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Diversification Effect of Commodity Futures on Financial Markets

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Abstract

This paper examines the portfolio diversification effect of commodity futures on financial market products introducing a comprehensive evaluation standard of risk standardization, robustly small correlations, and risk-return tradeoffs. Regarding risk standardization, we propose a definition of portfolio diversification as how much the distribution of portfolio returns is close to a normal distribution. It is shown by using α-stable distribution that if the commodity price return distribution has the opposite sign of skewness parameter β to financial portfolio's β, commodity diversification effect exists. The empirical studies using S&P500, U.S. 10-year bond and DJ-AIG commodity index are conducted to investigate the portfolio diversification effects. The parameter estimation results of portfolio return distributions, the conditional correlations using the dynamic conditional correlation model with financial exogenous variables, and the efficient frontier from the mean-CVaR portfolio optimization all suggest that commodity futures have a diversification effect on financial markets.

Keywords: Diversification effect, Commodity futures, α-stable distribution, Dynamic conditional correlation model with exogenous variables, Mean-CVaR portfolio optimization

JEL classification: C19, G19, Q40

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1. Introduction

Commodity markets are booming during the last decade against the backdrop of sluggish financial markets due to subprime loan problems and soaring real asset demand including demand for oil and iron ore from developing countries, in particular China and India. While commodity trading is generally classified in alternative investments, the classification stems from the alternation to traditional stocks and bonds in financial markets. The attractive characteristics of alternative assets may consist in the robustness including small correlations to financial asset price changes, i.e., the diversification effect. In reality, financial institutions like investment banks tend to increase the portion of commodities on their portfolios in an effort to diversify the portfolio risk during this commodity boom period. This paper highlights the diversification effect of commodities on financial assets.

Geman (2005) suggests that commodity trading produce high returns because of the diversification effects in the portfolio of stocks and bonds by showing the negative effect of oil prices on stock prices in the constant correlation coefficient over the periods from 1990 to 1999. While Gorton and Rouwenhorst (2006) examine the correlations between commodities futures and stock or bond price returns, they assume that the correlations are constant and it may not reflect the time varying property of the correlation. Geman and Kharoubi (2008) employ copula functions to analyze the diversification effects due to crude oil futures on equity portfolio. These studies focus on the linear and nonlinear relation between financial asset and commodity price returns. But they may miss the examination of the portfolio return changes due to the inclusion of commodities. In addition, they may not take into account time varying properties of the correlations between financial asset and commodity price returns. Büyüşahin, Haigh, and Robe (2010) find time varying correlations of equity-commodity returns using DCC model but they do not examine the time varying correlations affected by financial exogenous variables. Erb and Harvey (2006) explore the diversification effect of commodity futures on a financial portfolio by obtaining the efficient frontier with respect to the mean and variance as the return-risk relation. However, the return distribution of commodity prices may be skewed and the risk cannot be captured appropriately by the variance.

Regarding the recent advance of the dynamic correlation relationship among different asset classes, the relationship between commodities including crude oil and financial assets has recently been investigated by many researchers. Creti, Joëts, and Mignon (2013) employ the dynamic conditional correlation model of Engle (2002) to investigate the links between stock and commodity markets including crude oil, resulting in the key role of the 2007-2008 financial crisis in highlighting the financialzation of commodity markets. Büyükşahin and Robe (2014) empirically found that the correlation rises between commodities and equities after 2008 using the Engle's DCC model. Delatte and Lopez (2013) provide a thorough study on equity and commodity dependence and identify stylized facts of both asset classes using copula approaches, resulting in stronger equity-commodity comovement after 2008 financial crisis. Berger and Uddin (2016) provide a comprehensive survey on different dependence schemes between wavelet decomposed return series of stock indices, commodity futures including crude oil, and volatility indices, resulting in strong long-run dependence between S&P 500 and commodity futures returns. Kyrtsou, Mikropoulou, and Papana (2016) suggest a multivariate investigation of the dynamic linkage between the S&P 500 index and the petroleum products including the asymmetric Mackey-Glass causality test, resulting in the presence of nonlinear relationship from declining S&P 500 index returns to the crude oil futures-spot price spreads. Cifarelli and Paladino (2010) employ a behavioral ICAPM to investigate whether feedback trading strategies may cause oil price deviations from the fundamental values. Kolodziej, Kaufmann, Kulatilaka, Bicchetti, and Maystre (2014) empirically support the argument that it has been more beneficial to hold crude oil as a financial asset than as a real commodity after fall 2008. Cheng, Kirilenko, and Xiong (2015) study the joint responses of commodity futures prices and positions of financial traders and hedgers, resulting in convective risk flows that financial traders reduce their net long positions during the 2008 financial crisis while hedgers facilitate this by reducing their short positions. By using commodity-link notes, Henderson, Pearson, and Wang (2015) demonstrate the significantly positive and economically meaningful impacts of financial investors on commodity futures prices around the pricing dates of commodity-link notes when the hedge trades are executed while they demonstrate significant negative price impacts around the determination dates when the hedge trades are unwound. These studies are quite insightful to figure out the relationship between commodities including crude oil and financial assets. However these studies do not examine the diversification effect of commodities on financial asset portfolio.

The paper tries to deepen the understandings of the diversification effect of commodities on a financial portfolios shedding lights on the portfolio return distribution changes due to the addition of commodities to the portfolios. Then, the paper tries to fill the gap between existing literature regarding two angles from time varying commodity-financial asset price correlations with financial exogenous variables and the mean-conditional value at risk (CVaR) based efficient portfolio frontier. In the beginning, we propose a method to capture the diversification effect of commodities on financial portfolios using the portfolio return model with α-stable distribution. Then, the method is applied to the empirical studies using S&P500, U.S. 10-year bond, and DJ-AIG commodity index. We show that the commodity may have the diversification effect on financial markets according to the estimated βs. As the additional studies to fill the gap between the existing literature, we first address the diversification issue due to commodities by using the dynamic conditional correlation model of Engle (2002) with financial exogenous variables (see Vegas (2008)). It is shown that the diversification effects are supported by small and robust conditional correlations between S&P500 or U.S. 10-year bond and DJ-AIG commodity index using the DCC model with financial exogenous variables. Second, we compare the efficient frontiers of financial portfolios with and without a commodity obtained from the mean-CVaR portfolio optimization. The results demonstrate that the efficient frontier of the financial portfolio with a commodity can be improved more than the efficient frontier of the financial portfolio without a commodity.

This paper is organized as follows. Section 2 proposes a portfolio diversification definition from risk standardization using α-stable distribution, which is helpful to analyze the diversification effect of commodities on financial portfolios. Section 3 conducts empirical studies of the diversification effect of commodity indices on stocks and bonds using the diversification definition in Section 2 as well as using a dynamic conditional correlation model with financial exogenous variables and the mean-CVaR portfolio optimization by introducing a comprehensive evaluation standard of risk standardization as well as robustly small correlation and risk-return tradeoff, respectively. Section 4 concludes and offers the directions for our future research.

2. Portfolio Diversification from Risk Standardization

The central limit theorem is used to justify the assumption that most random variables which represent the numerical characteristics of real populations are approximately normally distributed as in e.g., Bickel and Doksum (2001). The theory gives us an idea that the return distribution of equally weighted portfolio made of random variables may be represented by a normal distribution. On the other hand, Ibragimov (2005) demonstrates that the equally weighted portfolio is most diversified than the others if return distributions are moderately heavy-tailed by employing majorization theory. To this end, portfolio diversification can be interpreted as the degree of the closeness of the portfolio return distribution to a normal distribution in the first order approximation. In addition, King and Young (1994) suggests that non normality of real estate returns can not attain efficient diversification within real estate asset class.

Non normality of price return distribution is mainly characterized by its third and fourth moments, i.e., skewness and kurtosis. In particular, the fat tail due to the skewness is problematic for portfolio risk management. We recognize that traditionally the concept of diversification is based on the second moment of the portfolio return: diversification of a portfolio is defined as the second moment approaches to zero. However we believe that the other moment like skewness than the second moment is also a key to the diversification effect taking into account that commodities are used for diversification of financial portfolios in practice although commodities have no correlations to financial assets' returns, which may cause the increase of portfolio return volatilities. We start with the definition of the diversification effect of a commodity to a financial portfolio shedding lights on the skewness of the portfolio return distribution.

Definition 1 *A commodity contributes to the diversification of a financial portfolio, if the return distribution of the integrated financial portfolio with a commodity approaches the normal distribution more than the return distribution of the original financial portfolio in the sense of the skewness.*

In order to examine the usefulness of commodity to help financial portfolio diversified, we need to model the distribution of the price returns. α-stable distribution is useful to demonstrate how much the distribution is close to a normal distribution by examining the parameters. The log-returns of commodity prices may have high skewness and kurtosis. In that case, it is difficult to model such time series appropriately by using the normal distribution. α-stable distribution is often introduced as a tool to model such high skewness and kurtosis. Unfortunately, α-stable distribution does not have its distribution function and density in its closed form. Stable distributions are introduced by their characteristic function as follows (see, e.g., Samorodnitsky and Taqqu (1994)):

$$E\exp(itX) = \begin{cases} \exp\{-\sigma^\alpha \mid t \mid^\alpha \left(1 - i\beta \mathrm{sgn}(t)\tan(\frac{\pi\alpha}{2})\right) + i\mu t\}, & \alpha \neq 1 \\ \exp\{-\sigma \mid t \mid \left(1 + i\beta\frac{2}{\pi}\mathrm{sgn}(t)\log \mid t \mid\right) + i\mu t\}, & \alpha = 1. \end{cases} \tag{1}$$

The parameter α describes the kurtosis of the distribution with $0 < \alpha \leq 2$. The smaller α is, the heavier the tail of the distribution is. The parameter β describes the skewness of the distribution, $-1 \leq \beta \leq 1$. If β is positive (negative), then the distribution is skewed to the right (left, respectively). μ and σ are the shift and scale parameters, respectively. If α and β equal 2 and 0, respectively, then the α-stable distribution is identical to a normal distribution.

We present the portfolio return model using α-stable distribution. Suppose that the asset price return X_i is i.i.d. and follows the stable distribution $S_\alpha(\sigma_i, \beta_i, \mu_i)$, i.e., each parameter α is approximately considered as a constant among the assets in the portfolio. Considering n assets that constitute the portfolio where each asset weight is denoted by w_i, the portfolio return X is given by

$$X \sim S_{\alpha}(\sigma, \beta, \mu), \tag{2}$$

$$\text{where } \sigma = \left(\sum_{i=1}^{n} \left(w_i \sigma_i \right)^{\alpha} \right)^{\frac{1}{\alpha}}, \tag{3}$$

$$\beta = \sum_{i=1}^{n} \left(\frac{\sigma_i w_i}{\sigma} \right)^{\alpha} \beta_i, \tag{4}$$

$$\mu = \sum_{i=1}^{n} \left(w_i \mu_i \right). \tag{5}$$

Note that we assume $w_i > 0$ for simplicity. The proof is obtained from Eq. (1). Then, we have the following proposition relevant to the portfolio diversification.

Proposition 1 *Suppose that a commodity whose price return distribution possesses the opposite sign of* β *to a financial portfolio is injected into the portfolio. Then, the commodity injection contributes to the diversification effect of the portfolio.*

Proof $(\frac{\sigma_i w_i}{\sigma})^{\alpha}$ in Eq. (4) is always positive by definition. It implies that if an asset with the opposite sign of β to a financial portfolio is injected into the financial portfolio, the ex post β of the portfolio approaches zero. Since it makes the distribution of the portfolio more symmetry, we can recognize that the diversification effect is valid injecting a commodity into a financial portfolio by Definition 1.||

The idea is consistent with Goetzmann and Kumar (2008) in the sense that the high volatility and high skewness in stock prices disturb the portfolio diversification. By using this proposition, we empirically examine the diversification effect of the commodity that is often recognized by financial market participants.

3. Empirical Studies for Commodity Futures

3.1. Data

We use Dow Jones-AIG commodity index (DJ-AIG CI), S&P 500, and the US 10 year treasury note rates to represent commodity, stock, and bond markets, respectively. The data start from

January 3, 1991 to May 25, 2007.[1] Note that we calculate hypothetical bond prices with 10 year maturity where a coupon is equal to the yield (e.g., see Jones, Lamont, and Lumsdaine (1998)). The data consist of the daily adjusted closing data provided by yahoo finance. The DJ-AIG CI is comprised of 19 commodity futures including energy, metal, and agricultural futures. The DJ-AIG CI is considered as an index to represent physical commodity futures markets. The basic statistics of the data are reported in Table 1. Table 1 implies that the price distribution of S&P 500 is skewed to the left, while that of the DJ-AIG CI is skewed to the right. We test whether the skewness of S&P 500 and 10-year bond rate are statistically significantly different from zero using D'Agostino skewness test. p-values for S&P 500 and 10-year bond rate are obtained as 2.103×10^{-5} and 6.465×10^{-12}, respectively, implying that the both have skewness. We also test whether the kurtosis of the DJ-AIG CI is statistically significantly different from three using Anscombe-Glynn kurtosis test. p-value for the DJ-AIG CI is obtained as 0.04393, implying that the kurtosis of the DJ-AIG CI is not equal to three.

[INSERT TABLE 1 ABOUT HERE]

3.2. Analyses of risk standardization

We estimate the parameters of stability α and skewness β for the DJ-AIG commodity index, US 10-year bond price, and S&P500 returns, respectively using the regression method of Koutrouvelis (1980). In order to compare the parameters among the three, we split the observations into 8 terms each of which comprises of 500 observations. The results are illustrated in Figure 1 and Table 2, respectively. Figure 1 shows that the estimated stability parameters (αs) of the three assets are located from 1.75 to 2.0 within the whole parameter range from 0 to 2. In contrast, the estimated skewness parameters (βs) are scattered from -1 to 1, which corresponds to the whole parameter range. Comparing to βs, αs are considered constant in the first order approximation. This is consistent with Table 2 where all three αs are close to 1.8571, the average of αs. In Figure 1, the corresponding skewness parameters (βs) for commodity tend to possess the opposite sign to the stock index βs. This is also observed from Table 2 such that the skewness parameter of β for the commodity index is -0.2955 while the skewness parameter for the stock index is 0.0916. Then setting the constant α as 1.8571, the average of αs in Table 2, we reestimate the skewness parameters. The results are reported in Table 3. The skewness parameter of β for the commodity index is -0.3081 while the stock index β is 0.0851. According to Proposition 1, β for commodity

[1] After May 2009, DJ-AIG CI was renamed as Dow Jones-UBS Commodity Index (DJ-UBS CI) due to the acquisition of U.S. insurer AIG's stake in the DJ-AIG CI by the Swiss bank USB.

index has the opposite sign to β for S&P500. Thus the injection of the commodity index into the financial portfolio of stock index can improve the diversification of the portfolio.

[INSERT FIGURE 1 ABOUT HERE]

[INSERT TABLE 2 ABOUT HERE]

[INSERT TABLE 3 ABOUT HERE]

Tables 2 and 3 imply that a commodity does not seem to contribute to the diversification of the bond portfolio because the both βs are negative. But this may come from the influence of the total data period. To examine how the commodity index return distribution simultaneously contributes to the portfolio distribution including bonds, Figure 2 illustrates the estimated β's for equally divided eight periods stemmed from the data. The commodity β's tend to have the opposite sign to stock βs in periods 1 and 5 during eight periods. The commodity β's tend to have the opposite sign to bond βs in periods 1, 3, 4, and 6 during eight periods. Thus, the results may suggest that the commodity be useful for the portfolio diversification both for stocks and bond in that the distribution skewness approaches zero from Definition 1. That is, as the implication of this study, the third moment of commodity price returns can help to reduce the distortion of the financial portfolio investment risk.

[INSERT FIGURE 2 ABOUT HERE]

3.3. Analysis of robustly small correlation

Portfolio diversification is often discussed using price correlations between various assets. Geman (2005) offers the negative effect of oil prices on stock prices in the constant correlation coefficient over the periods from 1990 to 1999 in an attempt to show the diversification effect. However, the correlation structure may demonstrate time-varying property. In addition, whether commodity-financial asset correlations may be affected by financial exogenous variables or not is a key to examine the robustness of the correlations. To examine the diversification effects of commodities on financial assets taking into account the time-varying correlation property affected by financial exogenous variables, we model the log return of the prices y_t using the dynamic conditional correlation (DCC) model of Engle (2002) with exogenous variables X_t (DCC-X model of Vegas (2008)).

$$y_t = \varepsilon_t \sim N(0, H_t), \tag{6}$$

$$\varepsilon_t = D_t \eta_t, \tag{7}$$

$$D_t = \text{diag}[h_{1,t}^{\frac{1}{2}} h_{2,t}^{\frac{1}{2}}], \tag{8}$$

where $y_t = (y_{1,t}, y_{2,t})'$, $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$, and $\eta_t = (\eta_{1,t}, \eta_{2,t})'$.

For $i = 1, 2$, we have

$$h_{i,t} = \omega_i + \alpha_{i1} \varepsilon_{i,t-1}^2 + \beta_{i1} h_{i,t-1}, \tag{9}$$

$$E[\varepsilon_t \varepsilon_t' \mid F_{t-1}] = D_t R_t D_t, \tag{10}$$

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1}, \tag{11}$$

$$Q_t = (1 - \theta_1 - \theta_2) Q - K\xi\bar{X} + \theta_1 \eta_{t-1} \eta_{t-1}' + \theta_2 Q_{t-1} + K\xi X_{t-1}, \tag{12}$$

where Q_t^* is the diagonal component of the square root of the diagonal elements of Q_t[2] and K is a matrix of ones. Eq. (9) represents the GARCH(1,1) effect for the price returns for each asset. The conditional correlations are calculated using Eq. (11) where Eq. (12) represents time varying conditional covariance. The scale parameters θ_1 and θ_2 represent the effects of previous standardized shock and conditional correlation persistence, respectively.[3] If either of the estimations of θ_1 or θ_2 in Eq. (12) is statistically significant, the correlation structure of the pairs demonstrates time varying. The estimation of parameters is conducted using the QMLE. The time-varying conditional correlations are empirically calculated using the errors (η_t) obtained from each GARCH(1,1) model.

We estimate the DCC-X model for DJ-AIG commodity index and S&P500 and calculate the conditional correlations to examine the diversification effect of the commodity on the stock portfolio. Note that U.S. 10-year bond price returns are employed as the exogenous variable X. The estimation results and conditional correlations are illustrated in Table 4 and Figure 3, respectively. According to the standard errors as in Table 4, α_1, β_1, α_2, and β_2 are statistically significant. It implies that there exist the GARCH effects both in the returns of DJ-AIG commodity index and S&P500. In addition, Table 4 demonstrates that θ_2 is statistically significant, meaning that there exist the dynamic conditional correlations. Thus, the conditional correlations between commodity

[2] Define $Q_t \equiv \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$. Then, $Q_t^* = \begin{pmatrix} \sqrt{q_{11}} & 0 \\ 0 & \sqrt{q_{22}} \end{pmatrix}$.

[3] θ_2 represents the persistence of the conditional covariance matrix. Since the standardized shock η_t is used for the calculation, θ_2 is approximately considered as the conditional correlation persistence.

and stock are time varying as in Figure 3. However, judging from Figure 3, the conditional correlations are range bound between the small correlations of 0.2 and -0.2. Thus, the correlations are considered as nearly zero while the correlations are heteroskedastic. This is consistent with the correlation persistency of 0.954 in Table 4, implying that the low correlation structure lives long. It may suggest that there exist the diversification effect due to a commodity on the stock portfolio in that almost no correlated asset of the commodity is combined with the stock index. More importantly, ξ is not statistically significant, implying that U.S. 10-year bond price returns do not affect the correlations between DJ-AIG commodity index and S&P500. It may support the robustness of small commodity-stock correlations in that they are not affected by exogenous U.S. 10-year bond price returns.

[INSERT TABLE 4 ABOUT HERE]

[INSERT FIGURE 3 ABOUT HERE]

As the next step, we estimate the DCC-X model for DJ-AIG commodity index and U.S. 10-year bond and then calculate the conditional correlations to examine the diversification effect of the commodity on the bond portfolio. Note that we assume that the 10 year U.S. treasury note price is calculated using an assumption of a coupon equivalence to the yield and $100 principal for simplicity because we only have the 10 year US treasury note rate r_t data. Here S&P500 returns are employed as the exogenous variable X. The estimation results and conditional correlations are illustrated in Table 5 and Figure 4, respectively. According to the standard errors as in Table 5, the parameters except ω_2, α_2, β_2, θ_1, and ξ are statistically significant. Thus, there is no GARCH effect in the US 10-year bond prices. However, the conditional covariances are still influenced by the previous covariances judging from the significance of estimated θ_2 as in Table 5. According to Figure 4, the conditional correlations are time varying and between about 0.2 and -0.2. Thus, we conclude that such a small correlation gives rise to the diversification effect by commodity on the bond portfolio. More importantly, ξ is not statistically significant, implying that S&P500 returns do not affect the correlations between DJ-AIG commodity index and U.S. 10-year bond prices. It may also support the robustness of small commodity-bond price correlations in that they are not affected by exogenous S&P500 returns.

[INSERT TABLE 5 ABOUT HERE]

[INSERT FIGURE 4 ABOUT HERE]

In the process to obtain Proposition 1, we assume that commodity, stock, and bond price returns are i.i.d.. If it is true, the correlations between price returns should be observed as nearly zero. The results obtained using the DCC-X model may partly correspond to the i.i.d assumption. As the implication of this study, commodities can help to work as robustly defensive assets in the financial portfolios, which stabilize the portfolios.

3.4. Analysis of risk-return tradeoff

Erb and Harvey (2006) explore the diversification effect of commodity futures in the financial portfolio by obtaining the mean-variance efficient frontier. However, since the returns of commodity futures do not necessarily follow the normal distribution, the variance cannot capture the risk of the portfolio including stocks, bonds, and commodities appropriately. This paper introduces the conditional value at risk (CVaR) as the risk measure. Rockafellar and Uryasev (2000) show that the mean-CVaR portfolio optimization is consistent with the following optimization:

$$\min_{x,\alpha} \; F_\beta(x,\alpha) \tag{13}$$

$$s.t. \;\; x_j \geq 0 \text{ for } j = 1,\ldots,n, \text{with } \sum_{j=1}^{n} x_j = 1, \tag{14}$$

$$\sum_{j=1}^{n} x_j E[y_j] \geq \mu, \tag{15}$$

$$\text{where } \; F_\beta(x,\alpha) = \alpha + \frac{1}{q(1-\beta)} \sum_{i=1}^{q} [-x^T y_i - \alpha]^+. \tag{16}$$

Note that β is the confident level often used as 90%, 95%, or 99%. If the efficient frontier of the financial portfolio with a commodity improves more than the efficient frontier of the financial portfolio without a commodity, there exists the diversification effect of the commodity as in Geman (2005). By using this evaluation criteria, we are going to examine the diversification effect of commodities on a financial portfolio empirically.

Following the technique of Rockafellar and Uryasev (2000), we conduct the mean-CVaR portfolio optimization of two portfolios: the portfolio of stock and bond and the portfolio of stock, bond, and commodity in order to observe the diversification effect of commodity on the stock and bond portfolio. We use a constant yield and then the 10 year US treasury rate for the bond.

In the beginning, for simplicity we assume risk free bond whose return is 0.0005 per trading day constant. Applying the procedure as in the above, we obtain the efficient frontier of the portfolio of stock and bond and the efficient frontier of the portfolio of stock, bond, and commodity, respectively. The result is illustrated as in Figure 5. Here, we employ the confidence level β as 90%. The result implies that the injection of commodity into financial portfolio may have the diversification effect in the sense of risk-return tradeoff because the efficient frontier of the financial portfolio with a commodity improves more than the efficient frontier of the financial portfolio without a commodity.

[INSERT FIGURE 5 ABOUT HERE]

We also conduct the mean-CVaR portfolio optimization by using two portfolio: the portfolio of S&P500 stock and U.S. 10-year treasury bond and the portfolio of the stock, bond, and commodity in order to observe the diversification effect of commodity on a stock and bond portfolio. The result is illustrated as in Figure 6. Here, we also employ the confidence level β as 90%. The result indicates that the injection of commodity into financial portfolio may have the diversification effect in the sense of risk-return tradeoff, even if we employ the U.S. 10-year treasury note. The results are identical to the diversification effect of commodities on the financial portfolio as in Geman (2005). One may think that if the efficient frontier of the portfolio conducted by various stocks can be improved by adding the commodity asset, then the results can be more persuasive. Taking into account that the portfolio conducted by various stocks is considered as the market portfolio of S&P500, we may safely say that the results are convincing.

[INSERT FIGURE 6 ABOUT HERE]

Finally, we calculate the stable tail adjusted risk ratio (STARR) which is defined by the mean over CVaR of the portfolio returns (see Martin, Rachev, and Siboulet (2003)).

$$STARR_{\beta} = \frac{E[r]}{CVaR_{\beta}(r)}, \tag{17}$$

where β denotes the confidence interval and here is set to be 90%. STARR ratio represents the ratio between the expected excess return and its CVaR. We illustrate the STARR ratios for the two portfolios as in Figure 7. Figure 7 implies that the STARR ratio with a commodity is greater than the STARR ratio without a commodity. Considering that the financial portfolio with a commodity can improve its efficient frontier more than the financial portfolio without a commodity, the STARR ratio may be useful to capture the diversification effect in the sense of risk-return tradeoff in the financial portfolios with commodities. As the implication of this study, commodities can help to improve the performance of financial portfolio regarding the advanced risk and return tradeoff.

[INSERT FIGURE 7 ABOUT HERE]

4. Conclusions and Directions for Future Research

This paper has examined the portfolio diversification effect of commodity futures on financial market products by introducing a comprehensive evaluation criterion of risk standardization, robustly small correlation, and risk-return tradeoff. Using the portfolio return model with α-stable distribution we propose a definition of the diversification from risk standardization as how much the distribution of portfolio returns is close to a normal distribution. It has been shown by using α-stable distribution that if the commodity has the opposite sign β's to the financial portfolio, the diversification effect of commodity exists in that the portfolio return distribution becomes more symmetry than the portfolio return distribution without a commodity. The empirical studies by using S&P500, U.S. 10-year bond, and DJ-AIG commodity index were conducted to investigate the portfolio diversification effects regarding risk standardization, robustly small correlation, and risk-return tradeoff. The parameter estimation results of α-stable portfolio return distributions have suggested that the commodity have the diversification effect on financial markets according to the estimated βs. Then, we have shown that the diversification effect is also supported by the conditional correlations between S&P500 or U.S. 10-year bond and DJ-AIG commodity index using the dynamic conditional correlation model with financial exogenous variables in that the correlations are small and robust to exogenous financial price returns. In addition, we have compared the efficient frontiers of the financial portfolios with and without commodity obtained from the mean-CVaR portfolio optimization. The results have illustrated that the efficient frontier of the financial portfolio with a commodity can improve the efficient frontier of the financial portfolio without a commodity. Thus, these results have been identical to the diversification effect of commodities on the financial portfolio as in Geman (2005) in the sense of risk-return tradeoff. Finally, we have shown that the stable tail adjusted risk ratio is useful to capture the diversification effect in the sense of risk-return tradeoff in the portfolio of financial assets and commodity.

The paper assumes that α of the α-stable distribution is a constant for simplicity. But we recognize that α causing the fat tail distribution also affects the portfolio diversification. In addition, while this paper only employs the DJ-AIG commodity index for the commodity because of the availability of data, the other indices such as Goldman Sachs commodity index and the Commodity Research Bureau index can be applied to this analysis. Furthermore, due to the data availability, we use the data until 2007 but we have to expand the examination using the recent data. We also recognize that introducing several macroeconomic variables exerting impact on the financial market might be of more interest for the examination of robustly small correlations between commodity and financial asset price returns. However macro data is difficult to be incorporated into the examination because price return data used in this study is daily while the macro data is typically longer period of time data like yearly data. We leave these items as the direction for future research.

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Figures & Tables

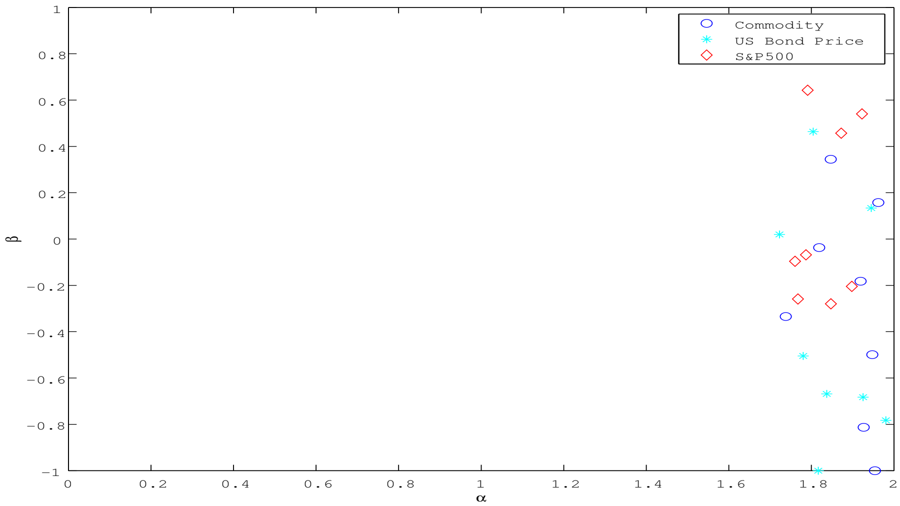

Figure 1. Scatter Plots of Indices of Stability α and Skewness Parameters β. The estimated stability parameters (αs) of the three assets are located from 1.75 to 2.0 within the whole parameter range from 0 to 2. In contrast, the estimated skewness parameters (βs) are scattered from -1 to 1, which corresponds to the whole parameter range. Comparing to βs, αs are considered constant in the first order approximation.

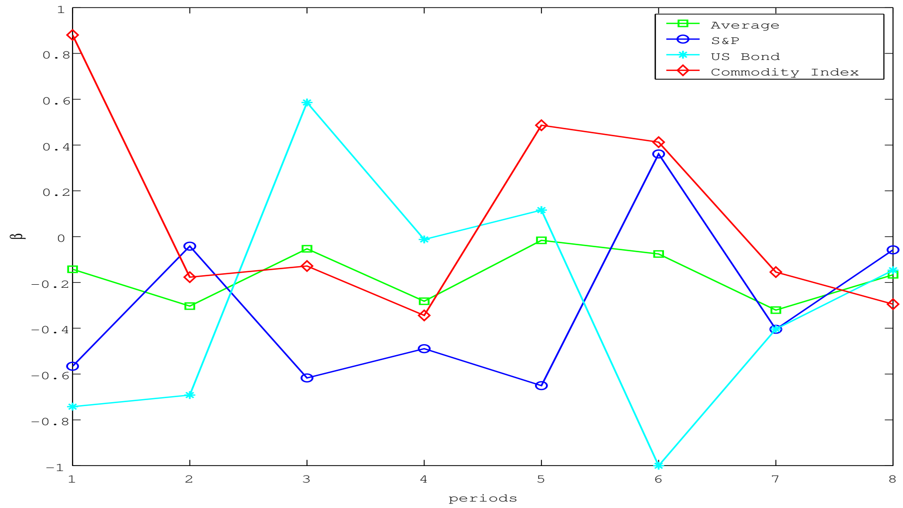

Figure 2. Skewness Parameter β. The commodity β's tend to have the opposite sign to stock βs in periods 1 and 5 during eight periods. The commodity β's tend to have the opposite sign to bond βs in periods 1, 3, 4, and 6 during eight periods.

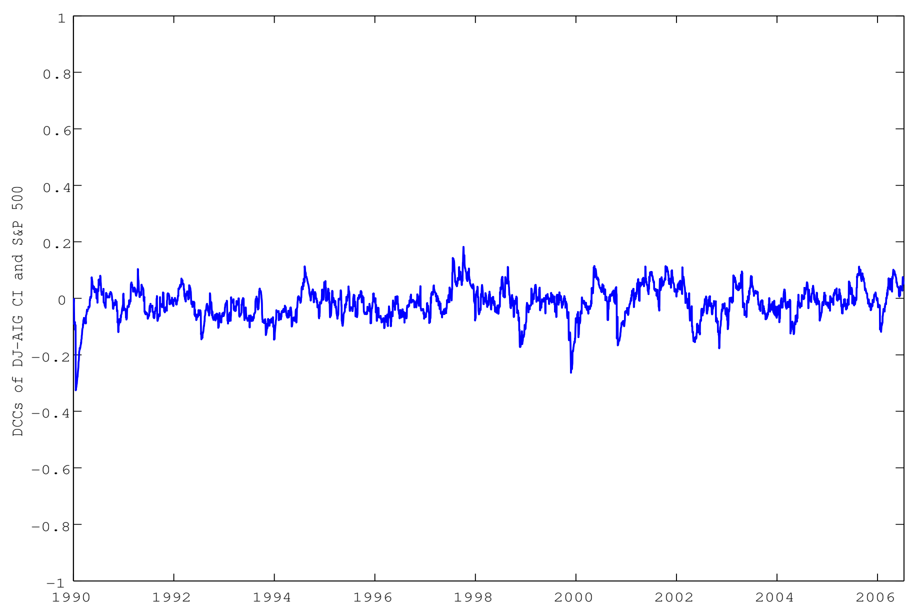

Figure 3. Dynamic Conditional Correlations between DJ-AIG CI and S&P 500. The conditional correlations are range bound between the small correlations of 0.2 and -0.2. The correlations are considered as nearly zero while the correlations are heteroskedastic.

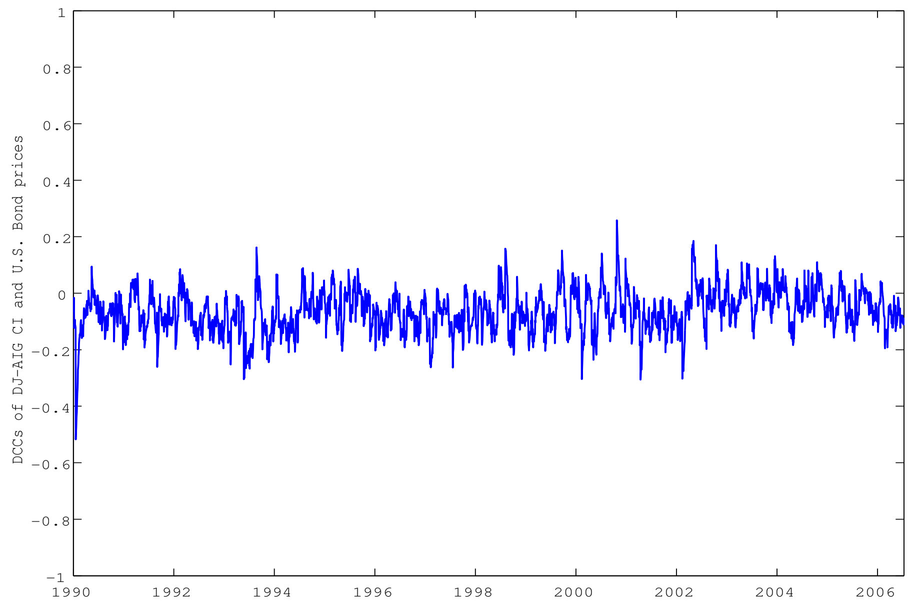

Figure 4. Dynamic Conditional Correlations of DJ-AIG CI and U.S. 10-Year Bond. The conditional correlations are time varying and between about 0.2 and -0.2.

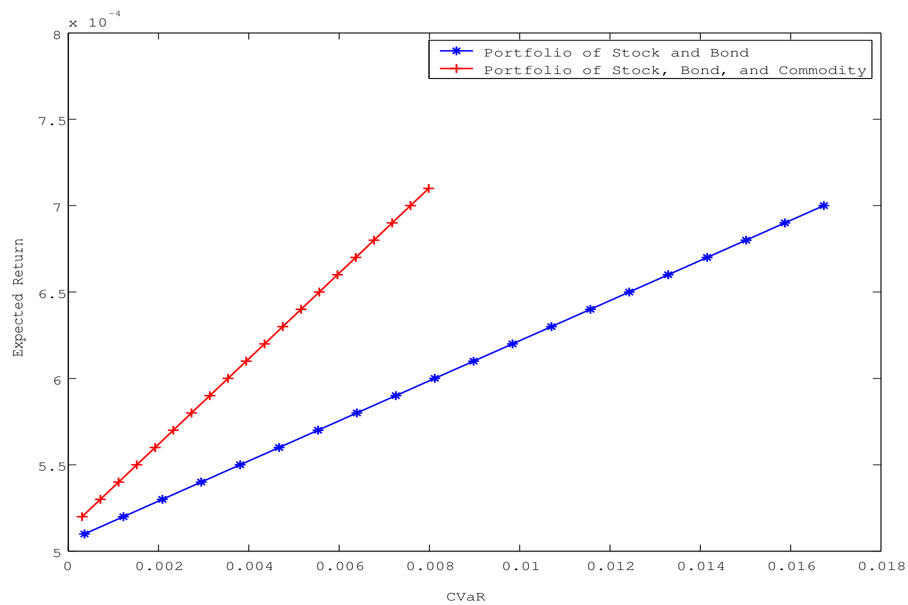

Figure 5. Mean-CVaR Portfolio Optimization with A Constant Yield. We employ the confidence level β as 90%. The result implies that the injection of commodity into financial portfolio may have the diversification effect in the sense of risk-return tradeoff because the efficient frontier of the financial portfolio with a commodity improves more than the efficient frontier of the financial portfolio without a commodity.

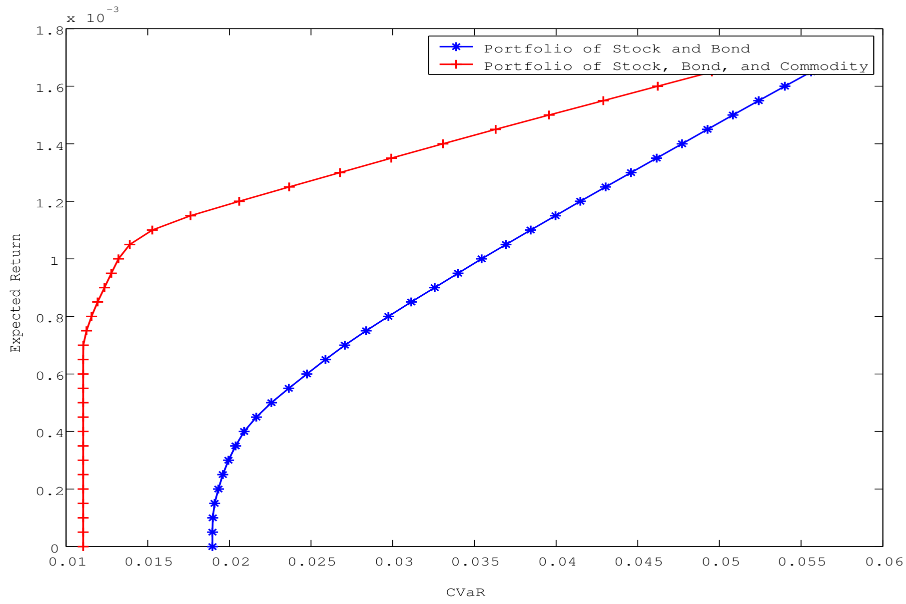

Figure 6. Mean-CVaR Portfolio Optimization with U.S. 10-Year Bond. We employ the confidence level β as 90%. The result indicates that the injection of commodity into financial portfolio may have the diversification effect in the sense of risk-return tradeoff, even if we employ the U.S. 10-year treasury note.

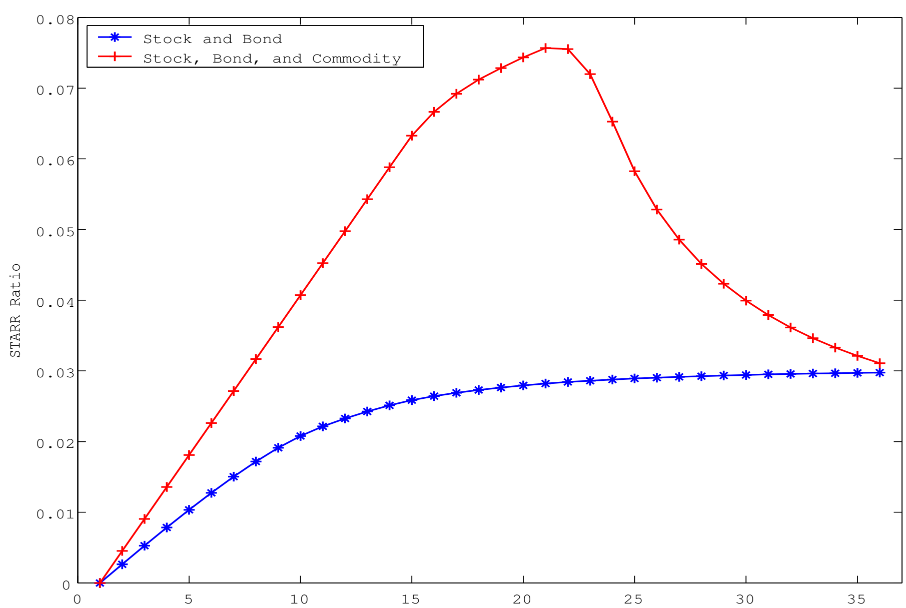

Figure 7. STARR Ratios for Financial Portfolios w./w.o. DJ-AIG CI. The STARR ratio with a commodity is greater than the STARR ratio without a commodity.

	DJ-AIGCI	S&P500	U.S. 10-Year Bond Rate
Mean	112.70	913.94	5.66
Median	102.50	989.09	5.66
Maximum	187.00	1527.46	8.36
Minimum	74.20	311.49	3.10
Std. Dev.	27.07	363.22	1.18
Skewness	1.03	-0.16	0.27
Kurtosis	2.85	1.57	2.17

Table 1. Basic Statistics of DJ-AIG Commodity Index, S&P500, and U.S. 10-Year Bond Rate. Note that the price distribution of S&P 500 is skewed to the left, while that of the DJ-AIG CI is skewed to the right. We test whether the skewness of S&P 500 and 10-year bond rate are statistically significantly different from zero using D'Agostino skewness test. p-values for S&P 500 and 10-year bond rate are obtained as 2.103×10^{-5} and 6.465×10^{-12}, respectively, implying that the both have skewness. We also test whether the kurtosis of the DJ-AIG CI is statistically significantly different from three using Anscombe-Glynn kurtosis test. p-value for the DJ-AIG CI is obtained as 0.04393, implying that the kurtosis of the DJ-AIG CI is not equal to three.

	DJ-AIG CI	U.S. 10-Year Bond	S&P500	Average
α	1.8891	1.8514	1.8307	1.8571
β	-0.2955	-0.3777	0.0916	-0.1938

Table 2. Average Stability and Skewness Parameters (α and β, resp.) We estimate the parameters of stability α and skewness β for the DJ-AIG commodity index, US 10-year bond price, and S&P500 returns, respectively. In order to compare the parameters among the three, we split the observations into 8 terms each of which comprises of 500 observations. The averages of the parameters are calculated. All three αs are close to 1.8571, the average of αs.

	DJ-AIG CI	U.S. 10-Year Bond	S&P500	Average
β	-0.3081	-0.2869	0.0851	-0.1700

Table 3. Average Skewness Parameter (β) at Constant Stability $\alpha = 1.8571$. Setting the constant α as 1.8571, the average of αs in Table 2, we reestimate the skewness parameters. β for commodity index has the opposite sign to β for S&P500.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2	ξ
Estimates	3.405E-07	0.051	0.945	5.935E-07	0.052	0.941	0.014	0.954	0.032
Std Errors	1.392E-07	0.013	0.012	2.465E-07	0.010	0.011	0.057	0.138	0.022
Logliklihood	2.816E+04								
AIC	-5.630E+04								
SIC	-5.624E+04								

Table 4. Parameter Estimates of DCC-X model for DJ-AIG CI and S&P 500s. According to the standard errors, α_1, β_1, α_2, and β_2 are statistically significant. It implies that there exist the GARCH effects both in the returns of DJ-AIG commodity index and S&P500. In addition, θ_2 is statistically significant, meaning that there exist the dynamic conditional correlations. More importantly, ξ is not statistically significant, implying that U.S. 10-year bond price returns do not affect the correlations between DJ-AIG commodity index and S&P500. It may support the robustness of small commodity-stock correlations in that they are not affected by exogenous U.S. 10-year bond price returns.

Parameters	ω_1	α_1	β_1	ω_2	α_2	β_2	θ_1	θ_2	ξ
Estimates	3.405E-07	0.051	0.945	8.000E-06	0.520	0.480	0.033	0.891	0.000
Std Errors	1.392E-07	0.013	0.012	1.272E-05	0.553	0.521	0.027	0.088	0.024
Logliklihood	2.941E+04								
AIC	-5.880E+04								
SIC	-5.875E+04								

Table 5. Parameter Estimates of DCC-X model for DJ-AIG CI and U.S. 10-Year Bond. According to the standard errors, the parameters except ω_2, α_2, β_2, θ_1, and ξ are statistically significant. Thus, there is no GARCH effect in the US 10-year bond prices. However, the conditional covariances are still influenced by the previous covariances judging from the significance of estimated θ_2. More importantly, ξ is not statistically significant, implying that S&P500 returns do not affect the correlations between DJ-AIG commodity index and U.S. 10-year bond prices. It may also support the robustness of small commodity-bond price correlations in that they are not affected by exogenous S&P500 returns.