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Corruption, Market Quality and Entry Deterrence in Emerging Economies
(Revised)

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Corruption, Market Quality and Entry Deterrence in Emerging Economies[1]

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Abstract

In many emerging economies corruption, poor quality of information and poor governance lead to restricted entry. In this paper we analyze the determinants of the .height.of entry barrier in a developing economy where established .rms often use dubious means to deter entry of other .rms. We analyse this scenario in a three-stage game of entry deterrence. The incumbent has incomplete information about the entrant.s costs but can increase this cost by resorting to unfair means (for example, bribing a politician who harms the entrant). Higher is the bribe, higher will be the entry cost and hence lower will be the incentive to enter. In our set-up bribe serves as a proxy for .height.of entry barrier. The entrant observes its cost and decides whether or not to enter. We completely characterise the optimal bribe and show that this depends on the market size, the .di¤erentiation.parameter (whether goods are substitutes or complement) and the extent of uncertainty. Uncertainty seems to increase bribe and decrease market quality. We also show that zero bribe need not maximise total surplus and market quality. Our results seem to be compatible with anecdotal evidences from an emerging economy like India.

Keywords: market quality, entry deterrence, bribe, Cournot
JEL Classification: L11, L13, O16, O17

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1 Introduction

How difficult it is for a potential entrant to enter a particular market is a very important factor that determines the 'quality' of the market. Entry barrier is of particular importance in an emerging economy in which information is far from complete. Consequently, uncertainty becomes an important determinant of the 'height' of the entry barrier. Greater is the height of the entry barrier, more difficult it becomes to gain access into the market. Despite this fact, most of the papers in the existing literature have treated entry barrier primarily as an exogenous object and have focused on whether or not a firm chooses to set up an entry barrier[1]. That is, entry barrier is typically analyzed as a zero-one choice (either it is chosen or not chosen). This paper focusses on the *extent* of entry barriers (i.e. how 'high' is the barrier) and demonstrates that incompleteness of information (together with market size and the differentiation parameter) is an important factor that determines the 'height' of entry barrier in the market.

In many emerging economies, corruption and poor governance have led to restricted entry. Established firms are wary about the technological superiority of potential entrants. Being uninformed of the technological efficiency of its potential rivals, many established firms adopt dubious means to deter entry of more efficient firms. Apart from spreading corruption and poor governance this also thwarts competition. The present study deals with such a phenomenon. A similar phenomenon is observed in developed economies where lobbying leads to restricted entry. The common perception is that incumbent firms in emerging economies, out of fear of competition from more efficient potential entrants, are more likely to pay bribes to get their desired objective, whereas firms in developed countries are more prone to lobby the government to change the rules. Note that lobbying is a legal and regulated activity in many countries, whereas bribery is not. However, all such activities tend to lower 'market quality'.[2]

[1] See the relevant chapters of Tirole (1988) and Vives (1999).

[2] Note that in a developed economy, incumbent firms often use lobbying to create entry barriers. An example which involves Airbnb, illustrating this aspect is as follows. In 2016, under pressure from the hotel industry and a populace concerned with the surge of foreigners in their neighborhoods, the government in Japan released guidelines for home sharing - called minpaku in Japanese - that made most Airbnb rentals in

We analyze this scenario in a three-stage game of entry deterrence. In our set-up the incumbent has incomplete information about the entrant's costs but can increase this cost by resorting to unfair means (for example, bribing a politician who harms the entrant). Higher is the bribe, higher will be the increase in the entry cost (entrant's MC) and hence lower will be the incentive to enter. That is, in our set-up bribe serves as a proxy for the 'height' of entry barrier. The entrant observes its cost and decides whether or not to enter. We completely characterize the optimal bribe ('height' of entry barrier) and show that the market size, the differentiation parameter and the extent of uncertainty affect the optimal 'height' and consequently, the entry decision by the potential entrant. Uncertainty plays a crucial role and we also analyze how it affects total surplus and 'market quality'. Our results seem to be compatible with anecdotal evidences from an emerging economy like India.

We now proceed to say a few words on the concept of 'market quality' and 'fairness'.

1.1 Efficiency, fairness and market quality

This study is the first attempt to deal with the endogenous determination of 'market quality' as defined by Yano (2009, 2016), who refers to 'market quality' as a measure of "efficiency in allocation" and "fairness in pricing" in a market.[3] Efficiency refers to Pareto efficiency. Fairness may be stated as fairness in dealing or in the process in which the terms of trade are formed. A price formed through fair dealing is a fair price.[4] Fair dealing should be measured against a set of rules and laws imposed so as to maintain the well functioning of a market.

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the country illegal. Under these guidelines Airbnb hosts were only allowed to rent to guests who would stay for a week or longer, a minuscule slice of the market. This is clearly a case where lobbying by the incumbent industry has successfully created entry barriers for the new entrant, Airbnb (for details see Nakamura, Y. and M. Takahashi in *Bloomberg*, February 19, 2016).

[3] There are a couple of papers that study the endogenous determination of market quality (although in a much looser sense). Dei (2011) analyses the dynamic development of a high quality labor market where unskilled and skilled workers are properly distinguished . The paper by Furukawa and Yano (2014) studies market quality by focusing on fairness in handling intellectual properties.

[4] "Actions in a particular market are competitively fair if they are conducted in compliance with the set of "generally accepted" rules. Moreover, a state of that market is competitively fair if it is formed through competitively fair actions and if there are no profit opportunities left available for competitively fair actions" (Yano, 2009).

According to Yano (2008a), one such rule may be the non-discriminatory treatment of actual and potential trading partners or, in other words, to ensure free entry and exit in the market. However, *such fairness cannot be guaranteed when one party (say one seller) has significant and unilateral powers (for instance, the power to use violence) enabling it to unilaterally set the terms and change the rules of the game.* This is often observed in emerging economies.

Three primary factors determine market quality. These primary factors are, "quality of competition", "quality of information", and "quality of products". In this paper we focus on two of these: (i) "quality of competition"- to what extent entry is restricted and (ii) "quality of information"- what is the extent of incompleteness of information affect 'fairness' and market quality'. Yano (2009 and 2016) postulate that a decline in the quality of competition and information often reduce 'fairness' and market quality. Broad pattern of historical events seem to support this idea.[5]

In our context, fairness, ϕ, is defined to be $\phi = -b$, where b is the bribe paid (= 'height' of entry barrier). In other words, fairness is the opposite of the 'height' of entry barrier. When $b > 0$, a cost is inflicted on the entrant through unfair (and illegal) means and the total fairness is negative. Higher is the bribe (i.e. more is the 'height' of entry barrier), lower will be the fairness. Lower is the fairness, higher will be the increase in the entrant's costs and hence lower will be the incentive to enter. In our set-up the maximum possible fairness is zero when no bribe is paid (absolutely no entry barrier). This will be true when the governance is perfect. Some of the Scandinavian countries (for example, Denmark) possibly have very high fairness as there is little bribery there. However, when the governance is

[5] One of the things that support this hypothesis is that a series of industrial revolutions and economic crises over the past two hundred years, tend to have a cyclical pattern and all such events were probably triggered by changes in market quality. The First Industrial Revolution gave rise to the exploitation of industrial workers, a major labor issue. The Second Industrial Revolution was followed by the formation of industrial monopolies, the Great Depression, and massive unemployment. The exploitation of workers and the monopolization of industries occurred because competition was imperfect, and the Great Depression occurred because information was not properly shared. The subprime loan crisis of 2008 was a result of poor quality information and greed that compelled people to take on debts they could never repay. There seems to be a common pattern of events. The advent of technological innovation is typically followed by a decline in the quality of competition and information, and this reduced market quality and this turn led to the economic crisis.

relatively poor (which is very likely in emerging economies like India, Pakistan, Bangladesh etc.), the fairness is likely to be negative. We define market quality to be the sum of ‘total surplus’ and ‘fairness’.[6]

In many emerging economies the law enforcement agencies are often weak and very corrupt and this means that big firms can get away by harming others illegally. Such an act increases the marginal cost of the potential entrant and the incumbent is able to unilaterally set the terms of the game to its advantage.[7]

In a country like India, the above scenario is often observed in many industries (for example, the real estate industry). Such an industry is dominated by a few large firms, who are, in general, close to powerful politicians and function under their patronage.[8]

Such firms take recourse to illegal means to discourage entry. Since the owners of such firms are well connected, they get away with their illegal actions. Even if the entrant (which may be a relatively small local firm or a new foreign firm, with little contacts with local politicians) were to lodge a complaint against the big incumbent firm, nothing is likely to happen. In countries like India, Pakistan, Bangladesh etc. it is almost taken for granted that the rich and powerful will not be touched even if they are on the wrong side of law. In fact, ordinary persons and entities are often hounded if they take on powerful organizations or persons. This fact is quite well known and there are several media reports on this as well.[9]

This implies that a potential entrant will not waste its time and resources in pursuing a

[6] ‘Total surplus’ is ‘consumer surplus’ plus ‘producer surplus’ plus ‘bribe’ (as bribe is just a mere tranfer and remains within the system). Essentially, this menas that ‘market quality’ is equal to consumer surplus’ plus ‘producer surplus’. In a more general setting, market quality is a function of "total surplus" and "fairness" and it is increasing in these two components. Here, for simplicity, we take an additive form.

[7] As noted earlier, the incumbent firm can bribe the local politician to harm the potential entrant. This can take the form of physical violence or some other forms of harassment. This bribe can also be interpreted as cost of hiring goons to threaten and even physically harm the entrant and since the law and order enforcing agencies do not function properly, this strategy is often very successful in countries like India.

[8] Sometimes politicians themselves or their family members are among the largest shareholders of such companies. The “Competition Commission of India” in its several annual reports have documented unfair practises by large Indian firms. All such reports are available on the website <<http://www.cci.gov.in/>>).

[9] The report by Ernst and Young (2012) provides details on bribery and corruption in the construction sector.

legal case against the big incumbent as this will simply increase its costs further without any possible benefit. In short, in such economies, if the incumbent harms the entrant through illegal means and increases the entrant's operating costs, the entrant just accepts this as fait acompli and then decides whether to enter or not. By analyzing this aspect, this study shows how the level of fairness in dealing and of efficiency in allocation, or in short, market quality, are determined simultaneously.

We now proceed to provide the following: (i) a brief overview of our framework (ii) a summary of our main results and (iii) a discussion of the literature related to our framework.

1.2 Overview and some definitions

We consider the following model in a differentiated product market. There is one incumbent and one potential entrant. The incumbent firm can bribe the local politician to harm the potential entrant. We capture the effect of such a bribe in the following way. If the incumbent firm pays bribe $b \geq 0$ then this increases the marginal cost of the entrant by b. This b, as noted before, serves as a proxy for the 'height' of entry barrier.

As noted before total fairness is defined to be $\phi = -b$. When $b > 0$, a cost is inflicted on the entrant through unfair (and illegal) means and the total fairness is negative. The maximum possible fairness is attained when bribe is zero (free entry is assured).

Market quality ($\mathbb{Q}$) is defined to be a sum of surplus (W) and fairness (ϕ). That is, we have

$$\mathbb{Q}(b) = W(b) + \phi \ = W(b) - b.$$

It is often the case that in the *absence of bribe* (zero 'height' of entry barrier) such potential entrants have lower marginal costs than the incumbent. Absence of any bribe means that total fairness is at its maximum. *In other words, in our framework, when the market is completely fair, the entrant has lower marginal cost than the incumbent.* We capture this in the following way. Let the incumbent firm's marginal cost be c. In the *absence of any bribe* (i.e. when $b = 0$) the potential entrant's marginal cost is $c - \alpha$, where α is the *efficiency* level of the entrant. Higher is the entrant's *efficiency*, lower will be its marginal cost. The incumbent cannot observe the entrant's type, α. We assume that α is distributed over $[0, \bar{\alpha}]$ with distribution function $F(.)$ and density function $f(.)$.

Note that $\bar{\alpha}$ *captures the extent of uncertainty.* When $\bar{\alpha} = 0$, there is no uncertainty and the players play a game of complete information. When $\bar{\alpha} > 0$, the game is that of incomplete information.

Some possible reasons behind such asymmetric costs and incomplete information are as follows. The potential entrant may be a foreign firm with a superior (low cost) technology. This may also be true in an emerging economy where the potential entrant is a small local firm. The incumbent firm is typically very large, has a large bureaucracy and it draws labour from the *formal sector*, where wages are higher as compared to the informal sector. This pushes up the per unit cost of the incumbent. On the other hand, the small local firm has access to the *informal labour market* and consequently it can pay lower wages. Also, the size of its bureaucracy is smaller and this means its per unit costs are lower as compared to the incumbent.[10]

Often, the incumbent firm does not know the true cost of operation of such an entrant. That is, α is private information to the entrant. If the potential entrant is a foreign firm with a superior technology the incumbent may not be aware of the extent of the technological superiority. When the potential entrant is a small local firm then also the incumbent may face incomplete information. As noted before the small local firm has access to the *informal labour market*. Such a labour market is completely unregulated and wages are often decided by informal bargaining. Consequently, wages are known only to the small local firm and the labourer. Moreover a small local firm is more likely to have a better idea of the cultural aspects like caste/community equations. Consequently, it may be able to get labour (and even other physical inputs like bricks, sand, cement) cheaply. Since the incumbent is an established entity, it is required (by law) to purchase inputs and hire labour from the formal sector where prices and wage rates are typically known. As such, its costs are generally known to everybody.

Since the entrant's marginal cost is lower, it means that if no bribe was paid by the

[10] In a country like India an overwhelming fraction of the labour force (about 90%) is employed in the informal sector. Often, small local firms have much better access to this labour force than the large established firms.

incumbent (i.e. $b = 0 \Leftrightarrow \ height = 0$), then entry would be certain. In this paper we try to capture this aspect, derive some results and offer some policy prescriptions to improve market quality.

1.3 Summary of our findings

We analyze the following three-stage game. In the first-stage the incumbent (firm 1) decides on a level of bribe, b ('height' of entry barrier). In the second stage the entrant (firm 2) observes its own marginal cost and then decides to enter or not to enter. In case it chooses to enter, 2 incurs an entry cost. If 2 enters, then in the third stage the firms play an incomplete information Cournot game in a differentiated good market. If 2 does not enter then 1 produces monopoly output.

We solve the three-stage game by backward induction. We first compute the Bayesian-Nash equilibrium of the third stage game.

Then we characterize firm 2's decision in the equilibrium of the second stage. We show that if the bribe (the 'height' of entry barrier) which has been chosen by firm 1 in the first stage is below a threshold ($\underline{b}$), then all types of firm 2 enter. That is, probability of entry is one. If the bribe (the 'height' of entry barrier) lies in between $\underline{b}$ and $\bar{b}$ (where $\bar{b} > \underline{b}$) then some types of firm 2 enter. We show specifically that 2 will enter iff it's efficiency (α) is higher than a critical type α^*. This α^* depends on the level of bribe (how high is the entry barrier). Higher is the level of bribe, higher will be α^*. In this case the probability of entry is $1 - F(\alpha^*)$. If the bribe chosen in the first period is greater than or equal to $\bar{b}$, then no type of firm 2 will enter (this is a case of blockaded entry). That is, a 'height' greater than or equal to $\bar{b}$ blocks entry completely.

Thereafter, we solve the first stage game when firm 1 chooses the optimal level of bribe. Note that 1 will choose a bribe to maximize its expected payoff after taking into account the possible equilibrium outcomes in the second and third stages. Since a bribe b ('height' of entry barrier) increases 2's per unit cost, 2's profit will be decreasing in b. This means that for b high enough 2's expected payoff will be zero and 2 will not enter. However bribing is also costly for 1 since b is like a sunk cost to firm 1.

We show that when goods are substitutes (i.e. $\gamma > 0$) then, if the differentiation parameter, γ, and the market size (denoted by A in our model) are small enough then the optimal bribe, b^*(optimal 'height' of entry barrier)), is zero. If γ and the market size are large enough then the optimal bribe (optimal 'height' of entry barrier) lies in the interval $\left[\underline{b}, \bar{b}\right]$. Under no circumstances will the optimal bribe lie in the interval $(0, \underline{b})$. This means, if there is a strictly positive bribe in equilibrium, then it must be greater than or equal to $\underline{b}$. When goods are complements (i.e. $\gamma < 0$), we show that the optimal bribe (optimal 'height' of entry barrier) is always zero (free entry is assured in equilibrium).

Thereafter, we proceed to discuss the case where there is no uncertainty ($\bar{\alpha} = 0$). Here the firms play a game of complete information. In this case we show that the optimal bribe (optimal 'height' of entry barrier) is either zero or some positive amount, $\hat{b}$ where $\hat{b} < \underline{b}$. The entrant is either clearly in (bribe paid is zero) or clearly out (blockaded entry with bribe equal to $\hat{b}$). Note that with complete information, if γ is small enough then the optimal bribe (optimal 'height' of entry barrier) is zero. This is similar to our result with incomplete information. However, if γ is large enough and market size, A, is above a critical level then optimal bribe (optimal 'height' of entry barrier) is $\hat{b}$ and there is completely blockaded entry. Note that when there is uncertainty ($\bar{\alpha} > 0$) and there is positive bribe in equilibrium (i.e. $b^* > 0$), it is at least $\underline{b}$. With no uncertainty ($\bar{\alpha} = 0$), the amount of equilibrium bribe ('height' of entry barrier), when positive, is equal to $\hat{b}$. *That is, equilibrium bribe ('height' of entry barrier), when positive, is more with incomplete information than with complete information (since $\hat{b} < \underline{b}$). This means, uncertainty increases bribe (or 'height' of entry barrier) and hence decreases probability of entry.* When goods are complements, with no uncertainty ($\bar{\alpha} = 0$) the optimal bribe is always zero. This is similar to the case where there is incomplete information.

Next we proceed to seek an answer to the following question. *Does an increase in the uncertainty lead to a increase in equilibrium bribe ('height' of entry barrier) and a decrease in market quality?* While a general answer to such a question is intractable, we produce an example, where 2's types are uniformly distributed, to demonstrate that an increase in uncertainty indeed leads to an increase in bribe ('height' of entry barrier) and a decrease in market quality. This seems to be a vindication of the idea in Yano (2009 and 2016)

A natural question that arises is the following: Does zero bribe (i.e. zero 'height' which implies certain entry) always maximize total surplus and market quality? When goods are substitutes, our answer is surprisingly negative. We provide an example to show that while zero bribe (no entry barrier) always maximizes 'fairness' (the maximum possible value of total fairness, $\phi = -b$ is zero), it need not maximize 'total surplus' or 'market quality'. This result is somewhat related to Mankiw and Whinston (1986), who introduced the concept of 'business stealing'. "The business-stealing effect exists when the equilibrium strategic response of existing firms to new entry results in their having a lower volume of sales - that is, when a new entrant "steals business" from incumbent firms. Put differently, a business-stealing effect is present if the equilibrium output per firm declines as the number of firms grows." (Mankiw and Whinston, 1986). In their paper it is shown that when entrants incur a fixed set-up cost of entry and when there is "business stealing effect" then free entry is not welfare maximizing. It may be noted that when $\gamma > 0$ (goods are substitutes) we have the "business stealing effect" in our model. In our model $b = 0$ implies certain entry but this need not maximize total surplus (as well as market quality) when goods are substitutes. It raises intriguing questions as to how bribe affects market quality.

However, when goods are complements, we show that under some parametric restrictions, zero bribe (zero 'height') maximizes total surplus and market quality. Note that the incumbent firm always choose zero bribe in the first-stage when goods are complements. An interesting policy prescription that emerges is as follows: In order to curb bribery (i.e. increase entry) and improve market quality, the government should foster competition in goods that are complements to each other. In the real estate sector an example of such complements would be where one firm provides residential housing and the other firm provides shopping malls in the same locality.

We now proceed to provide some remarks on our results.

Remarks

1. In our model firm 2's costs are private information (as α is known only to firm 2). With complete information ($\bar{\alpha} = 0$), when firm 1 takes a decision regarding bribe, b (height of entry barrier), it knows with certainty whether entry will be deterred or not (that is, probability of entry is either zero or one). However, when α is private information

then the equilibrium probability of entry lies between zero and one. In this case firm 1 is not exactly sure whether a bribe b (height of entry barrier) will deter entry or not (unless $b \geq \bar{b}$). There is an uncertainty regarding the impact of bribe on 2's entry decisions. This is compatible with evidences from real life. When an incumbent takes some irrevocable decision to deter entry, the incumbent is often not too sure whether entry will be successfully deterred or not. *It only knows that higher b (greater height) is more likely to deter entry.* We have tried to capture this aspect in our model.

2. It is often the case that although the incumbent knows that it possibly has an inferior technology (higher marginal cost) than the potential entrant, it does not know the extent of its own cost disadvantage. That is, the incumbent cannot observe the entrant's type (cost). This uncertainty (or incompleteness of information) is highlighted in the present study. It leads to three different findings:

   (a) First, if the differentiation parameter and the market size is low enough, the optimal bribe (optimal height of entry barrier) is zero regardless of whether there is incomplete information ($\bar{\alpha} > 0$) or complete information ($\bar{\alpha} = 0$).

   (b) If the differentiation parameter and the market size is high enough, the optimal bribe (optimal height of entry barrier) is positive under both complete and incomplete information. However, in this case, the amount of equilibrium bribe (height of entry barrier) is larger with incomplete information than with complete information. That is, incompleteness of information seems to be bribery-promotive rather than being bribery-preventive. This result stands somewhat in contrast to Maskin (1999).

   (c) Third, we analyze an example where the bribe is positive under incomplete information. That is, $b^* \in \left[\underline{b}, \bar{b}\right]$. We demonstrate that an increase in uncertainty leads to an increase in bribe (increase in height or a reduction in fairness) and a decrease in market quality.

3. Some of our results seem to be compatible with anecdotal evidences from an emerging economy like India. For example, in emerging economies like India, bribery is very

common. Our results also indicate that positive bribe in equilibrium (which in turn implies restricted entry) is more likely if the market size is large. It may be noted that the market size will be larger when income levels are higher. Anecdotal evidences suggest that bribes are higher in places that are relatively more prosperous. For example, the real estate business in the rich neighborhoods of major cities in India are marked by extreme corruption and bribery. Moreover, in such prosperous pockets the real estate industry is dominated by big firms. Only in less prosperous places one normally finds smaller local firms. Our results provide a possible theoretical explanation behind such evidences (see the report by Ernst and Young, 2012).

1.4 Related Literature

The idea of 'market quality' is borrowed from Yano (2009, 2016). Dastidar (2017) provides more details on this aspect.[11]

There is a huge literature on entry barriers. For a succinct survey on various aspects of entry deterrence see Tirole (1988), Shepherd (1997), Vives (1999) and Pepall et al (2008).

Salop and Scheffman (1983,1987) point out that imposing higher costs on a rival can tame or kill it as effectively as predatory pricing, and possibly at a lower cost to the dominant firm. In many emerging economies, where the law and order machinery is very weak, this increase in rivals' cost can be achieved through the abuse of government procedures, including sham litigation and the misuse of licensing and regulatory authorities. There may be many types of regulatory hurdles for new entrants in a market, including controls by licensing authorities, health and building inspectors and planning boards and an established firm often can bribe government officials to impose large costs on a potential entrant. In such economies big influential domestic firms may use sham proceedings under import relief laws, e.g. an unwarranted claim of dumping, to engage in non-price predation against a foreign

[11] Market quality economics is a field that was born directly out of Yano's research. Its basic idea is that, just as there are high-quality and low-quality products, there are also high-quality and low-quality markets. High-quality markets are enriching; low-quality markets are impoverishing. Yano (2006, 2008a and 2008b) provide some major results on several aspects on market quality. The recent paper by Dastidar and Dei (2014) provides a short introduction to the basic ideas on the concept of market quality.

rival. In our model the incumbent chooses bribe to inflict harm upon the entrant.[12]

Our approach is somewhat similar to Dixit (1980). In Dixit (1980) an irrevocable investment by the incumbent firm allows it to alter its own marginal cost curve and thereby the post entry equilibrium. The role of such an irrevocable investment in entry deterrence is to alter the initial conditions of the post-entry game to the advantage of the established firm. In our paper the incumbent pays some bribe which is similar to an irrevocable investment (since the bribe is like a sunk cost to the incumbent in the third stage). However, unlike Dixit (1980), in our model, such a bribe alters the marginal cost of the potential entrant and not the marginal cost of the incumbent. Moreover, unlike Dixit (1980) the post entry game in our exercise is an incomplete information game. It may be mentioned here that Dixit (1980) focusses entirely on the role of investment in impeding entry and his exercise is not linked to market quality. Our objective is to analyze how bribe ('height' of entry barrier) and uncertainty affects overall market quality.

The fact that zero bribe need not always maximize 'total surplus' is related to the work of Mankiw and Whinston (1986). Note that a business-stealing effect is present if the equilibrium output per firm declines as the number of firms grows. The presence of the business-stealing effect drives a wedge between the marginal entrant's evaluation of the desirability of entry and the social planner's. This means that free entry (which results in zero profit for all firms) need not be 'total surplus' maximizing. Similarly, in our model there is 'business stealing' when goods are substitutes. The bribe, $b = 0$ implies certain entry in our model, but this need not always be 'total surplus' maximizing (or 'market quality' maximizing).

Note that if an additional competitor reduces output per firm in a homogenous Cournot oligopoly (there is business stealing), market entry will be excessive (from a welfare viewpoint). Taxes can correct the so-called business stealing externality. Goerke (2017) investigates how evading a tax on operating profits affects the excessive entry prediction. Tax evasion raises the number of firms in market equilibrium and can alter their welfare maxi-

[12] It is interesting to note that even physical violence (bombings and beatings) had been used by incumbent firms in the past to tame rivals. In pages 253-255 of his book, Martin (2010) provides a summary of this theory of 'Raising Rival's Costs' and also gives specific real life examples of such cost-raising strategies chosen by the incumbent.

mizing number. Consequently, evasion can aggravate or mitigate excessive entry.

Maskin (1999) analyses a model where capacity installation by an incumbent firm serves to deter others from entering the industry. The uncertainty about demand or costs forces the incumbent to choose a higher capacity level than it would under certainty. This higher level diminishes the attractiveness of deterrence under uncertainty. In our model, we show that incompleteness of information may result in more bribes (which dis-incentivises entry) as compared to the case of complete information. Our results stands somewhat in contrast to the results in Maskin (1999).

There is a huge literature on corruption but very few papers are directly related to the approach taken in the present exercise. Some papers that are somewhat related to our paper are discussed below. It may be noted that the models and the focus of these papers are very different from ours.

The paper by Shleifer and Vishny (1993) shows that structure of government institutions and the political process are important determinants of corruption. In their framework government officials have some discretion over some economic activities (for example, issuing licence/permits to produce) and this enables them to collect bribes from private agents. In our model, weak law enforcement machinery allows the incumbent to harm potential entrants through unscrupulous means and deter entry.

Harstad and Svensson (2011) analyze a model where faced with a regulatory constraint, firms can either comply, bribe the regulator to get around the rule, or lobby the government to relax it. This model explains the common perception that bribery is relatively more common in poor countries, whereas lobbying is relatively more common in rich ones.

Campos, Estrin and Proto (2010) argue that corruption matters not so much because of the value of the bribe, but because of another less studied feature of corruption, namely bribe unavoidability. The paper by Emran and Shilpi (2000) constructs simple asymmetric information models to analyze the effects of bureaucratic corruption on entry conditions and output. Sequeria and Djankov (2013) analyze how the structure of public bureaucracies determines the way in which public officials set bribes. Broadman and Recanatini (2000) develop an analytical framework for examining the role basic market institutions play as determinants of rent-seeking and illicit behavior in transition economies. The papers by

Athreya and Majumdar (2005) and Lambert-Mogiliansky, Majumdar and Radner (2008) deal with some aspects of petty bureaucratic corruption and its efficiency implications. The paper by Kunieda and Shibata (2014) considers an economy with credit market imperfections (that typically characterize a low quality financial market with weak enforcements rules) and analyses how changes in the degree of credit constraints affect economic fluctuations.

Plan of the paper We provide the model of our exercise in section 2. Thereafter, in section 3 we derive the equilibrium of our three-stage game and provide the major results on optimal bribe. Section 4 provides some basic computations of total surplus and market quality. In section 5 we deal with uncertainty and market quality. Section 5 provides an analysis of bribe and market quality. In section 6 we give our concluding remarks. Lastly, in the appendix we provide the details of our equilibrium computations and the proofs of all the results.

2 The Model

We consider the following scenario in an emerging economy market. There is one incumbent (firm 1) and one potential entrant (firm 2). Firm 1's per unit cost is c. When firm 1 does not resort to any unfair means (it pays no bribe) firm 2's per unit cost is $c - \alpha$, where α is the efficiency level of firm 2. This α is private information to firm 2. Firm 1 cannot observe α. We assume that α is distributed over $[0, \bar{\alpha}]$ (where $\bar{\alpha} < c$) with distribution function $F(.)$ and density function $f(.)$. This means if firm 1 does not resort to any unfair means then firm 2 has a cost advantage.

However, the incumbent can increase the cost of operation of firm 2 by paying some bribe (say b) to the politician, who in turn can harm the potential entrant. If firm 1 pays b then this increases the marginal cost of firm 2 by b. As noted before, b is the 'height 'of entry barrier.

Let k^2 (where $k > 0$) be the entry cost of firm 2. We provide the cost function of the incumbent (firm 1) and the entrant (firm 2) below. Let firm 1 choose $b \geq 0$. This bribe is like a sunk cost for firm 1. Then we have the following:

$$
\begin{aligned}
C_1(q_1) &= cq_1 + b \\
C_2(q_2, \alpha) &= (c + b - \alpha) q_2 + k^2
\end{aligned}
$$

α can be interpreted as efficiency of the entrant. This is private information to firm 2. We consider the following three-stage game.

1. In the first-stage the incumbent (firm 1) decides on a level of bribe, b (= 'height 'of entry barrier).

2. In the second stage the entrant (firm 2) observes its own marginal cost and then decides to enter or not to enter. It may be noted that observing its own marginal cost is equivalent to observing α and b. Typically, in emerging economies when an entrant takes an entry decision, it is based on its cost which it knows clearly. In case it chooses to enter, 2 incurs an entry cost, k^2. 2 decides to enter iff it expects strictly positive profit in the third stage. This is a standard assumption in the IO literature (see Dixit, 1980 and Tirole, 1988).

3. If 2 enters, then in the third stage the firms play an incomplete information Cournot game in a differentiated good market. If 2 does not enter then 1 produces monopoly output. Note that in this stage b is sunk cost for 1 and k^2 is a sunk cost for 2.

For the third-stage competition we consider a representative consumer's utility function based on Dixit (1979). Scores of papers in the literature have used this. A small sample of such papers is as follows: Singh and Vives (1984), Hackner (2000), Bester and Petrakis (1993), Zanchettin (2006), Pal (2010) and Alipranti, Milliou and Petrakis (2014).

On the demand side of the market, the representative consumer's utility function of two differentiated products, q_1 and q_2, and a numeraire good, q_0, is given by the following:

$$
U = a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0.
$$

The parameter γ measures the degree of product differentiation and $\gamma \in [-1, 1]$. When $\gamma < 0$ the goods are complements and when $\gamma > 0$ the goods are substitutes. Note that when γ is unity then the products are homogeneous (perfect substitutes) and when γ is zero the products are independent. We will consider cases where $\gamma \neq 0$.

The utility function generates the following system of inverse demand functions:

$$\begin{aligned} p_1 &= a - q_1 - \gamma q_2 \\ p_2 &= a - \gamma q_1 - q_2 \end{aligned}$$

2.1 Notations and Assumptions

Let

$$A = a - c,\ B = 2 - \gamma \text{ and } D = 4 - \gamma^2$$

Note that since $\gamma \in [-1, 1]$ we have $B \in [1, 3]$ and $D \in [3, 4]$. *We can interpret A to be a proxy for market size.*

Let

$$\mu(x) = \frac{1}{1 - F(x)} \int_x^{\bar{\alpha}} \alpha f(\alpha)\, d\alpha$$

Note that $\mu(0)$ is the expected efficiency of the entrant and $\mu(x)$ is the expected efficiency given that it is more than x. We assume the following.

Assumption 1 $A > \mu(0)$ and $A \geq 3k$.

Assumption 2 $\mu'(x) \in (0, 1)$ and $\mu''(x) \geq 0$ for all $x \in (0, \bar{\alpha})$.

Remark The first assumption holds true if the market size 'A' is high enough relative to entry cost and the expected efficiency of the entrant. It may be noted that the first assumption ensures that in the absence of any bribe being paid (i.e. with zero height of entry barrier) all types of firm 2 will enter. That is, with $b = 0$ entry occurs with probability one (see Lemma 1). The second assumption puts some restrictions on the distribution function of 2's types. This is required for some of our major results on optimal bribe. It may be noted that the second assumption always holds if α is uniformly distributed over $[0, \bar{\alpha}]$. It also holds for many other distributions.

3 Equilibrium analysis

We will solve the game backwards. We will first analyze the third-stage game. We now proceed to provide the equilibrium derivations.

3.1 Third stage equilibrium

If firm 2 chooses to enter in the second stage, then in the third stage the firms play an incomplete information Cournot game and earn duopoly profits. If firm 2 had chosen not to enter in the second stage, then firm 1 chooses monopoly output.

Now suppose 2 has chosen to enter in the second stage. In the appendix we provide the computation the Bayesian-Nash equilibrium of the third stage. In this stage, bribe level b has been determined previously and known to both firms, whereas the entrant's efficiency, α, is known only to the entrant. For any given b, let the Bayesian-Nash equilibrium be $q_1(b)$ (quantity choice by firm 1, which does not know α) and equilibrium choice by firm 2 (with type α) be $q_2(\alpha, b)$.

Note that we construct an equilibrium such that in the second-stage 2 will enter iff it's efficiency (α) is higher than a critical type α^*. That is, firm 2 enters iff $\alpha \in (\alpha^*, \bar{\alpha}]$. If 2 enters, then in the third stage firms 1 and 2 play an incomplete information Cournot duopoly game. The equilibrium outcome is provided below. The computation details are provided in the appendix.

Third-stage Bayesian Nash equilibrium

$$q_1(b) = \begin{cases} \frac{AB+\gamma b-\gamma\mu(\alpha^*)}{D} \text{ if 2 enters} \\ \frac{A}{2} \text{ if 2 does not enter} \end{cases}$$

$$q_2(\alpha, b) = \begin{cases} \frac{2AB+\alpha D-4b+\gamma^2\mu(\alpha^*)}{2D} \text{ if } \alpha \in (\alpha^*, \bar{\alpha}] \\ 0 \text{ if } \alpha \in [0, \alpha^*] \text{ (2 does not enter for such } \alpha) \end{cases}$$

Equilibrium profits at a Bayesian Nash equilibrium Routine computations show that the equilibrium duopoly profits are as follows.

$$\begin{aligned}\pi_1(b) &= [q_1(b)]^2 - b \\ \pi_2(\alpha, b) &= [q_2(\alpha, b)]^2 - k^2 \text{ where } \alpha \in (\alpha^*, \bar{\alpha}]\end{aligned}$$

3.2 2nd stage equilibrium

Given the equilibrium outcome in the third stage, we next proceed to analyze the second stage game. In the second stage firm 2 makes a move and chooses either to enter or not to enter. We analyze the relationship between bribe paid (height of the nerdy barrier) in the first-stage and firm 2's entry decision in the second stage. The proofs are given in the appendix.

We first analyze the behavior of firm 2 when its type is $\alpha = 0$ (i.e. the lowest possible type). First suppose that $b = 0$. That is, firm 1 does not resort to any bribing. Then, we show that even the lowest possible type ($\alpha = 0$) will get strictly positive payoff if it enters.

Lemma 1 *$\pi_2(0,0) > 0$.*

Remark Note that $q_2(\alpha, b)$ is strictly increasing in α. This implies that $\pi_2(\alpha, b)$ is strictly increasing in α. Hence, $\pi_2(\alpha, 0) > \pi_2(0, 0)$ for all $\alpha > 0$. This means that if $b = 0$ (firm 1 does nothing), all types (even the lowest efficiency type) will choose to enter. This fact is also intuitively obvious.

We now analyze the case where $b > 0$. Let

$$\begin{aligned}\underline{b} &= \frac{2AB + \gamma^2 \mu(0) - 2Dk}{4} \\ \bar{b} &= \frac{2AB + 4\bar{\alpha} - 2Dk}{4}\end{aligned}$$

Since $\gamma^2 \leq 1$ and $\bar{\alpha} > \mu(0)$, we clearly get that $\bar{b} > \underline{b}$. We proceed to our major results in the second-stage equilibrium.

Proposition 1 *When $b \in (0, \underline{b})$ then the following is true. (i) All types enter and the probability of entry is one. (ii) $\frac{\partial \pi_2(\alpha,b)}{\partial b} < 0$ and $\frac{\partial \pi_2(\alpha,b)}{\partial \alpha} > 0$.*

Proposition 2 *When $b \in (\underline{b}, \bar{b})$ then the following is true. (i) $\alpha^* \in (0, \bar{\alpha})$ and when firm 2's type is $\alpha \in [0, \alpha^*]$ it does not enter. When firm 2's type is $\alpha \in (\alpha^*, \bar{\alpha}]$ it enters. The probability of entry is $1 - F(\alpha^*)$. (ii) $\frac{\partial \pi_2(\alpha,b)}{\partial b} < 0$ and $\frac{\partial \pi_2(\alpha,b)}{\partial \alpha} > 0$.*

Remark Note that firm 2 enters iff it expects strictly positive profit in the post-entry game. Suppose $b = \underline{b}$. Then, if firm 2 when type $\alpha = 0$ (the lowest type) enters, its payoff would be $\pi_2(0, \underline{b}) = 0$. This implies, when $b = \underline{b}$ the lowest type will not enter. Note that $\pi_2(\alpha, \underline{b}) > \pi_2(0, \underline{b})$ for all $\alpha \in (0, \bar{\alpha})$. This means if $b = \underline{b}$ then all types $\alpha > 0$ will enter. Now suppose $b = \bar{b}$. Then, if firm 2 when type $\alpha = \bar{\alpha}$ (the highest type) enters, its payoff would be $\pi_2(\bar{\alpha}, \bar{b}) = 0$. This means type $\bar{\alpha}$ will not enter if $b = \bar{b}$. Since $\pi_2(\alpha, \bar{b}) < \pi_2(\bar{\alpha}, \bar{b})$ for all $\alpha \in [0, \bar{\alpha})$, no type will enter if $b = \bar{b}$. In short, propositions 1 and 2 and the discussion above imply the following:

1. $b \in [0, \underline{b}]$ then $\alpha^* = 0$. All types enter. Here the probability of entry is one.
2. If $b \in (\underline{b}, \bar{b})$ then $\alpha^* \in (0, \bar{\alpha})$. Here α^* is such that $\pi_2(\alpha^*, b) = [q_2(\alpha^*, b)]^2 - k^2 = 0$. 2 enters iff its type is $\alpha \in (\alpha^*, \bar{\alpha}]$. Here The probability of entry is $1 - F(\alpha^*)$.
3. If $b \in [\bar{b}, \infty)$ then $\alpha^* = \bar{\alpha}$. No type will enter. Entry is completely blockaded and the probability of entry is zero.

The next result extends the points stated above. It identifies conditions under which $\alpha^* \in (0, \bar{\alpha})$.

Proposition 3 *$\alpha^* \in (0, \bar{\alpha})$ iff $\frac{4}{D} b \in \left(\frac{2A}{2+\gamma} - 2k + \frac{\gamma^2}{D} \mu(0), \frac{2A}{2+\gamma} - 2k + \frac{4\bar{\alpha}}{D} \right)$.*

3.2.1 Preliminary results

We now provide some preliminary results. These will help us in deriving the major results on optimal bribe (optimal height of entry barrier) in the next section.

Lemma 2 *$q_1(b)$ is continuous for all $b \in [0, \bar{b})$.*

Lemma 3

$$q_1'(b) = \begin{cases} \frac{\gamma}{D} \text{ if } b \in (0, \underline{b}) \\ \frac{\gamma\left(1-\frac{d\mu(\alpha^*)}{d\alpha^*}\right)}{D+\gamma^2\frac{d\mu(\alpha^*)}{d\alpha^*}} \text{ if } b \in \left(\underline{b}, \bar{b}\right). \end{cases}$$

Lemma 4 *If $f(0) > 0$ then*

$$\lim_{b \longrightarrow \underline{b}^-} \frac{dq_1(b)}{db} > \lim_{b \longrightarrow \underline{b}^+} \frac{dq_1(b)}{db} \text{ iff } \gamma > 0$$

and if $f(0) = 0$ then

$$\lim_{b \longrightarrow \underline{b}^-} \frac{dq_1(b)}{db} = \lim_{b \longrightarrow \underline{b}^+} \frac{dq_1(b)}{db}.$$

Lemma 5

$$\lim_{b \longrightarrow \bar{b}^-} q_1(b) = \frac{A - k\gamma}{2} \text{ and } \lim_{b \longrightarrow \bar{b}^-} \frac{dq_1(b)}{db} = \frac{\gamma}{D+4}.$$

Remark Lemma 2 shows that $q_1(b)$ is continuous for all $b \in \left[0, \bar{b}\right)$. Lemma 4 demonstrates that $q_1(b)$ is not differentiable at $\underline{b}$ unless $f(0) = 0$. However, $q_1(b)$ is differentiable at all $b \in (0, \underline{b}) \cup \left(\underline{b}, \bar{b}\right)$.

We now proceed to demonstrate that both $q_1(b)$ and $q_2(\alpha, b)$ are strictly positive for all $b \in \left[0, \bar{b}\right)$. We need this for non-triviality of our results. Note that assumption 1 implies that $A > \mu(0)$. Since $B \in [1, 2]$ and $\gamma \in [-1, 1]$ this implies that $q_1(0) = \frac{AB - \gamma\mu(0)}{D} > 0$. First take $\gamma > 0$ (goods are substitutes). Note that assumption 2 states $\frac{d\mu(\alpha^*)}{d\alpha^*} \in (0, 1)$. This implies from lemma 3 we get that $q_1'(b) > 0$ for all $b \in (0, \underline{b}) \cup \left(\underline{b}, \bar{b}\right)$ iff $\gamma > 0$. Also, lemma 2 shows that $q_1(b)$ is continuous for all $b \in \left[0, \bar{b}\right)$. Since $q_1(0) > 0$ and $q_1'(b) > 0$ when $\gamma > 0$ then we must have $q_1(b) > 0$ for all $b \in \left[0, \bar{b}\right)$. Now take $\gamma < 0$ (goods are complements). Lemma 5 shows that $\lim_{b \longrightarrow \bar{b}^-} q_1(b) = \frac{A - k\gamma}{2} > 0$ (since $\gamma < 0$). Note that from lemma 3 we have if $\gamma < 0$ then $q_1'(b) < 0$ for all $b \in (0, \underline{b}) \cup \left(\underline{b}, \bar{b}\right)$. This means that if $\gamma < 0$ then $q_1(b) > \lim_{b \longrightarrow \bar{b}^-} q_1(b)$ for all $b \in \left(0, \bar{b}\right)$. This ensures that $q_1(b) > 0$ for all for all $b \in \left(0, \bar{b}\right)$. Using a similar logic we can show that $q_2(\alpha, b) > 0$ for all $b \in \left[0, \bar{b}\right)$.

3.3 First stage equilibrium

We now analyze the first-stage game and solve for the equilibrium level of bribe (optimal height of entry barrier) chosen by firm 1. In the first stage firm 1 chooses the optimal level of bribe b to maximize its expected payoff (anticipating the equilibrium outcomes in stage 2 and 3). Let the optimal level of bribe be b^*.

Note that if 1 chooses $b \in [0, \underline{b}]$ then all types enter ($\alpha^* = 0$). In this case 1 gets duopoly payoff $\pi_1(b) = (q_1(b))^2 - b$. When 1 chooses $b \in (\underline{b}, \bar{b})$ then $\alpha^* \in (0, \bar{\alpha})$. 2 does not enter if its type is $\alpha \leq \alpha^*$. The probability of this event is $F(\alpha^*)$. In this event 1 will be monopolist and his payoff will be $\frac{A^2}{4} - b$. Note that 2 will enter if its type is $\alpha > \alpha^*$. The probability of this event is $(1 - F(\alpha^*))$. Here 1 will be a duopolist and his payoff will be $\pi_1(b) = (q_1(b))^2 - b$. When 1 chooses $b \in [\bar{b}, \infty)$ no type of 2 will enter ($\alpha^* = \bar{\alpha}$). In this event 1 will be monopolist and his payoff will be $\frac{A^2}{4} - b$.

Hence, 1's expected payoff, denoted by $E_1(b)$, is as follows:

$$
\begin{aligned}
E_1(b) &= \begin{cases} \pi_1(b) \text{ if } b \in [0, \underline{b}] \\ \left[F(\alpha^*(b))\left(\frac{A^2}{4} - b\right) + (1 - F(\alpha^*(b)))\,\pi_1(b)\right] \text{ if } b \in (\underline{b}, \bar{b}) \\ \frac{A^2}{4} - b \text{ if } b \in [\bar{b}, \infty) \end{cases} \\
\text{where } \pi_1(b) &= (q_1(b))^2 - b.
\end{aligned}
$$

Since $q_1(b)$ is continuous for all $b \in [0, \bar{b})$ and $\alpha^*(b)$ is continuous in b, we must have that $E_1(b)$ is also continuous for all $b \in [0, \bar{b})$. Note that if $b \in (\underline{b}, \bar{b})$

$$E_1(b) = F(\alpha^*)\left[\frac{A^2}{4} - (q_1(b))^2\right] + (q_1(b))^2 - b$$

Using computations derived in the appendix we get if $b \in (\underline{b}, \bar{b})$

$$E_1'(b) = \frac{d\alpha^*}{db}\left[f(\alpha^*)\left[\frac{A^2}{4} - (q_1(b))^2\right] + \frac{\gamma[1 - F(\alpha^*)]\,q_1(b)\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right)}{2} - 1\right]$$

Note that $\lim_{b \longrightarrow \underline{b}} \alpha^*(b) = 0$. Using the previous lemmas we get,

$$\lim_{b \longrightarrow \underline{b}^-} \frac{dE_1(b)}{db} = \frac{\gamma}{2D}[2A - \gamma\mu(0) - 2k\gamma] - 1$$

$$\lim_{b \longrightarrow \underline{b}^+} \frac{dE_1(b)}{db} = \frac{h\gamma}{4[D + \gamma^2 f(0)\,\mu(0)]}\left[\begin{array}{c} 4A - 4k\gamma - 2\gamma\mu(0) - 4f(0)\,k^2\gamma \\ +f(0)\,\gamma(\mu(0))^2 + 8Af(0)\,k \end{array}\right] - 1$$

Therefore,

$$\lim_{b \longrightarrow \underline{b}^{+}} \frac{dE_1(b)}{db} - \lim_{b \longrightarrow \underline{b}^{-}} \frac{dE_1(b)}{db}$$

$$= \frac{\gamma f(0)}{4D\left[f(0)\mu(0)\gamma^2 + D\right]} \left[\begin{array}{c} -4Dk^2\gamma + 4k\gamma^3\mu(0) + 8ADk + 2\gamma^3(\mu(0))^2 \\ -4A\gamma^2\mu(0) + D\gamma(\mu(0))^2 \end{array} \right]$$

Note that the above implies if $f(0) > 0$ then $E_1(b)$ may not be differentiable at $b = \underline{b}$. However, $E_1(b)$ is differentiable at all $b \in (0, \underline{b}) \cup (\underline{b}, \bar{b})$.

3.4 Optimal bribe (height of entry barrier)

We now provide the main results regarding the optimal level of bribe, b^*(optimal height of entry barrier), which is chosen by the incumbent. We will first consider the case when the goods are substitutes $(\gamma > 0)$.

Note that the bribe, b, is like a sunk cost for the incumbent. It chooses this sunk cost in the first stage. While higher b increases the sunk cost, it also increases 2's per unit cost of production. This in turn increases 1's profit and decreases 2's profit in the third stage (when they play a incomplete information quantity choice game). That is, higher b has two opposing effects. In equilibrium, therefore, 1 must choose an optimal b (optimal height of entry barrier) where these two effects balance each other out.

We noted earlier that if $b = \bar{b}$ and if firm 2's type is $\bar{\alpha}$ (highest possible type) and if this type enters then it will get zero payoff $(\pi_2(\bar{\alpha}, \bar{b}) = 0)$. Since 2 enters only when it gets strictly positive payoff it means that if $b = \bar{b}$ then no type of 2 will enter. Firm 1 is then a monopolist and produces output $\frac{A^2}{4}$ and gets a profit $\pi_1(\bar{b}) = \frac{A^2}{4} - \bar{b}$. Note that 1 will never choose a $b > \bar{b}$ as it will then get $E_1(b) = \frac{A^2}{4} - b < E_1(\bar{b})$. Hence, in all equilibria, we get $b^* \leq \bar{b}$.

3.4.1 Optimal bribe (height of entry barrier) when goods are substitutes $(\gamma > 0)$

We provide the following result regarding optimal bribe, b^*, when goods are substitutes $(\gamma > 0)$. The proof is given in the appendix.

Proposition 4 *(i) If* $\gamma A\left[f(\alpha)(2k+\bar{\alpha})+1\right] \leq D$ *for all* $\alpha \in [0,\bar{\alpha}]$ *then* $b^* = 0$*. (ii) If* $\gamma\left[A-\mu(0)\right] \geq 8$ *then* $b^* \geq \underline{b}$*. (iii) If* $\max\left\{\frac{12}{5f(\alpha)k\gamma A}, \frac{8}{\gamma[A-\mu(0)]}\right\} \leq 1$ *for all* $\alpha \in [0,\bar{\alpha}]$ *then* $b^* = \bar{b}$*.*

Remark Other things remaining the same, the market size, A, and the differentiation parameter, γ, play a crucial role in determining the optimal bribe, b^*(optimal height of entry barrier). If A and γ are small enough then the optimal bribe (optimal height) is zero (for small A and γ the inequality in Proposition 4(i) is likely to be satisfied). In this case entry is certain. If on the other hand, if A and γ are large enough then a positive b^* (or even the highest possible bribe level, $\bar{b}$) can be observed in equilibrium (Proposition 4 - (ii) and (iii)). In this case, entry is restricted (as height of entry barrier is positive).

The following proposition demonstrates that it is not possible to have $0 < b^* < \underline{b}$.

Proposition 5 $b^* \in \{0\} \cup \left[\underline{b}, \bar{b}\right]$.

Remark Proposition 5 is interesting. It states that if there is an equilibrium with strictly positive bribe, then the level of such a bribe will be at least $\underline{b}$. *Hence, in equilibrium we will observe either a 'zero' bribe or a bribe which is at least* $\underline{b}$*.* Since $\underline{b} = \frac{2AB+\gamma^2\mu(0)-2kD}{4}$, it is strictly increasing in the market size. *Therefore, the minimum possible bribe (height of entry barrier)is higher if the market size, A, is bigger. Note that if the distribution of 2's types is uniform, then $\mu(0) = \frac{1}{2}\bar{\alpha}$. In this case, higher is $\bar{\alpha}$ (uncertainty level) higher will be $\underline{b}$. That is, the minimum possible height of entry barrier increases with uncertainty.*

Note that $b^* \in \left(\underline{b}, \bar{b}\right)$ is possible. In section 5 we provide an example where this is demonstrated.

We now provide the main result on optimal bribe with complements ($\gamma < 0$).

3.4.2 Optimal bribe (height of entry barrier) when goods are complements ($\gamma < 0$)

Proposition 6 *If goods are complements (i.e.* $\gamma < 0$*) then* $b^* = 0$*.*

Remark *When goods are complements then regardless of the level of fairness index (or market quality), we observe zero bribe (zero height of entry barrier) in equilibrium.* The intuition is as follows. Bribe pushes up costs of entrants and discourages entry. However, with complements, it better for the incumbent firm to allow entry as it pushes up its demand and hence its profit. Consequently, the incumbent chooses to opt for zero bribe. This has interesting policy prescription in the following sense. In emerging economies like India, where market quality is poor, it is better for the government to foster competition in complementary goods. This is likely to reduce bribery. Since competition in substitutes tends to eat away profits, the incumbent finds it advantageous to resort to bribe and discourage entry. To counter this tendency, the governments should choose to encourage competition in complementary goods.

3.5 Optimal bribe (optimal height) with no uncertainty

Suppose $\bar{\alpha} = 0$. That is, there is no incompleteness of information. 2's marginal cost (which is equal to c in this case) is known to the incumbent. In this case, firms play a complete information Cournot game in the third stage. Routine computation yield the following.

$$q_1(b) = \begin{cases} \frac{AB+\gamma b}{D} \text{ if 2 enters} \\ \frac{A}{2} \text{ if 2 does not enter} \end{cases} \quad \text{and } q_2(b) = \begin{cases} \frac{2AB-4b}{2D} \text{ if 2 enters} \\ 0 \text{ if 2 does not enter} \end{cases}$$

Let $\hat{b} = \frac{2AB-2Dk}{4}$. It can be easily shown that 2 always enters if $b \in \left[0, \hat{b}\right)$ and does not enter, if $b \in \left[\hat{b}, \infty\right)$.

Remark First note that $\hat{b} < \underline{b} < \bar{b}$. Let optimal bribe (optimal height of entry barrier) for the case of complete information be b^{**}. Note it can be easily shown that $b^{**} \in \left\{0, \hat{b}\right\}$. That is, in equilibrium, firm 2 is either clearly in ($b^{**} = 0$) or clearly out $\left(b^{**} = \hat{b}\right)$. There cannot be any partial entry deterrence (as in the case of incomplete information).

Note that

$$b^{**} = \hat{b} \Longleftrightarrow \frac{A^2}{4} - \frac{2AB - 2Dk}{4} - \left(\frac{AB}{D}\right)^2 \geq 0.$$

Note that since $B = 2 - \gamma$ and $D = 4 - \gamma^2$, we get

$$b^{**} = \hat{b} \Longleftrightarrow A^2\left(\gamma^2 + 4\gamma\right) + 2A\left(-8 - 4\gamma + 2\gamma^2 + \gamma^3\right) + 2k\left(16 + 16\gamma - 4\gamma^3 - \gamma^4\right) \geq 0.$$

3.5.1 Case of substitutes

When goods are substitutes we have $\gamma \in (0,1]$. We now provide the following result with no uncertainty.

Proposition 7 *Let $\bar{\alpha} = 0$ and $\gamma \in (0,1]$. If $A \geq \frac{18}{5}$ then there exists $\underline{\gamma} \in (0,1)$ s.t. the following is true. (i) If $\gamma \geq \underline{\gamma}$ then $b^{**} = \hat{b}$. (ii) $\gamma < \underline{\gamma}$ then $b^{**} = 0$.*

Remark Note that if γ is small enough then the optimal bribe (optimal height of entry barrier) , $b^{**} = 0$. This resonates with our result with incomplete information (see proposition 4(i)). However, if γ is large enough and market size, A, is above a critical level, then $b^{**} = \hat{b}$ and we have completely blockaded entry. Note that when there is uncertainty ($\bar{\alpha} > 0$) and there is positive bribe in equilibrium (i.e. $b^* > 0$), it is at least $\underline{b}$ (see proposition 5). With no uncertainty ($\bar{\alpha} = 0$), the amount of equilibrium bribe (height of entry barrier), when positive, is $b^{**} = \hat{b}$. That is, equilibrium bribe, when positive, is larger with incomplete information than with complete information (since $\hat{b} < \underline{b}$). *That is, bribe (i.e., the height of entry barrier) is higher with uncertainty than it is with complete information.*

3.5.2 Case of complements

When goods are complements $\gamma \in [-1,0)$. We now provide the following result.

Proposition 8 *Let $\bar{\alpha} = 0$ and $\gamma \in [-1,0)$. Optimal bribe $b^{**} = 0$.*

Remark When goods are complements, with no uncertainty ($\bar{\alpha} = 0$) the optimal bribe (optimal height of entry barrier), $b^* = 0$. This is similar to the case where there is incomplete information (see proposition 6).

4 Total social surplus and market quality

We now provide some basics on social surplus and market quality.

4.1 Total social surplus

Note that on the demand side of the market, the representative consumer's utility function of two differentiated products, q_1 and q_2, and a numeraire good, q_0 is given by

$$U = a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0.$$

The inverse demand functions are

$$p_i = a - q_i - \gamma q_j;\ \ i, j = 1, 2;\ \ i \neq j.$$

Here consumer surplus is

$$\begin{aligned} CS &= U(q_1, q_2) - p_1 q_1 - p_2 q_2 \\ &= a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0 - p_1 q_1 - p_2 q_2 \end{aligned}$$

The producer surplus is 1's profit plus 2's profit.

$$PS = \pi_1 + \pi_2 = [p_1 q_1 - c q_1 - b] + \left[p_2 q_2 - (c + b - \alpha) q_2 - k^2\right]$$

Therefore, 'total social surplus' is $CS + PS + bribe$ (as bribe is just a transfer within the system):

$$\begin{aligned} CS + PS + bribe &= (a - c) q_1 + (a - c - hb + \alpha) q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0 - b - k^2 + b \\ &= A q_1 + (A - hb + \alpha) q_2 - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\gamma q_1 q_2\right) + q_0 - k^2 \end{aligned}$$

Note that in equilibrium 1's output is $q_1(b)$ and 2's output (when type α) is $q_2(\alpha, b)$. So we have to look for expected welfare. Note that when firm 2's type is $\alpha \leq \alpha^*$, firm 2 does not enter ($q_2 = 0$) and firm 1 is a monopolist $\left(q_1 = q_1^m = \frac{A}{2}\right)$. Such types ($\alpha \leq \alpha^*$) do not enter and do not incur the entry cost k^2. For this case,

$$\begin{aligned} CS + PS + bribe &= A q_1 - \frac{1}{2} q_1^2 + q_0 \\ &= \frac{3}{8} A^2 + q_0 \end{aligned}$$

The probability of the above case is $F(\alpha^*)$. Therefore the expected $(CS + PS + bribe)$, which is the expected total surplus, is as follows:

$$\begin{aligned} W(b) &= \left[\frac{3}{8}A^2 + q_0\right] F(\alpha^*) \\ &\quad + \int_{\alpha^*}^{\bar{\alpha}} \left[\begin{array}{c} Aq_1(b) + (A - hb + \alpha) q_2(\alpha, b) \\ -\frac{1}{2}\left\{q_1^2(b) + q_2^2(\alpha, b) + 2\gamma q_1(b) q_2(\alpha, b)\right\} + q_0 - k^2 \end{array} \right] dF(\alpha) \end{aligned}$$

4.2 Market quality

It may be noted that market quality ($\mathbb{Q}$) is defined to be sum of total surplus (W) and fairness ($-b$). That is, we have

$$\mathbb{Q}(b) = W(b) - b.$$

5 Uncertainty and market quality

As noted in the introduction, three primary factors determine market quality. These primary factors are, "quality of competition", "quality of information", and "quality of products". In this paper we focus on two of these: (i) "quality of competition"- to what extent entry is restricted? and (ii) "quality of information"- how the extent of incompleteness of information (captured by $\bar{\alpha}$) affect bribe (height of entry barrier) and market quality.

We now seek to answer the following question. Does an increase in the uncertainty lead to a increase in equilibrium bribe (optimal height of entry barrier) and decrease in market quality?

While a general answer to such a question may not be possible, we produce an example, where 2's types are uniformly distributed, to demonstrate that an increase in uncertainty indeed leads to a decrease in market quality. This seems to be a vindication of the idea in Yano (2009 and 2016).[13]

[13]Note that when $\bar{\alpha}$ changes, the disctribution unction $F(.)$ also changes. To take care of this problem we proceed as follows. Let for each $\bar{\alpha} > 0$, $F_{\bar{\alpha}}(.)$ denote a family of distribution functions over $[0, \bar{\alpha}]$. One example is $F_{\bar{\alpha}}(t) = \left[\frac{t}{\bar{\alpha}}\right]^{\theta}$ where $\theta > 0$. When $\theta = 1$ then it's a simple uniform distribution. In our example, we take $\theta = 1$.

5.1 An illustrative example

Let $k = 1 = \gamma,\ A = 5$. Let α be uniformly distributed over $[0, \bar{\alpha}]$. This means $F(\alpha) = \frac{\alpha}{\bar{\alpha}}$ and $f(\alpha) = \frac{1}{\bar{\alpha}}$. We assume that $\bar{\alpha} \in [2.5714,\ 3.5117]$. The following may be noted.

$$\underline{b} = \frac{1}{8}(8 + \bar{\alpha}),\ \bar{b} = 1 + \bar{\alpha}$$

$$\alpha^* = \begin{cases} 0 \text{ if } b \in [0, \underline{b}] \\ \frac{8b - \bar{\alpha} - 8}{7} \text{ if } b \in \left(\underline{b}, \bar{b}\right) \\ \bar{\alpha} \text{ if } b \in \left[\bar{b}, \infty\right] \end{cases},\ \mu(\alpha^*) = \begin{cases} \frac{\bar{\alpha}}{2} \text{ if } b \in [0, \underline{b}] \\ \frac{1}{2}[\alpha^* + \bar{\alpha}] = \frac{4b + 3\bar{\alpha} - 4}{7} \text{ if } b \in \left(\underline{b}, \bar{b}\right) \\ \bar{\alpha} \text{ if } b \in \left[\bar{b}, \infty\right] \end{cases}$$

$$q_1(b) = \begin{cases} \frac{13 - \bar{\alpha} + b}{7} \text{ if 2 enters} \\ \frac{A}{2} \text{ if 2 does not enter} \end{cases},\ q_2(\alpha, b) = \begin{cases} \frac{(22 + 7\alpha + \bar{\alpha} - 8b)}{14} \text{ if } \alpha \in (\alpha^*, \bar{\alpha}] \\ 0 \text{ if } \alpha \in [0, \alpha^*] \end{cases}$$

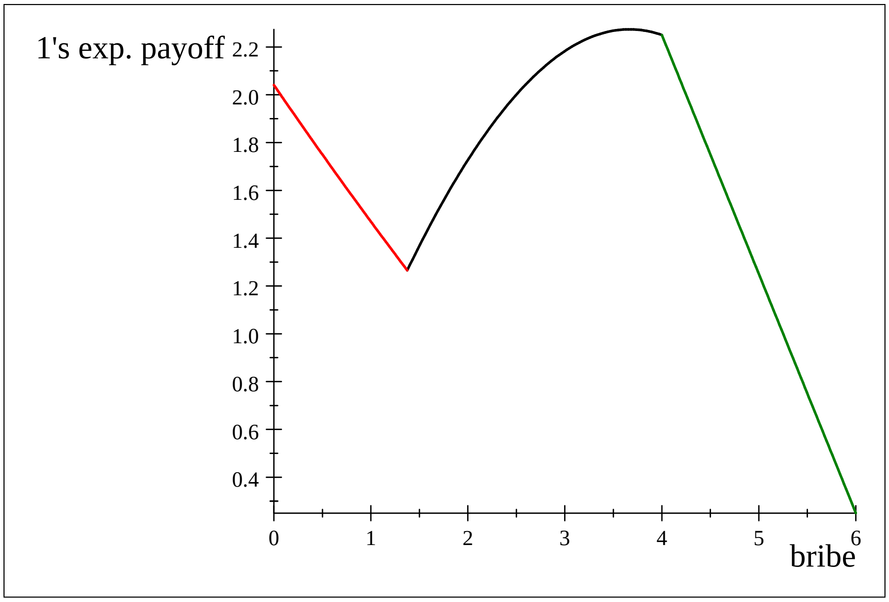
$$E_1(b) = \begin{cases} \left(\frac{13 - \bar{\alpha} + b}{7}\right)^2 - b \text{ if } b \in [0, \underline{b}] \\ \frac{8b - \bar{\alpha} - 8}{7\bar{\alpha}}\left[\frac{25}{4} - b\right] + \left(1 - \frac{8b - \bar{\alpha} - 8}{7\bar{\alpha}}\right)\left[\left(\frac{13 - \bar{\alpha} + b}{7}\right)^2 - b\right] \text{ if } b \in \left(\underline{b}, \bar{b}\right) \\ \frac{25}{4} - b \text{ if } b \in \left[\bar{b}, \infty\right) \end{cases}$$

5.1.1 Optimal bribe

Routine computations show that when $\bar{\alpha} \in [2.5714, 3.5117]$ then $b^* \in \left(\underline{b}, \bar{b}\right)$ and $b^* = \bar{\alpha} + \frac{7}{12}\sqrt{14(26 - 3\bar{\alpha})} - \frac{25}{3}$. To illustrate this point, take $\bar{\alpha} = 3$. Then

$$\begin{aligned} \underline{b} &= \frac{8 + 3}{8} = 1.375 \text{ and } \bar{b} = 4. \text{ Also,} \\ E_1(b) &= \begin{cases} \left(\frac{10 + b}{7}\right)^2 - b \text{ if } b \in [0, 1.375] \\ \frac{8b - 11}{21}\left[\frac{25}{4} - b\right] + \left(1 - \frac{8b - 11}{21}\right)\left[\left(\frac{10 + b}{7}\right)^2 - b\right] \text{ if } b \in (1.375, 4) \\ \frac{25}{4} - b \text{ if } b \in [4, \infty) \end{cases}. \end{aligned}$$

We plot $E_1(b)$ over $[0, 6]$ in figure 1 below to demonstrate our point. Note that when $\bar{\alpha} = 3$, $b^* = 3.6659$ maximizes $E_1(b)$.



Firgure 1: $E_1(b)$ when $\bar{\alpha} = 3$

5.1.2 Uncertainty and market quality

We noted that when $\bar{\alpha} \in [2.5714, 3.5117]$ then $b^* \in \left(\underline{b}, \bar{b}\right)$ and $b^* = \bar{\alpha} + \frac{7}{12}\sqrt{14(26 - 3\bar{\alpha})} - \frac{25}{3}$. At equilibrium (i.e. when $b = b^*$) we have the following.

$$\begin{aligned}
\alpha^*(b^*) &= \bar{\alpha} + \frac{2}{3}\sqrt{14(26 - 3\bar{\alpha})} - \frac{32}{3} \\
q_1(b^*) &= \frac{1}{12}\sqrt{14(26 - 3\bar{\alpha})} + \frac{2}{3} \\
q_2(\alpha, b^*) &= \begin{cases} \frac{1}{2}\alpha - \frac{1}{2}\bar{\alpha} - \frac{1}{3}\sqrt{14(26 - 3\bar{\alpha})} + \frac{19}{3} \text{ if } \alpha \in (\alpha^*, \bar{\alpha}] \\ 0 \text{ if } \alpha \in [0, \alpha^*] \end{cases}
\end{aligned}$$

The following may be noted.

1. It is clear that $q_1(b^*)$ is strictly decreasing in the uncertainty parameter $\bar{\alpha}$.

2. Also, for all $\bar{\alpha} \in [2.5714, 3.5117]$

$$\frac{\partial q_2(.)}{\partial \bar{\alpha}} = -\frac{1}{2}\frac{\sqrt{26 - 3\bar{\alpha}} - \sqrt{14}}{\sqrt{26 - 3\bar{\alpha}}} < 0.$$

   The above means that 2's output, conditional on entry, decreases with $\bar{\alpha}$ for each of 2's possible types.

3. Note that

$$\frac{\partial b^*}{\partial \bar{\alpha}} = \frac{1}{8}\frac{8\sqrt{26-3\alpha} - 7\sqrt{14}}{\sqrt{26-3\alpha}} > 0 \text{ for all } \bar{\alpha} \in [2.5714, 3.5117]$$

That is, optimal bribe, b^*((optimal height of entry barrier), is strictly increasing in $\bar{\alpha}$. As uncertainty goes up, the bribe paid also goes up (i.e. height of entry barrier increases). *That is, with increasing uncertainty, entry becomes more difficult.*

5.1.3 Market Quality

The above computations indicate that expected market quality is likely to decrease with an increase in $\bar{\alpha}$. We now demonstrate this possibility. Note that the expected market quality in our example is as follows:

$$\begin{aligned} MQ\left(b^*\right) &= W\left(b^*\right) - b^* \\ &= \left[\begin{array}{c} \left(\frac{3}{8}A^2\right)\frac{\alpha^*(b^*)}{\bar{\alpha}} - k^2\left(1 - \frac{\alpha^*(b^*)}{\bar{\alpha}}\right) + q_0 \\ + \int_{\alpha^*(b^*)}^{\bar{\alpha}} \left(Aq_1\left(b^*\right) + \frac{3}{2}q_2^2\left(t, b^*\right) - \frac{1}{2}q_1^2\left(b\right)\right)\frac{dt}{\bar{\alpha}} - b^* \end{array} \right] \\ &= \frac{1}{216\bar{\alpha}}\left(\begin{array}{c} 224\sqrt{14}\left(26-3\bar{\alpha}\right)^{\frac{3}{2}} - 12\,807\bar{\alpha} - 216\bar{\alpha}^2 - 126\bar{\alpha}\sqrt{14\left(26-3\bar{\alpha}\right)} \\ -15\,340\sqrt{14\left(26-3\bar{\alpha}\right)} + 987\bar{\alpha}\sqrt{14\left(26-3\bar{\alpha}\right)} + 182\,064 \end{array} \right) \end{aligned}$$

We now plot market quality over the range $\bar{\alpha} \in [2.5714, 3.5117]$ in figure 2 below to show that it is strictly decreasing with the uncertainty parameter $\bar{\alpha}$.

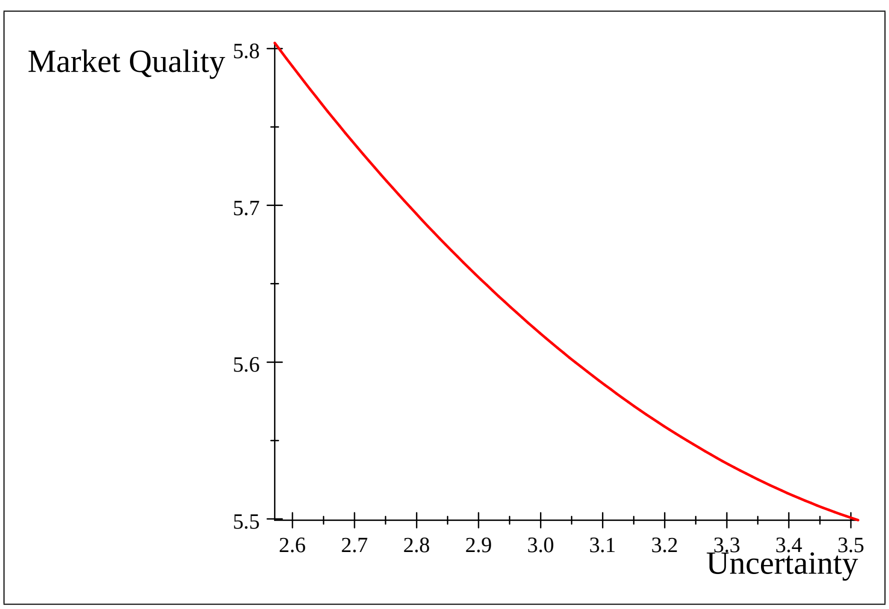


Figure 2: Uncertainty ($\bar{\alpha}$) and Market Quality

Remark The main takeaways of the above example is as follows:

1. *More uncertainty leads to an increase in the height of entry barrier (bribe amount) and a decrease in market quality.* This is a clear vindication of the Yano (2009, 2016) idea that market quality is inversely linked with the quality of information.

2. *Uncertainty is bribery-promotive since optimal bribe increases as the level of uncertainty increases. This result stands somewhat in contrast to Maskin (1999).*

6 Bribe (height of entry barrier) and market quality

Note that market quality $\mathbb{Q} = W(b) - b$. A natural question that arises is the following: Does zero bribe (which implies zero height of entry barrier and hence certain entry) always maximize total surplus and market quality? When goods are substitutes, our answer is surprisingly negative. However, when goods are complements, then zero bribe always maximizes total surplus and market quality.

6.0.4 Case of substitutes ($\gamma > 0$)

For substitutes we can show that $W'(b) < 0$ and $\mathbb{Q}'(b) < 0$ at $b = 0$ (see the appendix for the computations). This means $b = 0$ is a local maximizer for $W(b)$ and $\mathbb{Q}(b)$. We now demonstrate that $b = 0$ need not be a global maximizer for .

An illustrative example Note that when $b = 0$, all types of firm 2 enter (i.e. probability of entry is one). In this case $\alpha^* = 0$ and the expected total surplus is as follows:

$$W(0) = \int_0^{\bar{\alpha}} \begin{bmatrix} Aq_1(0) + (A+\alpha) q_2(\alpha, 0) \\ -\frac{1}{2}\{q_1^2(0) + q_2^2(\alpha, 0) + 2\gamma q_1(0) q_2(\alpha, 0)\} + q_0 - k^2 \end{bmatrix} dF(\alpha)$$

Note that market quality $\mathbb{Q}(b) = W(b) - b$. Hence, and $\mathbb{Q}(0) = W(0)$.

Note that when $b = \bar{b}$ then no type of firm 2 enters. This means $\alpha^* = \bar{\alpha}$ and firm 1 is a monopolist. In this case, we have

$$W(\bar{b}) = \frac{3}{8}A^2 + q_0 \text{ and } \mathbb{Q}(\bar{b}) = \frac{3}{8}A^2 + q_0 - \bar{b} - - - - (47)$$

Consider the following values of the parameters.

$$\begin{aligned} A &= 3,\ k=1,\ \gamma=1 \text{ and } \alpha \text{ is uniformly distributed over } [0,\bar{\alpha}]. \\ \bar{\alpha} &\in (0, 0.219) \end{aligned}$$

All the assumptions of our model are satisfied here and we have

$$\mu(0) = \frac{\bar{\alpha}}{2},\ \underline{b} = \frac{\bar{\alpha}}{8} \text{ and } \bar{b} = \bar{\alpha}.$$

Using routine computations we can show that

$$\begin{aligned} W(0) &= \mathbb{Q}(0) = \frac{53}{288}\bar{\alpha}^2 + \frac{2}{3}\bar{\alpha} + 3 + q_0 \text{ and} \\ W\left(\bar{b}\right) &= \frac{27}{8} + q_0,\ \mathbb{Q}\left(\bar{b}\right) = \frac{27}{8} - \bar{\alpha} + q_0 \end{aligned}$$

Note that from above we have

$$\begin{aligned} W\left(\bar{b}\right) - W(0) &= -\frac{53}{288}\bar{\alpha}^2 - \frac{2}{3}\bar{\alpha} + \frac{3}{8} \\ \mathbb{Q}\left(\bar{b}\right) - \mathbb{Q}(0) &= -\frac{53}{288}\bar{\alpha}^2 - \frac{5}{3}\bar{\alpha} + \frac{3}{8} \end{aligned}$$

It may be noted that

$$W\left(\bar{b}\right) - W(0) > 0 \text{ and } \mathbb{Q}\left(\bar{b}\right) - \mathbb{Q}(0) > 0 \text{ for all } \bar{\alpha} \in (0, 0.219)$$

Our example shows that zero bribe is **not** the global maximizer of $W(b)$ when $\bar{\alpha} \in (0, 0.219)$ (since $W\left(\bar{b}\right) - W(0) > 0$).

To illustrate it further take $\bar{\alpha} = 0.2$. Then, $\underline{b} = 0.025$ and $\bar{b} = 0.2$. Note that when $\bar{\alpha} = 0.2$ then

$$W(b) = \begin{cases} \frac{11}{18}b^2 - \frac{131}{90}b + \frac{22\,613}{7200} & \text{if } b \in [0, \underline{b}] \\ -\frac{300}{343}b^3 + \frac{1300}{343}b^2 + \frac{251}{343}b + \frac{42\,313}{13\,720} & \text{if } b \in \left(\underline{b}, \bar{b}\right] \end{cases}$$

$$\mathbb{Q}(b) = \begin{cases} \frac{11}{18}b^2 - \frac{221}{90}b + \frac{22\,613}{7200} & \text{if } b \in [0, \underline{b}] \\ -\frac{300}{343}b^3 + \frac{1300}{343}b^2 - \frac{92}{343}b + \frac{42\,313}{13\,720} & \text{if } b \in \left(\underline{b}, \bar{b}\right] \end{cases}$$

In figure 3 below we plot $W(b)$, in solid lines, and $\mathbb{Q}(b)$, in dash lines, over the range $\left[0, \bar{b}\right] = [0, 0.2]$

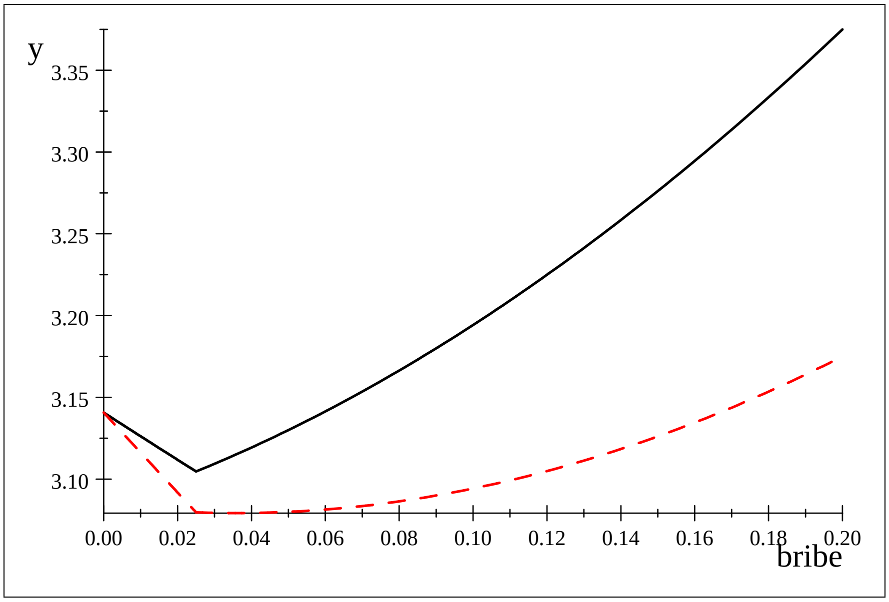


Figure 3: Total surplus and market quality

Clearly, when $\bar{\alpha} = 0.2$ the bribe that maximizes total surplus and market quality is $b = \bar{b} = 0.2$. [14] We state this result in terms of a proposition.

Proposition 9 *When goods are substitutes* ($\gamma > 0$) *zero bribe (no entry barrier) may not be maximize either total surplus or market quality..*

Remark As noted in the introduction, this result is somewhat related to Mankiw and Whinston (1986). In their paper it is shown that when entrants incur a fixed set-up cost of entry and when there is "business stealing effect" then free entry is not total surplus maximizing. We provide a discussion of this point below.

Business stealing in our model According to Mankiw and Whinston (1986) the business-stealing effect exists when the equilibrium strategic response of existing firms to new entry results in their having a lower volume of sales-that is, when a new entrant 'steals business' from incumbent firms. Put differently, a business-stealing effect is present if the equilibrium output per firm declines as the number of firms grows. Note that this does not mean that total output decreases with entry. In fact, in our model, where there are two

[14] When $b = \bar{b}$, firm 1 is a monopolist and chooses a monopoly output in equilibrium. Its net monopoly profit is $\frac{A^2}{4} - \bar{b} = \frac{9}{4} - \bar{\alpha} > 0$ when $\bar{\alpha} \in (0, 0.219)$.

firms, with complete information and substitutes ($\gamma > 0$) the incumbents' output decreases with entry while the total output goes up. With incomplete information the concept of business stealing is as follows:

When $b \in (\underline{b}, \bar{b})$ we have $\alpha^* \in (0, \bar{\alpha})$ and probability of entry is $1 - F(\alpha^*)$. Note that as b increases the probability of entry goes down. In our paper when $b \in (\underline{b}, \bar{b})$ then the incumbent's equilibrium output, $q_1(b)$, increases with b provided $\gamma > 0$ (substitutes). More b means lower probability of entry and incumbent's equilibrium output increases. In other words, more entry (lower $b \Leftrightarrow$ lower height of entry barrier) would imply that incumbent's equilibrium output decreases. This is 'business stealing effect' in our model with incomplete information.

Note that without entry the incumbent's output is $\frac{A}{2}$. If $b^* = 0$ then entry is certain. With zero bribe the incumbent's equilibrium output is $\frac{AB - \gamma\mu(0)}{D}$ and entrant's output is $\frac{2AB + \alpha D + \gamma^2\mu(0)}{2D}$. Now $\frac{A}{2} > \frac{AB - \gamma\mu(0)}{D}$ iff $\gamma > 0$. That is, with $b = 0$ and certain entry the incumbent's output goes down. However, total output (incumbent's output plus entrant's output) with entry is strictly greater than $\frac{A}{2}$. That is, total output increases with entry when there is zero bribe. Also note that both for $b \in (\underline{b}, \bar{b})$ we have $\frac{\partial}{\partial b}(q_1(b) + q_2(\alpha, b)) < 0$. That is, total output $q_1(b) + q_2(b)$ decreases with b. That is, lower entry (more $b \Leftrightarrow$ more height of entry barrier) implies lower total output. In other words, more entry (lower $b \Leftrightarrow$ lower height of entry barrier) implies greater total output.

Mechanism behind increase in total surplus with increase in bribe We have shown that zero bribe (certain entry) need not maximize total surplus. Take the case of $b^* = 0$ (incumbent's optimal bribe is zero). In our example both total surplus and market quality are maximized at $b = \bar{b}$ (maximum possible height and no-entry at all). If an entrant causes incumbent firms to reduce output, entry may be more desirable to the entrant than it is to society. In our model, lower b means more probability of entry but lower output for the incumbent. Also, incumbent's profit will be lower than its profit as a monopolist. While more entry (lower $b \Leftrightarrow$ lower height of entry barrier) means additional output of the entrant and some additional profits as well, it may be insufficient to make up for the loss in output and profit of the entrant. As a result, total surplus (and also market quality) may go down with lower b (lower height and hence more entry).

6.0.5 Case of complements ($\gamma < 0$)

Proposition 10 *When goods are complements ($\gamma < 0$), if $-A\gamma \geq k$ then zero bribe (zero height of entry barrier) maximizes both total surplus and market quality.*

Remark For complements $\gamma \in [-1, 0)$. Now $-A\gamma \geq k$ will hold true if the market size, A, is large relative to entry cost. Since $A \geq 3k$ (assumption 1), $-A\gamma \geq k$ will also hold if $\gamma \leq -\frac{1}{3}$.

In proposition 6 we had shown that when goods are complements then the incumbent optimally chooses zero bribe in equilibrium. In proposition 10 we demonstrated that if the market size is large enough zero bribe maximizes both total surplus and market quality when goods are complements. This reinforces our policy prescription that governments should foster competition in complements to reduce bribery (which increases fairness) and increase market quality.

7 Conclusion

In this paper we analyzed the determinants of the 'height' of entry barriers in a developing economy. The incumbent can raise the costs of the potential entrant by resorting to dubious means (such as bribes) that moves government officials to raise the entry barrier. We completely characterized the optimal bribe level (optimal height of entry barrier) in equilibrium and also showed zero bribe (no entry barrier) need not always be welfare maximizing. Our results seem to be compatible with anecdotal evidences from an emerging economy like India. A more rigorous empirical study is required to check whether our theoretical results hold true or not. Some simple questions that arise are as follows.

1. In our exercise if the incumbent pays bribe, b, it incurs a sunk cost equal to b. What happens to equilibrium outcomes if the the cost of paying the bribe (b) is some function $\beta(b)$, where $\beta(.)$ is strictly increasing? For example, suppose that the incumbent generates funds for the bribe amount by taking money away from the other profitable

activities. The optimal way to do this is to take it out from the least profitable alternatives first. The resulting opportunity cost of paying a bribe, b, may thus be convex in b. That is, we may have $\beta'' > 0$ (in our model $\beta'' = 0$). The analysis of equilibrium outcomes for such bribe costs will be an interesting exercise.

2. Will the results change if instead of Cournot competition we have Bertrand competition in the third stage? We know that in the industrial organization literature, there are papers that show that equilibrium outcomes depend crucially on the nature of competition (Cournot or Bertrand).[15] It would be interesting to recast our exercise with price competition in the third stage.

3. The result (zero bribes or no entry barrier is not always socially optimal) is possibly a direct consequence of the existence of an entry cost. In case the equilibrium bribe (equilibrium height of entry barrier) is positive, the optimal policy may be to set a tax on entry. A bribe is, at least partially, a social waste, but an entry tax probably represents a superior policy. Clearly more research is needed on this front.

[15] See Vives (1999) for a succinct summary of the classic results around this point. Alipranti et al (2014) provides some recent results.

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Appendix

In the appendix we provide the equilibrium computations and the proofs of all the results mentioned in the paper.

8 Computation of third stage equilibrium

If firm 2 chooses to enter in the second stage, then in the third stage the firms play an incomplete information Cournot game and earn duopoly profits. If firm 2 had chosen not to enter in the second stage, then firm 1 chooses monopoly output.

Now suppose 2 has chosen to enter in the second stage. We will now compute the Bayesian-Nash equilibrium of the third stage. In this stage, bribe level b has been determined previously and known to both firms, whereas the entrant's efficiency, α, is known only to the entrant. For any given b, let the Bayesian-Nash equilibrium be $q_1(b)$ (quantity choice by firm 1, which does not know α) and equilibrium choice by firm 2 (with type α) be $q_2(\alpha, b)$.

Note that in the second-stage 2 will enter iff it's efficiency (α) is higher than a critical type α^*. That is, firm 2 enters iff $\alpha \in (\alpha^*, \bar{\alpha}]$. If 2 enters, then in the third stage firms 1 and 2 play an incomplete information Cournot duopoly game.

Hence, in the third-stage equilibrium, 1 knows that 2 enters iff its type $\alpha \geq \alpha^*$. That is, in equilibrium, 1 knows that it is facing an opponent with type $\alpha \geq \alpha^*$. Since 1 knows it is facing an opponent with type $\alpha \geq \alpha^*$, then in equilibrium it computes 2's expected output to be $Exp.(q_2(\alpha, b) \mid \alpha \geq \alpha^*) = \int_{\alpha^*}^{\bar{\alpha}} q_2(\alpha, b) \frac{f(\alpha)}{1-F(\alpha^*)} d\alpha$.

In a Bayesian-Nash equilibrium, where $\alpha \in [\alpha^*, \bar{\alpha}]$ we have the following.

$$
\begin{aligned}
q_2(\alpha, b) &= \arg\max_{q_2 \geq 0} \left[q_2 \{A - \gamma q_1(b) - q_2 - b + \alpha\} - k^2\right], \quad ----(1a) \\
q_1(b) &= \arg\max_{q_1 \geq 0} \left[q_1 \left(A - q_1 - \gamma \int_{\alpha^*}^{\bar{\alpha}} q_2(\alpha, b) \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha\right) - b\right] ----(1b)
\end{aligned}
$$

Using (1a) and (1b) we get that

$$
\begin{aligned}
\frac{\partial}{\partial q_2} \left[q_2 \{A - \gamma q_1(b) - q_2 - b + \alpha\} - k^2\right] &= 0 \text{ at } q_2 = q_2(\alpha, b) \quad ---(2a) \\
\frac{\partial}{\partial q_1} \left[q_1 \left(A - q_1 - \gamma \int_{\alpha^*}^{\bar{\alpha}} q_2(\alpha, b) \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha\right) - b\right] &= 0 \text{ at } q_1 = q_1(b) ----(2b)
\end{aligned}
$$

Let

$$q_2^{Exp.}(\alpha^*) = \int_{\alpha^*}^{\bar{\alpha}} q_2(\alpha, b) \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha.$$

Using (2a) and (2b) we have

$$\begin{aligned} q_2(\alpha, b) &= \frac{1}{2}[A - \gamma q_1(b) - b + \alpha] - - - - (3a) \\ q_1(b) &= \frac{1}{2}\left[A - \gamma q_2^{Exp.}(\alpha^*)\right] - - - - (3b) \end{aligned}$$

Using (3b) in (3a) we get

$$\begin{aligned} q_2(\alpha, b) &= \frac{1}{2}\left[A - \frac{\gamma}{2}\left\{A - \gamma q_2^{Exp.}(\alpha^*)\right\} - b + \alpha\right] \\ &= \frac{1}{4}\left[AB + \gamma^2 q_2^{Exp.}(\alpha^*) - 2b + 2\alpha\right] - - - - (4) \end{aligned}$$

From (4) above we get

$$\begin{aligned} &\int_{\alpha^*}^{\bar{\alpha}} q_2(\alpha, b) \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha \\ = &\int_{\alpha^*}^{\bar{\alpha}} \frac{1}{4}\left[AB + \gamma^2 q_2^{Exp.}(\alpha^*) - 2b + 2\alpha\right] \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha \\ = &\frac{1}{4}\left[AB + \gamma^2 q_2^{Exp.}(\alpha^*) - 2b\right] + \frac{1}{2}\int_{\alpha^*}^{\bar{\alpha}} \alpha \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha \\ = &\frac{1}{4}\left[AB + \gamma^2 q_2^{Exp.}(\alpha^*) - 2b + 2\mu(\alpha^*)\right] - - (5) \end{aligned}$$

Since as per our definition $\int_{\alpha^*}^{\bar{\alpha}} q_2(\alpha, b) \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha = q_2^{Exp.}(\alpha^*)$ the above implies that

$$q_2^{Exp.}(\alpha^*) = \frac{[AB - 2b] + 2\mu(\alpha^*)}{4 - \gamma^2} = \frac{[AB - 2b] + 2\mu(\alpha^*)}{D} - - - - (6)$$

Using (6) in (3b) we get

$$q_1(b) = \frac{AB + \gamma b - \gamma\mu(\alpha^*)}{D} - - - - (7)$$

Using (7) in (3a) we get

$$q_2(\alpha, b) = \frac{2AB + \alpha D - 4b + \gamma^2 \mu(\alpha^*)}{2D} - - - - (8)$$

It may be noted that if firm 2 does not enter, then firm 1 is a monopolist in this stage and it produces $q_1^m = \frac{A}{2}$.

Third-stage Bayesian Nash equilibrium

$$
\begin{aligned}
q_1(b) &= \begin{cases} \frac{AB+\gamma b-\gamma\mu(\alpha^*)}{D} \text{ if 2 enters} \\ \frac{A}{2} \text{ if 2 does not enter} \end{cases} \\
q_2(\alpha,b) &= \begin{cases} \frac{2AB+\alpha D-4b+\gamma^2\mu(\alpha^*)}{2D} \text{ if } \alpha\in(\alpha^*,\bar{\alpha}] \\ 0 \text{ if } \alpha\in[0,\alpha^*] \text{ (2 does not enter for such } \alpha) \end{cases}
\end{aligned}
$$

Equilibrium profits at a Bayesian Nash equilibrium Routine computations show that the equilibrium duopoly profits are as follows.

$$
\begin{aligned}
\pi_1(b) &= [q_1(b)]^2 - b \\
\pi_2(\alpha,b) &= [q_2(\alpha,b)]^2 - k^2 \text{ where } \alpha\in(\alpha^*,\bar{\alpha}]
\end{aligned}
$$

9 Computation and proofs of 2nd stage equilibrium outcomes

Given the equilibrium outcome in the third stage, we next proceed to analyze the second stage game. In the second stage firm 2 makes a move and chooses either to enter or not to enter. We analyze the relationship between bribe paid in the first-stage and firm 2's entry decision in the second stage.

Proof of Lemma 1 Suppose $b=0$ and suppose that firm 2 when its type is $\alpha=0$ decides to enter. Then, following our earlier computations we get $\pi_2(0,0)=[q_2(0,0)]^2-k^2>0$ iff $q_2(0,0)>k$. Now

$$
\begin{aligned}
q_2(0,0) &= \frac{2AB+\gamma^2\mu(0)}{2D} > k \\
&\Longleftrightarrow 2AB+\gamma^2\mu(0) > 2kD ----(9)
\end{aligned}
$$

Note that $D=B(2+\gamma)$ and $\gamma\in[-1,1]$. Note $\gamma=0$ implies that $B=2$ and $D=4$. Then $2AB+\gamma^2\mu(0)=4A>2kD=8k$ (see assumption 1). Now suppose $\gamma\neq 0$. Then $\gamma^2>0$. Note that $2AB\geq 2kD \Longleftrightarrow A\geq k(2+\gamma)$. From assumption 1 we have $A\geq 3k$. This means $2AB+\gamma^2\mu(0)>2kD$.■

We now analyze the case where $b > 0$ but is small enough. Since $\pi_2(0,0) > 0$ and $\pi_2(0,b)$ is continuous in b, we have that for b small enough $\pi_2(0,b) > 0$. This means all types of firm 2 will enter when b is small enough. *That is, $\alpha^* = 0$ for b small enough.*

Note that for b *small enough* we have $\alpha^* = 0$ and hence

$$\begin{aligned}\pi_2(0,b) &= [q_2(0,b)]^2 - k^2 \\ &= \left[\frac{2AB - 4b + \gamma^2\mu(0)}{2D}\right]^2 - k^2 > 0. ----(10)\end{aligned}$$

(10) above clearly shows that $\pi_2(0,b)$ is strictly decreasing in b and there exists $\underline{b} > 0$ s.t. $\pi_2(0,\underline{b}) = 0$. This implies $\pi_2(0,b) > 0$ for all $b \in [0,\underline{b})$. This in turn means that if 1 chooses $b \in [0,\underline{b})$ all types of firm 2 will enter. Clearly when $b \leq \underline{b}$, we have $\alpha^* = 0$. Note that

$$\pi_2(0,\underline{b}) = 0 ----(11)$$

Now note that $\pi_2(0,\underline{b}) = 0 \Longleftrightarrow q_2(0,\underline{b}) = k$. This implies $\frac{2AB-4\underline{b}+\gamma^2\mu(0)}{2D} = k$.

Hence

$$\underline{b} = \frac{2AB + \gamma^2\mu(0) - 2kD}{4} ----(12)$$

Proof of Proposition 1 Straight forward and follows from discussion above. Note that when $b \in (0,\underline{b})$ all types enter. That is, $\alpha^* = 0$. Here

$$\pi_2(\alpha,b) = \left[\frac{2AB + \alpha D - 4b + \gamma^2\mu(0)}{2D}\right]^2 - k^2$$

Clearly $\frac{\partial\pi_2(.)}{\partial b} = -\frac{2}{D} < 0$ and $\frac{\partial\pi_2(.)}{\partial\alpha} = \frac{1}{2} > 0.\blacksquare$

We now provide some discussion that will lead to the proofs of propositions 2 and 3.

Since $\pi_2(0,\underline{b}) = 0$ it means that if $b > \underline{b}$ and if firm 2, when its type is $\alpha = 0$ were to enter, its payoff would be $\pi_2(0,b) < 0$. This implies that when $b > \underline{b}$ some types will choose not to enter. That is, $\alpha^* > 0$ for $b > \underline{b}$.

Now note that

$$\mu(\alpha^*) = \int_{\alpha^*}^{\bar{\alpha}} \alpha\frac{f(\alpha)}{1-F(\alpha^*)}d\alpha = \frac{\int_{\alpha^*}^{\bar{\alpha}} \alpha f(\alpha)\,d\alpha}{1-F(\alpha^*)}$$

This means

$$\begin{aligned}\frac{d\mu(\alpha^*)}{d\alpha^*} &= \frac{-(1-F(\alpha^*))\,\alpha^* f(\alpha^*) + f(\alpha^*)\int_{\alpha^*}^{\bar{\alpha}} \alpha f(\alpha)\,d\alpha}{(1-F(\alpha^*))^2} \\ &= \frac{f(\alpha^*)}{1-F(\alpha^*)}\left[\mu(\alpha^*)-\alpha^*\right] ----(13)\end{aligned}$$

Note that $\mu(\alpha^*)-\alpha^* \geq 0$ (Since $\mu(\alpha^*)$ is the expected value α given that it is more than α^*). This means $\frac{d\mu(\alpha^*)}{d\alpha^*} \geq 0$.

Now take $b > \underline{b}$. Note that for b close enough to $\underline{b}$ the critical type, α^* will be lower than the highest type $\bar{\alpha}$. Note that $\pi_2(\alpha^*, b) = [q_2(\alpha^*, b)]^2 - k^2 = 0$. Now $[q_2(\alpha^*, b)]^2 - k^2 = 0$ iff $q_2(\alpha^*, b) = k$. This means

$$\frac{2AB + \alpha^* D - 4b + \gamma^2 \mu(\alpha^*)}{2D} - k = 0$$

From above (and using (13)) we can compute that

$$\frac{d\alpha^*}{db} = \frac{4h}{D + \gamma^2 \frac{f(\alpha^*)}{1-F(\alpha^*)}\left[\mu(\alpha^*)-\alpha^*\right]} ----(14)$$

Note that $\frac{d\alpha^*}{db} > 0$ as $\mu(\alpha^*) \geq \alpha^*$.

For $b > \underline{b}$ and $\alpha \geq \alpha^*$ we have

$$\begin{aligned}\pi_2(\alpha, b) &= [q_2(\alpha, b)]^2 - k^2 \\ &= \left[\frac{2AB + \alpha D - 4b + \gamma^2 \mu(\alpha^*)}{2D}\right]^2 - k^2 ----(15)\end{aligned}$$

Note that

$$\begin{aligned}\frac{\partial q_2(\alpha, b)}{\partial b} &= \frac{1}{2D}\left[-4h + \gamma^2 \frac{d\mu(\alpha^*)}{d\alpha^*}\frac{d\alpha^*}{db}\right] \\ &= \frac{1}{2D}\left[-4h + \gamma^2 \frac{f(\alpha^*)}{1-F(\alpha^*)}\left[\mu(\alpha^*)-\alpha^*\right]\frac{d\alpha^*}{db}\right] ---(16)\end{aligned}$$

From (14) we get that

$$\begin{aligned}&-4h + \gamma^2 \frac{f(\alpha^*)}{1-F(\alpha^*)}\left[\mu(\alpha^*)-\alpha^*\right]\frac{d\alpha^*}{db} \\ &= -D\frac{d\alpha^*}{db} < 0. ----(17)\end{aligned}$$

Using (16), (17) and (14) we get that

$$\frac{\partial q_2(\alpha, b)}{\partial b} < 0 ----(18)$$

From (15) we know that $\frac{\partial \pi_2(\alpha,b)}{\partial b} = 2q_2(\alpha,b)\frac{\partial q_2(\alpha,b)}{\partial b}$. Hence from (18) we have when $b > \underline{b}$, $\pi_2(\alpha,b)$ is strictly decreasing in b. Also, in this case, $\pi_2(\alpha,b)$ is strictly increasing in α. That is, $\pi_2(\bar{\alpha},b) > \pi_2(\alpha,b)$ for $b > \underline{b}$.

Now note that using L'hospital's rule we get

$$\lim_{\alpha^* \longrightarrow \bar{\alpha}} \mu(\alpha^*) = \bar{\alpha} - - - - (19)$$

Since $D = 4 - \gamma^2$, by using (15) we get,

$$\begin{aligned}\pi_2(\bar{\alpha},b) &= \left[\frac{2AB - 4b + \bar{\alpha}D + \gamma^2\bar{\alpha}}{2D}\right]^2 - k^2 \\ &= \left[\frac{2AB - 4b + 4\bar{\alpha}}{2D}\right]^2 - k^2 - - - - (20)\end{aligned}$$

Note that $\pi_2(\bar{\alpha},b)$ is strictly decreasing in b and there exists $\bar{b}$ s.t. $\pi_2\left(\bar{\alpha},\bar{b}\right) = 0$. This means, for $b = \bar{b}$, $\alpha^* = \bar{\alpha}$.

Hence

$$\begin{aligned}\pi_2\left(\bar{\alpha},\bar{b}\right) &= \left[\frac{2AB - 4\bar{b} + 4\bar{\alpha}}{2D}\right]^2 - k^2 = 0 \\ \text{iff } \frac{2AB - 4\bar{b} + 4\bar{\alpha}}{2D} &= k\end{aligned}$$

From above we get that

$$\bar{b} = \frac{2AB + 4\bar{\alpha} - 2kD}{4} - - - - (21).$$

Since $\gamma^2 \leq 1$ and $\bar{\alpha} > \mu(0)$, comparing (21) with (12) we clearly get that

$$\bar{b} > \underline{b} - - - - (22)$$

Proof of Proposition 2 Straightforward and follows from discussion above (see equations 13-21).■

Proof of Proposition 3 Note that from the discussion in the main body of our paper it is clear that.

$$0 < \alpha^* < \bar{\alpha} \Longleftrightarrow \underline{b} < b < \bar{b}$$

From (12) and (21)) we get

$$\begin{aligned} \underline{b} &= \frac{2AB + \gamma^2\mu(0) - 2kD}{4} \\ \bar{b} &= \frac{2AB + 4\bar{\alpha} - 2Dk}{4h} \end{aligned}$$

Therefore,

$$\begin{aligned} \underline{b} &< b < \bar{b} \Longleftrightarrow \frac{2AB + \gamma^2\mu(0) - 2kD}{4} < b < \frac{2AB + 4\bar{\alpha} - 2kD}{4h} \\ &\Longleftrightarrow \frac{2AB + \gamma^2\mu(0) - 2kD}{D} < \frac{4b}{D} < \frac{2AB + 4\bar{\alpha} - 2kD}{D} \end{aligned}$$

Note that since $B = 2 - \gamma$ and $D = 4 - \gamma^2 = B(2+\gamma)$ the above is equivalent to the following:

$$\underline{b} < b < \bar{b} \Longleftrightarrow \frac{2A}{2+\gamma} - 2k + \frac{\gamma^2}{D}\mu(0) < \frac{4}{D}hb < \frac{2A}{2+\gamma} - 2k + \frac{4\bar{\alpha}}{D}$$

This completes the proof.■

10 Notes on preliminary results and their proofs

Note that $\pi_1(b) = [q_1(b)]^2 - b$. Hence,

$$\frac{d\pi_1(b)}{db} = 2q_1(b)\frac{dq_1(b)}{db} - 1 - - - - (23)$$

When $b \leq \underline{b}$ then $\alpha^* = 0$. This means for $b \in (0, \underline{b})$

$$\begin{aligned} q_1(b) &= \frac{AB + \gamma b - \gamma\mu(0)}{D} - - - - (24) \\ \frac{dq_1(b)}{db} &= \frac{\gamma}{D} - - - - (24a) \end{aligned}$$

This implies when $b \in (0, \underline{b})$

$$\frac{d\pi_1(b)}{db} = \frac{2[AB + \gamma b - \gamma\mu(0)]\gamma}{D^2} - 1 - - - - (25)$$

From (12) and (24) we get that

$$q_1(\underline{b}) = \frac{2A - \gamma\mu(0) - 2k\gamma}{4} - - - - (26)$$

Proof of lemma 2 Note that when $b \in (0, \underline{b})$ we have $q_1(b) = \frac{AB+\gamma b-\gamma\mu(0)}{D}$. Clearly $q_1(b)$ is continuous for all $b \in (0, \underline{b})$. When $b \in (\underline{b}, \bar{b})$ we have $q_1(b) = \frac{AB+\gamma b-\gamma\mu(\alpha^*)}{D}$. Since α^* is continuous in b, clearly $q_1(b)$ is continuous for all $b \in (\underline{b}, \bar{b})$. Note that as $b \longrightarrow \underline{b}$ we have $\alpha^* \longrightarrow 0$. Hence, $\lim_{b\to\underline{b}} \alpha^*(b) = 0$. Routine computation shows that $\lim_{b\to\underline{b}^-} q_1(b) = \lim_{b\to\underline{b}^+} q_1(b) = q_1(\underline{b})$. Hence, $q_1(b)$ is continuous for all $b \in [0, \bar{b})$.$\blacksquare$

Note that when $b \in (\underline{b}, \bar{b})$ we have

$$q_1(b) = \frac{AB + \gamma b - \gamma\mu(\alpha^*)}{D} ----(27)$$

where $\alpha^* \in (0, \bar{\alpha})$. Hence, for $b \in (\underline{b}, \bar{b})$ we have

$$\frac{dq_1(b)}{db} = \frac{\gamma}{D}\left[1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\frac{d\alpha^*}{db}\right]$$

$$\begin{aligned}
&= \frac{\gamma}{D}\left[1 - \frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]\frac{4}{D + \gamma^2\frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]}\right] \\
&= \frac{\gamma}{D\left(D + \gamma^2\frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]\right)}\left[D + \gamma^2\frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*] - \frac{4f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]\right] \\
&= \frac{\gamma}{D\left(D + \gamma^2\frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]\right)}\left[D\left(1 - \frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]\right)\right] \\
&= \frac{\gamma\left(1 - \frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]\right)}{\left(D + \gamma^2\frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]\right)} ----(28)
\end{aligned}$$

Since $\frac{d\mu(\alpha^*)}{d\alpha^*} = \frac{f(\alpha^*)}{1-F(\alpha^*)}[\mu(\alpha^*) - \alpha^*]$ using above we get that when $b \in (\underline{b}, \bar{b})$

$$\frac{dq_1(b)}{db} = \frac{\gamma\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right)}{D + \gamma^2\frac{d\mu(\alpha^*)}{d\alpha^*}} ----(28a)$$

Note that when $b = \underline{b}$ we have $\alpha^* = 0$. Hence, using (28) we get

$$\lim_{b\longrightarrow\underline{b}^+}\frac{dq_1(b)}{db} = \frac{\gamma(1 - f(0)\mu(0))}{(D + \gamma^2 f(0)\mu(0))} ----(29)$$

Proof of lemma 3 Note that when $b \in (0, \underline{b})$ we have $q_1(b) = \frac{AB+\gamma b-\gamma\mu(0)}{D}$. Hence, when $b \in (0, \underline{b})$ we have $q_1(b) = \frac{\gamma}{D}$. From (28a) we get that when $b \in (\underline{b}, \bar{b})$ we have $\frac{dq_1(b)}{db} = \frac{\gamma\left(1-\frac{d\mu(\alpha^*)}{d\alpha^*}\right)}{D+\gamma^2\frac{d\mu(\alpha^*)}{d\alpha^*}}$.$\blacksquare$

Proof of lemma 4 Note that from the previous lemma we have $\lim_{b \longrightarrow \underline{b}^-} \frac{dq_1(b)}{db} = \frac{\gamma}{D}$. From (29) we get that $\lim_{b \longrightarrow \underline{b}^+} \frac{dq_1(b)}{db} = \frac{\gamma(1-f(0)\mu(0))}{(D+\gamma^2 f(0)\mu(0))}$. Hence,

$$\begin{aligned} & \lim_{b \longrightarrow \underline{b}^-} \frac{dq_1(b)}{db} - \lim_{b \longrightarrow \underline{b}^+} \frac{dq_1(b)}{db} \\ = & \frac{\gamma}{D} \left[1 - \frac{D(1 - f(0)\mu(0))}{(D + \gamma^2 f(0)\mu(0))} \right] \end{aligned}$$

Note that since $\mu(0) > 0$ we get that if $f(0) > 0$

$$\frac{D(1 - f(0)\mu(0))}{(D + \gamma^2 f(0)\mu(0))} < 1 \Longrightarrow 1 - \frac{D(1 - f(0)\mu(0))}{(D + \gamma^2 f(0)\mu(0))} > 0.$$

This means when $f(0) > 0$

$$\lim_{b \longrightarrow \underline{b}^-} \frac{dq_1(b)}{db} > \lim_{b \longrightarrow \underline{b}^+} \frac{dq_1(b)}{db} \text{ iff } \gamma > 0.$$

Also, when $f(0) = 0$ then $\lim_{b \longrightarrow \underline{b}^-} \frac{dq_1(b)}{db} - \lim_{b \longrightarrow \underline{b}^+} \frac{dq_1(b)}{db} = 0.$ ■

Now let

$$J = \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{d\mu(\alpha^*)}{d\alpha^*} = \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{f(\alpha^*)}{1 - F(\alpha^*)} [\mu(\alpha^*) - \alpha^*] - - - - (30)$$

Note that

$$J = f(\bar{\alpha}) \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{\mu(\alpha^*) - \alpha^*}{1 - F(\alpha^*)} - - - - (31)$$

Using (19) and the fact that α is distributed over $[0, \bar{\alpha}]$ with distribution function $F(.)$ we know that

$$\begin{aligned} \lim_{\alpha^* \longrightarrow \bar{\alpha}} [\mu(\alpha^*) - \alpha^*] &= 0 \\ \text{and } \lim_{\alpha^* \longrightarrow \bar{\alpha}} [1 - F(\alpha^*)] &= 0 - - - - (32) \end{aligned}$$

Hence, by using L'hospital's rule we get

$$\begin{aligned}
J &= f(\bar{\alpha}) \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{\frac{d[\mu(\alpha^*)-\alpha^*]}{d\alpha^*}}{\frac{d[1-F(\alpha^*)]}{d\alpha^*}} \\
&= f(\bar{\alpha}) \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{\frac{d\mu(\alpha^*)}{d\alpha^*} - 1}{-f(\alpha^*)} \\
&= f(\bar{\alpha}) \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{\frac{f(\alpha^*)}{1-F(\alpha^*)} [\mu(\alpha^*) - \alpha^*] - 1}{-f(\alpha^*)} \quad \text{(using (13))} \\
&= f(\bar{\alpha}) \frac{\lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{f(\alpha^*)}{1-F(\alpha^*)} [\mu(\alpha^*) - \alpha^*] - 1}{-f(\bar{\alpha})} \\
&= 1 - \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{f(\alpha^*)}{1-F(\alpha^*)} [\mu(\alpha^*) - \alpha^*] \\
&= 1 - J \qquad \text{(see (30))} - - - - (32a)
\end{aligned}$$

(32a) implies that

$$J = \frac{1}{2} - - - - (33)$$

Using (28a) and routine computations show that

$$\lim_{b \longrightarrow \bar{b}^-} \frac{dq_1(b)}{db} = \frac{\gamma}{D+4} - - - - (34)$$

and

$$\lim_{b \longrightarrow \bar{b}^-} q_1(b) = \frac{A - k\gamma}{2} - - - - (35)$$

Proof of lemma 5 Directly follows from (34) and (35).■

11 Computations and proofs of first stage equilibrium outcomes

We now analyze the first-stage game and solve for the equilibrium level of bribe chosen by firm 1. In the first stage firm 1 chooses the optimal level of bribe b to maximize its expected payoff (anticipating the equilibrium outcomes in stage 2 and 3). Let the optimal level of bribe be b^*. We will first deal with substitutes ($\gamma > 0$).

11.1 Optimal bribe when goods are substitutes $(\gamma > 0)$

We now provide some preliminary computations for the proofs of propositions 4-9.

Note that combining (13) and (14) we get

$$\frac{d\alpha^*}{db} = \frac{4}{D + \gamma^2 \frac{d\mu(\alpha^*)}{d\alpha^*}} - - - - (36)$$

Since by assumption 2 $\frac{d\mu(\alpha^*)}{d\alpha^*} \geq 0$ we must have $\frac{d\alpha^*}{db} \leq \frac{4}{D}$. Again by assumption 2 $\frac{d\mu(\alpha^*)}{d\alpha^*}$ is non-decreasing in α^*. From (30) to (33) we showed $J = \lim_{\alpha^* \longrightarrow \bar{\alpha}} \frac{d\mu(\alpha^*)}{d\alpha^*} = \frac{1}{2}$. This means $\frac{d\mu(\alpha^*)}{d\alpha^*} \leq \frac{1}{2}$ for all $\alpha^* \in [0, \bar{\alpha}]$. Hence, from (36) we have $\frac{d\alpha^*}{db} \geq \frac{4}{D+\frac{1}{2}\gamma^2} = \frac{8}{2D+\gamma^2} = \frac{8}{8-\gamma^2} \geq 1$ (as $\gamma^2 \leq 1$). From the above discussion we get that

$$1 \leq \frac{d\alpha^*}{db} \leq \frac{4}{D} - - - - (37) .$$

From lemma 3 we get that when $b \in \left(\underline{b}, \bar{b}\right)$, $q_1'(b) = \frac{\gamma\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right)}{D+\gamma^2 \frac{d\mu(\alpha^*)}{d\alpha^*}}$. Using (36) we have

$$q_1'(b) = \frac{d\alpha^*}{db}\left(\frac{\gamma}{4}\right)\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right) - - - - (38)$$

From the main body of the paper we know that if $b \in \left(\underline{b}, \bar{b}\right)$

$$E_1(b) = F(\alpha^*)\left[\frac{A^2}{4} - (q_1(b))^2\right] + (q_1(b))^2 - b$$

This means if $b \in \left(\underline{b}, \bar{b}\right)$

$$E_1'(b) = f(\alpha^*)\frac{d\alpha^*}{db}\left[\frac{A^2}{4} - (q_1(b))^2\right] + 2q_1(b)\, q_1'(b)\left[1 - F(\alpha^*)\right] - 1$$

Using (38) we get for all $b \in \left(\underline{b}, \bar{b}\right)$

$$E_1'(b) = \frac{d\alpha^*}{db}\left\{\begin{array}{c} f(\alpha^*)\left[\frac{A^2}{4} - (q_1(b))^2\right] \\ +\frac{1}{2}q_1(b)\,\gamma\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right)\left[1 - F(\alpha^*)\right] \end{array}\right\} - 1 - - - (39)$$

Routine computations show that

$$\frac{A^2}{4} - \left(q_1(\underline{b})\right)^2 = \frac{\gamma}{16}\left[2k + \mu(0)\right]\left[4A - 2k\gamma - \gamma\mu(0)\right] - - - - (40)$$

Note that $\mu(0) < \bar{\alpha}$ and $\gamma \in [0, 1]$ as goods are substitutes. This means

$$\begin{array}{rcl} \text{If } \gamma & > & 0 \\ \frac{A^2}{4} - \left(q_1(\underline{b})\right)^2 & < & \frac{\gamma}{4}\left[2k + \bar{\alpha}\right]A - - - - (41) \end{array}$$

Proof of Proposition 4 (i) From previous lemmas and discussion we get that when $\gamma > 0$ (goods are substitutes) and $b \in (0, \underline{b})$ we have $q_1(b) = \frac{AB+\gamma b-\gamma\mu(0)}{D}$ and $\frac{dq_1(b)}{db} = \frac{\gamma}{D} > 0$. When $b \in (0, \underline{b})$ we have $E_1(b) = \pi_1(b) = (q_1(b))^2 - b$. From (26) we have $q_1(\underline{b}) = \frac{2A-\gamma\mu(0)-2k\gamma}{4}$. As $q_1(.)$ is strictly increasing in b we get for all $b \in (0, \underline{b})$

$$\begin{aligned} E_1'(b) &= \pi_1'(b) = 2q_1(b)\frac{\gamma}{D} - 1 < 2q_1(\underline{b})\frac{\gamma}{D} - 1 \\ &= \frac{2A - \gamma\mu(0) - 2k\gamma}{2}\frac{\gamma}{D} - 1 \le \frac{\gamma A}{D} - 1. \end{aligned}$$

The above shows that $E_1'(b) < 0$ if $1 \le \frac{D}{\gamma A}$. By hypothesis of the proposition we have $1 \le \frac{D}{\gamma A[f(\alpha)(2k+\bar{\alpha})+1]}$. This implies $1 \le \frac{D}{\gamma A}$. Hence, we get $E_1'(b) < 0$ for all $b \in (0, \underline{b})$.

Using (37) and (39) we get that for all $b \in \left(\underline{b}, \bar{b}\right)$

$$E_1'(b) \le \frac{4}{D}\left\{\begin{array}{c} f(\alpha^*)\left[\frac{A^2}{4} - (q_1(b))^2\right] \\ +\frac{1}{2}q_1(b)\gamma\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right)[1 - F(\alpha^*)] \end{array}\right\} - 1 - - - (42)$$

Note that since $q_1(b)$ is strictly increasing for all $b \in \left(\underline{b}, \bar{b}\right)$,$1 - F(\alpha^*) \le 1$ and $1 - \frac{d\mu(\alpha^*)}{d\alpha^*} \le 1$ (from assumption 2) we get from above

$$E_1'(b) < \frac{4}{D}\left\{f(\alpha^*)\left[\frac{A^2}{4} - (q_1(\underline{b}))^2\right] + \frac{1}{2}\gamma\lim_{b\to\bar{b}^-} q_1(b)\right\} - 1 - - - (43)$$

Since $\gamma > 0$ by using (35) and (41) in (43) we get that

$$\begin{aligned} E_1'(b) &< \frac{4}{D}\left\{f(\alpha^*)\frac{\gamma}{4}[2k + \bar{\alpha}]A + \frac{1}{2}\gamma\frac{A - k\gamma}{2}\right\} - 1 \\ &< \frac{4}{D}\left\{f(\alpha^*)\frac{\gamma}{4}[2k + \bar{\alpha}]A + \frac{1}{4}\gamma A\right\} - 1 \\ &= \frac{\gamma A}{D}\{f(\alpha^*)\gamma[2k + \bar{\alpha}] + 1\} - 1 - - - - (44) \end{aligned}$$

Note that $1 \le \frac{D}{\gamma A[f(\alpha)(2k+\bar{\alpha})+1]}$ for all $\alpha \in [0, \bar{\alpha}]$ implies $\frac{\gamma A}{D}\{f(\alpha^*)\gamma[2k + \bar{\alpha}] + 1\} - 1 \le 0$. From (44) this implies $E_1'(b) < 0$ for all $b \in \left(\underline{b}, \bar{b}\right)$. We have already demonstrated that $E_1'(b) < 0$ for all $b \in (0, \underline{b})$. Also, $E_1(b)$ is continuous for all $b \in \left[0, \bar{b}\right]$. All these together imply that $E_1(0) > E_1(b)$ for all $b \in \left(0, \bar{b}\right]$. Hence, if $1 \le \frac{D}{\gamma A[f(\alpha)(2k+\bar{\alpha})+1]}$ for all $\alpha \in [0, \bar{\alpha}]$ then $b^* = 0$.■

Proof of Proposition 4(ii) From (25) we get that when $b \in (0, \underline{b})$

$$\frac{d\pi_1(b)}{db} = \frac{2\left[AB + \gamma b - \gamma\mu(0)\right]\gamma h}{D^2} - 1$$

Since $\gamma > 0$, we get $\frac{2[AB+\gamma b-\gamma\mu(0)]\gamma}{D^2} > \frac{2[AB-\gamma\mu(0)]\gamma}{D^2}$. Since $\gamma \leq 1$, $D \in [3,4]$ and $B \in [1,3]$ we get $1 \geq \frac{8}{\gamma[A-\mu(0)]}$ implies that $1 \geq \frac{D^2}{2\gamma(AB-\gamma\mu(0))}$ and this turn implies $\frac{2[AB-\gamma\mu(0)]\gamma}{D^2} - 1 \geq 0$ and this means $\frac{d\pi_1(b)}{db} = \frac{2[AB+\gamma hb-\gamma\mu(0)]\gamma}{D^2} - 1 > 0$ for all $b \in (0, \underline{b})$. Note that $E_1(b) = \pi_1(b)$ for all $b \in (0, \underline{b})$. Hence we get that if $1 \geq \frac{8}{\gamma[A-\mu(0)]}$ then $E_1'(b) > 0$ for all $b \in (0, \underline{b})$. Also, $E_1(b)$ is continuous for all $b \in \left[0, \bar{b}\right]$. Consequently, $b^* \geq \underline{b}$.$\blacksquare$

Proof of Proposition 4(iii) Since $\gamma > 0$ implies that $q_1(b)$ is strictly increasing in all $b \in \left(\underline{b}, \bar{b}\right)$ we must have

$$\begin{aligned}
\frac{A^2}{4} - (q_1(b))^2 &> \frac{A^2}{4} - \lim_{b \to \bar{b}^-} (q_1(b))^2 \\
&= \frac{A^2}{4} - \left(\frac{A - k\gamma}{2}\right)^2 \\
&= \frac{k\gamma}{4}(2A - k\gamma) \\
&\geq \frac{k\gamma}{4}(2A - k) \text{ (since } \gamma \leq 1) - - - (45)
\end{aligned}$$

From (37) we have $1 \leq \frac{d\alpha^*}{db}$. Also, $1 - F(\alpha^*) \geq 0$. Using these facts and (45) in (39) we have for all $b \in \left(\underline{b}, \bar{b}\right)$

$$\begin{aligned}
E_1'(b) &= \frac{d\alpha^*}{db}\left\{\begin{array}{c} f(\alpha^*)\left[\frac{A^2}{4} - (q_1(b))^2\right] \\ +\frac{1}{2}q_1(b)\gamma\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right)\left[1 - F(\alpha^*)\right] \end{array}\right\} - 1 \\
&> f(\alpha^*)\frac{k\gamma}{4}(2A - k) - 1 - - - - (46).
\end{aligned}$$

Now note that from assumption 1 we have $A \geq 3k$. This means

$$2A - k \geq 2A - \frac{A}{3} = \frac{5A}{3} - - - - (47).$$

Using (47) in (46) we get for all $b \in \left(\underline{b}, \bar{b}\right)$

$$E_1'(b) > \frac{5k\gamma f(\alpha^*)}{12} - 1 - - - - (50).$$

If $1 \geq \max\left\{\frac{12}{5f(\alpha)k\gamma A}, \frac{8}{\gamma[A-\mu(0)]}\right\}$ for all $\alpha \in [0, \bar{\alpha}]$ then $1 \geq \frac{12}{5f(\alpha)k\gamma A}$. But this implies (from (50)) for all $b \in \left(\underline{b}, \bar{b}\right)$, $E_1'(b) > 0$.

From proposition we know that if $1 \geq \frac{8}{\gamma[A-\mu(0)]}$ then $b^* \geq \underline{b}$. Consequently, $1 \geq \max\left\{\frac{12}{5f(\alpha)k\gamma A}, \frac{8}{\gamma[A-\mu(0)]}\right\}$ implies that $b^* = \bar{b}$.■

Proof of Proposition 5 Note that from proposition 4 we get sufficient conditions under which $b^* = 0$ or $b^* \in \left[\underline{b}, \bar{b}\right]$. We now rule out the possibility that $b^* \in (0, \underline{b})$. On the contrary suppose that $0 < b^* < \underline{b}$. Note that when $b \in (0, \underline{b})$, $E_1(b) = \pi_1(b)$. From (25) we know that when $b \in (0, \underline{b})$

$$\frac{d\pi_1(b)}{db} = \frac{2\left[AB + \gamma b - \gamma\mu(0)\right]\gamma}{D^2} - 1$$

Clearly $\frac{d\pi_1(b)}{db}$ is strictly increasing in b when $b \in (0, \underline{b})$. This implies $\pi_1(b)$ cannot achieve a maximum in $(0, \underline{b})$. Hence, $b^* \notin (0, \underline{b})$.■

11.2 Optimal bribe when goods are complements $(\gamma < 0)$

Proof of Proposition 6 Note that when $b \in (0, \underline{b})$ we have $E_1(b) = \pi_1(b) = (q_1(b))^2 - b$. Therefore when $b \in (0, \underline{b})$ we get $\frac{dE_1(b)}{db} = 2q_1(b)\frac{dq_1(b)}{db} - 1$. From (24a) we get that for $b \in (0, \underline{b})$, $\frac{dq_1(b)}{db} = \frac{\gamma}{D} < 0$ (since $\gamma < 0$). Hence, $\gamma < 0$ implies that $E_1'(b) < 0$ for all $b \in (0, \underline{b})$.

Now note that if $b \in \left(\underline{b}, \bar{b}\right)$

$$E_1(b) = F(\alpha^*)\left[\frac{A^2}{4} - (q_1(b))^2\right] + (q_1(b))^2 - b$$

From (44) we know that for all $b \in \left(\underline{b}, \bar{b}\right)$

$$E_1'(b) < \frac{\gamma A}{D}\left\{f(\alpha^*)\gamma\left[2k + \bar{\alpha}\right] + 1\right\} - 1 - - - - (51)$$

Since $\gamma < 0$ from (61) we get that $E_1'(b) < 0$ for all $b \in \left(\underline{b}, \bar{b}\right)$. Also, $E_1(b)$ is continuous for all $b \in \left[0, \bar{b}\right]$. Hence, $b^* = 0$.■

11.3 Case of No uncertainty $(\bar{\alpha} = 0)$

Proof of Proposition 7 From the discussion in section 5 we know that

$$A^2\left(\gamma^2 + 4\gamma\right) + 2A\left(-8 - 4\gamma + 2\gamma^2 + \gamma^3\right) + 2k\left(16 + 16\gamma - 4\gamma^3 - \gamma^4\right) \geq 0 \Longrightarrow b^{**} = \hat{b} - -(51)$$

and

$$A^2\left(\gamma^2 + 4\gamma\right) + 2A\left(-8 - 4\gamma + 2\gamma^2 + \gamma^3\right) + 2k\left(16 + 16\gamma - 4\gamma^3 - \gamma^4\right) < 0 \Longrightarrow b^{**} = 0 - -(52)$$

Note that since $\gamma \in (0,1]$

$$\begin{aligned} & \frac{\partial}{\partial \gamma}\left[A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)\right] \\ = & \; 2(\gamma+2)\left(A^2+3A\gamma-2A-4k\gamma^2-4k\gamma+8k\right) \\ \geq & \; 0 \Longleftrightarrow \left(A^2+3A\gamma-2A-4k\gamma^2-4k\gamma+8k\right) \geq 0 ----(53) \end{aligned}$$

Again, since $\gamma \in (0,1]$ and $A \geq \frac{18}{5}$

$$\begin{aligned} & A^2+3A\gamma-2A-4k\gamma^2-4k\gamma+8k \\ = & \; A(A+3\gamma-2)+k\left(8-4\gamma^2-4\gamma\right) \\ \geq & \; A(A+3\gamma-2)>0 \;\; ----(54) \end{aligned}$$

(53) and (54) together imply that

$$\frac{\partial}{\partial \gamma}\left[A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)\right]>0----(55)$$

Now note that when $\gamma=0$ then

$$\begin{aligned} & \left[A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)\right] \\ = & \; -16A+32k<0 \text{ since } A \geq 3k \text{ (assumption 1)} ----(56) \end{aligned}$$

When $\gamma=1$ then since $A \geq \frac{18}{5}$ and $k>0$ we get

$$\begin{aligned} & \left[A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)\right] \\ = & \; 5A^2-18A+54k>0 \;\; ----(57). \end{aligned}$$

Note that $\left[A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)\right]$ is continuous in γ and is strictly increasing (see (54)). Using this fact, (55), (56) and the intermediate value theorem, we get that there exists a unique $\underline{\gamma} \in (0,1)$ such that

$$A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)=0 \Longleftrightarrow \gamma=\underline{\gamma}---(58)$$

Also,

$$\begin{aligned} \left[A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)\right] & > 0 \text{ if } \gamma>\underline{\gamma}----(59) \\ & \text{and} \\ \left[A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right)\right] & < 0 \text{ if } \gamma<\underline{\gamma}----(60) \end{aligned}$$

Hence, using (51) and (52) we get that if $A \geq \frac{18}{5}$ then $\gamma \geq \underline{\gamma} \Longrightarrow b^{**}=\hat{b}$ and $\gamma<\underline{\gamma} \Longrightarrow b^{*}=0$. ■

Proof of Proposition 8 Note that $(\gamma^2+4\gamma)$ is strictly increasing. This implies when $\gamma\in[-1,0)$, $(\gamma^2+4\gamma)\leq 0$. Since $|\gamma|\leq 1$, we have

$$\left(-8-4\gamma+2\gamma^2+\gamma^3\right)<0----(61)$$

Since $A\geq 3k$,

$$\begin{aligned}
& A^2\left(\gamma^2+4\gamma\right)+2A\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right) \\
\leq\ & 6k\left(-8-4\gamma+2\gamma^2+\gamma^3\right)+2k\left(16+16\gamma-4\gamma^3-\gamma^4\right) \\
=\ & -2k\left(\gamma+2\right)^2\left(\gamma-1\right)\left(\gamma-2\right)<0 \text{ for all } \gamma\in[-1,0)----(62).
\end{aligned}$$

Using (52) and (62) we get that when $\gamma\in[-1,0)$ we must have $b^{**}=0$.■

12 Computations and proofs on total surplus and market quality

Note that in section 4.1 we have already derived the total surplus function. We reproduce it below.

$$\begin{aligned}
W(b) &= \left[\frac{3}{8}A^2+q_0\right]F(\alpha^*) \\
&+\int_{\alpha^*}^{\bar{\alpha}}\left[\begin{array}{c} Aq_1(b)+(A-b+\alpha)\,q_2(\alpha,b) \\ -\frac{1}{2}\left\{q_1^2(b)+q_2^2(\alpha,b)+2\gamma q_1(b)\,q_2(\alpha,b)\right\}+q_0-k^2 \end{array}\right]dF(\alpha)--(63)
\end{aligned}$$

From above we can write the expected total surplus as follows:

$$\begin{aligned}
W(b) &= \left[\frac{3}{8}A^2\right]F(\alpha^*)+\left[Aq_1(b)-\frac{1}{2}q_1^2(b)-k^2\right](1-F(\alpha^*)) \\
&+[A-b-\gamma q_1(b)]\int_{\alpha^*}^{\bar{\alpha}}q_2(\alpha,b)\,dF(\alpha)+\int_{\alpha^*}^{\bar{\alpha}}\alpha q_2(\alpha,b)\,dF(\alpha) \\
&-\frac{1}{2}\int_{\alpha^*}^{\bar{\alpha}}q_2^2(\alpha,b)\,dF(\alpha)+q_0----(64)
\end{aligned}$$

Note that for $b\in[0,\underline{b})$ we have $\alpha^*=0$ (all types enter). This means $F(\alpha^*)=0$. Hence for $b \in [0, \underline{b})$

$$\begin{aligned} W(b)|_{b\in[0,\underline{b})} &= Aq_1(b) - \frac{1}{2}q_1^2(b) + [A - b - \gamma q_1(b)] \int_0^{\bar{\alpha}} q_2(\alpha, b)\, dF(\alpha) \\ &+ \int_0^{\bar{\alpha}} \alpha q_2(\alpha, b)\, dF(\alpha) - \frac{1}{2}\int_0^{\bar{\alpha}} q_2^2(\alpha, b)\, dF(\alpha) \\ &+ q_0 - k^2 - - - - (65) \end{aligned}$$

Hence, for $b \in [0, \underline{b})$

$$\begin{aligned} W'(b)|_{b\in[0,\underline{b})} &= (A - q_1(b))\, q_1'(b) - [h + \gamma q_1'(b)] \int_0^{\bar{\alpha}} q_2(\alpha, b)\, dF(\alpha) \\ &+ [A - hb - \gamma q_1(b)] \int_0^{\bar{\alpha}} \frac{\partial q_2(\alpha, b)}{\partial b} dF(\alpha) \\ &+ \int_0^{\bar{\alpha}} \alpha \frac{\partial q_2(\alpha, b)}{\partial b} dF(\alpha) - \int_0^{\bar{\alpha}} q_2(\alpha, b) \frac{\partial q_2(\alpha, b)}{\partial b} dF(\alpha) - - - - (66) \end{aligned}$$

To compute $W'(b)$ for $b \in [0, \underline{b})$, note the following. From our earlier derivations we know that

$$\int_{\alpha^*}^{\bar{\alpha}} q_2(\alpha, b) \frac{f(\alpha)}{1 - F(\alpha^*)} d\alpha = \frac{[AB - 2b] + 2\mu(\alpha^*)}{D}$$

When $b \in [0, \underline{b})$, $\alpha^* = 0$ and we have

$$\begin{aligned} q_1(b) &= \frac{AB + b\gamma - \gamma\mu(0)}{D} - - - - - - - - (67a) \\ q_2(\alpha, b) &= \frac{2AB + \alpha D - 4b + \gamma^2\mu(0)}{2D} - - - (67b) \\ \int_0^{\bar{\alpha}} q_2(\alpha, b)\, f(\alpha)\, d\alpha &= \frac{AB - 2b + 2\mu(0)}{D} - - - - - - - - (67c) \end{aligned}$$

From (67) we get that when $b \in [0, \underline{b})$

$$q_1'(b) = \frac{\gamma}{D} \text{ and } \frac{\partial q_2(\alpha, b)}{\partial b} = -\frac{2}{D} - - - - (68)$$

Hence, for $b \in [0, \underline{b})$ (using (66) and (68))

$$\begin{aligned} W'(b)|_{b\in[0,\underline{b})} &= (A - q_1(b)) \frac{\gamma}{D} - \left[1 + \frac{\gamma^2}{D}\right] \int_0^{\bar{\alpha}} q_2(\alpha, b)\, dF(\alpha) \\ &- [A - b - \gamma q_1(b)] \int_0^{\bar{\alpha}} \frac{2}{D} dF(\alpha) \\ &- \int_0^{\bar{\alpha}} \alpha \frac{2}{D} dF(\alpha) + \int_0^{\bar{\alpha}} q_2(\alpha, b) \frac{2}{D} dF(\alpha) - - - - (69) \end{aligned}$$

Using the fact that $D+\gamma^2=4$, $B=2-\gamma$ and using (69) we get that for $b\in[0,\underline{b})$

$$\begin{aligned} W'(b)\big|_{b\in[0,\underline{b})} &= \frac{1}{D}\left[\begin{array}{c} (A-q_1(b))\gamma-2\int_0^{\bar{\alpha}} q_2(\alpha,b)\,dF(\alpha) \\ -2\left[A-b-\gamma q_1(b)\right]-2\mu(0) \end{array}\right] \\ &= \frac{h}{D}\left[-AB+\gamma q_1(b)+2\left[hb-\mu(0)\right]-2\int_0^{\bar{\alpha}} q_2(\alpha,b)\,dF(\alpha)\right] ---(70) \end{aligned}$$

We now compute $W'(b)$ at $b=0$.

$$\begin{aligned} W'(b)\big|_{b=0} &= \frac{1}{D}\left[-AB+\gamma q_1(0)-2\mu(0)-2\int_0^{\bar{\alpha}} q_2(\alpha,0)\,dF(\alpha)\right] \\ &= \frac{1}{D}\left[-AB+\gamma\frac{AB-\gamma\mu(0)}{D}-2\mu(0)-2\frac{AB+2\mu(0)}{D}\right] ----(71) \end{aligned}$$

12.1 Case of substitutes $(\gamma>0)$

For substitutes since $\gamma\in(0,1]$ using (71) above we can show that $W'(b)<0$ at $b=0$. Since market quality, $\mathbb{Q}(b)=W(b)-b$. Hence, $\mathbb{Q}'(b)=W'(b)-1<0$ at $b=0$. This means $b=0$ is a local maximizer for $W(b)$ and $\mathbb{Q}(b)$. Now we will show that $b=0$ need not be the global maximizer of either $W(b)$ or $\mathbb{Q}(b)$.

Note that when $b=0$, all types of firm 2 enter (i.e. probability of entry is one). In this case $\alpha^*=0$ and the expected welfare is as follows:

$$W(0)=\int_0^{\bar{\alpha}}\left[\begin{array}{c} Aq_1(0)+(A+\alpha)\,q_2(\alpha,0) \\ -\frac{1}{2}\left\{q_1^2(0)+q_2^2(\alpha,0)+2\gamma q_1(0)\,q_2(\alpha,0)\right\}+q_0-k^2 \end{array}\right]dF(\alpha) ----(72)$$

Note that when $b=\bar{b}$ then no type of firm 2 enters. This means $\alpha^*=\bar{\alpha}$ and firm 1 is a monopolist. In this case, we have

$$W\left(\bar{b}\right)=\frac{3}{8}A^2+q_0 ----(73)$$

Now consider the following values of the parameters.

$$\begin{aligned} A &= 3,\ k=1,\ \gamma=1 \text{ and } \alpha \text{ is uniformly distributed over } [0,\bar{\alpha}]. \\ \bar{\alpha} &\in (0,0.219) \end{aligned}$$

All our assumptions are satisfied here and we have $\mu(0) = \frac{\bar{\alpha}}{2}$. In this specific example we also have

$$B = 1,\ D = 3,\ \underline{b} = \frac{\bar{\alpha}}{8} \text{ and } \bar{b} = \bar{\alpha}$$

Hence when $b = 0$, we have

$$\begin{aligned} q_1(0) &= \frac{3 - \frac{\bar{\alpha}}{2}}{3} \\ q_2(0, \alpha) &= \frac{6 + 3\alpha + \frac{\bar{\alpha}}{2}}{6} ---- (74) \end{aligned}$$

Using (72), (73) and (74) routine computations show that

$$\begin{aligned} W(0) &= \frac{53}{288}\bar{\alpha}^2 + \frac{2}{3}\bar{\alpha} + 3 + q_0 = \mathbb{Q}(0) ---- (75a) \\ W(\bar{b}) &= \frac{27}{8} + q_0,\ \mathbb{Q}(\bar{b}) = \frac{27}{8} - \bar{\alpha} + q_0 ---- (75b) \end{aligned}$$

Note that from (75a and 75b) we have

$$\begin{aligned} W(\bar{b}) - W(0) &= -\frac{53}{288}\bar{\alpha}^2 - \frac{2}{3}\bar{\alpha} + \frac{3}{8} \\ \mathbb{Q}(\bar{b}) - \mathbb{Q}(0) &= -\frac{53}{288}\bar{\alpha}^2 - \frac{5}{3}\bar{\alpha} + \frac{3}{8} \end{aligned}$$

It may be noted that $W(\bar{b}) - W(0) > 0$ t and $\mathbb{Q}(\bar{b}) - \mathbb{Q}(0)$ for all $\bar{\alpha} \in (0, 0.219)$.

12.2 Case of complements $(\gamma < 0)$

Note the following which follows from our earlier computations. Since $\gamma < 0$ (complements) and $\frac{d\mu(\alpha^*)}{d\alpha^*} < 1$ for all $\alpha^* \in [0, \bar{\alpha}]$ (assumption 2)

$$\frac{\partial q_2(\alpha, b)}{\partial b} = \begin{cases} -\frac{4h}{2D} < 0 \text{ for } b \in (0, \underline{b}) \\ -\frac{1}{2}\frac{d\alpha^*}{db} < 0 \text{ for } b \in (\underline{b}, \bar{b}) \end{cases} --- (76a)$$

$$\frac{dq_1(b)}{db} = \begin{cases} \frac{h\gamma}{D} < 0 \text{ for } b \in (0, \underline{b}) \\ \frac{h\gamma\left(1 - \frac{d\mu(\alpha^*)}{d\alpha^*}\right)}{D + \gamma^2 \frac{d\mu(\alpha^*)}{d\alpha^*}} < 0 \text{ for } b \in (\underline{b}, \bar{b}) \end{cases} --- (76b)$$

When $\gamma < 0$ by using (69) it is straightforward to show that $W'(b) < 0$ and $\mathbb{Q}'(b) < 0$ for all $b \in (0, \underline{b})$. Now suppose $b \in (\underline{b}, \bar{b})$. Then from (63) we have

$$\begin{aligned} W(b) &= \left[\frac{3}{8}A^2\right] F(\alpha^*) + q_0 \\ &+ \int_{\alpha^*}^{\bar{\alpha}} \left[\begin{array}{c} Aq_1(b) + (A - hb + \alpha) q_2(\alpha, b) \\ -\frac{1}{2}\{q_1^2(b) + q_2^2(\alpha, b) + 2\gamma q_1(b) q_2(\alpha, b)\} - k^2 \end{array} \right] f(\alpha)\, d\alpha -- (77) \end{aligned}$$

Note that $\pi_2(\alpha,b) = (A - hb + \alpha) q_2(\alpha,b) - k^2 = [q_2(\alpha,b)]^2 - k^2$. Using this in (77) we get that when $b \in (\underline{b}, \bar{b})$

$$\begin{aligned} W(b) &= \left[\frac{3}{8}A^2\right] F(\alpha^*) + q_0 \\ &\quad + \int_{\alpha^*}^{\bar{\alpha}} \left[\begin{array}{c} Aq_1(b) + \frac{1}{2}[q_2(\alpha,b)]^2 - k^2 \\ -\frac{1}{2}q_1^2(b) - \gamma q_1(b) q_2(\alpha,b) \end{array} \right] f(\alpha)\, d\alpha - -(78) \end{aligned}$$

From (78) we get that when $b \in (\underline{b}, \bar{b})$

$$\begin{aligned} W'(b) &= \frac{3}{8}A^2 f(\alpha^*) \frac{d\alpha^*}{db} \\ &\quad - \frac{d\alpha^*}{db} f(\alpha^*) \left[\begin{array}{c} Aq_1(b) + \frac{1}{2}[q_2(\alpha^*,b)]^2 - k^2 \\ -\frac{1}{2}q_1^2(b) - \gamma q_1(b) q_2(\alpha^*,b) \end{array} \right] \\ &\quad + \int_{\alpha^*}^{\bar{\alpha}} \left[\begin{array}{c} A\frac{dq_1(b)}{db} + q_2(\alpha,b)\frac{\partial q_2(\alpha,b)}{\partial b} \\ -q_1(b)\frac{dq_1(b)}{db} - \gamma q_1(b)\frac{\partial q_2(\alpha,b)}{\partial b} - \gamma q_2(\alpha,b)\frac{dq_1(b)}{db} \end{array} \right] f(\alpha)\, d\alpha - -(79) \end{aligned}$$

Note that $\pi_2(\alpha^*,b) = [q_2(\alpha^*,b)]^2 - k^2 = 0$ and $q_2(\alpha^*,b) = k$. Using these facts and (79) we have that when $b \in (\underline{b}, \bar{b})$

$$\begin{aligned} W'(b) &= \frac{3}{8}A^2 f(\alpha^*) \frac{d\alpha^*}{db} \\ &\quad - \frac{d\alpha^*}{db} f(\alpha^*) \left[Aq_1(b) - \frac{1}{2}k^2 - \frac{1}{2}q_1^2(b) - \gamma k q_1(b) \right] \\ &\quad + \int_{\alpha^*}^{\bar{\alpha}} \left[\begin{array}{c} \frac{dq_1(b)}{db}(A - q_1(b) - \gamma q_2(\alpha,b)) \\ +\frac{\partial q_2(\alpha,b)}{\partial b}(q_2(\alpha,b) - \gamma q_1(b)) \end{array} \right] f(\alpha)\, d\alpha - -(80) \end{aligned}$$

Note that from (76a) and (76b) we get that $\frac{dq_1(b)}{db}, \frac{\partial q_2(\alpha,b)}{\partial b} < 0$. This means

$$\frac{dq_1(b)}{db}(A - q_1(b) - \gamma q_2(\alpha,b)) + \frac{\partial q_2(\alpha,b)}{\partial b}(q_2(\alpha,b) - \gamma q_1(b)) < 0 - -(81)$$

Also note that

$$\begin{aligned} &\frac{d}{db}\left[Aq_1(b) - \frac{1}{2}k^2 - \frac{1}{2}q_1^2(b) - \gamma k q_1(b) \right] \\ &= \frac{dq_1(b)}{db}[A - q_1(b) - \gamma k] < 0 - -(82) \end{aligned}$$

(82) implies that

$$\begin{aligned} & Aq_1(b) - \frac{1}{2}k^2 - \frac{1}{2}q_1^2(b) - \gamma k q_1(b) \\ > \; & \lim_{b \to \bar{b}} \left[Aq_1(b) - \frac{1}{2}k^2 - \frac{1}{2}q_1^2(b) - \gamma k q_1(b) \right] \\ = \; & \frac{3}{8}(A - \gamma k)^2 - \frac{1}{2}k^2 - - - - (83) \end{aligned}$$

Now note that

$$\frac{3}{8}(A - \gamma k)^2 - \frac{1}{2}k^2 \geq \frac{3}{8}A^2 \iff -4A\gamma \geq k\left(4 - 3\gamma^2\right) - - (84)$$

Proof of Proposition 11 Note that since for complements $\gamma \in (-1, 0)$ we get that $-A\gamma \geq k \implies -4A\gamma \geq k(4 - 3\gamma^2)$. Now using (80) and (84) we get that $W'(b) < 0$ for all $b \in \left(\underline{b}, \bar{b}\right)$. This means $\mathbb{Q}'(b) = W'(b) - 1 < 0$ for all $b \in \left(\underline{b}, \bar{b}\right)$. We also know that $W'(b) < 0$ and $\mathbb{Q}'(b) < 0$ for $b \in (0, \underline{b})$. All these imply that when $\gamma \in (-1, 0)$ we have $b^* = 0$.■