RIETI Discussion Paper Series 16-E-079

# An Asymmetric Melitz Model of Trade and Growth

**NAITO Takumi**
Waseda University

Research Institute of Economy, Trade & Industry, IAA
The Research Institute of Economy, Trade and Industry
http://www.rieti.go.jp/en/

RIETI Discussion Paper Series 16-E-079

**August 2016**

# An Asymmetric Melitz Model of Trade and Growth*

NAITO Takumi

Waseda University

Abstract

To examine the effects of unilateral trade liberalization on growth and welfare of the liberalizing and partner countries through intraindustry reallocations, we formulate an asymmetric two-country Melitz model of trade and endogenous growth based on capital accumulation. We obtain two general results analytically. First, each country's mass of exported varieties, revenue share of exported varieties, and growth rate increase if and only if its domestic productivity cutoff increases. Second, compared with the old balanced growth path, a permanent fall in any import trade cost raises the growth rates of all countries for all periods, and welfare of all countries.

*Keywords*: Melitz model, Unilateral trade liberalization, Endogenous growth, Heterogeneous firms, Asymmetric countries

*JEL classification*: F13, F43

RIETI Discussion Papers Series aims at widely disseminating research results in the form of professional papers, thereby stimulating lively discussion. The views expressed in the papers are solely those of the author(s), and neither represent those of the organization to which the author(s) belong(s) nor the Research Institute of Economy, Trade and Industry.

*This study is conducted as a part of the Project "Analyses of Trade Costs" undertaken at Research Institute of Economy, Trade and Industry (RIETI). The author is grateful for helpful comments and suggestions by Discussion Paper seminar participants at RIETI.

# 1 Introduction

Trade liberalization in one country creates a greater export opportunity for more efficient firms in another country. This drives down the profits from domestic sales in the latter country, forcing the least efficient firms to exit. Such reallocation of resources through selection improves the average productivity of surviving firms, which brings about additional gains from trade. This idea, known as the Melitz (2003) model of heterogeneous firms, has become one of the standard theories of international trade since the beginning of this century. However, since users of the Melitz model largely keep the assumption of symmetric countries, nothing is known about how the reallocation process caused by *unilateral* trade liberalization such as illustrated above affects the growth paths of the liberalizing and partner countries in *asymmetric* ways.[1] Economic growth is not only important in itself, but it also delivers additional welfare effects that are ignored in static or stationary trade models. It is also important to consider asymmetric countries and unilateral trade liberalization because trade costs indeed vary across countries: developing countries, on average, have more room for liberalizing imports than developed countries.[2] To examine the effects of unilateral trade liberalization on growth and welfare of the liberalizing and partner countries through intraindustry reallocations, we formulate an asymmetric two-country Melitz model of trade and endogenous growth.

Some researchers combine the Melitz model with the R&D-based endogenous growth models synthesized by Grossman and Helpman (1991) to investigate the effects of symmetric trade liberalization on long-run growth (e.g., Baldwin and Robert-Nicoud, 2008; Haruyama and Zhao, 2008; Dinopoulos and Unel, 2011; Perla et al., 2015; Sampson, 2016). The literature starts from Baldwin and Robert-Nicoud (2008), who introduce heterogeneity in unit labor requirements for production in the standard variety expansion model. Trade liberalization affects each country's expected R&D cost through two channels. On one hand, it lowers the price of knowledge good if and only if it increases the degree of international spillovers of knowledge stocks. On the other hand, it makes it harder for potential entrants to survive in their domestic market (i.e., decreases the cutoff unit labor requirement for domestic sales), which increases the expected units of knowledge good required for successful entry. Overall, trade liberalization raises or lowers the long-run growth rate, depending on whether the former channel is stronger or weaker than the latter. Dinopoulos and Unel (2011) also report an ambiguous relationship between symmetric trade costs and long-run growth, whereas Haruyama and Zhao (2008), Perla et al. (2015), and Sampson (2016) find that the relationship is always negative.[3] It is true that these models successfully describe some firm dynamics under symmetric

[1] Helpman et al. (2004) and Melitz and Ottaviano (2008) deal with asymmetric countries by adding a homogeneous good sector, whose constant marginal product of labor fixes each country's wage. In contrast, Felbermayr et al. (2013), Demidova and Rodríguez-Clare (2013), and Segerstrom and Sugita (2015) allow wages to be endogenously determined in their asymmetric two-country models without a homogeneous good sector: Felbermayr et al. (2013) analyze the optimal tariff problem by assuming a Pareto distribution for productivity. Demidova and Rodríguez-Clare (2013) examine the effects of unilateral reduction in a country's iceberg trade cost under a general productivity distribution. Segerstrom and Sugita (2015) introduce multiple sectors to study the effects of unilateral and sector-specific trade liberalization. However, all of these models are either static or stationary. In their review of heterogeneous firm trade models, Melitz and Redding (2014) point out that: "(d)espite some work on dynamics, much of the literature on firm heterogeneity and trade remains static, and we have relatively little understanding of the processes through which large and successful firms emerge and the implications of these processes for the transitional dynamics of the economy's response to trade liberalization." (Melitz and Redding, 2014, p. 49)

[2] In their comprehensive survey of trade costs, Anderson and van Wincoop (2004) conclude that: "(t)rade costs also vary widely across countries. On average, developing countries have significantly larger trade costs, by a factor of two or more in some important categories." (Anderson and van Wincoop, 2004, p. 747)

[3] Haruyama and Zhao (2008) employ the standard quality ladder model with international knowledge spillovers of the second-highest quality goods. Dinopoulos and Unel (2011) deal with heterogeneity in product quality (i.e., marginal utility) instead of productivity in the variety expansion model. Perla et al. (2015) consider heterogeneous firms' choices between remaining in production and paying a cost to adopt a better technology from more productive domestic firms. Sampson (2016) assumes that productivity of each entrant is proportional to the average productivity of domestic incumbents, which in turn shifts the productivity distribution of entrants to the right over time.

countries, but they do not work under asymmetric countries due to the difficulty of evaluating future domestic and export profits possibly growing at different rates across countries and over time. For our purpose of studying the growth effects of unilateral trade liberalization on asymmetric countries with heterogeneous firms, we have to depart from the R&D-based endogenous growth models.

To allow for asymmetric countries, we build on the multi-country AK model of Acemoglu and Ventura (2002). Motivated by their own empirical findings that a one percentage point rise in a country's growth rate deteriorates its terms of trade by 0.6 percentage points, they construct a model where such terms of trade adjustments in the differentiated intermediate good sector lead the growth rates of capital in all countries to converge in the long run even without knowledge spillovers (and thus without scale effects).[4] We incorporate the variable-wage, asymmetric, and static Melitz framework developed by Felbermayr et al. (2013) and Demidova and Rodríguez-Clare (2013) into the intermediate good sector of the two-country Acemoglu-Ventura model. The variable terms of trade create new interactions that cannot be captured in symmetric growth models: a possible difference in countries' growth rates changes their terms of trade satisfying their zero balance of trade, which induces reallocations across firms. Conversely, such reallocations in turn change countries' terms of trade through their zero balance of trade, thereby influencing their growth rates. Our simple model enables us to examine how unilateral trade liberalization affects each country's productivity cutoffs, extensive margin of exports (i.e., mass of exported varieties), openness (i.e., revenue share of exported varieties), and growth rate not only in the long run but also during the transition, and welfare.

We obtain two general results analytically, without resorting to numerical simulations. First, each country's mass of exported varieties, revenue share of exported varieties, and growth rate increase if and only if its domestic productivity cutoff increases. Although it sounds natural that an increase in a country's domestic productivity cutoff and a decrease in its export productivity cutoff following from its free entry condition increase its mass and revenue share of exported varieties, it is far from trivial that such cutoff changes always raise the country's own growth rate because its growth equation contains all countries' productivity cutoffs as well as factor and trade costs. The nontrivial relationship can be understood by considering that a more productive domestic cutoff firm can survive at a higher rate of return to capital. Just like a country's domestic productivity cutoff is a sufficient statistic for its welfare in static or stationary heterogeneous firm trade models (e.g., Melitz and Redding, 2014), the cutoff is a sufficient statistic for its growth rate as well as its mass and revenue share of exported varieties in our model. Second, compared with the old balanced growth path (BGP), a permanent fall in any import trade cost raises the growth rates (as well as masses and revenue shares of exported varieties) of all countries for all periods, and welfare of all countries.[5] In the short run, a permanent fall in a country's import trade cost directly decreases the export productivity cutoff of the partner country, but it also indirectly decreases that of the liberalizing country itself because its capital becomes relatively cheaper for its trade deficit to be cleared. This increases the domestic productivity cutoffs and hence the growth rates of both countries. Furthermore, since terms of trade are gradually adjusted to equalize countries' growth rates in the long run, both countries grow faster than the old BGP

[4] Based on cross-country development accounting exercises, Caselli (2005) finds that 34–39% of cross-country income variation is explained by variation in physical and human capital, whereas the rest is attributed to variation in total factor productivity. This suggests that capital accumulation is still as important, if not more so, as technological change in explaining growth.

[5] This result itself is the same as Naito (2012), who introduces a continuum-good Ricardian structure in the intermediate good sector of the two-country Acemoglu-Ventura model. This does not mean that the present result is not original: as Arkolakis et al. (2012) point out that the heterogeneous firm monopolistic competition model of Melitz (2003) and the Ricardian perfect competition model of Eaton and Kortum (2002) share the same welfare formula under specific productivity distributions, we find that our dynamic Melitz model delivers the same growth effects of unilateral trade liberalization as a dynamic Ricardian model of Naito (2012), but through completely different mechanisms to be discussed later.

for all periods. One contribution of this result to the literature on heterogeneous firm models of trade and endogenous growth is that positive externalities from knowledge spillovers are unnecessary for trade liberalization to raise long-run growth. International spillovers are necessary for the case of growth-enhancing trade liberalization in Baldwin and Robert-Nicoud (2008), Haruyama and Zhao (2008), and Dinopoulos and Unel (2011), whereas domestic spillovers are the engine of endogenous growth in Perla et al. (2015) and Sampson (2016). Our model shows the positive growth effect of trade liberalization even without positive externalities. Another contribution is that even unilateral trade liberalization is sufficient for faster growth of all countries for all periods. In contrast to the literature, where cross-country symmetry needs to be imposed, our model is so flexible as to yield such a strong result. Finally, the positive relationship between trade liberalization and economic growth derived in our model is consistent with recent empirical evidence such as Wacziarg and Welch (2008), Estevadeordal and Taylor (2013), and Romalis (2007), who use sophisticated empirical methods to overcome Rodríguez and Rodrik's (2000) robustness critique.[6]

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 derives some basic properties. Section 4 examines the effects of unilateral trade liberalization. Section 5 concludes.

# 2 The model

Consider a two-country world economy. In country $i(= 1, 2)$, there is a nontradable final good, which is used for consumption and investment. The final good is produced from a continuum of tradable differentiated intermediate goods under constant returns to scale and perfect competition. Each variety of intermediate good is produced from nontradable capital under increasing returns to scale and monopolistic competition. Capital is the only primary factor, whose growth rate is endogenously determined. The two countries can be asymmetric in terms of all parameters except the elasticity of substitution across intermediate goods. In particular, the asymmetry of trade costs is necessary for expressing unilateral trade liberalization.

## 2.1 Households

The representative household in country $i$ maximizes its overall utility $U_i = \int_0^\infty \ln C_{it} \exp(-\rho_i t)dt$, subject to its budget constraint:

$$p^Y_{it}(C_{it} + \dot{K}_{it} + \delta_i K_{it}) = r_{it}K_{it}; \dot{K}_{it} \equiv dK_{it}/dt, \tag{1}$$

with $\{p^Y_{it}, r_{it}\}^\infty_{t=0}$ and $K_{i0}$ given, where $t(\in [0, \infty))$ is time, $C_i$ is consumption, $\rho_i$ is the subjective discount rate, $p^Y_i$ is the price of the final good, $K_i$ is the supply of capital, $\delta_i$ is the depreciation rate of capital, and $r_i$ is the rental rate of capital. The time subscript is omitted whenever no confusion arises. Dynamic optimization under the logarithmic instantaneous utility implies that capital always grows at the same rate as consumption given by the Euler equation:[7]

[6]Wacziarg and Welch (2008) runs fixed-effects regressions for a panel of 136 countries during 1950-1998 to find a positive relationship between the Sachs-Warner (1995) openness index and annual growth rate. Estevadeordal and Taylor (2013) apply difference-in-differences regressions to two country groups (i.e., liberalizers and nonliberalizers) and two periods (i.e., 1975-1989 and 1990-2004), and report that the change in the liberalizers' growth rate between the two periods is 0.72 percentage points larger than the nonliberalizers on average. Romalis (2007) uses tariffs of the USA as instruments for openness (i.e. trade/GDP ratio) of developing countries, and finds that reductions in US tariffs make developing countries more open, which causes their growth rates to rise.

[7]Integrating the budget constraint (1) from $s = t$ to infinity, and using the Euler equation and the transversality condition, we obtain $C_{it} = \rho_i K_{it} \forall t$.

$$\dot{K}_{it}/K_{it} = \dot{C}_{it}/C_{it} = r_{it}/p_{it}^Y - \delta_i - \rho_i \equiv \gamma_{it} \forall t. \tag{2}$$

## 2.2 Final good firms

The representative final good firm in country $i$ maximizes its profit $\Pi_i^Y = p_i^Y Y_i - \int_{\Omega_i} p_i(\omega)x_i(\omega)d\omega$, subject to its production function $Y_i = (\int_{\Omega_i} x_i(\omega)^\alpha d\omega)^{1/\alpha}; \alpha \equiv (\sigma-1)/\sigma \in (0,1)$, with $p_i^Y$ and $\{p_i(\omega)\}_{\omega \in \Omega_i}$ given, where $Y_i$ is the supply of the final good, $\Omega_i$ is the set of available varieties, $p_i(\omega)$ is the demand price of variety $\omega$, $x_i(\omega)$ is the demand for variety $\omega$, and $\sigma(>1)$ is the elasticity of substitution between any two varieties. Cost minimization yields the conditional input demand function for variety $\omega$:

$$x_i(\omega) = p_i(\omega)^{-\sigma} P_i^\sigma Y_i; P_i \equiv (\int_{\Omega_i} p_i(\omega)^{1-\sigma} d\omega)^{1/(1-\sigma)}, \tag{3}$$

where $P_i$ is the price index of the intermediate goods. Then the minimized expenditure for the intermediate goods is expressed as $\int_{\Omega_i} p_i(\omega)x_i(\omega)d\omega = P_i Y_i \equiv E_i$. Finally, perfect competition drives the price of the final good to its unit cost:

$$p_i^Y = P_i. \tag{4}$$

## 2.3 Intermediate good firms

Our description of the intermediate good sector is based on Felbermayr et al. (2013) and Demidova and Rodríguez-Clare (2013), who provide variable-wage, asymmetric, two-country, one-sector, and static versions of Melitz (2003). After paying a sunk fixed entry cost, an intermediate good firm in country $i$ draws its productivity $\varphi$ from a general distribution $G_i(\varphi)$ with its density $g_i(\varphi)$. If a firm's $\varphi$ is sufficiently high that the resulting gross profit covers its fixed overhead cost which is specific to each source-destination pair $(i,j), i,j = 1,2$, then it survives selling its variety to the destination country $j$; otherwise, it exits from market $j$ without having to pay the overhead cost. The fixed entry and overhead costs are specified as $r_i K_i f_i^e$ and $r_i K_i f_{ij}$, respectively, where $f_i^e$ and $f_{ij}$ are exogenous constants. This means that the richer country $i$ is in terms of GDP, the more difficult it is to create a new variety there and to start up a business in each market. This is not unrealistic: Bollard et al. (2014) estimate that the elasticity of the number of firms per worker with respect to the value added per worker is at most around 0.2 over time and across countries, suggesting that such fixed costs rise with development. Finally, we assume that all decisions of individual firms are static. More specifically, each variety has a product life of only one period, and so each intermediate good firm has to pay not only the overhead costs but also the entry cost in each period with no hysteresis in productivity draws.[8] Only by trading off within-firm dynamics in this way, can we describe rich macroeconomic dynamics of two asymmetric countries with heterogeneous firms.

An intermediate good firm in country $i$ with productivity $\varphi$ maximizes its profit in country $j$ $\pi_{ij}(\varphi) = p_{ij}^f(\varphi)y_{ij}(\varphi) - r_i k_{ij}(\varphi)$, subject to its cost function (measured in terms of capital) $k_{ij}(\varphi) = K_i f_{ij} + y_{ij}(\varphi)/\varphi$, the market-clearing condition for its variety $y_{ij}(\varphi) = \tau_{ij} x_{ij}(\varphi)$, and the conditional input demand function for its variety $x_{ij}(\varphi) = p_{ij}(\varphi)^{-\sigma} P_j^{\sigma-1} E_j = (\tau_{ij} p_{ij}^f(\varphi))^{-\sigma} P_j^{\sigma-1} E_j$ from Eq. (3), with $r_i, K_i, P_j$, and $E_j$ given,

[8] According to the Osiris database, in a total of 9,891 industrial companies, whose R&D expenses per operating revenue in 2010 is available, 8,497 firms pay no less than 0.1 percent of their revenue for R&D, of which 5,193 firms pay no less than 1 percent, for all available years during 2000-2010. This implies that the R&D cost, which can be regarded as the fixed entry cost in our model, is more likely to be paid recurrently than only one time.

where $p_{ij}^f(\varphi)$ is the FOB supply price of a variety, $y_{ij}(\varphi)$ is the supply of a variety, $k_{ij}(\varphi)$ is the demand for capital as overhead and variable costs, and $\tau_{ij}(\geq 1)$ is the iceberg trade cost factor of delivering one unit of a variety from country $i$ to country $j$, with $\tau_{ii} = 1$. Since $\sigma$ is common across countries, each firm sets the same supply price for all destinations:

$$(p_{ij}^f(\varphi) - r_i/\varphi)/p_{ij}^f(\varphi) = 1/\sigma \Leftrightarrow p_{ij}^f(\varphi) = r_i/(\alpha\varphi)\forall j. \tag{5}$$

Using Eq. (5), the resulting revenue and profit are calculated as:

$$\begin{aligned}
e_{ij}(\varphi) &\equiv p_{ij}^f(\varphi) y_{ij}(\varphi) = (\tau_{ij} r_i)^{1-\sigma} (\alpha\varphi P_j)^{\sigma-1} E_j, \\
\pi_{ij}(\varphi) &= e_{ij}(\varphi)/\sigma - r_i K_i f_{ij} = (\tau_{ij} r_i)^{1-\sigma} (\alpha\varphi P_j)^{\sigma-1} E_j/\sigma - r_i K_i f_{ij}.
\end{aligned}$$

Since $\pi_{ij}(\varphi)$ is increasing in $\varphi$, a firm in country $i$ survives in country $j$ if and only if $\varphi \geq \varphi_{ij}$, where the productivity cutoff $\varphi_{ij}$ is determined by:

$$\pi_{ij}(\varphi_{ij}) = 0 \Leftrightarrow e_{ij}(\varphi_{ij}) = (\tau_{ij} r_i)^{1-\sigma} (\alpha\varphi_{ij} P_j)^{\sigma-1} E_j = \sigma r_i K_i f_{ij}. \tag{6}$$

Dividing Eq. (6) by itself with $j = i$, we obtain $\varphi_{ij}/\varphi_{ii} = (P_i/P_j)(E_i/E_j)^{1/(\sigma-1)} \tau_{ij} (f_{ij}/f_{ii})^{1/(\sigma-1)}, i, j = 1, 2$. In line with Melitz (2003), we assume that the variable and fixed export costs are sufficiently large that country $i$'s export productivity cutoff is larger than its domestic productivity cutoff:

$$\varphi_{ij}/\varphi_{ii} > 1, i, j = 1, 2, j \neq i.$$

Let $\widetilde{\varphi}_{ij}(\varphi_{ij}) \equiv (\int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} \mu_{ij}(\varphi|\varphi_{ij}) d\varphi)^{1/(\sigma-1)} (> \varphi_{ij})$ be the average productivity of firms in country $i$ surviving in country $j$, where $\mu_{ij}(\varphi|\varphi_{ij})$ is the density of $\varphi$ conditional on survival:

$$\mu_{ij}(\varphi|\varphi_{ij}) = \begin{cases} g_i(\varphi)/(1 - G_i(\varphi_{ij})) & \text{if } \varphi \geq \varphi_{ij}; \\ 0 & \text{otherwise.} \end{cases}$$

Using $e_{ij}(\varphi) = (\varphi/\varphi_{ij})^{\sigma-1} e_{ij}(\varphi_{ij}) = (\varphi/\varphi_{ij})^{\sigma-1} \sigma r_i K_i f_{ij}$ from Eq. (6), the expected values of revenue and profit are calculated as:

$$\begin{aligned}
&\int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi) g_i(\varphi) d\varphi = (1 - G_i(\varphi_{ij})) \int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi) \mu_{ij}(\varphi|\varphi_{ij}) d\varphi = (1 - G_i(\varphi_{ij}))(h_{ij}(\varphi_{ij}) + 1) \sigma r_i K_i f_{ij}, \\
&\int_{\varphi_{ij}}^{\infty} \pi_{ij}(\varphi) g_i(\varphi) d\varphi = (1 - G_i(\varphi_{ij})) \int_{\varphi_{ij}}^{\infty} \pi_{ij}(\varphi) \mu_{ij}(\varphi|\varphi_{ij}) d\varphi = H_{ij}(\varphi_{ij}) r_i K_i f_{ij}; \\
&\qquad h_{ij}(\varphi_{ij}) \equiv (\widetilde{\varphi}_{ij}(\varphi_{ij})/\varphi_{ij})^{\sigma-1} - 1 > 0, \\
&\qquad H_{ij}(\varphi_{ij}) \equiv (1 - G_i(\varphi_{ij})) h_{ij}(\varphi_{ij}) > 0,
\end{aligned} \tag{7}$$

where $h_{ij}(\varphi_{ij}) = \int_{\varphi_{ij}}^{\infty} \pi_{ij}(\varphi) \mu_{ij}(\varphi|\varphi_{ij}) d\varphi/(r_i K_i f_{ij})$ is the ratio of the conditional expected profit of a firm in country $i$ surviving in country $j$ to its fixed overhead cost, which is multiplied by the probability of survival $1 - G_i(\varphi_{ij})$ to obtain its unconditional version $H_{ij}(\varphi_{ij})$. Free entry implies that the fixed entry cost of an entrant in country $i$ is equal to its total expected profit, that is, $r_i K_i f_i^e = \sum_j \int_{\varphi_{ij}}^{\infty} \pi_{ij}(\varphi) g_i(\varphi) d\varphi$.[9]

[9]What happens to the free entry condition if each firm has an infinite product life, so that it pays the fixed entry cost only

Noting that $r_i K_i$ appears in all terms of the free entry condition, it is simplified to:

$$f_i^e = \sum_j H_{ij}(\varphi_{ij}) f_{ij}. \tag{8}$$

Since Appendix A shows that $H_{ij}(\varphi_{ij})$ is decreasing in $\varphi_{ij}$, the simplified free entry condition (8) means that country $i$'s domestic productivity cutoff $\varphi_{ii}$ is always negatively related to its export productivity cutoff $\varphi_{ij}$: a decrease in $\varphi_{ij}$, for example, makes it easier for firms in country $i$ to survive in their export market. For the free entry condition to be satisfied, $\varphi_{ii}$ increases so that surviving in their domestic market should become more difficult.

Finally, let $M_i^e$ denote the mass of entrants in country $i$. Then the mass of entrants in country $i$ surviving in country $j$, or the mass of varieties sold from country $i$ to country $j$, is given by $M_{ij} = M_i^e(1 - G_i(\varphi_{ij}))$.

## 2.4 Markets

The market-clearing conditions for the final good, capital, and the intermediate goods are given by:

$$Y_i = C_i + \dot{K}_i + \delta_i K_i, i = 1, 2, \tag{9}$$

$$K_i = \sum_j M_{ij} \int_{\varphi_{ij}}^{\infty} k_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi + M_i^e K_i f_i^e, i = 1, 2, \tag{10}$$

$$y_{ij}(\varphi) = \tau_{ij} x_{ij}(\varphi), i, j = 1, 2. \tag{11}$$

On the other hand, summing up the household budget constraint (1), the zero profit condition in the final good sector (4), and the free entry condition in the intermediate good sector (8), we obtain Walras' law in country $i$:

$$\begin{aligned} 0 = {} & p_i^Y(C_i + \dot{K}_i + \delta_i K_i - Y_i) + r_i(\textstyle\sum_j M_{ij} \displaystyle\int_{\varphi_{ij}}^{\infty} k_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi + M_i^e K_i f_i^e - K_i) \\ & + \textstyle\sum_j M_{ji} \displaystyle\int_{\varphi_{ji}}^{\infty} \tau_{ji} p_{ji}^f(\varphi) x_{ji}(\varphi)\mu_{ji}(\varphi|\varphi_{ji})d\varphi - \textstyle\sum_j M_{ij} \displaystyle\int_{\varphi_{ij}}^{\infty} p_{ij}^f(\varphi) y_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi. \end{aligned}$$

Summing this up for all countries, we can confirm Walras' law in the world, implying that any one of the eight (i.e., (9) $\times$ 2, (10) $\times$ 2, (11) $\times$ 4) types of the market-clearing conditions is redundant, and that the price of any one of the eight types of goods and factors can be normalized to unity. Finally, Walras' law in country $i$, together with Eqs. (9), (10), and (11), implies that its balance of trade is zero: $M_{ij} \int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi = M_{ji} \int_{\varphi_{ji}}^{\infty} e_{ji}(\varphi)\mu_{ji}(\varphi|\varphi_{ji})d\varphi, i, j = 1, 2, j \neq i$.

## 2.5 Dynamic system

From now on, we derive the dynamic system, regarding the iceberg trade costs $\tau_{21}$ and $\tau_{12}$ as the only policy variables. Let capital in country 2 be the numeraire: $r_2 \equiv 1$, and let $\kappa \equiv K_1/K_2$ be the relative supply of capital in country 1 to country 2. From the zero cutoff profit condition (6) for all source-destination pairs,

once? Then the one-time fixed entry cost should be equal to the present value of the total expected profits for all future periods. However, since the latter includes the future paths of endogenous variables, whose growth rates are not always constant under asymmetric countries, the productivity cutoffs in period $t$ depend not only on period $t$ endogenous variables just like Eq. (14), but also on their future paths. To avoid such intractability, we are assuming a one-period product life.

the ratio of the foreign export productivity cutoff to the domestic productivity cutoff in each destination is obtained as:

$$\varphi_{21}/\varphi_{11} = (r_1^{\sigma}\kappa)^{-1/(\sigma-1)}\tau_{21}(f_{21}/f_{11})^{1/(\sigma-1)}, \tag{12}$$

$$\varphi_{12}/\varphi_{22} = (r_1^{\sigma}\kappa)^{1/(\sigma-1)}\tau_{12}(f_{12}/f_{22})^{1/(\sigma-1)}. \tag{13}$$

A fall in a country's import trade cost, and/or an increase in its relative rental rate and/or its relative supply of capital, makes more foreign firms survive relative to domestic firms. From Eqs. (8), (12), and (13), all productivity cutoffs can be solved as functions of $r_1^{\sigma}\kappa, \tau_{21}$, and $\tau_{12}$:

$$\varphi_{ij} = \varphi_{ij}(r_1^{\sigma}\kappa, \tau_{21}, \tau_{12}), i, j = 1, 2. \tag{14}$$

Using Eqs. (7), (8), and $M_{ij} = [(1 - G_i(\varphi_{ij}))/(1 - G_i(\varphi_{ii}))]M_{ii}$, the capital market-clearing condition (10) is solved for $M_{ii}$. Substituting the solution back into $M_{ij} = [(1-G_i(\varphi_{ij}))/(1-G_i(\varphi_{ii}))]M_{ii}$, we obtain:

$$M_{ij} = (1/\sigma)(1 - G_i(\varphi_{ij}))/\textstyle\sum_k(H_{ik}(\varphi_{ik}) + 1 - G_i(\varphi_{ik}))f_{ik} \equiv M_{ij}(\{\varphi_{ik}\}_{k=1}^2), i, j, k = 1, 2. \tag{15}$$

Our specification of the fixed entry and overhead costs implies that the mass of varieties sold from country $i$ to country $j$ does not grow proportionately with the source country's GDP, which is consistent with the empirical findings of Bollard et al. (2014).

Using Eqs. (7) and (15), country $i$'s revenue share of exported varieties $\beta_i \equiv M_{ij}\int_{\varphi_{ij}}^{\infty} e_{ij}(\varphi)\mu_{ij}(\varphi|\varphi_{ij})d\varphi/\sum_k M_{ik}\int_{\varphi_{ik}}^{\infty} e_{ik}(\varphi)\mu_{ik}(\varphi|\varphi_{ik})d\varphi$ is rewritten as:

$$\beta_i = (H_{ij}(\varphi_{ij}) + 1 - G_i(\varphi_{ij}))f_{ij}/\textstyle\sum_k(H_{ik}(\varphi_{ik}) + 1 - G_i(\varphi_{ik}))f_{ik} \equiv \beta_i(\{\varphi_{ik}\}_{k=1}^2) \in (0,1), i, j, k = 1, 2, j \neq i. \tag{16}$$

Considering the zero balance of trade, this is equal to country $i$'s expenditure share of imported varieties $\zeta_i \equiv M_{ji}\int_{\varphi_{ji}}^{\infty} e_{ji}(\varphi)\mu_{ji}(\varphi|\varphi_{ji})d\varphi/\sum_k M_{ki}\int_{\varphi_{ki}}^{\infty} e_{ki}(\varphi)\mu_{ki}(\varphi|\varphi_{ki})d\varphi$. Eq. (16) serves as a measure of country $i$'s openness.

Using Eqs. (7), (15), and (16), the zero balance of trade is rewritten as:

$$\beta_1(\{\varphi_{1k}\}_{k=1}^2)r_1\kappa = \beta_2(\{\varphi_{2k}\}_{k=1}^2). \tag{17}$$

The left- and right-hand sides of Eq. (17) correspond to country 1's exports and imports, respectively.

The intermediate good price index in Eq. (3) is rewritten as:[10]

$$P_i = \{\textstyle\sum_j M_{ji}(\{\varphi_{jk}\}_{k=1}^2)[\tau_{ji}r_j/(\alpha\widetilde{\varphi}_{ji}(\varphi_{ji}))]^{1-\sigma}\}^{1/(1-\sigma)} \equiv Q_i(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{ji}r_j\}_{j=1}^2), i, j, k = 1, 2. \tag{18}$$

Country $i$'s simplified intermediate good price index $Q_i(\cdot)$ depends on the four productivity cutoffs through $M_{ji}(\{\varphi_{jk}\}_{k=1}^2)$ and $\widetilde{\varphi}_{ji}(\varphi_{ji})$, whereas it is increasing and homogeneous of degree one in the suppliers' factor and trade costs $\{\tau_{ji}r_j\}_{j=1}^2$. From Eqs. (2), (4), and (18), country $i$'s growth rate is expressed as:

[10] The intermediate good price index in Eq. (3) is given by $P_i = (\sum_j M_{ji}\int_{\varphi_{ji}}^{\infty} p_{ji}(\varphi)^{1-\sigma}\mu_{ji}(\varphi|\varphi_{ji})d\varphi)^{1/(1-\sigma)}$. Using Eq. (5) and $\int_{\varphi_{ji}}^{\infty} \varphi^{\sigma-1}\mu_{ji}(\varphi|\varphi_{ji})d\varphi = \widetilde{\varphi}_{ji}(\varphi_{ji})^{\sigma-1}$, and considering Eq. (15), we obtain Eq. (18).

$$\gamma_i = 1/q_i(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{ji}r_j/r_i\}_{j=1}^2) - \delta_i - \rho_i, i, j, k = 1, 2; \quad (19)$$
$$q_i(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{ji}r_j/r_i\}_{j=1}^2) \equiv Q_i(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{ji}r_j\}_{j=1}^2)/r_i,$$

where $q_i(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{ji}r_j/r_i\}_{j=1}^2)$ is country $i$'s simplified intermediate good price index divided by its rental rate, meaning that $1/q_i(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{ji}r_j/r_i\}_{j=1}^2)$ is country $i$'s gross rate of return to capital. Country $i$'s growth rate depends on the productivity cutoffs whereas it is decreasing in $\tau_{ji}r_j/r_i, j \neq i$, country $i$'s import trade cost divided by its relative rental rate. From Eq. (19), the growth rate of $\kappa$ is simply given by:

$$\begin{aligned}\dot{\kappa}/\kappa &= \gamma_1 - \gamma_2 \\ &= 1/q_1(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{j1}r_j/r_1\}_{j=1}^2) - \delta_1 - \rho_1 - (1/q_2(\{\{\varphi_{jk}\}_{k=1}^2\}_{j=1}^2, \{\tau_{j2}r_j\}_{j=1}^2) - \delta_2 - \rho_2). \quad (20)\end{aligned}$$

With the initial condition $\kappa_0 = K_{10}/K_{20}$ and the trade costs $\tau_{21}$ and $\tau_{12}$ given, Eqs. (14), (17), and (20) characterize an equilibrium path $\{\varphi_{11t}, \varphi_{12t}, \varphi_{21t}, \varphi_{22t}, r_{1t}, \kappa_t\}_{t=0}^{\infty}$.

# 3 Basic properties

## 3.1 Productivity cutoffs, extensive margins, openness, and growth rates

First of all, the logarithmically differentiated form of the free entry condition (8) is given by:[11]

$$0 = (1 - \beta_i)\widehat{\varphi}_{ii} + \beta_i\widehat{\varphi}_{ij}, j \neq i, \quad (21)$$

where $\widehat{\varphi}_{ij} \equiv d\ln\varphi_{ij} \equiv d\varphi_{ij}/\varphi_{ij}$ represents the rate of change in $\varphi_{ij}$. Using Eq. (21), we derive important relationships among each country's domestic productivity cutoff, extensive margin of exports, openness, and growth rate:

**Proposition 1** *Each country's mass of exported varieties, revenue share of exported varieties, and growth rate increase if and only if its domestic productivity cutoff increases.*

**Proof.** See Appendix B. ■

An increase in country $i$'s domestic productivity cutoff is always followed by a decrease in its export productivity cutoff from the free entry condition. This encourages exports relative to domestic sales, thereby increasing both its mass and revenue share of exported varieties. On the other hand, since country $i$'s domestic cutoff firm becomes more productive, it can survive at a higher rental rate relative to the intermediate good price index, meaning that its gross rate of return to capital and hence its growth rate rise. An implication of this proposition is that we can predict the direction of change in each country's extensive margin of exports, openness, and growth rate by just looking at its domestic productivity cutoff as an indicator of firm selection.

Proposition 1 is closely related to the so-called ACR formula by Arkolakis et al. (2012), stating that a country's welfare is always negatively correlated with its expenditure share of domestic varieties in a wide

[11] This is obtained by logarithmically differentiating Eq. (8), and using $\eta_{ij} \equiv -d\ln H_{ij}/d\ln\varphi_{ij} \equiv [(h_{ij}+1)/h_{ij}](\sigma-1) = [(H_{ij}+1-G_{ij})/H_{ij}](\sigma-1)$ (see Appendix A) and Eq. (16), where $G_{ij} \equiv G_i(\varphi_{ij})$.

class of trade models including the Melitz model with a Pareto distribution for productivity. The present proposition implies that a country's gross rate of return to capital $r_i/p_i^Y = 1/q_i$, which serves as its welfare measure in the static version of our model (i.e., $\dot{K}_i + \delta_i K_i = 0$ in Eq. (1)), is always negatively correlated with its domestic expenditure share $1 - \zeta_i = 1 - \beta_i$ in line with Arkolakis et al. (2012). Our model produces the ACR relationship without requiring a Pareto distribution because of the two-country setting: for any country, there is only one export productivity cutoff, which is negatively related to its domestic productivity cutoff.

Substituting $\widehat{\varphi}_{12}$ and $\widehat{\varphi}_{21}$ from the logarithmically differentiated forms of Eqs. (13) and (12), respectively, into Eq. (21), $\widehat{\varphi}_{11}$ and $\widehat{\varphi}_{22}$ are solved as:

$$\widehat{\varphi}_{11} = [\beta_1/(1-\beta_1-\beta_2)][-(\sigma\widehat{r}_1+\widehat{\kappa})/(\sigma-1)+\beta_2\widehat{\tau}_{21}-(1-\beta_2)\widehat{\tau}_{12}], \tag{22}$$

$$\widehat{\varphi}_{22} = [\beta_2/(1-\beta_1-\beta_2)][(\sigma\widehat{r}_1+\widehat{\kappa})/(\sigma-1)-(1-\beta_1)\widehat{\tau}_{21}+\beta_1\widehat{\tau}_{12}]. \tag{23}$$

To obtain intuitions, consider the normal case where $1 - \beta_1 - \beta_2$ is positive (it turns out, however, that all propositions hold regardless of its sign).[12] In Eq. (23), for example, $\partial \ln \varphi_{22}/\partial \ln \tau_{21} < 0$ and $\partial \ln \varphi_{22}/\partial \ln(r_1^\sigma \kappa) > 0$ can be understood by remembering the discussion right after Eqs. (12) and (13) and country 2's free entry condition: a fall in country 1's import trade cost $\tau_{21}$, and/or an increase in $r_1^\sigma\kappa$, encourages country 2's exporters (i.e., $\varphi_{21}$ decreases), which in turn forces its least productive domestic suppliers to exit (i.e., $\varphi_{22}$ increases). On the other hand, $\partial \ln \varphi_{22}/\partial \ln \tau_{12} > 0$ can be interpreted as follows. A fall in country 2's import trade cost $\tau_{12}$ helps country 1's exporters (i.e., $\varphi_{12}$ decreases). From country 1's free entry condition, this induces its least efficient domestic firms to exit (i.e., $\varphi_{11}$ increases). Due to tougher competition in market 1, some firms in country 2 cease exporting (i.e., $\varphi_{21}$ increases). Then again from country 2's free entry condition, more firms enter its domestic market (i.e., $\varphi_{22}$ decreases). The last result, together with Proposition 1, implies that unilateral trade liberalization directly lowers the growth rate of the liberalizing country. However, the total growth effect of unilateral trade liberalization cannot be signed until changes in $r_1$ and $\kappa$ are endogenously determined in a general equilibrium.

### 3.2 Balanced growth path

By endogenizing all productivity cutoffs and country 1's rental rate from Eqs. (14) and (17), country $i$'s growth rate (19) is rewritten as $\gamma_i(\kappa; \tau_{21}, \tau_{12})$. This means that our dynamic system is reduced to a one-dimensional autonomous differential equation for $\kappa$ as $\dot{\kappa}/\kappa = \gamma_1(\kappa; \tau_{21}, \tau_{12}) - \gamma_2(\kappa; \tau_{21}, \tau_{12})$ from Eq. (20).

As usual, a balanced growth path (BGP) is defined as a path along which all variables grow at constant rates. In the present model, a BGP is simply characterized by:

$$0 = \gamma_1(\kappa^*; \tau_{21}, \tau_{12}) - \gamma_2(\kappa^*; \tau_{21}, \tau_{12}),$$

where an asterisk represents a BGP.

**Proposition 2** *There exists a unique* $\ln \kappa^* \in (\ln \underline{\kappa}, \ln \overline{\kappa})$ *which is globally stable if* $\gamma_1 - \gamma_2 > 0$ *at* $\ln \kappa = \ln \underline{\kappa}$ *and* $\gamma_1 - \gamma_2 < 0$ *at* $\ln \kappa = \ln \overline{\kappa} > \ln \underline{\kappa}$*, where* $\ln \underline{\kappa}$ *and* $\ln \overline{\kappa}$ *correspond to* $\ln(\varphi_{12}/\varphi_{11}) = 0$ *and* $\ln(\varphi_{21}/\varphi_{22}) = 0$*, respectively.*

[12] According to the World Development Indicators, the annual averages of the total trade/GDP ratios of the aggregate high-income countries and low- and middle-income countries during 2001-2010 were 52.0% and 58.0%, respectively. Under the assumption of zero balance of trade, dividing them by two gives estimates of $\beta_i$, which were below 30%, or 0.3.

**Proof.** See Appendix C. ■

Dynamics of our model is illustrated in Fig. 1, where $\kappa$ and $\gamma_i$ are measured on the horizontal and vertical axes, respectively. Country 1's growth rate is drawn by the downward-sloping dashed curve $\gamma_1(\kappa;\tau_{21},\tau_{12})$ because an increase in $\kappa$ increases $r_1^\sigma\kappa$ (cf. Eq. (C.2)), which increases $\varphi_{12}$ and hence decreases $\varphi_{11}$ (cf. Eq. (22)).[13] Similarly, since an increase in $r_1^\sigma\kappa$ arising from an increase in $\kappa$ decreases $\varphi_{21}$ and thus increases $\varphi_{22}$ (cf. Eq. (23)), country 2's growth rate is represented by the upward-sloping dashed curve $\gamma_2(\kappa;\tau_{21},\tau_{12})$. The two curves intersect at point A: $(\kappa^*,\gamma_i^*)$, a unique BGP. If $\kappa$ starts from a low value, then country 1 starts to grow faster than country 2. Since this pushes $\kappa$ toward $\kappa^*$, country 1's growth rate falls whereas country 2's growth rate rises. This process continues until the two growth rates are equalized at point A.

# 4 Effects of unilateral trade liberalization

General expressions for the growth rates are derived as (see Appendix C):

$$
\begin{aligned}
&d\gamma_1 = -[(\beta_1/q_1)/A]\{(\sigma-1)\widehat{\kappa} + [\sigma\chi_{21}(1-\beta_2)+(\sigma-1)\beta_2]\widehat{\tau}_{21} + (1-\beta_2)[\sigma\chi_{21}-(\sigma-1)]\widehat{\tau}_{12}\}, && (24)\\
&d\gamma_2 = -[(\beta_2/q_2)/A]\{-(\sigma-1)\widehat{\kappa} + (1-\beta_1)[\sigma\chi_{12}-(\sigma-1)]\widehat{\tau}_{21} + [\sigma\chi_{12}(1-\beta_1)+(\sigma-1)\beta_1]\widehat{\tau}_{12}\}; && (25)\\
&\chi_{ij} \equiv -d\ln\beta_i/d\ln\varphi_{ij} = \sigma-1+(1-\beta_i)g_{ij}\varphi_{ij}/(H_{ij}+1-G_{ij}) + \beta_i g_{ii}\varphi_{ii}/(H_{ii}+1-G_{ii}) > \sigma-1, j\neq i,\\
&g_{ij}\equiv g_i(\varphi_{ij}), G_{ij}\equiv G_i(\varphi_{ij}),\\
&A \equiv \sigma[\chi_{12}(1-\beta_1)+\chi_{21}(1-\beta_2)] - (\sigma-1)(1-\beta_1-\beta_2) > 0.
\end{aligned}
$$

Eqs. (24) and (25) mean that, with $\kappa$ given, a fall in any import trade cost raises the growth rates of both countries. At first sight Eq. (24) seems to contradict partly with Eq. (22): the latter suggests that a fall in $\tau_{21}$ directly decreases $\varphi_{11}$. In fact, a decrease in $\varphi_{11}$ and an increase in $\varphi_{22}$ caused by a fall in $\tau_{21}$ tend to create a trade deficit for country 1 (cf. Eq. (17) and Proposition 1). For the deficit to be cleared, its capital should become relatively cheaper (cf. Eq. (C.1)). This indirectly increases $\varphi_{11}$. Since the indirect effect outweighs the direct effect, $\gamma_1$ rises as a result.

To examine the long-run growth effects, we substitute Eqs. (24) and (25) into $d\gamma_1^* = d\gamma_2^*$ to solve for $\widehat{\kappa}^*$:

$$
\begin{aligned}
\widehat{\kappa}^* = {}& [1/(\sigma-1)][1/(\beta_1^*/q_1^*+\beta_2^*/q_2^*)]\\
&\times\{-\{(\beta_1^*/q_1^*)[\sigma\chi_{21}^*(1-\beta_2^*)+(\sigma-1)\beta_2^*] - (\beta_2^*/q_2^*)(1-\beta_1^*)[\sigma\chi_{12}^*-(\sigma-1)]\}\widehat{\tau}_{21}\\
&+\{(\beta_2^*/q_2^*)[\sigma\chi_{12}^*(1-\beta_1^*)+(\sigma-1)\beta_1^*] - (\beta_1^*/q_1^*)(1-\beta_2^*)[\sigma\chi_{21}^*-(\sigma-1)]\}\widehat{\tau}_{12}\}.
\end{aligned}
$$

Substituting this back into either Eq. (24) or (25), we obtain:

$$
d\gamma_1^* = d\gamma_2^* = -[(\beta_1^*/q_1^*)(\beta_2^*/q_2^*)/(\beta_1^*/q_1^*+\beta_2^*/q_2^*)](\widehat{\tau}_{21}+\widehat{\tau}_{12}).
$$

This implies that a fall in any import trade cost raises the balanced growth rate.

To consider the short-run growth effects of unilateral trade liberalization, e.g., a fall in $\tau_{21}$, we first see its growth effects in the initial period, where $\widehat{\kappa}_0 = 0$. Eqs. (24) and (25) immediately give:

[13] This intuition is true when $1-\beta_1-\beta_2>0$. However, even when $1-\beta_1-\beta_2<0$, an increase in $\kappa$ decreases $\varphi_{11}$ through a decrease in $r_1^\sigma\kappa$. Therefore, Fig. 1 is valid regardless of the sign of $1-\beta_1-\beta_2$.

$$\partial\gamma_{10}/\partial\ln\tau_{21} = -[(\beta_1^*/q_1^*)/A^*][\sigma\chi_{21}^*(1-\beta_2^*)+(\sigma-1)\beta_2^*] < 0,$$
$$\partial\gamma_{20}/\partial\ln\tau_{21} = -[(\beta_2^*/q_2^*)/A^*](1-\beta_1^*)[\sigma\chi_{12}^*-(\sigma-1)] < 0,$$

where all endogenous variables are evaluated at the old BGP.

Fig. 1 illustrates the case where $(\beta_1^*/q_1^*)[\sigma\chi_{21}^*(1-\beta_2^*)+(\sigma-1)\beta_2^*] > (\beta_2^*/q_2^*)(1-\beta_1^*)[\sigma\chi_{12}^*-(\sigma-1)]$. This is more likely, the more open country 1 is relative to country 2 at the old BGP.[14] Suppose that we are originally at point A, the old BGP, and that $\tau_{21}$ falls permanently to $\tau_{21}'(<\tau_{21})$. Then both dashed curve $\gamma_1(\kappa;\tau_{21},\tau_{12})$ and curve $\gamma_2(\kappa;\tau_{21},\tau_{12})$ shift up to solid curve $\gamma_1(\kappa;\tau_{21}',\tau_{12})$ and curve $\gamma_2(\kappa;\tau_{21}',\tau_{12})$, respectively, but the former shift is larger than the latter. The growth rates of country 1 and country 2 in the initial period are given by point B: $(\kappa^*,\gamma_{10}')$ and point C: $(\kappa^*,\gamma_{20}')$, respectively. Since country 1 starts to grow faster than country 2, $\kappa$ starts to increase. This continues to pull down country 1's growth rate whereas pushes up country 2's growth rate along curve $\gamma_1(\kappa;\tau_{21}',\tau_{12})$ and curve $\gamma_2(\kappa;\tau_{21}',\tau_{12})$, respectively. The growth rates are equalized at point D: $(\kappa^{*\prime},\gamma_i^{*\prime})$, the new BGP, which is to the northeast of point A. We observe that, compared with the old BGP, a permanent fall in $\tau_{21}$ raises the growth rates of all countries for all periods. Moreover, from Proposition 1, this is also true for masses and revenue shares of exported varieties. It can also be pointed out that the relatively more open country 1 experiences an overshooting in its growth rate.

In the other case where $(\beta_1^*/q_1^*)[\sigma\chi_{21}^*(1-\beta_2^*)+(\sigma-1)\beta_2^*] < (\beta_2^*/q_2^*)(1-\beta_1^*)[\sigma\chi_{12}^*-(\sigma-1)]$, which is possible if country 2 is relatively more open at the old BGP, the upward shift of curve $\gamma_2(\kappa;\tau_{21}',\tau_{12})$ is now larger than curve $\gamma_1(\kappa;\tau_{21}',\tau_{12})$, so point C is now higher than point B. Since $\kappa$ starts to decrease, country 1's growth rate goes up whereas country 2's growth rate goes down until they are equalized at point D, which is now to the northwest of point A. Even in this case, compared with the old BGP, a permanent fall in $\tau_{21}$ raises the growth rates as well as masses and revenue shares of exported varieties of all countries for all periods. We again observe an overshooting in the growth rate of the relatively more open country 2.

Finally, considering that country $i$'s consumption in period $t$ is given by $C_{it} = C_{i0}\exp(\int_0^t\gamma_{is}ds) = \rho_i K_{i0}\exp(\int_0^t\gamma_{is}ds)$ under the logarithmic instantaneous utility, the fact that $\gamma_{is}' > \gamma_{is}\forall s \geq 0$ implies that $C_{it}' > C_{it}\forall t > 0$, and hence $U_i' > U_i$. The following proposition summarizes our results:

**Proposition 3** *Compared with the old BGP, a permanent fall in any import trade cost raises the growth rates of all countries for all periods, and welfare of all countries.*

This result, together with Proposition 1, shows a striking coincidence with Naito (2012, Propositions 2 to 4), who embeds a continuum-good Ricardian framework with perfect competition in the intermediate good sector of the two-country Acemoglu-Ventura model. However, the mechanisms behind the result are quite different. In Naito (2012), a fall in a country's import trade cost directly raises its own growth rate, but the former indirectly lowers the latter through a fall in its relative rental rate due to the decreased demand for its capital. Since the direct effect outweighs the indirect effect, the growth rate of the liberalizing country goes up in total. On the other hand, the growth rate of the partner country also goes up because of a rise in its relative rental rate. In contrast, in the present model, a fall in a country's import trade cost directly lowers its own growth rate whereas raises that of the partner country through changes in domestic

[14] This case applies when the old BGP is symmetric because $(\beta_1^*/q_1^*)[\sigma\chi_{21}^*(1-\beta_2^*)+(\sigma-1)\beta_2^*]-(\beta_2^*/q_2^*)(1-\beta_1^*)[\sigma\chi_{12}^*-(\sigma-1)] = (\beta_1^*/q_1^*)(\sigma-1) > 0$ if $\beta_i^* = \beta_j^*, q_i^* = q_j^*$, and $\chi_{ij}^* = \chi_{ji}^*$.

productivity cutoffs. In addition, such trade liberalization lowers the relative rental rate of the liberalizing country satisfying its zero balance of trade, which has counteracting effects on countries' growth rates. The indirect effect is stronger than the direct effect for the liberalizing country, whereas the opposite is true for the partner country.

The robust positive growth effect of (even unilateral) trade liberalization is theoretically remarkable because it is obtained even with imperfect competition and without positive externalities from knowledge spillovers. In the literature, trade liberalization can raise long-run growth only under international spillovers (e.g., Baldwin and Robert-Nicoud, 2008; Haruyama and Zhao, 2008; Dinopoulos and Unel, 2011) or domestic spillovers (e.g., Perla et al., 2015; Sampson, 2016). This tempts us to suppose that freer trade cannot foster economic growth in the Melitz-type models of trade and growth without positive externalities. Our result shows that the conjecture is not true.

# 5 Concluding remarks

This paper makes both theoretical and policy contributions. On the theoretical side, we first provide an asymmetric Melitz model of trade and growth. In the literature on heterogeneous firm models of trade and growth (e.g., Baldwin and Robert-Nicoud, 2008; Haruyama and Zhao, 2008; Dinopoulos and Unel, 2011; Perla et al., 2015; Sampson, 2016), all papers consider R&D as the engine of endogenous growth, but they cannot drop the assumption of symmetric countries. By replacing R&D with capital accumulation, we successfully formulate an asymmetric Melitz model, where a possible difference in countries' growth rates itself creates asymmetric adjustments of their productivity cutoffs and hence their growth rates toward convergence. On the policy side, our model supports unilateral trade liberalization as a way of promoting growth globally. Naito (2012) obtains a similar result in his two-country Acemoglu-Ventura model with a perfectly competitive intermediate good sector. This paper complements his work by showing that the growth-enhancing effect of unilateral trade liberalization is valid even under imperfect competition.

Because of its flexibility, our theoretical framework can be applied to problems involving asymmetric countries. For example, by replacing the iceberg import trade costs with the revenue-generating import tariffs, we can study the optimal tariff of each country. Compared with the static optimal tariff model of Felbermayr et al. (2013), the growth effect will pull down a country's optimal tariff in our model. Another example is to increase the number of countries to more than two. By doing this, we can see the effects of preferential trade liberalization on both member and nonmember countries. Even the nonmember countries will partly gain from preferential trade liberalization due to the growth effect. Although these extensions might cause some technical difficulties, it is worth trying them.

# Appendix A. Elasticities of $h_{ij}(\varphi_{ij})$ and $H_{ij}(\varphi_{ij})$

Let $h_{ij}(\varphi_{ij}) = (\widetilde{\varphi}_{ij}(\varphi_{ij})/\varphi_{ij})^{\sigma-1} - 1 = N_{ij}(\varphi_{ij})/\varphi_{ij}^{\sigma-1} - 1$, where $N_{ij}(\varphi_{ij}) \equiv \widetilde{\varphi}_{ij}(\varphi_{ij})^{\sigma-1} = \int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1}\mu_{ij}(\varphi|\varphi_{ij})d\varphi = (\int_{\varphi_{ij}}^{\infty} \varphi^{\sigma-1} g_i(\varphi)d\varphi)/(1 - G_i(\varphi_{ij}))$. Simple differentiation of $N_{ij}(\varphi_{ij})$ gives:

$$\begin{aligned} &N'_{ij} = [\varphi_{ij}^{\sigma-1} g_{ij}/(1 - G_{ij})](-1 + N_{ij}/\varphi_{ij}^{\sigma-1}) = \varphi_{ij}^{\sigma-1} g_{ij} h_{ij}/(1 - G_{ij}) > 0; \\ &g_{ij} \equiv g_i(\varphi_{ij}), G_{ij} \equiv G_i(\varphi_{ij}). \end{aligned}$$

Using this, $h'_{ij}(\varphi_{ij})$ is calculated as:

$$h'_{ij} = g_{ij}h_{ij}/(1-G_{ij}) - (h_{ij}+1)(\sigma-1)/\varphi_{ij}.$$

Differentiating $H_{ij}(\varphi_{ij}) = (1-G_i(\varphi_{ij}))h_{ij}(\varphi_{ij})$, and using the above expression, we obtain:

$$H'_{ij} = -(1-G_{ij})(h_{ij}+1)(\sigma-1)/\varphi_{ij} < 0.$$

Multiplying this by $\varphi_{ij}/H_{ij}(\varphi_{ij})$, the elasticity of $H_{ij}(\varphi_{ij})$ is obtained as:

$$H'_{ij}\varphi_{ij}/H_{ij} = -\eta_{ij} < 0; \eta_{ij} \equiv -d\ln H_{ij}/d\ln\varphi_{ij} \equiv [(h_{ij}+1)/h_{ij}](\sigma-1) > \sigma-1 > 0. \tag{A.1}$$

Finally, multiplying the expression for $h'_{ij}(\varphi_{ij})$ by $\varphi_{ij}/h_{ij}(\varphi_{ij})$, and using Eq. (A.1), the elasticity of $h_{ij}(\varphi_{ij})$ is given by:

$$h'_{ij}\varphi_{ij}/h_{ij} = -[\eta_{ij} - g_{ij}\varphi_{ij}/(1-G_{ij})].$$

# Appendix B. Proof of Proposition 1

First of all, logarithmically differentiating Eq. (15), and using Eqs. (16) and (21), we obtain:

$$\begin{aligned} &\widehat{M}_{ij} = -\nu_{ij}\widehat{\varphi}_{ij}, j \neq i; \\ &\nu_{ij} \equiv -\frac{d\ln M_{ij}}{d\ln\varphi_{ij}} \equiv \frac{g_{ij}\varphi_{ij}}{1-G_{ij}}\frac{H_{ij}+(1-G_{ij})(1-\beta_i)}{H_{ij}+1-G_{ij}} + \frac{g_{ii}\varphi_{ii}\beta_i}{H_{ii}+1-G_{ii}} > 0. \end{aligned} \tag{B.1}$$

Next, logarithmically differentiating Eq. (16), and using Eqs. (16) and (21), we obtain:

$$\begin{aligned} &\widehat{\beta}_i = -\chi_{ij}\widehat{\varphi}_{ij}, j \neq i; \\ &\chi_{ij} \equiv -\frac{d\ln\beta_i}{d\ln\varphi_{ij}} \equiv (1-\beta_i)\left[-\frac{(H'_{ij}-g_{ij})\varphi_{ij}}{H_{ij}+1-G_{ij}}\right] + \beta_i\left[-\frac{(H'_{ii}-g_{ii})\varphi_{ii}}{H_{ii}+1-G_{ii}}\right] > 0. \end{aligned} \tag{B.2}$$

Moreover, since Eq. (A.1) implies that $-(H'_{ij}-g_{ij})\varphi_{ij}/(H_{ij}+1-G_{ij}) = \sigma-1+g_{ij}\varphi_{ij}/(H_{ij}+1-G_{ij})$, $\chi_{ij}$ is rewritten as:

$$\chi_{ij} = \sigma-1+(1-\beta_i)\frac{g_{ij}\varphi_{ij}}{H_{ij}+1-G_{ij}} + \beta_i\frac{g_{ii}\varphi_{ii}}{H_{ii}+1-G_{ii}} > \sigma-1 > 0.$$

Finally, considering that $E_i = P_iY_i = p_i^Y(C_i + \dot{K}_i + \delta_iK_i) = r_iK_i$ from Eqs. (1), (4), and (9), and remembering that $1/q_i = r_i/P_i$ from Eq. (19), the zero cutoff profit condition (6) for $j = i$ is rewritten as $[r_i/(\alpha\varphi_{ii}P_i)]^{1-\sigma} = \sigma f_{ii}$, or $1/q_i = (\sigma f_{ii})^{1/(1-\sigma)}\alpha\varphi_{ii}$. This immediately implies that:

$$\widehat{q}_i = -\widehat{\varphi}_{ii}. \tag{B.3}$$

Differentiating Eq. (19), and using Eq. (B.3), the amount of change in $\gamma_i$ is simply given by:

$$d\gamma_i = (1/q_i)(-\widehat{q}_i) = (1/q_i)\widehat{\varphi}_{ii}. \tag{B.4}$$

Eqs. (B.1), (B.2), and (B.4), together with Eq. (21), imply Proposition 1.

# Appendix C. Proof of Proposition 2

We first derive Eqs. (24) and (25). Logarithmically differentiating Eq. (17), and using Eqs. (21), (22), (23), and (B.2), $\widehat{r}_1$ is solved as:

$$\widehat{r}_1 = (1/A)\{-B\widehat{\kappa} + (\sigma-1)\{(1-\beta_1)[\chi_{21}(1-\beta_2)+\chi_{12}\beta_2]\widehat{\tau}_{21} - (1-\beta_2)[\chi_{12}(1-\beta_1)+\chi_{21}\beta_1]\widehat{\tau}_{12}\}\}; \tag{C.1}$$
$$B \equiv \chi_{12}(1-\beta_1)+\chi_{21}(1-\beta_2)-(\sigma-1)(1-\beta_1-\beta_2) > 0(\because \chi_{ij} > \sigma-1>0),$$
$$A \equiv \sigma[\chi_{12}(1-\beta_1)+\chi_{21}(1-\beta_2)]-(\sigma-1)(1-\beta_1-\beta_2) > B > 0.$$

Substituting Eq. (C.1) into $\sigma\widehat{r}_1+\widehat{\kappa}$, and noting that $A-\sigma B = (\sigma-1)^2(1-\beta_1-\beta_2)$, we obtain:

$$\begin{aligned}(\sigma\widehat{r}_1+\widehat{\kappa})/(\sigma-1) = (1/A)\{&(\sigma-1)(1-\beta_1-\beta_2)\widehat{\kappa}\\ &+\sigma\{(1-\beta_1)[\chi_{21}(1-\beta_2)+\chi_{12}\beta_2]\widehat{\tau}_{21} - (1-\beta_2)[\chi_{12}(1-\beta_1)+\chi_{21}\beta_1]\widehat{\tau}_{12}\}\}.\end{aligned} \tag{C.2}$$

Substituting Eq. (C.2) into Eqs. (22) and (23), and substituting them into Eq. (B.4), we obtain Eqs. (24) and (25).

We next study dynamics. With $\widehat{\tau}_{21} = \widehat{\tau}_{12} = 0$, Eqs. (24) and (25) give:

$$d\gamma_1 = -(\beta_1/q_1)[(\sigma-1)/A]\widehat{\kappa},$$
$$d\gamma_2 = (\beta_2/q_2)[(\sigma-1)/A]\widehat{\kappa}.$$

Substituting them into the differentiated form of Eq. (20), and noting that $\dot{\kappa}/\kappa = d\ln\kappa/dt$ and $\widehat{\kappa} = d\ln\kappa$, we obtain:

$$d(d\ln\kappa/dt)/d\ln\kappa = -(\beta_1/q_1+\beta_2/q_2)[(\sigma-1)/A] < 0.$$

This implies that $\ln\kappa^*$ determined by $d\ln\kappa/dt = 0$ is unique if exists. Also, since $d\ln\kappa/dt > 0$ if and only if $\ln\kappa < \ln\kappa^*$, a BGP is globally stable.

To ensure existence, we first find the lower and upper bounds of $\ln\kappa$. From Eqs. (21), (22), (23), and (C.2), $\ln(\varphi_{12}/\varphi_{11})$ is increasing, whereas $\ln(\varphi_{21}/\varphi_{22})$ is decreasing, in $\ln\kappa$ regardless of the sign of $1-\beta_1-\beta_2$. Considering the assumption that $\varphi_{12}/\varphi_{11} > 1$ and $\varphi_{21}/\varphi_{22} > 1$, $\ln\underline{\kappa}$, the lower bound of $\ln\kappa$, is determined by $\varphi_{12}/\varphi_{11} = 1$, or $\ln(\varphi_{12}/\varphi_{11}) = 0$. Similarly, $\ln\overline{\kappa}$, the upper bound of $\ln\kappa$, is determined by $\varphi_{21}/\varphi_{22} = 1$, or $\ln(\varphi_{21}/\varphi_{22}) = 0$. Then there exists $\ln\kappa^* \in (\ln\underline{\kappa}, \ln\overline{\kappa})$ if $d\ln\kappa/dt = \gamma_1-\gamma_2 > 0$ at $\ln\kappa = \ln\underline{\kappa}$ and $d\ln\kappa/dt = \gamma_1-\gamma_2 < 0$ at $\ln\kappa = \ln\overline{\kappa} > \ln\underline{\kappa}$.

# References

[1] Acemoglu, D., Ventura, J., 2002. The world income distribution. Quarterly Journal of Economics 117, 659–694.

[2] Anderson, J. E., Wincoop, E. v., 2004. Trade costs. Journal of Economic Literature 42, 691–751.

[3] Arkolakis, C., Costinot, A., Rodríguez-Clare, A., 2012. New trade models, same old gains? American Economic Review 102, 94–130.

[4] Baldwin, R. E., Robert-Nicoud, F., 2008. Trade and growth with heterogeneous firms. Journal of International Economics 74, 21–34.

[5] Bollard, A., Klenow, P. J., Li, H., 2014. Entry costs rise with development. unpublished.

[6] Caselli, F., 2005. Accounting for cross-country income differences, in: Aghion, P., Durlauf, S. N. (Eds.), Handbook of Economic Growth, Vol. 1A. North-Holland, Amsterdam, pp. 679–741.

[7] Demidova, S., Rodríguez-Clare, A., 2013. The simple analytics of the Melitz model in a small economy. Journal of International Economics 90, 266–272.

[8] Dinopoulos, E., Unel, B., 2011. Quality heterogeneity and global economic growth. European Economic Review 55, 595–612.

[9] Eaton, J., Kortum, S., 2002. Technology, geography, and trade. Econometrica 70, 1741–1779.

[10] Estevadeordal, A., Taylor, A. M., 2013. Is the Washington Consensus dead? growth, openness, and the Great Liberalization, 1970s-2000s. Review of Economics and Statistics 95, 1669–1690.

[11] Felbermayr, G., Jung, B., Larch, M., 2013. Optimal tariffs, retaliation, and the welfare loss from tariff wars in the Melitz model. Journal of International Economics 89, 13–25.

[12] Grossman, G. M., Helpman, E., 1991. Innovation and Growth in the Global Economy. MIT Press, Cambridge, MA.

[13] Haruyama, T., Zhao, L., 2008. Trade and firm heterogeneity in a quality-ladder model of growth. Discussion Paper Series No. 223, RIEB, Kobe University.

[14] Helpman, E., Melitz, M. J., Yeaple, S. R., 2004. Export versus FDI with heterogeneous firms. American Economic Review 94, 300–316.

[15] Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71, 1695–1725.

[16] Melitz, M. J., Ottaviano, G. I. P., 2008. Market size, trade, and productivity. Review of Economic Studies 75, 295–316.

[17] Melitz, M. J., Redding, S. J. 2014. Heterogeneous firms and trade, in: Helpman, E., Rogoff, K., Gopinath, G. (Eds.), Handbook of International Economics, Volume 4. Elsevier B. V., Amsterdam, pp. 1–54.

[18] Naito, T., 2012. A Ricardian model of trade and growth with endogenous trade status. Journal of International Economics 87, 80–88.

[19] Perla, J., Tonetti, C., Waugh, M. E., 2015. Equilibrium technology diffusion, trade, and growth. unpublished.

[20] Rodríguez, F., Rodrik, D., 2000. Trade policy and economic growth: a skeptic's guide to the cross-national evidence. NBER Macroeconomics Annual 15, 261–325.

[21] Romalis, J., 2007. Market access, openness and growth. NBER Working Paper No. 13048.

[22] Sachs, J. D., Warner, A., 1995. Economic reform and the process of global integration. Brookings Papers on Economic Activity 1995, 1–118.

[23] Sampson, T., 2016. Dynamic selection: an idea flows theory of entry, trade, and growth. Quarterly Journal of Economics 131, 315–380.

[24] Segerstrom, P. S., Sugita, Y., 2015. A new way of solving the Melitz model using simple and intuitive diagrams. unpublished.

[25] Wacziarg, R., Welch, K. H., 2008. Trade liberalization and growth: new evidence. World Bank Economic Review 22, 187–231.

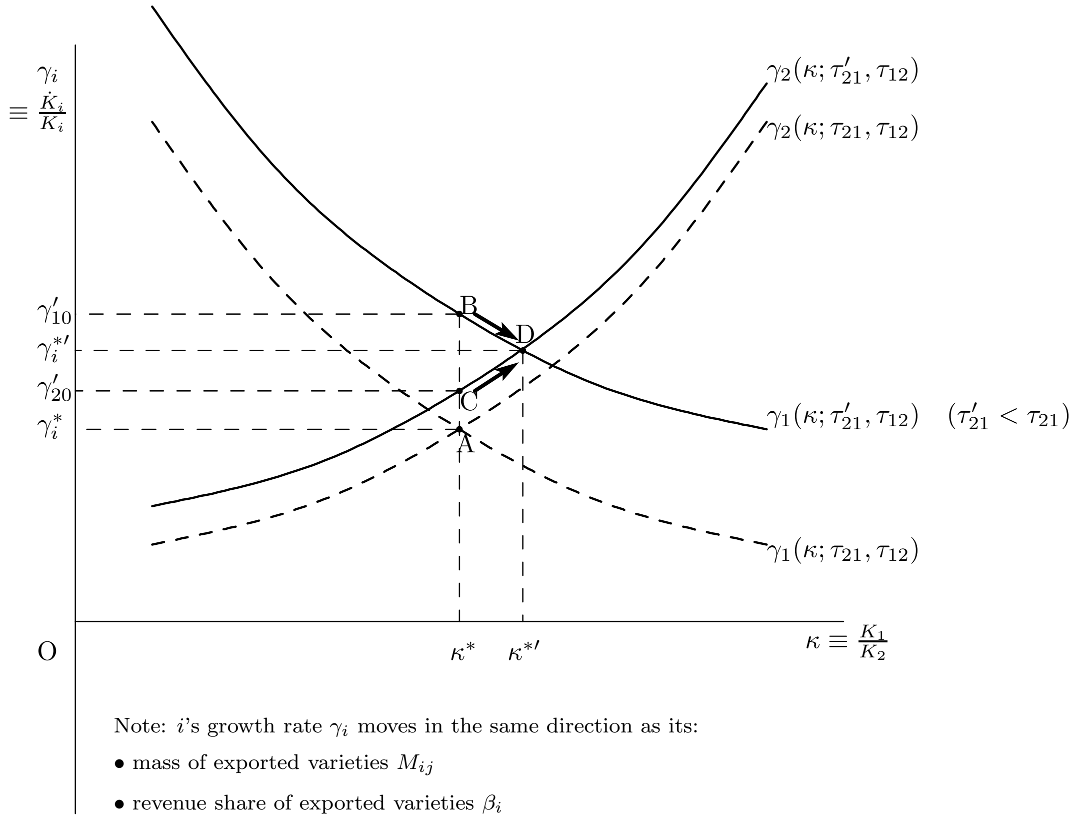


Fig. 1. Growth effects of a permanent fall in country 1's import trade cost $\tau_{21}$.