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Abstract

We construct and analyze a model of household debt with endogenous borrowing constraint, in which a large initial debt has a persistent adverse effect. In this economy, the redistribution shock that makes a percentage of people overly indebted can cause persistent shortages of aggregate demand and inefficiency due to labor-wedge deterioration. Overly accumulated debt may be a primal cause of persistent recessions. Debt forgiveness may be effective to restore aggregate demand and efficiency, and enhance welfare in a crisis-hit economy, where many households suffer from excessive debt burden.

Keywords: Endogenous borrowing constraint, Labor wedge, Mortgage loans, Secular stagnation. *JEL classification*: E21, E30, G01.

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1 Introduction

Severe household debt problem emerged during the Great Recession. Mian and Sufi (2014) emphasize that too much debt in the household sector may have some causal relationship with the severe and persistent recession, which seems to be caused by aggregate demand shortage. In this note, we show that overly accumulated household debt depresses the household demand for consumption for an extended period and thereby lowers the aggregate demand persistently. This model could be used as a simplistic framework for economics of recessions, and shed some light on the growing concerns about the "secular stagnation" thesis that economic growth in developed nations would slow down for good in the aftermath of the financial crisis (e.g., Summers 2013; and Eggertson and Mehrotra 2014). In the recent literature, household debt is modeled by Livshits, MacGee, and Tertilt (2007), Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), and Nakajima and Ríos-Rull (2014). In these models, household debt has the role to smooth consumption intertemporally in response to (idiosyncratic) income shocks. These papers focus mainly on causality from the aggregate business cycles to household debt and defaults, whereas our focus is on causality from household debt to aggregate fluctuations. Specifically, we show that too much debt for (some) households may depress aggregate demand persistently. In this paper, household debt plays two roles. It provides liquidity for payment in purchasing consumer goods, whereas it also has the role to smooth consumption intertemporally. Borrowers are subject to the borrowing constraint, which is a hybrid of those in Kiyotaki and Moore (1997) and Jermann and Quadrini (2012). In our model, a large initial debt in household sector depresses consumption demand and deteriorates the labor wedge persistently. A persistent shortage of aggregate demand and labor-wedge deterioration are both observed in the recessions in the aftermath of financial crises, such as the Great Recession. Our model shows that overly accumulated household debt can be a primal cause of these features. This result also implies that policy interventions that facilitate debt forgiveness of overly indebted agents may restore aggregate demand and efficiency.

The organization of the paper is as follows. In the next section, the model is described. The equilibrium is analyzed in Section 3. Section 4 concludes.

2 Model

To demonstrate the main results analytically, we make the model as simple as possible.

2.1 Setup

The economy is deterministic and closed. Time is discrete and goes from 0 to infinity: $t = 0, 1, 2, \dots$. There exists a unit mass of households, whose instantaneous utility in period t is given by

$$U(c_t, l_t, h_t) = \ln c_t + \gamma \ln(1 - l_t) + \eta \ln h_{t-1},$$

where c_t is consumption, l_t is labor supply $(1 - l_t$ is leisure), and h_{t-1} is housing services generated from the housing stock purchased in period t - 1. Housing price is q_t and a household purchases h_t units of houses. Total amount of housing stock is fixed: $H_t = 1$. A household may borrow a mortgage loan to purchase the houses and the outstanding debt in period t is d_t . The debt is made by (other) households. The lender is content with the repayment b_{t+1} , if $d_t = \frac{1}{1+r}[b_{t+1} + d_{t+1}]$. Consumer goods are produced competitively by the Cobb-Douglas technology: $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, where K_t is the capital stock, L_t is the total labor, and A_t is the aggregate productivity. The price of capital stock is q_t^K and the total supply of capital stock is fixed: $K_t = 1$.

2.2 Benchmark case where households have no debt

If all households have no debt, they solve

$$\max \sum_{t=0}^{\infty} \beta^{t} U(c_{t}, h_{t-1}, l_{t}),$$

s.t. $c_{t} \leq w_{t} l_{t} + r_{t}^{K} k_{t-1} + q_{t} (h_{t-1} - h_{t}) + q_{t}^{K} (k_{t-1} - k_{t}),$

where in equilibrium, $k_t = h_t = 1$, $c_t = A_t l_t^{1-\alpha}$, $r_t^K = \alpha A_t l_t^{1-\alpha}$, and $w_t = (1-\alpha)A_t l_t^{-\alpha}$. Given that $U(c, h, l) = \ln c + \gamma \ln(1-l) + \eta \ln h$, it is easily shown that the labor supply is

$$l = L^* = \frac{1 - \alpha}{1 - \alpha + \gamma}.$$

We use this case as a benchmark and L^* is the first-best labor supply.

2.3 Model without borrowing constraint

There exist the lending households with measure 1-p and the borrowing households with measure p. All households hold identical amount of houses, $h_0 = 1$, at the beginning of period 0. All borrowers owe the identical amount, (1 + r)d, as the initial debt at the beginning of period 0. We assume that lending households own all capital and lend consumption loans to the borrowing households. If there does not exist a borrowing constraint, the borrowing households solve

$$V(d_{t-1}, h_{t-1}) = \max U(c_t, h_{t-1}, l_t) + \beta V(d_t, h_t),$$

s.t.
$$\begin{cases} c_t + b_t \le w_t l_t + q_t (h_{t-1} - h_t), \\ d_t = (1 + r_{t-1}) d_{t-1} - b_t, \end{cases}$$

and the lending households solve

$$\max \sum_{t=0}^{\infty} \beta^{t} U(\hat{c}_{t}, \hat{h}_{t-1}, \hat{l}_{t}),$$

s.t. $\hat{c}_{t} \leq w_{t} \hat{l}_{t} + r_{t}^{K} \hat{k}_{t-1} + \hat{b}_{t} + q_{t} (\hat{h}_{t-1} - \hat{h}_{t}) + q_{t}^{K} (\hat{k}_{t-1} - \hat{k}_{t}),$

where in equilibrium, $\hat{k}_t = \frac{1}{1-p}$, $Y_t = (1-p)\hat{c}_t + pc_t$, $L_t = (1-p)\hat{l}_t + pl_t$, $Y_t = A_t L_t^{1-\alpha}$, $r_t^K = \alpha A_t L_t^{1-\alpha}$, $w_t = (1-\alpha)A_t L_t^{-\alpha}$, $q_t^K = \beta \frac{\hat{c}_t}{\hat{c}_{t+1}}[r_{t+1}^K + q_{t+1}^K]$, and $r_t = \frac{r_{t+1}^K + q_{t+1}^K}{q_t^K} - 1$. For any sequence of repayment path $\{b_t\}_{t=0}^{\infty}$, it is easily calculated that the labor supply is efficient:

$$L_t = L^*.$$

We have shown that production is efficient even if there are household debt outstanding, as long as there is no borrowing constraint.

Equilibrium repayment path: We can show the following lemma.

Lemma 1. When A_t is constant, there exists the steady-state equilibrium where the borrowers' consumption and labor supply are constant over time.

Proof. We guess and verify later that $1 + r_t = \beta^{-1}$ and (q_t, w_t) are constant over time. Given that $1 + r_t = \beta^{-1}$ and (q_t, w_t) are constant over time, the first-order conditions (FOCs) and the envelope conditions for the borrowing household imply that

$$V_d(d_{t-1}, h_{t-1}) = V_d(d_t, h_t) = U_c(c_{t+1}, h_t, l_{t+1}).$$

As U(c, h, l) is additively separable, it is easily shown from the FOCs that (c_t, l_t, h_t) are constant over time: $(c_t, l_t, h_t) = (c, l, h)$ for $t \ge 0$. Thus the equilibrium is a steady-state, and constant prices are justified. The repayment of debt is given by $b_0 = wl - c + q(h_0 - h)$ and $b_t = b \equiv wl - c$ for $t \ge 1$, where h_0 is the initial amount of housing asset for the borrower.

This feature also appears in the model with a borrowing constraint, as long as the amount of debt is so small that the borrowing constraint is always nonbinding.

2.4 Model with borrowing constraint

Now we consider the model of household debt with borrowing constraint (BC, hereafter). There exist the lending households with measure 1 - p and the borrowing households with measure p. All households hold identical amount of houses, $h_0 = 1$, at the beginning of period 0. All borrowers owe the identical amount, (1 + r)d, as the initial debt at the beginning of period 0. We assume that lending households own all capital and lend consumption loans to the borrowing households. The borrowing households are subject

to the BC which is endogenously derived from the no default condition in the following paragraph. The derivation is not exactly the same as Jermann and Quadrini (2012), but is in the same spirit. The borrowing households solve

$$V_t(d,h) = \max_{c,h_{\pm 1},l,b} U(c,h,l) + \beta V_{t\pm 1}(d_{\pm 1},h_{\pm 1}),$$
s.t.
$$\begin{cases}
c+b \le w_t l + q_t(h-h_{\pm 1}), \\
c+b \le \phi q_t h, \\
d_{\pm 1} = (1+r)d - b,
\end{cases}$$
(1)

where the second constraint is the borrowing constraint, and the lending households solve

$$\max \sum_{t=0}^{\infty} \beta^{t} U(\hat{c}, \hat{h}, \hat{l}),$$

s.t. $\hat{c} \leq w_{t} \hat{l} + r_{t}^{K} \hat{k} + \hat{b} + q_{t}^{K} (\hat{k} - \hat{k}_{+1}) + q_{t} (\hat{h} - \hat{h}_{+1}).$

Derivation of borrowing constraint: We consider what would happen if the borrower defaults on the debt counterfactually, and derive the no default condition. For simplicity, we assume that the borrowing household borrows inter-period loan and intraperiod loan from different households. We call them the inter-period lender and intraperiod lender, respectively. At the beginning of period t, the outstanding amount of inter-period debt is $(1 + r_{t-1})d_{t-1}$. At this point, the borrower has option to default on the inter-period debt and exit the economy, while she can obtain the outside value $G_t(h_{L,t-1})$, where $h_{L,t-1} = (1 - \phi)h_{t-1}$, if she exits. The outside value can be specified as follows. We assume that when the borrower defaults on the inter-period debt in period t, the inter-period lender can seize ϕh_{t-1} units of houses immediately, and also can exclude the defaulter from the loan market. Thus, if the borrower defaults on the inter-period debt, she cannot borrow funds in the future. Thus $G_t(h_{L,t-1})$ is defined by

$$G_{t}(h) = \max_{c_{t}, h_{t}, l_{t}} U(c_{t}, h, l_{t}) + \beta G_{t+1}(h_{t}),$$

s.t. $c_{t} \leq w_{t}l_{t} + q_{t}(h - h_{t}).$

The participation constraint for the borrowing household is

$$V_t(h_{t-1}, d_{t-1}) \ge G_t(h_{L, t-1}).$$
(2)

We assume in what follows that ϕ is sufficiently close to 1 so that (2) is always nonbinding. Then, the borrowing household does not default on $(1+r_{t-1})d_{t-1}$ and borrows $c_t + b_t$ from the intra-period lender. She uses the intra-period debt to purchase c_t units of the consumer goods and repay b_t to the inter-period lender so that the remaining inter-period debt equals $d_t = (1+r_{t-1})d_{t-1} - b_t$. At this point, there arrives a chance to default on the intra-period debt $c_t + b_t$. Note that the borrower owes $c_t + b_t$ to the intra-period lender, while she owes d_t to the inter-period lender. She can default only on the intra-period debt at this point. Once the borrowing household defaults, the intra-period lender unilaterally seizes $\phi q_t h_{t-1}$ as the collateral, while the intra-period lender can impose no additional penalty on the defaulting borrower. The critical distinction is this assumption that the inter-period lender can seize a fraction of the future wage income of the borrower, whereas the intraperiod lender cannot seize any wage income. Thus, the household who defaults on the intra-period debt can just live on in this economy (with remaining houses $(1 - \phi)h_{t-1}$ and inter-period debt d_t). Therefore, the intra-period lender makes the intra-period loan no greater than $\phi q_t h_{t-1}$, implying the following constraint:

$$c_t + b_t \le \phi q_t h_{t-1}.\tag{3}$$

This constraint is also the no-default condition, as the borrower optimally defaults on the intra-period debt if and only if the above condition is violated.¹

Note: Endogenous borrowing constraints are usually formulated as the participation constraint for the borrower in, e.g., Albuquerque and Hopenhayn (2004, AH hereafter), following the spirit of Kehoe and Levine (1993). Difference between the BC in our model and in AH model is as follows. The BC in AH model is derived from the participation constraint with respect to the total debt, i.e., the sum of intra- and inter-period debt, while in our model we make a distinction between the participation constraint with respect to the total debt to the inter-period debt. In our model, the former is always binding and gives the borrowing constraint, whereas the latter is basically nonbinding. The difference between the two constraints is caused by the difference in financial technology between the inter-period and intra-period banks. That is, the inter-period lender can exclude the borrower from the loan market in the future when she defaults, whereas the intra-period lender can only seize collateral and cannot impose any

$$\label{eq:max_linear_$$

whereas the payoff when she defaults on the intra-period debt is

$$\max_{h_{+1}} U(c,h,l) + \beta V(d_{+1},h_{+1}),$$

s.t. $0 \le wl + (1-\phi)q_th - q_th_{+1}.$

The no default condition is that the former is no smaller than the latter, which implies the borrowing constraint (3).

¹Note that the borrowing constraint (3) is interpreted as the participation constraint with respect to the intra-period borrowing. As we see below, this constraint implies that the payoff for the borrower when she defaults on the intra-period debt is no greater than the payoff when she does not default. After (c, l) is fixed, the payoff of borrowing household when she does not default on the intra-period debt is

further penalty on the defaulter. This technological difference seems a realistic setting that reflects, for example, the differences in organizational structures and agency problems in short-term and long-term lenders in reality.

3 Equilibrium of the model with borrowing constraint

The equilibrium path is characterized by the initial value of debt, d.

3.1 Equilibrium with small initial debt

Suppose that the initial debt d is small and satisfies (4), specified below, so that the BC is always nonbinding. In this case, the FOCs and the envelope conditions imply that there exists the steady-state equilibrium in which the macroeconomic variables are invariant over time. Given d and h_0 (= 1), the variables $(c, d_+, h, l, \hat{c}, \hat{h}, \hat{l}, L, Y, w, r^K, q, q^K, \hat{k}, \hat{b})$ are determined by $c+(1+r)d-d_+ = wl+qh_0-qh$, $c+rd_+ = wl$, $w = \frac{\gamma c}{1-l}$, $q = \beta \left[\frac{\eta c}{h} + q\right]$, $\hat{c} =$ $w\hat{l} + r^K\hat{k} + \hat{b}$, $w = \frac{\gamma \hat{c}}{1-\hat{l}}$, $q = \beta \left[\frac{\eta \hat{c}}{\hat{h}} + q\right]$, $\hat{k} = \frac{1}{1-p}$, $\hat{b} = \frac{p}{1-p}rd$, $ph + (1-p)\hat{h} = 1$, Y = $(1-p)\hat{c} + pc$, $L = (1-p)\hat{l} + pl$, $Y = AL^{1-\alpha}$, $r^K = \alpha AL^{1-\alpha}$, $w = (1-\alpha)AL^{-\alpha}$, and $q^K = \beta [r^K + q^K]$.² The equilibrium values of these variables are expressed as functions of $d: c = c(d), d_+ = d_+(d), q = q(d), h = h(d)$. To justify the assumption of nonbinding BC, the value d must satisfy $(1+r)d \leq \phi q(d)h_0 - c(d) + d_+$ and

$$c(d) + rd_+(d) \le \phi q(d)h(d).$$

This condition can be rewritten as

$$d \le d^n,\tag{4}$$

where d^n is defined as the solution to $d = \beta \left\{ \phi q(d)h_0 - c(d) + \frac{\beta}{1-\beta} [\phi q(d)h(d) - c(d)] \right\}.$

3.2 Equilibrium with a large initial debt

Now we consider the case where the initial debt d exceeds d_n . In this case, the BC is binding at least for the first several periods. We define the labor wedge τ_t by

$$\frac{\gamma C_t}{1 - L_t} = (1 - \tau_t) w_t.$$

Note that $\tau_t = 0$ if $L_t = L^*$. We can prove the following lemma.

Lemma 2. In the period when the BC is binding, the total labor supply, L_t , is inefficiently small: $L_t < L^*$, and the labor wedge is deteriorated: $\tau_t > 0$.

²One equation is redundant due to the Walras' law.

Proof. We denote the Lagrange multipliers associated with the budget constraint and the BC by λ_t and μ_t , respectively. We define $x_t = \mu_t/\lambda_t$. When the BC is binding, $x_t > 0$. The equilibrium variables in period t is characterized by $\frac{\gamma c_t}{1-l_t} = \frac{w_t}{1+x_t}$, $\frac{\gamma \hat{c}_t}{1-\hat{l}_t} = w_t$, $Y_t = C_t = pc_t + (1-p)\hat{c}_t$, $w_t = (1-\alpha)A_tL_t^{-\alpha}$, $Y_t = A_tL_t^{1-\alpha}$, $L_t = pl_t + (1-p)\hat{l}_t$. These equations imply that

$$L_t = (1 - X_t)L^*,$$

where $X_t \equiv (1 - l_t) \frac{px_t}{1 + x_t}$, and

$$\tau_t = 1 - \frac{1}{1 + \frac{X_t}{\gamma L^*}}.$$

We used $\frac{(1-L^*)(1-\alpha)}{\gamma L^*} = 1$ to derive the second equation. As $X_t > 0$ when the BC is binding, these equations imply $L_t < L^*$ and $\tau_t > 0$ if and only if the BC is binding.

3.3 Steady-state equilibrium with $\phi = 1$

To characterize equilibrium dynamics with binding BC is not easy in general. When we set $\phi = 1$, however, there exists a steady-state equilibrium where macroeconomic variables are constant over time.

Lemma 3. Suppose that prices satisfy that $1 + r_t = \beta^{-1}$ and $(q_t, w_t) = (q, w)$ for all t. Then, given that $d_0 > d_n$, the solution to the borrower's problem (1) is constant over time: $(c_t, l_t, h_t) = (c, l, h)$ for all $t \ge 0$.

Proof. Given the constant prices, the solution for (1) is specified by the budget and borrowing constraints:

$$c_t + b_t = wl_t + q(h_{t-1} - h_t),$$

$$c_t + b_t = \phi q h_{t-1},$$

the FOCs:

$$\frac{1}{c_t} = \lambda_t + \mu_t,$$

$$\beta V_h(t+1) = \lambda_t q,$$

$$\frac{\gamma}{1-l_t} = w\lambda_t,$$

$$V_d(t+1) = -\beta^{-1}(\lambda_t + \mu_t)$$

and the envelope conditions:

$$V_d(t) = -\beta^{-1}(\lambda_t + \mu_t) = V_d(t+1),$$

$$V_h(t) = \frac{\eta}{h_t} + (\lambda_t + \phi\mu_t)q.$$

The first envelope condition implies that $\lambda_t + \mu_t = \lambda_{t+1} + \mu_{t+1}$, which in turn implies that $c_t = c_{t+1} = c$. As $qh_{t+1} = \frac{\beta\eta c(1+x_t)}{1-\beta(1+x_t)(1+\phi x_{t+1})(1+x_{t+1})^{-1}}$, where $x_t = \mu_t/\lambda_t$, it is shown that h_{t+1} is a function of x_t when $\phi = 1$. Then it is shown that $c = \frac{w(1+x_t)^{-1}}{\frac{\beta\eta}{1-(1+x_t)\beta}+\gamma}$, which implies that x_t is constant over time. Then, $l_t = 1 - \frac{\gamma}{\frac{\beta\eta}{1-(1+x_t)\beta}+\gamma} = l(x)$ and $h_t = \frac{wl_t}{q} = h(x)$ are also both constant over time. The repayment of debt is given by $b_0 = b_0(x) \equiv qh_0 - c(x)$ and $b_t = b(x) \equiv qh(x) - c(x)$. The initial debt d must satisfy

$$d = \beta [b_0(x) + (\beta^{-1} - 1)^{-1} b(x)].$$
(5)

As the initial value of d is set by an exogenous shock, the value of x is given by the above equation and the other variables are also set accordingly.

The variables for the lending households are determined as before. Thus, this lemma implies that there exists the steady-state equilibrium when the initial debt is large. Note that in this equilibrium, the aggregate labor L_t and the labor wedge τ_t are at the inefficient level permanently: $L_t = L < L^*$ and $\tau_t = \tau > 0$ for all t.

Maximum repayable debt: In this paper we (implicitly) assume that the participation constraint for the borrower (2) is always nonbinding. As $\phi = 1$, it is the case that $h_{Lt} = 0$ and $G(h_{Lt}) = G(0)$ is a constant. The participation constraint for the borrower with a large initial debt d is

$$\frac{1}{1-\beta} [\ln c(x) + \gamma \ln(1-l(x))] + \eta \ln h_0 + \frac{\beta \eta}{1-\beta} \ln h(x) \ge G(0),$$
(6)

where x is the solution to (5), given d. The participation constraint is this form because the timing of default on the inter-period debt is at the beginning of period 0. The maximum repayable debt d_{max} is defined as d that solves (6) with equality.³

4 Conclusion

It is shown that in the economy with endogenous borrowing constraint, the redistribution shock that makes a certain portion of people overly indebted can cause persistent shortage of aggregate demand and inefficiency due to labor-wedge deterioration. Overly accumulated debt may be a primal cause of a persistent recession. Debt forgiveness may be effective to restore aggregate demand and efficiency. Thus, the policy intervention that facilitates debt forgiveness may be welfare enhancing in a crisis-hit economy, where various agents are debt-ridden.

³If we introduce stochastic productivity shocks that hits the economy at the beginning of every period, then the participation constraint (2) can be violated and the borrower defaults on the inter-period debt in equilibrium. In this way, we can easily generalize our model so that there exist equilibrium default.

References

- Albuquerque, Rui, and Hugo A. Hopenhayn (2004) "Optimal Lending Contracts and Firm Dynamics," *Review of Economic Studies*, Vol. 71, No. 2, pp. 285–315, April.
- Chatterjee, Satyajit, Dean Corbae, Makoto Nakajima, and José-Víctor Ríos-Rull (2007)
 "A Quantittive Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, Vol. 75, No. 6, pp. 1525–1589, November.
- Eggertsson, Gauti B., and Neil R. Mehrotra (2014) "A Model of Secular Stagnation," NBER Working Papers 20574, National Bureau of Economic Research, Inc.
- Jermann, Urban, and Vincenzo Quadrini (2012) "Macroeconomic Effects of Financial Shocks," American Economic Review, Vol. 102, No. 1, pp. 238–271, February.
- Kehoe, Timothy J., and David K. Levine (1993) "Debt-Constrained Asset Markets," Review of Economic Studies, Vol. 60, No. 4, pp. 865–888.
- Kiyotaki, Nobuhiro, and John Moore (1997) "Credit Cycles," Journal of Political Economy, Vol. 105, No. 2, pp. 211–248.
- Livshits, Igor, James MacGee, and Michèle Tertilt (2007) "Consumer Bankruptcy: A Fresh Start," *American Economic Review*, Vol. 97, No. 1, pp. 402–418, March.
- Mian, Atif, and Amir Sufi (2014) The House of Debt, University of Chicago Press: Chicago.
- Nakajima, Makoto, and José-Víctor Ríos-Rull (2014) "Credit, Bankruptcy, and Aggregate Fluctuations," NBER Working Paper No. 20617, National Bureau of Economic Research, Inc., October.
- Summers, Lawrence H. (2013) "IMF Economic Forum: Policy Responses to Crises." speech at the IMF Fourteenth Annual Research Conference, Washington, DC, 9 November.