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Loyalty and Consumption: A CES representation

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Loyalty and Consumption: A CES representation[i]

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Abstract

This paper offers a new interpretation of the elasticity of substitution in the constant elasticity of substitution (CES) utility function under discrete choice and separability. We model an economy with one discrete choice goods group and one composite good under diverse consumers. The results from our theoretical analysis illustrate the relation between the diversity of loyalty of each good and goods demands. Moreover, the origin of the elasticity of substitution of the CES utility function is described based on our assumptions. According to our results, the power index of the CES utility function does not change even if the diversity of loyalty differs by each good. On the other hand, coefficients of X_i^{σ} vary according to each good's attractiveness. We also consider the production under this economy, and find that an increase in productivity leads to a decrease in price. This effect is the same as the standard Melitz model (monopolistic competition).

Keywords: Discrete choice, Separability, Demand aggregation
JEL classification: D11; E20; E21

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1 Introduction

The theoretical and empirical analysis of consumer behavior has a long and rich history in economics and econometrics, and the utility function plays one of the most key role to describe micro- and macroeconomic consumer behavior. One of the most standard methodology to assume the representative consumer and its functional type of the utility functions, most typically a CES-type utility function (see for example Dixit and Stiglitz (1977)). But apart from analytic convenience, is there any justification for these assumptions? Also, is it really reasonable to assume the elasticity of substitution as constant across all goods? In this paper, we explore the answers to these questions by considering the micro-foundation of the CES utility function under the separability and discrete choice.

To begin with, we set our first focus on the allocation of the total expenditure; the separable problem. In principle, each consumer has to deal with the problem to allocate his/her income between saving and consumption, or purchasing durable and non-durable goods simultaneously. All of the parts of this allocation problem may interact, and therefore the change in future wage may cause the change in current saving plan or the purchase of the durable goods. However, if we allow all interaction at the same time, the allocation problem can not be solved because of its complexity and therefore the simplification is required, either by aggregation or separation. The separation of the decision making leads that the ultimate determinants like assets, wage rates, prices, interest rates are related to the total expenditure, but not to each group expenditures directly. An importance for detecting the separability for certain goods group to examine the structure of the utility function was recognized for a long time. It has its origin in Stotz (1957), Gorman (1959), Goldman and Uzawa (1964) and other related works in the same literature, and related developments are also available (see, for example, Deaton and Muellbauer (1980)).

Another focus shall be set in the probabilistic consumer choice and the discrete choice, especially a brands of commodity purchases. The major reason to set the focus is simple; after allocating the expenditure, consumer have only to consider the brand to be purchased, and most of goods in the real world is literally discrete and probabilistic. The probabilistic consumer theory, or the probabilistic choice system, describes the observable distribution of demands by a population of consumers, and assume the hypothesis of random preference maximization which postulates that the distribution of demands in a population is the results of individual preference maximization (see McFadden (1981) for details). The demand for differentiated products, discrete choice models and the characteristics approach (see for example Lancaster (1966)) are connected by Anderson et. al. (1989) and it was revealed that these literature has some common part both in technical and conceptual aspects.

Recent developments treat these two different literature simultaneously, like considering the separability under the discrete choice (see, for example, Smith et. al. (2010)). In this paper, we consider the aggregated demand under the separability and the discrete choice with micro-foundation. The reasons for considering discreteness under the separability could be summarized as the following two points. First point is that the introduction of this model enables us to consider the distribution of attractiveness of each goods, especially the relation between the diversity of demand of each consumer and the total demand of its good. In the real economy, the loyalty, or attraction consumers feel, for certain goods differs by consumers, and its difference can be described by a distribution. According to our result, the shape of the distribution of consumer's loyalty for each goods is essential for describing the demand of goods, and maintaining or increasing its loyalty plays key role for the better profits of firms. As a second point, this model also reveals another interpretation of the elasticity constant σ of the CES utility function with micro foundation, which is generally set as a deep paameter. Under our assumption, the elasticity constant σ has a micro foundation and has a meaning which closely relates to the distribution of loyalties.

In our model, each consumer purchases a product which is most attractive for him/her. In such case, the firm has to increase consumers who like their product the best and this may lead to the market strategy, i.e., customer segmentation, promotion strategy, etc. Under such literature, a strategy to provide a product which offers high loyalty only for certain consumer segment, but not for other usual segment may be able to provide better profit for certain market circumstances. In addition, this theoretical framework also enables us to analyze firm's strategies to increase R&D expenditure to achieve product innovation, or disruptive innovation which lead to rapid increase in loyalty and better profit.

The reminder of the paper is organized as follows: Section 2 firstly outline our model by defining micro foundations and introducing several approximations, followed by the application of the Houthakker (1955) into our model. Section 3 shows brief numerical calculation results for reference and section 4 concludes.

2 The Model

2.1 Definition of the Variables

Let us construct our economic model. In this paper, we first start from focusing a certain goods group with considering the separability. Let x_{iA} be m dimensional sub-vector of the consumption vector x_i of consumer i so that $x_i = (x_{iA}, x_{i\bar{A}})$. x_{iA} is then said to be strongly *separable* if the utility function takes in the form

$$u_i = f_i\left(u_{iA}(x_{iA}) + u_{i\bar{A}}(x_{i\bar{A}})\right) \tag{2.1}$$

where $u_{iA}(x_{iA})$ is the sub-utility function associated with x_{iA}, and f is some monotone increasing function. The consumer i chooses a good under the consumption vector x_i with the discrete choice, i.e., each consumer purchases one unit of the commodity which offers the greatest utility. In this case, $u_{iA}(x_{iA})$ can be written in the form;

$$u_{iA}(x_{iA}) = max\left[u_{iA1},\ u_{iA2},\ ...,\ u_{iAm}\right] \tag{2.2}$$

where u_{iAj} is the amount of utility consumer i obtains when the consumer purchases good j under the commodity subgroup.

If we consider a distribution $\varphi(u_{A1},\ u_{A2}, ...,\ u_{Am})$ of utility values of each goods in m dimensional phase space for all consumers, the potential demand of good j could be characterized without any budget constraints as;

$$X_j = \int_{u_j > u_k\ for\ \forall k \neq j} \varphi(u_{A1},\ u_{A2}, ...,\ u_{Am}) du_{A1} du_{A2} ... du_{Am} \tag{2.3}$$

Here we define the "potential demand" as the demand achieved when there is no budget constraint in the economy. However, in the real economy, we can not neglect the effect of pricing and budget constraint for each good and consumer.

2.2 Economy with 1 Goods Subgroup and a Composite Good

For the simplicity, let us consider the economy with one discrete choice goods subgroup x_{iA} $(m = 2)$ and one composite good. The effect of pricing can be taken into account by substitution between discrete goods and continuous good in case of the strong separability. The decision for purchasing good A1 or A2 can be described as follows;

Proposition 1. *Under the strong separability, the consumer i purchases good A1 if and only if;*
(1-1) The utility for purchasing the good A1 is greater than the utility for purchasing a composite good additionally, and

(1-2) The utility for purchasing good A1 is greater than the sum of (a) the utility purchasing good A2 and (b) the utility for purchasing a composite good with reserved money.

Proof. Firstly, the strong separability is generally defined as follows in this economy.

$$u(x_{iA1}, x_{iA2}, x_{iN}) = u(x_{iA1}, x_{iA2}) + u(x_{iN}).$$

Also, as the goods subgroup $x_{iA}$is discrete choice goods group, each consumer basically chooses one good from goods subgroup (A1, A2). However, in this economy, we also allow not to purchase anything from goods subgroup (A1, A2) and use all budget for purchasing the composite good for more generalization.

To describe these condition in equations, firstly let us define assume valuables. p_{A1}, p_{A2} and p_N are prices of good A1, A2 and a composite good, and ω_i is a budget for the consumer i. Also, the utility for purchasing q_{iN} amount of the composite good is described by $u_{iN}(q_{iN})$ for each consumer i. Then, the utility for purchasing (i) good A1, (ii) good A2 and (iii) purchasing nothing from goods subgroup (A1, A2) can be described as follows;

$$\begin{cases} u_{iA1} + u_{iN}(\omega_i - p_{A1}/p_N) & (i) \\ u_{iA2} + u_{iN}(\omega_i - p_{A2}/p_N) & (ii) \\ u_{iN}(\omega_i/p_N) & (iii) \end{cases} \tag{2.4}$$

According to the discrete choice model, the good A1 will be purchased only in the case that the utility in case of (i) is larger than that of (ii) and (iii), and this leads to the descriptions in the **Proposition 1.**

$$\begin{array}{llll} (1-1) & u_{iA1} & +u_{iN}(\omega_i - p_{A1}/p_N) > & u_{iN}(\omega_i/p_N) \\ (1-2) & u_{iA1} & +u_{iN}(\omega_i - p_{A1}/p_N) > & u_{iA2} + u_{iN}(\omega_i - p_{A2}/p_N) \end{array} \tag{2.5}$$

Also, we need a strong assumption regarding the income effect to proceed calculations.

Assumption 1. *The utility function for the composite good $u_{iN}(q_N)$ is same for all consumers in the whole economy, i.e., $u_{iN}(q_{iN}) = u_N(q_{iN})$ for $^\forall i$.*

These two assumptions lead us to describe the actual demand of good A1 as

$$X_1 = \int_0^\infty \rho(\omega)d\omega \int_{u_N(\omega/p_N) - u_N(\omega - p_{A1}/p_N)}^\infty du_{A1} \int_0^{u_{A1} + u_N(\omega - p_{A1}/p_N) - u_N(\omega - p_{A2}/p_N)} \varphi(u_{A1},\ u_{A2}) du_{A2} \tag{2.6}$$

where $\rho(\omega)$ is the income distribution in this economy, and the demand for the good 2 can be described symmetrically. Here we define $\delta_m(\omega, p_{Am}, p_N)$ as $u_N(\omega/p_N) - u_N(\omega - p_{Am}/p_N)$ and $\delta_{mn}(\omega, p_{Am}, p_{An}, p_N)$ as $-u_N(\omega - p_{Am}/p_N) + u_N(\omega - p_{An}/p_N)$ $(= -\delta_{nm}(\omega, p_{An}, p_{Am}, p_N))$ to describe equations simply.

The schematic image of the effect of introducing the pricing into this model is described in the Figure 2.2. Next we consider the whole utility in this economy. By using the distribution function $\varphi(u_1,\ u_2)$, the utility of whole economy could be described as;

$$\begin{array}{rcl} U & = & \int_0^\infty \rho(\omega)d\omega \int_{\delta_1}^\infty u_{A1} du_{A1} \int_0^{u_{A1} - \delta_{12}} \varphi(u_{A1},\ u_{A2}) du_{A2} \\ & + & \int_0^\infty \rho(\omega)d\omega \int_{\delta_2}^\infty u_{A2} du_{A2} \int_0^{u_{A2} - \delta_{21}} \varphi(u_{A1},\ u_{A2}) du_{A1} \end{array} \tag{2.7}$$

As we can describe demand of good i (X_i) and utility of whole economy U in terms of φ, we can specify utility function $U(X_1,\ X_2)$ by assuming some functional shape of φ.

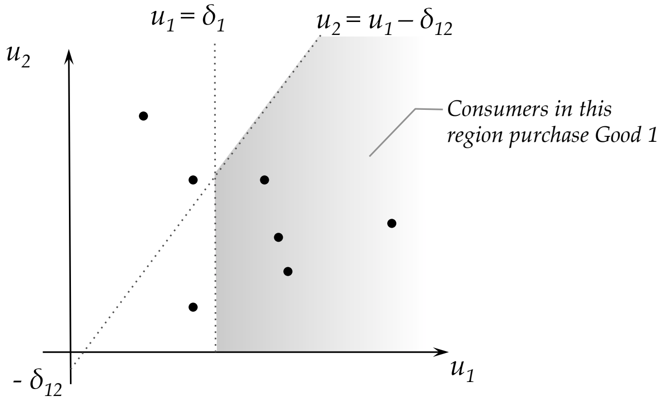

Figure 2.1: Distribution of Utility Values and Product Choice

2.3 CES

One of the most standard, frequently used functional type of the utility function in the macro economics would be the CES function. Therefore, in this section, we introduce the distribution function φ to retrieve aggregated utility function in the CES form as to be power function, just as assumed in Houthakker (1955).

Proposition 2. *The utility for holding discrete choice goods is described in CES type utility function with aggregated goods demands under the following Assumption 2-5.*

Proof. Firstly, let us assume the functional type of the distribution function $\varphi(u_1,\, u_2)$.

Assumption 2. *The distribution function $\varphi(u_1,\, u_2)$ follows the functional form:*

$$\varphi(u_{A1},\, u_{A2}) \quad = \begin{cases} A\,(u_{A1}+d)^{-\alpha}\left(\frac{u_{A2}+d+\delta_{12}}{u_{A1}+d}\right)^{\beta_1} & (u_{A1}>\delta_1,\ u_{A2}<u_{A1}+\delta_{12},\ \alpha>0,\ \beta_1>0) \\ \\ A\,(u_{A2}+d+\delta_{12})^{-\alpha}\left(\frac{u_{A1}+d}{u_{A2}+d+\delta_{12}}\right)^{\beta_2} & (u_{A2}>\delta_2,\ u_{A2}>u_{A1}+\delta_{12},\ \alpha>0,\ \beta_2>0) \end{cases} . \tag{2.8}$$

Assumption 3. *Income of all consumers are the same across the economy, i.e., $\rho(\omega) = D(\omega-\omega_0)$ where $D(\omega-\omega_0)$ is Dirac's delta-function.*

Under these assumptions, the demand of good m ($m = 1, 2$) could be straightly shown as;

$$
\begin{aligned}
X_1 &= A\int_0^\infty \rho(\omega)d\omega \int_{\delta_1}^\infty du_{A1}\int_0^{u_{A1}-\delta_{12}} du_{A2}\,(u_{A1}+d)^{-\alpha}\left(\tfrac{u_{A2}+d+\delta_{12}}{u_{A1}+d}\right)^{\beta_1}\\
&= \tfrac{A}{1+\beta_1}\left(\delta_1+d\right)^{-\alpha+2}\left\{\tfrac{1}{\alpha-2}-\tfrac{1}{\alpha+\beta_1-1}\left(\tfrac{d+\delta_{12}}{d+\delta_1}\right)^{1+\beta_1}\right\}\\
X_2 &= A\int_0^\infty \rho(\omega)d\omega \int_{\delta_2}^\infty du_{A2}\int_0^{u_{A2}+\delta_{12}} du_{A1}\,(u_{A2}+d+\delta_{12})^{-\alpha}\left(\tfrac{u_{A1}+d}{u_{A2}+d+\delta_{12}}\right)^{\beta_2}\\
&= \tfrac{A}{1+\beta_2}\left(\delta_2+\delta_{12}+d\right)^{-\alpha+2}\left\{\tfrac{1}{\alpha-2}-\tfrac{1}{\alpha+\beta_2-1}\left(\tfrac{d}{d+\delta_2+\delta_{12}}\right)^{1+\beta_2}\right\}
\end{aligned}
\tag{2.9}
$$

The normalization term A can be calculated by $X_1|_{\delta_1=\delta_{12}}+X_2|_{\delta_2=-\delta_{12}}=1$ and this leads to;

$$
\frac{1}{A}=\frac{(\delta_{12}+d)^{-\alpha+2}}{(\alpha-2)(\alpha+\beta_1-1)}+\frac{d^{-\alpha+2}}{(\alpha-2)(\alpha+\beta_2-1)} \tag{2.10}
$$

Also, the utility of the whole economy for purchasing discrete goods (defined as U_{d0}) could be also shown as;

$$
\begin{aligned}
U_{d0} &= A\sum_{m=1,2,\,m\neq n}\int_0^\infty \rho(\omega)d\omega\int_{\delta_m}^\infty du_{Am}\left[u_{Am}\int_0^{u_{Am}-\delta_{mn}}\varphi(u_{A1},\,u_{A2})du_{An}\right]\\
&= X_1|_{\alpha\to\alpha-1}+X_2|_{\alpha\to\alpha-1}-d(X_1+X_2)
\end{aligned}
\tag{2.11}
$$

The relation between the aggregated demand and utility in the whole economy can be obtained by eliminating the function δ (δ_1, δ_2 and δ_{12}) from the equation (2.9) and (2.11). Here let us employ two more assumptions to make equations simpler.

Assumption 4. *$d,\,\delta_{12}\ll\delta_1,\,\delta_2$ holds with good approximation.*

Under this assumption, the integration in (2.9) and (2.10) can be described simply as;

$$
\begin{aligned}
X_m &\sim \tfrac{A}{(1+\beta_m)(\alpha-2)}\delta_m^{-\alpha+2},\ (m=1,2)\\
U_{d0} &\sim \tfrac{A}{(1+\beta_1)(\alpha-3)}\delta_1^{-\alpha+3}+\tfrac{A}{(1+\beta_2)(\alpha-3)}\delta_2^{-\alpha+3}
\end{aligned}
$$

Deleting δ_m from X_m and U_{d0} to obtain the relation between aggregated demands and the utility for the discrete choice goods in the whole economy as:

$$
U_{d0}=A^{\frac{1}{\alpha-2}}\frac{(\alpha-2)^\sigma}{\alpha-3}\left\{(1+\beta_1)^{\frac{-1}{\alpha-2}}X_1^\sigma+(1+\beta_2)^{\frac{-1}{\alpha-2}}X_2^\sigma\right\} \tag{2.12}
$$

where $\sigma={}^{\alpha-3}/_{\alpha-2}$. Furthermore, let us assume further condition for the exponent in (2.8).

Assumption 5. *The exponent of the utility function defined in the equation (2.8) is the same across the good 1 and 2, i.e., $\beta_1=\beta_2=\beta$.*

The CES functional form can be obtained if we employ the **Assumption 5.** and re-define the utility in whole economy U_d as $U_d=U_{d0}^{1/\sigma}$.

2.4 Meaning of Assumptions for the CES

The necessary assumptions for the CES utility function to be a better approximation are that i) the distribution of consumer's utility can be well approximated in (2.8), ii) the price of each good itself is much higher than the price differences ($\delta_{12} \ll \delta_1, \delta_2$), and iii) the shape of the distribution of utilities are almost similar among goods in the choice set ($\beta_1 = \beta_2 = \beta$). The first assumption has to be confirmed with some marketing technology like a conjoint analysis. If the utilities for a certain good distributes in power low among potential consumers, this assumption can be regarded as rational for discussing approximations. This assumption reflects features of consumers under the discrete choice model, while the assumption ii) and iii) reflect features of goods of our interest. The assumption ii) and iii) becomes reasonable if the loyalties of goods are almost the same across the goods sub-vector. In other words, this assumption may not hold if there are appreciable product differentiation, especially in the field of the monopolistic competition with various goods loyalties.

3 Profit Maximization and Pricing under This Economy

3.1 Production under This Economy

Let us assume that each commodity is produced by one firm. Each firm attempts to maximize its profit under the given production function.

Profit Maximization
Firstly, assume the production by firm m as q_m. Also, the cost function to produce q_m amount of good m as cq_m^γ. Then, the firm m faces the following profit maximization problem;

$$\underset{p_m}{max}\ \pi_m = \underset{p_m}{max}\ \{p_m q_m - cq_m^\gamma\} \tag{3.1}$$

Market Clearing Condition
If we assume the good 1 and 2 to be non-storable goods, the market clearing condition for each goods is simply described as $X_m = q_m$.

Functional Type of the δ_1
If we employ all assumptions except for the **Assumption 5**, the profit maximization condition of the firm 1 can be written as

$$\frac{\partial}{\partial p_1}\pi_1 = q_1 + p_1\frac{\partial q_1}{\partial p_1} - c\gamma q_1^{\gamma-1}\frac{\partial q_1}{\partial p_1} \tag{3.2}$$

where

$$q_1 = \frac{A}{(1+\beta_1)(\alpha-2)}\delta_1(p_1)^{-\alpha+2}. \tag{3.3}$$

To calculate $\frac{\partial q_1}{\partial p_1}$, we need to assume the concrete functional type of the $\delta_1(p_1)$. As defined in the section 2.2., $\delta_1(p_1) = u_N({}^{\omega}/_{p_N}) - u_N({}^{\omega - p_{A1}}/_{p_N})$. The right hand side of this equation means the difference of the level of utility when the amount of the composite good is ${}^{\omega}/_{p_N}$ and ${}^{\omega-p_{A1}}/_{p_N}$. To consider its perceptual difference, it might be worth to employ the famous literature constructed by Weber and Fechner. If we employ the famous analogy, the utility could be described in the logarithm form.

Assumption 6. *$\delta_1(p_1)$ is described in the form $B \times log\frac{\omega}{\omega - p_{A1}}$ with some constant B.*

Under the **Assumption 1-4.** and the **Assumption 6.**, $\frac{\partial q_1}{\partial p_1}$ can be calculated as

$$\frac{\partial q_1}{\partial p_1} = \frac{-A}{(1+\beta_1)(\omega - p_{A1})}\left(B \times log\frac{\omega}{\omega - p_{A1}}\right)^{-\alpha+1}. \tag{3.4}$$

By plugging 3.3 and 3.4 into 3.2, this maximization condition is solved and equilibrium price p_1^* can be calculated.

3.2 Numerical Calculation

Although it is possible to calculate the general equilibrium with these assumptions, theoretical analysis may turn out to be complicated and hard to understand its functional features. To help its understandings on actual relations in this economy, it would be useful to calculate concretely with certain parameter value. The table 1 is the relation between parameters related to the goods loyalty (β_1, β_2) and equilibrium value (p_1^*, X_1^* and π_1^*) under the parameters set $\alpha = 3.1,\ \omega = 500,\ p_2 = 50,\ c = 10,\ \gamma = 0.8,\ B = 1.$

In this parameter setting, the equilibrium price p_1^* becomes lower when the loyalty of good 1 is higher ($\beta_1 = 1$). This means that if the productivity of the firm 1 is higher, the equilibrium price to maximize the profit becomes lower. This result is the same as that of the general monopolistic competition model like Melitz (2003). In the standard Melitz model, the demand of good m in industry l is described $X_{lm} = A_l p_{lm}^{-\frac{1}{1-\rho_l}}$ and firm's profit maximization condition yields $D_{lm} = p_{lm}X_{lm} = \rho_l^{\frac{\rho_l}{1-\rho_l}} A_l^{\frac{1}{1-\rho_l}} \omega_l^{-\frac{\rho_l}{1-\rho_l}} \theta_{lm}^{\frac{\rho_l}{1-\rho_l}}$ where $A_l = \beta_l Y P_l^{-\frac{\rho_l}{1-\rho_l}}$, ω_l is a wage and θ_{lm} is a productivity. Using these 2 relations to obtain $p_{lm}^{-\frac{\rho_l}{1-\rho_l}} = \rho_l^{\frac{\rho_l}{1-\rho_l}} A_l^{\frac{\rho_l}{1-\rho_l}} \omega_l^{-\frac{\rho_l}{1-\rho_l}} \theta_{lm}^{\frac{\rho_l}{1-\rho_l}}$, and as $\rho_l^{\frac{\rho_l}{1-\rho_l}} A_l^{\frac{\rho_l}{1-\rho_l}} \omega_l^{-\frac{\rho_l}{1-\rho_l}}$ is the same in given industry l, we can simply describe the relation of price and productivity as $p_{lm}^{-1} = c\theta_{lm}$. Under this condition, the increase in productivity leads the decrease in price.

Table 1: Relation of Equilibrium Price and Profit

β_1	1	1	5	5
β_2	1	5	1	5
p_1^*	92	87	137	129
X_1^*	5.3×10^{-3}	7.9×10^{-3}	1.6×10^{-3}	2.8×10^{-3}
π_1^*	1.2×10^{-1}	1.7×10^{-1}	5.7×10^{-2}	1.0×10^{-1}

4 Discussions

One of the major feature of our model deriving such results lies in the budget constraint in this economy. In case of the 2 stage budgeting, the budget allocated for the goods group A is regarded as rigid and determined without considering goods loyalties. On the other hand, our economy allows each consumer to consider flexible budgeting.

In this paper we established new methodology for deriving CES utility function with micro-foundation. As is already pointed out, there are 5 major assumptions required to retrieve CES utility function, and these assumptions would not be valid at least in case of the market with various level of goods loyalties. Taking this result into account, the validity to expand the CES utility function into the whole economy may change by industry, i.e., an industry with poor differentiate goods may obtain good approximation by CES utility function, however, an industry with strong differentiation may not. In our model the product differentiation affects mainly to the coefficient of X_1^σ just described in the equation (2.12). In case of the Dixit-Stiglitz lite (Dixit and Stiglitz (1977)), utility of the representative consumer is described in CES based form for the whole goods in the economy. However, if this CES assumption may not hold for several goods subgroups or industries, this may lead the model to differ from the reality as a macro economy.

Also, in our model, each consumer purchases a product which is most attractive for him/her. In such case, the firm has to increase number of consumers who like their product the best and this may lead to the market strategy of the firm, i.e., customer segmentation, promotion strategy, etc. Under such literature, we may be able to analyze that a strategy to provide a product which offers high loyalty for certain consumer segment, but not for other segment (like a good for only the professional, or say, the geek) might be able to provide better profit under certain market circumstances. In addition, this theoretical framework also enables us to analyze firm's strategies to increase R&D expenditure to achieve product innovation, or disruptive innovation which lead to rapid decrease in β.

Lastly, we would like to point out that our approach allows us to provide new methodology to clarify assumptions to retrieve the CES utility function, and also hope that this theoretical framework encourages future applications in empirical works.

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