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Endogenous Fluctuations and Social Welfare under Credit Constraints and Heterogeneous Beliefs[1]

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Abstract

This paper examines the relationship between the aggregate output level and social welfare in an overlapping generations (OLG) model of a financial economy with heterogeneous beliefs by focusing on the case of rational beliefs in the sense of Kurz (1994). The aggregate output level is affected by the endogenously determined net supply of the riskless asset, which in turn is affected by the distribution of beliefs; thus, there is a coordination issue. To measure the social welfare, we adopt a measure that is based on the *ex post* social welfare concept in the sense of Hammond (1981), instead of the standard *ex ante* criterion to reflect the heterogeneous beliefs. Simulation results indicate that there may be an inverse relationship between the aggregate output and the social welfare. The results suggest that commonly used macroeconomic variables such as gross domestic product (GDP) may not be a very appropriate measure when making policy recommendations.

Keywords: Credit constraints, Heterogeneous beliefs, Rational belief, Social welfare

JEL classification: D52, D60, D84, E32, G18

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1 Introduction

In the wake of an economic crisis, economic issues become central in the public debate, which is natural, considering the impacts on people's livelihoods. In contrast, an economic boom does not cause immediate public concerns, even though a boom is typically followed by a bust. Thus, there seems to be some asymmetry between booms and busts as far as the interests from the public are concerned, even though it is really a boom-bust cycle as a whole.

There is no consensus, however, what causes a boom-bust cycle as a whole, although macroeconomic fluctuation is one of the central topics in macroeconomics. Whilst typically assuming rational expectations, the real business cycle (RBC) school claims that it is the productivity shock that drives economic fluctuations. Other schools argue that various frictions cause fluctuations, e.g. search costs, price stickiness, asymmetric information, learning and so on.

A recent study by Schularick and Taylor (2012) showed empirically the possible causal relation between credit/leverage and financial crisis. Namely, a large expansion of credit/leverage precedes and possibly causes a financial crisis on its own, rather than the credit system merely acting as a shock amplifier as in the financial accelerator model. Also, Attanasio et al. (2000) showed a possible negative causality from investment to economic growth, suggesting a feedback from the financial side to the real side of the economy.

Numerous studies in financial economics attempted to explain the mechanism of asset price fluctuations. The literature on speculative trades initiated by Harrison and Kreps (1978) is such an example - e.g. Harris and Raviv (1993), Kandel and Pearson (1995), Scheinkman and Xiong (2003). Also, the works based on the concept of rational beliefs stemming from Kurz (1994) show that large fluctuations of economic variables including prices may be driven by heterogeneous beliefs – e.g. Kurz and Schneider (1996), Kurz and Motolese (2001, 2011), Wu and Guo (2003, 2004), Kurz, Jin and Motolese (2005), Nakata (2007, 2013), Guo et al. (2011), and Nielsen (2011).

When heterogenous beliefs are present, it is not straightforward how we should measure welfare. Note that the standard Pareto optimality criterion is an *ex ante* one, since it is based on the *ex ante* preferences that govern the choices of the agents. However, such a measure may not be very plausible as there seems to be asymmetry in the intensity of public debates between booms and busts, which suggests the difference between the *ex ante* and *ex post* views. In other words, *ex ante* preferences do not capture regrets or pleasure arising from the outcomes of decisions made in accord with incorrect subjective beliefs. Such arguments can be found in Diamond (1967) and Drèze (1970), while Starr (1973) introduces the notion of *ex post* optimality, which is based on realised allocations rather than prospects of future allocations, with which the standard *ex ante* optimality is defined. Starr (1973) shows that the two concepts do not coincide generically, when beliefs are heterogeneous.[1] Hammond (1981) introduces the notion of the *ex post* social welfare optimum, which is based on an expected social welfare function, where the expectation is with respect to the social planner's probability (or social probabilities) rather than with respect to the subjective beliefs of the agents.

Gilboa, Samuelson and Schmeidler (2014) provide a welfare criterion that requires unanimity of preferences and the existence of a social probability belief, while not requiring the social planner to know the probability beliefs of the agents. Their criterion makes a distinction between trades essentially due to difference in beliefs and those that are due to risk sharing.

[1] Harris (1978) introduces different concepts of *ex post* efficiency and generalises the results of Starr (1973).

The view against trades due to difference in beliefs or speculations makes an explicit distinction between trades based on tastes and those arising from speculations – e.g. Stiglitz (1989), and Posner and Weyl (2013) and Buss et al. (2016). Posner and Weyl (2013), for instance, argue that speculations may cause damage and restrictions on speculative trades should be introduced.[2] It is however not straightforward how we can distinguish speculative trades from trades for risk hedging purposes, since trades may take place because of the difference in beliefs, but purely for risk hedging/sharing purposes for each agent's perspective as Duffie (2014) points out.

This paper studies the relationship between the aggregate output and the social welfare for an economy whose aggregate output level is endogenously determined through the interactions between the financial side and the real side of the economy that are driven by the beliefs of the agents. In particular, we introduce an overlapping generations (OLG) model of a financial economy with heterogeneous beliefs, in which the aggregate output level is affected by endogenously determined net supply of a riskless asset (or bank deposits and loans), while restricting the class of beliefs to be *rational beliefs* in the sense of Kurz (1994). There is a co-ordination issue regarding the net supply of the riskless asset, since the portfolio choice of an agent is affected by the distribution of beliefs through prices. We also introduce short-sale constraints, and examine the welfare effects of the constraints.

In an economy with rational beliefs, all agents agree on the stationary measure (or the long-term frequencies) by assumption (that all agents hold a rational belief), but may hold diverse beliefs about the stochastic process that drives the economy. Nielsen (2009) argues that the stationary measure be used as the social probability for the social welfare function since this is the only measure that is agreed by all agents and is essentially objective. Note however that the stationary measure may or may not be the true probability in a rational beliefs economy, and the social planner does not need to know if this is the correct probability or not.

2 The Model

2.1 The structure of the economy

Consider a standard competitive OLG economy with H young agents born in every period t ($t = 1, 2, ...$), who live for two periods. We assume that there are H old agents in period 1, too; thus, there are H old agents in each period. We index each agent by $h = 1, 2, ..., H$. Each young agent h is a *replica* of the old agent h who preceded him, where a *replica* refers to tastes and beliefs. This makes us interpret the streams of agents as 'dynasties' or 'types'. There is a single perishable consumption good, whose price is normalised to unity in every period t. Every young agent h receives an endowment W_t^{1h} of this consumption good, and every old agent h (born in period $t-1$) receives W_t^{2h} for all t. Furthermore, young agent h consumes C_t^{1h} in period t, and he consumes C_{t+1}^{2h} when he becomes old in period $t+1$.

Also, there is a single infinitely lived 'tree' owned by the agents. Let P_t denote the price of the tree, and θ_t^h the shareholding of young agent h purchased in period t. We assume without loss of generality that the $\sum_{h=1}^{H} \theta_t^h = 1$ holds for all t. The tree bears a random stream of 'fruits' or returns $\{D_t\}_{t=1}^{\infty}$, and we call it the dividend stream. We assume that $D_t > 0$ for all

[2]There are works that focus on the roles of short sale constrains when heterogeneous beliefs are present — e.g. Jarrow (1980), Jouini and Napp (2007), and Gallmeyer and Hollifield (2008).

t and define the growth rate of dividends $d_{t+1} := D_{t+1}/D_t$. For the agents, shareholding yields income from the dividend as well as from capital gains or losses. Moreover, old agents in period 1 (born in period 0) receive some share of ownership of the tree as endowment in period 0 (θ_0^h with $\sum_{h=1}^{H} \theta_0^h = 1$).

Depending on the specification of the stochastic process $\{d_t\}_t$, the dividend stream may tend to rise or fall. Hence, it may be more convenient to focus on the variables relative to the dividend. To this end, we define the following variables:

$p_t := P_t/D_t$:	the price/dividend ratio;
$w_t^{1h} := W_t^{1h}/D_t$:	the endowment/dividend ratio of young agent h;
$w_t^{2h} := W_t^{2h}/D_t$:	the endowment/dividend ratio of old agent h (born in $t-1$);
$c_t^{1h} := C_t^{1h}/D_t$:	the consumption/dividend ratio of young agent h;
$c_t^{2h} := C_t^{2h}/D_t$:	the consumption/dividend ratio of old agent h (born in $t-1$).

In particular, by letting $\mathbf{w}_t^1 = (w_t^{11}, w_t^{12}, ..., w_t^{1H})$ and $\mathbf{w}_t^2 = (w_t^{21}, w_t^{22}, ..., w_t^{2H})$, the joint stochastic process $\{(d_t, \mathbf{w}_t^1, \mathbf{w}_t^2)\}_{t=2}^{\infty}$ is assumed to be a stable and ergodic Markov process with $d_t \in \mathcal{D}$ for all t, where $\mathcal{D}$ is the state space for d_t.[3]

In addition, young agents can invest in a one-period riskless asset or bank deposit, which we may call as 'vegetables'. Let B_t^h denote young agent h's investment in the bank deposit in period t. All agents take the deposit's return R_t as given. However, R_t is determined by the aggregate investment relative to the dividend of the stock so that $R_t := f(b_t)/b_t$ with $f(0) = 0$, where $b_t := \sum_{h=1}^{H} b_t^h$ with $b_t^h := B_t^h/D_t$. Although we do not model the mechanism explicitly, we may interpret that there is a banking sector that collects deposits from the agents and provides loans to various firms. By assuming that there is a sufficiently large number of firms so that the strong law of large numbers applies, the aggregate return on the loans is riskless. Also, we assume that the interest rate on loans and deposits are identical.

The market for the consumption good is cleared when the following equation is satisfied. For every t,

$$\sum_{h=1}^{H} \left(C_t^{1h} + C_t^{2h}\right) + B_t = \sum_{h=1}^{H} \left(W_t^{1h} + W_t^{2h}\right) + D_t + R_{t-1}B_{t-1}.$$

Note that the left hand side of the equation comprises the aggregate consumption and the aggregate investment (seeds), whilst its right hand side comprises the total endowment, the dividend (i.e. fruits), and the vegetables. Equivalently, the above equation can be expressed as follows:

$$\sum_{h=1}^{h} \left(c_t^{1h} + c_t^{2h}\right) + b_t = \sum_{h=1}^{H} \left(w_t^{1h} + w_t^{2h}\right) + 1 + R_{t-1}\frac{b_{t-1}}{d_t}.$$

The lefthand side of the equation is the aggregate demand - dividend ratio, i.e. aggregate consumption together with investment divided by dividend, or the gross domestic products (GDP) normalised by dividend, which we define accordingly as follows:

$$y_t := \sum_{h=1}^{h} \left(c_t^{1h} + c_t^{2h}\right) + b_t = \frac{\sum_{h=1}^{h} \left(C_t^{1h} + C_t^{2h}\right) + B_t}{D_t}.$$

[3]The definitions of stability and ergodicity are given in the appendix.

2.2 Young agent's problem

We now turn our attention to each young agent's optimisation problem. First, we make the following assumptions:

Assumption 1:

(a) *Each agent believes that the economy is Markovian.*

(b) *Each agent believes that no single agent can affect the equilibrium.* □

The optimisation problem of a young agent h in period t is described as follows:

$$\begin{aligned}
\max_{(\theta_t^h, B_t^h)} \quad & E_{Q_t^h} \left\{ u^h(C_t^{1h}, C_{t+1}^{2h}) \,\middle|\, P_t, D_t, R_t, \mathbf{W}_t^1, \mathbf{W}_t^2 \right\} \\
\text{s.t.} \quad & C_t^{1h} + P_t \theta_t^h + B_t^h = W_t^{1h}, && (1)\\
& C_{t+1}^{2h} = \theta_t^h \cdot (P_{t+1} + D_{t+1}) + R_t B_t^h + W_{t+1}^{2h}, && (2)\\
& B_t^h \geq \hat{\underline{b}} W_t^{1h},
\end{aligned}$$

where $\mathbf{W}_t^i = D_t \mathbf{w}_t^i$ for $i = 1, 2$, and $E_{Q_t^h}$ is the expectation with respect to effective (probability) belief Q_t^h, which we explain later. $\hat{\underline{b}}$ is a parameter that defines the short-sale constraint on the riskless asset, or the credit constraint.[4] The credit constraint is therefore set proportionately with each young agent's wealth level.

To make the model more tractable, we assume agent h's utility function to be of the CES form

$$u^h\left(C_t^{1h}, C_{t+1}^{2h}\right) = \frac{1}{1-\nu^h}\left(C_t^{1h}\right)^{1-\nu^h} + \frac{\beta}{1-\nu^h}\left(C_{t+1}^{2h}\right)^{1-\nu^h}, \quad \nu^h > 0,$$

where $\beta \in (0, 1)$ is the discount factor and ν^h is the parameter that indicates the degree of relative risk aversion of agent h. Then, the Euler equations for young agent h in period t will be

$$\begin{aligned}
P_t \cdot \left(C_t^{1h}\right)^{-\nu^h} &= \beta E_{Q_t^h} \left\{ \left(C_{t+1}^{2h}\right)^{-\nu^h} (P_{t+1} + D_{t+1}) \,\middle|\, P_t, D_t, R_t, \mathbf{W}_t^1, \mathbf{W}_t^2 \right\}, \\
\left(C_t^{1h}\right)^{-\nu^h} &= \beta R_t E_{Q_t^h} \left\{ \left(C_{t+1}^{2h}\right)^{-\nu^h} \,\middle|\, P_t, D_t, R_t, \mathbf{W}_t^1, \mathbf{W}_t^2 \right\} + \lambda_t^{hB},
\end{aligned}$$

where λ_t^{hB} is the Lagrange multiplier for the short-sale constraint of the riskless asset, which we call as the credit constraint hereafter.

By letting $x_t = (p_t, d_t, R_t, \mathbf{w}_t^1, \mathbf{w}_t^2)$, we can describe the optimality conditions by using ratios $(p_t, d_{t+1}, c_t^{1h}, c_{t+1}^{2h}, b_t^h)$ as follows:

$$\begin{aligned}
p_t \cdot \left(c_t^{1h}\right)^{-\nu^h} &= \beta E_{Q_t^h} \left\{ \left(c_{t+1}^{2h} d_{t+1}\right)^{-\nu^h} (p_{t+1} + 1) d_{t+1} \,\middle|\, x_t \right\}, && (3)\\
\left(c_t^{1h}\right)^{-\nu^h} &= \beta R_t E_{Q_t^h} \left\{ \left(c_{t+1}^{2h} d_{t+1}\right)^{-\nu^h} \,\middle|\, x_t \right\} + \tilde{\lambda}_t^{hB}, && (4)\\
c_t^{1h} &= -p_t \theta_t^h - b_t^h + w_t^{1h}, && (5)\\
c_{t+1}^{2h} &= \theta_t^h (p_{t+1} + 1) + \frac{R_t b_t^h}{d_{t+1}} + w_{t+1}^{2h}, && (6)
\end{aligned}$$

[4] Potentially, we can introduce short-sale constraints on the stock. However, as shown by Buss et al. (2016) and Nakata (2013), credit constraints are welfare improving in general; thus, we do not introduce stock short-sale constraints.

where $\tilde{\lambda}_t^{hB} = \lambda_t^{hB} D_t^{\nu^h - 1}$. It follows that the demand correspondences of the young will be a time-invariant map of Q_t^h and x_t: for every h, t,

$$\begin{aligned} \theta_t^h &= \theta_{Q_t^h}^h(x_t), \\ b_t^h &= b_{Q_t^h}^h(x_t). \end{aligned}$$

Observe that the demand is influenced by Q_t^h. This suggests that the distribution of the effective beliefs may have impacts on the equilibrium of the economy.

2.3 The structure of beliefs

Now we specify the structure of beliefs of the agents, in particular the effective beliefs. Let Σ denote the state space of data or observables, i.e. $\left(x_t, \langle \theta_t^h, b_t^h, c_t^{1h}, c_t^{2h} \rangle_{h=1}^H\right)$ for all t. Also, let Σ^∞ denote the state space for the entire infinite sequence of the data. On top of data, we introduce a random variable n_t^h, which is called a *generating variable* as in Kurz and Schneider (1996). Agent h forms a belief Q^h on $\left(\left(\Sigma \times \mathcal{N}^h\right)^\infty, \mathcal{B}\left(\left(\Sigma \times \mathcal{N}^h\right)^\infty\right)\right)$, where $\mathcal{N}^h := \{0, 1\}$ denotes the state space of n_t^h (so, n_t^h is a binary random variable), and $\mathcal{B}\left(\left(\Sigma \times \mathcal{N}^h\right)^\infty\right)$ is the Borel σ-field generated by $\left(\Sigma \times \mathcal{N}^h\right)^\infty$. Now, let $\mathbf{n}^{ht} := (n_1^h, n_2^h, ..., n_t^h)$, i.e. a history of generating variables n_t^h up to period t. Then, each finite history $\mathbf{n}^{ht}$ determines young agent h's effective belief in period t denoted by $Q_t^h(A) = Q^h\left(A|\mathbf{n}^{ht}\right)$ for $A \in \mathcal{B}\left(\Sigma^\infty\right)$, which is a probability measure on $\left(\Sigma^\infty, \mathcal{B}\left(\Sigma^\infty\right)\right)$. To simplify the analysis, we make the following assumption on the generating variables:

Assumption 2: *The marginal distribution for n_t^h with respect to Q^h is i.i.d. with $Q^h\{n_t^h = 1\} = \alpha^h$.* $\square$

By assumptions 1 and 2, the current generating variable n_t^h alone determines the effective belief Q_t^h, i.e. $Q_t^h(A) = Q^h(A|n_t^h)$ for $A \in \mathcal{B}(\Sigma^\infty)$. Consequently, one may interpret that a generating variable n_t^h is describing the *state of belief* of young agent h in period t. In particular, when a belief Q^h supports a regime switching model, n_t^h describes the regime in which agent h believes/perceives the economy is. For instance, $n_t^h = 1$ corresponds to an optimistic belief state or a bullish regime, and $n_t^h = 0$ to a pessimistic belief state or a bearish regime. As this example demonstrates, a generating variable n_t^h therefore is purely a convenient means to characterise the non-stationarity of the agent's belief Q^h. Moreover, it is meaningful only to young agent h in period t himself, but not to any other agents. It follows that any other agents do not form beliefs about agent h's generating variables, i.e. n_t^h is not measurable with respect to $Q^{(h)}$ for all $(h) \neq h$.

2.4 The equilibrium

We have so far described the optimisation problem of a young agent and characterised its solution by the optimality conditions. Also, we have explained the structure of the beliefs of the agents. In what follows, we define the equilibrium of the economy (including the rational belief equilibrium) by introducing the market clearing conditions in addition to the optimality conditions of the young agents' problems.

2.4.1 The definition of Markov rational belief equilibrium

In addition to the optimality conditions for young agents' problems, the equilibrium of the economy requires markets to clear: for every period t, the markets clear if

$$\sum_{h=1}^{H} \theta^h_{Q^h_t}(x_t) = 1, \tag{7}$$

$$\frac{f\left(\sum_{h=1}^{H} b^h_{Q^h_t}(x_t)\right)}{\sum_{h=1}^{H} b^h_{Q^h_t}(x_t)} = R_t. \tag{8}$$

Now, we define a Markov competitive equilibrium as follows:

Definition: *Sequences of probability measures $\{Q^1_t, Q^2_t, ..., Q^H_t\}_{t=1}^{\infty}$ constitute a* ***Markov competitive equilibrium*** *if $(x_t, \theta^h_t, b^h_t, c^{1h}_t, c^{2h}_t; h = 1, 2, ..., H)$ satisfy the conditions (3)—(6), the complementary slackness conditions for all h, t, and (7) and (8) for all t.* □

By construction, the equilibrium prices (or risk free rate) will be a sequence generated by a time-invariant map as follows:

$$\begin{bmatrix} p_t \\ R_t \end{bmatrix} = \Phi_{\mathbf{Q}_t}\left(d_t, \mathbf{w}^1_t, \mathbf{w}^2_t\right), \quad \forall t,$$

where $\mathbf{Q}_t = (Q^1_t, Q^2_t, ..., Q^H_t)$.[5] Alternatively, the equilibrium prices can be expressed in terms of generating variables instead of effective beliefs, since the effective beliefs are determined by the generating variables:

$$\begin{bmatrix} p_t \\ R_t \end{bmatrix} = \Phi\left(d_t, \mathbf{w}^1_t, \mathbf{w}^2_t, \mathbf{n}_t\right), \quad \forall t, \tag{9}$$

where $\mathbf{n}_t = (n^1_t, n^2_t, ..., n^H_t)$.

It is clear from the equilibrium map (9) that the stochastic primitives of the economy are the dividend growth rate d_t, the young's endowment/dividend ratios $\mathbf{w}^1_t$, the old's endowment/dividend ratios $\mathbf{w}^2_t$, and the generating variables $\mathbf{n}_t$ (or the effective beliefs $\mathbf{Q}_t$) given the preferences. It follows that although the economy appears to be a joint stochastic process of $(x_t, \mathbf{n}_t)$, which is assumed to be Markov, it is sufficient to describe the joint stochastic process of $(d_t, \mathbf{w}^1_t, \mathbf{w}^2_t, \mathbf{n}_t)$ since they determine the prices (p_t, R_t), and consequently $(\theta^h_t, b^h_t, c^{1h}_t, c^{2h}_t)$ for all h, t. In other words, the states of the prices and other endogenous variables are partitioned by the states of $(d_t, \mathbf{w}^1_t, \mathbf{w}^2_t, \mathbf{n}_t)$. This implies that the long term frequencies of all variables can be computed as long as we can specify the stationary transition probabilities of $(d_t, \mathbf{w}^1_t, \mathbf{w}^2_t, \mathbf{n}_t)$. Note, however, that this does not require knowledge of the true probability that governs the joint stochastic process of $(d_t, \mathbf{w}^1_t, \mathbf{w}^2_t, \mathbf{n}_t)$, but only requires that the stochastic process is stochastically stable.[6] By letting $\mathcal{Y}$ denote the state space of $(d_t, \mathbf{w}^1_t, \mathbf{w}^2_t, \mathbf{n}_t)$ for all t, we may then define a one-to-one map Φ^* such that

$$\Phi^* : \mathcal{Y} \mapsto \mathcal{S},$$

[5] The uniqueness of the map $\Phi_{\mathbf{Q}_t}$ is not guaranteed, since the short-sale constraints may give rise to multiple equilibria as Giménez (2003) showed. The subsequent argument assumes a fixed map.

[6] See appendix for the definition of stochastic stability.

where $\mathcal{S}$ is some finite set, i.e. $\mathcal{S} := \{1, 2, ..., S\}$. To better illustrate the structure of Φ^*, we provide the following example, which corresponds to the simulation model below.

Example: In the simulation model below, we assume $H = 2$, i.e. two agents in every generation, $\mathcal{D} = \{\bar{d}, \underline{d}\}$ with $\bar{d} > \underline{d}$, $w_t^{1h} = w^{1h}$ for all h, t, and for the old agent's endowment/dividend ratio, $\mathbf{w}_t^2 = (0, 0)$ for all t. It follows that the maximum number of distinct states in each period is $2 \times 2 \times 2 = 8$. By letting $\mathcal{S} := \{1, 2, ..., 8\}$, we may define a one-to-one map Φ^* as follows:

$$\Phi^*(d_t, n_t^1, n_t^2) = \begin{cases} 1 & \text{if } d_t = \bar{d}, n_t^1 = 1, n_t^2 = 1; \\ 2 & \text{if } d_t = \bar{d}, n_t^1 = 1, n_t^2 = 0; \\ 3 & \text{if } d_t = \bar{d}, n_t^1 = 0, n_t^2 = 1; \\ 4 & \text{if } d_t = \bar{d}, n_t^1 = 0, n_t^2 = 0; \\ 5 & \text{if } d_t = \underline{d}, n_t^1 = 1, n_t^2 = 1; \\ 6 & \text{if } d_t = \underline{d}, n_t^1 = 1, n_t^2 = 0; \\ 7 & \text{if } d_t = \underline{d}, n_t^1 = 0, n_t^2 = 1; \\ 8 & \text{if } d_t = \underline{d}, n_t^1 = 0, n_t^2 = 0; \end{cases} \tag{10}$$

We may call the first four states as the 'high dividend states', because the dividend growth rate is $\bar{d}$, and the latter four states the 'low dividend states'. Also, we may call the states in which $n_t^1 = n_t^2$ as the 'agreement states' (i.e. states 1, 4, 5 and 8), and those in which $n_t^1 \neq n_t^2$ as the 'disagreement states' (i.e. states 2, 3, 6 and 7). $\square$

Now, we focus on the cases in which the equilibrium dynamical system is ergodic and stable. Since a stochastically stable dynamical system has an associated stationary measure, we may then define a stationary transition probability matrix Γ on $\mathcal{S} \times \mathcal{S}$, rather than on $\mathcal{Y} \times \mathcal{Y}$. A typical element of Γ is $\Gamma(s, s^+)$, which is the stationary transition probability from state s to state s^+ for $s, s^+ \in \mathcal{S}$.

Recall that the effective beliefs are determined by the generating variables. To reflect this feature, we define pairs of transition probability matrices on $\mathcal{S} \times \mathcal{S}$ that correspond to the generating variables as follows: the effective belief Q_t^h of young agent h in period t is represented by a transition probability matrix F_t^h, which is determined by the following rule:

$$F_t^h = \begin{cases} F_H^h & \text{if } n_t^h = 1; \\ F_L^h & \text{if } n_t^h = 0. \end{cases}$$

We require the transition probability matrices to satisfy the following condition, which is essentially the same as the rationality condition in Kurz and Motolese (2001):

The rationality condition: *The transition probability matrices F_H^h and F_L^h of each agent h satisfies the following condition:*

$$\alpha^h \cdot F_H^h + (1 - \alpha^h) \cdot F_L^h = \Gamma. \qquad \square \tag{11}$$

Because the long-term frequency of the event $\{n_t^h = 1\}$ is α^h, agent h uses the transition probability matrix F_H^h with frequency α^h. Also, by construction, the identification of s is at least as fine as that of the data $\big(x_t, \langle \theta_t^h, b_t^h, c_t^{1h}, c_t^{2h} \rangle_{h=1}^H \big)$. Hence, the long-term frequency of all data generated by any pair (F_H^h, F_L^h) that satisfies the rationality condition (11) will be the same as the one generated by Γ. In other words, it is impossible for the agents to reject the sequence of effective beliefs $\{Q_t^h\}_{t=1}^{\infty}$ (or Q^h) by observing the data. This ensures that the belief Q^h is a rational belief, whose formal definition is given in the appendix.[7] Finally, we define a Markov rational belief equilibrium as follows:

Definition: *A* **Markov rational belief equilibrium** *(RBE) is a Markov competitive equilibrium in which the rationality condition (11) is satisfied for all h.* $\square$

Let m denote the probability measure on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ that corresponds to a stationary transition probability matrix Γ. Then, m is a stationary measure that governs a stationary dynamical system $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty), T, m)$, where T is the (left) shift transformation such that $T(x_t, x_{t+1}, x_{t+2}, ...) = (x_{t+1}, x_{t+2}, x_{t+3}, ...)$. As mentioned above, for any belief Q^h, whose corresponding pair of transition probability matrices (F_H^h, F_L^h) satisfies the rationality condition (11), it generates the same data as m does. Since m corresponds to the long-term frequencies of the economic variables/data, the true probability Π on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ and any Q^h that satisfies the rationality condition (11) will generate the same data. This implies that it is impossible to learn the true probability Π from the data. Note that the true stochastic process driven by Π may be non-stationary, but stochastically stable.

2.5 Welfare measure

In what follows, we examine how we should measure the welfare of the economy. We discuss the issue, because with heterogeneous beliefs, the standard *Pareto* optimality and/or social welfare criterion is problematic, since by allowing for heterogeneous beliefs some agents inevitably hold incorrect beliefs, and such incorrect beliefs cause 'mistakes', which may results in regrets or pleasure *ex post*, even if they act optimally *ex ante* in accord with their beliefs. We express as 'mistakes', since it is impossible to identify exactly if and how they made 'mistakes' by data when the beliefs are rational beliefs. Ignoring such regrets or pleasure calls for a significant value judgement, since it requires that the inability to hold the correct belief be penalised.

Instead of taking such a strong value judgement, and to take *ex post* regrets or pleasure into account, it is probably reasonable to measure the welfare of the individuals and the society as a whole with respect to an *ex post* measure. An *ex post* social welfare function for one generation is defined by

$$\hat{E}V(u^1, u^2, ..., u^H),$$

where $\hat{E}$ is the expectation operator with respect to a social probability measure, u^h is the *ex post* utility of agent h (a random variable), and V is a von Neumann-Morgenstern social welfare function, which is a function of the *ex post* utilities of the individuals. In particular, we assume that the *ex post* social welfare function takes the following form:

$$\hat{E}V(u^1, u^2, ..., u^H) = \hat{E}\sum_{h=1}^{H} \vartheta^h u^h = \sum_{h=1}^{H} \vartheta^h \hat{E} u^h, \tag{12}$$

where ϑ^h is some weight attached to agent h. Hammond (1981) shows that a socially optimal allocation based on an *ex post* social welfare function is not Pareto optimal in terms of *ex ante*

[7]The conditional stability theorem by Kurz and Schneider (1996), also given in the appendix, provides a more formal proof of this claim.

expected utilities of the agents unless all agents agree on the probability even if the *ex post* social welfare function takes the above form (12).

However, the choice of the social probability measure is not trivial, since there is no way to learn the true probability, and one can only *believe* that his probability belief is the true probability, although one may happen to hold the true probability as his belief. One easy resolution would be to *assume* that the modeller knows the true probability, while the agents in the model don't, and then specify the true probability as the social probability measure. However, such an assumption is not plausible, since apparently no objective justification can be given for the assumption. In other words, *we propose to take a view that the modeller and the agents in the model have equal knowledge and/or ability, rather than taking a paternalistic view that the modeller takes care of the agents in the model by assuming the modeller's possession of superior knowledge and/or ability.* We therefore follow the argument in Nakata (2013), which is based on the observation by Nielsen (2009) that the stationary measure is suitable for the social probability measure for the *ex post* social welfare function (12), since all agents agree on the stationary measure as long as the beliefs of the agents satisfy the rationality condition (11), and thus, it is objective.

Using the expected utility with respect to the stationary measure, each agent h's lifelong *ex post* welfare is

$$\hat{E}_m \left\{ \frac{1}{1-\nu^h} \left(\frac{C_t^{1h}}{D_t} \right)^{1-\nu^h} + \frac{\beta}{1-\nu^h} \left(\frac{C_{t+1}^{2h}}{D_t} \right)^{1-\nu^h} \right\},$$

where $\hat{E}_m$ denotes the expectation with respect to the stationary measure. The normalisation due to the multiplication of $D_t^{\nu^h-1}$ is to nullify the upward or downward trend. Then, we can define the *ex post* certainty equivalent of agent h's lifelong utility (denoted by CE^h) as follows:

$$CE^h := \left[\hat{E}_m \left\{ \frac{1}{1+\beta} \left(\frac{C_t^{1h}}{D_t} \right)^{1-\nu^h} + \frac{\beta}{1+\beta} \left(\frac{C_{t+1}^{2h}}{D_t} \right)^{1-\nu^h} \right\} \right]^{\frac{1}{1-\nu^h}}.$$

While the *ex post* certainty equivalent of the lifelong utility is central to measure the welfare of each agent, it may be useful to break it down into two parts, the young and the old, in order to see how the trade-off between gains and losses from flexibility works. To do so, we define the *ex post* certainty equivalent of young agent h by

$$CE^{1h} := \left[\hat{E}_m \left\{ \left(\frac{C_t^{1h}}{D_t} \right)^{1-\nu^h} \right\} \right]^{\frac{1}{1-\nu^h}},$$

and also, the *ex post* certainty equivalent of old agent h is defined by

$$CE^{2h} := \left[\hat{E}_m \left\{ \left(\frac{C_{t+1}^{2h}}{D_t} \right)^{1-\nu^h} \right\} \right]^{\frac{1}{1-\nu^h}}.$$

Since the appropriate values of the weights ϑ^h are not always very obvious, we just use these individual agents' certainty equivalents below.

3 Simulations

3.1 The simulation model

We assume that there are only two dynasties, i.e. $H = 2$. Also, in order to isolate the impacts of heterogeneity of beliefs as a possible source of randomness to the economy, we assume that $w_t^{1h} = w^{1h}$ are constant for all h, t. It follows that the young agents' aggregate endowment of the consumption good $\sum_{h=1}^{H} W_t^{1h}$ is proportional to the total dividend D_t in each period t. Also, the credit constraint $B_t^h \geq \hat{\underline{b}} W_t^{1h}$ can be rewritten as $b_t^h \geq \underline{b}^h$, where $\underline{b}^h := \hat{\underline{b}} w^{1h}$. The state space of the process $\{d_t\}_{t=2}^{\infty}$ is $\mathcal{D} = \{\bar{d}, \underline{d}\}$ with $\bar{d} = 1.054$ and $\underline{d} = 0.982$, while we assume that old agents receive no endowment.[8] Hence, there are at most eight price states, which are described by the map Φ^* in equation (10).

As discussed above, to compute the long-term frequencies of the economic variables, we need to specify the stationary transition matrix Γ for the stochastic process of the eight price states. To do so, we specify the stochastic process $\{d_t\}_t$ so that it is driven by an empirical transition probability matrix

$$\Psi = \begin{bmatrix} \psi & 1-\psi \\ 1-\psi & \psi \end{bmatrix}.$$

In particular, we set $\psi = 0.43$, following Mehra and Prescott (1985).

Then, the stationary transition probability matrix must satisfy the following:

- the empirical distribution for the joint process $\{d_t\}_t$ is specified by transition probability matrix Ψ,
- the marginal distribution for n_t^h is i.i.d. with frequency of $\{n_t^h = 1\} = \alpha^h$ for all h.

There are many matrices that satisfy these conditions. Amongst them, we use the one that is analogous the one in Kurz and Motolese (2001), since we know that the beliefs that are compatible with the stationary distribution in accord with the specification can generate large fluctuations of economic variables. We assume that the 8×8 stationary transition probability matrix Γ has the following structure:

$$\Gamma = \begin{bmatrix} \psi A & (1-\psi)A \\ (1-\psi)A & \psi A \end{bmatrix},$$

where A is a 4×4 matrix, which is characterised by six parameters (α^1, α^2) and (a_1, a_2, a_3, a_4):

$$A = \begin{bmatrix} a_1, & \alpha^1 - a_1, & \alpha^2 - a_1, & 1 + a_1 - \alpha^1 - \alpha^2 \\ a_2, & \alpha^1 - a_2, & \alpha^2 - a_2, & 1 + a_2 - \alpha^1 - \alpha^2 \\ a_3, & \alpha^1 - a_3, & \alpha^2 - a_3, & 1 + a_3 - \alpha^1 - \alpha^2 \\ a_4, & \alpha^1 - a_4, & \alpha^2 - a_4, & 1 + a_4 - \alpha^1 - \alpha^2 \end{bmatrix}.$$

Matrix A describes the transition probabilities of the states of effective beliefs. Namely, the ith row jth column element of A is the transition probability from effective beliefs state i (in

[8]The set up of $\mathcal{D}$, alongside the specification of its transition probability given later, is compatible with that of Mehra and Prescott (1985).

period t) to effective beliefs state j (in period $t+1$) with

$$\text{effective beliefs state } i = \begin{cases} 1 & \text{if } (n_t^1, n_t^2) = (1,1); \\ 2 & \text{if } (n_t^1, n_t^2) = (1,0); \\ 3 & \text{if } (n_t^1, n_t^2) = (0,1); \\ 4 & \text{if } (n_t^1, n_t^2) = (0,0). \end{cases}$$

As before, we may call effective beliefs states 1 and 4 as 'agreement states', where the generating variables of the two young agents have the same value, and effective beliefs states 2 and 3 as 'disagreement states', where the generating variables of the two young agents have different values. Note that when $a_1 = a_2 = a_3 = a_4 = 0.25$ and $\alpha^1 = \alpha^2 = 0.5$, the stationary transition probability matrix Γ corresponds to a jointly i.i.d. process of (d_t, n_t^1, n_t^2), which is fully represented by the transition probability matrix Ψ that drives $\{d_t\}_t$ corresponding to the economy in Mehra and Prescott (1985).

Next, we specify the transition probability matrices that represent the beliefs of the agents. As we noted above, young agent h in period t uses F_H^h when his generating variable is $n_t^h = 1$, and F_L^h when $n_t^h = 0$. Because the rationality condition (11) must be satisfied so that the agent's belief is a rational belief, F_L^h is determined by F_H^h and Γ. Hence, we only specify $F_H^h(\eta^h)$, which is parameterised by η^h as follows:

$$F_H^h = \begin{bmatrix} \eta^h \psi A & (1-\eta^h\psi)A \\ \eta^h(1-\psi)A & (1-\eta^h(1-\psi))A \end{bmatrix}.$$

It is clear that *the parameter η^h determines how much F_H^h deviates from* Γ, and so it is representing the degree of non-stationarity of beliefs. In particular, when $\eta^h > 1$, the conditional probability of $\{d_{t+1} = \bar{d}\}$ given the current state s with respect to $Q^h\left(\cdot \mid n_t^h = 1\right)$ is higher than the conditional probability of $\{d_{t+1} = \bar{d}\}$ specified in Γ for all current states $s \in \mathcal{S}$.

In the simulation results below, we choose the following parametric set-up, following Kurz and Motolese (2001):

$$\alpha^1 = \alpha^2 = 0.57, \quad (a_1, a_2, a_3, a_4) = (0.5, 0.14, 0.14, 0.14).$$

As for the function $f(b_t)$ that determines the risk-free rate $R_t = f(b_t)/b_t$, we assume that it has the following functional form:

$$f(b) = b^{0.8}.$$

Thus, $R_t = b_t^{-0.2}$. Also, we choose the following parametric set up for the rest of the parameters:

$$\beta = 0.96, \quad \nu^1 = \nu^2 = 2, w^1 = w^2 = 20.$$

Because we set $w^1 = w^2$, $\underline{b}^h = \underline{b}$ for $h = 1, 2$.

3.2 Simulation results

In what follows, we examine the effects of the diversity of beliefs (η) and those of the credit constraint ($|\underline{b}|$). The following notation is used.

σ_p:	the standard deviation of the price/dividend ratio p_t;
σ_f:	the standard deviation of the risk free rate R_t;
σ_y:	the standard deviation of the aggregate demand/dividend ratio y_t;
μ_f:	the mean risk free rate $(\hat{E}_m R_t)$;
μ_b:	the mean aggregate investment to dividend ratio $(\mu_b := \hat{E}_m b_t)$;
μ_y:	the mean aggregate demand/dividend ratio $\hat{E}_m y_t$;
max CR:	the maximal short-sale of the riskless asset, i.e. max CR $:= \max\{-b_s^h; s \in \mathcal{S}\}$;
$\underline{c}^{2h}$:	the minimal consumption/dividend ratio of old agent h $(\underline{c}^{2h} := \min\{c_s^{2h}; s \in \mathcal{S}\})$.

3.2.1 The effects of diversity of beliefs

First, we examine the effects of the diversity of beliefs η on the economy. In particular, we first look at the case in which there is no short-sale constraint. As it is clear from Table 1 that summarises the effects of η on various variables, the volatility of the economy represented by σ_p and σ_f is increasing in η. Namely, the economy is more volatile when the diversity of beliefs is greater. Meanwhile, the effects on the mean risk free rate are limited - it stays at around 6.50 to 6.98 percent. Thus, the higher volatility and a similar average return on the riskless investment are reflected in the lower welfare level CE^h for an economy with more diverse beliefs, i.e. an economy with a higher η. Moreover, the maximal level of short-sales of the riskless asset is clearly increasing in η except for very large η, while the lifelong welfare level as welfare level of the old are decreasing and that of the young is increasing in η.

Table 1: Effects of η on the economy without short-sale constraint

η	CE^h	CE^{1h}	CE^{2h}	σ_p	σ_f	μ_f	μ_b	Max CR
1.00	10.343	10.273	10.417	0.008	0.579%	6.976%	0.714	0.000
1.10	10.331	10.280	10.385	0.014	0.662%	6.979%	0.714	17.820
1.20	10.292	10.299	10.285	0.040	0.890%	6.985%	0.714	35.497
1.30	10.220	10.332	10.107	0.089	1.233%	6.989%	0.715	52.146
1.35	10.169	10.354	9.983	0.123	1.449%	6.986%	0.715	59.812
1.40	10.104	10.380	9.833	0.163	1.697%	6.979%	0.716	66.843
1.45	10.023	10.410	9.650	0.211	1.983%	6.964%	0.718	73.080
1.50	9.920	10.445	9.427	0.268	2.315%	6.940%	0.720	78.369
1.55	9.787	10.485	9.152	0.335	2.709%	6.904%	0.724	82.556
1.60	9.606	10.531	8.801	0.415	3.191%	6.857%	0.728	85.464
1.65	9.343	10.585	8.325	0.514	3.814%	6.797%	0.735	86.840
1.70	8.880	10.652	7.569	0.648	4.719%	6.717%	0.746	86.270
1.75	6.636	10.758	4.743	0.922	6.756%	6.500%	0.778	82.142

However, things become somewhat different, especially regarding the aggregate riskless investment, once credit constraints are introduced. Table 2 reports the effects of η on various variables for different levels of credit constraints $\underline{b}$.[9] The mean risk free rate is decreasing in η when $|\underline{b}| = 0$ or $|\underline{b}| = 10$, but is highest for an intermediate value of η in all other cases. Thus,

[9]Figures in the appendix report the results summarised in Table 2 graphically.

the aggregate output level would tend to be lower on average for higher η. The lower aggregate output level on top of the higher volatility therefore would imply that the social welfare of an economy with more diverse beliefs would be lower than the one with less diverse beliefs, which is consistent with the result on CE^h reported in the table.

Table 2: Effects of η on the economy with credit constraints

$\lvert\underline{b}\rvert$	CE^h	CE^{1h}	CE^{2h}	σ_p	σ_f	μ_f	μ_b
0	−	+	−	+	+	−	+
10	−	+	−	+	+	−	+
20	−	+	−	+	+	m.p.	+
30	−	+	−	+	+	m.p.	+
40	−	+	−	+	+	m.p.	+
50	−	+	−	+	+	m.p.	+
60	−	+	−	+	+	m.p.	+
70	−	+	−	+	+	m.p.	+
80	−	+	−	+	+	m.p.	+
90	−	+	−	+	+	m.p.	+

Note: 'm.p.' stands for 'middle peaked'.

3.2.2 The effects of credit constraints

Next, we examine the effects of credit constraints on the economy. As we have shown above, the effects of the diversity of beliefs η are different in accord with different levels of credit constraints. Table 3 reports the results. Note that in the case of rational expectations, i.e. $\eta = 1$, credit constraints never bind. The table indicates that the lifelong welfare level is higher with a tighter credit constraint, so does the welfare level of the old. However, the welfare level of the young is not monotonic in the credit constraint. The same applies to the price volatility, i.e. σ_p and σ_f.

Table 4 in the appendix reports the mean aggregate demand/dividend ratio μ_y and Table 5 the standard deviation of the aggregate demand/dividend ratio σ_y for different η and $|b|$. Observe that μ_y is highest when $\eta = 1.75$ and $|b| = 50$ in Table 4 and σ_y is lowest when $\eta = 1.75$ and $|b| = 40$ in table 5. Thus, the social welfare would appear to be highest when $\eta = 1.75$ and $|b|$ is around 40 to 50 if we focus on the aggregate output level. Table 6 and Figure 1 in the appendix, however, show that the *ex post* social welfare is very low when the credit constraint is looser than $|b| = 30$ in the case of $\eta = 1.75$.

The apparently inconsistent relationship between the welfare level and the aggregate output level may arise if the distribution of the consumption level across agents, in particular across old agents, may have an impact that exceeds an increase in the aggregate output level. To see if this indeed the case, we examine the consumption level of the old agents in the 'worst' case, i.e. $\underline{c}^{2h}$. Table 7 reports $\underline{c}^{2h}$ for various combinations of $|\underline{b}|$ and η. Clearly, the 'worst' case is not too damaging when either the credit constraint is very tight or the diversity of beliefs is limited. Namely, $\underline{c}^{2h}$ is less than 5 only when the credit constraint is loose ($|\underline{b}| \geq 40$) *and* the beliefs are diverse ($\eta \geq 1.55$).

Table 3: Effects of credit constraints $|\underline{b}|$ on the economy

η	CE^h	CE^{1h}	CE^{2h}	σ_p	σ_f	μ_f	μ_b
1	n.b.	n.b.	n.b.	n.b.	n.b.	n.b.	n.b.
1.1	$-$	m.p.	$-$	m.p.	?	$+$	$-$
1.2	$-$	m.p.	$-$	m.p.	?	$+$	$-$
1.3	$-$	m.p.	$-$	m.p.	?	$+$	$-$
1.35	$-$	m.p.	$-$	m.p.	?	$+$	$-$
1.4	$-$	m.p.	$-$	m.p.	?	$+$	$-$
1.45	$-$	m.p.	$-$	m.p.	?	$+$	$-$
1.5	$-$	m.p.	$-$	m.p.	?	$+$	$-$
1.55	$-$	m.p.	$-$	m.p.	m.p.	$+$	$-$
1.6	$-$	m.p.	$-$	m.p.	m.p.	t.m.	m.p.
1.65	$-$	m.p.	$-$	m.p.	m.p.	t.m.	m.p.
1.7	$-$	m.p.	$-$	m.p.	m.p.	t.m.	m.p.
1.75	$-$	m.p.	$-$	m.p.	m.p.	t.m.	m.p.

Note: 'm.p.' stands for 'middle peaked', and 't.m.' for 'trough in the middle'. '?' refers to non-unimodal cases, and 'n.b.' no binding constraints.

The results reported in Tables 6 and 7 as well as in Figures 1 and 3 indicate the effectiveness of credit constraints in keeping the level of social welfare in a very narrow range and at a high level regardless of the distribution of effective beliefs parametrised by η by eliminating disastrous states for the old agents. This result is analogous to the results found in Nakata (2013) for an exchange economy and in Blume et al. (2015). From the social planner's perspective, the result is very important since there is no need for the beliefs of the agents to be observable to achieve an allocation that attains a reasonably high level of social welfare, but it is only required to simply tighten the credit constraint or more generically the short sales constraints.

Moreover, there is no need to distinguish 'speculative' trades from trades due to hedging purposes in an objective way. Note that this distinction is not obvious in our model since the agents are all risk averse and they all are solving a constrained optimisation problem with respect to their own effective beliefs Q_t^h, which can be understood as hedging, while the volume of trades may well become substantially larger when the credit constraint is loose due to the differences in beliefs. Also, our results indicate that tighter credit constraints may be welfare improving even when the aggregate output becomes larger on average and its volatility lower.

4 Conclusion

We introduced an OLG model with financial assets under heterogeneous beliefs in which the net supply of the riskless asset (aggregate bank savings minus aggregate bank loans) is determined endogenously. The endogenous net supply of the riskless asset is equal to the aggregate investment that generates returns in the subsequent period; thus, the aggregate investment is endogenous in the model, where the banking sector is not explicitly modelled but is channelling the net savings (i.e. the net supply of the riskless asset) to finance capital investment. It follows that the endogenous aggregate investment may cause the fluctuations of the GDP and/or the aggregate consumption on top of the stochastic dividends on the stocks. Also, short-sale constraints on the riskless asset or the credit constraints are introduced to the model so that the positions of the financial assets held by the agents to be restricted.

By following the argument made by Nielsen (2009, 2011), we measure the social welfare of the economy by an *ex post* social welfare function that has an expected utility form based on the stationary measure rather than the *ex ante* social welfare function that adds up the each individual's *ex ante* (expected) utility function. Our *ex post* social welfare function is objective in the model, since all agents in the model hold rational beliefs, which require the knowledge of the stationary measure. The simulation results showed that credit constraints are generally welfare improving, matching the results by Blume et al. (2015), Buss et al. (2016) and Nakata (2013). The result arises from the fact that credit constraints prevent agents from taking extreme positions of financial assets, which could lead to a large negative return subsequently. Moreover, the simulations demonstrate that there may well be a disjoint between the GDP and the *ex post* social welfare, i.e. a larger output level does not necessarily associate with a higher social welfare level. This result has a significant policy implications – policy evaluations for macroeconomic policies are usually based on measures related to the output (level and growth rate) as well as on inflation currently, but it shows that a measure based on the output is not a good proxy for the social welfare.

A Stability and ergodicity

Let Ω denote a sample space, $\mathcal{F}$ a σ-field of subsets of Ω, T the (left) shift transformation such that $T(x_t, x_{t+1}, x_{t+2}) = (x_{t+1}, x_{t+2}, x_{t+3}, ...)$, and Π a probability measure.

Definition (Stability): *A dynamical system* $(\Omega, \mathcal{F}, T, \Pi)$ *is said to be* ***(stochastically) stable*** *if for all cylinders* $Z \in \mathcal{F}$ *the limit of* $m^i(Z)(x) = \frac{1}{i}\sum_{k=0}^{i-1} 1_Z(T^k x)$ *exists* Π *a.e., where*

$$1_Z(x) = \begin{cases} 1 & \text{if } x \in Z, \\ 0 & \text{if } x \notin Z. \end{cases}$$

Definition (Invariance): *A set* $Z \in \mathcal{F}$ *is said to be* ***invariant*** *with respect to* T *if* $T^{-1}Z = Z$. *A measurable function is said to be* ***invariant*** *with respect to* T *if for any* $x \in \Omega$, $f(Tx) = f(x)$.

Definition (Ergodicity): *A dynamical system* $(\Omega, \mathcal{F}, T, \Pi)$ *is said to be* ***ergodic*** *if* $\Pi(Z) = 0$ *or* $\Pi(Z) = 1$ *for all invariant sets* $Z \in \mathcal{F}$.

B Rational belief

In what follows, the definition of a rational belief is provided by following Kurz (1994). Suppose the true dynamical system $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty), T, \Pi)$ is stable, where Π is the true probability. Then, by definition, there is a limit for an empirical frequency $m(Z)(\sigma) = \lim_{i\to\infty} \frac{1}{i}\sum_{k=0}^{i-1} 1_Z(T^k\sigma)$ for all cylinders $Z \in \mathcal{B}(\Sigma^\infty)$ (with initial state $\sigma \in \Sigma$), where

$$1_Z(x) = \begin{cases} 1 & \text{if } x \in Z, \\ 0 & \text{if } x \notin Z. \end{cases}$$

Moreover, there exists a set function for all cylinders $Z \in \mathcal{B}(\Sigma^\infty)$ such that

$$\hat{m}_\Pi(Z) = \lim_{i\to\infty} \frac{1}{i}\sum_{k=0}^{i-1} \Pi(T^{-k}Z) \qquad \text{(proposition 2 in Kurz, 1994).}$$

Also, by proposition 3 in Kurz (1994) we can define a stationary measure m on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ as follows:

$$m(Z) = \int_{\Sigma^\infty} m(Z)(\sigma) m_\Pi(d\sigma), \forall Z \in \mathcal{B}(\Sigma^\infty),$$

where m_Π on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ is a unique extension of $\hat{m}_\Pi(\cdot)$. Moreover, $m(Z) = m_\Pi(Z)$ holds for all $Z \in \mathcal{B}(\Sigma^\infty)$. Hence, we say that the stable dynamical system $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty), T, \Pi)$ is associated with a unique stationary measure m. Note however that there are many stable dynamical systems that are associated with the same stationary measure m. The following definition is central to the concept of rational beliefs:

Definition: *A probability* Q *on* $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ *is said to be* ***compatible with the data*** *if*

(a) $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty), T, Q)$ *is stable with a stationary measure* m. *That is, for all cylinders* $Z \in \mathcal{B}(\Sigma^\infty)$,

$$\lim_{i\to\infty} \frac{1}{i}\sum_{k=0}^{i-1} Q(T^{-k}Z) = m(Z).$$

(b) *Q satisfies the tightness condition on* Π.[10] $\square$

Condition (a) requires that measure Q generates data whose long-term frequencies coincide with those under the true measure Π. Also, a probability measure P is said to be *tight* if for each $\varepsilon > 0$ there exists a compact set K such that $P(K) > 1 - \varepsilon$. Now, since Σ is a Euclidean space, it is separable and complete. Hence, by theorem 1.3 in Billingsley (1999), any measure on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ is tight. This implies that the true measure Π on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ is tight, and so is Q on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$. In other words, condition (b) naturally follows from the fact that all agents know that the true measure Π can be defined on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$, even if they don't know what Π is and instead form subjective beliefs on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$.

Now, we define a rational belief.

Definition: *A probability Q on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ is a* ***rational belief*** *if it satisfies the following conditions:*

(a) *It is compatible with the data.*

(b) *For $Z \in \mathcal{B}(\Sigma^\infty)$, $m(Z) > 0$ implies $Q(Z) > 0$* (i.e. $m \ll Q$). $\square$

Note that the condition that $m \ll Q$, i.e. m is absolutely continuous with respect to Q, does not imply that there will be a merging of conditional probabilities in the limit amongst agents with diverse beliefs. This is because by Lebesgue's decomposition theorem Q can be decomposed into two parts: a measure that is equivalent to m, and a measure that is singular with m.[11] Hence, there is a disagreement on the measures that are singular to m. Moreover, since the true probability is not necessarily m, it is in general impossible to learn the truth. Indeed, *the crucial feature of a rational belief is that no learning concerning the true probability measure is possible.*

C Conditional stability theorem

In what follows, the conditional stability theorem (Theorem 2 in Kurz and Schneider (1996)) is provided in our context. In the construction of effective beliefs, we introduced sequences of generating variables $\{n_t^h\}_{t=1}^\infty$. Thus, each agent h's belief Q^h is defined as a probability measure on $(\Omega^h, \mathcal{B}(\Omega^h))$ rather than on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$, where $\Omega^h := (\Sigma \times \mathcal{N}^h)^\infty$. It follows that we need to establish the conditions that conditional measures of Q^h given $\mathbf{n}^h \in (\mathcal{N}^h)^\infty$ on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ constitute a rational belief. To do so, let $\hat{\Pi}^h$ denote the true probability measure on the space $(\Omega^h, \mathcal{B}(\Omega^h))$, whose marginal measure for Σ^∞ is Π and that for $(\mathcal{N}^h)^\infty$ is $\bar{\mu}^h$. Then, the expanded true stochastic process is described as a dynamical system $(\Omega^h, \mathcal{B}(\Omega^h), T, \hat{\Pi}^h)$.

Now, we introduce some notation to be more precise concerning the construction of the probability space(s). Let $\hat{\Pi}^h_{\mathbf{n}^h}$ denote the conditional probability of $\hat{\Pi}^h$ given a particular sequence of generating variables $\mathbf{n}^h \in (\mathcal{N}^h)^\infty$:

$$\hat{\Pi}^h_{\mathbf{n}^h}(\cdot) : (\mathcal{N}^h)^\infty \times \mathcal{B}(\Sigma^\infty) \mapsto [0, 1].$$

[10]The definition of tightness of measures given below is from Billingsley (1999).
[11]See Kurz (1994) for more details, in particular the main theorem.

For each $A \in \mathcal{B}(\Sigma^\infty)$, $\hat{\Pi}^h_{\mathbf{n}^h}$ is a measurable function of $\mathbf{n}^h$, and for every sequence $\mathbf{n}^h$ given, it is a probability on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$. For $A \in \mathcal{B}(\Sigma^\infty)$ and $B \in \mathcal{B}((\mathcal{N}^h)^\infty)$, we have

$$\hat{\Pi}^h(A \times B) = \int_B \hat{\Pi}^h_{\mathbf{n}^h}(A)\bar{\mu}^h(d\mathbf{n}^h).$$

Also, as we noted above,

$$\begin{aligned} \Pi(A) &= \hat{\Pi}^h(A \times (\mathcal{N}^h)^\infty), \quad \forall A \in \mathcal{B}(\Sigma^\infty), \\ \bar{\mu}^h(B) &= \hat{\Pi}^h(\Sigma^\infty \times B), \quad \forall B \in \mathcal{B}((\mathcal{N}^h)^\infty). \end{aligned}$$

When $(\Omega^h, \mathcal{B}(\Omega^h), T, \hat{\Pi}^h)$ is a stable dynamical system with a stationary measure $m^{\hat{\Pi}^h}$, we define the two marginal measures of $m^{\hat{\Pi}^h}$ as follows:

$$\begin{aligned} m(A) &:= m^{\hat{\Pi}^h}(A \times (\mathcal{N}^h)^\infty), \quad \forall A \in \mathcal{B}(\Sigma^\infty), \\ m^{\mathbf{n}^h}(B) &:= m^{\hat{\Pi}^h}(\Sigma^\infty \times B), \quad \forall B \in \mathcal{B}((\mathcal{N}^h)^\infty). \end{aligned}$$

Also, the stationary measure of $\hat{\Pi}^h_{\mathbf{n}^h}$ is denoted by $\hat{m}_{\mathbf{n}^h}$, which is a measure on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$.

Since $\mathcal{N}^h$ is finite by assumption, the following conditional stability theorem holds:

Theorem (Kurz and Schneider, 1996; Theorem 2): *Let* $\left(\Omega^h, \mathcal{B}(\Omega^h), T, \hat{\Pi}^h\right)$ *be a stable and ergodic dynamical system. Then,*

(a) $\left(\Sigma^\infty, \mathcal{B}(\Sigma^\infty), T, \hat{\Pi}^h_{\mathbf{n}^h}\right)$ *is stable and ergodic for* $\hat{\Pi}^h$ *a.a.* $\mathbf{n}^h$.

(b) $\hat{m}^h_{\mathbf{n}^h}$ is independent of $\mathbf{n}^h$, and $\hat{m}^h_{\mathbf{n}^h} = m$.

(c) *If* $\left(\Omega^h, \mathcal{B}(\Omega^h), T, \hat{\Pi}^h\right)$ *is stationary, then the stationary measure of* $\hat{\Pi}^h_{\mathbf{n}^h}$ *on* $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ *is* Π. *That is*
$$\hat{m}^h_{\mathbf{n}^h} = m = \Pi. \qquad \square$$

Suppose Q^h is a probability measure on $\left(\Omega^h, \mathcal{B}(\Omega^h)\right)$, and that, a dynamical system $\left(\Omega^h, \mathcal{B}(\Omega^h), T, Q^h\right)$ is stable and ergodic. Then the above theorem states that the dynamical system governed by measure Q^h is associated with a stationary measure $m^h_{\mathbf{n}^h} = m^h$ for all $\mathbf{n}^h \in (\mathcal{N}^h)^\infty$, where m^h is the marginal measure of m^{Q^h} on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$, while m^{Q^h} is the stationary measure of Q^h on $\left(\Omega^h, \mathcal{B}(\Omega^h)\right)$.[12] Recall that the definition of rational belief restricts the class of stable measures Q^h to satisfy the property such that $m^h = m$. Hence, *the above theorem ensures that Q^h is a rational belief as long as its stationary measure m^h satisfies the condition such that $m^h = m$.*

D Simulations with inequality constraints

The Euler equation (4) reflects inequality constraints, i.e. the short-sale constraints on the riskless asset, which make direct computations of these equations troublesome. To overcome

[12]We need the superscript h for m^h here while there is none for m in claim (b) of the theorem, because the marginal measure of Q^h on $(\Sigma^\infty, \mathcal{B}(\Sigma^\infty))$ is subjective, while that of $\hat{\Pi}^h$ is the true probability.

the difficulty, we replace the Lagrange multiplier $\tilde{\lambda}_t^{hB}$ with the following functions of a new variable ξ_t^h: given ξ_t^h,

$$\begin{aligned}
\xi_t^{h+} &:= \left(\max\{0, \xi_t^h\}\right)^2; \\
\xi_t^{h-} &:= \left(\max\{0, -\xi_t^h\}\right)^2.
\end{aligned}$$

Then, by applying the result in section 4.2 of Zangwill and Garcia (1981), equation (4) together with the complementary slackness conditions can be replaced with the following equations:

$$\begin{aligned}
(c_t^{1h})^{-\nu^h} - \beta \cdot \frac{1}{R_t} \cdot E_{Q_t^h}\left\{(c_{t+1}^{2h} d_{t+1})^{-\nu^h} \,\middle|\, p_t, R_t, d_t\right\} - \xi_t^{h+} &= 0, \\
\xi_t^{h-} - b_t^h + \underline{b} &= 0.
\end{aligned}$$

Namely, instead of finding the optimal $\left(\theta_t^h, b_t^h, \tilde{\lambda}_t^{hB}\right)$ directly, we try to find the optimal $\left(\theta_t^h, b_t^h, \xi_t^h\right)$.

The main advantage of this method lies in its ability to eliminate the possible discontinuities arising from the inequality constraints, and to make the system of equations that characterise the equilibrium smooth with respect to the endogenous variables; thus, the computation becomes much easier.

E Figures and Tables

Table 4: Mean aggregate demand/dividend ratio μ_y

$\lvert\underline{b}\rvert$	η												
	1.00	1.10	1.20	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75
0	42.12	42.12	42.14	42.15	42.15	42.16	42.16	42.16	42.17	42.17	42.18	42.18	42.19
10	42.12	42.12	42.13	42.14	42.15	42.15	42.16	42.16	42.17	42.17	42.18	42.18	42.19
20	42.12	42.12	42.13	42.13	42.14	42.15	42.15	42.16	42.16	42.17	42.18	42.19	42.19
30	42.12	42.12	42.12	42.13	42.13	42.14	42.15	42.15	42.16	42.17	42.18	42.19	42.20
40	42.12	42.12	42.12	42.12	42.13	42.13	42.14	42.15	42.15	42.16	42.18	42.20	42.24
50	42.12	42.12	42.12	42.12	42.12	42.13	42.13	42.14	42.15	42.16	42.17	42.20	42.25
60	42.12	42.12	42.12	42.12	42.12	42.12	42.13	42.13	42.14	42.15	42.16	42.18	42.22
70	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.13	42.13	42.14	42.15	42.16	42.20
80	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.13	42.13	42.14	42.15	42.17
90	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.13	42.13	42.14	42.17
∞	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.12	42.13	42.13	42.14	42.17

Table 5: Volatility of aggregate demand/dividend ratio σ_y

$\lvert\underline{b}\rvert$	η												
	1.00	1.10	1.20	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75
0	0.708	0.708	0.708	0.708	0.708	0.707	0.707	0.707	0.707	0.707	0.707	0.707	0.707
10	0.708	0.707	0.706	0.705	0.704	0.703	0.703	0.702	0.701	0.700	0.699	0.698	0.697
20	0.708	0.708	0.706	0.703	0.702	0.700	0.698	0.697	0.695	0.693	0.691	0.689	0.687
30	0.708	0.708	0.706	0.702	0.700	0.698	0.695	0.693	0.690	0.687	0.683	0.679	0.674
40	0.708	0.708	0.706	0.703	0.701	0.697	0.694	0.690	0.686	0.682	0.676	0.667	0.635
50	0.708	0.708	0.706	0.704	0.701	0.698	0.695	0.691	0.686	0.681	0.674	0.664	0.649
60	0.708	0.708	0.706	0.704	0.702	0.699	0.696	0.692	0.688	0.682	0.675	0.666	0.650
70	0.708	0.708	0.706	0.704	0.702	0.700	0.697	0.693	0.689	0.684	0.678	0.669	0.655
80	0.708	0.708	0.706	0.704	0.702	0.700	0.697	0.694	0.691	0.686	0.680	0.673	0.660
90	0.708	0.708	0.706	0.704	0.702	0.700	0.697	0.694	0.691	0.687	0.682	0.675	0.661
∞	0.708	0.708	0.706	0.704	0.702	0.700	0.697	0.694	0.691	0.687	0.682	0.675	0.661

Table 6: Lifelong *ex post* certainty equivalent CE^h

$\lvert\underline{b}\rvert$	η												
	1.00	1.10	1.20	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75
0	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.34
10	10.34	10.34	10.34	10.34	10.34	10.34	10.34	10.33	10.33	10.33	10.33	10.33	10.33
20	10.34	10.33	10.32	10.32	10.32	10.32	10.31	10.31	10.31	10.30	10.30	10.30	10.29
30	10.34	10.33	10.30	10.29	10.28	10.28	10.27	10.26	10.25	10.24	10.22	10.20	10.16
40	10.34	10.33	10.29	10.25	10.23	10.22	10.20	10.18	10.15	10.11	10.04	9.90	9.37
50	10.34	10.33	10.29	10.23	10.20	10.16	10.11	10.06	10.00	9.89	9.69	9.38	8.06
60	10.34	10.33	10.29	10.22	10.17	10.13	10.07	10.00	9.90	9.76	9.56	9.19	7.44
70	10.34	10.33	10.29	10.22	10.17	10.10	10.03	9.95	9.84	9.69	9.46	9.04	6.99
80	10.34	10.33	10.29	10.22	10.17	10.10	10.02	9.92	9.80	9.63	9.39	8.93	6.69
90	10.34	10.33	10.29	10.22	10.17	10.10	10.02	9.92	9.79	9.61	9.34	8.88	6.64
∞	10.34	10.33	10.29	10.22	10.17	10.10	10.02	9.92	9.79	9.61	9.34	8.88	6.64

Table 7: Consumption of the old agent in the "worst" state $\underline{c}^{2h}$

$\lvert\underline{b}\rvert$	η												
	1.00	1.10	1.20	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75
0	10.06	10.04	10.03	10.01	10.01	10.00	10.00	9.99	9.98	9.97	9.96	9.96	9.95
10	10.06	9.62	9.61	9.59	9.58	9.57	9.57	9.56	9.55	9.54	9.53	9.52	9.51
20	10.06	9.30	9.16	9.12	9.10	9.08	9.05	9.03	9.00	8.97	8.89	8.78	8.66
30	10.06	9.30	8.72	8.61	8.57	8.51	8.36	8.15	7.92	7.67	7.40	7.09	6.73
40	10.06	9.30	8.50	8.11	8.00	7.83	7.49	7.13	6.73	6.28	5.74	5.04	3.69
50	10.06	9.30	8.50	7.72	7.53	7.32	6.87	6.37	5.83	5.21	4.52	3.96	1.49
60	10.06	9.30	8.50	7.65	7.19	6.92	6.63	6.21	5.59	4.96	4.20	3.21	1.11
70	10.06	9.30	8.50	7.65	7.19	6.71	6.29	5.88	5.21	4.45	3.59	2.57	0.85
80	10.06	9.30	8.50	7.65	7.19	6.71	6.21	5.55	4.78	3.98	3.09	2.08	0.62
90	10.06	9.30	8.50	7.65	7.19	6.71	6.21	5.55	4.68	3.75	2.80	1.83	0.59
∞	10.06	9.30	8.50	7.65	7.19	6.71	6.21	5.55	4.68	3.75	2.80	1.83	0.59

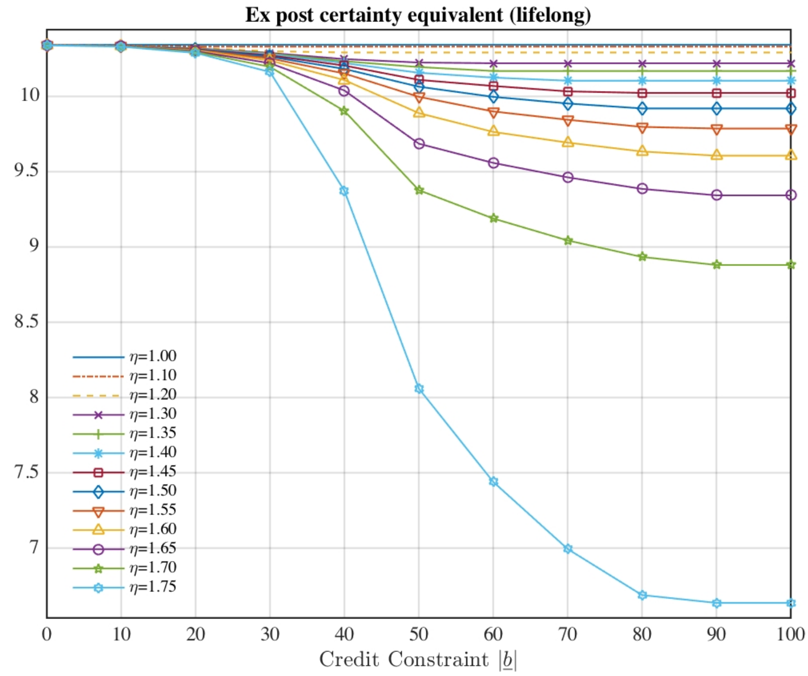

Figure 1: Effects on Welfare

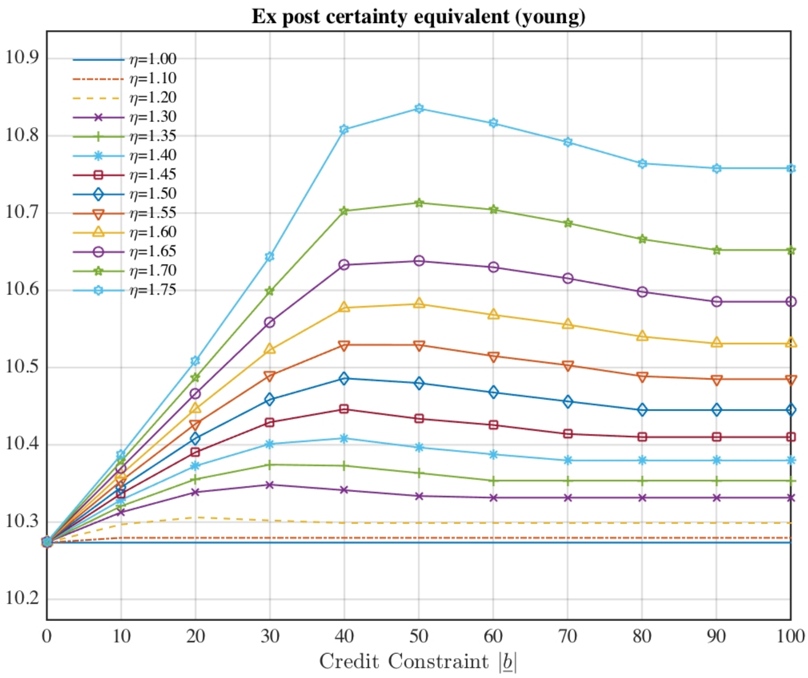

Figure 2: Effects on Welfare when Young

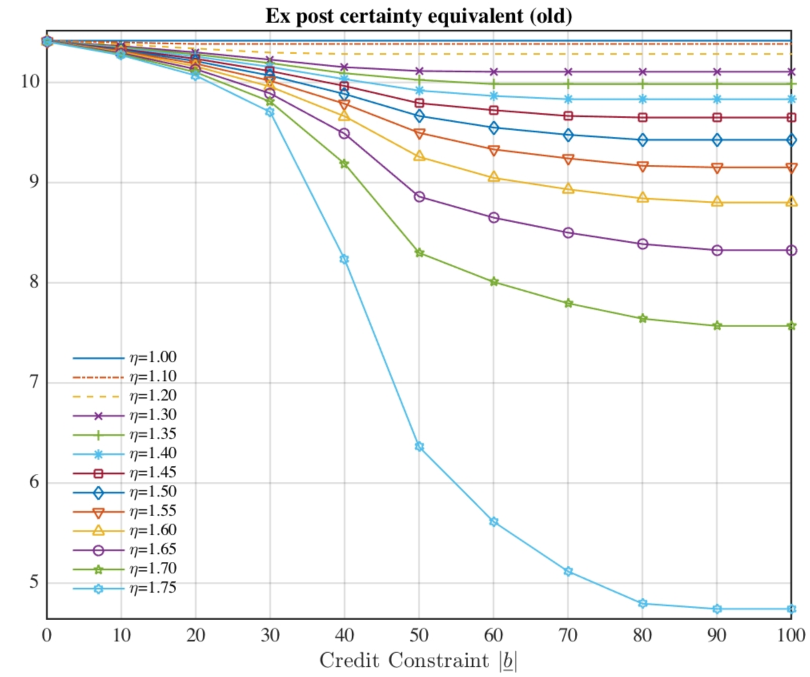

Figure 3: Effects on Welfare when Old

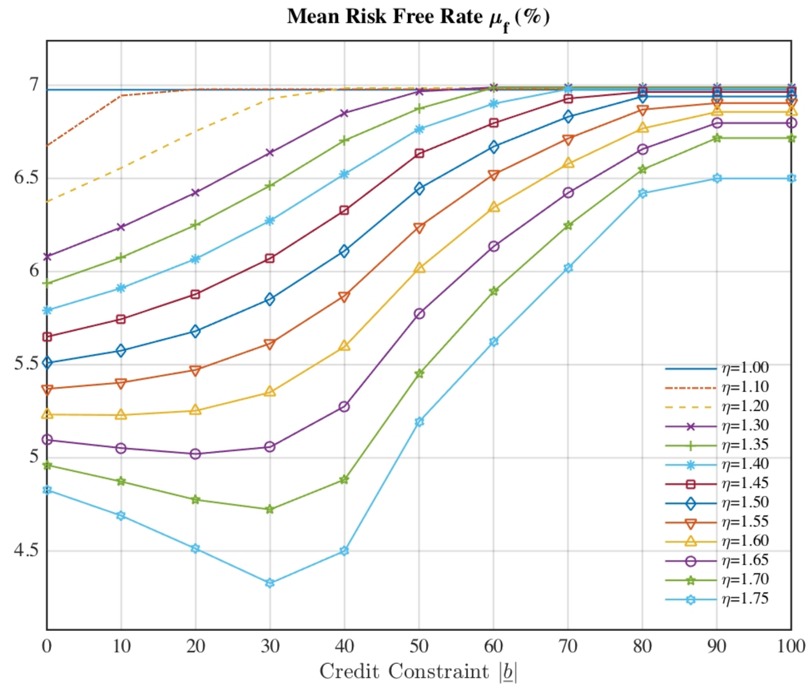

Figure 4: Effects on the Mean Risk Free Rate (%)

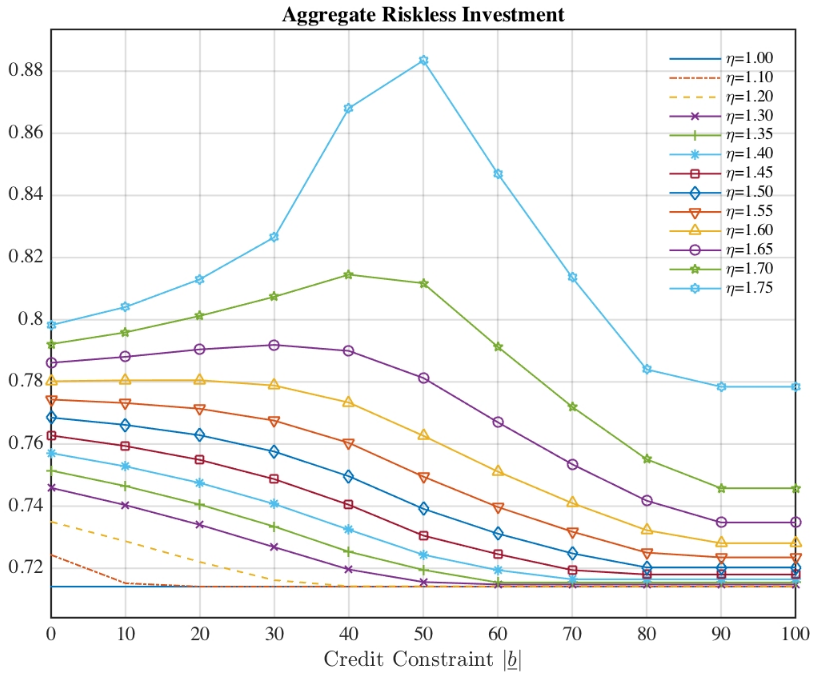

Figure 5: Effects on Aggregate Riskless Investment

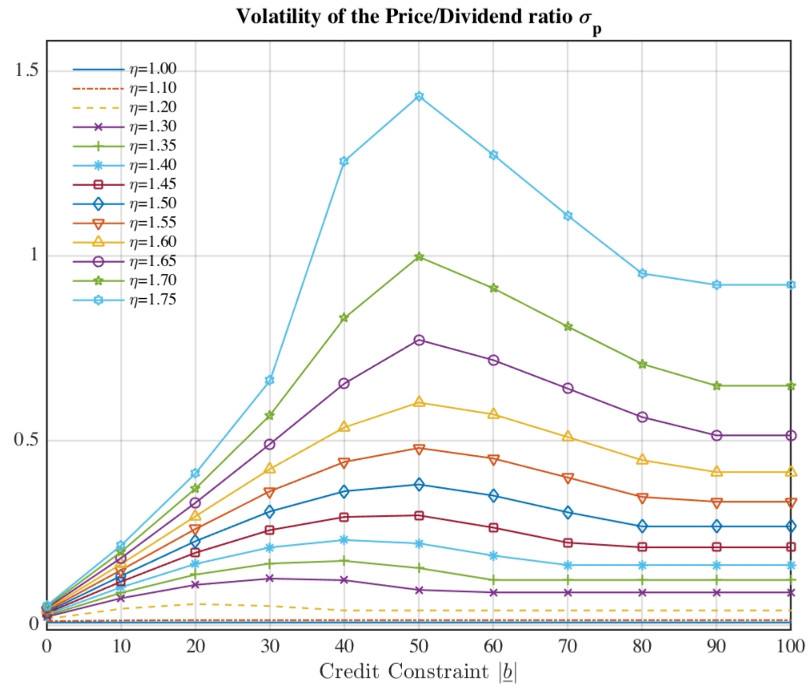

Figure 6: Effects on Price/Dividend ratio Volatility

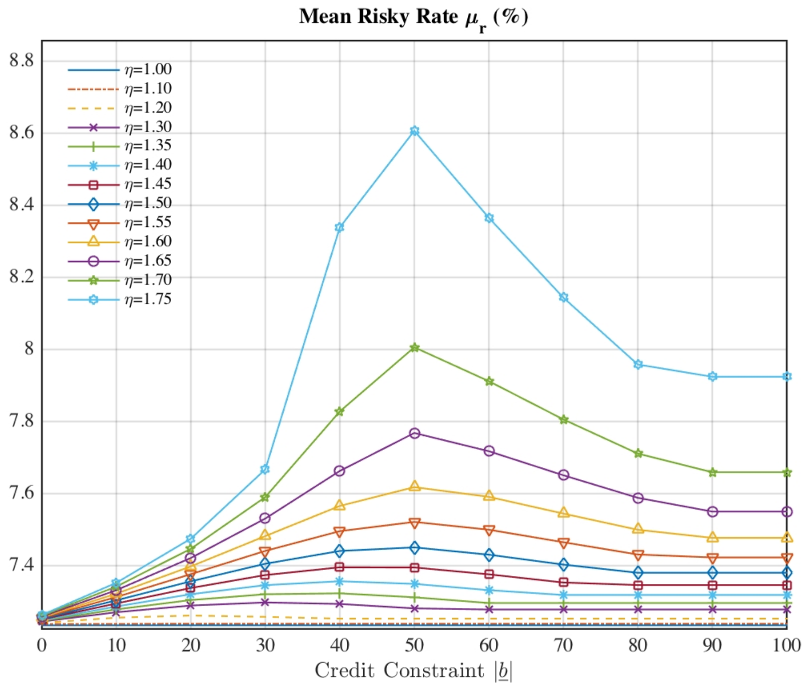

Figure 7: Effects on the Mean Risky Rate (%)

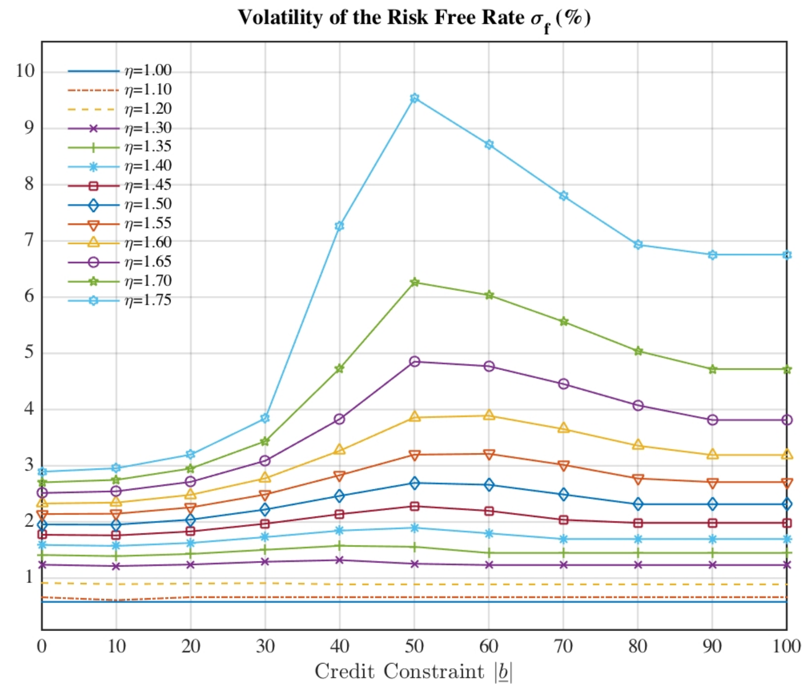

Figure 8: Effects on the Volatility of the Mean Risk Free Rate (%)

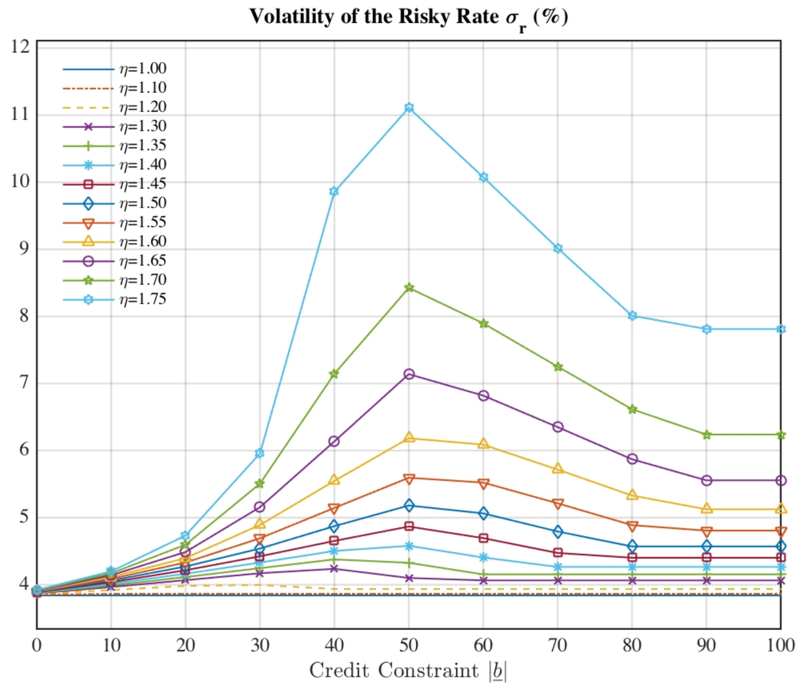

Figure 9: Effects on the Volatility of the Mean Risky Rate (%)

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