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Tariffs, Vertical Oligopoly, and Market Structure

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Abstract

We study the impact of market thickness on the optimal tariff in vertical specialization. We show that, in the exogenous market structure where the extensive margin is fixed and only the intensive margin responds to trade policy, when the Home optimal tariff is higher, the thicker is the Home final-good market (relative to the Foreign intermediate-good market). In the endogenous market structure where both extensive and intensive margins respond to trade policy, this relationship is overturned and as the Home optimal tariff is higher, the thinner is the Home final-good market. We also show that our analysis has an advantage of separately deriving the impact of tariffs on the extensive and intensive margins of homogeneous goods.

Keywords: Tariffs, Vertical oligopoly, Free entry, Extensive margin, Intensive margin
JEL classification: F12, F13

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1 Introduction

Recent years have witnessed faster growth in intermediate-good trade than final-good trade. It is often argued that rapid growth in intermediate goods has triggered by vertical specialization that allows countries to provide particular stages of good's production sequence by fragmenting production processes spread across the globe.[1] It is also argued that vertical specialization takes place largely by foreign outsourcing relative to foreign direct investment.[2] When domestic final-good producers outsource some intermediate goods to foreign intermediate-good suppliers under contractual agreements, these vertical structures of this kind are often represented by bilateral oligopoly models with bargaining over the terms of contracts that specify the price and quantity of intermediate goods that are negotiated. In practice, however, a large fraction of intermediate goods are internationally traded through spot markets among anonymous final-good producers and intermediate-good suppliers rather than vertical negotiations within matched pairs.[3]

There is established evidence that documents the difference between contractual agreements and spot markets in the international trade literature. Rauch (1999) divides manufactured final goods that are internationally traded into three groups (sold on organized exchanges, reference priced, or neither) and finds that differentiated final goods tend to be internationally traded by proximity and preexisting "ties" (including contractual agreements), whereas homogeneous final goods tend to be internationally traded through spot markets. Using Rauch's (1999) classification to intermediate goods, Nunn (2007) also finds that differentiated intermediate goods tend to be exchanged by non-market mechanisms (including contractual agreements), whereas homogeneous intermediate goods tend to be exchanged through spot markets. In addition to this stylized fact, there is another kind of evidence that vertical specialization tend to produce homogeneous intermediate goods especially in China. For example, applying Schott's (2008) export similarity index to vertical specialization, Dean et al. (2011) find that the similarity between Chinese and OECD intermediate-good exports is generally very low. Dai et al. (2016) also find that Chinese intermediate-good exports in vertical specialization (i.e., processing exports) are less skill and R&D intensive. This evidence suggests that a large fraction of intermediate goods produced in vertical specialization should be homogeneous in nature and, together with the first evidence, they should be internationally traded through spot markets.

Building on these pieces of evidence that contractual agreements are not the only means of procuring intermediate goods in vertical specialization, we develop a bilateral oligopoly model to capture an impact that market-based interactions between vertically related industries have on trade policy. Following the evidence that vertical specialization allows countries to produce homogeneous goods and to exchange these goods through markets, we assume that one country

[1]See Yi (2003) for the quantitative importance of vertical specialization in the growth of world trade. According to his calibrated model, vertical specialization can explain over 50% of non-linear trade growth since the mid-1980s.

[2]Kimura and Ando (2005) find that the share of arm's length transactions increased from 52% to 65%, but the share of intra-firm transactions decreased from 44% to 33% for Japanese multinationals in East Asia during 1995-1998.

[3]Noting that the term "outsourcing" refers to the procurement of intermediate goods that occurs through *both* contractual arrangements and spot markets, Spencer (2005) emphasizes that this distinction is important.

(Home) specializes in producing homogeneous final goods, whereas another country (Foreign) specializes in producing homogeneous intermediate goods. Further, the price and quantity of final goods (resp. intermediate goods) are determined at the market-clearing levels in the Home (resp. Foreign) market, where the number of entrants into each market is either exogenous or endogenous. A Home government imposes a tariff on intermediate goods imported from Foreign to maximize its welfare, even though Home final-good producers require Foreign intermediate goods. In this setting, we derive the relationship between the optimal tariff and market thickness (i.e., the number of firms in the Home and Foreign markets) in both exogenous and endogenous market structures. In so doing, we explore the impact of trade policy on the extensive margin (number of firms) and the intensive margin (average output per firm) of homogeneous final goods and homogeneous intermediate goods that are produced in vertical specialization.

In the exogenous market structure where the extensive margin is fixed and only the intensive margin responds to trade policy, we find that the Home optimal tariff is higher, the thicker is the Home market (relative to the Foreign market). An increase in the Home tariff rate leads to a less-than-proportionate increase in the price of intermediate goods for all demand functions that are strictly logconcave. Since Home imports intermediate goods from Foreign, this works as a terms-of-trade improvement. Counteracting this welfare gain is a welfare loss due to the tariff-induced reduction in the intensive margin. While these two opposing forces impact a characterization of the optimal tariff, the strength of the two forces is influenced by the relative number of firms in each market. Suppose that the number of Home firms is arbitrary large that the Home market is perfectly competitive. If the Home government faces perfect competition in its domestic market, the rationale of imposing tariff is only the terms-of-trade gain at the expense of Foreign, which induces the Home government to impose a positive tariff. Suppose another extreme case where the number of Foreign firms is arbitrary large that the Foreign market is perfectly competitive. Then, the situation is like a single-stage Cournot oligopoly in Home, in which case a positive subsidy (i.e., a negative tariff) increases Home welfare by narrowing the wedge between the price and marginal cost. As the number of Home firms is greater and the Home market is thicker, the reduction in the intensive margin is smaller relative to the terms-of-trade improvement, and the higher tariff is more likely to be optimal for Home. This yields a policy implication that, in the markets with fixed entry, the Home government should set higher tariff on Foreign intermediate goods if the Home final-good market is more competitive. In other words, the degree of domestic competition should be *inversely* related to freeness of input trade in the short run.

In the endogenous market structure where both extensive and intensive margins respond to trade policy, we find that this relationship is overturned and the Home optimal tariff is higher, the thinner is the Home final-good market. Though the optimal tariff is dictated by the reduction in the intensive margin and terms-of-trade gain (as in the exogenous market structure), these two forces do not always occur for all logconcave demand functions in the endogenous market structure, since in addition to directly increasing prices, tariff also indirectly increases prices by reducing the number of firms (extensive margin). By virtue of this additional adjustment in the extensive margin, the sign of the optimal tariff depends on the elasticity of demand and the optimal tariff is positive (negative) for strictly concave (convex) demand. In this setup with free entry where the number of firms is endogenous, the key exogenous parameter that shapes the market thickness is the fixed entry cost. Suppose that Home firms have the low entry cost and find it easy to start a business in Home, which makes the Home market thicker than the Foreign market. Then, in vertical specialization where Home output and Foreign input are complements, this encourages not only entry of Home firms but also entry of Foreign firms into the respective market, and since each market is more competitive, it is optimal for Home to adopt freer trade for all logconcave demand functions. Thus, in the markets with fluid entry, the Home government should set lower tariff if the Home final-good market is more competitive; the degree of domestic competition should be *directly* related to freeness of input trade in the long run. This suggests that only when the market structure is endogenous, do entry liberalization in the Home market and trade liberalization in the Foreign market give rise to complementarity on welfare.

As indicated earlier, our analysis of the endogenous market structure has an advantage of separately deriving the impact of tariff on the extensive and intensive margins of homogeneous goods. Although it is not surprising that tariff reduces the aggregate volume of final goods and intermediate goods, this reduction is largely accounted for by exit of firms (extensive margin). In contrast, net change in average output per firm (intensive margin) is generally ambiguous. This distinctive impact on the two margins is well-known in the literature of monopolistic competition and heterogeneous firms (see, e.g., Arkolakis et al., 2008). Our contribution is to demonstrate that this impact holds even for oligopolistic competition in which to produce homogeneous goods, a key feature in processing trade. Note importantly that, in the presence of vertical linkages, Home tariff leads to not only the reduction in the extensive margin of Foreign firms but also the reduction in the extensive margin of Home firms. We refer to this as the "firm-colocation" effect in vertical specialization, which is used as an antonym to the "firm-delocation" effect that arises in horizontal specialization (see, e.g., Bagwell and Staiger, 2012a, b). These predictions about the two margins are empirically testable, and we plan to investigate them by using the dataset of processing trade between Japan (Home) and China (Foreign).

A handful of papers have considered trade policy in the context of vertical oligopolies. In an international vertical oligopoly setting, Ishikawa and Lee (1997), Ishikawa and Spencer (1999), and Chen, Ishikawa and Yu (2004) analyze the strategic interaction between Foreign firms and Home firms, but these papers consider the exogenous market structure only. The current paper is most closely related to a companion paper (Ara and Ghosh, 2016). While that paper also studies the optimal tariff in vertical oligopolies in both exogenous and endogenous market structures, we assume that Home and Foreign firms make use of contractual agreements (rather than spot markets), and focus on the impact of bargaining power on the optimal tariff. One of drawbacks is that bargaining power is not directly observable and measurable, and thus it is not possible to empirically test our theoretical prediction. In contrast, since the number of firms is available in data, this paper enables us to potentially address the link between our theory and evidence.

2 Model

Consider a setting with two countries, Home and Foreign, specializing respectively in a final good and an intermediate input. Foreign has n identical intermediate-input suppliers, $F_1, F_2, ..., F_n$. Home has m identical final-good producers, $H_1, H_2, ..., H_m$. In the intermediate-input market in Foreign, a homogeneous intermediate input is produced with constant marginal cost c and shipped to Home with a specific tariff rate t. In the final-good market in Home, the imported intermediate input is transformed into a homogeneous final good with constant marginal cost c_d, which is normalized to zero for simplicity. Production of one unit of the final good requires one unit of the intermediate input. In addition to these production costs, upon entry, Home and Foreign firms incur fixed entry costs K_H and K_F respectively.

There is a unit mass of identical consumers with a quasi-linear utility function, $U(Q) + y$, where Q is a imperfectly competitive final good produced by using an intermediate input and y is a perfectly competitive numeraire good.[4] Assuming income to be high enough, maximizing $U(Q)+y$ subject to the budget constraint gives demand for the homogeneous product: $Q = Q(P)$. Assume the preferences are such that (i) $Q(P)$ is twice continuously differentiable and $Q'(P) < 0$ for all $P \in (0, \bar{P})$ where $\bar{P} \equiv \lim_{Q\to 0} P^{-1}(Q)$ and (ii) $Q(P) = 0$ for $P \geq \bar{P}$. These assumptions guarantee the existence of the Cournot equilibrium. We will often work with inverse demand functions. These assumptions regarding $Q(P)$ imply that the inverse demand function $P = P(Q)$ is twice continuously differentiable and $P'(Q) < 0$ for all $Q \geq 0$. For a sharper characterization, we assume that the final goods are consumed only in Home and that the Foreign government does not undertake trade policy, but none of the key results relies on these assumptions.

We consider the three-stage game. In the first-stage, the Home government sets a specific tariff rate, t, to maximize Home welfare which consists of consumer surplus, aggregate Home profits and tariff revenues. In the second stage, upon paying the fixed entry cost K_F, Foreign firms enter the market and engage in a Cournot competition in the intermediate-input market where profit-maximizing Foreign firms commit to choose the quantity of the intermediate input taking rival firms' quantity as given. In the third stage, upon paying the fixed entry cost K_H, Home firms enter the market and engage in a Cournot competition in the final-good market where profit-maximizing Home firms commit to choose the quantity of the final good taking rival firms' quantity and the input price (denoted by r) as given.[5] The input price r is determined at the market clearing level which equals the total amount of the intermediate input demanded by Home firms to the total amount of the intermediate input supplied by Foreign firms.

In order to illustrate important policy implications and empirically testable predictions, we conduct both the "short-run" analysis and the "long-run" analysis in a unified framework. In the short-run analysis in Section 3, we bypass entry considerations in both sectors of production

[4]The model is presented as partial equilibrium analysis, but, as is well-known, if this numeraire is freely tradable across Home and Foreign, the model can be interpreted in general-equilibrium terms.

[5]Following the successive vertical oligopoly literature (e.g, Ishikawa and Spencer, 1999; Ghosh and Morita, 2007), we assume that Home firms have no oligopsony power over the Foreign input market.

and assume that the numbers of Home and Foreign firms are fixed. Thus, in this section, tariff has no effect on the market structure. In Section 4, by contrast, we assume that after observing tariff rates, firms enter the market. Thus, in this section, the market structure is endogenous in that tariff changes the numbers of Home and Foreign firms as well as the quantities of them.

3 Exogenous Market Structure

This section considers an environment where the market structure is given. The entry costs K_H and K_F have been sunk and entry of Home and Foreign firms has taken place. Thus, we treat the numbers of these firm m, n as fixed and invariant to the tariff rate. In what follows, we derive the Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model described in the previous section. Formal proofs for all propositions and lemmas are relegated to the Appendix.

3.1 Cournot Competition

We first analyze the third-stage Cournot competition among Home firms in the final-good market. Each Home firm H_i chooses q_i to maximize $\left(P\left(q_i + \sum_{j\neq i}^{m} q_j\right) - r\right) q_i$ taking other Home firms' quantity and input price $r(< \bar{P})$ as given. If $q_i > 0$ for all $i = 1, 2, ..., m$, the first-order conditions are

$$P\left(q_i + \sum_{j\neq i}^{m} q_j\right) - r + P'\left(q_i + \sum_{j\neq i}^{m} q_j\right) q_i = 0.$$

The assumption below ensures that the solution to the maximization problem is unique.

Assumption 1 The demand function $Q(P)$ is logconcave.

The equivalent assumption in terms of inverse demand function is:

Assumption 1' $P'(Q) + QP''(Q) \leq 0$ for all $Q \geq 0$.

Assumption 1 holds if and only if marginal revenue is steeper than demand. In the trade literature, this assumption is first introduced in Brander and Spencer (1984a, b) who show that when the Home country imports from a Foreign monopolist with constant marginal cost, a small tariff improves welfare if and only if Assumption 1' holds.

In our framework, in addition to guaranteeing uniqueness, Assumption 1' ensures that the optimal tariff is non-negative at least for some $m > 1$ and $n > 1$. A convenient way to state Assumption 1' is in terms of elasticity of slope which is defined as $\epsilon(Q) \equiv \frac{QP''(Q)}{P'(Q)}$. Observe that $\epsilon(Q) \geq -1$ if and only if $P'(Q) + QP''(Q) \leq 0$. This condition is sufficient to prove the main results. For analytical simplicity, we focus on a class of demand functions which not only satisfy Assumption 1 but also satisfy the following:

Assumption 2 $\epsilon(Q) \equiv \frac{QP''(Q)}{P'(Q)} = \epsilon$ for all $Q \geq 0$.

Note, if ϵ is constant for all $Q(\geq 0)$, ϵ is greater than -1 and Assumption 1' or Assumption 1 is satisfied as well. Although this assumption is admittedly restrictive, any well-known inverse demand function (e.g., linear, constant elasticity, semi-log) satisfies Assumption 2.[6]

Now back to the Cournot competition in the final-good market. If $r \in (0, \bar{P})$, Assumption 1 or 1' guarantees that there exists a unique symmetric equilibrium $q_1 = q_2 = ... = q_m \equiv \hat{q}(> 0)$ where $\hat{q}$ is given by the equation below:

$$\hat{q} = -\frac{P(\hat{Q}) - r}{P'(\hat{Q})},$$

where $\hat{Q} = m\hat{q}$ is uniquely solves the following equation:

$$mP(\hat{Q}) + \hat{Q}P'(\hat{Q}) = mr. \tag{3.1}$$

Let $\pi_H(q, \hat{q}) \equiv [P(q + (m-1)\hat{q}) - r]q$ denote the post-entry profit of a Home firm that chooses q as its quantity given all other $m-1$ firms choose $\hat{q}$. Suppose $\pi_H(q, \hat{q})$ is pseudoconcave in q at $q = \hat{q}$. If $r \in (0, \bar{P})$, we have that $q_1 = q_2 = ... = q_m \equiv \hat{q}(> 0)$ constitutes the Stage 3 equilibrium. On the other hand, if $r \in [\bar{P}, \infty)$, each Home firm i chooses $q_i = 0$ in the Stage 3 equilibrium (see Ghosh and Morita (2007) for details).

Let X denote the aggregate input demanded at any given input price $r \in (0, \bar{P})$. Since one unit of final good requires one unit of intermediate input, we have that $X = Q$. Further since the input price is determined at the market clearing level and the aggregate amount of final good produced at any given $r \in (0, \bar{P})$ is Q, it follows from (3.1) that the inverse demand function for intermediate good X faced by Foreign firms is given by

$$r = P(Q) + \frac{QP'(Q)}{m} \equiv g(X).^{7} \tag{3.2}$$

From $P'(Q) + QP''(Q) \leq 0$ (by Assumption 1'), we have that

$$g'(X) = \frac{(m+1)P'(Q) + QP''(Q)}{m} = \frac{P'(Q)(m+1+\epsilon)}{m} < 0. \tag{3.3}$$

Moreover, from $\epsilon(Q) = \frac{QP''(Q)}{P'(Q)} = \epsilon$ for all $Q \geq 0$ (by Assumption 2), we also have that

$$\frac{Xg''(X)}{g'(X)} = \frac{\frac{QP''(Q)(m+1+\epsilon)}{m}}{\frac{P'(Q)(m+1+\epsilon)}{m}} = \epsilon.$$

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[6] It is crucial for welfare analysis of imperfect competition models to assume that the curvature of inverse demand is greater than the curvature of slope of inverse demand (see, e.g., Cowen, 2007): $\frac{P''(Q)Q}{P'(Q)} \geq \frac{P'''(Q)Q}{P''(Q)} \iff \frac{\epsilon'(Q)Q}{\epsilon(Q)} \leq 1$. Note that if $\epsilon = \epsilon(Q)$ for for all $Q \geq 0$, this inequality holds.

[7] From (3.2), it follows that r depends not only on X but also on m and thus it is more precise to define $r \equiv g(X, m)$. While we apply this short-hand definition $r \equiv g(X)$ for the short-run analysis (since we mainly focus on comparative statics with respect to n), this distinction becomes important in the long-run analysis.

Thus, the inverse demand function for intermediate good is downward-sloping and the elasticity of slope of input demand is the same as that of final-good demand. Note that Assumption 1 implies $\epsilon \geq -1$ which in turn implies for all $X \geq 0$ that

$$g'(X) + Xg''(X) \leq 0. \tag{3.4}$$

Now consider the second-stage Cournot competition among Foreign firms in the intermediate-good market. The inverse demand function faced by Foreign firms at Stage 2 is given by (3.2). Each Foreign firm F_i chooses x_i to maximize $\left[g\left(x_i + \sum_{j\neq i}^{n} x_j\right) - c - t\right] x_i$ taking other Foreign firms' quantity as given. If $x_i > 0$ for all $i = 1, 2, ..., n$, the first-order conditions are

$$g\left(x_i + \sum_{j\neq i}^{n} x_j\right) - c - t + g'\left(x_i + \sum_{j\neq i}^{n} x_j\right)x_i = 0.$$

Given that $\lim_{X\to 0} g(X) = \bar{P}$ from (3.2), condition (3.4) guarantees that there exists a unique symmetric equilibrium $x_1 = x_2 = ... = x_n \equiv \hat{x}(> 0)$ such that

$$\hat{x} = -\frac{g(\hat{X}) - c - t}{g'(\hat{X})},$$

where $\hat{X} = n\hat{x}$ uniquely solves the following equation:

$$ng(\hat{X}) + \hat{X}g'(\hat{X}) = n(c + t). \tag{3.5}$$

Let $\pi_F(x, \hat{x}) \equiv [g(x + (n-1)\hat{x}) - c - t]x$ denote the post-entry profit of an Foreign firm that chooses x as its quantity given all other $n - 1$ firms choose $\hat{x}$. Since $\pi_F(x, \hat{x})$ is strictly concave in x for all $x > 0$ (by virtue of (3.4)) and $r \in (0, \bar{P})$, we have that $x_1 = x_2 = ... = x_n \equiv \hat{x}(> 0)$ constitutes the Stage 2 equilibrium.

To summarize, in the Cournot competition with given m, n and t, we have an output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$ and a price vector $(\hat{P}, \hat{r})$ where

- $\hat{Q}$ solves (3.1);
- $\hat{X}$ solves (3.5);
- $\hat{Q} = \hat{X}$;
- $\hat{q} = \frac{\hat{Q}}{m}$, $\hat{x} = \frac{\hat{X}}{n}$;
- $\hat{P} \equiv P(\hat{Q})$, $\hat{r} \equiv g(\hat{X})$.

The following lemma records some comparative statics results with respect to n and t.[8]

[8] In this section, we omit comparative statics with respect to m just for simplicity. In the Appendix, we also show these comparative statics results as well.

Lemma 3.1

(i) *For a given tariff rate* t*, the aggregate output* $\hat{Q}$ *and aggregate input* $\hat{X}$ *are increasing in* n*; while the final-good price* $\hat{P}$ *and input price* $\hat{r}$ *are decreasing in* n*; i.e.,* $\partial \hat{Q}/\partial n = \partial \hat{X}/\partial n > 0$*,* $\partial \hat{P}/\partial n < 0$*,* $\partial \hat{r}/\partial n < 0$*.*

(ii) *For a given number of firms* m, n*, the aggregate output* $\hat{Q}$ *and aggregate input* $\hat{X}$ *are decreasing in* t*; while the final-good price* $\hat{P}$ *and input price* $\hat{r}$ *are increasing in* t*; i.e.,* $\partial \hat{Q}/\partial t = \partial \hat{X}/\partial t < 0$*,* $\partial \hat{P}/\partial t > 0$*,* $\partial \hat{r}/\partial t > 0$*.*

(iii) *Let* $r^* \equiv \hat{r} - t$ *denote the price received by a Foreign firm in equilibrium (for each unit of the intermediate input). Then,*

$$\frac{\partial r^*}{\partial t} \lesseqgtr 0 \quad \Leftrightarrow \quad \frac{\partial \hat{r}}{\partial t} \lesseqgtr 1 \quad \Leftrightarrow \quad 1 + \epsilon \gtreqless 0.$$

Not surprisingly, $\hat{r}$ increases as t increases. However, $\frac{\partial \hat{r}}{\partial t} - 1 \leq 0$ or equivalently $\frac{\partial r^*}{\partial t} \leq 0$ as long as the demand is logconcave. For all such demand functions, the pass-through of tariff to an intermediate-input price faced by Home producers is less than complete. Foreign firms absorb part of the tariff increase which acts like a terms-of-trade gain for Home. While r^* is an input price internal to the firms, a reduction in r^* hurts Foreign firms and benefits Home firms. Hence, we refer to a decrease in r^* as an improvement in terms-of-trade in the paper, though we are aware that r^* is more like firms' terms-of-trade (rather than countries' terms-of-trade). The terms-of-trade improvement creates a rationale for Home to set a positive tariff.

At the same time, $\hat{Q}, \hat{X}$ decrease as t increases. Though this is also not a surprising result, it is important to stress that in the exogenous market structure where the number of firms m, n (extensive margin) is fixed, an increase in t decreases aggregate outputs $\hat{Q}, \hat{X}$ only through the average output per firm $\hat{q}, \hat{x}$ (intensive margin). In particular, applying Lemma 3.1(ii) to the equilibrium relationships $\hat{Q} = m\hat{q}$ and $\hat{X} = n\hat{x}$ yields $\frac{\partial \hat{Q}}{\partial t} = m\frac{\partial \hat{q}}{\partial t}$ and $\frac{\partial \hat{X}}{\partial t} = n\frac{\partial \hat{x}}{\partial t}$, indicating that the average outputs $\hat{q}, \hat{x}$ necessarily decrease by tariff in the exogenous market structure:

$$\frac{\partial \hat{q}}{\partial t} < 0, \quad \frac{\partial \hat{x}}{\partial t} < 0.$$ [9]

Note, like the vertical oligopoly models developed by Ishikawa and Lee (1997) and Ishikawa and Spencer (1999), there is a "double marginalization" effect at work in our model: imperfect competition in the final-good market creates a wedge between the price of final good and its marginal cost $\hat{P} - \hat{r}$, while imperfect competition in the intermediate-input market creates a wedge between the price of intermediate input and its marginal cost $\hat{r} - c - t$. The inefficiency associated with this double marginalization effect also influences the optimal tariff set by Home.

[9] From Lemma 3.1(i), we also have that $\frac{\partial \hat{q}}{\partial n} > 0$ and $\frac{\partial \hat{x}}{\partial n} < 0$, which imply that an increase in n leads to the "business-creating" ("business-stealing") effect in the final-good (intermediate-input) market.

3.2 Tariffs

In the first stage, the Home government chooses a tariff rate t to maximize Home welfare (W_H), anticipating the output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$ and the price vector $(\hat{P}, \hat{r})$ in the Cournot competition. In the SPNE of the Stage 1 subgame, W_H is given by

$$W_H \equiv \underbrace{\left[\int_0^{\hat{Q}} P(y)\mathrm{d}y - P(\hat{Q})\hat{Q}\right]}_{\text{Consumer surplus } (CS)} + \underbrace{\left[P(\hat{Q}) - \hat{r}\right]\hat{Q}}_{\text{Home profits } (\hat{\Pi}_H)} + \underbrace{t\hat{X}}_{\text{Tariff revenue } (TR)},$$ [10]

where $\hat{\Pi}_H \equiv m\hat{\pi}_H = (\hat{P} - \hat{r})\hat{Q}$. Using $r^* = \hat{r} - t$ and simplifying the above expression gives

$$W_H \equiv \int_0^{\hat{Q}} P(y)\mathrm{d}y - r^*\hat{X}.$$

Differentiating W_H with respect to t and using $\frac{\partial \hat{Q}}{\partial t} = \frac{\partial \hat{X}}{\partial t}$, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - r^*)\frac{\partial \hat{Q}}{\partial t} - \frac{\partial r^*}{\partial t}\hat{X},$$

The first term captures the welfare loss due to the tariff-induced output reduction ($\frac{\partial \hat{Q}}{\partial t} < 0$). Home consumers value the final-good at $P(\hat{Q})$ while effectively it costs $r^*(< P(\hat{Q}))$ to produce (from Home's perspective). This price-cost margin $P(\hat{Q}) - r^*$ multiplied by the amount of output lost $\frac{\partial \hat{Q}}{\partial t}$ is the magnitude of welfare loss. The second term captures the welfare gains arising from the terms-of-trade improvement ($\frac{\partial r^*}{\partial t} < 0$). The optimal tariff rate strikes a balance between the two competing effects – welfare gains from the terms-of-trade improvement and welfare losses from the reduction in output. As we show below, the numbers of firms in each sector of production m, n play an important role in delineating the relative importance of the two effects, which in turn helps to determine the sign of the optimal tariff.

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ and solving for t gives the expression for the optimal tariff which is presented later in Proposition 3.1. Here we first focus on the sign of the optimal tariff. Using $\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1$ and $\frac{\partial \hat{Q}}{\partial t} = m\frac{\partial \hat{q}}{\partial t}$, we can express $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ as follows:

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - \hat{r})m\frac{\partial \hat{q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} + t\frac{\partial \hat{X}}{\partial t}. \tag{3.6}$$

Using (3.6) and noting that $\frac{\partial \hat{X}}{\partial t} < 0$, the optimal tariff is strictly positive (negative) if and only if

$$(P(\hat{Q}) - \hat{r})m\frac{\partial \hat{q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} > (<)0. \tag{3.7}$$

[10]Using the comparative statics results in Lemma 3.1, we can show that W_H is strictly concave in t so that the second-order condition is satisfied, i.e., $\frac{\partial^2 W_H}{\partial t^2} < 0$.

Equation (3.7) indicates that the numbers of firms play a key role in determining the sign of the optimal tariff. To see this, suppose that for a given m, the number of Foreign firms n is arbitrarily large so that the intermediate-input market becomes perfectly competitive. Then, the input price equals its marginal cost ($\hat{r} = c + t$) and, as a result, $\left(1 - \frac{\partial \hat{r}}{\partial t}\right) \hat{X} = -\frac{\partial r^*}{\partial t} \hat{X} = 0$, i.e., the terms-of-trade motive vanishes. Only the harmful effect of the tariff – output reduction – remains. An import subsidy raises Home welfare by increasing output and indeed the optimal tariff is negative. More generally, when n is arbitrarily large, Home captures all profits in Cournot competition of the final-good market. The situation is like a domestic, single-stage, Cournot oligopoly with m firms. A positive subsidy increases welfare in an oligopoly setup by narrowing the wedge between price and marginal cost, which explains why an import subsidy is optimal.

In the other extreme case, suppose that for a given n, the number of Home firms m is arbitrarily large so that the final-good market becomes perfectly competitive. Then, the final-good price equals its marginal cost ($P(\hat{Q}) = \hat{r}$) and, as a result, $(P(\hat{Q}) - \hat{r}) m \frac{\partial \hat{q}}{\partial t} = 0$, i.e., the welfare loss due to the tariff-induced output reduction vanishes. This is equivalent for Home to importing the final good from Foreign and its welfare is composed of consumer surplus and tariff revenues. In such a case, the reduction in profits is borne only by Foreign producers and the sign of the optimal tariff is determined exclusively by the terms-of-trade motive, or by the sign of $1 - \frac{\partial \hat{r}}{\partial t} = -\frac{\partial r^*}{\partial t}$. As the pass-through from tariff to domestic prices is incomplete for all logconcave demand functions, $1 - \frac{\partial \hat{r}}{\partial t} > 0$ holds, which implies that the optimal tariff is strictly positive.

The above intuition suggests that Home's optimal tariff is positive when the number of Foreign firms (n) is relatively smaller than the number of Home firms (m), and it is negative when n is relatively larger than m. This comes out more cleanly in terms of the price-cost margin ratio $\frac{\hat{P} - \hat{r}}{\hat{r} - c - t}$. Note that, in the presence of the double marginalization effect, when this ratio is small (large), the final-good market is more (less) competitive relative to the intermediate-input market. Using (3.2) and (3.5), this ratio can be rewritten as

$$\frac{\hat{P} - \hat{r}}{\hat{r} - c - t} = \frac{-\frac{\hat{Q} P'(\hat{Q})}{m}}{-\frac{\hat{X} g'(\hat{X})}{n}} = \frac{n}{m + 1 + \epsilon}, \tag{3.8}$$

which is increasing in n. Further, invoking the standard continuity argument, there is a range of values such that the optimal tariff is strictly decreasing in n. Analyzing (3.6) further gives a more precise characterization.

Proposition 3.1

Let $\hat{t}$ denote the optimal tariff. At $t = \hat{t}$ the following holds:

$$\hat{t} = -\hat{Q} P'(\hat{Q}) \left(\frac{(1 + \epsilon)(m + 1 + \epsilon) - n}{mn} \right), \tag{3.9}$$

where $\hat{Q}$ is the aggregate output evaluated at $t = \hat{t}$. Furthermore,

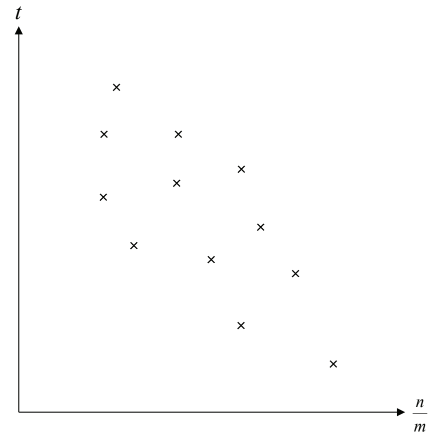

FIGURE 3.1 – Optimal tariff in short run

(i) *There exists* n^* *such that*

$$\hat{t} \gtreqless 0 \quad \Leftrightarrow \quad n \lesseqgtr n^* \equiv (1+\epsilon)(m+1+\epsilon).$$

(ii) $\hat{t}$ *is monotonically decreasing in* n.

As an illustrative example, consider the following class of inverse demand functions: $P(Q) = a - Q^b, b > 0$. Observe that $b = 1$ for linear demand and $b > (<)1$ for strictly concave (convex) demand. The elasticity of slope is constant and denoted by $\epsilon = b - 1$. Applying (3.8) yields

$$\hat{t} = \left[\frac{b(m+b) - n}{mn + b(b+1)(m+b)}\right](a-c)b.$$

Note the property of the optimal tariff in Proposition 3.1 holds for this specific demand function.

While we focus on how the number of Foreign firms n affect the optimal tariff t in Proposition 3.1, it is straightforward to show that the similar result holds for the number of Home firms m. The above intuition indeed tells us that the optimal tariff is increasing in m. This in turn helps consider how the relative number of firms $\frac{n}{m}$ – which is hereafter referred to as "relative market thickness" – affects the optimal tariff. Since the optimal tariff is increasing (decreasing) in m (n), our model predicts that the optimal tariff should be decreasing in $\frac{n}{m}$. Thus, if $\frac{n}{m}$ varies across industries, there would exist a negative relationship between $\frac{n}{m}$ and t in the short run.[11] Figure 3.1 illustrates our prediction when the optimal tariff is positive. Note that the optimal tariff is higher (i.e., higher t), the more competitive is the Home final-good market relative to the Foreign intermediate-input market (i.e., lower $\frac{n}{m}$). In this sense, the degree of freeness of trade policy is inversely related to the degree of competition in the domestic market in the short run.

[11] Empirical evidence on market-thickness effects has been documented in law and economics. For instance, Pirrong (1993) provide evidence that thicker markets tend to have lower transactions costs.

3.3 Profits

Home and Foreign aggregate profits respectively are given by

$$\hat{\Pi}_H \equiv m\hat{\pi}_H = [P(\hat{Q}) - \hat{r}]\hat{Q}, \quad \hat{\Pi}_F \equiv n\hat{\pi}_F = (r(\hat{X}) - c - \hat{t})\hat{X},$$

where $\hat{\pi}_H \equiv (P(\hat{Q}) - \hat{r})\hat{q}$ and $\hat{\pi}_F \equiv (\hat{r} - c - \hat{t})\hat{x}$ denote respectively the post-entry profit of each Home and Foreign firm in the SPNE of the Stage 1 subgame. Here we examine the impact of an increase in n on $\hat{\Pi}_H$ and $\hat{\Pi}_F$. Note that $\hat{\pi}_H$ and $\hat{\pi}_F$ are strictly increasing and decreasing in n respectively. Differentiating $\hat{\Pi}_H$ with respect to n yields

$$\frac{\mathrm{d}\hat{\Pi}_H}{\mathrm{d}n} = -(2+\epsilon)\hat{q}P'(\hat{Q})\frac{\mathrm{d}\hat{Q}}{\mathrm{d}n} > 0,$$

where $\frac{\mathrm{d}\hat{Q}}{\mathrm{d}n} = \frac{\partial \hat{Q}}{\partial n} + \frac{\partial \hat{Q}}{\partial t}\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} > 0$. An increase in n lowers the imported input price $\hat{r}$ in the final-good market and raises Home profits. On the other hand, differentiating $\hat{\Pi}_F$ with respect to n yields

$$\frac{\mathrm{d}\hat{\Pi}_F}{\mathrm{d}n} = \underbrace{(n-1)\hat{x}g'(\hat{X})\frac{\mathrm{d}\hat{X}}{\mathrm{d}n}}_{\text{competition effect}} \quad \underbrace{-\frac{\mathrm{d}\hat{t}}{\mathrm{d}n}\hat{X}}_{\text{tariff-reduction effect}},$$

where $\frac{\mathrm{d}\hat{X}}{\mathrm{d}n} = \frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{X}}{\partial t}\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} > 0$. An increase in n has two opposing effects on $\hat{\Pi}_F$. First, an increase in n amplifies competition in the intermediate-input market and lowers Foreign profits (*competition effect*). This effect exists even when the tariff is exogenously set. Second, an increase in n lowers the optimal tariff $\hat{t}$ and raises Foreign profits (*tariff-reduction effect*). Surprisingly, we find that for arbitrarily large m, the latter effect dominates the former effect and $\hat{\Pi}_F$ rises as n increases if the number of Foreign firms is small or the inverse demand is sufficiently concave.

Proposition 3.2

An increase in the number of Foreign firms might lead to higher Foreign profits. For arbitrarily large m, $\frac{\mathrm{d}\hat{\Pi}_F}{\mathrm{d}n}\big|_{m=\infty} > 0$ if

$$n < \frac{1+\sqrt{1+4(1+\epsilon)(2+\epsilon)}}{2}.$$

Proposition 3.2 suggests that an indirect increase in Foreign profits due to a lower tariff might outweigh a direct decrease in Foreign profits due to more competition in the Foreign market.[12] This situation is more likely when the number of Foreign firms n is small or the curvature of the inverse demand ϵ is big. To see this clearly, consider $P(Q) = a - Q^b$ for which $\epsilon = b - 1$. For linear demand ($b = 1$), $\lim_{m\to\infty} \frac{\mathrm{d}\hat{\Pi}_F}{\mathrm{d}n} \gtreqless 0$ if and only if $n \lesseqgtr 2$. This implies that for arbitrary large m, Foreign profits increase as the number of Foreign firms increases from one to two. As demand functions become more concave, this counter-intuitive outcome becomes more likely.

[12] This applies only for aggregate Foreign profits $\hat{\Pi}_F$, and per-firm Foreign profits $\hat{\pi}_F$ are always decreasing in n.

4 Endogenous Market Structure

In Section 3, we have assumed that the numbers of Home and Foreign firms are fixed. Since m and n are fixed, these numbers do not vary with tariff rates. Now we consider an environment where m and n are endogenously determined and tariffs are set prior to entry decisions. Here, in addition to the the direct effect on quantities and prices, tariffs also indirectly affect quantities and prices by influencing the market structure. In this setting, we address the following questions: How do entry considerations affect the optimal trade policy? Should the Home government liberalize entry in its final-good market in order to enhance the effectiveness of trade policy in input trade?

In the context of single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985) and more recently Etro (2011) and Bagwell and Staiger (2012a, b) all have shown that the endogenous market structure can drastically alter the optimal trade policy obtained from the exogenous market structure. Like these preceding papers, we also find that free entry can affect the optimal tariff. Recall in the short-run equilibrium that the optimal tariff is higher, the thicker is the final-good market in Home (relative to the intermediate-input market in Foreign). In the long-run equilibrium, by contrast, we show that this relationship is overturned and the optimal tariff is higher, the thinner is the Home final-good market. This finding suggests that a reduction of import tariff for Foreign input has its greater effect on welfare when accompanied by liberalization of entry in the Home final-good market in longer-term perspectives.

The timing of events is as outlined in the last paragraph of Section 2. In Stage 1, the Home government chooses a tariff rate t, following which entry occurs. In Stage 2, upon paying a fixed entry cost K_F, Foreign firms enter in the intermediate-input market and engage in Cournot competition taking other Foreign firms' quantity as given. In Stage 3, upon paying a fixed entry cost K_H, Home firms enter in the final-good market and engage in Cournot competition taking other Home firms' quantity and input price r as given. As before, we derive the Subgame Perfect Nash Equilibria (SPNE) in pure strategies of the model and focus on a class of demand functions which satisfy Assumptions 1 and 2.

4.1 Cournot Competition

Let us start with analyzing Stage 3. The Cournot competition in the final-good market works exactly the same way as before and the unique symmetric equilibrium in this stage is characterized by $q_1 = q_2 = ... = q_m \equiv \hat{q}$ such that

$$\hat{q} = -\frac{P(\hat{Q}) - r}{P'(\hat{Q})},$$

where $\hat{Q}$ satisfies the following for any given m:

$$mP(\hat{Q}) + \hat{Q}P'(\hat{Q}) = mr. \tag{4.1}$$

In addition to (4.1), the number of Home firms m is endogenously determined as there is free entry of firms. Recall from Section 2 that the entry cost of a Home firm is K_H. In the long run where entry is unrestricted, entry occurs until the post-entry profit of Home firms equals the entry cost. Let $\pi_H(m) \equiv (P(mq) - r)q$ denote the post-entry profit of Home firms in the SPNE of the Stage 3. Then the free entry condition in the final-good market is given by $\pi_H(\hat{m}) = K_H$:

$$[P(\hat{m}q) - r]q = K_H.$$

Aggregating this condition for all $\hat{m}$ Home firms, $\hat{m}$ satisfies the following for any given Q:

$$[P(Q) - r]Q = \hat{m}K_H. \tag{4.2}$$

We assume that $K_H \leq \pi_H(1) \equiv \bar{K}_H$, which guarantees that at least one Home firm enters in the equilibrium.

Assumption 3 $K_H \leq \bar{K}_H$.

Since π_H is continuous in m and strictly decreasing in m for all $m > 1$, Assumption 3 also ensures that $\hat{m}$ uniquely exists in the SPNE of the Stage 3 subgame.

Now let us turn to analyzing Stage 2. Note that, given the SPNE of Stage 3 subgame above, we have that the inverse demand function for intermediate good X faced by Foreign firms (3.2) holds in both the short-run and long-run equilibria. Thus,

$$r = P(Q) + \frac{QP'(Q)}{m} \equiv g(X, m),$$ [13]

which satisfies

$$\begin{aligned}
g_x(X, m) &\equiv \frac{\partial g(X, m)}{\partial X} = \frac{(m + 1 + \epsilon)P'(Q)}{m} < 0, \\
g_m(X, m) &\equiv \frac{\partial g(X, m)}{\partial m} = -\frac{QP'(Q)}{m^2} > 0, \\
g_{xm}(X, m) &\equiv \frac{\partial^2 g(X, m)}{\partial X \partial m} = -\frac{(1 + \epsilon)P'(Q)}{m^2} > 0.
\end{aligned}$$

In Stage 2, the Cournot competition in the intermediate-input market works almost the same way as before except for the expression of input price $r = g(X, m)$, and the unique symmetric equilibrium in this stage is characterized by $x_1 = x_2 = ... = x_n \equiv \hat{x}$ such that

$$\hat{x} = -\frac{g(\hat{X}, m) - c - t}{g_x(\hat{X}, m)},$$

[13] We define $r \equiv g(X)$ in the short run (see (3.2)) since the main focus is on comparative statics with respect to n. Here we explicitly define r as a function of m as well as X since m is endogenous in the long run.

where $\hat{X}$ satisfies the following for any given n:

$$ng(\hat{X}, m) + \hat{X} g_x(\hat{X}, m) = n(c + t). \tag{4.3}$$

In addition to (4.3), the number of Foreign firms n is also endogenously determined by equaling entry occurs until the post-entry profit of Foreign firms to the entry cost. Let $\pi_F(n) \equiv (g(nx) - c - t)x$ denote the post-entry profit of Foreign firms in the SPNE of the Stage 2. Then the free entry condition in the intermediate-input market is given by $\pi_F(\hat{n}) = K_F$:

$$[g(\hat{n}x, m) - c - t]x = K_F.$$

Aggregating this condition for all $\hat{n}$ Foreign firms, $\hat{n}$ satisfies the following for any given X:

$$[g(X, m) - c - t]X = \hat{n}K_F. \tag{4.4}$$

We assume that $K_F \leq \pi_F(1) \equiv \bar{K}_F$, which guarantees that at least one Foreign firm enters in the equilibrium. By applying the similar claim, this also ensures that $\hat{n}$ uniquely exists in the SPNE of the Stage 2 subgame.[14]

Assumption 4 $K_F \leq \bar{K}_F$.

To summarize, in the Cournot competition with given K_H, K_F and t, we have an output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$, a price vector $(\hat{P}, \hat{r})$ and a number vector $(\hat{m}, \hat{n})$ where

- $\hat{Q}$ solves (4.1);
- $\hat{X}$ solves (4.3);
- $\hat{m}$ solves (4.2);
- $\hat{n}$ solves (4.4);
- $\hat{Q} = \hat{X}$;
- $\hat{q} = \frac{\hat{Q}}{\hat{m}}, \hat{x} = \frac{\hat{X}}{\hat{n}}$;
- $\hat{r} \equiv g(\hat{X}, \hat{m}),\ \hat{P} \equiv P(\hat{Q})$.

Note that (4.1) and (4.3) are the *market clearing* (*MC*) conditions that hold even in the short run, whereas (4.2) and (4.4) are the *free entry* (*FE*) conditions that hold only in the long run. These two conditions jointly pin down the number of firms as well as the output of these firms in the long-run equilibrium.

[14] Following the previous section, we can show that $q_1 = q_2 = ... = q_m = \hat{q}(> 0)$ constitutes the Stage 3 equilibrium whereas $x_1 = x_2 = ... = x_n = \hat{x}(> 0)$ constitutes the Stage 2 equilibrium.

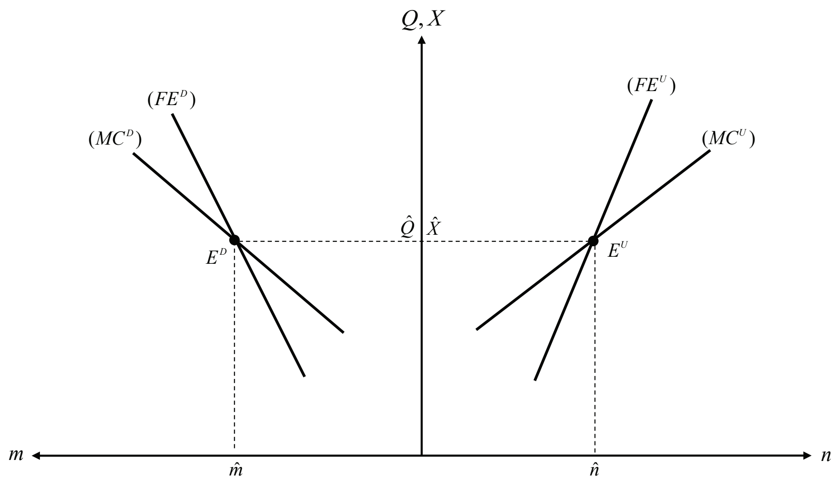

FIGURE 4.1 – Equilibrium outcomes

Figure 4.1 illustrates the equilibrium outcomes which can be solved from the ***MC*** and ***FE*** conditions. An equilibrium in the SPNE of the Stage 3 subgame is a vector $(\hat{Q}, \hat{m})$, which solves (4.1) and (4.2) in the final-good market in Home. The second quadrant of Figure 4.1 depicts the relationship between Q and m, where (4.1) and (4.2) are given by MC^D and FE^D respectively. The fact that FE^D is steeper than MC^D follows from noting that

$$\left.\frac{\mathrm{d}Q}{\mathrm{d}m}\right|_{FE^D} = \frac{2q}{2+\epsilon} > \left.\frac{\mathrm{d}Q}{\mathrm{d}m}\right|_{MC^D} = \frac{q}{m+1+\epsilon}.$$

Point E^D, the intersection of MC^D and FE^D, uniquely determines the equilibrium vector $(\hat{Q}, \hat{m})$. From (4.2) and $\hat{Q} = \hat{m}\hat{q}$, it follows that $\hat{m}$ and $\hat{q}$ are given by

$$\hat{m} = \sqrt{-\frac{P'(\hat{Q})\hat{Q}^2}{K_H}}, \qquad \hat{q} = \sqrt{-\frac{K_H}{P'(\hat{Q})}}.$$

Similarly, an equilibrium in the SPNE of the Stage 2 subgame is a vector $(\hat{X}, \hat{n})$, which solves (4.3) and (4.4) in the intermediate-input market in Foreign. The first quadrant of Figure 4.1 depicts the relationship between X and n, where (4.3) and (4.4) are given by MC^U and FE^U respectively. The fact that FE^U is steeper than MC^U follows from noting that

$$\left.\frac{\mathrm{d}X}{\mathrm{d}n}\right|_{FE^U} = \frac{2x}{2+\epsilon} > \left.\frac{\mathrm{d}X}{\mathrm{d}n}\right|_{MC^U} = \frac{x}{n+1+\epsilon}.$$

Point E^U, the intersection of MC^U and FE^U, uniquely determines the equilibrium vector $(\hat{X}, \hat{n})$, where

$$\hat{n} = \sqrt{-\frac{g_x(\hat{X}, \hat{m})\hat{X}^2}{K_F}}, \qquad \hat{x} = \sqrt{-\frac{K_F}{g_x(\hat{X}, \hat{m})}}.$$

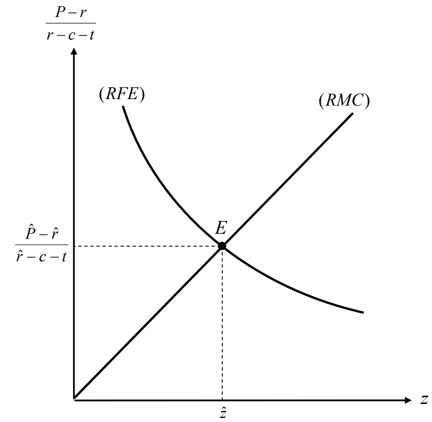

FIGURE 4.2 – Market thickness and price-cost margin

It is useful to work with the *relative market clearing* (*RMC*) condition and *relative free entry* (*RFE*) condition. Dividing (4.1) by (4.3) yields the *RMC* condition:

$$\frac{P(Q)-r}{g(X,m)-c-t} = \left(\frac{P'(Q)}{g_x(X,m)}\right) z, \tag{4.5}$$

which holds even in the short run.[15] Further, dividing (4.2) by (4.4) yields the *RFE* condition:

$$\frac{P(Q)-r}{g(X,m)-c-t} = \frac{k}{z}, \tag{4.6}$$

where $z \equiv \frac{n}{m}$ is the relative thickness of markets and $k \equiv \frac{K_H}{K_F}$ is the relative fixed cost of entry. This latter condition (4.6) holds only in the long run.

Figure 4.2 illustrates the equilibrium outcome which can be solved from the *RMC* and *RFE* conditions. Noting $r = g(X,m)$, the figure depicts the relationship between z and $\frac{P-r}{r-c-t}$, where (4.5) and (4.6) are given by *RMC* and *RFE* respectively. The fact that *RMC* is upward-sloping and *RFE* is downward-sloping directly follows from (4.5) and (4.6). Point E, the intersection of *RMC* and *RFE*, uniquely determines the equilibrium vector $(\hat{z}, \frac{\hat{P}-\hat{r}}{\hat{r}-c-t})$. Noting that the system of equations (4.5) and (4.6) can be solved for this vector, we get

$$\hat{z} = \sqrt{k\frac{g_x(\hat{X},\hat{m})}{P'(\hat{Q})}},$$
$$\frac{\hat{P}-\hat{r}}{\hat{r}-c-t} = \sqrt{k\frac{P'(\hat{Q})}{g_x(\hat{X},\hat{m})}}.$$

Given this equilibrium outcome, we next examine comparative statics with respect to t and K_H.

[15]Since $\frac{P'(Q)}{g_x(X,m)} = \frac{m}{m+1+\epsilon}$ from (3.3), the *RMC* condition (4.5) is also expressed as $\frac{P(Q)-r}{g(X,m)-c-t} = \frac{n}{m+1+\epsilon}$, which is the same as (3.8).

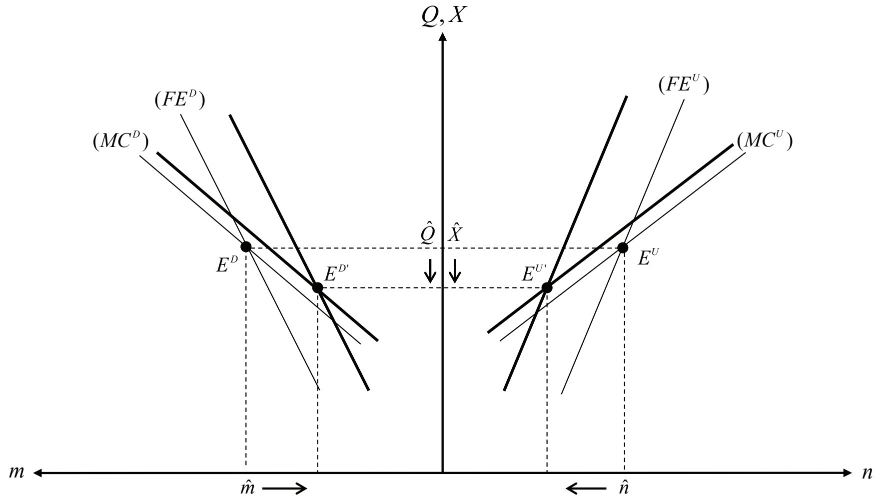

FIGURE 4.3 – Effects of an increase in t

Effect of a change in tariff rate: First we consider the effect of a change in tariff rate t. Observe that t only appears in (4.3) and (4.4). Hence, only MC^U and FE^U are affected by a change in t. From (4.3) and (4.4) it follows that as t increases, n must decrease for any given X and both MC^U and FE^U curves shift to the left; however this shift is greater for FE^U than MC^U. Consequently, as illustrated in Figure 4.3, n must decrease for any given X and both $\hat{X}$ and $\hat{n}$ decline. Further, since $Q = X$, a decline in X implies a decline in Q, which successively induces changes in MC^D and FE^D. From (4.1) and (4.2), as Q decreases, m must decrease and both MC^D and FE^D curves shift to the right; however this shift is greater for FE^D than MC^D. As a result, both $\hat{Q}$ and $\hat{n}$ decline.

Recall from Section 3, a tariff lowers equilibrium outputs $\hat{Q}, \hat{X}$ and raises equilibrium prices $\hat{P}, \hat{r}$ even when the numbers of Home and Foreign firms are exogenously given. Here, a tariff discourages entry in both sectors of production and lowers the numbers of Home and Foreign firms $\hat{m}, \hat{n}$. This effect on entry lowers outputs and raises prices even further.

It is important to emphasize that trade policy has a crucial impact not only on Foreign firms, but also on Home firms through "firm-colocation" effects. In vertical specialization, Home firms' output and Foreign firms' input are *complements*. Thus, when tariff on intermediate input from Foreign discourages entry of Foreign firms, it also discourages entry of Home firms ($\frac{\partial \hat{m}}{\partial t} < 0, \frac{\partial \hat{n}}{\partial t} < 0$). Note the firm-colocation effect occurs only in vertical specialization. If we consider horizontal specialization where Home and Foreign firms' outputs are *substitutes*, a "firm-delocation" effect arises: when tariff on final good from Foreign discourages entry of Foreign firms, it encourages entry of Home firms ($\frac{\partial \hat{m}}{\partial t} > 0, \frac{\partial \hat{n}}{\partial t} < 0$).[16]

[16]For example, Bagwell and Staiger (2012a, b) study long-run effects of trade policy in which two countries trade a homogeneous final good. The markets are segmented and firms compete in a Cournot fashion, whereby two-way trade occurs in a homogeneous good. In the long-run setup with fixed entry costs, they show that higher tariff increases the number of firms in the importing country and decreases the number of firms in the exporting country.

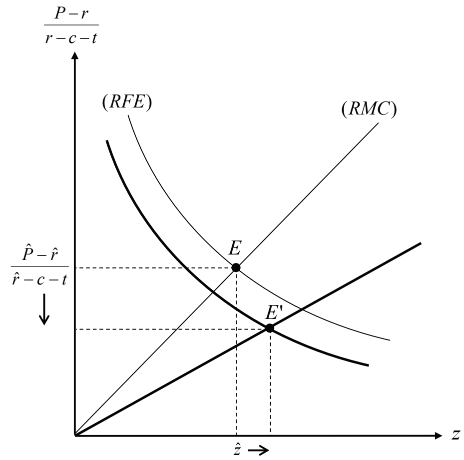

FIGURE 4.4 – Effects of an increase in t

Figure 4.4 illustrates an impact of an increase in t in terms of ***RMC*** and ***RFE*** given by (4.5) and (4.6). As t increases, both ***RMC*** and ***RFE*** curves shift down in $(z, \frac{P-r}{r-c-t})$ space, whereby $\hat{z}$ increases but $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ decreases in long-run equilibrium. The fact that $\hat{z}$ increases with t implies that, although both $\hat{m}$ and $\hat{n}$ are lowered by an increase in t, $\hat{m}$ declines relatively more than $\hat{n}$.[17]

The following lemma summarizes some important comparative statics results that arise from Figures 4.3 and 4.4.

Lemma 4.1

(i) *For given entry costs K_H and K_F, the aggregate output $\hat{Q}$ and aggregate input $\hat{X}$ are decreasing in t; while the final-good price $\hat{P}$ and input-price $\hat{r}$ are increasing in t; i.e., $\partial\hat{Q}/\partial t = \partial\hat{X}/\partial t < 0$, $\partial\hat{P}/\partial t > 0$, and $\partial\hat{r}/\partial t > 0$*

(ii) *For given entry costs K_H and K_F, the number of firms $\hat{m}$, $\hat{n}$ is decreasing in t and the market thickness $\hat{z} \equiv \hat{n}/\hat{m}$ is increasing in t; i.e., $\partial\hat{m}/\partial t < 0$, $\partial\hat{n}/\partial t < 0$ and $\partial\hat{z}/\partial t > 0$.*

(iii) *Let $r^* \equiv \hat{r} - t$ denote the price received by a Foreign firm. Then, there exists $\epsilon^* \in (0,1)$ such that*

$$\frac{\partial r^*}{\partial t} \lesseqqgtr 0 \quad \Leftrightarrow \quad \frac{\partial \hat{r}}{\partial t} \lesseqqgtr 1 \quad \Leftrightarrow \quad \epsilon^* \gtreqqless 0.$$

Lemma 4.1 (iii) says that an increase in tariff improves the terms-of-trade, i.e., lowers r^*, if and only if the demand is concave. Recall that when the market structure is exogenous, tariff reduces r^* for all logconcave demand functions ($\epsilon \geq -1$). When the market structure is endogenous, in contrast, tariff reduces r^* only for concave demand functions ($\epsilon \geq \epsilon^*$). This suggests that terms-of-trade improvement is less likely with endogenous market structure. The reasoning goes

[17] Note that if we consider horizontal specialization, it follows from the firm-delocation effect that $\frac{\partial \hat{z}}{\partial t} < 0$.

as follows. Differentiating the implicit terms-of-trade $r^* = \hat{r} - t$ and using $\frac{\partial \hat{X}}{\partial t} = \hat{n}\frac{\partial \hat{x}}{\partial t} + \hat{x}\frac{\partial \hat{n}}{\partial t}$,

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X}, \hat{m})\hat{n}\frac{\partial \hat{x}}{\partial t} + g_x(\hat{X}, \hat{m})\hat{x}\frac{\partial \hat{n}}{\partial t} + g_m(\hat{X}, \hat{m})\frac{\partial \hat{m}}{\partial t} - 1.$$

Note when the market structure is exogenous, the second and third terms are absent. In other words, when the market structure is endogenous, tariff gives rises to additional adjustments through the exit of Home and Foreign firms. Further substituting $\frac{\partial \hat{m}}{\partial t}$ and $\frac{\partial \hat{n}}{\partial t}$ in Lemma 4.1(ii), the above expression can be simplified as

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X}, \hat{m})\frac{\partial \hat{x}}{\partial t}.$$

Thus, the terms-of-trade improvement occurs ($\frac{\partial r^*}{\partial t} < 0$) if and only if the average imported input of Foreign firms $\hat{x}$ increases by tariff ($\frac{\partial \hat{x}}{\partial t} > 0$). Although this is less likely to occur at first glance, we find that whether the average outputs $\hat{q}, \hat{x}$ decrease by tariff depends on the elasticity of slope of demand $\epsilon = \frac{QP''(Q)}{P'(Q)}$ in our model:

$$\frac{\partial \hat{q}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless 0, \quad \frac{\partial \hat{x}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless \epsilon^*.$$

Intuitively, while an increase in t decreases aggregate outputs $\hat{Q}, \hat{X}$, it also discourages entry of firms $\hat{m}, \hat{n}$, which reduces the degree of competition. Consequently, surviving firms might find it profitable to increase their outputs, which is caused by the exit of rival firms. More generally, our model suggests that the decrease in aggregate outputs is largely accounted for by the decrease in the numbers of firms $\hat{m}, \hat{n}$, whereas net changes in the average outputs $\hat{q}, \hat{x}$ are ambiguous.[18]

Effect of a change in entry cost: Recall in the short run that we examine comparative statics with respect to the number of Foreign firms n (in addition to t). In the long run, however, since the number of firms is an endogenous variable, we cannot conduct these comparative statics. A natural candidate of an exogenous variable that shapes the numbers of Home and Foreign firms m, n (and hence the relative market thickness $\frac{n}{m}$) would then be firms' entry costs, K_H and K_F. Note that these costs can be interpreted as competition policy broadly defined, or policies in general – as well as other institutional features of an economy – that make it difficult to start a business. While we focus on the effect of Home's entry cost K_H, the effect of Foreign's entry cost K_F is qualitatively similar. These comparative statics allow us to show that the optimal tariff can affect market thickness as well, but the thickness is still constrained by the limits given by K_H and K_F in the next subsection.

Observe that K_H only appears in (4.2). Hence, only FE^D is affected by a change in K_H. From (4.2) it follows that as K_H increases, m must decrease for any given Q and FE^D curves shift to

[18]This finding is similar with that of Arkolakis et al. (2008), who find that an increase in transport cost decreases the export volume mainly through the numbers of varieties in the Dixit-Stiglitz model of monopolistic competition. The difference is that we analyze homogeneous goods in the Cournot model of oligopolistic competition.

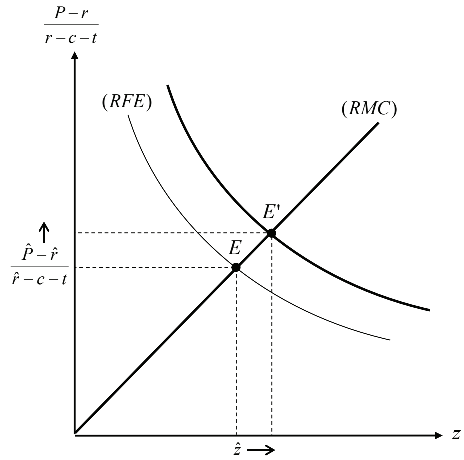

FIGURE 4.5 – Effects of an increase in K_H

the right. Consequently, using a similar diagram in Figure 4.3, we find that m must decrease for any given Q and both $\hat{Q}$ and $\hat{m}$ decline. Further, since $Q = X$, a decline in Q implies a decline in X, which successively induces changes in MC^U and FE^U. From (4.3) and (4.4), as X decreases, m must decrease and both MC^U and FE^U curves shift to the left; however this shift is greater for FE^U than MC^U. As a result, both $\hat{X}$ and $\hat{m}$ decline. Note importantly that, as in trade policy, competition policy also has the "firm-colocation" effects: when it is difficult to start a business in Home, this discourages entry of Foreign firms as well as Home firms. In that sense, there is a complementarity between competition policy and trade policy.

Figure 4.5 illustrates an impact of an increase in K_H in terms of ***RMC*** and ***RFE***. As K_H increases, it follows from (4.5) and (4.6) that only the ***RFE*** curve shifts up in $(z, \frac{P-r}{r-c-t})$ space, whereby both $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ increase in long-run equilibrium. As in the case of t, the fact that $\hat{z}$ increases with K_H implies that, although both $\hat{m}$ and $\hat{n}$ are lowered by an increase in K_H, $\hat{m}$ declines relatively more than $\hat{n}$.

The following lemma summarizes comparative statics results with respect to K_H.[19]

Lemma 4.2

(i) *For a given tariff rate t and Foreign entry cost K_F, the aggregate output $\hat{Q}$ and aggregate input $\hat{X}$ are decreasing in K_H; while the final-good price $\hat{P}$ and input price $\hat{r}$ are increasing in K_H; i.e., $\partial\hat{Q}/\partial K_H = \partial\hat{X}/\partial K_H < 0$, $\partial\hat{P}/\partial K_H > 0$, $\partial\hat{r}/\partial K_H > 0$.*

(ii) *For a given tariff rate t and Foreign entry cost K_F, the number of firms $\hat{m}$, $\hat{n}$ is decreasing in K_H and the market thickness $\hat{z} \equiv \hat{n}/\hat{m}$ is increasing in K_H; i.e., $\partial\hat{m}/\partial K_H < 0$, $\partial\hat{n}/\partial K_H < 0$ and $\partial\hat{z}/\partial K_H > 0$.*

[19] Comparative statics with respect to K_F are almost the same as those of K_H, due to the firm-colocation effects. Only the difference is that an increase in K_F decreases both $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$, as is expected from Figure 4.5.

4.2 Tariffs

In the first stage, the Home government chooses a tariff rate t to maximize Home welfare (W_H), anticipating the output vector $(\hat{q}, \hat{Q}, \hat{x}, \hat{X})$, the price vector $(\hat{P}, \hat{r})$ and the number vector $(\hat{m}, \hat{n})$ in the Cournot competition. As profits are zero under free entry, Home welfare effectively consists of consumer surplus and tariff revenues only. In the SPNE of Stage 1 subgame, W_H is given by

$$W_H \equiv \underbrace{\left[\int_0^{\hat{Q}} P(y)\mathrm{d}y - P(\hat{Q})\hat{Q}\right]}_{\text{Consumer surplus}} + \underbrace{t\hat{X}}_{\text{Tariff revenue}} . \tag{4.7}$$

Differentiating W_H with respect to t, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \left(1 - \frac{\partial P(\hat{Q})}{\partial t}\right)\hat{Q} + t\frac{\partial \hat{X}}{\partial t}.$$

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ and solving for t gives the expression for the optimal tariff which is presented later in Proposition 4.1. Since $\frac{\partial \hat{X}}{\partial t} < 0$, the optimal tariff is strictly positive (negative) if and only if $1 - \frac{\partial P(\hat{Q})}{\partial t} > (<)0$. In the short-run analysis, we argue that tariff induces the welfare loss due to the tariff-induced output reduction but the welfare gain arising from the terms-of-trade improvement. However, the above expression is not directly related to how the terms-of-trade r^* improves by tariff.

To better connect the optimal tariff in the short-run and long-run equilibria, noting that the aggregate Home profit is zero under free entry, i.e., $(P(\hat{Q}) - \hat{r})\hat{Q} = \hat{m}K_H$ in the SPNE of Stage 1 subgame, we have $P(\hat{Q})\hat{Q} = \hat{r}\hat{Q} + \hat{m}K_H$. Substituting this equality into (4.7) yields

$$W_H = \int_0^{\hat{Q}} P(y)\mathrm{d}y - g(\hat{X}, \hat{m})\hat{Q} - \hat{m}K_H + t\hat{X}. \tag{4.8}$$

The expression (4.8) implies that Home welfare is total surplus defined as gross benefit to the consumers less the sum of production costs and entry costs (from Home's perspectives). Further, using $r^* \equiv \hat{r} - t$ and simplifying (4.8), we have that

$$W_H = \int_0^{\hat{Q}} P(y)\mathrm{d}y - r^*\hat{X} - \hat{m}K_H.$$

This expression is similar with that in the short run except for the extra term $\hat{m}K_H$: in the long run, $\hat{m}$ entering Home firms pay the entry cost K_H and Home welfare takes into account the total entry cost $\hat{m}K_H$. Noting $\hat{Q} = \hat{X}$ and differentiating this W_H with respect to t, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - r^*)\frac{\partial \hat{Q}}{\partial t} - \frac{\partial r^*}{\partial t}\hat{X} - \frac{\partial \hat{m}}{\partial t}K_H.$$

As in the short run, the first term captures the welfare loss due to the tariff-induced output reduction ($\frac{\partial \hat{Q}}{\partial t} < 0$), and the second term captures the welfare gain arising from the terms-of-trade improvement ($\frac{\partial r^*}{\partial t} < 0$). In contrast to the short run, however, the third term captures the welfare loss due to the tariff-induced reduction of Home firms ($\frac{\partial \hat{m}}{\partial t} < 0$), which arises only in the long run due to the firm-colocation effect. In addition, the terms-of-trade improvement does not always occur for all logconcave demand functions and tariff reduces r^* only for concave demand functions ($\frac{\partial r^*}{\partial t} < 0$ if and only if $\epsilon > \epsilon^*$).

Using the expression for $\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1$, we can express $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ as follows:

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} - \frac{\partial \hat{m}}{\partial t}K_H + t\frac{\partial \hat{X}}{\partial t}. \tag{4.9}$$

Following the previous section, we first focus on the optimal tariff. Noting that $\frac{\partial \hat{X}}{\partial t} < 0$ in (4.9), the optimal tariff is strictly positive (negative) if and only if

$$(P(\hat{Q}) - \hat{r})\frac{\partial \hat{Q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} - \frac{\partial \hat{m}}{\partial t}K_H > (<)0. \tag{4.10}$$

Contrary to Section 3, the sign of the optimal tariff cannot be argued by the numbers of Home and Foreign firms $\hat{m}, \hat{n}$ as these numbers are not parameters in the long run. Since $\frac{\partial \hat{Q}}{\partial t} = \hat{m}\frac{\partial \hat{q}}{\partial t} + \hat{q}\frac{\partial \hat{m}}{\partial t}$ and $(P(\hat{Q}) - \hat{r})\hat{q} - K_H = 0$ in the SPNE of Stage 1 subgame, condition (4.10) is rewritten as

$$(P(\hat{Q}) - \hat{r})\hat{m}\frac{\partial \hat{q}}{\partial t} + \left(1 - \frac{\partial \hat{r}}{\partial t}\right)\hat{X} > (<)0. \tag{4.11}$$

It is important to note that the expression of (4.11) is exactly the same as that of (3.7) in the exogenous market structure. This implies that the sign of the optimal tariff does not depend on whether the profits are positive or zero. Instead, it depends on whether there arise the tariff-induced reduction in average output ($\frac{\partial \hat{q}}{\partial t} < 0$) and terms-of-trade improvement ($1 - \frac{\partial \hat{r}}{\partial t} = -\frac{\partial r^*}{\partial t} > 0$), which in turn depends on whether the market structure is exogenous or endogenous.[20]

The comparative statics in Section 4.1 tell us that the sign of $\frac{\partial \hat{q}}{\partial t}$ and $1 - \frac{\partial \hat{r}}{\partial t} = -\frac{\partial r^*}{\partial t}$ depends on the elasticity of slope of demand ϵ. Together with (4.11), it turns out that the sign of the optimal tariff also depends on ϵ. In particular, the optimal tariff is positive for concave demand functions ($\epsilon \geq \epsilon^*$) since we have that $\frac{\partial \hat{q}}{\partial t} > 0$ and $1 - \frac{\partial \hat{r}}{\partial t} \geq 0$. On the other hand, the optimal tariff is negative for convex demand functions ($\epsilon \leq 0$) since $\frac{\partial \hat{q}}{\partial t} \leq 0$ and $1 - \frac{\partial \hat{r}}{\partial t} < 0$. For a special case of linear demand ($\epsilon = 0$), $\frac{\partial \hat{q}}{\partial t} = 0$ and $1 - \frac{\partial \hat{r}}{\partial t} < 0$ and thus the optimal tariff is always negative for any $\hat{Q}, \hat{X}, \hat{m}, \hat{n}$. This suggests that there exists a cutoff $\epsilon^{**} \in (0, 1)(< \epsilon^*)$ at which the sign of the optimal tariff is determined: the optimal tariff is positive for $\epsilon > \epsilon^{**}$, whereas the optimal tariff is negative for $\epsilon < \epsilon^{**}$.

[20]More specifically, it follows from Lemma 3.1 that $\frac{\partial \hat{q}}{\partial t} < 0$ and $\frac{\partial r^*}{\partial t} < 0$ in the exogenous market structure, and from Lemma 4.1 that $\frac{\partial \hat{q}}{\partial t} \gtreqless 0$ and $\frac{\partial r^*}{\partial t} \gtreqless 0$ in the endogenous market structure.

In contrast to the short run, we cannot use the numbers of firms to examine the impact of the market thickness on the optimal tariff in the long run. As in Section 4.1, we instead focus on Home's entry cost K_H. Suppose that this entry cost decreases and Home firms find it easier to start a business in the final-good market. Then it follows from Lemma 4.2 that while this increases the number of Home firms $\hat{m}$ as well as the number of Foreign firms $\hat{n}$, $\hat{m}$ increases relatively more than $\hat{n}$, making the Home market thicker. As a result, $P(\hat{Q})$ decreases relatively more than $\hat{r}$, and the price-cost margins in the Home final-good market $P(\hat{Q}) - \hat{r}$ and in the Foreign intermediate-input market $\hat{r} - c - t$ decrease by new entry of these firms. Further, (4.11) shows that the smaller price-cost margins induce the Home government to set the lower positive (negative) tariff if $\epsilon > (<)\epsilon^{**}$. Therefore, the lower is Home's entry cost, the thicker is the Home market relative to the Foreign market, and the lower is the optimal tariff (in absolute terms). Proposition 4.1 presents these findings and provides a sharper characterization.

Proposition 4.1 *Let $\hat{t}$ denote the optimal tariff. At $t = \hat{t}$, the following holds:*

$$\hat{t} = -\hat{Q}P'(\hat{Q}) \left(\frac{2(\hat{m} + \hat{n})\epsilon + (\epsilon + 1)(\epsilon - 2)}{4\hat{m}\hat{n}} \right), \tag{4.12}$$

where $\hat{Q}$ is the aggregate output evaluated at $t = \hat{t}$. Furthermore,

(i) *There exists $\epsilon^{**} \in (0, 1)$ such that*

$$\hat{t} \gtreqless 0 \iff \epsilon \gtreqless \epsilon^{**}.$$

(ii) *$\hat{t}$ is monotonically increasing (decreasing) in K_H if $\epsilon > (<)\epsilon^{**}$.*

Recall that there exists a *negative* relationship between $\frac{n}{m}$ and t in the short-run equilibrium: the optimal tariff t is higher, the thicker is the Home final-good market relative to the Foreign intermediate-input market (i.e., lower $\frac{n}{m}$), as in Figure 3.1. In the long-run equilibrium, however, this relationship is overturned and our model predicts that there exists a *positive* relationship between $\frac{n}{m}$ and t. As noted above, in the long run where firms can freely enter and exit, the market thickness is constrained by the limits of the entry cost K_H. This implies that the greater K_H makes it more difficult to start a business not only for Home firms but also for Foreign firms through the firm-colocation effect, thereby leading to the higher market thickness $\hat{z} = \frac{\hat{n}}{\hat{m}}$, as seen in Lemma 4.2(ii). At the same time, due to a complementarity between competition policy and trade policy, the greater K_H also induces the higher optimal tariff t, as seen in Proposition 4.1(ii). Combining these two observations establishes the positive relationship between $\frac{n}{m}$ and t, as depicted in Figure 4.6. Note that the optimal tariff is higher (i.e., higher t), the less competitive is the Home final-good market relative to the Foreign intermediate-input market (i.e., higher $\frac{n}{m}$). In this sense, the degree of freeness of trade policy is directly related to the degree of competition in the domestic market in the long run. This finding suggests that a reduction of import tariff

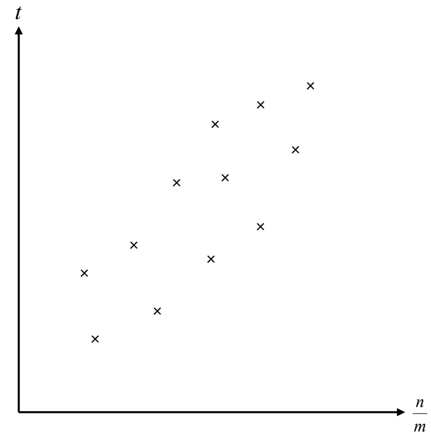

FIGURE 4.6 – Optimal tariff in long run

for Foreign input has its greater effect on welfare when accompanied by liberalization of entry in the Home final-good market in longer-term perspectives.

What should we make of the fact that (a) demand curvature matters for the sign of the optimal tariff and (b) the relationship between market thickness and tariff differs between the two cases – endogenous and exogenous market structures? Our reading of the literature suggests that, in terms of the dependence of optimal policy on demand curvature, our results have a similar flavor to some of the existing results in the trade literature. For example, the classic result that the sign of the optimal tariff in the presence of a Foreign monopoly depends on whether there is incomplete pass-through, which in turn depends on whether the demand curve is flatter than the marginal revenue curve (Brander and Spencer, 1984a,b; Helpman and Krugman, 1989, Chapter 4). Concerning the difference in results between endogenous and exogenous market structures, our finding is in the line with Horstmann and Markusen (1986) and Venables (1985), who have shown that in the single-stage oligopoly models, entry can alter optimal trade policy due to the firm-delocation effect (although our result is derived by firm-colocation effects). This point has also recently been made by Etro (2011) and Bagwell and Staiger (2012a, b) in the contexts of strategic trade policy and trade agreements respectively. We do not necessarily view (b) as a shortcoming. Depending on the industry characteristics, such as industry-specific fixed costs or stability of demand, some industries fit an exogenous market structure description better, while for some other industries with fluid entry and volatile demand, an endogenous market structure is more apt.

As an illustrative example, consider again the following class of inverse demand functions: $P(Q) = a - Q^b$ for which $\epsilon = b - 1$. Applying (4.13), the optimal tariff is given by

$$\hat{t} = \left[\frac{2(b-1)(\hat{m}+\hat{n}) + b(b-3)}{4\hat{m}\hat{n} + b(1+b)[2(\hat{m}+\hat{n}) + b]} \right] (a-c)b.$$

While $\hat{m}$ and $\hat{n}$ are endogenous in this setting, observe that if demand is concave (convex) with

$b > 2$ ($b < 1$), the optimal tariff is positive (negative) for any $\hat{m}, \hat{n}(> 1)$. This implies there exists $b^{**} \in (1, 2)$ at which the sign of the optimal tariff is determined (note, if demand is linear ($b = 1$), the optimal tariff is negative). Further, since $\hat{m}$ and $\hat{n}$ are decreasing in K_H, it follows from the above expression that $\hat{t}$ is increasing (decreasing) in K_H for $b > (<) b^{**}$.

5 Conclusion

With reductions in trade costs, firms from various countries are increasingly specializing in different but complementary stages of production. In such environments of vertical specialization, under what conditions might a welfare maximizing government impose a tariff? We demonstrate that the market thickness of a domestic final-good market (relative to foreign intermediate-input market) might prompt a domestic government to impose a tariff on foreign firms. We find that, in the short-run equilibrium, the optimal tariff is higher, the thicker is the Home final-good market. Surprisingly, we find that an increase in the number of foreign firms not only benefits domestic firms but it can also benefit foreign firms by lowering tariff rates. In the long run where the market structure is endogenously determined by firms' entry and exit, the relationship between relative market thickness and tariff is overturned and the optimal tariff is higher, the thinner is the Home final-good market. This is because in vertical specialization where domestic firms' output and foreign firms' input are complements, if tariff on intermediate input from foreign discourages entry of foreign firms, it also discourages entry of domestic firms, due to the "firm-colocation" effect. This finding suggests that a reduction of import tariff for Foreign input has its greater effect on welfare when accompanied by liberalization of entry in the Home final-good market in longer-term perspectives.

The structure of vertical oligopoly in our paper is admittedly simplistic. Each foreign firm has no choice but to sell homogeneous input through spot markets, whereas each domestic firm has no choice but to purchase homogeneous input through spot markets. Yet in reality each domestic firm often also negotiates with foreign firms over the terms of contracts, whereas each foreign firm often also has contractual relationships with domestic firms in the global production chains. If we introduce the possibility of bargaining in foreign outsourcing, the market thickness has an impact on the optimal tariff not only through the double-marginalization effect but also through matching and search between domestic firms and foreign firms. Although we have addressed the second channel in a separate paper (Ara and Ghosh, 2016), a full-fledged analysis of trade policy that includes both contractual arrangements and spot markets is challenging. Since the numbers of Home and Foreign firms as well as tariffs are measurable and observable, however, our analysis of spot markets has an advantage of being able to empirically test our theoretical prediction. In future work we plan to investigate the link between our theory and evidence.

Appendix

A Proofs for Section 3

A.1 Equivalence between Assumptions 1 and 1'

The assumption $Q(P)$ is logconcave implies

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{\mathrm{d}\ln Q(P)}{\mathrm{d}P}\right] &= \frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{Q'(P)}{Q(P)}\right] \\ &= \frac{Q(P)\cdot Q''(P) - [Q'(P)]^2}{[Q(P)]^2} \leq 0,\end{aligned}$$

which can be expressed as

$$\frac{Q(P)Q''(P)}{[Q'(P)]^2} \leq 1. \tag{A.1}$$

Differentiating $P = P(Q(P))$ with respect to P, we get

$$1 = P'(Q(P))Q'(P).$$

Differentiating this once again with respect to P gives

$$0 = P''[Q'(P)]^2 + P'Q''(P).$$

Rewriting this equation, we get

$$\frac{Q''(P)}{[Q'(P)]^2} = -\frac{P''}{P'}.$$

Substituting this relationship into (A.1), we find that

$$-\frac{QP''(Q)}{P'(Q)} \leq 1,$$

which implies $P'(Q) + QP''(Q) \leq 0$. □

A.2 Proof of Lemma 3.1

(i) Differentiating (3.5) with respect to n, rearranging and using (3.5) subsequently, we get

$$\frac{\partial \hat{X}}{\partial n} = -\frac{g(\hat{X}) - c - t}{(n+1)g'(\hat{X}) + \hat{X}g''(\hat{X})} = \frac{\hat{x}}{n+1+\epsilon}.$$

Note $\hat{Q} = \hat{X}$ implies that $\frac{\partial \hat{Q}}{\partial n} = \frac{\partial \hat{X}}{\partial n}$. Since $\hat{Q} = m\hat{q}$ and $\hat{X} = n\hat{x}$, we get

$$\frac{\partial \hat{q}}{\partial n} = \frac{\hat{q}}{n(n+1+\epsilon)}, \qquad \frac{\partial \hat{x}}{\partial n} = -\frac{(n+\epsilon)\hat{x}}{n(n+1+\epsilon)}.$$

Using the expression for $\frac{\partial \hat{X}}{\partial n}$ we get

$$\begin{aligned}
\frac{\partial \hat{r}}{\partial n} &= \frac{\partial g(\hat{X})}{\partial n} = g'(\hat{X})\frac{\partial \hat{X}}{\partial n} = \frac{\hat{x}g'(\hat{X})}{n+1+\epsilon} = \frac{\hat{x}P'(\hat{Q})(m+1+\epsilon)}{m(n+1+\epsilon)}, \\
\frac{\partial \hat{P}}{\partial n} &= \frac{\partial P(\hat{Q})}{\partial n} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial n} = \frac{\hat{x}P'(\hat{Q})}{n+1+\epsilon}.
\end{aligned}$$

The results follow from noticing that $P'(\hat{Q}) < 0$, $m+1+\epsilon > 0$ and $n+1+\epsilon > 0$.

(ii) Differentiating (3.5) with respect to t, we get

$$\frac{\partial \hat{X}}{\partial t} = \frac{n}{g'(\hat{X})(n+1+\epsilon)} = \frac{mn}{P'(\hat{Q})(n+1+\epsilon)(m+1+\epsilon)}.$$

Note $\hat{Q} = \hat{X}$ implies that $\frac{\partial \hat{Q}}{\partial t} = \frac{\partial \hat{X}}{\partial t}$. Since $\hat{Q} = m\hat{q}$ and $\hat{X} = n\hat{x}$, we get

$$\frac{\partial \hat{q}}{\partial t} = \frac{n}{P'(\hat{Q})(m+1+\epsilon)(n+1+\epsilon)}, \qquad \frac{\partial \hat{x}}{\partial t} = \frac{m}{P'(\hat{Q})(m+1+\epsilon)(n+1+\epsilon)}.$$

Using the expression for $\frac{\partial \hat{X}}{\partial t}$ we get

$$\frac{\partial \hat{r}}{\partial t} = g'(\hat{X})\frac{\partial \hat{X}}{\partial t} = \frac{n}{n+1+\epsilon}, \qquad \frac{\partial \hat{P}}{\partial t} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} = \frac{mn}{(m+1+\epsilon)(n+1+\epsilon)}.$$

The results follow from noticing that $P'(\hat{X}) < 0$, $n+1+\epsilon > 0$ and $n+1+\epsilon > 0$.

(iii) We have that

$$\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1 = -\frac{1+\epsilon}{n+1+\epsilon}.$$

The claim follows from observing that $n+1+\epsilon > 0$.

Although we have focused on comparative statics with respect to n, it is straightforward to examine comparative statics with respect to n. From (3.1), we have that

$$\frac{\partial \hat{Q}}{\partial m} = -\frac{P(\hat{Q}) - r}{(m+1)P'(\hat{Q}) + \hat{Q}P''(\hat{Q})} = \frac{\hat{q}}{m+1+\epsilon}.$$

Since $\hat{Q} = m\hat{q}$ and $\hat{X} = n\hat{x}$, we get

$$\frac{\partial \hat{q}}{\partial m} = -\frac{(m+\epsilon)\hat{q}}{m(m+1+\epsilon)}, \qquad \frac{\partial \hat{x}}{\partial m} = \frac{\hat{x}}{m(m+1+\epsilon)}.$$

Regarding the prices, note in particular that the input price r depends on m as well as X (see (3.2)). While we apply the short-hand definition $r \equiv g(X)$ for the short-run analysis (since we mainly focus on comparative statics with respect to n), we need to explicitly define $r \equiv g(X, m)$

when we conduct comparative statics with respect to m. Thus

$$\frac{\partial \hat{r}}{\partial m} = \frac{\partial g(\hat{X}, m)}{\partial m} = g_x(\hat{X}, m)\frac{\partial \hat{X}}{\partial m} + g_m(\hat{X}, m) = 0,$$
$$\frac{\partial \hat{P}}{\partial m} = \frac{\partial P(\hat{Q})}{\partial m} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial m} = \frac{\hat{q}P'(\hat{Q})}{m+1+\epsilon},$$

where

$$g_x(X, m) \equiv \frac{\partial g(X, m)}{\partial X}, \quad g_m(X, m) \equiv \frac{\partial g(X, m)}{\partial m}.$$

The results follow from noticing that $P'(\hat{Q}) < 0$ and $m + 1 + \epsilon > 0$. □

A.2 Proof of Proposition 3.1

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ in (3.6) and rearranging, we immediately get (3.9). Concerning the properties of the optimal tariff $t = \hat{t}$, consider (i) first. It directly follows from (3.9) that

$$\hat{t} \gtreqless 0 \;\;\Leftrightarrow\;\; n \lesseqgtr n^* \equiv (1+\epsilon)(m+1+\epsilon).$$

(ii) Differentiating $\frac{\mathrm{d}W_H}{\mathrm{d}t}\big|_{t=\hat{t}} = 0$ with respect to n gives

$$\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} = -\frac{\frac{\partial^2 W_H}{\partial n \partial t}}{\frac{\partial^2 W_H}{\partial t^2}}.$$

From the comparative statics results in Lemma 3.1, the second-order condition is satisfied, i.e.,

$$\frac{\partial^2 W_H}{\partial t^2} = \left[\frac{mn + (1+\epsilon)(2+\epsilon)(m+1+\epsilon)}{(m+1+\epsilon)(n+1+\epsilon)}\right]\frac{\partial \hat{Q}}{\partial t} < 0.$$

Then it follows that

$$\mathrm{sgn}\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} = \mathrm{sgn}\frac{\partial^2 W_H}{\partial n \partial t}. \tag{A.2}$$

Differentiating (3.6) with respect to n gives

$$\frac{\partial^2 W_H}{\partial n \partial t} = (1+\epsilon)\left(-\frac{\hat{X}g'(\hat{X})}{n}\right)\left[\frac{m+2+\epsilon}{(m+1+\epsilon)(n+1+\epsilon)}\right]\frac{\partial \hat{Q}}{\partial t} < 0.$$

Since $\frac{\partial^2 W_H}{\partial n \partial t} < 0$, (A.2) implies that $\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} < 0$.

We can also show that $t = \hat{t}$ is increasing in m. Differentiating (A.3) with respect to m gives

$$\frac{\partial^2 W_H}{\partial m \partial t} = \left(\frac{\hat{Q}P'(\hat{Q})}{m}\right)\left(\frac{1}{m+1+\epsilon}\right)\frac{\partial \hat{Q}}{\partial t} > 0.$$

Since $\mathrm{sgn}\frac{\mathrm{d}\hat{t}}{\mathrm{d}m} = \mathrm{sgn}\frac{\partial^2 W_H}{\partial m \partial t}$, this implies that $\frac{\mathrm{d}\hat{t}}{\mathrm{d}m} > 0$.

While we focus on the sign of $\frac{\partial^2 W_H}{\partial n \partial t}$, it is possible to show for future reference that $\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} < 0$ also holds by directly differentiating the optimal tariff in (3.9) with respect to n and m. Using $P'(Q) = \frac{m}{m+1+\epsilon} g'(X)$ from (3.3), rewrite the optimal tariff as $\hat{t} = -\hat{X} g'(\hat{X}) \Phi$ where $\Phi \equiv \frac{(1+\epsilon)(m+1+\epsilon)-n}{(m+1+\epsilon)n}$. Differentiating this $\hat{t}$ with respect to n,

$$\begin{aligned}\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} &= -\left[g'(\hat{X}) + \hat{X} g''(\hat{X})\right] \frac{\mathrm{d}\hat{X}}{\mathrm{d}n} \Phi - \hat{X} g'(\hat{X}) \frac{\mathrm{d}\Phi}{\mathrm{d}n} \\ &= -g'(\hat{X})(1+\epsilon)\Phi \left(\frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{X}}{\partial t} \frac{\mathrm{d}\hat{t}}{\mathrm{d}n} \right) + \hat{X} g'(\hat{X}) \left(\frac{1+\epsilon}{n^2} \right).\end{aligned}$$

Substituting $\frac{\partial \hat{X}}{\partial n}$ and $\frac{\partial \hat{X}}{\partial t}$ from Lemma 3.1(i)-(ii) and solving for $\frac{\mathrm{d}\hat{t}}{\mathrm{d}n}$ yields

$$\frac{\mathrm{d}\hat{t}}{\mathrm{d}n} = -(1+\epsilon) \left(-\frac{\hat{X} g'(\hat{X})}{n} \right) \left[\frac{m+2+\epsilon}{mn + (1+\epsilon)(2+\epsilon)(m+1+\epsilon)} \right].$$

The claim follows from noting that $g'(X) < 0$ and $1+\epsilon > 0$. Following the similar steps, differentiating (3.9) with respect to m yields

$$\frac{\mathrm{d}\hat{t}}{\mathrm{d}m} = \left(-\frac{\hat{Q} P'(\hat{Q})}{m} \right) \left[\frac{n+1+\epsilon}{mn + (1+\epsilon)(2+\epsilon)(m+1+\epsilon)} \right].$$

The claim follows from noting that $P'(Q) < 0$. □

A.3 Proof of Proposition 3.2

We first show that the impact of n on $\hat{\Pi}_F$ is decomposed into the competition effect and tariff-reduction effect. Differentiating $\hat{\Pi}_F = (\hat{r} - c - \hat{t})\hat{X}$ with respect to n, we have that

$$\begin{aligned}\frac{\mathrm{d}\hat{\Pi}_F}{\mathrm{d}n} &= (\hat{r} - c - \hat{t}) \frac{\mathrm{d}\hat{X}}{\mathrm{d}n} + \frac{\mathrm{d}\hat{r}}{\mathrm{d}n} \hat{X} - \frac{\mathrm{d}\hat{t}}{\mathrm{d}n} \hat{X} \\ &= -\hat{x} g'(\hat{X}) \frac{\mathrm{d}\hat{X}}{\mathrm{d}n} + n\hat{x} g'(\hat{X}) \frac{\mathrm{d}\hat{X}}{\mathrm{d}n} - \frac{\mathrm{d}\hat{t}}{\mathrm{d}n} \hat{X} \\ &= (n-1)\hat{x} g'(\hat{X}) \frac{\mathrm{d}\hat{X}}{\mathrm{d}n} - \frac{\mathrm{d}\hat{t}}{\mathrm{d}n} \hat{X},\end{aligned}$$

where the second equality comes from (3.5) and $\hat{X} = n\hat{x}$.

Next we show that the size effect can dominate the competition effect. Differentiating $\hat{\Pi}_F = (\hat{r} - c - \hat{t})\hat{X} = -\frac{\hat{X}^2 g'(\hat{X})}{n}$ with respect to n gives

$$\frac{\mathrm{d}\hat{\Pi}_F}{\mathrm{d}n} = \frac{\hat{X}^2 g'(\hat{X})}{n^2} \left[1 - (2+\epsilon)\delta \right], \tag{A.3}$$

where $\delta \equiv \frac{n}{\hat{X}} \frac{\mathrm{d}\hat{X}}{\mathrm{d}n} = \frac{n}{\hat{X}} \left(\frac{\partial \hat{X}}{\partial n} + \frac{\partial \hat{X}}{\partial t} \frac{\mathrm{d}\hat{t}}{\mathrm{d}n} \right)$. Substituting $\frac{\partial \hat{X}}{\partial n}$ and $\frac{\partial \hat{X}}{\partial t}$ from Lemma 3.1(i)-(ii) and $\frac{\mathrm{d}\hat{t}}{\mathrm{d}n}$ from

Proposition 3.1(ii) into δ, we have that

$$\delta = \frac{1}{n+1+\epsilon}\left[\frac{mn + (1+\epsilon)(2+\epsilon)(m+1+\epsilon) + n(1+\epsilon)(m+2+\epsilon)}{mn + (1+\epsilon)(2+\epsilon)(m+1+\epsilon)}\right].$$

Since $\frac{\hat{X}^2 g'(\hat{X})}{n^2} < 0$ in (A.3), $\frac{\mathrm{d}\hat{\Pi}_F}{\mathrm{d}n} > 0 \Leftrightarrow \delta > \frac{1}{2+\epsilon}$. Evaluating $\lim_{m\to\infty}\delta$ and solving the last inequality for n establishes the result. Regarding per-firm Foreign profits $\hat{\pi}_F$, we have that

$$\frac{\mathrm{d}\hat{\pi}_F}{\mathrm{d}n} = \frac{\hat{X}^2 g'(\hat{X})}{n^3}\left[2 - (2+\epsilon)\delta\right],$$

which is always negative for $\lim_{m\to\infty}\delta$, and the counter-intuitive outcome never occurs for $\hat{\pi}_F$. □

B Proofs of Section 4

B.1 Proof of Lemma 4.1

Let a dot represent proportional rates of change (e.g. $\dot{Q} \equiv \frac{Q'}{Q}$) and totally differentiating (4.3), (4.2) and (4.4) respectively gives

$$(n+1+\epsilon)\dot{X} = \left(\frac{n+1+\epsilon}{m+1+\epsilon}\right)\dot{m} + \dot{n} + \frac{mnt}{(m+1+\epsilon)QP'(Q)}\dot{t}, \tag{B.1}$$

$$(2+\epsilon)\dot{Q} = 2\dot{m}, \tag{B.2}$$

$$(2+\epsilon)\dot{X} = \left(\frac{1+\epsilon}{m+1+\epsilon}\right)\dot{m} + 2\dot{n}, \tag{B.3}$$

where K_H and K_F hold constant. (B.1), (B.2) and (B.3) are three equations that have three unknowns $\dot{X}(=\dot{Q})$, $\dot{m}$ and $\dot{n}$, which can be solved explicitly as a function of $\dot{t}$:

$$\begin{aligned}
\dot{X} &= \left(\frac{4}{\Omega}\right)\frac{mnt}{QP'(Q)}\dot{t},\\
\dot{m} &= \left(\frac{2(2+\epsilon)}{\Omega}\right)\frac{mnt}{QP'(Q)}\dot{t},\\
\dot{n} &= \left(\frac{2+\epsilon}{\Omega}\right)\left(\frac{2m+1+\epsilon}{m+1+\epsilon}\right)\frac{mnt}{QP'(Q)}\dot{t},
\end{aligned}$$

where $\Omega \equiv (2m+\epsilon)(2n+\epsilon) - (2+\epsilon) > 0$ for $m > 1$ and $n > 1$. Evaluated at $X = \hat{X}, m = \hat{m}, n = \hat{n}$,

$$\frac{\partial \hat{X}}{\partial t} = \left(\frac{4}{\hat{\Omega}}\right)\frac{\hat{m}\hat{n}}{P'(\hat{Q})} < 0, \tag{B.4}$$

$$\frac{\partial \hat{m}}{\partial t} = \left(\frac{2(2+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}^2\hat{n}}{\hat{Q}P'(\hat{Q})} < 0, \tag{B.5}$$

$$\frac{\partial \hat{n}}{\partial t} = \left(\frac{2+\epsilon}{\hat{\Omega}}\right)\left(\frac{(2\hat{m}+1+\epsilon)\hat{n}^2}{\hat{X}g_x(\hat{X},\hat{m})}\right) < 0. \tag{B.6}$$

Further, since $\hat{Q} = \hat{X}$, $\hat{Q} = \hat{m}\hat{q}$ and $\hat{X} = \hat{n}\hat{x}$, we have $\frac{\partial \hat{Q}}{\partial t} = \hat{m}\frac{\partial \hat{q}}{\partial t} + \hat{q}\frac{\partial \hat{m}}{\partial t}$ and $\frac{\partial \hat{X}}{\partial t} = \hat{n}\frac{\partial \hat{x}}{\partial t} + \hat{x}\frac{\partial \hat{n}}{\partial t}$, and using (B.4), (B.5) and (B.6) yields

$$\frac{\partial \hat{q}}{\partial t} = -\frac{2\hat{n}\epsilon}{\hat{\Omega} P'(\hat{Q})}, \tag{B.7}$$

$$\frac{\partial \hat{x}}{\partial t} = -\frac{2\hat{m}\epsilon + (\epsilon+1)(\epsilon-2)}{\hat{\Omega} g_x(\hat{X}, \hat{m})}, \tag{B.8}$$

which suggests that

$$\frac{\partial \hat{q}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless 0,$$
$$\frac{\partial \hat{x}}{\partial t} \gtreqless 0 \iff \epsilon \gtreqless \epsilon^*,$$

where $\epsilon^* \in (0,1)$ satisfies $2\hat{m}\epsilon^* + (\epsilon^*+1)(\epsilon^*-2) = 0$. Using the expressions of $\frac{\partial \hat{X}}{\partial t}$ and $\frac{\partial \hat{m}}{\partial t}$ in (B.4) and (B.5), we also have that

$$\frac{\partial \hat{P}}{\partial t} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} = \frac{4\hat{m}\hat{n}}{\hat{\Omega}} > 0,$$
$$\frac{\partial \hat{r}}{\partial t} = g_x(\hat{X}, \hat{m})\frac{\partial \hat{X}}{\partial t} + g_m(\hat{X}, \hat{m})\frac{\partial \hat{m}}{\partial t} = \frac{2\hat{n}(2\hat{m}+\epsilon)}{\hat{\Omega}} > 0.$$

Next, differentiating $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \hat{X}}{\partial t}$ and $\frac{\partial \hat{m}}{\partial t}$ in (B.4) and (B.5), we have that

$$\frac{\partial \hat{z}}{\partial t} = -\left(\frac{(1+\epsilon)(2+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}k}{\hat{Q}P'(\hat{Q})} > 0,$$
$$\frac{\partial\left(\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}\right)}{\partial t} = \left(\frac{(1+\epsilon)(2+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}k}{\hat{X}g_x(\hat{X}, \hat{m})} < 0.$$

Finally, using the expression of $\frac{\partial \hat{r}}{\partial t}$, it directly follows that

$$\frac{\partial r^*}{\partial t} = \frac{\partial \hat{r}}{\partial t} - 1 = -\frac{2\hat{m}\epsilon + (\epsilon+1)(\epsilon-2)}{\hat{\Omega}}. \tag{B.9}$$

Comparing (B.8) and (B.9) suggests that

$$\frac{\partial r^*}{\partial t} = g_x(\hat{X}, \hat{m})\frac{\partial \hat{x}}{\partial t}.$$

The claim that $\frac{\partial r^*}{\partial t} \lesseqgtr 0 \Leftrightarrow \frac{\partial \hat{x}}{\partial t} \gtreqless 0$ follows from noting that $g_x(X,m) < 0$. □

B.2 Proof of Lemma 4.2

Using a dot representation once again (e.g. $\dot{Q} \equiv \frac{Q'}{Q}$) and totally differentiating (4.3), (4.2) and (4.4) respectively gives

$$(n+1+\epsilon)\dot{X} = \left(\frac{n+1+\epsilon}{m+1+\epsilon}\right)\dot{m} + \dot{n}, \tag{B.10}$$

$$(2+\epsilon)\dot{X} = 2\dot{m} + \dot{K}_H, \tag{B.11}$$

$$(2+\epsilon)\dot{X} = \left(\frac{1+\epsilon}{m+1+\epsilon}\right)\dot{m} + 2\dot{n}, \tag{B.12}$$

where t and K_F hold constant. (B.10), (B.11) and (B.12) are the three equations that have the three unknowns $\dot{X}(=\dot{Q})$, $\dot{m}$ and $\dot{n}$, which can be solved explicitly as a function of $\dot{K}_H$:

$$\dot{X} = -\left(\frac{2n+1+\epsilon}{\Omega}\right)\dot{K}_H,$$
$$\dot{m} = -\left(\frac{(m+1+\epsilon)(2n+\epsilon)}{\Omega}\right)\dot{K}_H,$$
$$\dot{n} = -\left(\frac{n+1+\epsilon}{\Omega}\right)\dot{K}_H,$$

where Ω is exactly the same as before. Evaluated at $X = \hat{X}, m = \hat{m}, n = \hat{n}$, we have that

$$\frac{\partial \hat{X}}{\partial K_H} = -\left(\frac{2\hat{n}+1+\epsilon}{\hat{\Omega}}\right)\frac{\hat{X}}{K_H} < 0, \tag{B.13}$$

$$\frac{\partial \hat{m}}{\partial K_H} = -\left(\frac{(\hat{m}+1+\epsilon)(2\hat{n}+\epsilon)}{\hat{\Omega}}\right)\frac{\hat{m}}{K_H} < 0, \tag{B.14}$$

$$\frac{\partial \hat{n}}{\partial K_H} = -\left(\frac{\hat{n}+1+\epsilon}{\hat{\Omega}}\right)\frac{\hat{n}}{K_H} < 0. \tag{B.15}$$

Further, since $\hat{Q} = \hat{X}$, $\hat{Q} = \hat{m}\hat{q}$ and $\hat{X} = \hat{n}\hat{x}$, we have $\frac{\partial \hat{Q}}{\partial K_H} = \hat{m}\frac{\partial \hat{q}}{\partial K_H} + \hat{q}\frac{\partial \hat{m}}{\partial K_H}$ and $\frac{\partial \hat{X}}{\partial K_H} = \hat{n}\frac{\partial \hat{x}}{\partial K_H} + \hat{x}\frac{\partial \hat{n}}{\partial K_H}$, and using (B.13), (B.14) and (B.15) yields

$$\frac{\partial \hat{q}}{\partial K_H} = \left(\frac{(\hat{m}+\epsilon)(2\hat{n}+\epsilon)-1}{\hat{\Omega}}\right)\frac{\hat{q}}{K_H} > 0,$$
$$\frac{\partial \hat{x}}{\partial K_H} = -\left(\frac{\hat{n}}{\hat{\Omega}}\right)\frac{\hat{x}}{K_H} < 0.$$

Using the expressions of $\frac{\partial \hat{X}}{\partial K_H}$ and $\frac{\partial \hat{m}}{\partial K_H}$ in (B.13) and (B.14), we also have that

$$\frac{\partial \hat{P}}{\partial K_H} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial K_H} = -\left(\frac{2\hat{n}+1+\epsilon}{\hat{\Omega}}\right)\frac{\hat{Q}P'(\hat{Q})}{K_H} > 0,$$
$$\frac{\partial \hat{r}}{\partial K_H} = g_x(\hat{X},\hat{m})\frac{\partial \hat{X}}{\partial K_H} + g_m(\hat{X},\hat{m})\frac{\partial \hat{m}}{\partial K_H} = -\left(\frac{1}{\hat{\Omega}}\right)\frac{\hat{X}g_x(\hat{X},\hat{m})}{K_H} > 0.$$

Next, differentiating $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \hat{X}}{\partial K_H}$ and $\frac{\partial \hat{m}}{\partial K_H}$ in (B.13) and (B.14), we have that

$$\frac{\partial \hat{z}}{\partial K_H} = \left(\frac{(\hat{m}+\epsilon)(2\hat{n}+\epsilon)+(\hat{n}-1)}{\hat{\Omega}} \right) \frac{\hat{z}}{K_H} > 0,$$
$$\frac{\partial \left(\frac{\hat{P}-\hat{r}}{\hat{r}-c-t} \right)}{\partial K_H} = \left(\frac{(\hat{m}-1)(\hat{n}+\epsilon)+(\hat{m}\hat{n}-1)}{\hat{\Omega}} \right) \frac{k}{\hat{z}K_H} > 0.$$

These derivations establish the desired results.

Although we have focused on comparative statics with respect to K_H, it is straightforward to examine comparative statics with respect to K_F. Holding t and K_H constant, totally differentiating (4.3), (4.2) and (4.4) respectively gives

$$\frac{\partial \hat{X}}{\partial K_F} = -\left(\frac{2(\hat{m}+1+\epsilon)}{\hat{\Omega}} \right) \frac{\hat{X}}{K_F} < 0,$$
$$\frac{\partial \hat{m}}{\partial K_F} = -\left(\frac{(2+\epsilon)(\hat{m}+1+\epsilon)}{\hat{\Omega}} \right) \frac{\hat{m}}{K_F} < 0,$$
$$\frac{\partial \hat{n}}{\partial K_F} = -\left(\frac{(2\hat{m}+\epsilon)(\hat{n}+1+\epsilon)}{\hat{\Omega}} \right) \frac{\hat{n}}{K_F} < 0.$$

Note that the signs of $\frac{\partial \hat{X}}{\partial K_F}$, $\frac{\partial \hat{m}}{\partial K_F}$ and $\frac{\partial \hat{n}}{\partial K_F}$ are the same as those of $\frac{\partial \hat{X}}{\partial K_H}$, $\frac{\partial \hat{m}}{\partial K_H}$ and $\frac{\partial \hat{n}}{\partial K_H}$ respectively. Intuitively, this equivalence comes from the firm-colocation effect. The signs of $\frac{\partial \hat{P}}{\partial K_F}$ and $\frac{\partial \hat{r}}{\partial K_F}$ are also the same as those of $\frac{\partial \hat{P}}{\partial K_H}$ and $\frac{\partial \hat{r}}{\partial K_H}$:

$$\frac{\partial \hat{P}}{\partial K_F} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial K_F} = -\left(\frac{2(\hat{m}+1+\epsilon)}{\hat{\Omega}} \right) \frac{\hat{Q}P'(\hat{Q})}{K_F} > 0,$$
$$\frac{\partial \hat{r}}{\partial K_F} = g_x(\hat{X},\hat{m})\frac{\partial \hat{X}}{\partial K_F} + g_m(\hat{X},\hat{m})\frac{\partial \hat{m}}{\partial K_F} = -\left(\frac{2\hat{m}+\epsilon}{\hat{\Omega}} \right) \frac{\hat{X}g_x(\hat{X},\hat{m})}{K_F} > 0.$$

However, the signs of $\frac{\partial \hat{z}}{\partial K_F}$ and $\frac{\partial \left(\frac{\hat{P}-\hat{r}}{\hat{r}-c-t} \right)}{\partial K_F}$ are opposite to those of $\frac{\partial \hat{z}}{\partial K_H}$ and $\frac{\partial \left(\frac{\hat{P}-\hat{r}}{\hat{r}-c-t} \right)}{\partial K_H}$. Differentiating $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ that are derived from (4.5) and (4.6) and using the expressions of $\frac{\partial \hat{X}}{\partial K_F}$ and $\frac{\partial \hat{m}}{\partial K_F}$ above,

$$\frac{\partial \hat{z}}{\partial K_F} = -\left(\frac{\hat{m}(\hat{n}+\epsilon)+\hat{n}(\hat{m}+\epsilon)-2(1+\epsilon)}{\hat{\Omega}} \right) \frac{\hat{z}}{K_F} < 0,$$
$$\frac{\partial \left(\frac{\hat{P}-\hat{r}}{\hat{r}-c-t} \right)}{\partial K_F} = -\left(\frac{\hat{m}(\hat{n}+\epsilon)+\hat{n}(\hat{m}+\epsilon)+\epsilon(1+\epsilon)}{\hat{\Omega}} \right) \frac{k}{\hat{z}K_F} < 0.$$

The last comparative statics results can easily be seen in Figure 4.5. As K_F increases, only the ***RFE*** curve shifts down in $(z, \frac{P-r}{r-c-t})$ space, whereby both $\hat{z}$ and $\frac{\hat{P}-\hat{r}}{\hat{r}-c-t}$ decrease in long-run equilibrium. In contrast to the case of K_H, the fact that $\hat{z}$ decreases with K_F implies that, although both $\hat{m}$ and $\hat{n}$ are lowered by an increase in K_F, $\hat{n}$ declines relatively more than $\hat{m}$. □

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