RIETI Discussion Paper Series 17-E-124

Does Sales Factor Apportionment Benefit the Welfare of State?

MIYOSHI Yoshiyuki
RIETI

RIETI
Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 17-E-124
December 2017

Does Sales Factor Apportionment Benefit the Welfare of State?[1]

MIYOSHI Yoshiyuki
Ministry of Land, Infrastructure, Transport and Tourism
Research Institute of Economy, Trade and Industry

Abstract

U.S. states have raised the sales apportionment weight and lowered the payroll weight to stimulate local labor demand. However, the policy discussions often ignore the negative effect of sales apportionment tax; the tax distorts the sales allocation of firms across states and causes an increase in the local price level in the state. This study examines the optimal tax policy from the perspective of a single state and predicts the Nash equilibrium with a quantitative model that incorporates the effects of an apportionment formula both on local labor demand and on the price level. The calibration suggests that the sales weight should be zero for the optimal state tax policy because the negative effect on the price level outweighs the positive effect on the local labor demand under a range of plausible parameters.

Keywords: State corporate income tax, Formula apportionment, Optimal taxation, Tax competition
JEL classification: H21, H25, H71, R58

[1]This study is conducted as a part of the project "Dynamics of Inter-organizational Network and Geography" undertaken at Research Institute of Economy, Trade and Industry (RIETI). The author is grateful for helpful comments and suggestions by participants at Discussion Paper seminar, RIETI.

1 Introduction

Formula apportionment constitutes a major difference in the systems of corporate income tax between the federal and state governments in some developed countries, including the U.S., Canada and Japan. The apportionment formula determines what fraction of the taxable domestic profit of a multi-state firm is subject to state corporate income tax in each state. Adjusting this formula can significantly change the distribution of tax liability of a firm across states and then can affect the firm's decision about location, production and sales. The previous studies have found apportionment formula has impacts on the welfare and aggregate economic variables, including the prices and employment.

Most of the U.S. states use the three-factor apportionment formula, which defines the tax liability of multi-state firm j for state n as follows:

$$T^n(j) = t_C^n \left(\gamma_W^n \frac{W^n(j)}{W(j)} + \gamma_K^n \frac{K^n(j)}{K(j)} + \gamma_S^n \frac{S^n(j)}{S(j)} \right) \pi^T(j), \tag{1}$$

in which $W^n(j)$, $K^n(j)$, and $S^n(j)$ represent payroll, property, and sales of firm j in state n, respectively; $W(j)$, $K(j)$, and $S(j)$ represent total domestic payroll, property, and sales of firm j respectively; γ_W^n, γ_K^n, and γ_S^n are the weights for payroll, property, and sales factors respectively set by state n and $\sum_h \gamma_h^n = 1$; t_C^n is the statutory rate of corporate income tax in state n; $\pi^T(j)$ is the taxable domestic profit for firm j. In the U.S., states can choose these factor weights independently as long as the weights sum up to one.[1] From this formula, state corporate income tax can be viewed as a combination of three separate taxes on payroll, property, and sales of firms, as McLure (1981) points out.

Most states used to opt for the equally weighted formula, which puts the equal weight on three factors, because the Multistate Tax Compact recommended the formula. After the Supreme Court upheld the right of states to use other formulas than the equal weight formula in 1978, many states started to lessen the weights on payroll and property factors and put more weight on the sales factor to stimulate the demand for local employment (Mazerov 2001). This trend in state tax policy still continues, if not accelerates, these days. Now 19 states have even adopted the single sales factor apportionment, which put the full weight on the sales factor. The conventional wisdom in policy discussions is that larger sales weight benefits the state, especially through an increase in the demand for local labor, but the policy is not desirable from the perspective of nation because the competition of tax policy among state governments eventurelly leads to a "prisoner's dilemma" type equilibrium.

It is often overlooked, however, that taxing the sales factor has its own cost. If a state

[1] Unlike in the U.S., the formula is uniformly set by the national government in Canada and Japan.

sets a higher tax rate for sales factor ($t_C^n \gamma_S^n$ in equation 1) than the other states, the tax liability of a firm increases as the firm sells more in the state.[2] Thus the higher tax rate for the sales factor distorts the sales of firms in the way that firms sell less in the state. At the aggregate level, this leads to relatively less sales and higher prices in the state compared to other states with a lower tax rate for the sales factor. In sum, a greater sales weight will not only raise local real income, either through improved employment or through a higher wage rate, but it will also reduce local real income through a higher price level at the same time. As a whole, the total effect of change in the apportionment formula is ambiguous.

The goal of this study is to derive the optimal corporate income tax policy for state governments, including the optimal approtionment weights, with a quantitative model that is calibrated plausibly. In particular, the model formalizes the trade-off between local labor demand and price distortion caused by the apportionment formula. The literature on formula apportionment has successfully illuminated the former effect, especially by Goolsbee and Maydew (2000). However, no theoretical studies have been done to examine the optimal apportionment formula with a model that incorporates this trade-off and is able to evaluate the combined effect.[3]

Another key feature of my model is that states use personal income tax as well as corporate income tax as a source of revenue. If states only considered the effect of corporate income tax on the local labor market and if personal income tax were available to them with no cost, then the best tax policy would be choosing zero (or even negative) corporate income tax and collect the necessary revenue by personal income tax. On the contrary, it is a conspicuous observation from the actual state tax policy in the U.S. that most states impose corporate income tax and, on average, the corporate income tax rate is close to the top marginal tax rate of personal income: these rates are 7.33% and 7.58% respectively in 2014, if weighted by state GDP.[4] Since the revenue from personal income tax acounts for much larger share of the total state tax revenue than corporate income tax, it is important to examine states' choice about personal income tax when a study wants to derive implications for the optimal state corporate income tax. However, the previous theoretical studies on the

[2] In this paper, I assume the sales in apportionment formula is defined as sales at destination rather than at origin. Gordon and Wilson (1986) follows the same definition because it matches the provisions of tax code of most U.S. states.

[3] As examples of studies regarding the effect of sales factor, Gordon and Wilson (1986) theoretically analyze the effect of sales weight on output prices separately from the effects of property or payroll weight, and derive the result of "cross-hauling" caused by uneven tax rates for sales factor. The empirical study of Edmiston and Arze del Granado (2006) reports that the increase in sales weight led to a drop in local sales of multi-state firms as well as a rise in payroll of them in State of Georgia. Fajgelbaum et al. (2015) focuses on the improvement of welfare that can be attained by harmonizing state tax policy, including apportionment formula, rather than the optimal tax policy from the perspective of a single state.

[4] This relation is also found in federal corporate and personal income taxes.

optimal apportionment formula do not allow states to use other sources for revenue than corporate income tax.[5]

States need both personal and corporate income taxes in my model because of income shifting behavior among workers. Workers can shift their income from labor income to capital income if their effective tax rate for labor income is higher than the rate for capital income. Thus if a state sets personal and corporate income tax rates being wildly different, the gap in the tax rates severely distorts such income shifting behavior of workers. In short, state govenrments use corporate income tax as the "backstop" for implementation of personal income tax system (Mirrlees, ed 2011).

The model used in this study is a general equilibrium trade model in which multiple states set their tax policy strategically in the setting of static game. Firms respond to the state tax policy by choosing location in which to operate and the vector of price and output for each state. Basically, the tax rate for the payroll factor affects the location choice of firms while the tax rate for the sales factor does the prices and output that firms choose. In the calibration exercises, I look for Nash equilibrium of the model. Contrary to the popular policy recommendation and the implications from the existing literature, the model suggests that zero sales weight should be chosen in equilibrium under plausible parameters because the negative effect of sales apportionment on the price level outweighs its positive effect on the local labor demand. In addition, even if the current equilbrium is distorted due to some exogenous restriction on apportionment weights, say that the sales weight should equal one third, the best response of a state is to reduce sales weight to zero once it starts to have the right to adjust the weights on its own.

Section 2 develops the model and defines the objective of state governments. Section 3 describes the specific features of the model: the optimal balance between corporate and personal income taxes and the effects of apportioment weights on the aggregate economic variables. Section 4 presents the results of calibration and simulation. Section 5 discusses the validity of the model, including comparing the model's prediction with the existing empirical study. Section 6 concludes.

2 Model

2.1 Overview and discussions

This study use a model to derive implications about the optimal tax policy from the perspective of a single state government and Nash equilibrium of state tax policy of multiple

[5]For examples, see Anand and Sansing (2000), Pinto (2007), and Runkel and Schjelderup (2011).

states. Therefore it is critical to define the objective of state government plausibly. State governments in this model aim to maximize the social welfare in the state, which is defined as the purely utilitarian welfare function, by using a restricted set of policy instruments. A state government collects its revenue through personal and corporate income taxes, and the schedules of those taxes are constrained to be linear. It produces services that the private sector does not produce, and provides an equal amount of the services (hereafter referred to as the state government services) to each of its residents. The federal government also imposes personal and corporate income taxes. Federal personal income tax has a progressive schedule while federal corporate income tax a linear one. Another important assumption in my model is that workers supply their endowment of labor inelastically regardless of the state and federal tax policies.

The key feature of my model is that state governments set personal and corporate income tax rates at the same time and keep a balance between these two tax rates. The mechanism to detemine the optimal balance for state govenrments is centered on income shifting behavior of high income workers. In this model, workers can choose to receive the payment for the labor they supply as capital income rather than as labor income at no cost if they want to. Capital income is subject to corporate income tax as part of firm's taxable profit but not subject to any tax at the personal level. In addition, workers are heterogeneous in labor efficiency and then in income. For workers with high labor efficiency, it may be advantageous to shift their income from labor income to capital income since federal personal income tax has a progressive schedule while federal and state corporate income tax has a linear one. Thus given federal and state tax rates, there is the cut-off value of income; workers whose income is above the cutoff value undertake income shifting.

The objective function of state governments in this model is defined as it is not sensitive to the allocation of income across workers. Although the utility function of workers is a Cobb-Douglas function that is *quasi*-concave in the private goods and state government services, it is defined to be linear in private consumption. Thus the social welfare function that adds up the utility of workers with an equal weight is linear in the aggregate private consumption, or equivalently in the sum of real after-tax income across workers in the state.

Given this objective function, state governments do not care about the distribution of effective tax rates across workers as the result of income shifting. State governments, however, do care about the aggregate income of workers after federal taxes are collected. A state can change the cutoff income for income shifting in the state by setting different tax rates for personal and corporate income taxes, and, as the cutoff income changes, federal tax collection also changes. As I argue in the next section in detail, if state tax policy did not affect the economic variables, including the wage rate and the price level, equal tax rates for state corporate and personal income taxes would minimize federal tax collection. In other words, it would be best for states not to use uneven tax rates for corporate and personal income taxes because such tax policy distorts income shifting behaviour of workers. The combination of inelastic labor supply and the social welfare function that is linear in private consumption enables the model to incorporate the balance between corporate and personal income taxes as an endogenous choice of state governments in a simple way.

Although my model makes states choose the optimal mix of corporate and personal income taxes endogenously, the limited policy instruments for the government sector and inelastic labor supply of workers are restrictive and different from the standard optimal taxation literature, in which the government searches for the flexible tax schedule that maximizes the social welfare defined with a certain degree of redistributive taste under the constraint of private information.[6] However, the structure of my model makes the focus of this study—the trade-off between local labor demand and the price level—transparent. Moreover, it approximates the principal features of actual state government finances to a considerable extent, and can readily offer quantitative policy implications for state governments within the restriction of policy tools.

In practice, the progressivity of state personal income tax is moderate on average: among the 43 states that impose personal income tax (including the District of Columbia), 10 states have a linear tax schedule in 2015.[7] In addition, in the 18 states out of the 33 states that have progressive tax schedules, the top rate is reached at a household income level below the national average, and, in most cases, below $20,000. The average tax rate for the household with the national average income is 5.34%, if weighted by state GDP, while the weighted average tax rate for the highest bracket is 7.33% in 2014. As for the effective tax rates rather than the statutory rates, Gordon and Cullen (2012) report the effective marginal tax rates of federal and state personal income tax that are calculated based on the data including individual tax returns. The effective marginal tax rates of state income tax range between 3.5% and 6.2% across the top four income groups out of the five income groups, excluding the bottom group, while the federal rates range between 15.0% and 31.2% across the same groups.[8] As for corporate income tax, two thirds of states have linear tax schedules.

[6] Notable studies with policy implications in the standard optimal taxation literature are Saez (2001) and Diamond and Saez (2011).

[7] The states that do not impose personal income tax collect a large fraction of their revenue from general sales tax, except for Alaska and Wyoming. General sales tax with a single tax rate is considered to be equivalent to a linear personal income tax in my static model, although the model does not accomodate the option of sales tax as an endogeneous choice for state governments.

[8] Gordon and Cullen (2012) report a very high marginal tax rate of state personal income tax for the bottom income group. They attribute it to transfer income that the households in the group lose as their income goes up.

Therefore, it appears a plausible approximation of the actual U.S. state tax policy to restrict the choice of state personal and corporate income tax schedules to being linear.

On the spending side of state government finances, the largest share is spent on education (35.6%). State governments are also involved in provision of large-scale public infrastructure, most notably highway projects (6.7%). These facts motivate my model to have state governments produce services that the private sector does not and provide the services equally to each of their residents.[9]

The model is a general equilibrium trade model of multiple states. In the following two sections, the economy is assumed to consist of two states, A and B, for the purpose of presentation although it is relatively straightforward to extend the model to an economy with more than two states. State governments are players of a static game who aim to maximize their objective function taking account of general equilibrium outcome. There are continuum of firms of fixed measure in the economy. Each of them produces a differentiated good. Every firm is perfectly mobile and chooses one of the states to operate in to maximize its profit, considering the wage rates, local productivity, and state tax policies. On the contrary, workers are immobile in this model and consume a set of the differentiated goods and the state government services. Only state governments can produce the state government services, using the set of differentiated goods as inputs. The differentiated goods can be transported between states with no cost, but the state government services cannot be transported to the other state.

2.2 Production technology and firms

Firms of measure M produce the differentiated goods using labor as the only input. The production function for firm j is

$$x(j) = z_n(j)l(j),$$

in which $z_n(j)$ is the productivity of firm j if it operates in state n. A perfectly competitive sector in each state produces the final good from the differentiated goods according to the

[9] The treatment of state expenditure here is almost equivalent to Fajgelbaum et al. (2015), in which the level of state government spending affects the level of local amenity that enters consumers' utility equally, although the level of state government spending also affects the productivity of firms in their model. On the contrary, state governments also engage in income redistribution not through a highly progressive income tax, but through the expenditure on public welfare programs, including Medicaid. That type of expenditure accounts for 30.8% of total expenditure in 2013. Gordon and Cullen (2012) focus on this role of state governments and provide a theoretical framework for the optimal taxation in which each of multi-level governments tries to maximize thier own welfare function by nonlinear labor income tax.

following technology:

$$X = \left(\int_M x(j)^{\frac{\sigma-1}{\sigma}} \mathrm{d}j \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

in which the substitution parameter $\sigma > 1$. The final good is either consumed by workers in the state or used to produce the state government services. The price of the final good in state n is expressed as

$$P^n = \left(\int_M p^n(j)^{1-\sigma} \mathrm{d}j \right)^{\frac{1}{1-\sigma}},$$

in which $p^n(j)$ is the price that firm j sets for its good sold in state n.

Firms maximize their profits as they choose their production levels, the prices for their goods, and the locations of production. A firm operates in a single state and is not allowed to merge with another firm in this model. All the firms are subject to federal and state corporate income tax and not allowed to choose any organizational forms to which corporate income tax is not applied.

The state corporate income tax system follows a two-factor formula apportionment.[10] In addition to their corporate income tax rates, state governments set their apportionment weights on the payroll and sales factors. Let γ_S^n be the sales weight of state n, and then its payroll weight equals $1 - \gamma_S^n$. If firm j chooses to locate in state n, the tax liability for the firm is

$$T_n(j) = \left(t_C^F + \bar{t}_n(j) \right) \pi_n^T(j),$$

in which t_C^F is federal corporate income tax rate; $\bar{t}_n(j)$ is the effective state corporate income tax rate for firm j, which is calculated based on the apportionment formula; $\pi_n^T(j)$ is the taxable profit for firm j.[11] The apportionment formula defines $\bar{t}_n(j)$ as

$$\bar{t}_n(j) = t_C^n(1 - \gamma_S^n) + \frac{t_C^A \gamma_S^A p^A(j) x^A(j) + t_C^B \gamma_S^B p^B(j) x^B(j)}{p^A(j) x^A(j) + p^B(j) x^B(j)}, \tag{3}$$

in which t_C^n is the statutory rate for corporate income tax of state n; $x^m(j)$ is the amount of

[10] Production does not require capital as an input in this model. However, if production function is homogeneous of degree one in labor and capital and if capital is also immobile (like land), the following results remain true.

[11] To make the model simple and transparent, the federal government in this model does not allow firms to deduct the payment of state corporate income tax from their federal taxable income, unlike the U.S. tax code.

firm j's good that is sold in state m.

The payment to workers is the only cost of production for firms. In this model, a worker can negotiate with the firm to receive the payment either as labor income or as capital income. The payment of labor wage is deductible from the taxable profit of firms. But if a firm pays capital income to a worker rather than labor wage, the payment adds to the firm's taxable profit and is subject to federal and state corporate income tax. Thus the *wage rate* for capital income payment to the *income-shifters* at firm j that operates in state n is determined as

$$\tilde{w}_C^n(j) = \left(1 - t_C^F - \bar{t}_n(j)\right) w_N^n, \tag{4}$$

in which w_N^n is the prevailing market wage rate for labor income in state n. The tilde on a variable in this paper means that the variable is the after-tax one.

The after-tax profit of firm j, when it chooses to locate in state n, is represented as

$$\begin{aligned} \tilde{\pi}_n(j) =& (1 - t_C^F - \bar{t}_n(j)) \left[p_n^A(j) x_n^A(j) + p_n^B(j) x_n^B(j) - w_N^n (1 - \theta_n) \frac{x_n^A(j) + x_n^B(j)}{z_n(j)} \right] \\ & - \tilde{w}_C^n \theta_n \frac{x_n^A(j) + x_n^B(j)}{z_n(j)}, \end{aligned} \tag{5}$$

in which θ is the share of efficiency unit of labor that is provided by income-shifters; the subscripts n on some of the variables represent being conditional on operating in state n. In this model, θ_n is not a choice variable of each firm, but every firm in a state is assumed to employ the same fraction of income shifters. On the other hand, the effective state tax rate $\bar{t}_n(j)$ is an endogenous variable that is determined by the firm's decision on the allocation of sales among the states. Nonetheless, all the firms in the same state choose the same allocation of sales shares as the solution to its profit maximization problem, even if they are heterogeneous in productivity. Refer to Section 7.1 for the proof.[12] Since it implies all the firms in the same state are subject to the same effective state tax rate $\bar{t}_n$, the wage rate for income-shifters $\tilde{w}_C^n$ is uniquely determined by equation 4. Note that this makes all the firms in state n indifferent about θ_n from equation 5. Finally, firm j chooses to locate in the state where the maximum of $\tilde{\pi}_n(j)$ is the larger.

Since firms engage in monopolistic competition, firms earn positive profits after paying out capital income to income-shifters. Those profits are pooled and distributed to workers by

[12] Fajgelbaum et al. (2015) first prove this result of constant $\bar{t}_n(j)$ across all the firms in the same state for the standard Dixit-Stiglitz model of monopolistic competition. The proof in Section 7.1 shows their result can be extended to the model with income shifting with minor adjustments.

a perfectly competitive financial sector. ρ^A fraction of the total after-tax profit is distributed to workers in state A while the remaining, the fraction of $\rho^B = 1 - \rho^A$, to state B.

2.3 Workers

There are N^n workers in state n. Workers are heterogeneous in terms of their endowment of efficiency unit of labor, $l^n(i)$ for worker i, and they supply labor inelastically. L^n stands for the mean efficiency unit of labor of workers in state n, and then the aggregate labor supply in state n equals $N^n L^n$. In addition to labor income and capital income for income-shifters, workers receive dividend from the financial sector. Workers in state n own shares for dividend in proportion to their efficiency unit of labor.[13] Thus the dividend payment for worker i in state n is

$$\tilde{d}^n(i) = \frac{\rho^n l(i)\tilde{\Pi}}{N^n L^n},$$

in which $\tilde{\Pi}$ is the aggregate profit of firms after the payments for income shifters and for the federal and state corporate tax liabilities.

Labor income is subject to federal and state personal income taxes while capital income for income-shifters and dividend income are not. Since federal personal income tax has a progressive schedule, the liability of that tax is larger for high income workers. Let $t_N^F(i,n)$ be a shorthand notation for the *average* tax rate for worker i in state n if the worker chooses not to be an income-shifter: $t_N^F(i,n) = T_N^F(w_N^n l^n(i))/(w_N^n l^n(i))$, in which $T_N^F(\centerdot)$ is the function for the schedule of federal personal income tax liability. Thus the total after-tax income of worker i in state n is

$$\tilde{y}^n(i) = \begin{cases} \tilde{w}_C^n l^n(i) + \tilde{d}^n(i) & \text{if } i \text{ is an income-shifter,} \\ (1 - t_N^F(i,n) - t_N^n) w_N^n l^n(i) + \tilde{d}^n(i) & \text{otherwise.} \end{cases} \tag{6}$$

in which t_N^n is state personal income tax rate in state n. Since the cost of income shifting for workers is zero in this model, they choose the type of income that makes $\tilde{y}^n(i)$ the larger. Thus equation 4 implies that worker i in state n becomes an income-shifter if the personal income tax liability is larger than the incidnce of federal and state corporate income tax, or $t_N^F(i,n) + t_N^n > t_C^F + \bar{t}_n$.

Workers consume the state government service as well as the private final good. Worker

[13] I make this assumption regarding the distribution of financial income being motivated by the well-known fact that the distribution of wealth is significantly skewed. However, the following results are not affected by the assumption because, as I explain below, the state welfare function is linear in aggregate income.

i in state n has the following utility function:

$$u^n(i) = \frac{\tilde{y}^n(i)}{P^n}(g^n)^{\alpha_G}, \tag{7}$$

in which g^n is the state government service provided to worker i by the state government in n; $\alpha_G > 0$ is the preference parameter for the state government services. Note that this utility function is a standard Cobb-Douglas function and exhibits the quasi-concave property although it is linear in (real) income.

2.4 Governments

State governments maximize the social welfare of residents that is defined as the purely utilitarian welfare function:

$$V^n = \int_{i \in n} u^n(i) f^n(i) N^n \mathrm{d}i, \tag{8}$$

in which $f^n(i)$ is the probability density function for the distribution of heterogeneous workers in state n. The budget constraint for state government is

$$N^n g^n \leq z_G^n (T^n / P^n), \tag{9}$$

in which z_G^n is the productivity for state government services in state n; T^n is the aggregate tax revenue for state n.[14] State governments take into account only how the federal tax collection affects the utility of their residents, but not how the changes in federal revenue caused by the state's fiscal policy affect the utility.[15] The federal government takes the final good from the economy as its tax collection.

Since the utility function defined as equation 7 is linear in real income for worker i and g^n is constant across all the workers in state n, the social welfare function for state n is linear in the aggregate income of the state. Therefore state governments in this model are not sensitive to the allocation of income among their residents. This feature isolates the aggregate effects of formula apportionment on the welfare of state from the issue of income redistribution, still keeping the heterogeneity across workers, which is needed to incorporate the choice of state governments about the optimal mix of personal and corporate income

[14]Since the state welfare function preserves the property of Cobb-Douglas utility function, z_G^n is irrelevant for the optimal tax policy.

[15]This assumption can be justified, for example, when the federal government covers a lot more states with which the two states have no economic interaction, and when the two states are too small to have a significant effect on the total federal revenue.

taxes.

3 State tax policy

3.1 Optimal tax policy in autarky

I use a variant of my baseline model, which turns the trading economy into the autarky economy, to illustrate how the environment of this model determines the optimal mix of state corporate and personal income taxes. In this case, state A has no economic interaction with state B: there is no trade between the two states, and the profits of firms are not pooled nationally, but are distriburted within the state. The federal government, however, still collects tax from both states. Since the apportionment weights are irrelevant in this case, the variables the state governments use are personal and corporate income tax rates.

From equation 2, the demand for good j is:

$$x(j) = p(j)^{-\sigma} \frac{Y^n}{(P^n)^{1-\sigma}}, \tag{10}$$

in which Y^n is the before-tax aggregate income in state n. The nominal price level is fixed in such a way that Y^n is normalized to one. The optimization problem for firm j is:

$$\max_{p(j),x(j)} (1 - t_C^F - t_C^n) \left(p(j)x(j) - \frac{w_N^n}{z(j)} x(j) \right),$$

subject to equation 10. This objective function is simplified from equation 5 by substituting equation 4. It can be shown that the equilibrium before-tax wage rate and price level do not depend on either federal or state tax policy:

$$w_N^n = \left(\frac{\sigma - 1}{\sigma} \right) \frac{1}{N^n L^n};$$

$$P^n = (\bar{z}^n)^{-1},$$

in which

$$\bar{z}^n = \left[\int_M z^n(j)^{\sigma-1} g^n(j) dj \right]^{\frac{1}{\sigma-1}},$$

in which $g^n(j)$ is the probability density function for firms in state n. The implication that state tax policy does not distort the production in the autarky economy makes the problem of optimal state tax policy simple as Proposition 1 shows:[16]

Proposition 1. [The optimal state tax rates in autarky] *In the case of autarky, the optimal rates for state personal and corporate income tax are equal:* $t_S^n = t_C^n$.

The proof for Proposition 1 is presented in Section 7.2. This result can be interpreted as follows. If there were no state taxes, the federal personal and corporate income taxes would completely determine the cutoff efficiency unit of labor $\bar{l}$ in state n such that $T_N^F(w_N^n \bar{l}) = t_C^F w_N^n \bar{l}$. Those who are more efficient than $\bar{l}$ engage in income shifting while those who are less efficient do not. With this being said for the no state tax case, if the state sets equal rates for both income taxes, the cutoff efficiency does not change. However, if the state tax rates do not match, it changes the cutoff efficiency and distort the behavior of workers that originally minimizes the *federal* tax burden. Therefore uneven state tax rates increase the federal tax collection, which flows out of the state economy. Since the welfare function for state government is linear in the aggregate after-tax income and is not sensitive to the allocation of after-tax income among the residents, the state government does not want to distort the cutoff efficiency.

The interpretation can be extended to a broader perspective. My model abstracts from elastic labor supply and the benefits and costs regarding firms' choice of organizational form. Although this feature makes the model tractable, its cost is that the optimal federal personal and corporate income tax schedules should be taken as exogenous.[17] If the federal government implements its tax policy, especially the arrangement of personal and corporate income taxes, considering factors that my model cannot capture, Proposition 1 suggests state governments respect the choice of federal government except for a uniform linear tax that does not distort firms' choice of organizational form in the case of autarky. As the following sections argue, however, when the two states interact, state governments are faced with the trade-off between the opportunity for boosting local labor demand and the distortion to income shifting behaviour. In addition, it complicates the state governments' problem that part of the tax burden can be exported through taxing the profits of firms in the other state.

3.2 Effects of taxing apportionment factors

In the model of trading states, state corporate income tax affects the location and production choices of firms since firms are perfectly mobile across the states. It requires specifiying the

[16] Although monopolistic competition is present in this model, the production activity is not distorted because workers supply labor inelastically and all the profits of firms are distributed to workers.

[17] Gordon and Mackie-Mason (1994) and Mackie-Mason and Gordon (1997) provide the empirical evidence that the difference between corporate and personal income tax distorts the choice of organizational form. Piketty et al. (2014) propose a model for the optimal labor income tax in the presence of income shifting.

distribution of productivity among firms to analyze the effect of state corporte tax on firms' location choice. I choose the assumption of homogeneous firms, in which $z^n(j) = z$ for all j and n in the rest of this section and the following calibration exercises. Section 5 discussses the implications of this assumption.

The corpoarte tax rate affects firms' choice, interacting with the apportionment weights. If the effective tax rate on the payroll factor in state A is higher than in state B, it reduces the demand for labor in state A and then lowers the before-tax wage rate there. If the effective tax rate on the sales factor in state A is higher, it raises output prices in the state since firms have an incentive to reduce the corporate income tax burden by selling less in state A and more in state B. The rest of this section examines the effects of state corporate income tax in the two polar cases regarding the apportionment weights to illustrate how the choice of the weights affects firms' behavior and the aggregate variables.

3.2.1 Payroll factor

First assume the states put the full weight on payroll factor and zero on sales factor. Since firms are homogeneous and mobile, they are indifferent about the location in equilibrium:

$$\tilde{\pi}_A(j) = \tilde{\pi}_B(j) \quad \text{for all } j, \tag{11}$$

in which

$$\tilde{\pi}_n(j) = \max_{p_n(j), x_n(j)} (1 - t_C^F - t_C^n) \left[p_n^A(j) x_n^A(j) + p_n^B(j) x_n^B(j) - w_N^n \frac{x_n^A(j) + x_n^B(j)}{z} \right].$$

The first order conditions imply that frims sell their goods at the same price in both states:

$$p_n^A(j) = p_n^B(j) = \left(\frac{\sigma}{\sigma - 1} \right) \frac{w_N^n}{z}.$$

It is possible to derive the relation between the corporate income tax rates and the equilibrium wage rates:

$$\frac{w_N^A}{w_N^B} = \left(\frac{1 - t_C^F - t_C^A}{1 - t_C^F - t_C^B} \right)^{\frac{1}{\sigma - 1}}. \tag{12}$$

This implies the elasticity of wage rate with respect to net-of-tax rate, $1 - t_C^F - t_C^n$, is $1/(\sigma - 1)$ if the tax rate in the other state is fixed.

Equation 12 means that state governments can increase the local wage rate by reducing its corporate income tax rate, or its effective tax rate for the payroll factor more precisely.[18] This result fits the previous empirical studies: for example, Goolsbee and Maydew (2000) and Edmiston and Arze del Granado (2006).

However, there are two considerations that offset this positive effect of lower corporate income tax rate. First, if firms earn economic profits and state tax policy does not affect the distribution of ownerships of firms, as this model presumes, the incidence of state corporate income tax partly falls on the owners outside the state. This makes taxing the profits of firms a less costly way to raise revenue. Second, if a state government taxes firms lightly, the gap between personal and corporate tax rates changes the income shifting behavior from one that federal tax policy intends and, as a result, it increases the federal tax collection, which is taken away from the state economy. Therefore, the quantitative implications for optimal state tax policy in this case depends not only on the substitution parameter σ, but also on the distribution of firm ownerships and federal tax policy.

3.2.2 Sales factor

Next assume the states put the full weight on the sales factor and zero on the payroll factor. In this case, the effective corporate tax rate for firms is not affected by the location of firms, but it is completely determined by the ratio of sales between the two states.[19] State corporprate income tax does not affect the production cost, and homogeneous firms locate in whichever state that offers the lower production cost. Thus, $w_N^A = w_N^B = w_N$ in equilibrium.

The optimization problem for firm j is:

$$\max_{p_n(j),x_n(j)} (1 - t_C^F - \bar{t}_n) \left[p_n^A(j)x_n^A(j) + p_n^B(j)x_n^B(j) - (1-\theta_n)w_N^n \frac{x_n^A(j) + x_n^B(j)}{z} \right] - \theta_n w_C^n \frac{x_n^A(j) + x_n^B(j)}{z},$$

in which

$$\bar{t}_n(j) = \frac{t_C^A p_n^A(j)x_n^A(j) + t_C^B p_n^B(j)x_n^B(j)}{p_n^A(j)x_n^A(j) + p_n^B(j)x_n^B(j)}.$$

The closed-form solution for equilium is not derived in this case, but the qualitative effect of taxing sales factor on the price level can be shown as follows:

[18] In the limiting case, state corporate tax does not affect the local wage as σ goes to infinity. For the market is perfectly competitive and firms earn no economic proft in this case. Corporate income tax revenue only comes from income shifters in the state and the full incidence falls on those workers without distortion.

[19] In practice, the effective tax rate also depends on the location of firms because of the nexus rules in the U.S. tax code. Section 5 discusses this issue further.

Proposition 2. [Distortion by unequal sales factor taxation] *When states use the single sales factor, namely $\gamma_S^A = \gamma_S^B = 1$ in equation 3, the following statements concerning the prices of final good, P^A and P^B, hold.*

1. *If $t_C^A = t_C^B$, then $P^A = P^B = \bar{P}$, in which*

$$\bar{P} = \left(\frac{\sigma}{\sigma - 1}\right)\frac{w_N}{z}.$$

2. *If $t_C^A > t_C^B$, then $P^A > \bar{P} > P^B$.*

Refer to Section 7.3 for the proof. Proposition 2 shows a greater weight on the sales factor carries some cost to the state economy as the local price level rises. Note that the structure for the optimal tax policy is similar to the one in the case of the full payroll weight; a low corporate tax rate may improve the state's welfare through a lower price level, which leads to higher real income because w_N is equal among states. But there are two offsetting effects: the distortion caused by the gap between corporate and personal income tax rates and the possible benefit of exporting tax burden through taxing the corporate profits of firms in the other state.

4 Calibration

The arguments in the previous section illustrate the effects of taxing the payroll and sales factors separately by factors. It shows state governments are faced with the trade-off when they consider apportionment weights: if a state raises its weight on sales, then the local wage will go up, but so will the price of final good in the state at the same time, which harms the state welfare by reducing the real income. Calibration exercises are necessary to evaluate the total effect and find the optimal tax policy for states. I first explain the parameters for the calibration, and then report the results.

4.1 Parameters

The income distribution among workers is important because it determines how many workers undertake income shifting. Considering labor income is linear in efficiency unit of labor and only the highest part of the distribution matters for income shifting behavior in the model, I assume the distribution of efficiency unit of labor follows a Pareto distribution.[20]

[20]In particular, the distribution of middle and low income workers does not affect the state welfare because the welfare function is linear in aggregate income in this model. For the highest part of the distribution,

The density function of efficiency in state n is defined as

$$f^n(l(i)) = \frac{\eta(l_m^n)^\eta}{(l(i))^{\eta+1}},$$

in which l_m^n is the efficiency unit of the least efficient workers in state n and $\eta > 1$. I set $\eta = 5/3$ following Jones (2015). Since the mean efficiency in state n is L^n, $l_m^n = L^n(\eta - 1)/\eta$.

Federal personal income tax rate is the other determinant for income shifting behaviour. I replicate its progressive schedule following Gouveia and Strauss (1994).[21] In their model, the effective average tax rate for federal personal income tax is expressed as:

$$t_N^F(y_N) = b_2 - b_2(b_1(y_N)^{b_0} + 1)^{-1/b_0}, \tag{13}$$

in which y_N is labor income. They estimate the parameters in equation 13 and report values of $b_0 = 0.768$, $b_1 = 0.031$, and $b_2 = 0.258$ for year 1989. Note that the limit of t_N^F equals b_2 as labor income goes to infinity. Since the highest marginal tax rate is lower in 1989 than today, I use value of $b_2 = 0.396$, which is the highest marginal tax rate in the current tax schedule, in the following calibration exercises. Since equation 13 is not linear in labor income, I have to adjust the nominal price level to replicate the tax schedule properly. The nominal price level is set such that the mean before-tax labor income in the model equals \$76,000 when the two states are symmetric.[22] The effective tax rate for federal personal income tax by income level is presented in Figure 1. Combined with the Pareto distribution of labor efficiency, the share of shifted income out of total labor income (θ) equals 0.137 when two identical states use a symmetric tax policy.[23]

The elasticity of substitution σ in the final good production function is the key parameter in this model since it affects the elasticity of wage rate with respect to tax rates and the taxable profits of firms. I use the value of $\sigma = 4$ following Fajgelbaum et al. (2015) as the benchmark. The preference for state government services α_G is set at 0.116 so as to make

Piketty and Saez (2013) show the U.S. before-tax income closely follows a Pareto distribution especially above the income of \$400,000.

[21] Conesa et al. (2009) and Conesa and Krueger (2006) also use this function to parametrize the optimal federal personal income tax schedule.

[22] The mean household income in the U.S. is \$75,738 in 2014.

[23] As a related statistic, the share of state corporate income tax revenue out of the sum of state personal and corprate income tax revenue is 12.7%. Thus the value of θ here does not seem extreme. Moreover, the following result does not change much if I use the parametric function for the effective federal personal income tax schedule that is adopted by Benabou (2002) and Heathcote et al. (2014):

$$t_N^F(y_N) = \frac{y_N - \lambda(y_N)^{1-\tau}}{y_N}.$$

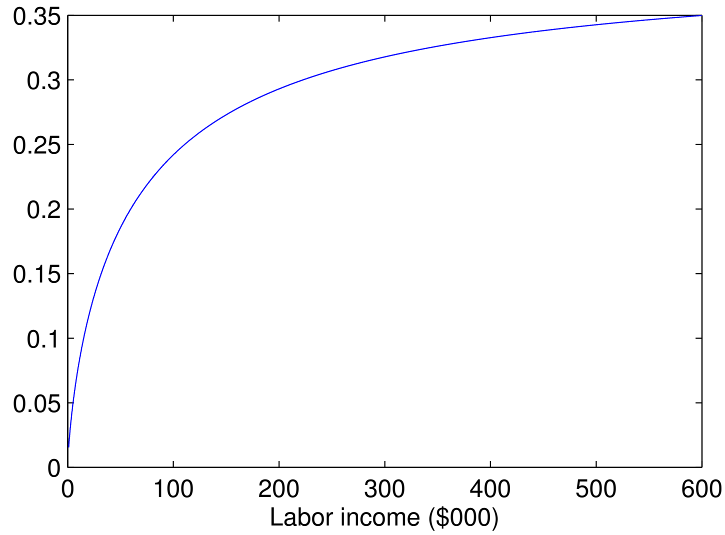


Figure 1: The Effective Average Federal Personal Income Tax Rate

the optimal state corporate income tax rate in the autarky case equal the average tax rate of U.S. states weighted by state GDP, which is 7.58% in 2014.

4.2 Results

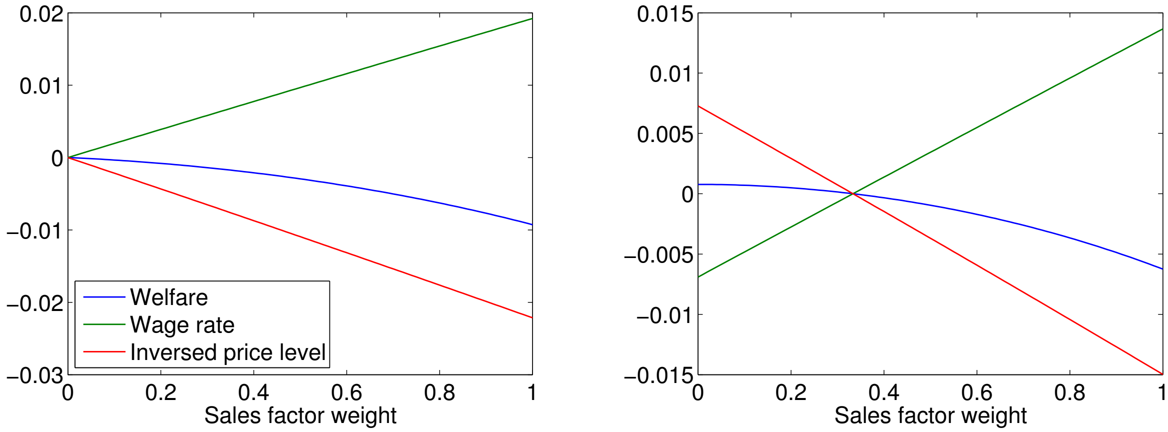
In the calibration exercises, I derive Nash equilibrium of state tax policy by iteration. I focus on the case of symmetric states and homogeneous firms although my model allows heterogeneity in the key variables, including the distribution of productivity of firms across the states. I start with the model of two states, and then extend the model to the economy of 50 states to calibrate the optimal tax policy for an average state in the U.S.

The first column of Table 1 reports the optimal state tax policy in Nash equilibrium of the model of two symmetric states. The model suggests the optimal sales apportionment weight is zero: the state governments should use the full payroll weight.

Table 1: Optimal State Tax Rates in Nash Equilibrium of the Two-State Model

σ	4	3	2	1.5
t_C^n	6.79%	6.50%	6.65%	7.01%
γ_S^n	0	0	0.603	1
t_N^n	7.75%	7.77%	7.04%	5.79%

To examine this result, Panel (a) of Figure 2 reports how state welfare, the local wage rate, and the local price level change as the sales factor weight in one of the states increases from zero through 100% while the tax policy in the other state is fixed. The y-axis represents



(a) From the Equilibrium of Unrestricted γ

(b) From the Equilibrium of $\gamma = 1/3$

Figure 2: The Effects of Sales Factor Weights on Welfare and Aggregate Variables in the Two-State Model

the rate of change in the variables. As expected, the local wage rate increases as the state puts more weight on the sales factor. If the state adopts the single sales factor, the local wage rate will go up by 1.9%. However, the local price level increases at a faster pace than wage rate at the same time. For example, when the single sales factor is used, the price level goes up by 2.2%. As a whole, the state welfare keeps decreasing as the state raises its sales factor weight. The degree of decrease in welfare is equivalent to a 0.93% decrease in private consumption for the change in sales weight from zero to 100%.

The optimal state corporate income tax rate is lower than personal inecome tax rate due to the negative effect of corporate income tax on local wage rate. However, t_C^n does not deviate far from t_N^n since the distortion in income shifting behavior and the resulting increase in federal tax payment discourage states from reducing t_C much.

Equation 12 implies that the local wage rate is more sensitive to t_C^n, or the tax rate on the payroll factor $t_C^n(1 - \gamma_S^n)$ in this context, when σ is lower. This suggests the positive effect of sales apportionment on the local wage may outweigh its negative effect on the price level in case of a lower σ. The second through fourth columns of Table 1 report the optimal state tax policy under a few different values of σ that are lower than 4. While the optimal sales weight is still zero even if $\sigma = 3$, the optimal weight increases rapidly to 0.6 and then to 1 as σ goes down to 2 and then to 1.5 respectively. Nonetheless, to obtain a positive γ_S^n as the optimum requires a considerably smaller value of σ than the standard values in the trade literature.

Another observation for the various values of σ is that the relationship between the tax rates for corporate and personal income taxes gets reversed as σ goes down. The optimal corporate income tax rate is higher than the personal income tax rate when $\sigma = 1.5$. When σ is close to one, the markup rate $\sigma/(\sigma-1)$ is very high, which leads to large corporate profits of firms. In this case, a state can export a significant portion of corporate tax liability to the owners of firms who live in the other state. Therefore when σ is close to one, corporate income tax becomes an attractive tool for revenue for states.

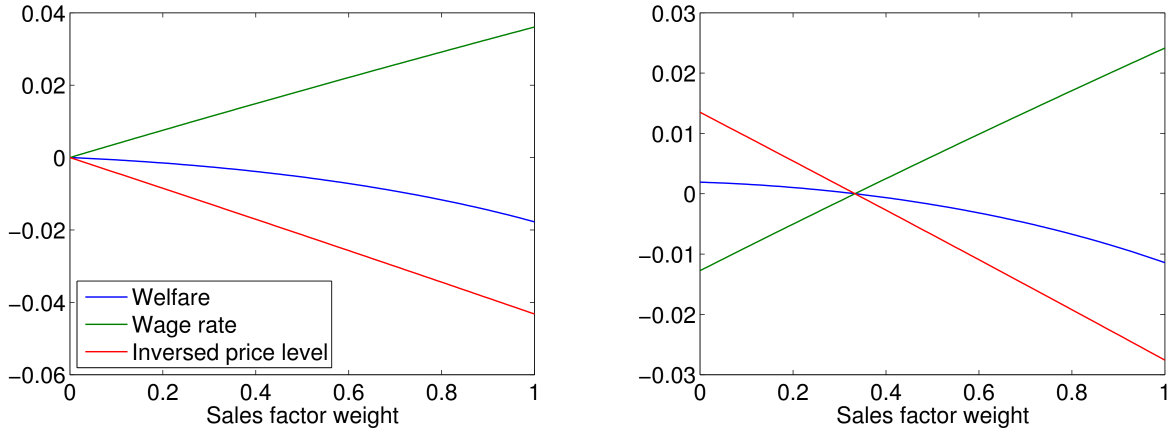
Most of the states in the U.S. used to follow the Multistate Tax Compact that recommended states should adopt the equally weighted formula. Thus it is worth examining how Nash equilibirum looks if γ_S^n is fixed at one third rather than considered as one of the choice variables of states. In this case, $t_C^n = 7.16\%$ and $t_N^n = 7.34\%$ in equilibrium. If states start to be allowed to freely choose the value of γ_S^n suddenly, for example due to the ruling by the Court, adopting zero sales weight is the best response in this case too as Panel (b) of Figure 2 shows. The panel presents the result of same simulation as Panel (a) but starting from the Nash equilibrium of fixed γ_S^n being equal to one third. Even though the local wage rate will decrease if state lowers γ_S^n from one third, the positive effect of decrease in the price level outweighs the effect on wage rate.

Although the current model has described the economy of two states, it is readily extended to the economy of many states if they are symmetric. To simulate results for an average state in the U.S., I also calibrate the Nash equilibrium of the model of 50 symmetric states. Table 2 reports the optimal tax policy in Nash equilibrium of the 50-state economy for various values of σ. When σ equals 4 or 3, the optimal sales weight is zero again while t_C^n is lower and t_N^n is slightly higher than the two-state model. In this economy, the optimal sales weight is zero even when σ equals as low as 2. If the sales apportionment weight is exogenously fixed at one third, the two tax rates come closer as in the two-state economy: $t_C^n = 6.70\%$ and $t_N^n = 7.22\%$.

Table 2: Optimal State Tax Rates in Nash Equilibrium of the 50-State Model

σ	4	3	2	1.5
t_C^n	6.18%	5.82%	5.16%	7.82%
γ_S^n	0	0	0	1
t_N^n	7.88%	8.02%	8.59%	3.22%

Figure 3 examines how state welfare, local wage rate, and local price level change as sales weight deviates from the equilibrium value. The shape of curves in both panels look very similar to those in Figure 2. However, the magnitude of changes is almost doubled compared to the two-state case. For example, when the sales apportionment is not restricted, raising the sales weight from zero to one increases wage rate by 3.6% and price level by 4.3%.



(a) From the Equilibrium of Unrestricted γ

(b) From the Equilibrium of $\gamma = 1/3$

Figure 3: The Effects of Sales Factor Weights on Welfare and Aggregate Variables in the 50-State Model

5 Discussions

Goolsbee and Maydew (2000) is one of the first rigorous empirical studies that estimate the impact of payroll apportionment tax on the local labor demand. They use extensive panel data to find the statistically significant impact of apportionment weights on the local labor demand: if the sales weight is raised from one third to one half and the payroll and property factors are lowered accordingly, manufacturing employment in the state goes up by 1.1%.[24] Their result provides some test of how relevant the model of this study is to the U.S. economy. The dataset of Goolsbee and Maydew (2000) covers the period from 1978 through 1994. Most states used the equally weighted formula at the beginning of the period, and then more and more states started putting a greater weight on the sales factor towards the end of the period. Thus I choose Nash equilibrium of the model of 50 states in which all the states are required to set $\gamma_S^n = 1/3$ and look at the growth rate of local wage rate when one of the states deviates from the equilibrium by raising γ_S^n from one third to one half. My model predicts the growth rate equals 0.62%. This value is not equal to 1.1%, the point estimate of Goolsbee and Maydew (2000), but it is in the same order and within one standard error of their point estimate.[25] Basically, the prediction of my model is not far from their estimate

[24] Goolsbee and Maydew (2000) calculate this result based on their estimate for the elasticity of manufacturing employment with respect to state payroll tax burden (-1.92) and the mean corprate income tax rate in their dataset (7.3%). Although the mean tax rate predicted in the equilibrium of my model is slightly different (6.7%), the result of calculation replacing their mean tax rate with the predicted rate is not significantly different from the value of 1.1%.

[25] Moreover, Goolsbee and Maydew (2000) find that the estimate for elasticity of non-manufacturing employment is smaller than manufacturing. If the average of those estimates are taken, their estimate will be

even though the model is meant to be parsimonious for transparency and tractability.

The use of some continuous distribution to represent heterogeneity in locations and firms is standard in the recent trade literature.[26] Although the calibration in the previous section assumes homogeneous productivity across states and firms, the model can accommodate such distributions in productivity. Firms may have some idiosyncratic preferences about their location as in Fajgelbaum et al. (2015) and Suárez Serrato and Zidar (2016); for example, an oil company may be more profitable in Alaska, and some kind of agricultural production more suitable in Iowa. If such heterogeneity is brought into my model, the elasticity of the number of firms in a state with respect to its corporate income tax rate will be lessened compared to the case of perfectly homogeneous firms. This leads to a smaller positive effect of reducing the payroll factor tax rate on local labor demand, and the positive effect is less likely to outweigh the negative effect of higher price level. Therefore the assumption of homogeneous firms used in the calibration can be considered as the most favorable for a positive sales weight. Since the calibration suggests there is no point of using a positive sales weight under the assumption of homogeneous firms, a positive sales weight will not be justified in the model of heterogeneous firms either, at least under the baseline value of $\sigma = 4$.

Another restrictive assumption of the model is that workers cannot move across state borders. But this assumption is also the most favorable for a positive sales weight. The mobility of workers can be introduced in the model, for example by heterogeneous preferences of workers regarding locations. The mobility, however, will reduce the gap in local wage rates between states with different tax rates. Thus a decrease in payroll apportionment tax will raise the wage rate less than the model of immobile workers. Moreover, the assumption of immobile workers has another advantage; the model can avoid the issue of how to define the state welfare with changing population.

The complexity of U.S. state tax rules that is beyond the scope of the current study includes tax nexus and throwback rules. The U.S. state tax rules do not allow a state to impose corporate income tax on a firm unless the firm establishes nexus in the state; for exapmle, a firm does not establish nexus if it has no contact with a state except for soliciting sales of tangible products in the state. In addition, the majority of U.S. states have the throwback rule; under the rule, the sales of tangible goods are counted as sales in the origin state if the seller does not establish nexus in the destination state.

If a perfectly competitive retail industry exists in the economy and if producing firms do not have to perform any business activity in the destination states, then producing firms sell

closer to the prediction of my model.

[26] The classic examples from this literature include Eaton and Kortum (2002) and Melitz (2003).

all the goods to retail companies in a state that has zero tax rate for the sales factor. The retail companies distribute the goods to the final consumers across states. If this story is true, firms never report positive sales to states with a positive tax rate for the sales factor as long as there exists a state that adopts zero rate for the sales factor. However, Edmiston and Arze del Granado (2006) examine the data set of corporate income tax returns filed to the State of Georgia by multistate firms and report the share of sales in Georgia is 4.4% in 1992, when the corporate income tax rate and sales apportionment weight in the state were 6% and 1/3 respectively. Moreover, they estimate the sales share in Georgia decreased by 6.3% when the state raised the sales apportionment weight to 1/2. Thus firms do not seem to simply minimize the tax burden on the sales factor by concentrating their nexus in a zero-rate state, but they have nexus in various states for some reasons and respond to a change in apportionment formula of a state in a nontrivial way.

In this context, my model can be interpreted in such a way that firms establish nexus in all the states that they sell goods to, for example by setting up an office at an infinitesimal cost. In reality, probably it is possible for firms in some industries to sell in states without establishing nexus, for example online retail companies, but it is hard for some industries to do so, for example, car makers. Therefore it is necessary to extend the model to incorporate nexus choice of firms and heterogeneity across industries in terms of production locations if one wants to include these aspects of U.S. tax rules in the analysis.

6 Conclusion

Recently, increasing the sales apportionment weight, including even adopting the single sales factor, is a popular policy choice among U.S. states, and its effect of stimulating the local labor demand is well recognized in the literature. However, its possible negative effect on the local price level is often overlooked in policy discussions. This paper searches for the optimal state tax policy including apportionment weights and predicts Nash equilibrium of multiple states with a model that incorporates these effects of apportionment formula. Contrary to the popular argument, the model suggests the optimal sales weight should be zero even from the perspective of a single state because a positve sales weight increases the local price level more than the local wage rate. This result is consistent under a wide range of plausible parameters.

Two explanations are possible to reconcile the model's suggestion and the dominant trend among U.S. states. First, the negative effect of sales apportionment tax may actually be overlooked by state policy makers and constituents. The positive effect on local labor demand is often visible; for example, some firms may open new plants or cancel layoffs because of a change in tax policy. However, changes in the price level are harder to detect. It requires a rigorous empirical study to find out the causality between state tax policy and the local price level.

The second possibility is that this model may not capture how tax rules affect firms' decision about production and sales perfectly and overestimate the negative effect. Since the tax rules of U.S. states are complex as discussed in the previous section, firms may be actively avoiding the tax burden on sales apportionment by adjusting nexus, though not so perfectly as the simple story predicts. To verify these hypotheses, careful empirical studies are needed to find out how firms allocate and report sales shares across states in practice.

7 Appendix

7.1 Profit maximization problem for firms

This subsection proves that all the firms in the same state choose the same allocation of sales shares among the states. The proof is a variant of the one by Fajgelbaum et al. (2015). If firm j chooses to operate in state n, the profit maximization problem for firm j is:

$$\begin{aligned} \max_{\{p_n(j), x_n(j)\}} \quad & \tilde{\pi}_n(j) \\ \text{subject to} \quad & x_n^m(j) = p_n^m(j)^{-\sigma} \frac{Y^m}{(P^m)^{1-\sigma}} \quad \text{for } m = \{A, B\}, \end{aligned}$$

in which $\tilde{\pi}_n(j)$ is defined in equation 5. Dividing the first order condition of $\tilde{\pi}_n(j)$ with respect to $p_n^A(j)$ by $p_n^A(j)^{-\sigma-1} Y^A (P^A)^{\sigma-1}$ and using the constraint give:

$$\begin{aligned} (1 - t_C^F - \bar{t}_n(j)) \left[(1-\sigma) p_n^A(j) + \sigma(1-\theta_n) \frac{w_N^n}{z_n(j)} \right] + \sigma \theta_n \frac{\tilde{w}_C^n}{z_n(j)} \\ - (1-\sigma) \left[t_S^A - (t_S^A s_n^A(j) + t_S^B s_n^B(j)) \right] \frac{p_n^A(j)}{S_n(j)} \pi_n^T(j) = 0, \quad (14) \end{aligned}$$

in which $t_S^m = t_C^m \gamma_S^m$, the effective tax rate for the sales factor in state m; $S_n(j)$ is the total sales of firm j; $s_n^m(j)$ is the share of sales in state m out of $S_n(j)$. Solving equation 14 for $p_n^A(j)$ by using $\tilde{w}_C^n = (1 - t_C^F - \bar{t}_n(j)) w_N^n$ gives:

$$p_n^A(j) = \frac{1}{1 - \tilde{t}_n^A (\pi_n^T(j)/S_n(j))} \frac{\sigma}{\sigma - 1} \frac{w_N^n}{z_n(j)}, \quad (15)$$

in which

$$\tilde{t}_n^A = \frac{t_S^A - (t_S^A s_n^A(j) + t_S^B s_n^B(j))}{1 - t_C^F - \bar{t}_n(j)}. \tag{16}$$

The taxable profit can be expressed as:

$$\pi_n^T = S_n(j) \sum_{m=\{A,B\}} s_n^m(j) \left[1 - (1-\theta_n)\frac{w_N^n}{z_n(j) p_n^m(j)}\right]. \tag{17}$$

Substituting equation 15 into equation 17 gives $\pi_n^T = S_n(j)[1 + \theta_n(\sigma - 1)]/\sigma$. This implies

$$p_n^A(j) = \frac{\sigma}{\sigma - \tilde{t}_n^A[1 + \theta_n(\sigma-1)]} \frac{\sigma}{\sigma - 1} \frac{w_N^n}{z_n(j)}. \tag{18}$$

Finally, note that the sales shares are independent of productivity, $z_n(j)$:

$$\begin{aligned} s_n^A(j) &= \frac{p_n^A(j)^{1-\sigma}\bar{Y}^A}{\sum_{m=\{A,B\}} p_n^m(j)^{1-\sigma}\bar{Y}^m} \\ &= \frac{\{\sigma - \tilde{t}_n^A[1+\theta_n(\sigma-1)]\}^{\sigma-1}\bar{Y}^A}{\sum_{m=\{A,B\}}\{\sigma - \tilde{t}_n^m[1+\theta_n(\sigma-1)]\}^{\sigma-1}\bar{Y}^m}, \end{aligned} \tag{19}$$

in which $\bar{Y}^m = Y^m(P^m)^{\sigma-1}$. The symmetric equation holds for $s_n^B(j)$. Equations 16 and 19 and the corresponding equations for state B define a system for $\{\tilde{t}_n^m\}$ and $\{s_n^m(j)\}$ whose solution is independent from $z_n(j)$. Threfore $s_n^m(j) = s_n^m$ and $\bar{t}_n(j) = \bar{t}_n$ for all the firms in state n. This result can easily be extended to the case of more than two states.

7.2 Proof for Proposition 1

From equations 7, 8 and 9, the problem for the government of state n is:

$$\max_{t_C^n, t_N^n} \quad \int_{i \in n} \frac{\tilde{y}^n(i)}{P^n}(g^n)^{\alpha_G} f^n(i) N^n \mathrm{d}i$$
$$\text{subject to} \quad N^n P^n g^n = z_G^n T^n.$$

If the before-tax aggregate income is normalized to one, as in Section 3.1, it can be shown $\Pi = 1/\sigma$ and

$$\tilde{d}^n(i) = (1 - t_C^F - t_C^n)\frac{l(i)}{\sigma N^n L^n}, \tag{20}$$

in which Π is the before-tax aggregate markup. By substituting equations 4, 6 and 20, dropping the variables that are exogeneous for the state government, and changing the variable for integration from i to $l(i)$, the state government's problem can be rewritten:

$$\max_{t_C^n, t_N^n} \quad (T^n)^{\alpha_G} \left[\int_{\bar{l}}^{\infty} (1 - t_C^F - t_C^n) w_N^n l f_L^n(l) \mathrm{d}l \right. \\ \left. + \int_0^{\bar{l}} (1 - t_N^F(l) - t_N^n) w_N^n l f_L^n(l) \mathrm{d}l + \frac{1 - t_C^F - t_C^n}{\sigma N^n} \right]$$

$$\text{subject to} \quad T^n = t_C^n \int_{\bar{l}}^{\infty} w_N^n N^n l f_L^n(l) \mathrm{d}l + t_N^n \int_0^{\bar{l}} w_N^n N^n l f_L^n(l) \mathrm{d}l + t_C^n \frac{1}{\sigma},$$

in which $f_L^n(l)$ is the probability density function of labor efficiency in state n; $\bar{l}$ is the cutoff labor efficiency for income shifting, which is defined implicitly as $t_N^F(\bar{l}) + t_N^n = t_C^F + t_C^n$. I assume the federal personal income tax schedule $t_N^F(l)$ is differentiable and $\partial t_N^F / \partial l$ is strictly positive everewhere. Dividing the first order condition with respect to t_N^n by that with respect to t_C^n gives:

$$\frac{\partial T^n / \partial t_N^n}{\partial T^n / \partial t_C^n} = \frac{\int_0^{\bar{l}} w_N^n N^n l f_L^n(l) \mathrm{d}l}{\int_{\bar{l}}^{\infty} w_N^n N^n l f_L^n(l) \mathrm{d}l + 1/\sigma}. \tag{21}$$

The partial derivatives of T^n with respect to t_N^n and t_C^n are:

$$\frac{\partial T^n}{\partial t_N^n} = \int_0^{\bar{l}} w_N^n N^n l f_L^n(l) \mathrm{d}l + (t_N^n - t_C^n) \frac{\partial \bar{l}}{\partial t_N^n} w^n N^n \bar{l} f_L^n(\bar{l}); \tag{22}$$

$$\frac{\partial T^n}{\partial t_C^n} = \int_{\bar{l}}^{\infty} w_N^n N^n l f_L^n(l) \mathrm{d}l + \frac{1}{\sigma} + (t_N^n - t_C^n) \frac{\partial \bar{l}}{\partial t_C^n} w^n N^n \bar{l} f_L^n(\bar{l}). \tag{23}$$

From equations 22 and 23, note that equation 21 holds when $t_N^n = t_C^n$. Therefore the optimal rates for state personal and corporate income taxes are equal in the case of autarky.

7.3 Proof for Proposition 2

Since the assumptions made in Proposition 2 constitute a special case of Section 7.1, equation 18 still holds:

$$p_n^A(j) = \frac{\sigma}{\sigma - \tilde{t}_n^A [1 + \theta_n(\sigma - 1)]} \frac{\sigma}{\sigma - 1} \frac{w_N}{z}. \tag{24}$$

From equation 16, note $\tilde{t}_n^A = 0$ if $t_C^A = t_C^B$. Then equation 24 implies:

$$p_n^A(j) = \left(\frac{\sigma}{\sigma - 1}\right)\frac{w_N}{z}. \tag{25}$$

Since equation 25 holds for all j and does not depend on the state, the first statement of Proposition 2 holds.

Equation 16 implies $\tilde{t}_n^B < 0 < \tilde{t}_n^A$ if $t_C^A > t_C^B$. It is straightforward from equation 24 to show

$$p_n^B < \left(\frac{\sigma}{\sigma - 1}\right)\frac{w_N}{z} < p_n^A.$$

Therefore the second statement of Proposition 2 holds.

References

Anand, Bharat N. and Richard Sansing, "The Weighting Game: Formula Apportionment as an Instrument of Public Policy," *National Tax Journal*, 2000, *53* (2), 183–199.

Benabou, Roland, "Tax and Education Policy in a Heterogeneous Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?," *Econometrica*, 2002, *70* (2), 481–517.

Conesa, Juan Carlos and Dirk Krueger, "On the Optimal Progressivity of the Income Tax Code," *Journal of Monetary Economics*, 2006, *53* (7), 1425–1450.

_ , Sagiri Kitao, and Dirk Krueger, "Taxing Capital? Not a Bad Idea After All!," *American Economic Review*, 2009, *99* (1), 25–48.

Diamond, Peter A. and Emmanuel Saez, "The Case for a Progressive Tax: From Basic Research to Policy Recommendations," *Journal of Economic Perspectives*, 2011, *25* (4), 165–190.

Eaton, Jonathan and Samuel Kortum, "Technology, Geography, and Trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Edmiston, Kelly D. and F. Javier Arze del Granado, "Economic Effects of Apportionment Formula Changes: Results from a Panel of Corporate Income Tax Returns," *Public Finance Review*, 2006, *34* (5), 483–504.

Fajgelbaum, Pablo D., Eduardo Morales, Juan Carlos Suárez Serrato, and Owen Zidar, "State Taxes and Spatial Misallocation," *NBER Working Paper*, 2015.

Goolsbee, Austan and Edward L. Maydew, "Coveting Thy Neighbor's Manufacturing: the Dilemma of State Income Apportionment," *Journal of Public Economics*, 2000, *75* (1), 125–143.

Gordon, Roger H. and Jeffrey K. Mackie-Mason, "Tax Distortions to the Choice of Organizational Form," *Journal of Public Economics*, 1994, *55* (2), 279–306.

_ **and John D. Wilson**, "An Examination of Multijurisdictional Corporate Income Taxation under Formula Apportionment," *Econometrica*, 1986, *54* (6), 1357–1373.

_ **and Julie Berry Cullen**, "Income Redistribution in a Federal System of Governments," *Journal of Public Economics*, 2012, *96* (11-12), 1100–1109.

Gouveia, Miguel and Robert P. Strauss, "Effective Federal Individual Income Tax Functions: An Exploratory Empirical Analysis," *National Tax Journal*, 1994, *47* (2), 317–339.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante, "Consumption and Labor Supply with Partial Insurance: An Analytical Framework," *American Economic Review*, 2014, *104* (7), 2075–2126.

Jones, Charles I., "Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality," *Journal of Economic Perspectives*, 2015, *29* (1), 29–46.

Mackie-Mason, Jeffrey K. and Roger H. Gordon, "How Much Do Taxes Discourage Incorporation?," *Journal of Finance*, 1997, *52* (2), 477–505.

Mazerov, Michael, "The Single-Sales-Factor Formula: A Boon to Economic Development or a Costly Giveaway?," Technical Report, Center on Budget and Policy Priorities 2001.

McLure, Charles E., "The Elusive Incidence of the Corporate Income Tax: the State Case," *Public Finance Review*, 1981, *9* (4), 395–413.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

Mirrlees, James A., ed., *Tax by Design*, Institute for Fiscal Studies. Oxford, 2011.

Piketty, Thomas and Emmanuel Saez, "Optimal Labor Income Taxation," in "Handbook of Public Economics," Vol. 5, Elsevier B.V., 2013, pp. 392–474.

_ , _ , **and Stefanie Stantcheva**, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities," *American Economic Journal: Economic Policy*, 2014, *6* (1), 230–271.

Pinto, Santiago M., "Corporate Profit Tax, Capital Mobility, and Formula Apportionment," *Journal of Urban Economics*, 2007, *62* (1), 76–102.

Runkel, Marco and Guttorm Schjelderup, "The Choice of Apportionment Factors under Formula Apportionment," *International Economic Review*, 2011, *52* (3), 913–934.

Saez, Emmanuel, "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 2001, *68* (1), 205–229.

Suárez Serrato, Juan Carlos and Owen Zidar, "Who Benefits from State Corporate Tax Cuts? A Local Labor Markets Approach with Heterogeneous Firms," *American Economic Review*, 2016, *106* (9), 2582–2624.