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# Firm-to-Firm Trade: Imports, exports, and the labor market[*]

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Abstract

Customs data and firm-level production data reveal both the heterogeneity and the granularity of individual buyers and sellers. We seek to capture these firm-level features in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production that is broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

*Keywords*: Firm, Intermediate, Outsource, Search, Trade

*JEL Classification*: D2, F1, L2



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# 1 Introduction

International economists have begun to exploit data generated by customs records, which describe the finest unit of trade transactions. These records expose the activity of individual buyers and sellers that underlie the aggregate trade flows, which had been the object of earlier quantitative analysis in international trade.

Some striking regularities emerge. One that has received attention previously (e.g., Eaton, Kortum, and Kramarz (2011), Eaton, Kortum, and Sotelo (2013)) is the tight connection between market size, market share, and the number of individual exporters. Figures 1 and 2 illustrate this relationship for French manufacturing exports to other members of the European Union (EU) in 2005.[1] Figure 1 reports a destination's market size, as measured by its manufacturing absorption, on the $x$-axis, and the number of French manufacturing firms selling there, on the $y$-axis. The slope of 0.52 (standard error 0.064) is well above zero but also well below one. Figure 2 repeats the exercise, only dividing the number of exporters by French market share in that destination. The relationship is tighter, with a slope of around 0.49 (standard error 0.045).

While previous work has documented regularities among exporters, the data reveal some interesting patterns among importers as well. Figure 3 reports the average number of buyers per French exporter across the other EU members, again with market size on the $x$-axis. The relationship is positive, but with a slope of only 0.20 (standard error 0.051).

While international trade theory has now incorporated exporter heterogeneity, most analysis has continued to treat demand as monolithic. But, as Figure 3 reveals, the average exporter

[1] Data sources are described in Appendix A.

has only a small number of buyers. Moreover, there is a lot of heterogeneity across exporters in terms of their number of buyers. Table 1 reports on the customers of French exporters in four EU destinations of diverse size. Note that the modal number remains below five even in Germany, the largest EU market, but numbers at the top end soar into the hundreds.[2]

The theory has also taken a monolithic approach to modeling technology, with all firms in a sector employing factors and intermediate inputs in the same way. But the data reveal substantial heterogeneity with respect to inputs as well. Figure 4 portrays the distribution of the share of production labor in total costs (including the cost of intermediates) across French manufacturing firms (the $y$-axis show the percent of firms with a share of production labor less than or equal to the value shown on the $x$-axis).

We seek to capture both the heterogeneity and the granularity in individual buyer-seller relationships in a general equilibrium model that is also consistent with observations at the aggregate level. Our model is one of product trade through random meetings. Buyers, who may be households looking for final products or firms looking for inputs, connect with sellers randomly. At the firm level, the model generates predictions for imports, exports, and the share of labor in production broadly consistent with observations on French manufacturers. At the aggregate level, firm-to-firm trade determines bilateral trade shares as well as labor's share of output in each country.

In contrast to standard production theory, we model a firm's technology as combining a set of tasks. Each task can be performed by labor, which can be of different types appropriate for different tasks. But labor competes with intermediate goods produced by other firms which

[2] Our findings in Table 1 and Figure 3 on buyers per firm match evidence from Norwegian exporters reported in Bernard, Moxnes, and Ultveit-Moe (2016), their Figures 1 and 2 in particular.

can also perform these tasks. Firms may thus look very different from one another in terms of their production structure, depending on the sellers of intermediate goods that they happen to encounter. A firm's cost in a market thus depends not only on its underlying efficiency, but also on the costs of its suppliers. An implication is that an aggregate change, such as a reduction in trade barriers, can reduce the share of labor in production by exposing producers to more and cheaper sources of supply.

Our model is complementary to recent work of Oberfield (2013) in which a producer's cost depends not only on its own efficiency but the efficiencies of its upstream suppliers. It is also complementary to recent work of Chaney (2014) and Eaton, Eslava, Jinkins, Krizan, and Tybout (2014), with trade the consequence of individual links formed between buyers and sellers over time. In order to embed the framework into general equilibrium, however, our analysis here remains static, more in line with the two-stage model of production in Bernard, Moxnes, and Ultveit-Moe (2016).[3] Our model also relates to Garetto (2013), in that firms and workers compete directly to provide inputs for firms.[4]

[3] Bernard, Moxnes, and Saito (2015) apply this model to micro data from Japan to evaluate the effects of a new high-speed train line on firms' supplier networks.

[4] In addition to the work already cited, our paper relates closely to a number of active areas. One is recent work on exports and the labor market, including Caliendo and Rossi-Hansberg (2012), Egger and Kreickemeier (2009), Felbermayr, Prat, and Schmerer (2008), Helpman, Itskhoki, and Redding (2010), and Hummels, Jørgenson, Munch, and Xiang (2014). Another is quantitative work focussing on firm-level imports, including Biscourp and Kramarz (2007), Blaum, Lelarge, and Peters (2014), Bricongne, Lionel, Gaulier, Taglioni, and Vicard (2012), Caliendo, Monte, and Rossi-Hansberg (2015), Frías, Kaplan, and Verhoogen (2009), Helpman, Itskhoki, Muendler, and Redding (2012), Irarrazabal, Moxnes, and Ulltveit-Moe (2013), Klein, Moser, and Urban (2010), Kramarz (2009), and Kramarz, Martin, and Mejean (2015). A third is other theories of networks or input-output interactions, including Acemoglu and Autor (2011), Acemoglu, Carvalho, Ozdaglar,

We proceed as follows. Section 2 develops our model. Section 3 analyzes its theoretical and quantitative implications for aggregate outcomes such as the distribution of wages. Section 4 turns to the model's firm-level implications. Section 5 concludes.

# 2 A Model of Production through Random Encounters

Consider a world with a set of $i = 1, 2, ..., \mathcal{N}$ countries. Each country has an endowment of $L_i^l$ workers of type $l = 1, 2, ..., L$.

## 2.1 Technology

A producer $j$ in country $i$ can make a quantity of output $Q_i(j)$ by combining a set of $k = 1, ..., K$ *tasks* according to the production function

$$Q_i(j) = z_i(j) \prod_{k=1}^{K} b_k^{-1} \left( \frac{m_{k,i}(j)}{\beta_k} \right)^{\beta_k}$$

where $z_i(j)$ is the overall efficiency of producer $j$, $m_{k,i}(j)$ is the input of task $k$, $b_k$ is a constant, and $\beta_k$ is the Cobb-Douglas share of task $k$ in production. The Cobb-Douglas parameters satisfy $\beta_k > 0$ and

$$\sum_{k=1}^{K} \beta_k = 1.$$

A task can be performed either by the unique type of labor appropriate for that task, denoted $l(k)$, or with an input produced by a firm. We allow $K \geq L$, so that one type of labor might be able to perform several different tasks. We denote the set of tasks that labor of type $l$ can perform as $\Omega_l$.

and Tahbaz-Salehi (2012), Lucas (2009), and Luttmer (2015).

Worker productivity performing a task for a given firm is $q_{k,i}(j)$. If the firm hires labor it pays the wage for workers of type $l(k)$. The producer also is in contact with a set of suppliers of an intermediate good that can also perform the task. From producer $j$'s perspective, labor and the available inputs are perfect substitutes for performing the task. Hence it chooses whatever performs the task at lowest cost.

We assume that producers can hire labor in a standard Walrasian labor market at the market wage $w_{k,i} = w_i^{l(k)}$. In finding intermediates, however, buyers match with only an integer number of potential suppliers, either because of search frictions or because only a handful of producers make an input appropriate for this particular firm. We could make various assumptions about the price at which the intermediate is available. Because it yields the simplest set of results, we assume Nash bargaining in which the buyer has all the bargaining power, so that the price is pushed down to unit cost.[5]

Let $c_{k,i}^{\min}(j)$ denote the lowest price available to firm $j$ for an intermediate to perform task $k$. The price it pays to perform task $k$ is thus:

$$c_{k,i}(j) = \min\left\{\frac{w_{k,i}}{q_{k,i}(j)}, c_{k,i}^{\min}(j)\right\}$$

and the firm's unit cost of delivering a unit of its output to destination $n$ is:

$$c_{ni}(j) = \frac{d_{ni}}{z_i(j)} \prod_{k=1}^{K} \left(\frac{c_{k,i}(j)^{\beta_k}}{b_k}\right). \tag{1}$$

where $d_{ni} \geq 1$ is the iceberg transport cost of delivering a unit of output from source $i$ to

[5] An implication is that there are no variable profits. Our model thus cannot accommodate fixed costs, either of market entry as in Melitz (2003) or in accessing markets for inputs, as in Antras et al. (2014). An alternative, which would allow for variable profits and hence fixed costs, is Bertrand pricing. While we found this alternative analytically tractable, we deemed the added complexity not worth the benefit.

destination $n$, with $d_{ii} = 1$ for all $i$. In order to derive a closed form solution we impose specific distributions for producer efficiency, the efficiency of labor in performing a task, and the distribution of the prices of intermediate inputs.

First, following Melitz (2003) and Chaney (2008), each country has a measure of potential producers. The measure of potential producers in country $i$ with efficiency $z_i(j) \geq z$ is:

$$\mu_i^Z(z) = T_i z^{-\theta}, \tag{2}$$

where $T_i \geq 0$ is a parameter reflecting the magnitude of country $i$'s endowments of technology and $\theta \geq 0$ their similarities.

Second, worker productivity performing a task for a given producer $q_{k,i}(j)$ is a random variable $Q$ drawn from the distribution:

$$F(q) = \Pr[Q \leq q] = e^{-q^{-\phi}}, \tag{3}$$

where $\phi \geq 0$ reflects the similarity of labor productivities across tasks and firms. For purposes that will become apparent below we restrict $\phi \leq \theta$.

Third, the measure of producers who can supply country $i$ at a unit cost below $c$ is given by:

$$\mu_i(c) = \Upsilon_i c^{\theta}, \tag{4}$$

where $\Upsilon_i \geq 0$. These suppliers could be located in country $i$ or anywhere else.

Our specifications of the heterogeneity in producer efficiency given in (2) and the distribution of labor productivity given in (3) are primitives of the model, with $T_i, \theta$, and $\phi$ exogenous parameters. We show below, however, that the resulting heterogeneity in unit costs $c$ given by (4) arises endogenously from our other assumptions, with $\Upsilon_i$ determined by underlying technology, labor market conditions, and access to intermediates in different countries of the world, as well as to trade barriers between countries.

## 2.2 Matching Buyers and Sellers

In contrast with standard Walrasian models, we assume that matching between buyers and sellers is random. Even though there are a continuum of possible sellers and buyers, an individual seller matches with only an integer number of potential buyers and an individual buyer matches with only an integer number of potential sellers. The matching literature (e.g., Mortenson and Pissarides, 1994) typically posits that in a market with more potential buyers and sellers, the likelihood of a match between any given potential buyer and potential seller is smaller.[6]

In our case, however, the measure of potential sellers implied by (4) is unbounded. But for a seller with unit cost $c$, the measure of sellers with unit cost below $c$ is always bounded. So instead we treat the likelihood of a match involving a seller with unit cost $c$ as limited by the measure of sellers with unit cost below $c$.

We thus posit that the intensity with which a seller with unit cost $c$ in country $n$ encounters a buyer seeking to fulfill purpose $k$ is:

$$e_{k,n}(c) = \lambda_{k,n}\mu_n(c)^{-\gamma}. \tag{5}$$

The key new parameters are $\lambda_{k,n}$, which governs how easy it is for a seller to come into contact

[6] Matching in our framework can be interpreted literally as coming into contact with each other, but it also could relate to the appropriateness of a seller's product for the buyer's purpose. In this sense we can think of products as differentiated not only by seller, but by buyer as well.

with a buyer for task $k$, and $\gamma$, which captures the extent to which lower cost sellers impede the ability of a seller to match with a buyer.

Aggregating across the measure of potential suppliers with different costs, the number of potential suppliers ("quotes") that a buyer receives for task $k$ with a price below $c$ is distributed Poisson with parameter

$$\begin{aligned}\rho_{k,n}(c) &= \int_0^c e_{k,n}(x)d\mu_n(x) \\ &= \theta\lambda_{k,n}\Upsilon_n^{1-\gamma}\int_0^c x^{\theta(1-\gamma)-1}dx \\ &= \frac{1}{1-\gamma}\lambda_{k,n}\Upsilon_n^{1-\gamma}c^{\theta(1-\gamma)}. \end{aligned} \tag{6}$$

where we require $\gamma < 1$. With this restriction this Poisson parameter grows arbitrarily large with $c$, so that many potential suppliers are available to serve any given buyer.

The firm can perform task $k$ at a cost below $c_k$ unless the cost of hiring workers directly and the lowest quote both exceed $c_k$. From the Poisson density, we know that with probability $\exp\left[-\rho_{k,i}(c_k)\right]$ the buyer will encounter no quotes below $c_k$. It will cost more than $c_k$ to hire workers to perform the task if $w_{k,i}/Q > c_k$, which occurs with probability $F(w_{k,i}/c_k)$. Since the two events are independent the distribution of the lowest cost to fulfill task $k$ is:

$$G_{k,i}(c_k) = 1 - F(w_{k,i}/c_k)e^{-\rho_{k,i}(c_k)}.$$

To work out the implications of this distribution for the resulting distribution of production costs, we restrict:

$$\gamma = \frac{\theta - \phi}{\theta}.$$

With this restriction, the parameter governing heterogeneity in the distribution of costs of intermediates is the same as the parameter governing heterogeneity in the distribution of worker efficiency (3) at a given task for a given buyer. In particular, the distribution of the cost to the buyer of fulfilling task $k$ becomes:

$$G_{k,i}(c_k) = 1 - e^{-\Xi_{k,i} c_k^{\phi}}, \tag{7}$$

where

$$\Xi_{k,i} = \nu_{k,i} + w_{k,i}^{-\phi} \tag{8}$$

and

$$\nu_{k,i} = \frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i^{\phi/\theta}. \tag{9}$$

With probability $\upsilon_{k,i} = w_{k,i}^{-\phi}/\Xi_{k,i}$ the buyer hires workers to perform task $k$ while with probability $1 - \upsilon_{k,i} = \nu_{k,i}/\Xi_{k,i}$ it purchases an intermediate from the lowest-cost supplier. Notice that these probabilities are independent of the unit cost $c$.

While $\upsilon_{k,i}$ is the probability that task $k$ is performed by labor in country $i$, since there are a continuum of producers, it is also the aggregate share of labor in performing task $k$ in country $i$.[7] The aggregate share of labor of type $l$ in total production costs is consequently:

$$\beta_i^l = \sum_{k \in \Omega_l} \beta_k \upsilon_{k,i}$$

and the overall labor share in production costs is:

$$\beta_i^L = \sum_l \beta_i^l.$$

Note that, even though our basic technology is Cobb-Douglas, the labor share depends on wages and other factors.

[7]Similarly, in Eaton and Kortum (2002) the probability $\pi_{ni}$ that destination $n$ buys a good from a source $i$ is also source $i$'s share in destination $n$'s spending.

We proceed by showing first how the cost measure (4) arises from our model of firm-to-firm trade. We then turn to consumer demand and then to intermediate demand before closing the model in general equilibrium.

## 2.3 Deriving the Cost Distribution

Each $c_k$ is distributed independently according to (7). From (2) and (1), the measure of potential producers from source $i$ that can deliver to destination $n$ at a unit cost below $c$ is:

$$\begin{aligned}
\mu_{ni}(c) &= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \int_0^{\infty} b_k^{\theta} c_k^{-\theta\beta_k} dG_{k,i}(c_k) \\
&= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \int_0^{\infty} b_k^{\theta} c_k^{-\theta\beta_k} \phi \Xi_{k,i} c_k^{\phi-1} \exp\left(-\Xi_{k,i} c_k^{\phi}\right) dc_k \\
&= T_i d_{ni}^{-\theta} c^{\theta} \prod_k \Xi_{k,i}^{\frac{\theta}{\phi}\beta_k} \\
&= T_i \Xi_i d_{ni}^{-\theta} c^{\theta} \qquad (10)
\end{aligned}$$

where:

$$\Xi_i = \prod_{k=1}^{K} (\Xi_{k,i})^{\frac{\theta}{\phi}\beta_k},$$

and we have defined:

$$b_k = \left[\Gamma(1 - \frac{\theta}{\phi}\beta_k)\right]^{-1/\theta}.$$

to eliminate the multiplicative constant emerging from integration. We require that parameter values satisfy $\frac{\theta}{\phi}\beta_k < 1$.

Aggregating across all sources of supply, the measure of potential producers that can deliver a good to market $n$ at a cost below $c$ is:

$$\mu_n(c) = \sum_{i=1}^{\mathcal{N}} \mu_{ni}(c) = \Upsilon_n c^{\theta}$$

where:

$$\Upsilon_n = \sum_i T_i \Xi_i d_{ni}^{-\theta}, \tag{11}$$

showing how the parameter $\Upsilon_n$ posited in (4) relates to deeper parameters of technology, search, and trade costs, as well as to wages, to which we turn below.

Substituting in (9), we can solve for the vector of $\Upsilon_n$ from the system of equations:

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left( \frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i^{\phi/\theta} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k} \tag{12}$$

for $n = 1, 2, ..., \mathcal{N}$. Given wages and exogenous parameters of the model, the $\Upsilon_n$ are thus the solution to the set of equations (12). Appendix B provides sufficient conditions for a unique solution to the $\Upsilon_n$'s and an iterative procedure to compute them.

The measure of potential producers from source $i$ with unit cost below $c$ in destination $n$ is $T_i \Xi_i d_{ni}^{-\theta} c^{\theta}$. The total measure of potential producers with unit cost below $c$ in $n$ is $\Upsilon_n c^{\theta}$. Hence the probability that a potential producer selling in $n$ with unit cost below $c$ is from $i$ is just:

$$\pi_{ni} = \frac{T_i \Xi_i d_{ni}^{-\theta}}{\sum_{i'} T_{i'} \Xi_{i'} d_{ni'}^{-\theta}} \tag{13}$$

regardless of $c$. Just as in Eaton and Kortum (2002), with our continuum of producers, in the aggregate $\pi_{ni}$ is the share of source $i$ in the purchases of destination $n$.

## 2.4 The Aggregate Production Function

Before finishing our specification of the model and turning to its solution, we take a moment to show how our assumptions about technology are consistent with an aggregate production function for output $Q_i$ of the form:

$$Q_i = \prod_{k=1}^{K} \left[ \tilde{\varphi} \left( L_{k,i} \right)^{\phi/(\phi+1)} + (1-\tilde{\varphi}) \left( I_{k,i} \right)^{\phi/(\phi+1)} \right]^{\beta_k(\phi+1)/\phi}, \tag{14}$$

where $L_{k,i}$ is the labor force employed in performing task $k$, $I_{k.i}$ are intermediates used for task $k$, and:

$$\tilde{\varphi} = \frac{1}{1+\varphi^{\phi/(1+\phi)}},$$

where:

$$\varphi = \Gamma(1+1/\phi).$$

To see this implication, note that, since the distribution of the price for an intermediate to perform task $k$ in country $i$ is:

$$H_{k,i}(p) = 1 - e^{-\nu_{k,i}p^{\phi}},$$

the average of such prices across firms in $i$ is:

$$\begin{aligned}
\bar{p}_{k,i} &= \int_0^\infty p dH_{k,i}(p) \\
&= \int_0^\infty p\phi\nu_{k,i}e^{-\nu_{k,i}p^{\phi}}p^{\phi-1}dp \\
&= \int_0^\infty \left(\frac{x}{\nu_{k,i}}\right)^{1/\phi} e^{-x}dx \\
&= \varphi \left(\nu_{k,i}\right)^{-1/\phi}.
\end{aligned}$$

We can then write the share in total production costs of type $k$ labor in performing task $k$ as:

$$\begin{aligned}
\beta^{L,k} &= \beta_k \upsilon_{k,i} \\
&= \beta_k \frac{w_{k,i}^{-\phi}}{\nu_{k,i} + w_{k,i}^{-\phi}} \\
&= \beta_k \frac{w_{k,i}^{-\phi}}{\left(\bar{p}_{k,i}/\varphi\right)^{-\phi} + w_{k,i}^{-\phi}}
\end{aligned} \tag{15}$$

For each task $k$ the representative firm can hire labor $L_{k,i}$ at wage $w_{k,i}$ and purchase a composite intermediate $I_{k,i}$ at price $\bar{p}_{k,i}$.

The first-order-conditions for cost minimization deliver:

$$\frac{L_{k,i}}{I_{k,i}} = \left(\frac{(1-\tilde{\varphi})w_{k,i}}{\tilde{\varphi}\bar{p}_{k,i}}\right)^{-(1+\phi)} = \left(\varphi^{\phi/(1+\phi)}\right)^{-(1+\phi)} \left(\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-(1+\phi)}.$$

Hence

$$\frac{w_{k,i}L_{k,i}}{\bar{p}_{k,i}I_{k,i}} = \left(\varphi^{\phi/(1+\phi)}\right)^{-(1+\phi)} \left(\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi} = \left(\varphi\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi}$$

Thus the share in total production costs of labor of type $k$ in performing task $k$ in country $i$ is:

$$\begin{aligned}
\beta^{L,k} &= \beta_k \frac{w_{k,i}L_{k,i}}{w_{k,i}L_{k,i} + \bar{p}_{k,i}I_{k,i}} \\
&= \beta_k \frac{\left(\varphi\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi}}{\left(\varphi\frac{w_{k,i}}{\bar{p}_{k,i}}\right)^{-\phi} + 1} \\
&= \beta_k \frac{(w_{k,i})^{-\phi}}{(w_{k,i})^{-\phi} + (\bar{p}_{k,i}/\varphi)^{-\phi}},
\end{aligned}$$

just as above.

## 2.5 Preferences

Final demand is by different types of workers spending their wage income (since there are no profits in our model). We model their preferences in parallel to our assumptions about production. Consumers have an integer number $K$ of needs, with each need having a Cobb-Douglas share $\alpha_k$ in preferences, with $\alpha_k > 0$ and

$$\sum_{k=1}^{K} \alpha_k = 1.$$

In parallel with the tasks of a producer, need $k$ of consumer $j$ can be satisfied either directly with the services of an appropriate type of labor $l(k)$ at wage $w_{k,i} = w_i^{l(k)}$ with efficiency $Q$ drawn from the distribution (3) or with a good produced by a firm. Final buyers match with potential sellers with the same intensity as firms, as given by (5).

Proceeding as above, a consumer faces a distribution of costs for fulfilling need $k$ given by (7). The probability that need $k$ is fulfilled by labor is again $\upsilon_{k,i}$, which, with our continuum of consumers, is the share of labor in fulfilling need $k$. The share of labor of type $l$ used by consumers in their total spending is thus:

$$\alpha_i^l = \sum_{k \in \Omega_l} \alpha_k \upsilon_{k,i}$$

and the share of labor in consumer spending in country $i$ is:

$$\alpha_i^L = \sum_l \alpha_i^l.$$

As with the share of labor in production costs, the share of labor in final spending depends on wages and other factors.

When a consumer in country $n$ fulfills a need by purchasing a good, the probability that the good come from country $i$ is given by $\pi_{ni}$ in expression (13). With our continuum of consumers $\pi_{ni}$ thus represents the share of country $i$ in country $n$'s final spending.

## 2.6 Consumer Welfare

Two worker's with the same income won't typically have the same level of utility as they encounter different goods and worker productivities in satisfying their needs. We can write the indirect utility of a consumer $j$ in $n$ spending $y_n(j) = y$ and facing costs of performing each need $k$ given by $\boldsymbol{c(j)} = (c_1, c_2, ..., c_K)$ as:

$$V(j) = V(y(j), \boldsymbol{c(j)}) = \frac{y(j)}{\prod_{k=1}^{K} c_k^{\alpha_k}/a_k}.$$

where $a_k$ is a constant that will be chosen to eliminate the effect of $K$ on utility. The expenditure $Y(V)$ needed to obtain expected utility $V$ in market $n$ is thus:

$$Y(V) = V \prod_{k=1}^{K} \left( \frac{1}{a_k} \int_0^{\infty} c_k^{\alpha_k} dG_{k,n}(c_k) \right).$$

In parallel to the derivation of the cost distribution, the term in parentheses above can be expressed as:

$$\begin{aligned}
& \frac{1}{a_k} \int_0^{\infty} (c_k)^{\alpha_k} \, dG_{k,n}(c_k) \\
= & \frac{1}{a_k} \int_0^{\infty} c_k^{\alpha_k} \phi \Xi_{k,n} c_k^{\phi-1} \exp\left(-\Xi_{k,n} c_k^{\phi}\right) dc_k \\
= & \frac{1}{a_k} \int_0^{\infty} \left( \frac{x}{\Xi_{k,n}} \right)^{\frac{1}{\phi}\alpha_k} e^{-x} dx \\
= & \Xi_{k,n}^{-\frac{1}{\phi}\alpha_k}
\end{aligned}$$

where: $a_k = \Gamma\left(1 + \frac{1}{\phi}\alpha_k\right)$.

The expected expenditure function is thus:

$$Y(V) = V \prod_{k=1}^{K} (\Xi_{k,n})^{-\frac{1}{\phi}\alpha_k}.$$

We can write the result more compactly as:

$$Y(V) = P_n^C V,$$

where

$$P_n^C = \prod_{k=1}^{K} (\Xi_{k,n})^{-\frac{1}{\phi}\alpha_k}$$

is the consumer price index.

# 3 Aggregate Equilibrium

We now have in place the assumptions we need to solve for the aggregate equilibrium. We first solve for equilibrium in the production of intermediates, given wages, and then turn to labor-market equilibrium, which determines those wages.

## 3.1 Production Equilibrium

With balanced trade, total final spending $X_n^C$ is labor income:

$$X_n^C = \sum_{l=1}^{L} w_n^l L_n^l = \sum_{k=1}^{K} w_{k,n} L_{k,n}. \tag{16}$$

Total production in country $i$ equals total revenue in supplying consumption goods and intermediates around the world:

$$Y_i = \sum_{n=1}^{\mathcal{N}} \pi_{ni} \left[ \Phi_n^C X_n^C + \Phi_n^I Y_n \right]$$

where $\Phi_n^C = 1 - \alpha_n^L$ and $\Phi_n^I = 1 - \beta_n^L$, the shares of goods in final spending and in production spending, respectively.

We can write this result in matrix form as:

$$\boldsymbol{Y} = \boldsymbol{\Pi} \left( \boldsymbol{\Phi}^C \boldsymbol{X}^C + \boldsymbol{\Phi}^I \boldsymbol{Y} \right)$$

where:

$$\boldsymbol{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_{\mathcal{N}} \end{bmatrix}, \quad \boldsymbol{X}^C = \begin{bmatrix} X_1^C \\ X_2^C \\ \cdot \\ \cdot \\ \cdot \\ X_{\mathcal{N}}^C \end{bmatrix}$$

$$
\mathbf{\Phi}^j = \begin{bmatrix} \Phi_1^j & 0 & \dots & 0 & 0 \\ 0 & \Phi_2^j & \dots & 0 & 0 \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ 0 & 0 & \dots & \Phi_{\mathcal{N}-1}^j & 0 \\ 0 & 0 & \dots & 0 & \Phi_{\mathcal{N}}^j \end{bmatrix} \quad j = C, I
$$

and:

$$
\mathbf{\Pi} = \begin{bmatrix} \pi_{11} & \pi_{21} & \dots & \pi_{\mathcal{N}-1,1} & \pi_{\mathcal{N}1} \\ \pi_{12} & \pi_{22} & \dots & \pi_{\mathcal{N}-1,2} & \pi_{\mathcal{N}2} \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ . & . & . \; . \; . & . & . \\ \pi_{1,\mathcal{N}-1} & \pi_{2,\mathcal{N}-1} & \dots & \pi_{\mathcal{N}-1,\mathcal{N}-1} & \pi_{\mathcal{N},\mathcal{N}-1} \\ \pi_{1\mathcal{N}} & \pi_{2\mathcal{N}} & \dots & \pi_{\mathcal{N}-1,\mathcal{N}} & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}
$$

We can then solve for $\boldsymbol{Y}$:

$$
\boldsymbol{Y} = (\boldsymbol{I}_{\mathcal{N}} - \mathbf{\Pi}\mathbf{\Phi}^I)^{-1}\mathbf{\Pi}\mathbf{\Phi}^C\mathbf{X}^C
$$

where $\boldsymbol{I}_{\mathcal{N}}$ is the $\mathcal{N} \times \mathcal{N}$ identity matrix.

## 3.2 Labor-Market Equilibrium

With balanced trade, final spending in country $i$, $X_i^C$ is given by (16). Equilibrium in the market for labor of type $l$ in country $i$ solves the expression:

$$
w_i^l L_i^l = \alpha_i^l X_i^C + \beta_i^l Y_i.
$$

where the first term on the right-hand side corresponds to labor demanded directly by households and the second term to labor demanded by firms. These sets of equations, for each type of labor $l$ in each country $i$, determine the wage $w_i^l$.

## 3.3 Some Quantitative Aggregate Implications

We can now investigate some quantitative implications of the model for aggregate outcomes. Table 2 provides a parameterization with two types of labor, which we call service (nonproduction) and production. The labor force in each country is divided into nonproduction workers (60 percent) and production workers (40 percent). Nonproduction workers can perform 4 tasks or fulfill 4 needs each with Cobb-Douglas shares $\beta^N = \alpha^N = .1$. Production workers can perform 12 tasks each with $\beta^P = \alpha^P = .05$. In our base case the iceberg cost is $d_{ni} = 1.2$ for all $i, n,\ i \neq n$. Finally $\lambda^N = 0$ for each nonproduction task and $\lambda^P = 0.2$ for production tasks. The world labor force, normalized at 1, is divided into 6 countries with the sizes given along the top of Table 3. The countries are identical to each other except for the sizes of their labor forces.

Note from Table 3 that the differences in relative size induce several systematic differences in outcomes across countries. Not surprisingly, the import share declines as country size increases. Because less has to be imported, intermediates are on average cheaper in larger countries. Hence more purposes are fulfilled with goods rather than labor. Since production labor competes with goods in fulfilling purposes, production workers earn relatively lower wages in larger countries, so that the "skill premium" (defined as the ratio of the wage of nonproduction to the wage of production workers) increases with size. Even though prices of intermediates are lower in large countries, the higher wage for non-production workers can lead to a higher cost of living there, as in the numerical results in Table 3. Thus, while welfare and the real wage of non-production workers is higher in large countries, the real wage of production workers declines with country size.

Table 4 reports the results (for just the second smallest and second largest countries) of varying the iceberg trade costs between all countries. A $d$ of 10 trade makes trade nearly prohibitive. A decline in trade costs, making goods more competitive with production workers, leads to a decline in the relative and real wage of production workers, even though total welfare rises.

# 4 Implications for Individual Producers

While our analysis so far has allowed us to investigate the implications of various changes in exogenous variables on equilibrium aggregate outcomes, we have more work to do to find out what happens to individual producers. We have not yet solved for the measure of active producers or sellers in an economy or for the distributions of the number of final and intermediate customers a firm has.

We first examine what our model implies about the distribution of buyers per firm, and then for the measure of firms selling and producing in a market. We conclude by examining what it predicts about the distribution of firm size.

## 4.1 The Conditional Distribution of Buyers

How many buyers a firm has depends not only on its efficiency $z$, but on its luck in finding low-cost suppliers and its luck in running into buyers who don't have better alternatives.

We start with a firm's contacts with final buyers. Consider a supplier with unit cost $c$ in market $n$ and final buyers for need $k$. The number of such customers it connects with is distributed Poisson with parameter:

$$e_{k,n}(c)L_n = \lambda_{k,n}\left(\Upsilon_n c^{\theta}\right)^{-\gamma} L_n.$$

Having met a final buyer, this supplier will make the sale with probability $e^{-\Xi_{k,n}c^{\phi}}$, the probability that that there is no lower quote. Combining these two results the number of final consumers in $n$ buying from a supplier with unit cost $c$ for need $k$ is distributed Poisson with parameter $\eta^{C}_{k,n}(c)$, given by:

$$\eta^{C}_{k,n}(c) = \lambda_{k,n} L_n \left(\Upsilon_n c^{\theta}\right)^{-\gamma} e^{-\Xi_{k,n}c^{\phi}},$$

where, recall,

$$\Xi_{k,n} = \nu_{k,n} + w_{k,n}^{-\phi}.$$

Note that $\eta^{C}_{k,n}(c)$ is decreasing in the producer's unit cost $c$ for two reasons. First, as long as $\gamma > 0$, a low-cost producer typically finds more potential customers. Second, each potential customer is more likely to have no better option. Note also that, given $\Upsilon_n$ and $w_{k,n}$, the Poisson parameter is at first increasing and then decreasing in $\lambda_{k,n}$. If it's impossible to meet customers ($\lambda_{k,n} = 0$) then it's impossible to make a sale. Thus, starting from 0, an increase in $\lambda_{k,n}$ increases the likelihood of a sale. But an increase in $\lambda_{k,n}$ also means that a potential buyer is more likely to have found another seller with a lower cost. At some point (which is earlier for a firm with a high $c$), as $\lambda_{k,n}$ rises, this second effect dominates, so that further increases reduce expected sales.

Since purchases are independent across $k$, the number of total purchases by consumers in $n$ from a producer with unit cost $c$ is distributed Poisson with parameter:

$$\eta^{C}_{n}(c) = \sum_{k=1}^{K} \eta^{C}_{k,n}(c).$$

By the properties of the Poisson distribution, $\eta_n^C(c)$ is also the expected number of customers for a potential producer selling a product at unit cost $c$ in market $n$.

In the case of final sales the set of potential customers in a market is exogenously given by the set of workers. For intermediate demand, however, the set of customers is given by the endogenous measure of local producers that actually make a sale. Let $M_n$ denote the measure of active producers in country $n$, the determination of which we turn to below. Analogous to our reasoning above, a supplier in country $n$ with unit cost $c$ encounters a number of buyers wanting to perform task $k$ that is distributed Poisson with parameter:

$$e_{k,n}(c)M_n = \lambda_{k,n}\left(\Upsilon_n c^\theta\right)^{-\gamma} M_n.$$

and its number of sales is distributed Poisson with parameter:

$$\eta_{k,n}^I(c) = \lambda_{k,n} M_n \left(\Upsilon_n c^\theta\right)^{-\gamma} e^{-\Xi_{k,n} c^\phi}.$$

Summing across tasks, the total number of sales by a seller with unit cost $c$ in country $i$ is distributed Poisson with parameter:

$$\eta_n^I(c) = \sum_{k=1}^{K} \eta_{k,n}^I(c).$$

By the properties of the Poisson distribution, $\eta_n^I(c)$ is also the expected number of customers for a potential producer selling an intermediate at unit cost $c$ in market $n$.

Combining these results, the number of buyers for a firm selling in $n$ at cost $c$ is distributed Poisson with parameter:

$$\eta_n(c) = \eta_n^C(c) + \eta_n^I(c) = (L_n + M_n)\left(\Upsilon_n c^\theta\right)^{-\gamma} \sum_{k=1}^{K} \lambda_{k,n} e^{-\Xi_{k,n} c^\phi}.$$

Now consider worldwide sales of a producer in country $i$ with local cost $c$. Its unit cost in country $n$ is $cd_{ni}$. The total number of customers around the world for this producer is distributed Poisson with parameter:

$$
\begin{aligned}
\eta_i^W(c) &= \sum_{n=1}^{\mathcal{N}} \eta_n(cd_{ni}) \\
&= \sum_{n=1}^{\mathcal{N}} (L_n + M_n)(d_{ni})^{-\gamma\theta} \left(\Upsilon_n c^\theta\right)^{-\gamma} \sum_{k=1}^{K} \lambda_{k,n} e^{-\Xi_{k,n}(d_{ni})^{-\phi} c^\phi}.
\end{aligned}
$$

## 4.2 The Measures of Producers and Sellers

In an open economy, the measure of firms making sales in country $n$, denoted $N_n$, need not be the same as the measure actually producing there, denoted $M_n$.

To appear as a firm a seller has to sell somewhere. The probability that a potential producer from source $i$ with unit cost $c$ fails to make a sale anywhere is $\exp(-\eta_i^W(c))$. Integrating over the cost distribution of potential producers in source $i$ (those from $i$ that can deliver to $i$ at cost $c$):

$$
\begin{aligned}
M_i &= \int_0^\infty (1 - e^{-\eta_i^W(c)}) d\mu_{ii}(c) \\
&= T_i \Xi_i \int_0^\infty (1 - e^{-\eta_i^W(c)}) \theta c^{\theta-1} dc. \qquad (17)
\end{aligned}
$$

Since $\eta_i^W(c)$ itself depends on the measure of customers for intermediates $M_n$ in each market $n$, we need to iterate to find a solution for all the $M_i$'s.

Having solved for the $M_i$'s, the measure of firms selling in $n$, $N_n$, can be calculated as

$$
\begin{aligned}
N_n &= \int_0^\infty (1 - e^{-\eta_n(c)}) d\mu_n(c) \\
&= \Upsilon_n \int_0^\infty (1 - e^{-\eta_n(c)}) \theta c^{\theta-1} dc. \qquad (18)
\end{aligned}
$$

We can evaluate this integral numerically to determine the relationship between entry $N_n$ and market size, $L_n + M_n$.

The measure of firms from $i$ exporting to $n$ is

$$N_{ni} = \pi_{ni} N_n = \int_0^\infty (1 - e^{-\eta_n(c)}) d\mu_{ni}(c). \tag{19}$$

Thus the fraction of firms from $i$ that export to $n$ is $N_{ni}/M_i$. The fraction of firms from $i$ that sell domestically is $N_{ii}/M_i$.

While equations (17) and (18) don't have closed form solutions, we can compute their solutions for numerical parameter values.

## 4.3 The Distribution of Buyers

So far we've considered only the distribution of a seller's customers in market $n$ conditional on its $c$ there, Let $S_n$ be the integer-valued random variable for the number of customers in $n$ that a firm sells to. From the Poisson distribution, the probability that a firm with cost $c$ has $s$ customers is

$$\Pr[S_n = s|c] = \frac{e^{-\eta_n(c)} \left[\eta_n(c)\right]^s}{s!},$$

for $s = 0, 1, ....$ We can integrate over the cost distribution and condition on $S_n > 0$ (since if $S_n = 0$ the firm would not be among those observed to sell in $n$) to get

$$\begin{aligned} \Pr[S_n &= s|S_n > 0] = \frac{1}{N_n} \int_0^\infty \frac{e^{-\eta_n(c)} \left[\eta_n(c)\right]^s}{s!} d\mu_n(c) \\ &= \frac{\Upsilon_n}{N_n s!} \int_0^\infty e^{-\eta_n(c)} \left[\eta_n(c)\right]^s \theta c^{\theta-1} dc, \end{aligned} \tag{20}$$

for $s = 1, 2, ....$

The expected number of buyers per active firm is thus simply:

$$
\begin{aligned}
E\left[S_n | S_n > 0\right] &= \frac{1}{N_n}\int_0^\infty \eta_n(c)d\mu_n(c) \\
&= \frac{L_n + M_n}{N_n}\int_0^\infty \left(\Upsilon_n c^\theta\right)^{-\gamma}\sum_{k=1}^{K}\lambda_{k,n}e^{-\Xi_{k,n}c^\phi}\Upsilon_n\theta c^{\theta-1}dc \\
&= \frac{L_n + M_n}{N_n}\int_0^\infty \theta\left(\Upsilon_n\right)^{\phi/\theta}\left(\sum_{k=1}^{K}\lambda_{k,n}e^{-\Xi_{k,n}c^\phi}\right)c^{\phi-1}dc \\
&= \frac{L_n + M_n}{N_n}\sum_{k=1}^{K}\frac{\nu_{k,n}}{\Xi_{k,n}}
\end{aligned}
$$

Since $\nu_{k,n}/\Xi_{k,n}$ is the probability that a potential customer purchases a good to carry out task $k$ (rather than hiring labor), the summation on the right hand side is then expected purchases per potential customer. Thus, expected sales per firm is the product of the measure of potential customers, $L_n + M_n$, in market $n$ and the expected number of goods purchased per potential customer, all divided by the measure of sellers in that market, $N_n$.

## 4.4 Some Quantitative Firm-Level Results

Using the same parameterization as in Table 2, we show in Tables 5 and 6 the firm-level results underlying the aggregate results shown in Tables 3 and 4. Note from Table 5 that the simulation mimics the patterns in the distribution of buyers per firm shown in Table 1. For Figure 5 we calculate the measures of sellers to each of our six hypothetical countries (labelled a through f, in increasing size) from country a, adjusting (as in Figure 3) by country a's market share in each destination. The figure illustrates how we capture the increasing but less than proportional relationship between market size and number of exporters, albeit with a somewhat greater slope of 0.80 (standard error 0.018).

Table 6 reports the effects of varying trade costs on the measures of active suppliers and producers in a market, with lower barriers tending to reduce each.

Figure 6 reports the average number of buyers per seller across our hypothetical markets. Notice that the pattern mimics that in Figure 3, with a similar slope of 0.23 (standard error 0.022).

Finally, Figure 7 is the model analog of the observed distribution of production labor shares (in total costs) shown in Figure 4. This figure is based on the cumulative binomial distribution with 12 trials (the number of production tasks) and a probability 0.47 of a "success" (a task not being outsourced) based on the fraction of tasks outsourced ($0.53 = 1 - 0.47$) shown in the second to last column of Table 3. The two figures resemble each other although the model grosssly underestimates the fraction of firms that employ no production labor. To match the data more closely would require differences in task shares (the $\beta_k$'s), which for parsimony we made symmetric.

# 5 Conclusion

Taking into account the granularity of individual buyer-seller relationships expands the scope for firm heterogeneity in a number of dimensions. Aside from differences in raw efficiency, firms experience different luck in finding cheap inputs. These two sources of heterogeneity combine to create differences in the firm's cost to deliver to different markets around the world. But within each market firms have different degrees of luck in connecting with buyers. We can thus explain why a firm may happen to sell in a small, remote market while skipping over a large one close by. It also explains why one firm may appear very successful in one market and sell very little in another, while another firm does just the opposite.

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# 6 Appendix A: Data Source

The empirical analysis is conducted using detailed export data covering the universe of French exporting firms. The data have been provided by the French Customs, and have been used by Kramarz, Martin, and Mejean (2014). The full data set covers all export transactions that involve a French exporter and an importing firm located in the European Union. In this paper, we use only the data for the year 2005.

Many researchers before us have used individual trade data from the French Customs. Typically, the data used in such empirical analyses are annual measures disaggregated at the level of the exporting firm, as in Eaton, Kortum, and Kramarz (2011) among others. Some papers, such as Biscourp and Kramarz (2007) and Blaum, Lelarge, and Peters (2014), also use data at the level of the importer. An exception is Bricongne, Fontagné, Gaulier, Taglioni, and Vicard (2012) who use data that record, for each exporting firm, each transaction in each month, although not identifying the exact buyer. In this respect, the data we use are more precise since they not only record the transaction but also the exact identity of the buyer. For each transaction, the dataset gives us the identity of the exporting firm (its name and its SIREN identifier), the identification number of the importer (an anonymized version of its VAT number), the date of the transaction (month and year), the product category of the transaction (at the 8-digit level of the combined nomenclature), the value and the quantity of the shipment. For the analysis here, records will be aggregated across transactions within a year, for each exporter-importer-product triplet. Such measurement is possible because, whereas goods are perfectly free to move across countries within the European Union, firms selling goods outside France are still compelled to fill a Customs form. Such forms are used to repay VAT for transactions on intermediate consumptions. Hence, our data are exhaustive. However, small exporters are allowed to fill a "simplified" form that does not require the product category of exported goods. The "simplified" regime can be used by firms with total exports in the EU below 100,000 euros in 2005 (and 150,000 euros thereafter). In 2005, we have data for 46,928 French firms exporting 7,807 8-digit products to 571,149 buyers located in the EU. Total exports by these firms amounts to 207 billions of euros. Such exports account for 58 percent of French total exports. The total number of observations is 3,983,909.

# 7 Appendix B: Computing $\Upsilon$

We derive conditions under which there is a unique solution for $\Upsilon$, given wages, that can be computed by simple iteration. To ensure a solution it helps to have a sufficient share of tasks in which outsourcing is not possible ($\lambda_k = 0$). Denote the set of such tasks as $\Omega^0$ and its complement (among the set of all tasks $\{1, 2, ..., K\}$) as $\Omega^P$. We require:

$$\beta^P = \sum_{k \in \Omega^P} \beta_k < 1.$$

As a warm-up exercise, we start with the case of a single country ($\mathcal{N} = 1$), so that $\Upsilon$ is a scalar. We then turn to the general case with multiple countries, in which $\Upsilon$ is an $\mathcal{N} \times 1$ vector.

## 7.1 The Case of a Single Country

With a single country, the solution for $\Upsilon$ is a fixed point

$$\Upsilon = f(\Upsilon)$$

of the function $f$ defined as:

$$f(x) = T\prod_{k=1}^{K}\left(\frac{\theta}{\phi}\lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi}\right)^{\frac{\theta}{\phi}\beta_k}.$$

Employing our assumption that $\lambda_k = 0$ for all tasks $k \in \Omega^0$, we can write:

$$f(x) = T\left(\prod_{k\in\Omega^0}(w_k)^{-\theta\beta_k}\right)\prod_{k\in\Omega^P}\left(\frac{\theta}{\phi}\lambda_k x^{\frac{\phi}{\theta}} + w_k^{-\phi}\right)^{\frac{\theta}{\phi}\beta_k}.$$

It is convenient to work in logs. Thus $\ln\Upsilon$ is the fixed point

$$\ln\Upsilon = F(\ln\Upsilon)$$

of the function:

$$F(y) = A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right),$$

where

$$A = \ln T - \sum_{k\in\Omega^0}\theta\beta_k \ln w_k,$$

and

$$u_k = \frac{\theta}{\phi}\lambda_k$$

There exists a unique fixed point of $F$ if it is a contraction. To show that it is, we can check Blackwell's sufficient conditions, monotonicity and discounting. For monotonicity, note that $x \leq y$ implies:

$$F(x) = A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}x} + w_k^{-\phi}\right) \leq A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right) = F(y).$$

For discounting, $a > 0$ implies:

$$\begin{aligned}
F(y+a) &= A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}(y+a)} + w_k^{-\phi}\right) = A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(e^{\frac{\phi}{\theta}a}u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right) \\
&= A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k\left[\frac{\phi}{\theta}a + \ln\left(u_k e^{\frac{\phi}{\theta}y} + e^{-\frac{\phi}{\theta}a}w_k^{-\phi}\right)\right] = A + \beta^P a + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + e^{-\frac{\phi}{\theta}a}w_k^{-\phi}\right) \\
&\leq A + \sum_{k\in\Omega^P}\frac{\theta}{\phi}\beta_k \ln\left(u_k e^{\frac{\phi}{\theta}y} + w_k^{-\phi}\right) + \beta^P a = F(y) + \beta^P a.
\end{aligned}$$

We can thus compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with $y^{(0)} = 0$. This method is justified, since the contraction mapping theorem guarantees that:

$$\lim_{t\to\infty} y^{(t)} = \ln \Upsilon.$$

This result also give us the comparative statics. We see directly that $\ln \Upsilon$ is increasing in technology $T$, decreasing in any task-specific wage $w_k$, and increasing in any task-specific arrival of price quotes $\lambda_k$.

## 7.2 Multiple Countries

Consider generalizing the argument above to a world of many countries, trading intermediates and final goods with each other. Now $\Upsilon$ is an $\mathcal{N} \times 1$ vector satisfying

$$\Upsilon_n = \sum_i T_i d_{ni}^{-\theta} \prod_k \left( \frac{\theta}{\phi} \lambda_{k,i} \Upsilon_i^{\frac{\phi}{\theta}} + w_{k,i}^{-\phi} \right)^{\frac{\theta}{\phi}\beta_k},$$

for $n = 1, ..., \mathcal{N}$.

Let $\ln \Upsilon$ be the corresponding vector with $\ln \Upsilon_n$ in place of $\Upsilon_n$ for $n = 1, ..., \mathcal{N}$. Thus $\ln \Upsilon$ is the fixed point

$$\ln \Upsilon = F(\ln \Upsilon)$$

of the mapping $F$, whose $n$'th element is:

$$\begin{aligned} F_n(y) &= \ln \left[ \sum_i \exp \left( \ln \left( T_i d_{ni}^{-\theta} \right) + \sum_k \frac{\theta}{\phi} \beta_k \ln \left( u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right] \\ & \ln \left[ \sum_i \exp \left( A_{ni} + \sum_{k \in \Omega^P} \frac{\theta}{\phi} \beta_k \ln \left( u_{k,i} e^{\frac{\phi}{\theta} y_i} + w_{k,i}^{-\phi} \right) \right) \right], \end{aligned}$$

where

$$A_{ni} = \ln\left(T_i d_{ni}^{-\theta}\right) - \sum_{k\in\Omega^0} \theta\beta_k \ln w_{k,i}$$

and

$$u_{k,i} = \frac{\theta}{\phi}\lambda_{k,i}.$$

We can check Blackwell's conditions again. For monotonicity, it is readily apparent that for a vector $x \le y$ we have $F_n(x) \le F_n(y)$ for all $n = 1, ..., \mathcal{N}$. For discounting, consider $a > 0$ so that

$$\begin{aligned}
F_n(y+a) &= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}(y_i+a)} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \left[\frac{\phi}{\theta}a + \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right]\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + e^{-\frac{\phi}{\theta}a}w_{k,i}^{-\phi}\right)\right)\right] \\
&\le \ln\left[\sum_i \exp\left(A_{ni} + \beta^P a + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] \\
&= \ln\left[\sum_i \exp\left(A_{ni} + \sum_{k\in\Omega^P} \frac{\theta}{\phi}\beta_k \ln\left(u_{k,i}e^{\frac{\phi}{\theta}y_i} + w_{k,i}^{-\phi}\right)\right)\right] + \beta^P a \\
&= F_n(y) + \beta^P a.
\end{aligned}$$

Thus, even with multiple countries, we can still compute the fixed point by iterating on:

$$y^{(t)} = F(y^{(t-1)}),$$

starting with an $\mathcal{N} \times 1$ vector $y^{(0)}$ (which could simply be a vector of zeros). This method is justified, since the contraction mapping theorem guarantees (just as in the scalar case) that:

$$\lim_{t\to\infty} y^{(t)} = \ln\Upsilon.$$

This result also give us the comparative statics. We see directly that each element of $\ln \Upsilon$ is increasing in technology anywhere $T_i$, decreasing in any task-specific wage $w_{k,i}$ in any country, and increasing in any task-specific arrival of price quotes $\lambda_{k,i}$ in any country. An important caveat, however, is that these comparative statics take task-specific wages as given, so do not predict general-equilibrium outcomes.

Table 1: Customers per French Exporter

| | Destination Market | | | |
|---|---|---|---|---|
| | Lithuania | Denmark | UK | Germany |
| Market Size ($billions) | 18 | 94 | 882 | 1480 |
| Customers per Exporter: | | | | |
| Mean | 4.2 | 7.1 | 17.9 | 24.9 |
| Percentiles: | | | | |
| 25th | 1 | 1 | 1 | 2 |
| 50th | 2 | 2 | 3 | 4 |
| 75th | 4 | 5 | 9 | 12 |
| 90th | 9 | 12 | 25 | 35 |
| 95th | 15 | 21 | 48 | 70 |
| 99th | 40 | 77 | 224 | 329 |

Data are for 2005.

Table 2: Baseline Parameter Settings for Simulation

| Parameter | symbol | value |
|---|---|---|
| Pareto parameters: | | |
| efficiency distribution | theta | 5 |
| price distribution | phi | 2 |
| Technology level per person | T_i/L_i | 3.6 |
| World labor force | L | 1 |
| Labor by type (fractions of labor force): | L^l | |
| nonproduction (service) | | 0.6 |
| production | | 0.4 |
| Iceberg trade cost | d | 1.2 |
| Tasks, by type: | | |
| service tasks: | | |
| number of tasks | K | 4 |
| total share | beta | 0.4 |
| production tasks: | | |
| number of tasks | K | 12 |
| total share | beta | 0.6 |
| Task shares in consumption (same as for production) | alpha | |
| Outsourcing parameters: | lambda | |
| service | | 0 |
| production | | 0.2 |

Table 3: Aggregate Results of Simulation

| | Country Size | | | | | |
|---|---|---|---|---|---|---|
| | L=0.001 | L=0.009 | L=0.09 | L=0.2 | L=0.3 | L=0.4 |
| Production value added: | | | | | | |
| Share of GDP | 0.126 | 0.126 | 0.128 | 0.130 | 0.131 | 0.132 |
| Share of gross production | 0.31 | 0.31 | 0.30 | 0.29 | 0.28 | 0.28 |
| Fraction of production tasks outsourced: | 0.48 | 0.48 | 0.50 | 0.51 | 0.53 | 0.54 |
| Import share of production | 1.00 | 0.97 | 0.79 | 0.61 | 0.49 | 0.39 |
| Wage: | | | | | | |
| service | 0.87 | 0.87 | 0.91 | 0.94 | 0.98 | 1.00 |
| production | 1.02 | 1.02 | 1.03 | 1.03 | 1.04 | 1.05 |
| Skill premium (service/production) | 0.85 | 0.86 | 0.88 | 0.91 | 0.94 | 0.96 |
| Real wage: | | | | | | |
| service | 1.45 | 1.46 | 1.50 | 1.55 | 1.58 | 1.62 |
| production | 1.71 | 1.71 | 1.70 | 1.69 | 1.69 | 1.69 |
| Welfare (real per capita consumption) | 1.55 | 1.56 | 1.58 | 1.61 | 1.63 | 1.64 |

1. Production value added does not include service tasks (i.e. purchased services)

2. Wage is normalized so that labor income of the World is 1

Table 4: Aggregate Results with Different Trade Costs

| | Trade Cost (small country, L=.009) | | | | | Trade Cost (large country, L=0.3) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 10.00 | 1.80 | 1.20 | 1.05 | 1.00 | 10.00 | 1.80 | 1.20 | 1.05 | 1.00 |
| Production value added: | | | | | | | | | | |
| Share of GDP | 0.06 | 0.09 | 0.13 | 0.13 | 0.13 | 0.12 | 0.13 | 0.13 | 0.13 | 0.13 |
| Share of gross production | 0.49 | 0.43 | 0.31 | 0.27 | 0.26 | 0.32 | 0.31 | 0.28 | 0.26 | 0.26 |
| Fraction of prod. tasks outsourced: | 0.19 | 0.29 | 0.48 | 0.55 | 0.57 | 0.47 | 0.48 | 0.53 | 0.56 | 0.57 |
| Import share of production | 0.00 | 0.76 | 0.97 | 0.99 | 0.99 | 0.00 | 0.11 | 0.49 | 0.65 | 0.70 |
| Wage: | | | | | | | | | | |
| service | 0.73 | 0.62 | 0.87 | 0.98 | 1.02 | 0.93 | 0.94 | 0.98 | 1.00 | 1.02 |
| production | 1.34 | 1.00 | 1.02 | 0.99 | 0.97 | 1.11 | 1.11 | 1.04 | 0.99 | 0.97 |
| Skill premium (service/production) | 0.55 | 0.62 | 0.86 | 0.99 | 1.04 | 0.83 | 0.85 | 0.94 | 1.01 | 1.04 |
| Real wage: | | | | | | | | | | |
| service | 0.98 | 1.10 | 1.46 | 1.66 | 1.74 | 1.42 | 1.45 | 1.58 | 1.69 | 1.74 |
| production | 1.78 | 1.76 | 1.71 | 1.68 | 1.67 | 1.71 | 1.71 | 1.69 | 1.68 | 1.67 |
| Welfare (real per capita cons.) | 1.30 | 1.36 | 1.56 | 1.67 | 1.71 | 1.54 | 1.55 | 1.63 | 1.69 | 1.71 |

1. Production value added does not include service tasks (i.e. purchased services)

2. Wage is normalized so that labor income of the World is 1

Table 5: Firm-Level Results of Simulation

| | Country Size | | | | | |
|---|---|---|---|---|---|---|
| | L=0.001 | L=0.009 | L=0.09 | L=0.2 | L=0.3 | L=0.4 |
| Measures of firms: | | | | | | |
| producing | 0.02 | 0.14 | 1.60 | 3.95 | 6.32 | 8.80 |
| selling | 0.08 | 0.61 | 3.93 | 7.38 | 10.15 | 12.67 |
| Measures normalized by Labor: | | | | | | |
| producing | 15.7 | 15.9 | 17.8 | 19.8 | 21.1 | 22.0 |
| selling | 84.9 | 67.6 | 43.7 | 36.9 | 33.8 | 31.7 |
| Fraction of firms selling domestically: | 0.02 | 0.11 | 0.53 | 0.74 | 0.83 | 0.88 |
| Mean # customers per firm: | 1.13 | 1.44 | 2.56 | 3.47 | 4.12 | 4.68 |
| Size distribution (percentiles): | | | | | | |
| 25th | 1 | 1 | 1 | 1 | 1 | 1 |
| 50th | 1 | 1 | 1 | 1 | 1 | 1 |
| 75th | 1 | 1 | 2 | 2 | 3 | 3 |
| 90th | 1 | 2 | 4 | 5 | 6 | 7 |
| 95th | 2 | 3 | 7 | 10 | 12 | 14 |
| 99th | 3 | 8 | 22 | 34 | 43 | 51 |

Table 6: Firm-Level Results with Different Trade Costs

| | Trade Cost (small country, L=.009) | | | | | Trade Cost (large country, L=0.3) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 10.00 | 1.80 | 1.20 | 1.05 | 1.00 | 10.00 | 1.80 | 1.20 | 1.05 | 1.00 |
| Measures of firms: | | | | | | | | | | |
| producing | 0.29 | 0.09 | 0.14 | 0.19 | 0.22 | 11.4 | 9.6 | 6.3 | 6.8 | 7.3 |
| selling | 0.29 | 0.21 | 0.61 | 0.91 | 1.07 | 11.4 | 10.8 | 10.1 | 11.7 | 12.7 |
| Measures normalized by Labor: | | | | | | | | | | |
| producing | 31.9 | 9.5 | 15.9 | 21.6 | 24.4 | 37.9 | 32.0 | 21.1 | 22.6 | 24.4 |
| selling | 31.9 | 23.7 | 67.6 | 101.6 | 118.5 | 37.9 | 36.1 | 33.8 | 38.9 | 42.4 |
| Fraction of firms selling domestically: | 1.00 | 0.60 | 0.11 | 0.06 | 0.04 | 1.00 | 1.00 | 0.83 | 0.61 | 0.52 |
| Mean # customers per firm: | 2.33 | 1.52 | 1.44 | 1.47 | 1.48 | 5.73 | 5.23 | 4.12 | 4.08 | 4.14 |
| Size distribution (percentiles): | | | | | | | | | | |
| 25th | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 50th | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 75th | 2 | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 3 |
| 90th | 4 | 2 | 2 | 2 | 2 | 9 | 8 | 6 | 6 | 6 |
| 95th | 6 | 3 | 3 | 3 | 3 | 17 | 16 | 12 | 12 | 12 |
| 99th | 19 | 9 | 8 | 8 | 8 | 66 | 59 | 43 | 43 | 44 |

# Figure 1: French Exporters and Market Size

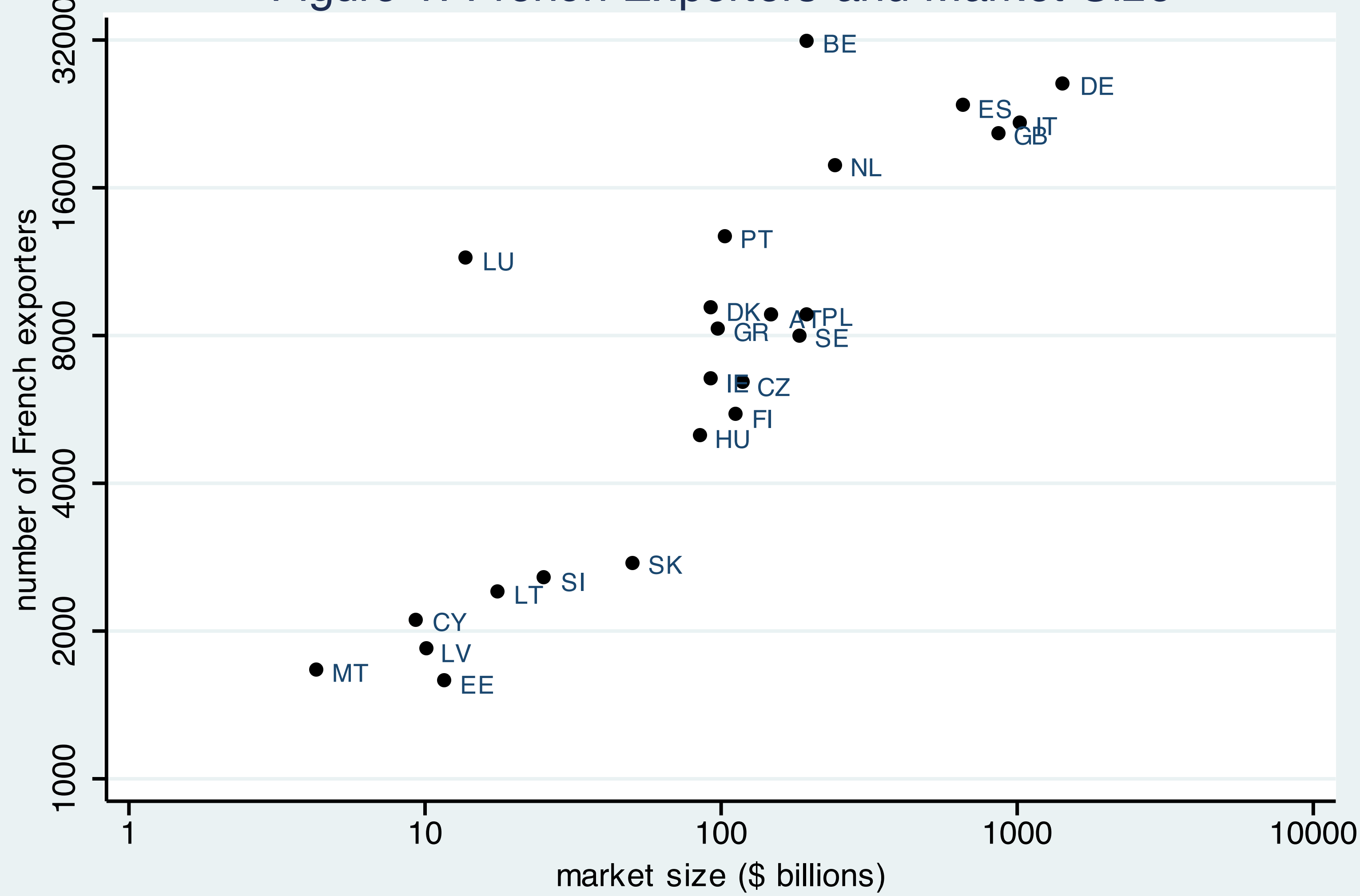
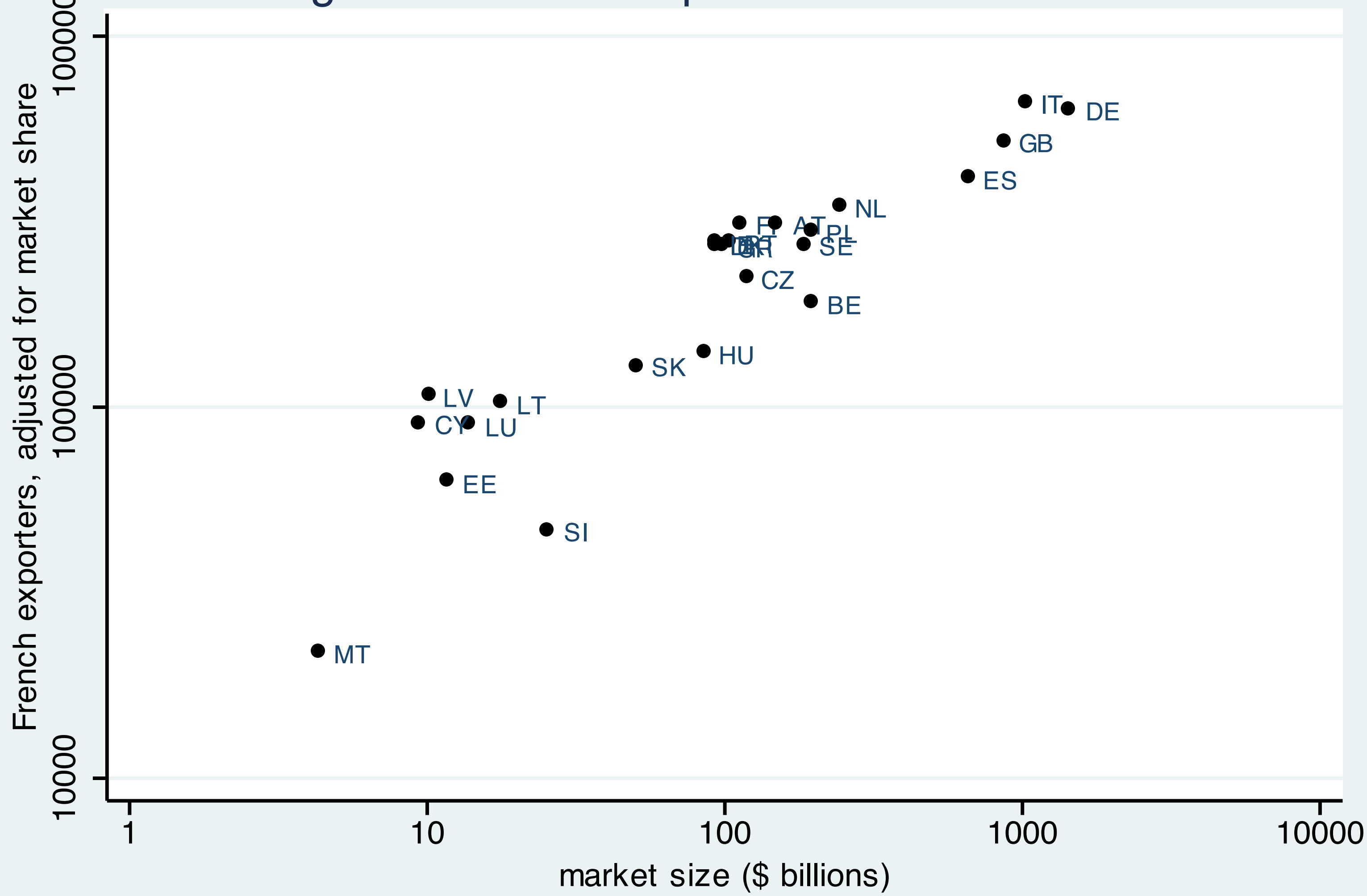


Figure 2: French Exporters and Market Size



# Figure 3: Buyers per French Exporter, by Destination

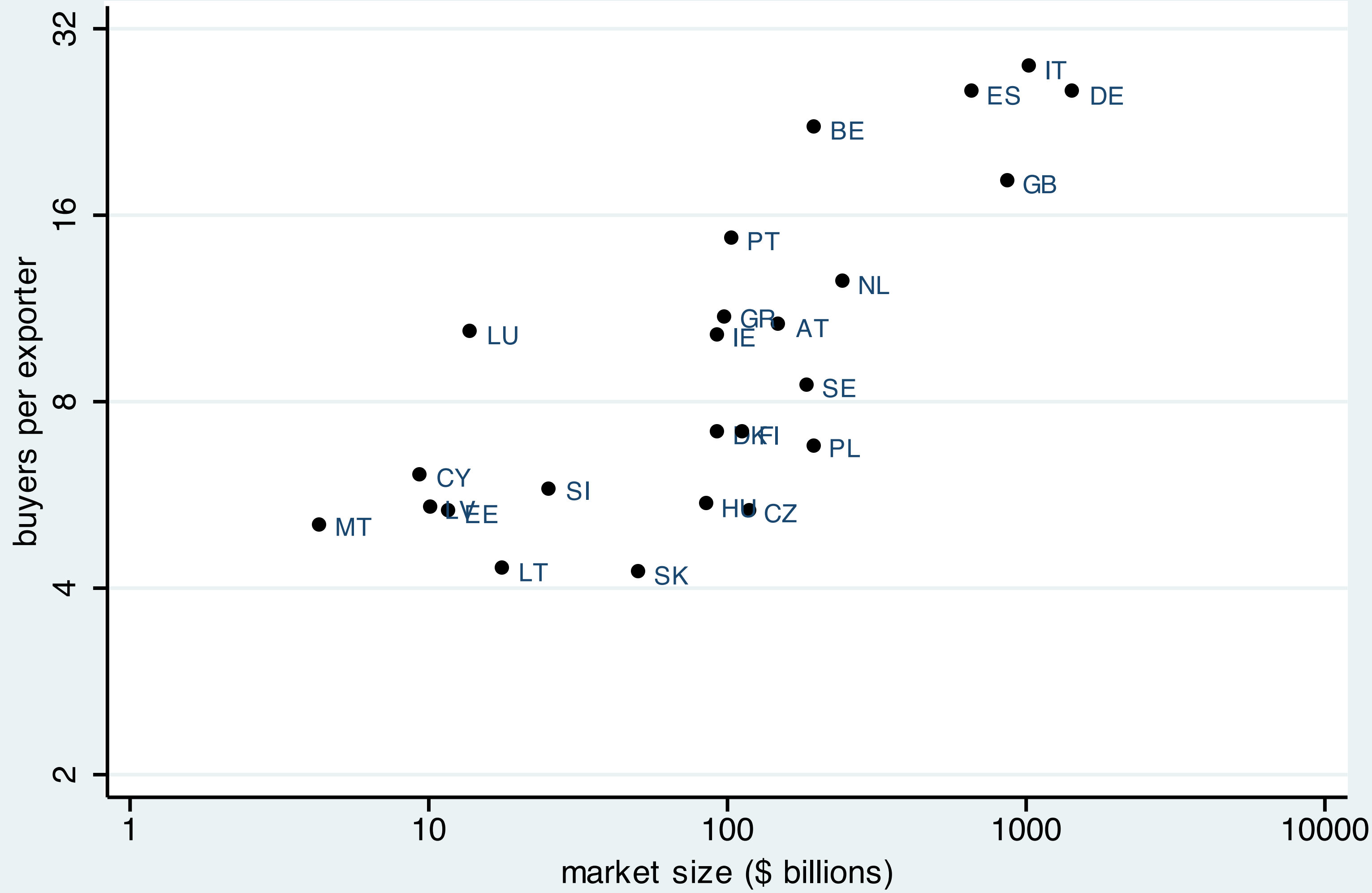
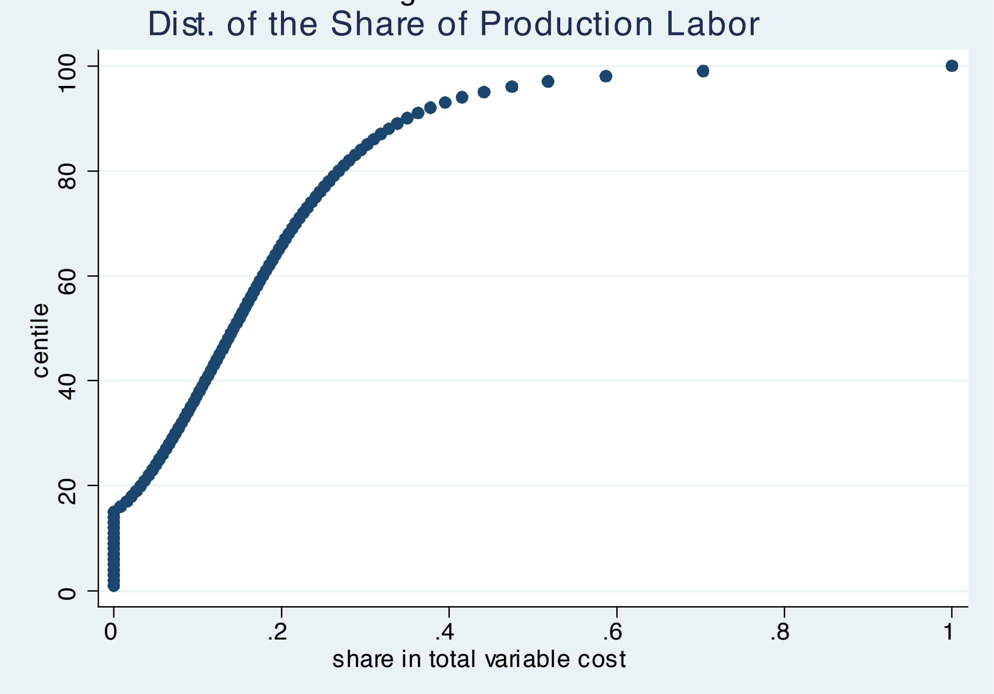


Figure 4


Dist. of the Share of Production Labor
centile
100
80
60
40
20
0
0
.2
.4
.6
.8
1
share in total variable cost

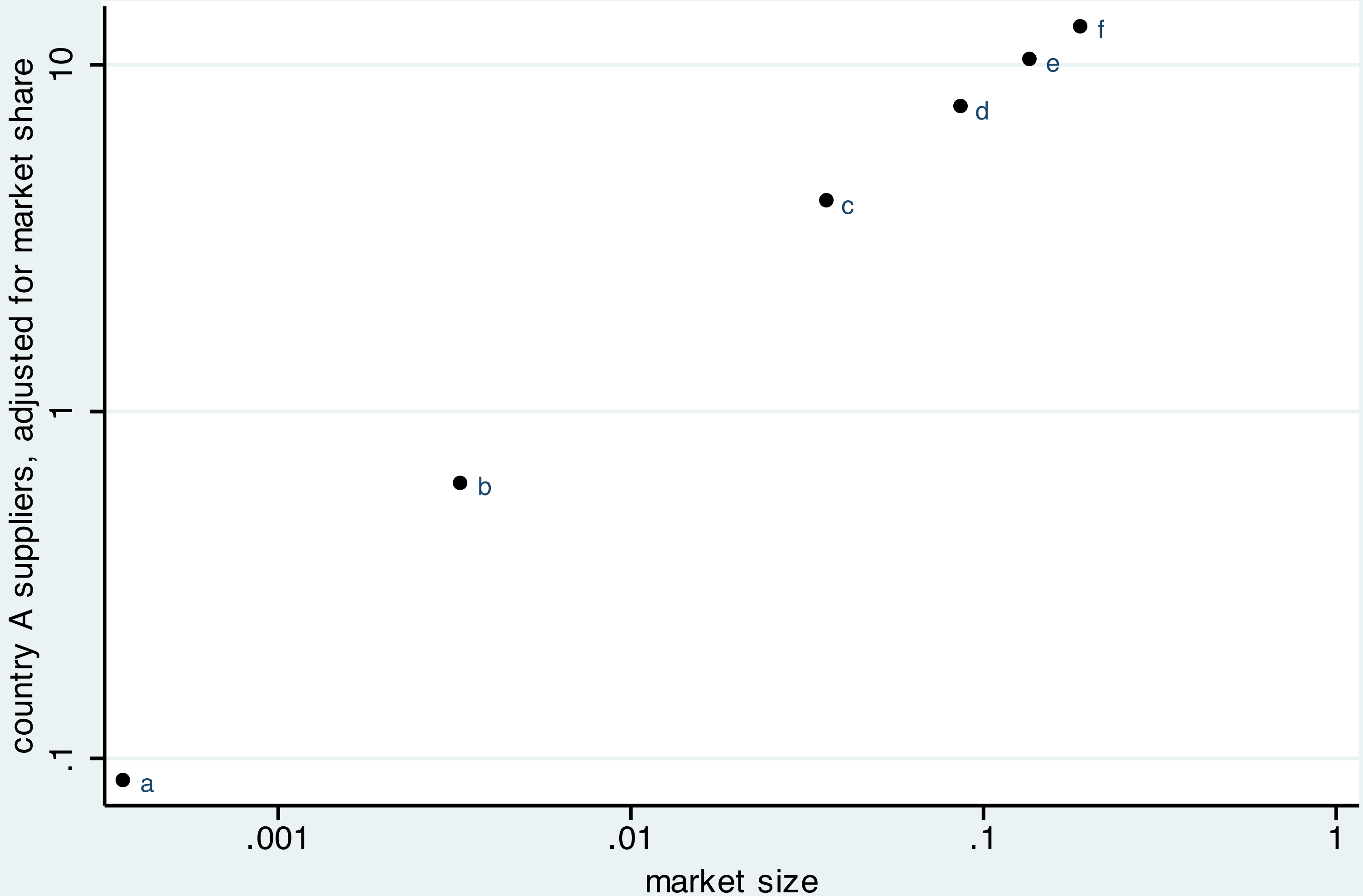


Figure 5: Suppliers and Market Size

# Figure 6: Buyers per Supplier, by Destination

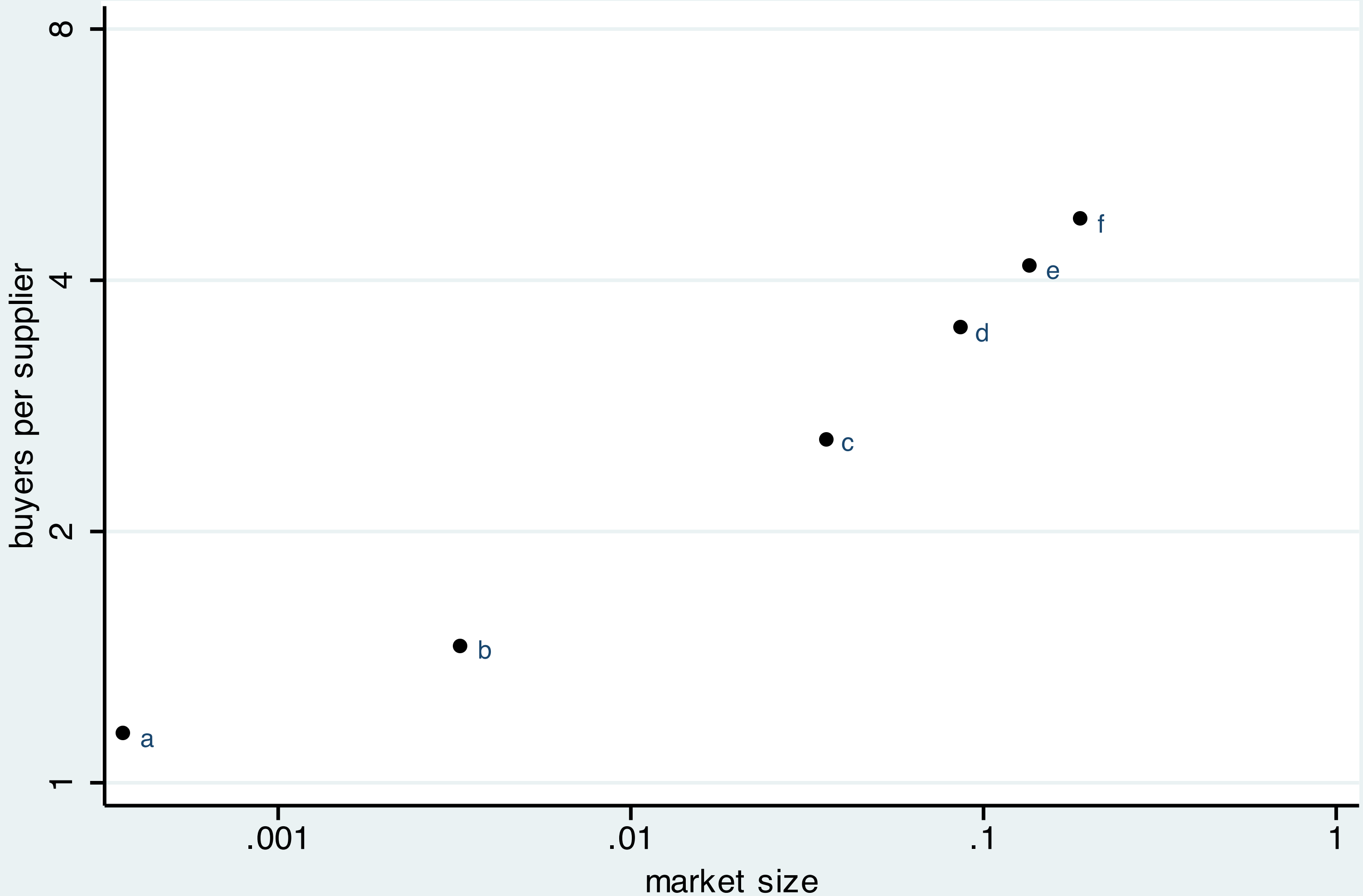


Figure 7: Predicted Share of Production Labor