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Relationship Specificity, Market Thickness, and International Trade

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Abstract

We develop a dynamic, search-theoretical, general-equilibrium model to investigate the effect of trade liberalization in vertically-related industries, emphasizing differential impacts depending on the degree of relationship specificity of components that are traded within the vertical relationships. The paper in particular unveils the role of search-and-matching and the resulting market structures of vertically-related industries in evaluating the impact of trade liberalization. We find that the higher the relationship specificity, the thinner is the components market; and that a reduction in trade costs, either in final-goods trade or in components trade, makes the components market thinner and enhances social welfare. We also examine how trade liberalization changes the trade volume of final goods and components, and whether they exhibit complementarity.

JEL classification: F12, F14

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1 Introduction

The volume of international trade has been steadily rising over the past fifty years. It is often argued that "offshoring" or "vertical specialization" in production processes contributes to this trade growth. For instance, using input-output tables, Hummels et al. (2001) estimate that offshoring in components accounts for around 30 percent of countries' export growth of final goods between 1970 and 1990. Similarly, Yi (2003) models international vertical specialization and demonstrates that vertical specialization amplifies the effect of trade liberalization on the growth of trade in goods.[1] While the evidence suggests that trade in final goods and trade in components have a complementary effect on the trade volume, few theoretical work has investigated this complementary effect and its consequences on welfare gains from trade.

This paper proposes a dynamic general-equilibrium model in which matching with a partner in vertically-related production and heterogeneous product quality that arises from matching status play an important role in explaining the interaction between trade in final goods and components. In particular, we build a search-theoretical framework where downstream firms search for upstream firms, while firms are matched randomly between these two types of firms. Matched firms will split the surplus that arises from the partnership through a bilateral negotiation, while the upstream partner carries out a relationship-specific investment to raise the quality of final goods. Unmatched upstream firms produce generic components and sell them in the market, while unmatched downstream firms buy these generic components to produce final goods, whose product qualities are poorer than those produced by matched downstream firms that use customized components procured from their own partners.[2] In this model setting, we examine the effects of international trade both in final goods and in components on matching environment for both upstream and downstream firms, trade volumes, and social welfare. The importance of matching varies across industries due to the differential degree of relationship specificity of components. So those effects of international trade naturally vary with the relationship specificity of the industry's components.

We begin our analysis with a closed-economy model to establish the interactive relationship between the components market and matching environment, which also affects bargaining over surplus between partners. Customized components are traded between matched pairs, while generic components, which can be inferior substitutes for the customized components, are traded in the market. To see how the interaction between the components market and matching environment varies with the industries, we pay special attention to cross-sectoral difference in component customization (or "relationship specificity") devised by Nunn (2007). Nunn constructs a variable that measures the importance of relationship-specific investments by calculating the proportion of components whose

[1]Bernard et al. (2017) show more recent evidence on firm export of final goods and firm import of components.

[2]Our theoretical framework attempts to capture some important features of vertically-related industries. Apple and Samsung are leading producers of high-end smartphones. There are also many producers that produce low-end smartphones, especially in China, mostly selling their products only domestically; "Teeming firms means vicious price competition, especially for cheaper phones" (*The Economist*, 2017). Apple, for example, outsources smartphone components and its assembly to its carefully chosen partners to enhance product quality; "$\cdots$ zealous pursuit of quality would be expected of factories that produce phones for Apple – the world-class facilities run by Taiwan's Foxconn in nearby Shenzhen $\cdots$" (*The Economist*, 2017), whereas many of low-end smartphone producers assemble components that are produced by small local firms (e.g., those in Shenzhen) and reference-priced.

markets are thin; the less the components are traded in the organized market or reference priced, the more relationship specific is the component. We build a theoretical foundation of Nunn's construction of the relationship-specificity measure. We show that the more important the relationship-specific investment in the determination of product quality, the thinner the components market *in equilibrium*. We also establish the negative relationship between the preferable matching environment for downstream firms and the thickness of the components market. Moreover, our model predicts that the higher the relationship specificity of the industry, the higher the extensive margin of firms that use customized components and the higher the intensive margin of their sales in equilibrium.

Then, we examine the impact of costly final-goods trade on the matching environment and social welfare. Melitz's (2003) effect is present also in our model: trade facilitates resource reallocation from unmatched firms, which produce low-quality products, to matched firms, which produce high-quality products. We also find that costly final-goods trade will induce more entry to the upstream sector than to the downstream sector, so that matching environment improves for downstream firms. That leads to an increase in the proportion of matched downstream firms relative to unmatched downstream firms, which entails an improvement of social welfare. That is, we identify another channel of favorable resource reallocation through a change in matching environment. Final-goods trade benefits exporting firms, which are matched downstream firms in our model, while it leads to foreign firms' penetration to domestic firms and gives a negative impact on domestic, unmatched, downstream firms as a result. Whereas trade gives such a mixed impact on downstream firms, it gives an unambiguously positive impact on upstream firms. Upstream firms' profits are associated with those of partner downstream firms through the profits sharing, so an increase in the partners' profits by final-goods trade only benefits upstream firms. Consequently, final-goods trade benefits upstream firms relatively more than downstream firms, inviting more entry to the upstream market than the downstream market. As for welfare gains, while every industry experiences gains from trade, these gains are not equally distributed across industries. We find that the profit reallocation and market restructuring through a change in matching environment are greater, the higher is relationship specificity of the industry.

We extend the model to allow downstream firms to search the foreign country as well as the home country for potential upstream partners. If matched with a foreign upstream firm, the downstream firm imports a customized component from the partner and components trade also takes place. In this extended model setting, we still find that a reduction in variable transport costs of final goods or components will lead to a market restructuring in favor of downstream firms, so that countries gain from this additional source of welfare improvement. In contrast, whether final-goods trade and components trade are complementary for trade volumes depends on whether only the strongest firms export their products, which in turn depends on the level of trade costs. If trade costs are relatively low, both domestically-matched and internationally-matched firms export their products. We find that a reduction in trade costs of either final goods or components unambiguously increase exports of both final goods and components. If trade costs are sufficiently high, only domestically-matched firms, which produce high-quality final goods without incurring the costs of importing components, export their products. We find that in this case, a reduction in final-goods transport costs entails an increase in the trade of final goods, while its impact on the trade volume of components is ambiguous.

The effect of a reduction in components transport costs is similar. The components trade increases as a result, while the final-goods trade may or may not expand.

Our paper contributes to the two strands of the recent literature of international trade. The first strand is firm heterogeneity and resource reallocation as a result of opening to trade. While our model also predicts a similar trade-induced reallocation emphasized by Melitz (2003), it is different from the existing work in that the firm distribution is endogenously determined through matching between upstream and downstream firms such that it varies across the industries. This is worth emphasizing because recent empirical studies have pointed out the underlying difference in the firm distribution across the industries.[3] In our model, firm heterogeneity arises from the difference in the matching status of firms, and this varies systematically across industries depending on the relationship specificity of the industry's components. Consequently, the model can explain heterogeneous impacts of trade liberalization in final goods and components on industrial structures and social welfare in various industries. In addition, our model establishes a new sufficient statistic for welfare evaluation in trade models in which vertical specialization plays an important role, namely the ratio of upstream firms and downstream firms that search for their partners, which in turn is negatively related to the thickness of the components market. We show that all endogenous variables of the model (including welfare) are expressed as a function of the single sufficient statistic in a search-and-matching model. Although the "sufficient statistic approach" is parallel to that in Arkolakis et al. (2012) and Melitz and Redding (2014), our new sufficient statistic is particularly informative when evaluating welfare impact of various policies for industries in which vertical linkage is important. In this respect, our model is closely related to the work that incorporates search and matching between downstream and upstream sectors into international trade models and examines the impact of trade on market thickness; see McLaren (2000) and Grossman and Helpman (2002, 2005). These studies, however, do not explicitly explore the impact of market thickness on welfare.

The second strand is firms' exporting and importing activities. A growing body of firm-level evidence allows trade researchers to investigate various aspects of final-goods trade and components trade. In particular, a series of work by Bernard et al. (2007, 2012, 2017) reveal that importing firms exhibit many of the same features as exporting firms, e.g., only a small fraction of firms import and importers are more productive than non-importers.[4] Finding of such similarities between firms' exporting and importing naturally leads them to further investigate whether trade liberalization in components gives rise to a similar impact as trade liberalization in final goods. For instance, Kasahara and Lapham (2013) find empirically that, because of import and export complementarities, policies that inhibit import of foreign components can have a large adverse effect on export of final goods. Bernard et al. (2017) also show both theoretically and empirically that there is a complementarity between exporting and importing. However, these preceding papers do not shed light on search and matching between downstream and upstream firms, which we believe is one of the most important aspects of industries with vertical specialization.

[3] See for example Helpman et al. (2004) and Del Gatto et al. (2006) as for differential firm distributions across industries in the United States and Western Europe.

[4] This evidence is also confirmed by Amiti and Konings (2007), Goldberg et al. (2010), Halpern et al. (2015), Kasahara and Lapham (2013), Kasahara and Rodrigue (2008), and Topalova and Khandelwal (2011) for different data from various countries.

2 Model

We consider a two-country, dynamic, general-equilibrium model in which firms in a downstream sector buy components from those in an upstream sector to produce final goods in each industry. Upon paying relevant fixed entry costs, firms enter either the upstream or downstream sector, and search for their input-transaction partners at each instance. If a firm is matched with its potential partner, they bargain over profit sharing and make a relationship-specific investment, if they have reached an agreement in the bargaining stage, before trading a customized component within the pairs. By contrast, unmatched firms in the upstream and downstream sectors sell and buy generic components in a competitive market, so that unmatched downstream firms can still produce their products. In this setting, we derive a stationary equilibrium when (i) there is no trade between the countries; (ii) when only final goods are traded; and (iii) when both components and final goods are traded.

2.1 Demand

There are multiple industries, indexed by $j \in \{1, 2, ..., J\}$, that produce differentiated final goods. Preferences of a representative consumer in the economy are given by

$$U = \prod_{j=1}^{J} X_j^{\delta_j}; \quad \sum_{j=1}^{J} \delta_j = 1,$$

where $\delta_j > 0$ is the expenditure share on differentiated goods in industry j. Within industry j, the consumer's preferences take a standard form of constant elasticity of substitution (CES):

$$X_j = \left[\int_{\omega \in \Omega_j} \alpha_j(\omega)^{\frac{1}{\sigma_j}} x_j(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} \mathrm{d}\omega \right]^{\frac{\sigma_j}{\sigma_j - 1}}, \quad \sigma_j > 1,$$

where $\alpha_j(\omega)$ is the quality of variety ω in industry j, such that the greater is $\alpha_j(\omega)$, the higher is the quality and the larger is the demand for variety ω.

It follows from the upper-tier Cobb-Douglas preferences that the consumer allocates expenditure $E_j \equiv \delta_j \bar{E}$ to differentiated goods in industry j, where $\bar{E}$ represents the aggregate expenditure. Moreover, from the lower-tier CES preferences, the consumer allocates sectoral expenditure E_j across varieties to maximize the aggregate consumption X_j within industry j. This generates the following consumption level for variety ω in industry j:

$$x_j(\omega) = E_j P_j^{\sigma_j - 1} \alpha_j(\omega) p_j(\omega)^{-\sigma_j}, \tag{1}$$

where

$$P_j = \left[\int_{\omega \in \Omega_j} \alpha_j(\omega) p_j(\omega)^{1-\sigma_j} \mathrm{d}\omega \right]^{\frac{1}{1-\sigma_j}} \tag{2}$$

is the price index associated with the aggregate consumption X_j. In what follows, we focus on a particular firm in a particular industry at a time and drop a variety index ω and an industry subscript j from the variables for notational simplicity.

2.2 Production

Each differentiated final good is produced by a unique factor, labor, and each country is endowed with L units of labor. We choose labor, which is completely mobile across industries, as a numeraire of the model so that the wage rate is normalized to one.

Every industry is composed of upstream and downstream sectors. The upstream sector specializes in producing components while the downstream sector specializes in producing final goods. Firms in each sector are either unmatched or matched with firms in the other sector. Regardless of the matching status, one unit of final goods requires one unit of components, and the unit cost of producing components is c^U, while the unit cost of transforming components into final goods is c^D. For a matched pair of firms, the upstream firm carries out a relationship-specific investment of K to produce customized components with which the matched downstream firm can produce final goods of high quality.

We assume that if firms are unmatched, upstream firms sell components in a perfectly competitive market, while downstream firms purchase those components from the market. Letting q denote the price of components in the market, the profits of unmatched firms in the upstream and downstream sectors are $\pi^U = (q - c^U)x$ and $\pi^D = (p - q - c^D)x$, respectively. Perfect competition in the upstream sector, however, yields $q = c^U$ which in turn yields $\pi \equiv \pi^D = (p - c)x$, where $c \equiv c^D + c^U$. Furthermore, due to the lack of partnerships, unmatched upstream firms inevitably manufacture generic components, so that unmatched downstream firms are unable to produce high-quality goods. We formalize this aspect by assuming that the product quality of unmatched downstream firms is $\alpha = 1/\gamma$, where $\gamma \in (1, \infty)$ denotes the degree of relationship specificity that varies across industries. The greater γ, the higher the relationship specificity of the industry, so that the quality of final goods produced with generic components is lower.

If firms are matched, by contrast, components are traded within pairs (not through the market). Matched upstream firms produce customized components with which matched downstream firms can produce final goods with high quality $\alpha = 1$. Note that the difference in the product quality between matched and unmatched firms is greater in industries that are featured with higher relationship specificity γ. Since the partnership yields excess profits to the downstream firms, not only matched downstream firms but also matched upstream firms have bargaining power and earn non-zero profits. We adopt the Nash bargaining to split the joint profits $\tilde{\pi} \equiv (\tilde{p} - c)\tilde{x}$ into the matched downstream firm's profits $\tilde{\pi}^D$ and the matched upstream firm's profits $\tilde{\pi}^U$.

Matched and unmatched downstream firms choose prices for their products to maximize their respective profits, $\tilde{\pi}$ and π. Given the CES preferences and the identical unit cost between matched and unmatched firms, both types of firms charge a common price, i.e., $\tilde{p} = p$, with a constant markup $\sigma/(\sigma - 1)$ over the unit cost c. Let M and N denote the numbers of upstream and downstream firms, and n denote the number of matched pairs. Then, n and $N - n$ represent the numbers of matched and unmatched firms in the downstream sector. Noting $\alpha = 1$ and $\alpha = 1/\gamma$ for their respective product qualities, the price index P in (2) is rewritten as

$$P = \left[n\tilde{p}^{1-\sigma} + (N-n)\frac{p^{1-\sigma}}{\gamma} \right]^{\frac{1}{1-\sigma}} = \frac{\sigma c}{\sigma - 1}\left(n + \frac{N-n}{\gamma} \right)^{\frac{1}{1-\sigma}}. \tag{3}$$

It follows directly from the constant markup that the profit of unmatched firms is

$$\pi = \frac{E}{\sigma\gamma}\left(\frac{P}{p}\right)^{\sigma-1} = \frac{E}{\sigma\gamma\left(n + \frac{N-n}{\gamma}\right)}. \tag{4}$$

Moreover, since the markup is the same between unmatched and matched firms, the ratios of the outputs and profits are nothing but the relationship-specificity parameter:

$$\frac{\tilde{x}}{x} = \frac{\tilde{\pi}}{\pi} = \gamma. \tag{5}$$

The greater the relationship specificity, the wider the gap in profits between matched and unmatched firms.[5] As described later, both $\tilde{\pi}$ and π represent the instantaneous profit levels (excluding any fixed costs) in the stationary equilibrium of our dynamic model.

2.3 Search and matching

There is free entry to both upstream and downstream sectors in every industry. Entrants to the upstream and downstream sectors pay one-time fixed entry costs of F^U and F^D (hiring respective units of labor), respectively, and enter the respective unmatched pool. Unmatched downstream firms also pay a per-period search cost of g (hiring labor) to search for their upstream partners at every instance. One-to-one matching occurs randomly between upstream and downstream firms. In the meantime, the fraction λ of firms, either in the upstream or downstream sector regardless of their matching status, are randomly chosen to go bankrupt.

Search technology denoted by $\nu(M-n, N-n)$ assigns the number of newly matched pairs to $M-n$, the number of unmatched upstream firms, and $N-n$, the number of unmatched downstream firms. Following the literature (e.g., Grossman and Helpman, 2002), search technology is characterized by an increasing and concave function of homogeneous of degree 1. These properties of search technology jointly imply complementarity, or supermodularity, in matching.[6]

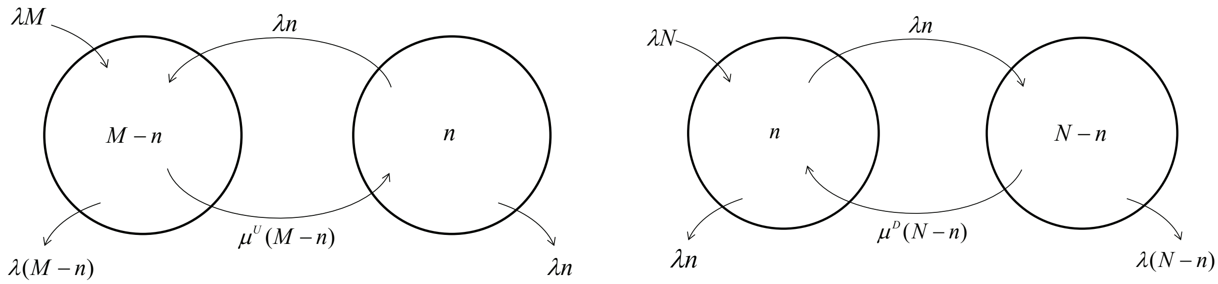
Due to the constant-returns-to-scale search technology, the hazard rate of matching for down-

[5] The results of our analysis remain the same qualitatively even if we consider a type of the relationship-specific investment that lowers the unit cost c (rather than raising the product quality). Suppose that $\tilde{c} = c\gamma^{\frac{1}{1-\sigma}}$ for matched firms while $\alpha = 1$ for both types of firms, i.e., matched firms have the lower unit cost than unmatched firms but have the same product quality. Then the price index (3) is given by

$$P = \frac{\sigma c}{\sigma - 1}(\gamma n + N - n)^{\frac{1}{1-\sigma}}.$$

With this specification, (4) remains the same as before, while (5) is $\frac{\tilde{x}}{x} = \gamma^{\frac{\sigma}{\sigma-1}}$ and $\frac{\tilde{\pi}}{\pi} = \gamma$, for example.

[6] As is originally studied by Shimer and Smith (2000), supermodular functions are routinely used in search and matching environments.



(a) Upstream sector (b) Downstream sector

FIGURE 1 – Search processes

stream and upstream firms are respectively written as

$$\begin{aligned} \mu^D &\equiv \frac{\nu(M-n, N-n)}{N-n} = \nu\left(\frac{M-n}{N-n}, 1\right), \\ \mu^U &\equiv \frac{\nu(M-n, N-n)}{M-n} = \nu\left(1, \frac{N-n}{M-n}\right). \end{aligned} \tag{6}$$

Letting $z \equiv (M-n)/(N-n)$ denote the endogenously-determined ratio of the number of unmatched upstream firms to that of unmatched downstream firms, the hazard rates represented in (6) are expressed in terms of z only:

$$\mu^D = s(z), \quad \mu^U = \frac{s(z)}{z}. \tag{7}$$

It follows from the properties of the search technology that the hazard rate for downstream firms, $\mu^D = s(z)$, is increasing and concave in z, while that for upstream firms, $\mu^U = s(z)/z$, is decreasing and convex in z.

We study a simple dynamic model in which matching, dissolving, and exiting of firms occur stochastically over the continuous time. Figure 1 illustrates the search processes in both sectors. Consider first the upstream sector in panel (a). At every instance, the fraction μ^U of $M-n$ unmatched firms find their partner, so that $\mu^U(M-n)$ firms enter the matched pool. Every firm is also faced with an exogenous shock at a hazard rate of λ, such that firms that are hit by the shock go bankrupt. Since a fraction λ of $M-n$ unmatched firms are hit by this shock, $\lambda(M-n)$ firms exit the unmatched pool and leave the market. Matched firms are also faced with the shock, so that, at every instance, λn matched upstream firms exit from the market as they themselves are hit by the shock, while additional λn matched upstream firms exit from the matched pool and enter the unmatched pool as their downstream partners are hit by the shock and hence the partnership is dissolved. Altogether, λM firms go bankrupt and the same number of firms enter the market in the stationary equilibrium. The same search process applies to the downstream sector as shown in panel (b).

We consider stationary equilibrium in which all endogenous variables (including the number of matched and unmatched firms) remain constant over time. This implies in particular that the number of pairs that are newly formed must be equal to the number of pairs that dissolve the partnership and leave the market at every instance. Thus, we have

$$\begin{cases} \mu^U(M-n) = 2\lambda n, \\ \mu^D(N-n) \ = 2\lambda n. \end{cases}$$

Solving the above steady-state relationships for n yields

$$n = \left(\frac{\mu^U}{\mu^U + 2\lambda}\right) M = \left(\frac{\mu^D}{\mu^D + 2\lambda}\right) N. \tag{8}$$

Equation (8) describes how the number of matched firms n in each sector is tied to the hazard rate of matching, μ^U and μ^D, and the total number of firms, M and N, in the dynamics.

3 Closed-economy equilibrium

This section considers a closed-economy version to derive some key features of the model. Then the model will be extended to an open-economy version in the next section to examine the impact of components trade as well as final-goods trade.

3.1 Equilibrium conditions

To find the stationary equilibrium, we first derive equilibrium value functions for upstream and downstream firms of different matching status, taking account of bargaining within the matched pairs over the surplus to be split. Then we characterize the equilibrium with free entry.

Let $\tilde{V}^D$ and V^D respectively denote the value functions of matched and unmatched firms in the downstream sector; similarly let $\tilde{V}^U$ and V^U denote those of matched and unmatched firms in the upstream sector. These value functions must satisfy the following no-arbitrage conditions:

$$\begin{aligned} r\tilde{V}^D &= \tilde{\pi}^D - \lambda\left(\tilde{V}^D - V^D\right) - \lambda\tilde{V}^D + \dot{\tilde{V}}^D, \\ rV^D &= \pi - g + \mu^D\left(\tilde{V}^D - V^D\right) - \lambda V^D + \dot{V}^D, \\ r\tilde{V}^U &= \tilde{\pi}^U - \lambda\left(\tilde{V}^U - V^U\right) - \lambda\tilde{V}^U + \dot{\tilde{V}}^U, \\ rV^U &= \mu^U\left(\tilde{V}^U - K - V^U\right) - \lambda V^U + \dot{V}^U, \end{aligned}$$

where r is a common discount rate. These four equations represent how matched and unmatched firms gain and lose from matching, dissolving and exiting in the dynamics. The first equation, for example, shows that matched downstream firms have (i) instantaneous profits of $\tilde{\pi}^D$; (ii) losses of $\tilde{V}^D - V^D$ from dissolving, which occurs at a hazard rate of λ; (iii) losses of $\tilde{V}^D$ from exiting the market, which occurs at a hazard rate of λ; and (iv) capital gains of $\dot{\tilde{V}}^D$ from remaining matched. Note that the second equation indicates that unmatched downstream firms pay the search cost g while they seek potential partners, and that the last equation indicates that upstream firms make a relationship-specific investment of K when matched with their partners.

There are no capital gains in the stationary equilibrium so that we have $\dot{\tilde{V}}^D = \dot{V}^D = \dot{\tilde{V}}^U = \dot{V}^U = 0$. For simplicity, we assume that the discount rate is zero ($r = 0$).[7] Solving the no-arbitrage conditions for $\tilde{V}^D$ and V^D yields

$$\begin{aligned} \tilde{V}^D &= \frac{\pi}{\lambda} - \frac{g}{\lambda} + \left(\frac{\mu^D + \lambda}{\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} + \frac{g}{\lambda}\right), \\ V^D &= \frac{\pi}{\lambda} - \frac{g}{\lambda} + \left(\frac{\mu^D}{\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} + \frac{g}{\lambda}\right). \end{aligned} \tag{9}$$

Because the hazard rate λ works as a discount rate in our dynamic model, $\frac{\tilde{\pi}^D}{\lambda}$ and $\frac{\pi}{\lambda}$ are the present values of profit flows of matched and unmatched firms respectively.

Similarly, the value functions of upstream firms can be rewritten as

$$\begin{aligned} \tilde{V}^U - K &= \left(\frac{\mu^U + \lambda}{\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right), \\ V^U &= \left(\frac{\mu^U}{\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right), \end{aligned} \tag{10}$$

where K is subtracted from $\tilde{V}^U$ in order to represent the *net* expected present value of matched upstream firms.[8]

To derive the instantaneous profits for matched firms, we consider bargaining over profit sharing between the matched pairs of upstream and downstream firms. We characterize the outcome of this bargaining as a symmetric Nash bargaining solution where downstream and upstream firms have the same bargaining power over ex-post gains from the relationship. Disagreement would force the pair to be dissolved, so the threat point of the Nash bargaining is (V^D, V^U). Letting $\tilde{\pi}^{D'}$ and $\tilde{\pi}^{U'}$ denote the respective parties' profits to be determined by the bargaining, and $\tilde{V}^{D'}$ and $\tilde{V}^{U'}$ denote the corresponding values given in (9) and (10), the equilibrium profits for the matched firms are

$$\begin{aligned} (\tilde{\pi}^D, \tilde{\pi}^U) \ \in \ \arg \max_{\tilde{\pi}^{D'}, \tilde{\pi}^{U'}} & \left(\tilde{V}^{D'} - V^D\right)\left(\tilde{V}^{U'} - K - V^U\right), \\ \text{s.t.} \quad & \tilde{\pi}^{D'} + \tilde{\pi}^{U'} = \tilde{\pi}. \end{aligned}$$

Using (9) and (10), the solution to this problem gives us the optimal profit sharing rule that satisfies

$$\begin{aligned} \frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} + \frac{g}{\lambda} &= \beta\left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K + \frac{g}{\lambda}\right), \\ \frac{\tilde{\pi}^U}{\lambda} - K &= (1-\beta)\left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K + \frac{g}{\lambda}\right), \end{aligned} \tag{11}$$

[7] The assumption that $r = 0$ would not qualitatively affect the main results. As in Melitz (2003), the aggregate profits equal the total sunk costs paid in individual industries at each instance with this assumption, so the aggregate expenditure equals the aggregate labor income (see the Appendix), helping to simplify the general-equilibrium analysis.

[8] Note that $2K$ is subtracted from $\tilde{\pi}^U/\lambda$ because the value that is lost when moving out of the matched pool is greater than $\tilde{V}^U - K$ by K as K is sunk, and this happens in two occasions (when it goes bankrupt and its partner is forced to exit the market).

where

$$\beta \equiv \frac{\mu^D + \lambda}{\mu^D + \mu^U + 2\lambda}. \tag{12}$$

In this model, $\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K + \frac{g}{\lambda}$ can be considered as the present value of economic rents generated by the relationship. Thus (11) indicates the distribution of the rents, determined by the effective bargaining power β.[9] Substituting (11) into (9) and (10), we can confirm that $\tilde{V}^D > V^D$ and $\tilde{V}^U - K > V^U$ if and only if the economic rents are positive. We hereafter assume $\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K + \frac{g}{\lambda} > 0$ to focus on the case where the relationship specific investment is meaningful.

The number of firms in each sector is endogenously determined by free entry, given the aforementioned search technology. Since new firms enter the unmatched pools, we can write the free entry conditions as

$$V^D = F^D, \quad V^U = F^U.$$

We rewrite them using (8), (9), (10) and (11) as

$$\begin{aligned} \pi - g + \frac{n}{N}\beta\left(\tilde{\pi} - \pi - k + g\right) &= f^D, \\ \frac{n}{M}(1-\beta)\left(\tilde{\pi} - \pi - k + g\right) &= f^U, \end{aligned} \tag{13}$$

where $f^D \equiv \lambda F^D$, $f^U \equiv \lambda F^U$ and $k \equiv 2\lambda K$. The fraction $\frac{n}{N}$ of matched downstream firms earn instantaneous rents of $\beta(\tilde{\pi} - \pi - k + g)$ over the instantaneous profit of $\pi - g$. Under free entry, the expected instantaneous profits equal the instantaneous entry cost f^D. The similar interpretation applies to upstream firms, except that they earn nothing when unmatched.

Together with search technology (6), the free entry conditions (13) simultaneously determine the total numbers of firms, M and N, as well as the number of matched pairs n. Moreover, free entry drives expected profits to zero, and hence the aggregate instantaneous profits equal the aggregate of instantaneous search costs and fixed costs of entry and investment. Thus, the aggregate income is equal to the aggregate payments to labor L, so that the aggregate expenditure in each industry j is given by $E_j = \delta_j L$ in the stationary equilibrium (see the Appendix).

3.2 Equilibrium characterizations

Having described the equilibrium conditions, we now characterize the closed-economy equilibrium. The numbers of firms M, N, n are determined by search technology (6) and the free entry conditions (13). We find it more convenient, however, to work with the profits of unmatched downstream firms

[9] It is important to note that the effective bargaining power β is influenced by the numbers of upstream and downstream firms, even though Nash bargaining power itself is equal between these firms; using the firm dynamics (8), we have

$$\beta \gtreqless \frac{1}{2} \iff \mu^D \gtreqless \mu^U \iff M \gtreqless N.$$

In addition, from the properties of search technology, β is increasing in $z = (M-n)/(N-n)$. This implies that the effective bargaining power of downstream firms increases with the ratio of the unmatched upstream firms to the unmatched downstream firms. This influence of market thickness plays a key role in understanding the impact of trade in the next section.

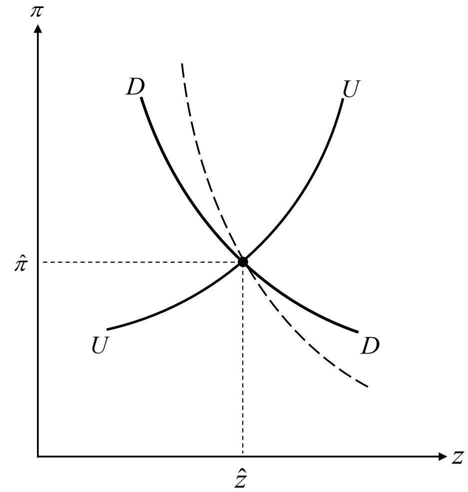


FIGURE 2 – Equilibrium in the closed economy

π and the ratio of unmatched firms $z = (M-n)/(N-n)$. Using (5), (7), (8), and (12), we can solve (13) for π to define it as a function of z:

$$\pi(z) = \frac{f^D + g + (k-g)\phi^D(z)}{1 + (\gamma - 1)\phi^D(z)}, \tag{14}$$

$$\pi(z) = \frac{f^U + (k-g)\phi^U(z)}{(\gamma - 1)\phi^U(z)}, \tag{15}$$

where we define $\phi^D(z)$ and $\phi^U(z)$ as follows.

$$\phi^D(z) \equiv \frac{n}{N}\beta = \frac{zs(z)}{s(z)+2\lambda}\frac{s(z)+\lambda}{s(z)+zs(z)+2\lambda z}, \quad \phi^U(z) \equiv \frac{n}{M}(1-\beta) = \frac{s(z)}{s(z)+2\lambda z}\frac{s(z)+\lambda z}{s(z)+zs(z)+2\lambda z}.$$

Since $s(z)$ is an increasing function of z, $\phi^{D'}(z) > 0$ and $\phi^{U'}(z) < 0$ for any z.

Figure 2 illustrates the relationship between z and π depicted in equations (14) and (15), labeled as DD and UU, respectively. The DD curve is downward-sloping, whereas the UU curve is upward-sloping.[10] For downstream firms, the greater z implies the higher probability of a match $\mu^D = s(z)$ and hence the higher expected profits. This induces entry into the downstream sector, which drives down the ex-post profits of downstream firms, offsetting the initial increase in the expected profits. Thus, the DD curve is downward-sloping. It is clear that the opposite is true for upstream firms. These features of the DD and UU curves ensure the existence and uniqueness of the closed-economy equilibrium; the intersection of these two curves determines the two endogenous variables $(\hat{z}, \hat{\pi})$ as illustrated in Figure 2.

Other endogenous variables can be written as a function of $\hat{z}$. The hazard rates of matching are $\mu^D = s(\hat{z})$ and $\mu^U = s(\hat{z})/\hat{z}$. Having derived $\hat{\pi} = \pi(\hat{z})$, we can calculate the profit for matched pairs as $\tilde{\pi} = \gamma\pi(\hat{z})$. We also derive the number of matched pairs n from (4) with $E = \delta L$ and $\frac{N-n}{n} = \frac{2\lambda}{s(\hat{z})}$

[10] The DD curve is downward-sloping if $\phi^{D'}(z) > 0$ and $(\gamma-1)(f^D+g) > k-g$, while the UU curve is upward-sloping if $\phi^{U'}(z) < 0$. Substituting (14) and $\tilde{\pi} = \gamma\pi$ into the assumption that $\tilde{\pi} - \pi - k + g > 0$ gives $(\gamma - 1)(f^D + g) > k - g$.

(which can be derived from (8)):

$$n = \left[\frac{s(\hat{z})}{\sigma\pi(\hat{z})(\gamma s(\hat{z}) + 2\lambda)}\right]\delta L. \tag{16}$$

Then, we can compute the total numbers of downstream and upstream firms, M and N, from (6), (8), and (16) as

$$M = \left[\frac{s(\hat{z}) + 2\lambda\hat{z}}{\sigma\pi(\hat{z})(\gamma s(\hat{z}) + 2\lambda)}\right]\delta L, \quad N = \left[\frac{s(\hat{z}) + 2\lambda}{\sigma\pi(\hat{z})(\gamma s(\hat{z}) + 2\lambda)}\right]\delta L. \tag{17}$$

Moreover, it follows from (4) and $E = \delta L$ that the price index P can be written as a function of $\pi(\hat{z})$:

$$P(\pi(\hat{z})) = \frac{\sigma c}{\sigma - 1}\left(\frac{\sigma\gamma\pi(\hat{z})}{\delta L}\right)^{\frac{1}{\sigma-1}}. \tag{18}$$

Social welfare in this economy is defined by

$$W(\pi(\hat{\mathbf{z}})) = \prod_{j=1}^{J} P_j(\pi_j(\hat{z}_j))^{-\delta_j}, \text{ where } \pi(\hat{\mathbf{z}}) = (\pi_1(\hat{z}_1), \cdots \pi_J(\hat{z}_J)), \tag{19}$$

which is proportional to the representative consumer's utility U. Note that $\hat{\mathbf{z}} = (\hat{z}_1, \cdots, \hat{z}_J)$ is a sufficient statistic for welfare (through its effect on profits for unmatched firms) since the price index in each industry depends solely on $\hat{z}$.[11] This feature plays a key role in evaluating gains from trade. This completes the characterization of the unique closed-economy equilibrium.

It is important to emphasize that market size δL has no effect on the key equilibrium variable $\hat{z}$. The free entry conditions (14) and (15) do not involve δL, so the intersection of DD and UU curves is independent of δL. Intuitively, when δL increases, the numbers of upstream and downstream firms both increase proportionately, and so do matched firms under the constant-returns-to-scale matching technology. Since all of M, N, and n grow proportionately, $z = (M - n)/(N - n)$ remains fixed. The equilibrium profits for unmatched firms, $\hat{\pi}$, also remain fixed because an increase in the market expenditure is exactly offset by a fall in the price index caused by new entry (see (1) and (3)). Despite that an increase in δL does not affect the equilibrium values of these endogenous variables, it contributes to welfare gains since the number of varieties increases and hence the price index falls with δL. Since the price index decreases with δL, social welfare increases with δL.

3.3 Relationship specificity and market thickness

Building on the above equilibrium characterization, we examine how the market characteristics vary with the relationship specificity of the components, measured by γ. We focus on the analysis on γ here, relegating other comparative statics to the Appendix.

[11] This "sufficient statistic approach" is reminiscent of those in Arkolakis et al. (2012) and Melitz and Redding (2014) who find that sufficient statistics for welfare are (i) the share of expenditure on domestic goods and the trade elasticity and (ii) the domestic productivity cutoffs, respectively. Our sufficient statistic, however, is different from theirs since we spotlight search and matching in vertical specialization which are absent in their models.

Inspection of (14) and (15) reveals that an increase in γ shifts down the DD and UU curves in Figure 2, and entails a fall in $\hat{\pi}$. This in turn raises $\hat{z}$, because we have the following equilibrium relationship derived from canceling out $\tilde{\pi} - \pi - k + g$ from (13) using (7), (8), and (12):

$$\hat{\pi} - g = f^D - \frac{\phi^D(\hat{z})}{\phi^U(\hat{z})} f^U. \tag{20}$$

As illustrated by the dotted curve in Figure 2, $\hat{z}$ and $\hat{\pi}$ are negatively related since $\phi^{D'}(z) > 0$ and $\phi^{U'}(z) < 0$ for any z. Given that the only difference across the industries is the level of γ in this model, we have thus found that the industry with higher γ is associated with higher $\hat{z}$ and lower $\hat{\pi}$. The key to understanding this result is a differential impacts of an increase in γ on upstream and downstream sectors. For both upstream and downstream firms, the resulting increase in economic rents, $\tilde{\pi} - \pi - k + g = (\gamma - 1)\pi - k + g$ contributes to greater expected profits, inducing further entry to both sectors. Increases in the numbers of upstream and downstream firms lead to downward shifts of the UU and DD curves. But this effect is weaker in the downstream sector, since unmatched *downstream* firms' profit π decreases, which dampens the entry incentive, while unmatched *upstream* firms' profits remain the same. Thus, the UU curve shifts down more than the DD curve does, hence decreasing $\hat{\pi}$ and increasing $\hat{z}$.

Our model also allows us to examine how market thickness varies with the relationship specificity. Following Nunn (2007), the market is thicker if more products are traded through the market relative to non-market mechanisms. In our model, the components market is thicker if the aggregate value of generic components $(N - n)px$ is higher relative to the aggregate value of customized components $n\tilde{p}\tilde{x}$. From the equilibrium characterizations, we compute these aggregate values as

$$(N - n)px = \left(\frac{2\lambda}{\gamma s(\hat{z}) + 2\lambda}\right)\delta L, \quad n\tilde{p}\tilde{x} = \left(\frac{\gamma s(\hat{z})}{\gamma s(\hat{z}) + 2\lambda}\right)\delta L,$$

and their ratio is given by

$$\frac{(N - n)px}{n\tilde{p}\tilde{x}} = \frac{2\lambda}{\gamma s(\hat{z})}. \tag{21}$$

This ratio decreases with γ since $\hat{z}$ and hence $s(\hat{z})$ increases with γ. Thus, we have shown as an equilibrium outcome that the higher the relationship specificity, the thinner the market. We can also examine the impacts of an increase in γ on market thickness through the effects on the extensive and intensive margins. The effect on the extensive margin can be measured by a change in $(N - n)/n$. We see from (16) and (17) that this relative extensive margin of the market transaction is written as

$$\frac{N - n}{n} = \frac{2\lambda}{s(\hat{z})}. \tag{22}$$

Since $\hat{z}$ increases with γ and $s(z)$ is an increasing function of z, we find that the components market is thinner at the extensive margin, the higher is the relationship specificity. As for the intensive margin, it follows from $px = \sigma\pi$, $\tilde{p}\tilde{x} = \gamma\sigma\pi$ that

$$\frac{px}{\tilde{p}\tilde{x}} = \frac{1}{\gamma}, \tag{23}$$

which implies that the market is thinner also at the intensive margin, the higher is the relationship specificity. Therefore, the market is thinner for the components of higher relationship specificity at either margin.

Proposition 1. *The higher the relationship specificity γ_j of industry j, the smaller the market transaction is relative to the bilateral transaction within the vertically related pairs.*

Our model provides a theoretical foundation of Nunn's (2007) characterizing association of market thickness to relationship specificity. Nunn (2007) measures the proportion of components sold in the market to components traded in other non-market mechanisms across a variety of industries, and use this measure of market thickness to define relationship specificity. Our model shows as an equilibrium feature that market thickness is decreasing in the relationship specificity, thereby laying a theoretical foundation of his measure.

Thus far we have characterized the closed-economy equilibrium and investigated some of its important properties. Our main interest of this paper, however, is to understand whether or not an increase in final-goods trade stimulates components trade and *vice versa*, a key question that will be addressed in the next section. One may think that components trade naturally stimulates final-goods trade. But it is possible that components trade substitutes final-goods trade by enabling countries to produce final goods for their own sake with the imported components.

4 Open-economy equilibrium

This section explores a global economy in which two symmetric countries previously described engage in international trade. We focus on trade in final goods in Section 4.1 before extending the model in Section 4.2 to also allow firms to import components in order to discuss the complementarity between trade in final goods and trade in components.

Recall that market size δL has no impact on the key endogenous variables, $\hat{z}$ and $\hat{\pi}$. If firms incur no trade costs, therefore, opening to trade has the same impact as the growth in the market size, without affecting $\hat{z}$ and $\hat{\pi}$. As a result, countries gain from trade solely from a decline in the price index caused by an increase in varieties in consumption (see Section 3.2). We show that if firms do incur trade costs, in contrast, trade in final goods and trade in components both affect $\hat{z}$ and $\hat{\pi}$ in such a way as to reinforce welfare gains from trade.

4.1 Trade in final goods

This section considers trade in final goods, in which exporting firms incur an iceberg transport cost τ_x and a fixed export-entry cost F_x (both hiring labor). There are three possible types of equilibrium associated with different levels of τ_x and F_x: (i) no firm exports; (ii) only matched firms export; and (iii) all firms export. For expositional purposes, we focus on the most realistic case where τ_x and F_x fall in a range such that only matched firms export. This assumption, however, is not crucial in that our results essentially remain valid even when all firms export, as will be discussed later.

We begin with the consideration of the optimal firm behavior at each instance. Recall that the wage rate is normalized to one in both countries (due to symmetry). The equilibrium price for final goods produced by unmatched firms and that for those produced by matched firms in the domestic country are both given by $p = \tilde{p}_d = \sigma c/(\sigma - 1)$. The price for final goods produced by matched firms from the foreign country is higher due to the transport costs and is given by $\tilde{p}_x = \tau_x \sigma c/(\sigma - 1) = \tau_x p$. Consequently, the price index P in each country is written as

$$P = \left[n\tilde{p}_d^{1-\sigma} + (N-n)\frac{p^{1-\sigma}}{\gamma} + n\tilde{p}_x^{1-\sigma} \right]^{\frac{1}{1-\sigma}} = \frac{\sigma c}{\sigma - 1}\left[n(1+\tau_x^{1-\sigma}) + \frac{N-n}{\gamma} \right]^{\frac{1}{1-\sigma}}. \tag{24}$$

The outputs of unmatched firms are given by $x = Ep^{-\sigma}P^{\sigma-1}/\gamma$, and the outputs of matched firms in the domestic and foreign markets are $\tilde{x}_d = Ep^{-\sigma}P^{\sigma-1} = \gamma x$ and $\tilde{x}_x = E(\tau_x p)^{-\sigma}P^{\sigma-1} = \tau_x^{-\sigma}\gamma x$, respectively. Similarly, the profits of unmatched firms are

$$\pi = \frac{E}{\sigma\gamma}\left(\frac{P}{p}\right)^{\sigma-1} = \frac{E}{\sigma\gamma\left[n(1+\tau_x^{1-\sigma}) + \frac{N-n}{\gamma} \right]}, \tag{25}$$

and the profits of matched firms from domestic and foreign sales are given by $\tilde{\pi}_d = \frac{E}{\sigma}\left(\frac{P}{p}\right)^{\sigma-1} = \gamma\pi$, and $\tilde{\pi}_x = \frac{E}{\sigma}\left(\frac{P}{\tau_x p}\right)^{\sigma-1} = \tau_x^{1-\sigma}\gamma\pi$, respectively. Thus, letting $\tilde{x} \equiv \tilde{x}_d + \tilde{x}_x$ and $\tilde{\pi} \equiv \tilde{\pi}_d + \tilde{\pi}_x$, the ratios of outputs and profits include not only relationship-specificity γ but also trade cost τ_x:

$$\frac{\tilde{x}}{x} = \left(1 + \tau_x^{-\sigma}\right)\gamma, \quad \frac{\tilde{\pi}}{\pi} = \left(1 + \tau_x^{1-\sigma}\right)\gamma. \tag{26}$$

The firm dynamics is still described by (8). The free-entry conditions require some modification, however, because downstream firms incur the one-time fixed cost F_x when they start exporting their products. Similarly to the treatment of investment of K, we define $f_x \equiv 2\lambda F_x$ and write the free entry conditions as follows (see the Appendix for the derivation):

$$\begin{aligned} \pi - g + \frac{n}{N}\beta\left(\tilde{\pi} - \pi - k - f_x + g\right) &= f^D, \\ \frac{n}{M}(1-\beta)\left(\tilde{\pi} - \pi - k - f_x + g\right) &= f^U, \end{aligned} \tag{27}$$

where the economic rents of matched firms $\tilde{\pi} - \pi - k - f_x + g = [(1+\tau_x^{1-\sigma})\gamma - 1]\pi - k - f_x + g$ now include trade costs, τ_x and f_x, while the effective bargaining power β remains the same as before (see (12)).[12] Using (7), (8) and (26), we solve (27) for $\pi = \pi(z)$:

$$\pi(z) = \frac{f^D + g + (k + f_x - g)\phi^D(z)}{1 + [(1+\tau_x^{1-\sigma})\gamma - 1]\phi^D(z)}, \tag{28}$$

$$\pi(z) = \frac{f^U + (k + f_x - g)\phi^U(z)}{[(1+\tau_x^{1-\sigma})\gamma - 1]\phi^U(z)}. \tag{29}$$

[12]The economic rents derived from the relationship specific investment and exporting are positive, i.e., $\tilde{V}^D - F_x > V^D$ and $\tilde{V}^U - K > V^U$, if and only if $\tilde{\pi} - \pi - k - f_x + g > 0$.

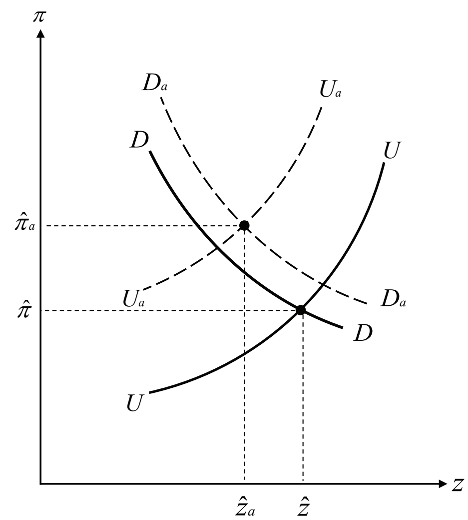


FIGURE 3 – Equilibrium in the open economy

Figure 3 depicts (28) and (29), labeled as DD and UU. The same argument as in the autarkic case applies here to establish the existence of the unique equilibrium $(\hat{z}, \hat{\pi})$, represented by the intersection of the two curves. The question that remains is how the equilibrium is affected by opening to costly final-goods trade. To see the impact of this trade, we note that the free entry conditions in autarky are a special case of (28) and (29) when $\tau_x = \infty$ and $f_x = 0$, so that we need only to examine how these equilibrium conditions are affected by the appropriate changes in τ_x and f_x. To compare the equilibria of the two regimes, we invoke the assumption that only matched firms can export, i.e., matched firms gain from exporting while unmatched firms do not:

$$(1 + \tau_x^{1-\sigma})\gamma\pi - f_x > \gamma\pi,$$
$$(1 + \tau_x^{1-\sigma})\pi - f_x < \pi.$$

These two inequalities simultaneously hold if and only if $\pi = \pi(z)$ satisfies the following condition:

$$\frac{\tau_x^{\sigma-1} f_x}{\gamma} < \pi(z) < \tau_x^{\sigma-1} f_x. \tag{30}$$

In equilibrium where only matched firms export, π should be sufficiently high so that matched firms are willing to incur the export costs to obtain the export profits, while it should be sufficiently low so that unmatched firms do not consider exporting to be profitable.

The intersection of the solid curves in Figure 3 depicts the equilibrium of costly final-goods trade, while that of the dotted curves depicts the autarkic equilibrium. Comparing (14) and (28) for the DD curve or (15) and (29) for the UU curve, we find that both DD and UU curves in the case of costly final-goods trade are located below the autarkic counterparts. In addition, simple inspection of (27) reveals that the equilibrium relationship indicated in (20) holds exactly also in the trade equilibrium, so that a decrease in $\hat{\pi}$ leads to an increase in $\hat{z}$. Costly final-goods trade not only enlarges a profit differential between matched and unmatched firms, but also entails market restructuring between the upstream and downstream sectors.

The downward shifts of both curves are caused by an increase in the economic rents from $(\gamma - 1)\pi - k + g$ to $[(1+\tau_x^{1-\sigma})\gamma - 1]\pi - k - f_x + g$, which is positive when the first inequality of (30) holds. While final-goods trade leads to higher profits for matched downstream firms, it leads to lower profits for unmatched downstream firms due to the foreign firms' penetration to the domestic market. Due to these mixed effects, final-goods trade has an ambiguous impact on the total number of downstream firms N. It always raises the total number of upstream firms M, however, since a resulting increase in matched downstream firms' profits also benefits upstream partners while unmatched upstream firms' profits are zero whether or not the final goods (produced by matched downstream firms) are internationally traded. Consequently, $z = (M-n)/(N-n)$ increases when opening the countries to final-goods trade. We also infer from this argument that the assumption that unmatched firms do not export is immaterial for this result. That is, even if both matched and unmatched firms export, the impact of final-goods trade on $\hat{z}$ and $\hat{\pi}$ is similar to what is depicted in Figure 3.[13] Opening to costly final-goods trade not only benefits downstream firms by enabling them to earn export profits but also causes some harms to them due to foreign firms' penetration to their own final-goods markets. In contrast, the upstream firms unambiguously benefit from opening to trade because their expected profits hinge on the profits when they are matched, which necessarily increase by opening to trade because international trade unambiguously benefits the most-competitive firms.

Now that we know the impact of trade on $\hat{z}$ and $\hat{\pi}$, we can analyze the impact on other endogenous variables. We first derive the number of matched pairs and that of the downstream firms and examine the impacts of trade on the price index and welfare. Similarly to the case of autarky, the number of matched pairs is readily obtained from (25):

$$n = \left[\frac{s(\hat{z})}{\sigma\pi(\hat{z})[(1+\tau_x^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]}\right]\delta L. \tag{31}$$

Substituting (31) into (8), we also obtain

$$N = \left[\frac{s(\hat{z}) + \lambda}{\sigma\pi(\hat{z})[(1+\tau_x^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]}\right]\delta L. \tag{32}$$

As for the price index and social welfare, the free entry conditions lead to $E = \delta L$ in the trade equilibrium (see the Appendix). It then follows from $E = \delta L$ and (25) that the formula for the price index (18) applies generally to all cases of our model. Since $\hat{z}$ is a sufficient statistic for social welfare (19), a fall in $\hat{\pi} = \pi(\hat{z})$ as a result of opening to costly final-goods trade implies that each country enjoys welfare gains from trade as a result of a decrease in the price index.

We next turn to examining the impact of costly final-goods trade on the market thickness, which equals the ratio of the aggregate value of final goods produced with generic components, $(N-n)px$, to that of final goods produced with customized components, $n\tilde{p}\tilde{x} + n\tilde{p}_x\tilde{x}_x = (1+\tau_x^{1-\sigma})n\tilde{p}_d\tilde{x}_d$. Using

[13] A supplementary note, available upon request, shows that the main result essentially holds in the case where both matched and unmatched firms export their products.

(31) and (32), we compute these aggregate values as

$$(N-n)px = \left[\frac{2\lambda}{(1+\tau_x^{1-\sigma})\gamma s(\hat{z})+2\lambda}\right]\delta L,$$
$$(1+\tau_x^{1-\sigma})n\tilde{p}_d\tilde{x}_d = \left[\frac{(1+\tau_x^{1-\sigma})\gamma s(\hat{z})}{(1+\tau_x^{1-\sigma})\gamma s(\hat{z})+2\lambda}\right]\delta L,$$

and obtain

$$\frac{(N-n)px}{(1+\tau_x^{1-\sigma})n\tilde{p}_d\tilde{x}_d} = \frac{2\lambda}{(1+\tau_x^{1-\sigma})\gamma s(\hat{z})}. \tag{33}$$

Comparing (21) and (33), we find that opening to costly final-goods trade makes the components market thinner since $\hat{z} > \hat{z}_a$ and $\tau_x^{1-\sigma} > 0$. Moreover, this finding can also be extended to trade liberalization of a smaller scale, i.e., a reduction in variable or fixed cost. As we can see in (28) and (29), a decrease in τ_x or f_x leads to downward shifts of both DD and UU curves, which in turn raises $\hat{z}$. Then, it follows from (33) that such trade liberalization results in a thinner components market.

We also find that the components market becomes thinner both at the extensive and intensive margins. The measures of market thickness at the extensive and intensive margins are

$$\frac{N-n}{n} = \frac{2\lambda}{s(\hat{z})}, \quad \frac{px}{(1+\tau_x^{1-\sigma})\tilde{p}_d\tilde{x}_d} = \frac{1}{(1+\tau_x^{1-\sigma})\gamma},$$

respectively. Comparing with the autarkic counterparts, shown in (22) and (23), immediately reveals that costly final-goods trade makes the component market thinner at both margins. The ratio of the unmatched to the matched in the downstream sector decreases dues to the improvement of the matching environment for downstream firms. The revenue of the matched firms increases relative to the unmatched since the matched firms are enabled to earn export revenue.

While we have examined the impact of final-goods trade on an industry with given relationship specificity γ, it is possible to examine how this effect varies across industries with different γ. Since the economic rents $[(1+\tau_x^{1-\sigma})\gamma - 1]\pi - k - f_x + g$ increases with γ, downward shifts of the DD and UU curves, caused by trade liberalization of final goods, are greater for the industries with high γ. As a result, the higher the relationship specificity γ, the greater is the impact of final-goods trade on $\hat{z}$ and $\hat{\pi} = \pi(\hat{z})$, and thus the greater is the gain from trade (see (19)).

The following proposition summarizes the main findings about the impact of final-goods trade.

Proposition 2. *Liberalization of final-goods trade entails a thinner components market (i.e., the trade volume through a market is smaller relative to the trade volume between matched pairs) and enhances social welfare. This impact is greater, the higher is the relationship specificity γ_j of the liberalized industry j.*

It is worthwhile to emphasize that final-goods trade leads to a thinner components market and to welfare improvement. Recent empirical evidence shows the importance of the direct impact of components import on productivity gains (see, for example, Amiti and Konings, 2007; Goldberg et al., 2010). Components trade is a key to understanding the current world trade, so we shall now turn to the investigation of its effects.

4.2 Offshoring

Having established that final-goods trade makes the component markets thinner and entails welfare gains, we now turn to the analysis of offshoring (i.e., the import of components) in its effects on final-goods trade and welfare of trading countries. To this end, we extend the final-goods trade model, developed in the last subsection, to the one in which downstream firms also search the foreign country for their partners. A downstream firm that has been matched with a foreign upstream firm imports a customized component from the foreign partner.[14]

Firms are matched internationally as well as domestically and that in addition to final goods, components are traded between internationally-matched pairs. To import components, firms incur an iceberg transport cost of τ_m, but do not incur any fixed cost of importing. Unmatched downstream firms pay a search cost of g^* for matching with foreign upstream firms in addition to g for matching with domestic upstream firms. As before, we assume that only unmatched *downstream* firms pay these search costs. We investigate the case in which firms stop searching once they are matched with some firms, domestic or foreign. We examine the case in which τ_x, τ_m and f_x fall in the range such that both domestically-matched and internationally-matched downstream firms export their products while unmatched firms do not, before analyzing the case in which only domestically-matched firms export their products.

Let us consider the optimal firm behavior at each instance. The price that internationally-matched firms charge in their domestic markets is higher than the price for other firms' products due to the import transport cost τ_m: $\tilde{p}^* = \tau_m \sigma c/(\sigma - 1) = \tau_m p$. In the foreign market, internationally-matched firms set an even higher price that also accounts for the export transport cost: $\tilde{p}_x^* = \tau_x \tau_m p$. Letting n^* represent the number of internationally-matched firms, the price index P in this case is

$$
\begin{aligned}
P &= \left[n\tilde{p}^{1-\sigma} + n^* \tilde{p}_d^{*1-\sigma} + (N - n - n^*) \frac{p^{1-\sigma}}{\gamma} + n\tilde{p}_x^{1-\sigma} + n^* \tilde{p}_x^{*1-\sigma} \right]^{\frac{1}{1-\sigma}} \\
&= \frac{\sigma c}{\sigma - 1} \left[n(1 + \tau_x^{1-\sigma}) + n^*(1 + \tau_x^{1-\sigma})\tau_m^{1-\sigma} + \frac{N - n - n^*}{\gamma} \right]^{\frac{1}{1-\sigma}}.
\end{aligned} \tag{34}
$$

Let $\tilde{x}_d^*$, $\tilde{x}_x^*$, $\tilde{\pi}_d^*$, and $\tilde{\pi}_x^*$ denote the output and profit levels of internationally-matched firms in the domestic and foreign markets, respectively. Then, we have $\tilde{x}_d^* = \tau_m^{-\sigma} \tilde{x}_d$, $\tilde{x}_x^* = (\tau_x \tau_m)^{-\sigma} \tilde{x}_d$, $\tilde{\pi}_d^* = \tau_m^{1-\sigma} \tilde{\pi}_d$ and $\tilde{\pi}_x^* = (\tau_x \tau_m)^{1-\sigma} \tilde{\pi}_d$. Also letting $\tilde{x}^* = \tilde{x}_d^* + \tilde{x}_x^* = (1 + \tau_x^{-\sigma}) \tau_m^{-\sigma} \tilde{x}_d$ and $\tilde{\pi}^* = \tilde{\pi}_d^* + \tilde{\pi}_x^* = \left(1 + \tau_x^{1-\sigma}\right) \tau_m^{1-\sigma} \tilde{\pi}_d$ represent the total output and profit levels of internationally-matched firms, we see that the differences across the three types of firms, characterized by their matching status, reflect

[14] It has been documented that importing firms display many of the same features as exporting firms. In particular, only a small fraction of firms import and importing firms are more productive than non-importing firms. For the detailed evidence of firm importing, see Amiti and Konings (2007), Goldberg et al. (2010), Halpern et al. (2015), Kasahara and Lapham (2013), Kasahara and Rodrigue (2008), and Topalova and Khandelwal (2011). Bernard et al. (2007, 2012, 2017) overview the recent literature of importing firms.

the differences in both export and import opportunities:

$$\begin{aligned}\frac{\tilde{x}}{\tilde{x}^*} &= \tau_m^{\sigma}, \quad \frac{\tilde{x}^*}{x} = \left(1+\tau_x^{-\sigma}\right)\tau_m^{-\sigma}\gamma, \\ \frac{\tilde{\pi}}{\tilde{\pi}^*} &= \tau_m^{\sigma-1}, \quad \frac{\tilde{\pi}^*}{\pi} = \left(1+\tau_x^{1-\sigma}\right)\tau_m^{1-\sigma}\gamma.\end{aligned} \tag{35}$$

Note that the internationally-matched firms produce less and earn lower profits than the domestically-matched due to the transport costs of components.

The firm dynamics must be revised such that firms may be matched with foreign firms as well as domestic ones. We assume for simplicity that the same matching technology applies to domestic and international matching, so that the number of domestically-matched pairs and that of internationally-matched pairs that are newly formed at each instance are both given by $\nu(M-n-n^*, N-n-n^*)$. Then the hazard rates of matching (6) are respectively redefined as

$$\begin{aligned}\mu^D &\equiv \frac{\nu(M-n-n^*, N-n-n^*)}{N-n-n^*} = \nu\left(\frac{M-n-n^*}{N-n-n^*}, 1\right), \\ \mu^U &\equiv \frac{\nu(M-n-n^*, N-n-n^*)}{M-n-n^*} = \nu\left(1, \frac{N-n-n^*}{M-n-n^*}\right).\end{aligned}$$

These hazard rates can still be described as (7) with $z \equiv (M-n-n^*)/(N-n-n^*)$. In addition, we assume $\mu^D(\tilde{V}^D - \tilde{V}^{D*}) < g$, so that downstream firms stop searching for domestic partners once they are matched with foreign partners despite that domestic matching is more profitable than international matching.[15]

In the stationary equilibrium, the number of newly-matched firms is equal to the number of exiting firms at every instance, so we have

$$\begin{cases}\mu^U(M-n-n^*) = 2\lambda n, \\ \mu^D(N-n-n^*) = 2\lambda n,\end{cases} \qquad \begin{cases}\mu^U(M-n-n^*) = 2\lambda n^*, \\ \mu^D(N-n-n^*) = 2\lambda n^*,\end{cases}$$

where the first and second systems of equations describe the firm dynamics of domestically-matched and internationally-matched firms, respectively. Solving these relationships for n and n^* yields

$$n = n^* = \left(\frac{\mu^U}{2(\mu^U+\lambda)}\right)M = \left(\frac{\mu^D}{2(\mu^D+\lambda)}\right)N. \tag{36}$$

The free entry conditions that equate the expected instantaneous profits and the instantaneous values of entry costs in the downstream and upstream sectors are given as follows (see the Appendix):

$$\begin{aligned}\pi - g - g^* + \frac{n}{N}\beta(\tilde{\pi} - \pi - k - f_x + g + g^*) + \frac{n^*}{N}\beta(\tilde{\pi}^* - \pi - k - f_x + g + g^*) = f^D, \\ \frac{n}{M}(1-\beta)(\tilde{\pi} - \pi - k - f_x + g + g^*) + \frac{n^*}{M}(1-\beta)(\tilde{\pi}^* - \pi - k - f_x + g + g^*) = f^U,\end{aligned} \tag{37}$$

[15] The condition $\mu^D(\tilde{V}^D - \tilde{V}^{D*}) < g$ also ensures that ex-post gains from the partnership relationship are positive (i.e., $\tilde{V}^{D^*} - F_x - V^D > 0$), as shown in the Appendix.

where $\tilde{\pi}^* - \pi - k - f_x + g + g^* = (1 + \tau_x^{1-\sigma})\tau_m^{1-\sigma}\gamma\pi - \pi - k - f_x + g + g^*$ represents the economic rents of international matching, which include both export costs of τ_x and f_x and the import cost of τ_m, while the effective bargaining power (the counterpart of (12) in the absence of offshoring) is given by

$$\beta \equiv \frac{\frac{3}{2}\mu^D + \lambda}{\frac{3}{2}\mu^D + \frac{3}{2}\mu^U + 2\lambda}.$$

Using (7), (35) and (36), we can solve (37) for $\pi = \pi(z)$:

$$\pi(z) = \frac{f^D + g + g^* + 2(k + f_x - g - g^*)\phi^D(z)}{1 + \left[(1 + \tau_x^{1-\sigma})(1 + \tau_m^{1-\sigma})\gamma - 2\right]\phi^D(z)}, \tag{38}$$

$$\pi(z) = \frac{f^U + 2(k + f_x - g - g^*)\phi^U(z)}{\left[(1 + \tau_x^{1-\sigma})(1 + \tau_m^{1-\sigma})\gamma - 2\right]\phi^U(z)}, \tag{39}$$

where $\phi^D(z) \equiv \frac{n}{N}\beta$ and $\phi^U(z) \equiv \frac{n}{M}(1 - \beta)$ are now given by

$$\phi^D(z) = \frac{zs(z)}{2[s(z) + \lambda]} \frac{\frac{3}{2}s(z) + \lambda}{\frac{3}{2}s(z) + \frac{3}{2}zs(z) + 2\lambda z}, \quad \phi^U(z) = \frac{s(z)}{2[s(z) + \lambda z]} \frac{\frac{3}{2}s(z) + \lambda z}{\frac{3}{2}s(z) + \frac{3}{2}zs(z) + 2\lambda z}.$$

While $\phi^D(z)$ and $\phi^U(z)$ are the counterparts of the corresponding ones in the cases of autarky and final-goods trade, the functional forms are different due to the possibility of international matching, although we continue to use the same notations for simplicity. In spite of this difference, $\phi^{D'}(z) > 0$ and $\phi^{U'}(z) < 0$ for any z, and the graph of $\pi(z)$ represented by (38) is downward-sloping, whereas that by (39) is upward-sloping, so the intersection of the two curves uniquely determines $(\hat{z}, \hat{\pi})$.

Now, we examine how a reduction in transport costs for final goods or components changes the market structures of upstream and downstream sectors and whether it yields differential impacts on trade volumes of final goods and components. We will also assess resulting welfare gains.

First, we find that even in the case where some firms offshore customized components, trade liberalization in the form of a decrease in τ_x or τ_m reduces $\hat{\pi}$ and raise $\hat{z}$. It immediately follows from (38) and (39) that both DD curve and UU curve shifts down and hence $\hat{\pi}$ decreases if either τ_x or τ_m falls. As for the effect on $\hat{z}$, we have from (37) that

$$\hat{\pi} - g - g^* = f^D - \frac{\phi^D(\hat{z})}{\phi^U(\hat{z})} f^U, \tag{40}$$

which is the counterpart of (20). As in (20), this equation shows a negative relationship between $\hat{z}$ and $\hat{\pi}$, as $\phi^{D'}(z) > 0$ and $\phi^{U'}(z) < 0$ for any z. Consequently, a fall in $\hat{\pi}$ caused by a decrease in τ_x or τ_m leads to an increase in $\hat{z}$. It also follows from (18) and (19), which are valid in this extended model with offshoring, that such trade liberalization enhances social welfare of each trading country. In this sense, international trade both in final goods and in components has a complementary effect on welfare gains from trade.

To examine trade liberalization in τ_x or τ_m has also a complementary effect on the trade volumes of final goods and components, we first derive the equilibrium number of matched firms and total number of downstream firms in the market as

$$
\begin{aligned}
n = n^* &= \left[\frac{s(\hat{z})}{\sigma\pi(\hat{z})((1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z})+2\lambda)}\right]\delta L,\\
N &= \left[\frac{2[s(\hat{z})+\lambda]}{\sigma\pi(\hat{z})((1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z})+2\lambda)}\right]\delta L.
\end{aligned}
\tag{41}
$$

Then, using $\tilde{p}_d\tilde{x}_d = \gamma\sigma\pi$ and (41), we calculate $n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^* = \tau_x^{1-\sigma}(1+\tau_m^{1-\sigma})n\tilde{p}_d\tilde{x}_d$, the aggregate value of final goods exported by both domestically-matched and internationally-matched firms as

$$
n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^* = \left[\frac{\tau_x^{1-\sigma}(1+\tau_m^{1-\sigma})\gamma s(\hat{z})}{(1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z})+2\lambda}\right]\delta L. \tag{42}
$$

As for the value of total components trade for each country, we use the marginal costs $\tau_m c$ to evaluate the trade volume since the components are traded between each pair of internationally-matched firms so that there is no explicit price for those components to be used for the evaluation. Given that, the aggregate value of components is $n^*\tau_m c\tilde{x}^* = n^*(1+\tau_x^{1-\sigma})\tau_m^{1-\sigma}\gamma(\sigma-1)\pi$, which is computed as

$$
n^*\tau_m c\tilde{x}^* = \left(\frac{\sigma-1}{\sigma}\right)\left[\frac{(1+\tau_x^{1-\sigma})\tau_m^{1-\sigma}\gamma s(\hat{z})}{(1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z})+2\lambda}\right]\delta L. \tag{43}
$$

It is easy to see that a reduction in τ_x raises not only the aggregate value of final goods, expressed in (42), but also the aggregate value of components, represented by (43). Similarly, a reduction in τ_m increases these two aggregate values. Thus, we conclude that components trade and final-goods trade are complementary in the sense that a reduction in trade costs of either trade encourages trade in that sector, which in turn stimulates trade in the other sector.

We summarize these results in the following proposition (see the Appendix for proof).

Proposition 3. *In any industry $j \in J$, if both domestically-matched and internationally-matched firms export, a reduction in trade barriers gives rise to the following:*

(i) A reduction in τ_x makes the components market thinner and enhances social welfare. Exports of final goods and those of components both increase as a result.

(ii) A reduction in τ_m also makes the components market thinner and enhances social welfare. Exports of final goods and those of components both increase as a result.

Let us turn to another case in which internationally-matched firms are not efficient enough (due to incurring transport costs of components τ_m) to export, so that only domestically-matched firms profitably export their products.

The no-arbitrage equations are almost the same as in the previous case except that internationally-matched firms do not incur the fixed export cost F_x. This means that the free entry conditions (37) are modified as follows:

$$\pi - g - g^* + \frac{n}{N}\beta(\tilde{\pi} - \pi - k - f_x + g + g^*) + \frac{n^*}{N}\beta(\tilde{\pi}^* - \pi - k + g + g^*) = f^D,$$
$$\frac{n}{M}(1-\beta)(\tilde{\pi} - \pi - k - f_x + g + g^*) + \frac{n^*}{M}(1-\beta)(\tilde{\pi}^* - \pi - k + g + g^*) = f^U.$$

Since the ratios of the profits (35) are now given by

$$\frac{\tilde{\tilde{\pi}}}{\tilde{\pi}^*} = \frac{1+\tau_x^{1-\sigma}}{\tau_m^{1-\sigma}}, \quad \frac{\tilde{\pi}^*}{\pi} = \tau_m^{1-\sigma}\gamma,$$

we solve the above free entry conditions for $\pi = \pi(z)$:

$$\pi(z) = \frac{f + g + g^* + (2k + f_x - 2g - 2g^*)\phi^D(z)}{1 + [(1+\tau_x^{1-\sigma} + \tau_m^{1-\sigma})\gamma - 2]\phi^D(z)}, \tag{44}$$
$$\pi(z) = \frac{f^U + (2k + f_x - 2g - 2g^*)\phi^U(z)}{[(1+\tau_x^{1-\sigma} + \tau_m^{1-\sigma})\gamma - 2]\phi^U(z)}. \tag{45}$$

As before, these equations uniquely determine $\hat{z}$ and $\hat{\pi} = \pi(\hat{z})$. Furthermore, we can easily verify that the key comparative statics results still obtain also in this case, i.e., $\partial\hat{z}/\partial\tau_x < 0$, $\partial\hat{z}/\partial\tau_m < 0$, $\partial\hat{\pi}/\partial\tau_x > 0$, $\partial\hat{\pi}/\partial\tau_m > 0$. After all, a reduction in trade barriers, whether in final-goods trade or in components trade, increases final-goods market competition while benefiting most efficient downstream firms and hence all upstream firms, which in turn leads to a change in matching environment in favor of downstream firms.

Although trade liberalization in final-goods trade or in components trade enhances social welfare as in the previous equilibrium, it does not always have a complementary effect on the trade volumes of final goods and components in this equilibrium. The numbers of matched firms, domestically and internationally, are given by

$$n = n^* = \left[\frac{s(\hat{z})}{\sigma\pi(\hat{z})[(1+\tau_x^{1-\sigma} + \tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]}\right]\delta L.$$

Since per-firm exports of final goods are $\tilde{p}_x\tilde{x}_x = \tau_x^{1-\sigma}\gamma\sigma\pi$, the aggregate value of final goods exported from each country is computed as

$$n\tilde{p}_x\tilde{x}_x = \left[\frac{\tau_x^{1-\sigma}\gamma s(\hat{z})}{(1+\tau_x^{1-\sigma} + \tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda}\right]\delta L. \tag{46}$$

Similarly, since per-firm imports of components equal $\tau_m c\tilde{x}^* = \tau_m^{1-\sigma}\gamma(\sigma-1)\pi$, the aggregate value of components imported by each country is computed as

$$n^*\tau_m c\tilde{x}^* = \left(\frac{\sigma-1}{\sigma}\right)\left[\frac{\tau_m^{1-\sigma}\gamma s(\hat{z})}{(1+\tau_x^{1-\sigma} + \tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda}\right]\delta L. \tag{47}$$

In contrast to the equilibrium in Proposition 3, we find that a reduction in τ_m raises components trade flows, expressed in (47), but it does not necessarily raise final-goods trade, described in (46), and that the opposite is true for a reduction in τ_x (see the Appendix for proof). As we have seen, a reduction in τ_x or τ_m entails an increase in $\hat{z}$, which in turn increases the number of domestically-matched firms. Thus the impact through the extensive margin always works in favor of trade. However, as represented by a resulting decrease in $\hat{\pi}$, competition in each final-goods market becomes fiercer so trade in the intensive margin shrinks unless a reduction in trade barriers directly affects the trade, e.g., a reduction in τ_x on trade of final goods.

Proposition 4. *In any industry $j \in J$, if only domestically-matched firms export, a reduction in trade barriers gives rise to the following:*

(i) A reduction in τ_x makes the components market thinner and enhances social welfare. Exports of final goods increase as a result while those of components may or may not increase.

(ii) A reduction in τ_m also makes the components market thinner and enhances social welfare. Exports of final goods may or may not increase while those of components unambiguously increase as a result.

5 Conclusion

This paper has investigated the effect of trade liberalization in vertically-related industries, emphasizing differential impacts depending on the degree of relationship specificity of components that are traded within the vertical relationships. We find that the higher is the relationship specificity, the thinner is the market; and that a reduction in trade costs, either in final-goods trade or in components trade, makes the components market thinner and enhances social welfare. We also show that in the case where all matched final-goods producers, whether matched domestically or internationally, export their products, a reduction in trade costs in either final-goods trade or components trade entails an increase in trade in both final goods and components, i.e., trade in final goods and trade in components are complementary. We want to emphasize that these effects of international trade are not only through resource reallocation from low-productivity firms to high-productivity ones as emphasized by Melitz (2003) but also through a change in matching environments and market restructuring in upstream and downstream sectors.

To analyze vertical relationships, we resort to a rather strong assumption that while the downstream sector is monopolistically competitive, the upstream sector is perfectly competitive in trading generic components in anonymous markets, which we believe is a reasonable approximation of the reality. Relaxing the assumption of perfect competition in the upstream sector would change the impact of trade liberalization on market thickness and on trade volumes of final goods and components. Even in such circumstances, however, we believe it is possible to show that our key results continue to hold as long as the market competition is tougher in the upstream sector than in the downstream sector.

It would be interesting to analyze the effect of country asymmetry on the location of production of final goods and components. Will a larger country host disproportionally more final-goods producers or components producers than a smaller country? Which country, large or small, benefits relatively more from final-goods/component trade liberalization in the vertically-related world? We shall address such questions in future research.

Appendix

A Proofs of the closed-economy equilibrium

A.1 Proof of the Nash bargaining solution

The Nash bargaining problem within matched pairs is

$$\max_{\tilde{\pi}^{D'},\tilde{\pi}^{U'}} \left(\tilde{V}^{D'} - V^D\right)\left(\tilde{V}^{U'} - K - V^U\right),$$
$$\text{s.t.} \quad \tilde{\pi}^{D'} + \tilde{\pi}^{U'} = \tilde{\pi}.$$

Substituting $\tilde{V}^{D'}$ and $\tilde{V}^{U'} - K$ from (9) and (10) as well as the constraint $\tilde{\pi}^{U'} = \tilde{\pi} - \tilde{\pi}^{D'}$ into the above problem, we obtain

$$\max_{\tilde{\pi}^{D'}} \left(\frac{\pi}{\lambda} - \frac{g}{\lambda} + \left(\frac{\mu^D + \lambda}{\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^{D'}}{\lambda} - \frac{\pi}{\lambda} + \frac{g}{\lambda}\right) - V^D\right)\left(\left(\frac{\mu^U + \lambda}{\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi} - \tilde{\pi}^{D'}}{\lambda} - 2K\right) - V^U\right).$$

The solution to this problem gives us the optimal profit sharing rule between $\tilde{\pi}^{D'}$ and $\tilde{\pi}^{U'}$. Substituting V^D and V^U from (9) and (10) and evaluating the sharing rule at $\tilde{\pi}^{D'} = \tilde{\pi}^D$ and $\tilde{\pi}^{U'} = \tilde{\pi}^U = \tilde{\pi} - \tilde{\pi}^D$, we have

$$\begin{aligned}
\tilde{\pi}^D &= \frac{1}{\mu^D + \mu^U + 2\lambda}\left[\left(\mu^D + \lambda\right)\tilde{\pi} + \left(\mu^U + \lambda\right)(\pi - g) - 2\lambda\left(\mu^D + \lambda\right)K\right], \\
\tilde{\pi}^U &= \frac{1}{\mu^D + \mu^U + 2\lambda}\left[\left(\mu^U + \lambda\right)\tilde{\pi} - \left(\mu^U + \lambda\right)(\pi - g) + 2\lambda\left(\mu^D + \lambda\right)K\right].
\end{aligned}$$

Rearranging these equations establishes the result.

A.2 Proof of the labor market clearing condition

We first show that the aggregate profits equal the aggregate fixed costs in every industry. Let L_e^D denote the aggregate labor used for entry and search by downstream firms. In the downstream sector, since λN new entrants pay the fixed entry cost F^D and $N - n$ unmatched firms pay the search cost g at every instance, the aggregate labor used for entry and search at every instance is $L_e^D = \lambda N F^D + (N - n)g$. Using the free entry condition ($V^D = F^D$), instantaneous aggregate labor used for entry and search in the downstream sector is given by

$$\begin{aligned}
L_e^D &= \lambda N V^D + (N - n)g \\
&= \lambda N\left[\frac{\pi}{\lambda} - \frac{g}{\lambda} + \left(\frac{\mu^D}{\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} + \frac{g}{\lambda}\right)\right] + (N - n)g \qquad \text{(using (9))} \\
&= \left(\frac{\mu^D N}{\mu^D + 2\lambda}\right)\tilde{\pi}^D + \left(\frac{2\lambda N}{\mu^D + 2\lambda}\right)(\pi - g) + (N - n)g.
\end{aligned}$$

From the firm dynamics (8), we have $\frac{\mu^D N}{\mu^D + 2\lambda} = n$ and $\frac{2\lambda N}{\mu^D + 2\lambda} = N - n$. Substituting these equalities into the above equation, we see that the aggregate labor is equal to the aggregate profit in the downstream sector:

$$L_e^D = n\tilde{\pi}^D + (N - n)\pi. \tag{48}$$

Similarly, letting L_e^U denote the corresponding labor employed in the upstream sector, the aggregate labor used for entry is $\lambda M F^U$ and the aggregate labor for investment is $2\lambda n K$, and thus $L_e^U = \lambda M F^U + 2\lambda n K$. Using (8), (10) and (13) and rearranging, we have

$$L_e^U = n\tilde{\pi}^U. \tag{49}$$

Then, it immediately follows from (48) and (49) that

$$\begin{aligned} L_e^D + L_e^U &= n\tilde{\pi} + (N-n)\pi \\ &= \frac{E}{\sigma}. && \text{(using (4))} \end{aligned}$$

Next, we show that the aggregate expenditure equals the aggregate payments to labor in every industry. Let L_p^D and L_p^U denote the aggregate labor used for production by downstream and upstream firms respectively. Recall that the unit cost of producing final goods and components is $c = c^D + c^U$, which is identical between matched and unmatched firms. Since there are n matched firms and $N-n$ unmatched firms that earn $\tilde{\pi} = (\tilde{p}-c)\tilde{x}$ and $\pi = (p-c)x$ respectively, we have $L_p^D + L_p^U = nc\tilde{x} + (N-n)cx$. Using the optimal output level, we obtain

$$\begin{aligned} L_p^D + L_p^U &= \gamma\left(n + \frac{N-n}{\gamma}\right)cx && \text{(using (5))} \\ &= \left(\frac{\sigma-1}{\sigma}\right)E. && \text{(using (1) and (3))} \end{aligned}$$

Summing up the aggregate labor for production, investment, and search establishes the desired result that

$$\begin{aligned} L &\equiv (L_p^D + L_p^U) + (L_e^D + L_e^U) \\ &= E, \end{aligned}$$

where L represents the aggregate payments to labor since we choose labor as a numeraire of the model. This equation means that the aggregate payments to labor equal the aggregate expenditure on the final goods for each industry. Summing up both sides of the equation over all the industries, we obtain that the aggregate wage payments equal the aggregate expenditure of each country.

A.3 Proof of Proposition 1

It follows immediately from (14) with (15) that $\partial\hat{\pi}/\partial\gamma < 0$. Since $\hat{\pi}$ and $\hat{z}$ are negatively related as described by (20), we find that $\partial\hat{z}/\partial\gamma > 0$. Comparative statics for the other endogenous variables are straightforward. In particular, $\partial(\tilde{\pi}-\pi)/\partial\gamma > 0, \partial n/\partial\gamma > 0, \partial M/\partial\gamma > 0, \partial N/\partial\gamma \gtreqless 0$ (from (16) and (17)). The discussion that precedes Proposition 1 in the main text shows that the components market becomes thinner if γ rises, which holds true through both extensive and intensive margins. As for the extensive margin, since $s'(z) > 0$ for any z, (22) shows that a rise in γ increases the number of matched firms n relatively more than the number of unmatched firms $N-n$. As for the intensive margin, on the other hand, (23) directly shows that a rise in γ increases the revenue per matched firm $\tilde{p}\tilde{x}$ relatively more than the revenue per unmatched firm px.

Similarly, we can conduct comparative statics analyses with respect to other exogenous variables $(k, f^D, f^U, \lambda, g)$. Since its derivation is similar, we only report the results here:

$$\begin{gathered} \frac{\partial\hat{z}}{\partial k} < 0,\ \frac{\partial\hat{\pi}}{\partial k} > 0,\ \frac{\partial(\tilde{\pi}-\pi)}{\partial k} < 0,\ \frac{\partial n}{\partial k} < 0,\ \frac{\partial M}{\partial k} < 0,\ \frac{\partial N}{\partial k} \gtreqless 0; \\ \frac{\partial\hat{z}}{\partial f^D} > 0,\ \frac{\partial\hat{\pi}}{\partial f^D} < 0,\ \frac{\partial(\tilde{\pi}-\pi)}{\partial f^D} > 0,\ \frac{\partial n}{\partial f^D} \gtreqless 0,\ \frac{\partial M}{\partial f^D} \gtreqless 0,\ \frac{\partial N}{\partial f^D} \gtreqless 0; \\ \frac{\partial\hat{z}}{\partial f^U} < 0,\ \frac{\partial\hat{\pi}}{\partial f^U} > 0,\ \frac{\partial(\tilde{\pi}-\pi)}{\partial f^U} > 0,\ \frac{\partial n}{\partial f^U} < 0,\ \frac{\partial M}{\partial f^U} < 0,\ \frac{\partial N}{\partial f^U} \gtreqless 0; \\ \frac{\partial\hat{z}}{\partial\lambda} < 0,\ \frac{\partial\hat{\pi}}{\partial\lambda} > 0,\ \frac{\partial(\tilde{\pi}-\pi)}{\partial\lambda} > 0,\ \frac{\partial n}{\partial\lambda} < 0,\ \frac{\partial M}{\partial\lambda} < 0,\ \frac{\partial N}{\partial\lambda} \gtreqless 0; \\ \frac{\partial\hat{z}}{\partial g} > 0,\ \frac{\partial\hat{\pi}}{\partial g} < 0,\ \frac{\partial(\tilde{\pi}-\pi)}{\partial g} > 0,\ \frac{\partial n}{\partial g} \gtreqless 0,\ \frac{\partial M}{\partial g} \gtreqless 0,\ \frac{\partial N}{\partial g} \gtreqless 0. \end{gathered}$$

B Proofs of the open-economy equilibrium

B.1 Proof of the free entry condition in final-goods trade

We first derive equation (27). While the no-arbitrage conditions for upstream firms are the same as before, those for downstream firms are

$$
\begin{aligned}
0 &= \tilde{\pi}^D - \lambda \left(\tilde{V}^D - V^D \right) - \lambda \tilde{V}^D + \dot{\tilde{V}}^D, \\
0 &= \pi - g + \mu^D \left(\tilde{V}^D - F_x - V^D \right) - \lambda V^D + \dot{V}^D,
\end{aligned}
$$

where $\tilde{\pi}^D = \tilde{\pi}_d^D + \tilde{\pi}_x^D$. Setting $\dot{\tilde{V}}^D = \dot{V}^D = 0$, the value functions for each type of downstream firms are given by

$$
\begin{aligned}
\tilde{V}^D - F_x &= \frac{\pi}{\lambda} - \frac{g}{\lambda} + \left(\frac{\mu^D + \lambda}{\mu^D + 2\lambda} \right) \left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} \right), \\
V^D &= \frac{\pi}{\lambda} - \frac{g}{\lambda} + \left(\frac{\mu^D}{\mu^D + 2\lambda} \right) \left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} \right),
\end{aligned}
\tag{50}
$$

where F_x is subtracted from $\tilde{V}^D$ to represent the *net* expected present value of a matched downstream firm.

As in the closed economy, the profit sharing is uniquely determined by the symmetric Nash bargaining. Given that ex-post gains from the relationship for downstream firms are given by $\tilde{V}^D - F_x - V^D$ in the open economy, $(\tilde{\pi}^D, \tilde{\pi}^U) = (\tilde{\pi}^{D'}, \tilde{\pi}^{U'})$ uniquely solves the following maximization problem:

$$
\begin{aligned}
&\max_{\tilde{\pi}^{D'}, \tilde{\pi}^{U'}} \left(\tilde{V}^{D'} - F_x - V^D \right) \left(\tilde{V}^{U'} - K - V^U \right), \\
&\text{s.t.} \quad \tilde{\pi}^{D'} + \tilde{\pi}^{U'} = \tilde{\pi}.
\end{aligned}
$$

Substituting (50) and (10) into the above problem and evaluating them at $\tilde{\pi}^{D'} = \tilde{\pi}^D$ and $\tilde{\pi}^{U'} = \tilde{\pi}^U = \tilde{\pi} - \tilde{\pi}^D$ gives us the following profit sharing rule:

$$
\begin{aligned}
\tilde{\pi}^D &= \frac{1}{\mu^D + \mu^U + 2\lambda} \left\{ \left(\mu^D + \lambda \right) \tilde{\pi} + \left(\mu^U + \lambda \right) (\pi - g) - 2\lambda \left[\left(\mu^D + \lambda \right) K - \left(\mu^U + \lambda \right) F_x \right] \right\}, \\
\tilde{\pi}^U &= \frac{1}{\mu^D + \mu^U + 2\lambda} \left\{ \left(\mu^U + \lambda \right) \tilde{\pi} - \left(\mu^U + \lambda \right) (\pi - g) + 2\lambda \left[\left(\mu^D + \lambda \right) K - \left(\mu^U + \lambda \right) F_x \right] \right\}.
\end{aligned}
$$

Then, we rewrite them as

$$
\begin{aligned}
\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} &= \beta \left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K - 2F_x + \frac{g}{\lambda} \right), \\
\frac{\tilde{\pi}^U}{\lambda} - 2K &= (1 - \beta) \left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K - 2F_x + \frac{g}{\lambda} \right),
\end{aligned}
\tag{51}
$$

where β is the same as in the closed economy. Finally, substituting firm dynamics (8) into (50) and (10), the free entry conditions $(V^D = F^D, V^U = F^U)$ can be written as

$$
\begin{aligned}
\frac{\pi}{\lambda} - \frac{g}{\lambda} + \frac{n}{N} \left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} \right) &= F^D, \\
\frac{n}{M} \left(\frac{\tilde{\pi}^U}{\lambda} - 2K \right) &= F^U.
\end{aligned}
$$

Substituting the profit sharing rule (51) into the above equations and rearranging them gives us the free entry condition in the open-economy equilibrium, given by (27).

Next we show that in each industry, the aggregate wage payments equal total expenditure $E \equiv \delta L$. The aggregate labor used for investment and search in the downstream sector is $L_e^D = \lambda N F^D + \lambda n F_x + (N - n)g$. Using (8), (50), and (51), we find that (48) still holds in this setup. Furthermore, (49) also holds since the free entry condition is the same for upstream firms. These implies that

$$L_e^D + L_e^U = \frac{E}{\sigma}.$$

On the other hand, labor demands for production can be written as

$$\begin{aligned}
L_p^D + L_p^U &= nc\tilde{x}_d + n\tau_x c\tilde{x}_x + (N-n)cx \\
&= n(1+\tau_x^{1-\sigma})c\tilde{x}_d + (N-n)cx && \text{(using } \tilde{x}_x = \tau_x^{-\sigma}\tilde{x}_d\text{)} \\
&= \gamma\left[\left(1+\tau_x^{1-\sigma}\right)n + \frac{N-n}{\gamma}\right]cx && \text{(using (5))} \\
&= \left(\frac{\sigma-1}{\sigma}\right)E. && \text{(using (1) and (24))}
\end{aligned}$$

Given that the wage rate is normalized to one, we have thus established that total wage payments equal total expenditure in each industry (and hence in each country as a whole).

B.2 Proof of Proposition 2

We first show that $\partial\hat{z}/\partial\tau_x < 0$, $\partial\hat{z}/\partial f_x < 0$, $\partial\hat{\pi}/\partial\tau_x > 0$ and $\partial\hat{\pi}/\partial f_x > 0$. (The comparative statics results of the closed-economy equilibrium continue to obtain in the open-economy equilibrium.) It follows from (28) and (29) that

$$[\gamma(1+\tau_x^{1-\sigma})-1]\left[(f^D+g)\phi^U(\hat{z}) - f^U\phi^D(\hat{z})\right] = f^U + (k+f_x-g)\phi^U(\hat{z}).$$

where $(f^D+g)\phi^U(\hat{z}) - f^U\phi^D(\hat{z}) > 0$ as the right-hand side is positive. Differentiating the above equality with respect to γ and f_x yields

$$\begin{aligned}
\frac{\partial\hat{z}}{\partial\tau_x} &= \frac{\gamma(\sigma-1)\tau_x^{-\sigma}\left[(f^D+g)\phi^U(\hat{z}) - f^U\phi^D(\hat{z})\right]}{\Omega}, \\
\frac{\partial\hat{z}}{\partial f_x} &= \frac{\phi^U(\hat{z})}{\Omega},
\end{aligned} \tag{52}$$

where

$$\Omega \equiv \left\{[\gamma(1+\tau_x^{1-\sigma})-1](f^D+g) - k - f_x + g\right\}\phi^{U'}(\hat{z}) - [\gamma(1+\tau_x^{1-\sigma})-1]f^U\phi^{D'}(\hat{z})$$

is negative since $\phi^{D'}(z) > 0$, $\phi^{U'}(z) < 0$, and $[\gamma(1+\tau_x^{1-\sigma})-1](f^D+g) - k - f_x + g > 0$ (i.e., the economic rents are positive). The result directly follows from $\sigma > 1$ and $(f^D+g)\phi^U(\hat{z}) - f^U\phi^D(\hat{z}) > 0$. The discussion that precedes Proposition 2 in the main text shows that the components market becomes thinner if τ_x falls. As the main text has also shown that a fall in τ_x enhances social welfare, what remains to be shown is that these impacts are larger, the greater is γ. It immediately follows from (52) that the impact on $\hat{z}$ is greater, the larger is γ. This in turn implies from (20) that the impacts on $\hat{\pi}$ and hence on welfare are also greater.

B.3 Proof of the free entry condition in components trade

We show detailed derivations of equation (37). The no-arbitrage conditions in this case are

$$\begin{aligned}
0 &= \tilde{\pi}^D - \lambda\left(\tilde{V}^D - V^D\right) - \lambda\tilde{V}^D + \dot{\tilde{V}}^D, \\
0 &= \tilde{\pi}^{D^*} - \lambda\left(\tilde{V}^{D^*} - V^D\right) - \lambda\tilde{V}^{D^*} + \dot{\tilde{V}}^{D^*}, \\
0 &= \pi - g - g^* + \mu^D\left(\tilde{V}^D - F_x - V^D\right) + \mu^D\left(\tilde{V}^{D^*} - F_x - V^D\right) - \lambda V^D + \dot{V}^D, \\
0 &= \tilde{\pi}^U - \lambda\left(\tilde{V}^U - V^U\right) - \lambda\tilde{V}^U + \dot{\tilde{V}}^U, \\
0 &= \tilde{\pi}^{U^*} - \lambda\left(\tilde{V}^{U^*} - V^U\right) - \lambda\tilde{V}^{U^*} + \dot{\tilde{V}}^{U^*}, \\
0 &= \mu^U\left(\tilde{V}^U - K - V^U\right) + \mu^U\left(\tilde{V}^{U^*} - K - V^U\right) - \lambda V^U + \dot{V}^U,
\end{aligned}$$

where $\tilde{\pi}^{D^*} + \tilde{\pi}^{U^*} = \tilde{\pi}^*$. Setting $\dot{\tilde{V}}^D = \dot{\tilde{V}}^{D^*} = \dot{V}^D = 0$ in the first three equations gives us the value functions for each type of downstream firms:

$$
\begin{aligned}
\tilde{V}^D - F_x &= \frac{\pi}{\lambda} - \frac{g}{\lambda} - \frac{g^*}{\lambda} + \left(\frac{\lambda + \frac{3}{2}\mu^D}{2(\mu^D + \lambda)}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right) + \left(\frac{\frac{1}{2}\mu^D}{2(\mu^D + \lambda)}\right)\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right), \\
\tilde{V}^{D^*} - F_x &= \frac{\pi}{\lambda} - \frac{g}{\lambda} - \frac{g^*}{\lambda} + \left(\frac{\frac{1}{2}\mu^D}{2(\mu^D + \lambda)}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right) + \left(\frac{\lambda + \frac{3}{2}\mu^D}{2(\mu^D + \lambda)}\right)\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right), \\
V^D &= \frac{\pi}{\lambda} - \frac{g}{\lambda} - \frac{g^*}{\lambda} + \left(\frac{\mu^D}{2(\mu^D + \lambda)}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right) + \left(\frac{\mu^D}{2(\mu^D + \lambda)}\right)\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right).
\end{aligned}
\tag{53}
$$

Similarly, the last three equations can be solved for $\tilde{V}^U, \tilde{V}^{U^*}$ and V^U:

$$
\begin{aligned}
\tilde{V}^U - K &= \left(\frac{\lambda + \frac{3}{2}\mu^U}{2(\mu^U + \lambda)}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \left(\frac{\frac{1}{2}\mu^U}{2(\mu^U + \lambda)}\right)\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right), \\
\tilde{V}^{U^*} - K &= \left(\frac{\frac{1}{2}\mu^U}{2(\mu^U + \lambda)}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \left(\frac{\lambda + \frac{3}{2}\mu^U}{2(\mu^U + \lambda)}\right)\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right), \\
V^U &= \left(\frac{\mu^U}{2(\mu^U + \lambda)}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \left(\frac{\mu^U}{2(\mu^U + \lambda)}\right)\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right).
\end{aligned}
\tag{54}
$$

Both domestically and internationally matched pairs determine the profit sharing by the symmetric Nash bargaining to maximize ex-post gains from the relationship shown in (53) and (54). While the former pairs determine $(\tilde{\pi}^D, \tilde{\pi}^U)$ as before, the latter pairs determine $(\tilde{\pi}^{D^*}, \tilde{\pi}^{U^*}) = (\tilde{\pi}^{D^*\prime}, \tilde{\pi}^{U^*\prime})$ by solving the following maximization problem:

$$
\begin{aligned}
&\max_{\tilde{\pi}^{D^*\prime}, \tilde{\pi}^{U^*\prime}} \left(\tilde{V}^{D^*\prime} - F_x - V^D\right)\left(\tilde{V}^{U^*\prime} - K - V^U\right), \\
&\text{s.t.} \quad \tilde{\pi}^{D^*\prime} + \tilde{\pi}^{U^*\prime} = \tilde{\pi}^*.
\end{aligned}
$$

While ex-post gains for domestically-matched pairs are always positive ($\tilde{V}^D - F_x - V^D > 0$, $\tilde{V}^U - K - V^U > 0$), we need to assume that those for internationally-matched pairs are also positive, i.e., $\tilde{V}^{D^*} - F_x - V^D > 0$ and $\tilde{V}^{U^*} - K - V^U > 0$. Using (53) and (54), we write the last two inequalities as

$$
\begin{aligned}
\tilde{\pi}^{D^*} - \pi - 2\lambda F_x + g + g^* - \frac{\mu^D}{2}\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\tilde{\pi}^{D^*}}{\lambda}\right) &> 0, \\
\tilde{\pi}^{U^*} - 2\lambda K - \frac{\mu^U}{2}\left(\frac{\tilde{\pi}^U}{\lambda} - \frac{\tilde{\pi}^{U^*}}{\lambda}\right) &> 0.
\end{aligned}
\tag{55}
$$

For downstream firms to stop searching for domestic partners once they are matched with foreign firms, we also assume that

$$\mu^D(\tilde{V}^D - \tilde{V}^{D^*}) < g,$$

which can be written as

$$\frac{\mu^D}{2}\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\tilde{\pi}^{D^*}}{\lambda}\right) < g. \tag{56}$$

Under condition (56) and the assumptions that $\tilde{\pi}^{D^*} - \pi - 2\lambda F_x + g + g^* > 0$ and $\tilde{\pi}^{U^*} - 2\lambda K > 0$, we can readily verify that condition (55) is satisfied, so the Nash bargaining problem is well-defined for internationally-matched pairs.

It follows from the solutions for domestically-matched and internationally-matched pairs that

$$
\begin{aligned}
&\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda} + \frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2F_x + \frac{g}{\lambda} + \frac{g^*}{\lambda} \\
&\quad = \beta\left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2F_x - 2K + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right) + \beta\left(\frac{\tilde{\pi}^*}{\lambda} - \frac{\pi}{\lambda} - 2F_x - 2K + \frac{g}{\lambda} + \frac{g^*}{\lambda}\right)
\end{aligned}
\tag{57}
$$

where

$$\beta \equiv \frac{\lambda + \frac{3}{2}\mu^D}{2\lambda + \frac{3}{2}\mu^D + \frac{3}{2}\mu^U}.$$

Finally, the free entry conditions in this case are again the same as before ($V^D = F^D,\ V^U = F^U$). Substituting firm dynamics (36) into (53) and (54), these conditions can be written as

$$\begin{aligned}
\frac{\pi}{\lambda} - \frac{g}{\lambda} - \frac{g^*}{\lambda} + \frac{n}{N}\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2F_x\right) + \frac{n^*}{N}\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2F_x\right) &= F^D, \\
\frac{n}{M}\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \frac{n^*}{M}\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right) &= F^U.
\end{aligned} \tag{58}$$

By substituting the profit sharing rule (57) into (58) and rearranging, we have (37), the free entry condition in the case with offshoring. Moreover, it is readily shown that the total wage payments equal the total expenditure also in this case.

B.4 Proof of Proposition 3

It follows immediately from (38) with (39) that $\partial\hat{\pi}/\partial\tau_x > 0$ and $\partial\hat{\pi}/\partial\tau_m > 0$. Since $\hat{\pi}$ and $\hat{z}$ are negatively related as described by (40), we find that $\partial\hat{z}/\partial\tau_x < 0$ and $\partial\hat{z}/\partial\tau_m < 0$. Since $\partial\hat{\pi}/\partial\tau_x > 0$ and $\partial\hat{\pi}/\partial\tau_m > 0$, the welfare expression (19) suggests that a reduction in τ_x or τ_m enhances social welfare.

Next, we show that $\partial(n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^*)/\partial\tau_x < 0$, $\partial(n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^*)/\partial\tau_m < 0$, $\partial(n^*\tau_m c\tilde{x}^*)/\partial\tau_x < 0$ and $\partial(n^*\tau_m c\tilde{x}^*)/\partial\tau_m < 0$. Differentiating the total exports (42) and (43) with respect to τ_x and τ_m yields

$$\begin{aligned}
\frac{\partial(n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^*)}{\partial\tau_x} &= \frac{-(1+\tau_m^{1-\sigma})\gamma[(\sigma-1)\tau_x^{-\sigma}\{(1+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda\}s(\hat{z}) - s'(\hat{z})\frac{\partial\hat{z}}{\partial\tau_x}2\lambda]}{[(1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L, \\
\frac{\partial(n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^*)}{\partial\tau_m} &= \frac{-2\lambda\gamma[(\sigma-1)\tau_m^{-\sigma}s(\hat{z}) - (1+\tau_m^{1-\sigma})s'(\hat{z})\frac{\partial\hat{z}}{\partial\tau_m}]}{[(1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L, \\
\frac{\partial(n^*\tau_m c\tilde{x}^*)}{\partial\tau_x} &= \left(\frac{\sigma-1}{\sigma}\right)\frac{-2\lambda\gamma[(\sigma-1)\tau_x^{-\sigma}s(\hat{z}) - (1+\tau_x^{1-\sigma})s'(\hat{z})\frac{\partial\hat{z}}{\tau_x}]}{[(1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L, \\
\frac{\partial(n^*\tau_m c\tilde{x}^*)}{\partial\tau_m} &= \left(\frac{\sigma-1}{\sigma}\right)\frac{-2\lambda\gamma(1+\tau_x^{1-\sigma})[(\sigma-1)\tau_m^{-\sigma}s(\hat{z}) - s'(\hat{z})\frac{\partial\hat{z}}{\tau_m}]}{[(1+\tau_x^{1-\sigma})(1+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L.
\end{aligned}$$

The result follows from applying $s'(\hat{z}) > 0$, $\partial\hat{z}/\partial\tau_x < 0$ and $\partial\hat{z}/\partial\tau_m < 0$ to the above equations.

B.5 Proof of Proposition 4

Here, we need only to show that $\partial(n\tilde{p}_x\tilde{x}_x)/\partial\tau_x < 0$ and $\partial(n^*\tau_m c\tilde{x}^*)/\partial\tau_m < 0$, while the signs of $\partial(n\tilde{p}_x\tilde{x}_x)/\partial\tau_m$ and $\partial(n^*\tau_m c\tilde{x}^*)/\partial\tau_x$ are ambiguous.

Differentiating the total exports, represented by (46) and (47), with respect to τ_x and τ_m yields

$$\begin{aligned}
\frac{\partial(n\tilde{p}_x\tilde{x}_x)}{\partial\tau_x} &= \frac{-\gamma[(\sigma-1)\tau_x^{-\sigma}\{(1+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda\}s(\hat{z}) - \tau_x^{1-\sigma}s'(\hat{z})\frac{\partial\hat{z}}{\partial\tau_x}2\lambda]}{[(1+\tau_x^{1-\sigma}+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L, \\
\frac{\partial(n\tilde{p}_x\tilde{x}_x)}{\partial\tau_m} &= \frac{\gamma\tau_x^{1-\sigma}[(\sigma-1)\tau_m^{-\sigma}\gamma s(\hat{z})^2 + s'(\hat{z})\frac{\partial\hat{z}}{\partial\tau_m}2\lambda]}{[(1+\tau_x^{1-\sigma}+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L, \\
\frac{\partial(n^*\tau_m c\tilde{x}^*)}{\partial\tau_x} &= \left(\frac{\sigma-1}{\sigma}\right)\frac{\gamma\tau_m^{1-\sigma}[(\sigma-1)\tau_x^{-\sigma}\gamma s(\hat{z})^2 + s'(\hat{z})\frac{\partial\hat{z}}{\tau_x}2\lambda]}{[(1+\tau_x^{1-\sigma}+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L, \\
\frac{\partial(n^*\tau_m c\tilde{x}^*)}{\partial\tau_m} &= \left(\frac{\sigma-1}{\sigma}\right)\frac{-\gamma[(\sigma-1)\tau_x^{-\sigma}\{(1+\tau_x^{1-\sigma})\gamma s(\hat{z}) + 2\lambda\}s(\hat{z}) - \tau_m^{1-\sigma}s'(\hat{z})\frac{\partial\hat{z}}{\tau_m}2\lambda]}{[(1+\tau_x^{1-\sigma}+\tau_m^{1-\sigma})\gamma s(\hat{z}) + 2\lambda]^2}\delta L.
\end{aligned}$$

Inspection of the free entry conditions (44) with (45) reveals that $\partial\hat{\pi}/\partial\tau_x > 0$ and $\partial\hat{\pi}/\partial\tau_m > 0$. Since $\hat{\pi}$ and $\hat{z}$ are negatively related as described by (40), we find that $\partial\hat{z}/\partial\tau_x < 0$ and $\partial\hat{z}/\partial\tau_m < 0$. Together with the fact that $s'(\hat{z}) > 0$, the desired results directly follow from noting the above equations.

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