, $E_t^Q[\sigma_{t+T}] - E_t^P[\sigma_{t+T}]$ is called volatility risk premium (VRP).

As can be seen, the larger VRP is, the more averse the investor about variation of future volatility. In this sense, VRP is sometimes interpreted to indicate investors' sentiment on future asset returns.

3. Data

In the empirical analysis, we have to estimate the risk-neutral expected future volatility $E_t^Q[\sigma_{t+T}]$ and the expected future volatility $E_t^P[\sigma_{t+T}]$ to calculate VRP. The former can be estimated from option prices and hence is called the (option) implied volatility (IV). On the other hand, estimating the latter is not immediate. Hence, in this paper, we approximate the expected future volatility by the realized volatility (RV) and obtain VRP as the difference between IV and RV i.e., $VRP \equiv IV - RV$. [6]

More precisely, we calculate the daily VRP of stock (VRP_{sp}), as the difference of the VIX published by the Chicago Board of Trade (CBOE), [7] which measures the 30-day implied volatility of S&P 500 stock index options, and the daily realized volatility of the S&P 500 stock index provided by the Oxford-Man Institute of Quantitative Finance, which is calculated from 5 minutes returns of the index. [8] As noted above, while the former is the risk-neutral expected future volatility, the latter is not the expected future volatility, but the daily realized volatility. Thus, by this choice of variable, we assume that the daily realized volatility of Oxford-Man Institute approximates well the expected future volatility of stock.

To obtain the daily VRP of oil (VRP_{oil}), we use the OVX published by the CBOE, [9] which measures the 30-day implied volatility, i.e., the risk-neutral expected future volatility, of crude oil prices by applying the CBOE Volatility Index methodology to options on the United States Oil Fund (USO). Since we do not have the high frequency data of the USO returns to calculate its daily realized volatility, we estimate the daily realized volatility of oil by applying a stochastic volatility model to its returns. [10] Then, we obtain the daily VRP of oil (VRP_oil) as the difference of

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[6] Ornelas and Mauad (2017) explain what kind of realized volatility is used in the literature to approximate the expected future volatility.

[7] http://www.cboe.com/products/vix-index-volatility/vix-options-and-futures/vix-index/vix-historical-data

[8] http://realized.oxford-man.ox.ac.uk/

[9] http://www.cboe.com/products/vix-index-volatility/volatility-on-etfs/cboe-crude-oil-etf-volatility-index-ovx

[10] Appendix explains how we estimate the realized volatility of oil.

the OVX and the daily realized volatility of oil so estimated. Again, by doing so, we assume that the daily realized volatility of oil approximates well the expected future volatility of oil.

As CBOE publishes the data of OVX after the middle of 2007, we use the daily VRP of stock and oil returns from May 10, 2007 to May 16, 2017. In this period, however, the global financial markets and the world economy got through several different phases such as the global financial crisis, recovery from the crisis, and plunge of oil prices, which may affect the relation of stock and oil prices.

This can be seen, for example, from the graphs of the daily indices of stock (S&P 500) and oil (USO) in Fig. 1 where index_sp represents S&P 500 price and index_oil represents USO price multiplied by 50, the vertical axis is measured in U.S. dollars, time in the horizontal axis represents the date where time 1 corresponds to May 10, 2007, time 500 to May 13, 2009, time 1000 to May 6, 2011, time 1500 to May 3 2013, time 2000 to April 29, 2015, and time 2500 to April 24, 2017, respectively.

Fig 1. index_sp (S&P 500) and index_oil (USO multiplied by 50)
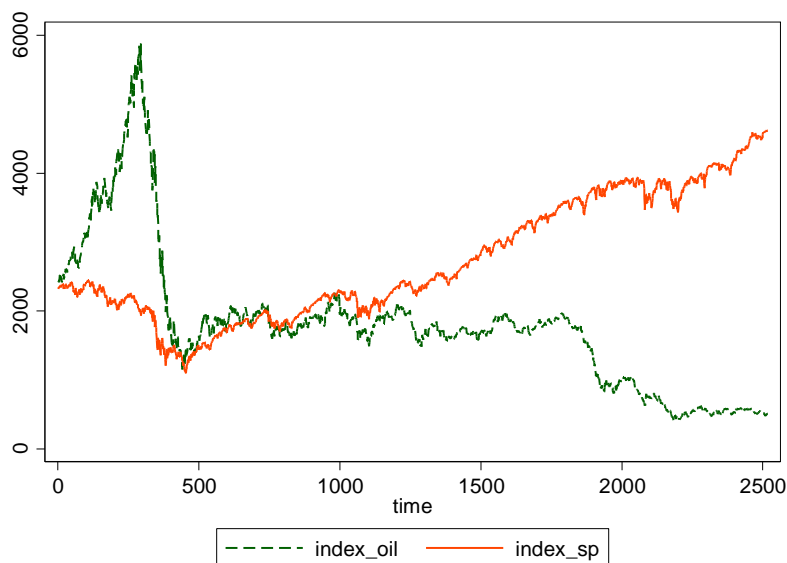
Thus, in this paper, while we use the daily VRP of stock and oil returns between May 10, 2007 and May 16, 2017, we divide the whole period into 5 sub-periods and investigate whether and how the VRP of stock and oil are related in each period. The sub-periods are tabulated in Table 1.

Table 1. Periods and corresponding dates/time

Whole period	2007/05/10 - 2017/05/16 (time = 1 - 2516)
Period 1 (Pre-crisis period)	2007/05/10 - 2008/05/30 (time = 1 - 266)
Period 2 (Crisis outbreak period)	2008/06/01 - 2009/06/30 (time = 267 - 533)
Period 3 (Post-crisis recovery period 1)	2009/07/01 - 2012/07/31 (time = 534 - 1311)
Period 4 (Post-crisis recovery period 2)	2012/08/01 - 2014/09/30 (time = 1312 - 1855)
Period 5 (Plunging oil price period)	2014/10/01 - 2017/05/16 (time = 1856 - 2516)

To select those periods, we refer to Liu, Ji, and Fan (2013) who investigate dynamic relation among the implied volatilities of stock (VIX), oil (OVX), euro/dollar exchange rate (EVZ), and gold (GVZ) between June 3, 2008 and July 20, 2012. Indeed, periods 2 and 3 of this paper roughly correspond to Liu, Ji, and Fan (2013)'s crisis outbreak period and post-crisis recovery period so that we can compare for those periods the interaction among implied volatilities shown by Liu, Ji, and Fan (2013) with that of VRP analyzed by this paper.

Fig. 2 shows the relation between VRP_{sp} and VRP_{oil} from May 10, 2007 to May 16, 2017. As can be seen from this graph, both stock and oil VRP look volatile especially in the crisis outbreak period (period 2) and are calming in the post-crisis recovery periods (periods 3 and 4). But, then, the oil VRP becomes slightly volatile in accordance with the recent plunging oil prices (in period 5).

The descriptive statistics of the daily VRP of stock and oil are given in Table 2. It confirms our observation about their volatility as the standard deviation of VRP_{sp} is very high in the crisis outbreak period (10.148 in period 2) but becomes low after the crisis (4.168, 2.115, and 2.801 in periods 3, 4, and 5, respectively). Also, the standard deviation of VRP_{oil} is high in the crisis outbreak period (9.754 in period 2), becomes low in the post-crisis recovering period (4.619 in period 4), but returns slightly higher in the plunging oil price period (6.724 in period 5).

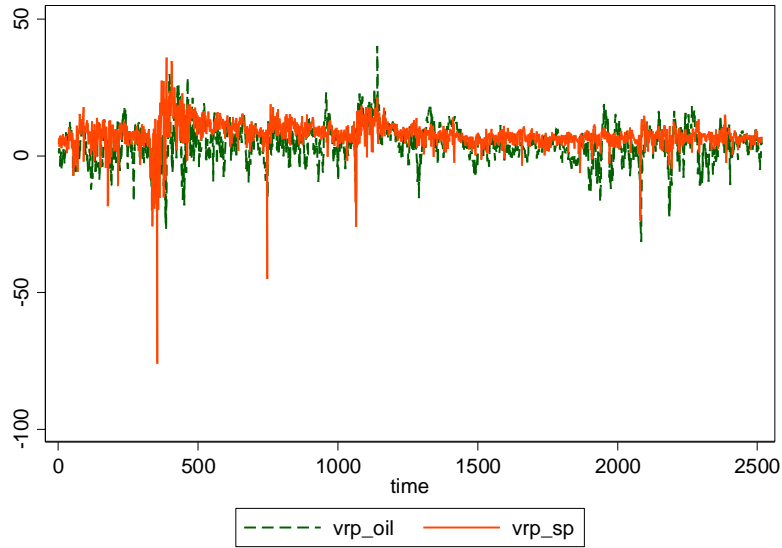Fig. 2. VRP_{sp} and VRP_{oil} from May 10, 2007 to May 16, 2017

Table 2. Descriptive statistics of VRP_{stock} and VRP_{oil}

	Mean	St. dev.	Skew.	Kurt.	Corr.	#Obs.
VRP_{sp}(Whole)	7.785	4.891	-3.141	49.116	0.273	2,516
VRP_{sp}(Period 1)	6.509	4.794	-1.502	7.724	0.097	266
VRP_{sp}(Period 2)	9.583	10.148	-2.778	22.344	0.278	267
VRP_{sp}(Period 3)	9.536	4.168	-4.291	49.595	0.344	778
VRP_{sp}(Period 4)	6.441	2.115	-0.722	5.156	0.212	544
VRP_{sp}(Period 5)	6.616	2.801	-2.395	24.814	0.222	661
VRP_{oil}(Whole)	4.202	6.627	-0.230	5.047	0.273	2,516
VRP_{oil}(Period 1)	2.529	5.842	0.161	2.596	0.097	266
VRP_{oil}(Period 2)	3.623	9.754	-0.061	3.518	0.278	267
VRP_{oil}(Period 3)	5.846	6.317	0.135	4.315	0.344	778
VRP_{oil}(Period 4)	4.231	4.619	0.491	3.134	0.212	544
VRP_{oil}(Period 5)	3.151	6.724	-0.929	5.407	0.222	661

The means of VRP_{sp} are between 6.4 and 9.6 and high in the crisis outbreak and just after crisis (9.583 and 9.536 in periods 2 and 3, respectively), while those of VRP_{oil} are between 2.5 and 5.8 and highest just after the crisis (5.846 in period 3). The stock VRP is slightly negatively skewed, while that of the oil VRP can be positive or negative. Kurtosis of the stock VRP is much larger than that of the oil VRP. Finally, the correlation of VRP_{sp} and VRP_{oil} is stable and between 0.2 and 0.3 in all, but the 1st, sub-periods.

4. Model selection

4.1. Unit root tests

To select an appropriate model, we start with unit root tests for VRP_{sp} and VRP_{oil}. The results are reported in Table 3. The null hypothesis of Augmented Dickey-Fuller (ADF), Dickey-Fuller-GLS (DF-GLS) and Phillips-Perron (PP) tests is that there is unit root in the variable.

Table 3. Unit root tests of VRP_{stock} and VRP_{oil}

	ADF	DF-GLS	PP
VRP_{sp}(Whole)	-11.991***	-9.888***	-1485.051***
VRP_{sp}(Period 1)	-4.928***	-4.815***	-130.293***
VRP_{sp}(Period 2)	-3.623***	-3.938***	-180.399***
VRP_{sp}(Period 3)	-8.022***	-7.041***	-517.148***
VRP_{sp}(Period 4)	-7.744***	-7.827***	-443.083***
VRP_{sp}(Period 5)	-8.864***	-8.651***	-450.466***
VRP_{oil}(Whole)	-11.445***	-11.071***	-370.718***
VRP_{oil}(Period 1)	-4.447***	-4.450***	-43.848***
VRP_{oil}(Period 2)	-4.476***	-4.656***	-62.622***
VRP_{oil}(Period 3)	-5.895***	-5.917***	-111.345***
VRP_{oil}(Period 4)	-3.628***	-3.864***	-32.445***
VRP_{oil}(Period 5)	-5.982***	-5.598***	-92.555***

Note: *** represents significance at 1% level.

As Table 3 shows, all tests reject the null hypothesis at 1 % level significance in all periods. Thus, we regard both VRP_{sp} and VRP_{oil} to be stationary in the whole and all sub-periods. Note that this is in contrast to the unit root test results on implied volatilities by Liu, Ji, and Fan (2013) where all implied volatilities of stock, oil, gold, and foreign exchange rate have unit roots. Unlike implied volatilities, VRP of stock and oil are stationary.

4.2. VAR model

Since there is no unit root in the whole and all sub-periods, we apply the following VAR model to investigate the dynamic relation between the VRP of stock and oil.

$$VRP_t = \alpha + \sum_{i=1}^{P} A_i\, VRP_t + e_t$$

where $VRP_t = (VRP_{sp_t},\ VRP_{oil_t})'$, $\alpha = (\alpha_{sp}, \alpha_{oil})'$, A_i is a 2×2 matrix, P is lag length, and $e_t = (e_{sp_t}, e_{oil_t})'$ are jointly normally distributed disturbances.

4.3. Choice of lag length

We choose the lag length P by comparing AIC (Akaike information criterion), HQIC (Hannan and Quinn information criterion), and SBIC (Schwartz's Bayesian information criterion) for each period in the analysis. The results are as follows:

Table 4. Optimal lag length by AIC, HQIC, and SBIC

	AIC	HQIC	SBIC	Selected length
Whole Period	18	5	2	5
Period 1	1	1	1	1
Period 2	2	2	2	2
Period 3	3	3	3	3
Period 4	2	2	2	2
Period 5	7	2	2	2

For the sub-periods 1, 2, 3, and 4, all criteria have the same results, which we choose as the lag length in the analysis. On the other hand, for the whole period and the sub-period 5, the optimal lag length given by the criteria do not match. Especially, AIC tends to provide the larger optimal lag length. Nonetheless, since the values of AIC are close (i.e., just 0.14% larger than the optimal one) and it is in the middle of the three criteria, we set the lag length to be 5 or equal to the one by HQIC in the whole period. Likewise, since the AIC values are very close (i.e., just 0.014% larger than the optimal one) and HQIC and SBIC give the same length, we set the lag length to be 2 in the analysis of sub-period 5.[11]

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[11] Analyses with different lag length provide the results quite similar to those in this paper.

5. Empirical results

5.1. Results for the whole period

The results of Granger causality tests for the whole period are reported in Table 5. Both test statistics are significant at 1% level. Thus, for the whole period, the VRP of stock and oil are dynamically influencing each other in the sense of Granger causality.

Table 5. Granger causality test (Whole period)

| Null Hypothesis	Period	Chi 2	# of lags
VRP_{sp} does not GC VRP_{oil}	Whole	21.174***	5
VRP_{oil} does not GC VRP_{sp}	Whole	26.076***	5

Note: *** represent significance at 1% level.

Figure 3 shows the orthogonalized impulse response functions of stock and oil with 95% confidence interval where we order VRP_{oil} before VRP_{sp}.[12] As the graphs in the above-right and below-left show, if we see the whole period, shocks in both VRP_{sp} and VRP_{oil} have, though small, significantly positive effects on each other for most of all 20 trading days (about 1 month) after the shock. For example, in the above-right graph, the impulse response functions of VRP_{sp} to VRP_{oil} are more than 0.2 till date 14 and then gradually decreasing to 0.118 at date 20. Meanwhile, in the below-left graph, the impulse response functions of VRP_{oil} to VRP_{sp} are more than 0.2 till date 17, except date 3 and 4, and then slightly decreases to 0.157 at date 20.

Fig. 3. Impulse response functions (Whole period)

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[12] We obtain the similar result if we reverse the order of VRP_{oil} and VRP_{sp}. We select this ordering since the results of Granger causality tests show more persistent Granger causality from oil to stock than from stock to oil for most of the sub-periods. For more detail, see next subsection.
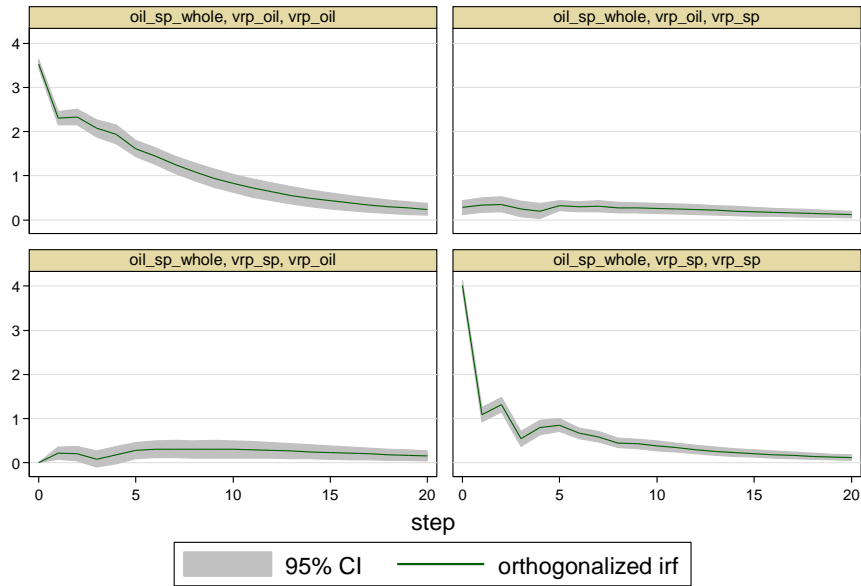
Graphs by irfname, impulse variable, and response variable

The results of variance decomposition are summarized in Table 6. Shocks to VRP_{sp} explained by innovations in VRP_{oil} are shown in the 4th column, which indicates that 5.2% of the forecast-error variance of VRP_{sp} is explained by innovations in VRP_{oil} at date 20. Meanwhile, shocks to VRP_{oil} explained by innovations in VRP_{sp} are shown in the 3rd column, which indicates that 2.7% of the forecast-error variance of VRP_{oil} is explained by innovations in VRP_{sp} at date 20.

Table 6. Variance decompositions (Whole period)

Impulse	VRP_{oil}	VRP_{sp}	VRP_{oil}	VRP_{sp}
Response	VRP_{oil}		VRP_{sp}	
1	1	0	0.005	0.995
5	0.996	0.004	0.020	0.980
10	0.986	0.014	0.037	0.963
15	0.977	0.023	0.048	0.952
20	0.973	0.027	0.052	0.948

Note: The numbers in the first column show the length of horizon.

In sum, seen in the whole period, VRP of stock and oil seem to have similar (i.e., small but significantly positive) effects on each other.

5.2. Results for the sub-periods

If we see the relations between VRP of stock and oil in the sub-periods, however, we have rather different pictures. First, Table 7 shows the results of Granger causality tests for the sub-periods.

Table 7. Granger causality tests for sub-periods

Null hypothesis	Period	Chi 2	# of lags
VRP_{sp} does not GC VRP_{oil}	Period 1	1.214	1
	Period 2	1.011	2
	Period 3	35.073***	3
	Period 4	1.439	2
	Period 5	4.786*	2
VRP_{oil} does not GC VRP_{sp}	Period 1	0.024	1
	Period 2	6.861**	2
	Period 3	27.029***	3
	Period 4	18.452***	2
	Period 5	9.066**	2

Note: *, **, and *** represent significance at 10%, 5%, and 1% level, respectively.

It is interesting to see that the stock VRP Granger causes the oil VRP very strongly at 1% significance level in the post-crisis recovery period 1 (period 3), but not in the other sub-periods except for the recent plunging oil price period (period 5) in which VRP_{sp} Granger causes VRP_{oil} only at 10% significance level. On the other hand, the oil VRP Granger causes the stock VRP strongly at 1% significance level in the post-crisis recovery periods (periods 3 and 4) and relatively strongly at 5% significance level in the crisis outbreak and the plunging oil price periods (periods 2 and 5, respectively). Thus, the Granger causality from VRP_{sp} to VRP_{oil} in the whole period is mainly from that in the post-crisis recovery period 1 (period 3), while the Granger causality from VRP_{oil} to VRP_{sp} is persistent after the outbreak of the crisis.

Now, since the results of Granger causality test suggest that Granger causality is stronger and more persistent from oil to stock than from stock to oil, we order VRP_{oil} before VRP_{sp} in the following analyses of orthogonalized impulse response functions and variance decomposition. The impulse response functions for each sub-periods are as follows:

Fig. 4. Impulse Response Function (Period 1)
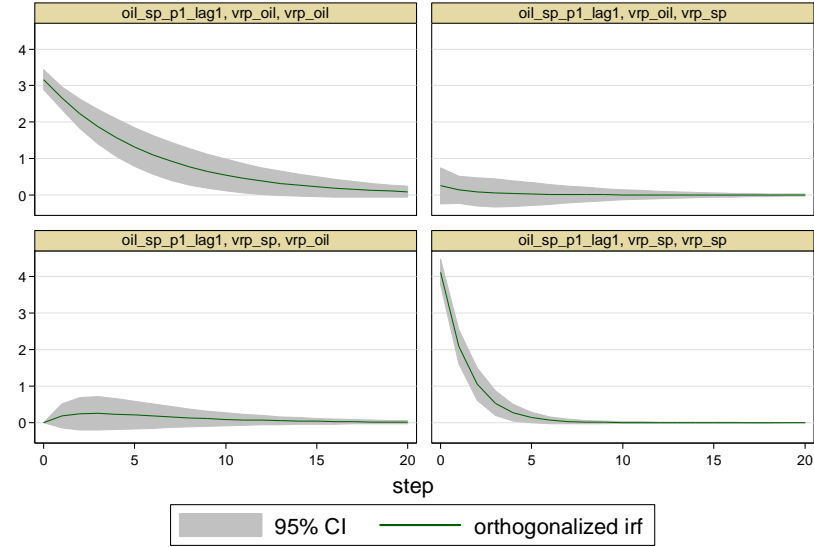
Graphs by irfname, impulse variable, and response variable
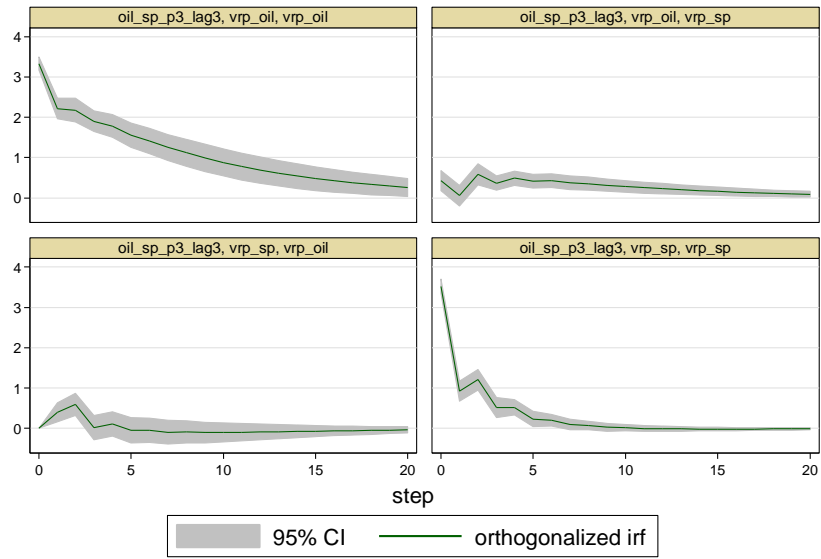
Fig. 5. Impulse Response Function (Period 2)
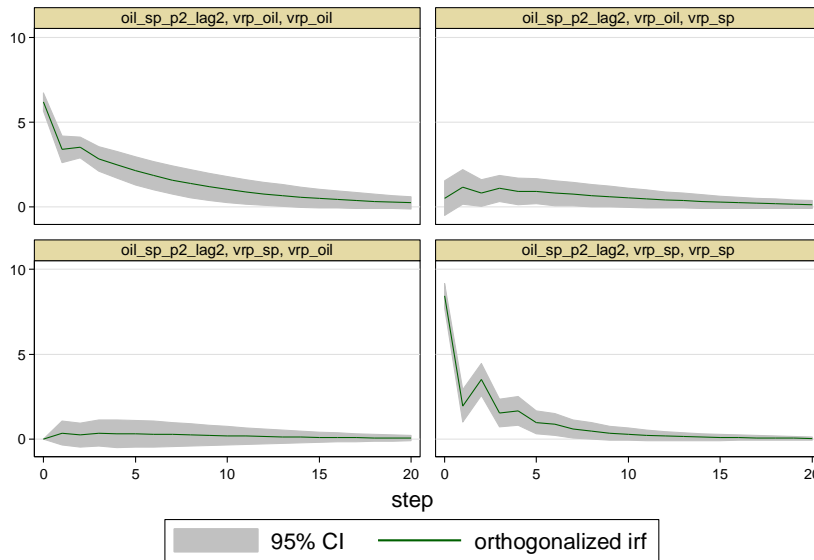
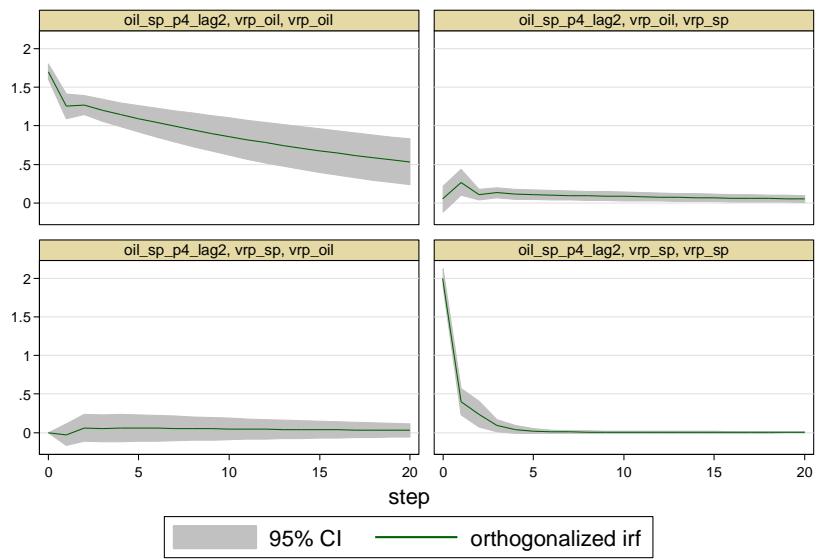Graphs by irfname, impulse variable, and response variable
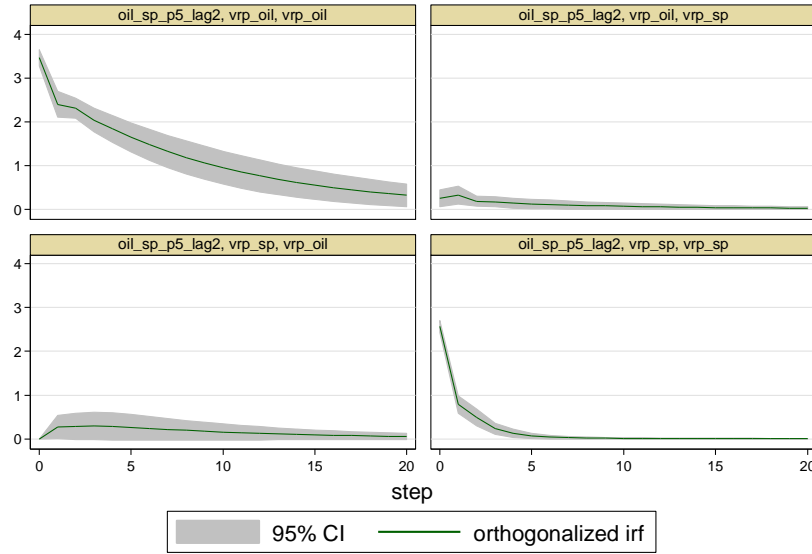
Fig. 6. Impulse Response Function (Period 3)

Fig. 7. Impulse Response Function (Period 4)

Fig. 8. Impulse Response Function (Period 5)

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| | oil_sp_p5_lag2, vrp_oil, vrp_oil | oil_sp_p5_lag2, vrp_oil, vrp_sp |
| oil_sp_p5_lag2, vrp_sp, vrp_oil | oil_sp_p5_lag2, vrp_sp, vrp_sp | |

95% CI   orthogonalized irf

Graphs by irfname, impulse variable, and response variable

As the graphs in the below-left of the figures show, in the sub-periods, the stock VRP has little effects on the oil VRP. Indeed, there are no significant effects from VRP_{sp} to VRP_{oil} except in the post-crisis recovery period 1 (period 3), but even in that period, the effects are very temporary and significant only up to the 2nd trading day after the shock.]

On the other hand, as the graphs in the above-right of the figures show, the oil VRP has, though small, significantly positive effects on the stock VRP after the outbreak of the crisis i.e., in the sub-periods 2, 3, 4, and 5. For example, in the crisis outbreak period (period 2), the orthogonalized impulse response functions from VRP_{oil} to VRP_{sp} are significantly positive from the 1st to the 8th trading days after the shock. In the post-crisis recovery period 1 (period 3), they are significantly positive in all, but the 1st, trading days. In the post-crisis recovery period 2 (period 4), they are significantly positive in all trading days after the shock. Finally, in the plunging oil price period (period 5), they are significantly positive up to the 7th trading days. Consistently with the Granger causality tests, those results of the impulse response functions in the sub-periods, suggest that the oil VRP dynamically affects the stock VRP, but not the other way around.

Table 7 shows the results of variance decomposition for the sub-periods. The shocks to VRP_{oil} explained by innovations in VRP_{sp} are shown in the 3rd column, and the shocks to VRP_{sp} explained by innovations in VRP_{oil} are shown in the 4th column. Similarly to the results of Granger causality tests and impulse response

functions, the forecast-error variances of the oil VRP explained by innovations in the stock VRP are much smaller than those of the stock VRP explained by the oil VRP in all sub-periods but the pre-crisis period (period 1). For example, at date 20, the forecast-error variances of VRP_{oil} (VRP_{sp}) explained by innovations in VRP_{sp} (VRP_{oil}) are 0.011 (0.005), 0.009 (0.080), 0.015 (0.114), 0.002 (0.048), and 0.015 (0.040) in periods 1, 2, 3, 4, and 5, respectively. Again, the oil VRP dynamically affects the stock VRP much more than the stock VRP dynamically affects the oil VRP after the financial crisis.

Table 7. Variance decompositions (Sub-periods)

Impulse	VRP_{oil}	VRP_{sp}	VRP_{oil}	VRP_{sp}
Response	VRP_{oil}		VRP_{sp}	
Period1				
1	1	0	0.004	0.996
5	0.992	0.008	0.005	0.995
10	0.989	0.011	0.005	0.995
15	0.989	0.011	0.005	0.995
20	0.989	0.011	0.005	0.995
Period2				
1	1	0	0.004	0.996
5	0.995	0.005	0.044	0.956
10	0.992	0.008	0.070	0.930
15	0.991	0.009	0.078	0.922
20	0.991	0.009	0.080	0.920
Period3				
1	1	0	0.015	0.985113
5	0.981	0.019	0.056	0.944
10	0.985	0.015	0.095	0.905
15	0.985	0.015	0.110	0.890
20	0.985	0.015	0.114	0.886
Period4				
1	1	0	0.001	0.999
5	0.999	0.001	0.027	0.973
10	0.998	0.002	0.037	0.963
15	0.998	0.002	0.044	0.956

20	0.998	0.002	0.048	0.952
Period5				
1	1	0	0.009	0.991
5	0.990	0.010	0.032	0.968
10	0.986	0.014	0.037	0.963
15	0.985	0.015	0.039	0.961
20	0.985	0.015	0.040	0.960

Thus, the analyses on the sub-periods reveal that the dynamic relation between the stock and oil VRP depends on the economic situation and, more importantly, that the VRP of oil has long-lasting and significantly positive effects on that of stock after the outbreak of financial crisis, while the effects of the stock VRP on the oil VRP are limited and, if any, much short-lived.

6. Robustness Analysis

To check the robustness of the results in orthogonalized impulse response functions and variance decomposition, we repeat the analysis by reversing the order of VRP to place VRP_{sp} before VRP_{oil}. The results are quite similar to the above, though the effects of VRP_{sp} to VRP_{oil} (resp. VRP_{oil} to VRP_{sp}) become slightly stronger (resp. weaker) than those obtained above.

For example, Fig. 9 shows the orthogonalized impulse response functions for the sub-periods. We can see that the figures are qualitatively the same as those in the above.

Moreover, Table 8 shows the results of variance decomposition focusing on the 20th trading day after the shocks. We can see from this table that the forecast-error variances of the oil (resp. stock) VRP explained by innovation in the stock (resp. oil) VRP are now generally larger (resp. smaller) than those in the previous case. For example, in period 3, the forecast-error variance of VRP_{oil} (resp. VRP_{sp}) explained by VRP_{sp} (resp. VRP_{oil}) is 0.039 (resp. 0.084) at the 20th day after the shock, while the corresponding ones in the case above are 0.015 (resp. 0.114).
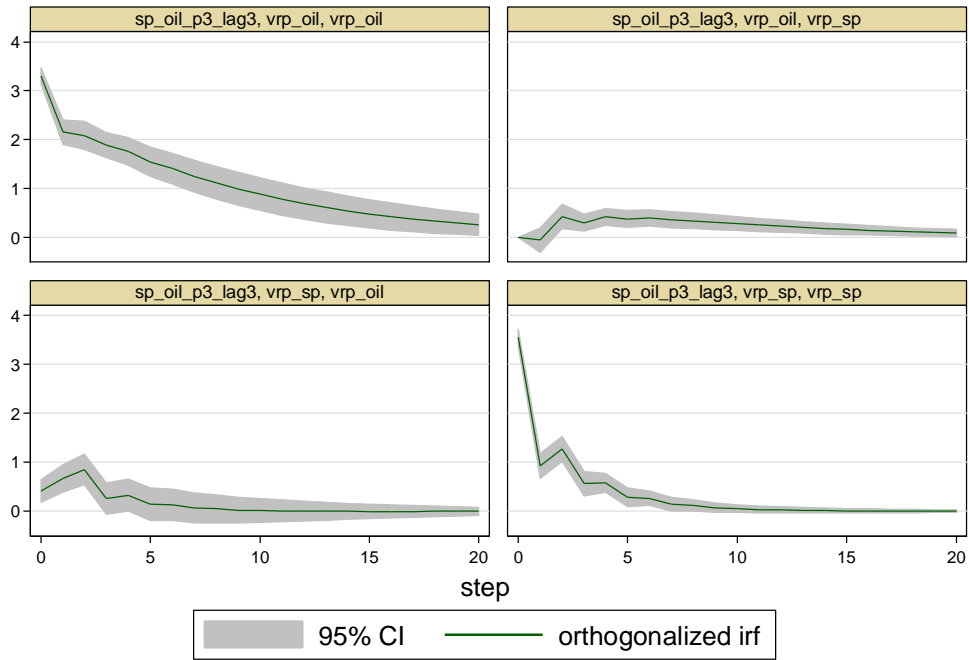
Fig. 9. Impulse Response Function with Order VRP_{sp} before VRP_{oil}
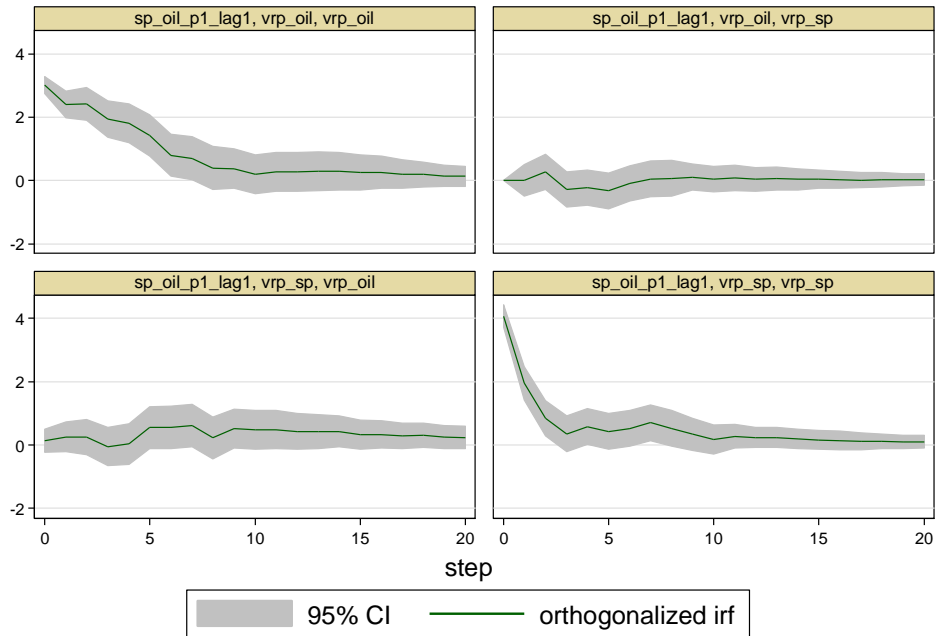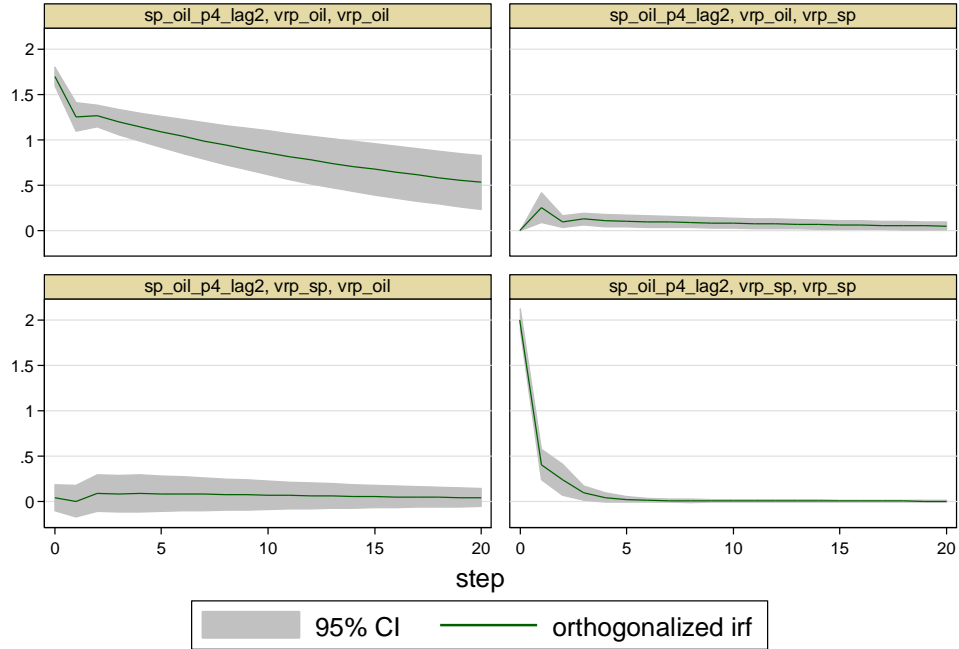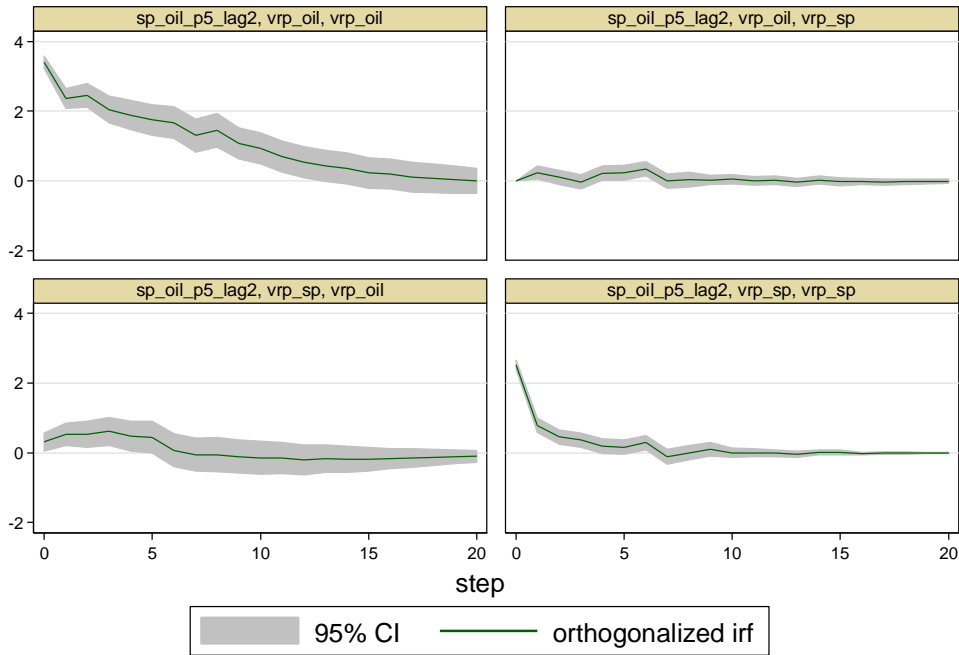

Graphs by irfname, impulse variable, and response variable
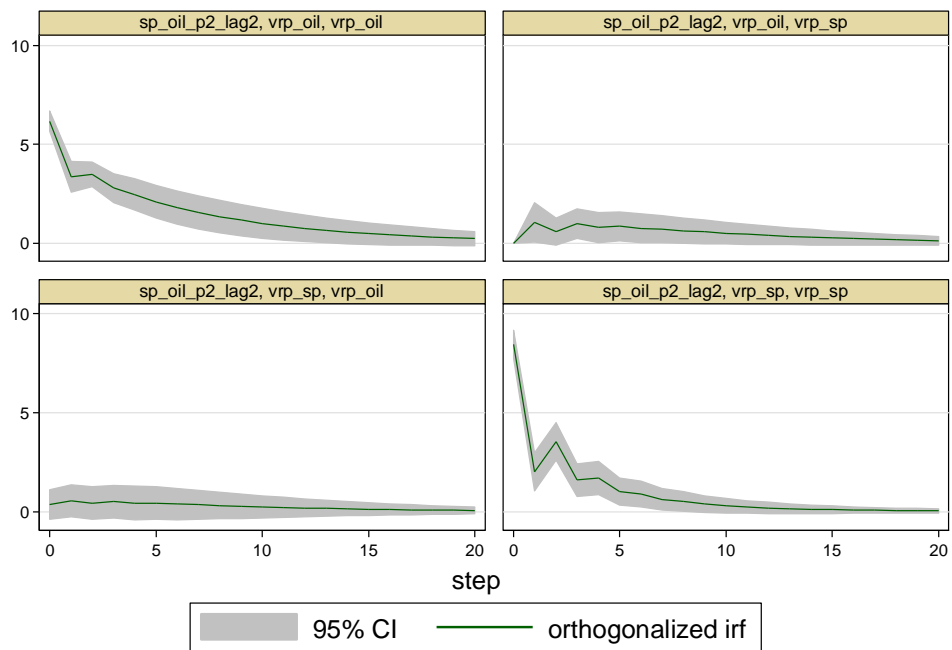
Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable

Graphs by irfname, impulse variable, and response variable

sp_oil_p5_lag2, vrp_oil, vrp_oil   sp_oil_p5_lag2, vrp_oil, vrp_sp

sp_oil_p5_lag2, vrp_sp, vrp_oil   sp_oil_p5_lag2, vrp_sp, vrp_sp

step

95% CI   ——— orthogonalized irf

Graphs by irfname, impulse variable, and response variable

Table 8. Variance decompositions with Order VRP_{sp} before VRP_{oil}
(Effects at the 20th trading day after the shock.)

Impulse	VRP_{oil}	VRP_{sp}	VRP_{oil}	VRP_{sp}
Response	VRP_{oil}		VRP_{sp}	
Period1				
20	0.975	0.025	0	1
Period2				
20	0.979	0.021	0.065	0.935
Period3				
20	0.961	0.039	0.084	0.916
Period4				
20	0.995	0.005	0.045	0.955
Period5				
20	0.956	0.044	0.023	0.977

Similarly, we repeat the analysis by using different lag length, though it is not optimal, to see how the results can change. We choose the lag length equal to 4 and 9 since the former corresponds roughly to 1 week (5 trading days) coverage and the

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latter to 2 weeks (10 trading days). We do not report the results here, but they are qualitatively the same as those obtained above while the effects of stock on oil seem slightly stronger.

7. Summary and Concluding Remarks

VRP is found to have predictive power on returns in many different assets. While most of the extant researches analyze such predictability of VRP on asset returns, in this paper, we investigate how the VRP of different assets, specifically those of stock and oil, are dynamically related to each other. To this end, we obtain the stock VRP as the difference of the VIX published by the CBOE and the realized volatility of the S&P 500 stock index provided by the Oxford-Man Institute of Quantitative Finance. On the other hand, to construct the oil VRP, we estimate the realized volatility of the USO by a stochastic volatility model and subtract it from the OVX published by the CBOE.

Using a daily data from May 10, 2007 to May 16, 2017 so obtained, we conduct VAR analyses on the stock and oil VRP for the whole period and 5 sub-periods that represent the pre-crisis, crisis outbreak, post-crisis recovery (the first and the second half), and plunging oil price periods.

The analysis on the whole period shows that the VRP of stock and oil have the similar (i.e., small but significantly positive) effects on each other. However, the analyses on the 5 sub-periods reveal a different picture. The dynamic relation between the stock and oil VRP depends on the economic situation and that contrary to the results for the whole period, the effects of the stock and oil VRP on their counterpart are quite different: The effects of the stock VRP on the oil VRP are limited mainly in the first half of the post-crisis recovery period and short-lived. On the other hand, the VRP of oil has significantly positive and long-lasting effects on that of stock in all sub-periods after the outbreak of financial crisis.

It is worth pointing out that those results suggest that the investors' sentiments (measured by volatility risk premia) are transmitted from the oil market to the stock market over time, but not the other way around. While Christoffersen and Pan (2017) find predictability of oil implied volatility on stock returns and implied volatility, this relation between oil and stock VRP is still an unexplored point in the extant literature and is a rather unexpected finding because *financialization* of commodities means the massive increase of investment in commodities by the investors in the traditional stock and bond markets and hence the direction of effects is thought to be from the

stock market to the commodity market, not from the commodity market to the stock market.

The mechanism of such transmission of VRP from oil to stock is, however, still unclear. One possible channel is funding constraints of financial intermediaries. As Christoffersen and Pan (2017) find that increases in oil implied volatility predict tightening funding constraints of financial intermediaries, which can affect stock price and implied volatility, the oil VRP may affect the stock VRP through funding constraints of institutional investors.[13] One may also speculate that the tendency of declining oil prices after the financial crisis and the emergence of shale oil/gas may make the investments into the U.S. stocks by the petroleum-exporting countries (i.e., "petrodollars") more sensitive to the future uncertainty of oil prices, to which other investors, too, become more sensitive. Investigating what causes the oil VRP to affect the stock VRP is an important target for the future research.

Also, due to the constraint on data availability, we have to estimate the realized volatility by applying a stochastic volatility model to the daily data of the USO returns. This makes our estimation of the oil VRP, which is the difference between the OVX published by the CBOE and the realized volatility so calculated, prone to misspecification of the used model. To utilize the same estimation method as the stock VRP, or more specifically as the stock realized volatility, it is desirable to estimate the realized volatility of the USO by using an intraday 1 minute return data. Such method also enables us to investigate the relation among the returns and VRP of stock and oil. Thus, this is the immediate next step for this research.

Finally, analyzing the relation among the returns and VRP of stock and oil helps investors to understand how the returns and sentiment of one market affects those of the other market and hence helps them to manage asset allocation and risk management. In this respect, the direction of this research is potentially fruitful.

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[13] Using weekly data, Hattori, Ilhoyock, and Sugihara (2018) find that increases in the U.S. stock VRP tend to reduce the fund flow into stocks of the emerging economy countries and suggest that this can be the cause of the spillover of VRP among countries.

Appendix: Estimation of realized volatility by a stochastic volatility model.

We estimate the daily realized volatility of oil by applying the following stochastic volatility model to its return.

$$r_{\text{oil } t} = \mu_{\text{oil}} + \beta_{\text{oil}}\left(e^{\frac{h_t}{2}} - \text{OVX}_t\right) + e^{\frac{h_t}{2}}\epsilon_t,$$

$$h_t = \mu_h + \beta_h(h_{t-1} - \mu_h) + \sigma_h\eta_t,$$

$$\epsilon_t \equiv \sqrt{\frac{\nu-2}{\nu}}\frac{\xi_t}{\sqrt{\zeta_t}}, \qquad \zeta_t \sim \Gamma(\nu/2, \nu/2),$$

$$\begin{pmatrix} \xi_t \\ \eta_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$$

where $r_{\text{oil } t}$ denotes the daily return of the USO, $N(\ )$ denote the bivariate standard normal distribution with correlation ρ, and $\Gamma(\nu/2, \nu/2)$ denotes the Gamma-distribution with shape and scale parameters equal to $\nu/2$. The realized volatility here is estimated as $RV_{\text{oil } t} = e^{\frac{h_t}{2}}$.

We estimate the model for the 5 sub-periods by Bayesian statistical inference according to the WAIC (widely applicable information criterion). The estimates of mean, 5% and 95% percentile points (in parenthesis) are as follows:

Table A. Estimates of mean and standard deviation of parameters

Parameter	Period 1	Period 2	Period 3	Period 4	Period 5
μ_{oil}	0.305 (0.192, 0.415)	-0.336 (-0.496, -0.178)	-0.017 (-0.03, -0.01)	0.007 (0.003, 0.010)	-0.194 (-0270, -0.122)
β_{oil}	0.041 (-0.039, 0.123)	0.068 (-0.017, 0.151)	0.032 (-0.044, 0.109)	0.028 (-0.046, 0.101)	0.085 (0.004, 0.164)
μ_h	1.232 (0.936, 1.530)	2.472 (2.084, 2.820)	1.278 (1.134, 1.428)	0.292 (0.095, 0.509)	1.675 (1.413, 1.939)

β_h	0.849	0.891	0.809	0.846	0.914
	(0.774, 0.915)	(0.835, 0.940)	(0.733, 0.873)	(0.771, 0.903)	(0.876, 0.946)
σ_h	0.380	0.369	0.363	0.368	0.349
	(0.336, 0.431)	(0.325, 0.417)	(0.325, 0.406)	(0.328, 0.413)	(0.311, 0.391)
ρ	-0.488	-0.4649	-0.467	-0.457	-0.413
	(-0.630, -0.335)	(-0.608, -0.310)	(-0.587, -0.331)	(-0.595, -0.308)	(-0.550, -0.269)
ν	8.047	8.083	8.455	8.014	8.534
	(6.455, 9.753)	(6.520, 9.757)	(6.972, 10.116)	(6.476, 9.664)	(6.994, 10.210)

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