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Competition between Cities and Their Spatial Structure

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Abstract

Spatial competition has dealt with a single city over which firms compete. This paper extends it to a model with multiple cities. Specifically, we construct a spatial Cournot model with circular cities where consumers can choose which city to go under the spatial distribution of firms as determined by a location-quantity game. As a result, firms can agglomerate at a point even in the circular cities in equilibrium due to their ability to enhance consumer surplus at their locating city and rob more consumers of the rival city if they agglomerate to commit to the lower price. A welfare analysis shows excess agglomeration.

Keywords: Spatial Cournot, Intercity competition, Agglomeration, Spatial regulation, Welfare
JEL classification: L13, L52, R12

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1 Introduction

Since Hotelling's (1929) seminal paper, spatial competition has been a popular topic in economic theory. In economic literature, economists have assumed that there is only one space (city) and have ignored any effects from outside the city. In the real world, transportation networks have continually progressed and the linkage between cities is strengthening. Therefore, firms that consider the location problem care not only about the spatial distribution of rivals inside their locating city but also about competition from other cities. The objective of this paper is to clarify such global effects in spatial competition by incorporating an additional city. In other words, we study a spatial competition with twin cities that have market interactions with each other. More specifically, we address twin circular spaces that have several competing firms when each consumer chooses one of the spaces as her shopping place.[1]

In this paper, we consider a location-quantity game (spatial Cournot) to highlight the role of multiplicity of space. Spatial Cournot competition was developed by Hamilton *et al.* (1989) and Anderson and Neven (1991). As a result, they found spatial agglomeration in equilibrium.[2] Pal (1998) developed a spatial Cournot duopoly model with circular space and showed maximal differentiation (i.e., one firm located at 12 a.m. and one at 6 a.m.). Matsushima (2001) considered an arbitrary even number of firms in the framework and showed that half of the firms agglomerate at 12 a.m. and the rest do so at 6 a.m. These studies showed that the spatial Cournot models enable firms to agglomerate, but circular space makes firms separate to some extent.[3]

One of the interesting questions is whether all firms agglomerate at one point in a circular space (full agglomeration). For the full agglomeration to appear in equilibrium, we need additional components in the standard setting. Sun (2010) assumed directional delivery constraints and showed that two firms agglomerate at a point in a circular city when they deliver their products in different directions. Ago (2013) incorporated a demand-enhancing effect at a firm's location, which led to such an agglomeration.

In this study, we also observe agglomeration as a result of competition between cities. Agglomeration in a city reduces prices there owing to fiercer competition, which is a negative effect for firms. In contrast, a positive feature of agglomeration is that reduced prices lead to increased demand by reducing consumption in a rival city. The link between cities is positively correlated with the effect level, which establishes agglomeration in equilibrium (Proposition 1).

Early in the study, we assume that firms cannot move between the cities. After that, the restriction is relaxed: Firms can both move inside a city and between cities. In other words, there are two dimensions in which firms differentiate: where to locate inside a city and in which city to locate. Hence, the paper is closely related with multidimensional models of spatial competition. Tabuchi (1994) and Irmen and Thisse (1998) analyzed such models and showed that firms differentiate in only one dimension.[4] Interestingly, the title of the latter paper is "Hotelling was

[1] See Henkel *et al.* (2000) for a model with two marketplaces and consumers' choice of shopping place. In contrast to our model, their marketplaces are just points and lack internal spatial structure.

[2] Gupta *et al.* (1997) showed dispersed locations in a non-uniform distribution of consumers in a linear city.

[3] See Shimizu and Matsumura (2003), Gupta *et al.* (2004), and Matsumura and Matsushima (2012) for further examples.

[4] Anderson *et al.* (1989) and De Fraja and Norman (1993) also showed a similar result. When the good is sufficiently differentiated, firms agglomerate. This can be interpreted as differentiation in one dimension.

almost right," implying that the principle of minimal differentiation led by Hotelling (1929) is almost reproduced against the famous refute by d'Aspremont *et al.* (1979). Unlike their results, our equilibria contain differentiation in both dimensions (Proposition 5). Further, in contrast to their duopolistic models, we address more than two firms, although the spatial structure of our models and theirs differ.

We conduct a welfare analysis and show the possibility of excess agglomeration because the agglomerating incentive causes harmful fierce competition and a very small producer surplus.

The remainder of the paper is organized as follows. In Section 2, our model is presented. Section 3 leads to the equilibrium. Section 4 is devoted to a welfare analysis. Section 5 addresses some extensions: We allow the firms to move between cities, and we address an n-firm oligopoly. Section 6 concludes.

2 The model

We extend the typical spatial Cournot structure developed by Pal (1998) to a model with two cities. In the twin city economy, there are four firms, a homogeneous good, and consumers whose total mass is normalized to 1. The cities, indexed using r ($r = A, B$), are symmetric and circular, and their circumferences are both equal to 1. Let $L_r = [0, 1)$ denote the space of city r. Before Section 5, we assume that each city has two firms that cannot move between the cities. Each firm only serves its own city. The market size (mass of consumers) of city r, $N_r > 0$ ($N_A + N_B = 1$), is endogenously determined in the model, as explained later.

There are three stages in the game. In the first stage, each firm simultaneously chooses its location in L_r. In the second stage, each firm determines its supply amount simultaneously at each $z \in L_r$. That is, Cournot competition appears in each z. Finally, in the third stage, consumers determine which city to go to. Our equilibrium concept is subgame-perfection. Further details for each stage are presented in the following subsections.

2.1 Spatial Cournot game

First, we present our spatial Cournot competition through the first and second stages. For notational simplicity, subscript r is omitted here owing to the symmetry of the cities. Consumers are evenly distributed over each city. Hence, the density at each location $z \in L$ is N. As explained in the next subsection, N is a function of firms' spatial distribution. Therefore, firms decide their location considering consumers' reactions in the third stage.

Each consumer has the same inverse demand function for the homogeneous good as follows:

$$P = 1 - Q, \quad Q = q_1 + q_2, \tag{1}$$

where P, Q, and q_i are the price, the total supply amount, and the supply by firm i ($i = 1, 2$ is the firm index), respectively.[5] From (1), consumer surplus for a consumer is given by

$$cs = \frac{Q^2}{2}. \tag{2}$$

[5] If we adopt $P = a - bQ$ as a more general demand function, the essential results remain unchanged.

The firms have the same production technology, and their marginal costs are assumed to be constant and normalized to zero. Meanwhile, they bear transport costs and can set an independent supply amount at each location because arbitrage between consumers is assumed to be prohibitively costly. The transport cost function is linear with regard to shipping distance. Specifically, let $x \in L$ denote a firm's location. Then, to ship a unit of product from x to z, the transport cost is given by

$$T(x,z) = \min\{t|x-z|, t(1-|x-z|)\} \tag{3}$$

because the city is circular with a unit circumference, where $t > 0$ is a parameter and is assumed to be sufficiently low such that

$$t < 1. \tag{4}$$

This ensures that all firms serve the entire city irrespective of their locations.[6] Consequently, the local profit at z for firm i locating at x_i is

$$\pi_i(z) = Nq_i(z)\left[P(z) - T(x_i, z)\right] \tag{5}$$

Summing over the entire city, the total profit is given by

$$\Pi_i = \int_0^1 \pi_i(z)dz \quad \text{for } i = 1, 2. \tag{6}$$

2.2 The market size

Next, we define the market size in the third stage. Our consumers must choose which city to shop in. Let u_r be a payoff function for a consumer if she chooses city $r \in \{A, B\}$. Then, she maximizes u_r. After the second stage, all consumers know the spatial distribution of the firms and the supply schedule at any location. Hence, they can compute their consumer surplus from (2). We assume that the surplus is a component of the payoff because it is an appropriate measure of the benefit from shopping.

Furthermore, we assume that the consumers have heterogeneous preferences for cities. To present this heterogeneity, we adopt another spatial structure a lá Hotelling (1929): Consider a line segment of unit length $\Phi = [0,1]$, where cities A and B are exogenously allocated at $\phi_A = 0$ and $\phi_B = 1$, respectively. The consumers are uniformly distributed over Φ, and the density at each point $\phi \in \Phi$ is thus 1. Each consumer must pay the mismatch cost given by kd, where $k > 0$ is a parameter and d is the distance between her location and the city to which she goes. Consequently, we define the payoff function as follows:

$$u_r = cs_r - k\left|\phi - \phi_r\right|,$$

where cs_r is the consumer surplus in city r. Throughout the paper, we also assume that each consumer always buys the good.[7] Furthermore, we exclude the possibility that all consumers go to one city before Section 5. To do so, we require a sufficient condition under which each city

[6] This threshold depends on the number of firms. In Section 3.1, we provide a similar threshold for a case with an arbitrary number of firms (16).

[7] It is sufficient that we assume that the payoff is less than $-k$ when she does not buy the good.

always has a positive market size. The condition is given later as (14). As a result, the market size is given by

$$N_A = \frac{1}{2} + \frac{cs_A - cs_B}{2k}, \ N_B = 1 - N_A. \tag{7}$$

Thus far, we have not defined the consumers' distribution *inside* a city. In our model, we assume the following.

Assumption 1 *Consumers are uniformly distributed inside each city (i.e., the density function $g(z) = N_r$ for all $z \in L_r$), and this is common knowledge.*

Assumption 2 *No consumer knows her exact location inside the city.*

From these assumptions, we assume that consumers care about their expected values from decision-making. Let $cs_r(z)$ denote consumer surplus generated at market $z \in L_r$ in city r. Then, our consumers benefit not from *a specific* consumer surplus, $cs_r(z)$, but from *the average (expected)*consumer surplus, cs_r, which is defined as $cs_r = \int cs(z)dz$. In other words, cs_r represents homogeneous preferences for the respective cities, while the mismatch cost represents heterogeneous ones.

This type of assumption is typical in the literature on product differentiation. Suppose that our space is not geographical but characteristic. Then, the cities correspond to some product categories (e.g., orange juice vs. grapefruit juice), and location space corresponds to product characteristics (e.g., the degree of sweetness). It is often assumed that consumers do not know their exact valuation of the product until they actually buy and use it.[8] Another reason for this formulation is to better compare our results with those of previous studies, where any location in a circular city is featureless and consumers are uniformly distributed on the city (Pal, 1998). Another interpretation may be as follows. Suppose that relocation costs for firms are sufficiently low and firms change their locations often. Furthermore, consumers move less owing to high relocation costs. Then, consumers must make decisions as if the firms' locations may change often, and they may then evaluate a city using average values in the long run.

3 Equilibrium

We solve our three-stage game using backward induction. The problem in the third stage is solved in Section 2.2 as (7). Then, we proceed to the second stage.

3.1 Quantity equilibrium

From (5), the market size is just a multiplier and has no effects on determining equilibrium quantity. Owing to symmetry, we omit subscript r. The first-order conditions $\partial\pi_i(z)/\partial q_i(z) = 0$ yield the equilibrium quantity for firm i at z as follows (the asterisk represents the equilibrium value):

$$q_i^*(z) = [1 - 2T(x_i, z) + T(x_j, z)]/3 \quad \text{for } i, j \in \{1, 2\}, \ i \neq j. \tag{8}$$

[8] See Takahashi (2013) for a recent example of a paper with a similar structure to ours. Ben-Akiva et al. (1989) used a cylinder space to describe a two-dimensional space (i.e., location and brand specification). Such a cylinder can represent our model if we provide an appropriate assumption for consumer distribution.

The total supply amount at z is

$$Q^*(z) = \left[2 - \sum_{i=1}^{2} T(x_i, z)\right] /3. \tag{9}$$

Substituting (8) into (5), the local profit is

$$\pi_i^*(z) = Nq_i^*(z)^2$$

and the total profit is obtained by

$$\Pi_i^*(x_i) = \int_0^1 \pi_i^*(z)dz. \tag{10}$$

From (2), the (average) consumer surplus is

$$cs^* = \int_0^1 \frac{Q^*(z)^2}{2} dz. \tag{11}$$

3.2 Location equilibrium

We consider the location equilibrium[9] in the first stage when the quantities are given as in Section 3.1. Owing to symmetry, we can assume without loss of generality that $0 \leq x_1 \leq 1/2$ and $x_2 = 0$ in each city. For notational convenience, we omit the subscript representing two firms. Instead, let $0 \leq x_r \leq 1/2$ ($r = A, B$) denote the location of firm 1 in city r. Then, from (7), (8), (9), (10), and (11), we have the total profits of both firms[10] and the average consumer surplus in city r as

$$\begin{aligned} \Pi^*(x_r) &= \frac{N_r}{108}\left[12 - 6t + t^2(1 + 24x_r^2 - 32x_r^3)\right], \\ cs_r &= \frac{1}{54}\left[12 - 6t + t^2\left(1 - 3x_r^2 + 4x_r^3\right)\right], \end{aligned} \tag{12}$$

respectively, where

$$N_r = \frac{1}{2} + \frac{t^2}{108k}\left(4x_r^3 - 3x_r^2 - 4x_\zeta^3 + 3x_\zeta^2\right) \quad \text{for } r, \zeta \in \{A, B\},\ r \neq \zeta. \tag{13}$$

Because $-1/4 \leq 4x_r^3 - 3x_r^2 - 4x_\zeta^3 + 3x_\zeta^2 \leq 1/4$ for all $x_r, x_\zeta \in [0, 1/2]$, we have

$$k > \frac{t^2}{216} \tag{14}$$

as a sufficient condition for $0 < N_r < 1$.

The function in the square brackets of (12) increases through the firms' separation (greater x_r), which implies that a dispersion force exists because firms dislike fierce competition. Meanwhile, N_r increases by their approach (smaller x_r), which shows an agglomeration force in the

[9] A location pair (x_1^*, x_2^*) is a subgame-perfect Nash equilibrium if and only if $\Pi_i^*(x_i^*) \geq \Pi_i^*(x_i)$ for $\forall i$ and $\forall x_i \in [0, 1)$.

[10] Owing to symmetry, the profit function is the same for both firms.

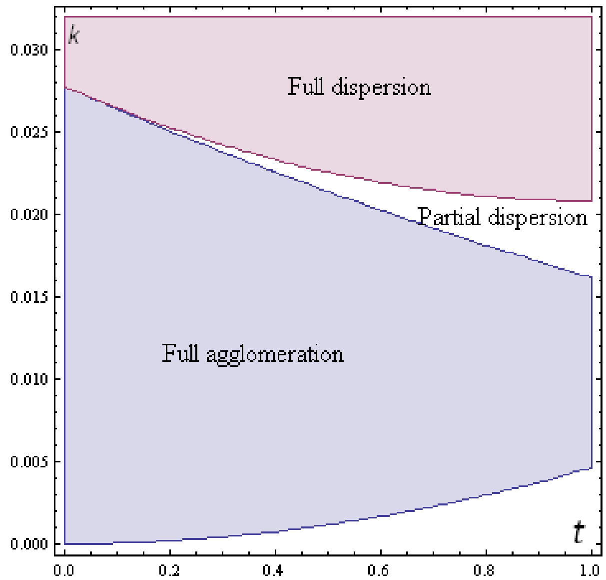

Figure 1: Classification of the equilibrium.

sense that firms can attract more consumers through enhanced consumer surplus when firms approach each other (cf. $\partial cs_r/\partial x_r < 0$). This trade-off determines the location equilibria.

Calculations yield three types of equilibria: i) an *agglomerated configuration (full agglomeration)*, where two firms agglomerate in each city; ii) a *fully dispersed configuration (full dispersion)*, where two firms are located at a maximal distance between them (1/2) in each city; iii) a *partially dispersed configuration (partial dispersion)*, where the distance between the firms is less than 1/2 and is the same between the cities. The formal result is as follows.

Proposition 1 *Let (4) and (14) hold. Then, there exists a unique location equilibrium: When* $k \leq (t^2 - 6t + 12)/432$*, the equilibrium is an agglomerated configuration. When* $k \geq (t^2 - 2t + 4)/144$*, the equilibrium is a fully dispersed configuration. Otherwise, if* $(t^2 - 6t + 12)/432 < k < (t^2 - 2t + 4)/144$*, the equilibrium is a partially dispersed configuration.*

Proof. See Appendix A. ■

Figure 1 shows the classification of an equilibrium in the parameter space. The intuition behind the proposition is as follows: When k is low, "consumer-surplus elasticity" between cities is high. Hence, the agglomeration incentive that leads to market expansion is stronger than that of dispersion to avoid competition.[11] When k is high, the elasticity is low. In other words, two cities become more independent against each other. Recall that Pal's (1998) one-city model has already shown that a fully dispersed configuration is the unique equilibrium.

[11] Our model addresses a homogeneous product. If we incorporate product differentiation into our structure, we can show that agglomeration can be reproduced. In other words, this main result is robust.

4 Welfare

To determine whether the location equilibrium is desirable, we conduct a welfare analysis.[12] Let a social welfare function be the sum of consumer surplus and profits as

$$SS = \sum_{r\in\{A,B\}} \left(N_r cs_r + \sum_{i\in\{1,2\}} \Pi_{ir} \right),$$

where N_r, cs_r, and Π_{ir} are the market size, consumer surplus, and total profit, respectively, given in a similar manner as in the previous sections. The social planner maximizes SS. We focus on the second-best scenario[13], in which the planner can only control the location and the supply amount is determined under Cournot equilibrium, as analyzed in Section 3.1 (the conditions (4) and (14) hold here). The number of firms in each city is maintained at 2 for a comparison with the equilibrium. Then, we have the following result.

Proposition 2 *In the second-best scenario, the social planner locates the firms as a fully dispersed configuration (the distance between the firms is* $1/2$*) in each city.*

Proof. See Appendix B. ■

We find *excess agglomeration* when the condition for agglomerated configuration or the partially dispersed configuration holds in Proposition 1 in the decentralized economy. As discussed before, consumers benefit from agglomeration (lower price by competition), while firms do from separation (relaxed competition) without a rival city. In this model, firms must take care of another city or consumer surplus of their own city. That incentive turns agglomeration into an equilibrium. The loss of producer surplus by agglomeration is greater than the consumer surplus gain; thus, excessive agglomeration may occur.

5 Extensions: Intercity move and an n-firm oligopoly

Thus far, we have excluded the possibility of intercity moves by firms. Here, we relax this restriction. That is, the firms can choose any location in both cities. Let n_r be the number of firms in city r ($n_A + n_B = n$, where n is the total number of the firms in the economy). The other structure is essentially preserved: For example, the demand schedule (1) is rewritten using $P = 1 - Q$, where $Q = \sum_{i=1}^{n_r} q_i$ and the linear transport costs (3) remain the same.

First, we show a spatial Cournot equilibrium. As in Section (3.1), the first-order conditions yield the equilibrium supply amount for firm i at z as follows:

$$q_i^*(z) = \frac{1}{n_r + 1} \left(1 - n_r T(x_i, z) + \sum_{j\neq i} T(x_j, z) \right) \quad \text{for } i = 1, \ldots, n_r. \tag{15}$$

[12] See Matsumura and Shimizu (2005) for a welfare analysis in spatial Cournot.

[13] The first-best scenario is obvious. The planner should set a fully dispersed configuration and marginal-cost pricing, because it minimizes total transport costs and removes the distortion.

Each firm always serves the entire city irrespective of firms' locations. In (15), because $\max T(x_i, z) = t/2$ and $\min \sum_{j \neq i} T(x_j, z) = 0$, $\min q_i^*(z) = (1 - n_r t/2)/(n_r + 1)$. Hence, we assume that

$$0 < t < \frac{2}{n} \tag{16}$$

as a sufficient condition. Summing (15), we have the total supply amount at z as

$$Q_r^*(z) = \frac{1}{n_r + 1} \left(n_r - \sum_{i=1}^{n_r} T(x_i, z) \right).$$

Then, consumer surplus is

$$cs_r = \int_0^1 \frac{Q_r^*(z)^2}{2} dz \tag{17}$$

The total profit for firm i is given by

$$\Pi_r^*(x_i) = N_r \int_0^1 q_i^*(z)^2 dz, \tag{18}$$

where N_r is the market size that is to be endogenously determined.

Furthermore, we should define what happens under $n_r = 0, 1$. When $n_r = 0$, we assume that all consumers go to the other city for shopping. That is, $N_r = 0$. When $n_r = 1$, the standard monopoly structure is applied. In the monopoly, location does not matter owing to symmetry, and we assume that the monopolist is located at $x_i = 0$ without loss of generality. Then, straightforward calculations yield the equilibrium supply schedule, consumer surplus, and total profits, as follows:

$$q_{mono}(z) = \frac{1}{2}\left(1 - T(0, z)\right),\ cs_{mono} = \frac{1}{96}\left(t^2 - 6t + 12\right),\ \Pi_{mono} = \frac{N_r}{48}\left(t^2 - 6t + 12\right).$$

Next, we consider the market size, which is determined in a manner similar to Section 2.2. We do not exclude the case where a city loses all consumers (corner solutions). The market size for city r, N_r, is defined as

$$N_A = \min\{\max\{0, \frac{1}{2} + \frac{cs_A - cs_B}{2k}\}, 1\},\ N_B = 1 - N_A \tag{19}$$

for $n_r \neq 0, 1$, where cs_r is given by (17). When $n_r = 0$, then $N_r = 0$. When $n_r = 1$, we must rewrite cs_r using cs_{mono} in (19). Substituting the market size into (18), we have total profits as a function of firms' locations.

Including all equilibria may be too complicated, because there are some equilibrium candidates like the partially dispersed configuration derived in Section 3.2. Therefore, we only focus on two equilibrium candidates: *symmetric configuration*, where firms locate equidistantly in each city ($x_i = (i - 1)/n$ for $i = 1, \ldots, n$), or *agglomerated configuration*, where firms agglomerate at a point in each city ($x_i = 0$ for all i). Recall that these configurations appear in equilibrium in the case where firms cannot move between the cities (Proposition 1).

5.1 Four-firm case

First, let the total number of firms be unchanged at four ($n = 4$) until the next subsection. This requires $t < 1/2$ to maintain a positive supply at any location for any firm, irrespective of the spatial distribution. There are three combinations of n_A-n_B (omitting symmetric cases) for the intercity firms' distributions: 2-2, 3-1, 4-0, which we analyze in the order 3-1, 4-0, and 2-2.

We first consider a case where a city has three firms and the other has one. Then, our question is whether symmetric or agglomerated configurations can be an equilibrium. The answer is negative, shown as follows.

Proposition 3 *Neither symmetric nor agglomerated configurations can be an equilibrium under which one city has three firms and the other has one.*

Proof. The proof details are available upon request. Its outline is as follows. In any configuration, some calculations show that no firm in the three-firm city has an incentive to change its location if $k \leq \check{k}(t)$ and that the firm in one-firm city also does not have this incentive if $k \geq \hat{k}(t)$. Yet, $\check{k}(t) < \hat{k}(t)$, which proves the proposition. ■

Recall that a lower k increases the importance of consumer surplus for consumers and thus for firms as well. Therefore, a city with many firms is sustainable under small k, but a city with few firms is sustainable when k is large. Thus, asymmetric distribution between the cities is incompatible.

Second, we consider the case where all (four) firms are located in a city. Clearly, agglomerated configuration cannot be an equilibrium because we can readily show that the profits of a firm monotonically increase with regard to the distance from the agglomeration. Therefore, we have to focus only on the symmetric configuration in the four-firm city. Furthermore, it is sufficient to check whether a firm has an incentive to change its location to the other city (empty city), because none of the firms have an incentive to move inside the city under this symmetric configuration.[14] We obtain the following result.

Proposition 4 *An agglomerated configuration is never an equilibrium while a symmetric configuration is an equilibrium if*

$$k \leq \frac{25\left(t^4 - 16t^3 + 92t^2 - 240t - 240\right)}{128\left(-31t^2 - 102t + 204\right)}$$

under which one city has all four firms.

Proof. Calculations show that the profit at the symmetric configuration in the four-firm city is $\pi_s = (7t^2 - 6t + 12)/300$. When a firm deviates to the empty city, it becomes a monopolist and its profit is $\pi_d = (t^2 - 6t + 12)[1/2 - (t^2 - 10t + 20)/256k]/48$ if $k > (t^2 - 10t + 20)/128$ and $\pi_d = 0$ if $0 < k \leq (t^2 - 10t + 20)/128$. Reducing $\pi_s \geq \pi_d$, we obtain the inequality in the proposition. ■

When k is small, more consumers prefer the four-firm city, where consumer surplus is high under fierce competition. Therefore, no firm has an incentive to move to the empty city because only limited consumers travel to the isolated firm.

[14] See Gupta et al. (2004) for the results of the one-city model.

Third and last, we consider a case in which each city has two firms. This case has been partially analyzed up to Section 3.2, and hence, we can utilize some part of the result in Proposition 1. Specifically, our question is whether agglomerated configuration is preserved under $k \leq (t^2 - 6t + 12)/432$ and whether symmetric configuration holds under $k \geq (t^2 - 2t + 4)/144$. Here, these preconditions for k ensure that no firm has an incentive to move inside its locating city from Proposition 1. Thus, we must consider only intercity moves.

Proposition 5 *The equilibrium is never an agglomerated configuration, while the equilibrium is a symmetric configuration if*

$$k \geq \frac{9\left(t^2 - 10t + 20\right)}{896}$$

under which each city has two firms.

Proof. First, we address an agglomerated configuration under $k \leq (t^2 - 6t + 12)/432$. Calculations show that the profit in the agglomerated configuration is $\pi_a = (t^2 - 2t + 4)/72$. Suppose that a firm deviates to the other city. We can readily show that the firm maximizes its profit when it locates as far as possible from the agglomeration in the destination city. Then, the profit using this deviation is $\pi_d = (t^2 - 6t + 12)/192$. Because π_a is always less than π_d, the former part of this proposition is established. Next, we proceed to a symmetric configuration under $k \geq (t^2 - 2t + 4)/144$. The profit at the symmetric configuration is given by $\pi_s = (t^2 - 2t + 4)/72$. When a firm deviates to the other city, the maximal profit becomes $\pi_{dd} = (t^2-2t+4)(t^2-10t+128k+20)/16384k$ if $k \geq (t^2-10t+20)/128$ or $\pi_{dd} = (t^2-2t+4)/64$ if $(t^2-2t+4)/144 \leq k < (t^2-10t+20)/128$. Reducing $\pi_s \geq \pi_{dd}$, we have $k \geq 9\left(t^2 - 10t + 20\right)/896$. Summarizing these, we obtain our result. ■

This shows the effect of an intercity move. In the case with no intercity moves, a symmetric configuration exists when $k \geq (t^2 - 2t + 4)/144$ (Proposition 1). Because $(t^2 - 2t + 4)/144 < 9\left(t^2 - 10t + 20\right)/896$, the intercity move breaks down the symmetric configuration when $(t^2 - 2t + 4)/144 \leq k < 9\left(t^2 - 10t + 20\right)/896$.

5.2 n-firm oligopoly with intercity migration

Next, we consider arbitrary oligopolies by $n > 4$ firms with an intercity move. Because it is difficult to have all equilibria, similar to previous cases, we focus on some interesting candidates as location equilibria: symmetric and agglomerated configurations. In addition, we only address the case in which all firms are located in one city, because there are too many combinations of n_A-n_B. We obtain the following proposition.

Proposition 6 *In any oligopoly with more than four firms in the economy, the equilibrium is never an agglomerated configuration in one city, while it may be a symmetric configuration in one city if k is sufficiently small.*

Proof. The former part of the proposition is straightforward. Suppose that all firms agglomerate at 0 in city A. Then, the profit of each firm is maximized if it relocates to $x_i = 1/2$. On the latter part, as Gupta et al. (2004) have shown, no firm has an incentive to relocate inside a circular city when a symmetric configuration is achieved. Hence, we need to consider only intercity migration. In a city of n firms with a symmetric configuration, we obtain consumer surplus as $cs_s = n^2(4-t)^2/32(n+1)^2$. If a firm deviates to become a monopolist at the other (empty) city, the consumer surplus in the monopolized city is given by $cs_m = (12-6t+t^2)/96$. When

$$0 < k < \frac{2n^2(t^2-9t+18) - 2n(t^2-6t+12) - t^2 + 6t - 12}{96(n+1)^2},$$

we find that $cs_s - k\phi > cs_m - k(1-\phi)$ for all $\phi \in [0,1]$. Then, the profit of the deviating firm becomes zero, which implies that the deviation is not profitable. Summarizing this, we obtain our desired result. ■

When k is small, consumer surplus matters more for consumers than transport cost does. When one city has all firms, no firm can earn enough profits by exiting from the city and being a monopolist in the other city. *Interestingly, this can be interpreted as a reproduction of multidimensional spatial competition (Tabuchi, 1994; Irmen and Thisse, 1998), that is, differentiation occurs in only one dimension.* On one hand, firms require product differentiation (i.e., the agglomerated configuration is not an equilibrium). On the other hand, under the symmetric configuration, the dispersed distribution of firms is considered a differentiation in one dimension (inside a city), but an additional differentiation in another dimension (intercity migration) is not necessarily required.

6 Concluding remarks

Spatial competition models have addressed a single city (space) and have shown that spatial Cournot with a circular city does not lead to agglomerated configuration. Yet, in the real world, competition is hierarchical: Firms compete inside the city (*inner-city competition*) and against firms in other cities (*intercity competition*). The paper sheds light on this phenomenon, and we have constructed a model of spatial Cournot with twin circular cities. By doing so, we observe agglomerated configuration as an equilibrium of the case with no intercity migration when mismatch (transport) costs are low because of higher price elasticity between cities, which is in a sharp contrast with the literature. This might be one reason why retail firms often agglomerate in a small district.

When we allow intercity move by firms, we reproduce the results of multidimensional spatial competition: differentiation should appear in only one dimension. Furthermore, our welfare analysis has shown that such agglomeration is not desirable from the viewpoint of the second-best. This may be why governments rationalize locational restrictions on concentrated retail districts. Finally, we assume a special form of hierarchy in the inner- and inter-city structure. Other formulations may be considered in future research.

Appendix

A. Proof of Proposition 1

We consider the best response of firm 1, x_r^*, in city r. From (12) and (13), we have

$$\frac{\partial \Pi^*(x_r)}{\partial x_r} = \frac{t^2}{1944k} x_r(1-2x_r)f(x_r), \tag{20}$$

where

$$f(x_r) = (16x_r^2(4x_r-3) - 8x_\zeta^2(4x_\zeta-3) - 1)t^2 + 6t - 12 + 432k \quad \text{for } r \neq \zeta.$$

Differentiating $f(x_r)$ with respect to x_r, we have

$$\frac{\partial f(x_r)}{\partial x_r} = 96t^2 x_r(2x_r - 1). \tag{21}$$

First, we seek an interior solution, that is, a partially dispersed configuration ($0 < x_r^* < 1/2$). From (20), the first-order condition requires $f(x_r^*) = 0$. $f(x_r)$ is decreasing in $[0, 1/2]$ from (21), which implies that the equation $f(x_r) = 0$ has a unique solution in $(0, 1/2)$ if and only if $f(0) > 0$ and $f(1/2) < 0$ (i.e., the solution satisfies the second-order condition). Furthermore, we have

$$f(x_A) - f(x_B) = 24t^2(x_A - x_B)g(x_A, x_B),$$

where $g(x_A, x_B) = 4x_Ax_B + 4x_A^2 + 4x_B^2 - 3x_A^2 - 3x_B^2$. We have $g(x_A, x_B) = 0 \iff (x_A, x_B) = (0, 0), (1/2, 1/2)$. Because $f(x_A) - f(x_B) = 0$ must hold from the first-order condition, a partially dispersed configuration, if any, must be symmetric ($x_A = x_B$). We can readily show that the conditions of $f(0) > 0$, $f(1/2) < 0$ with $x_A = x_B$ are equivalent to $(t^2 - 6t + 12)/432 < k < (t^2 - 2t + 4)/144$. Then, the best response of firm 1 is uniquely determined as x_r^* such that $f(x_r^*) = 0$ in each city. Next, we proceed to firm 2. Because of the symmetry of the profit function, the optimal distance from firm 1 is also x_r^* for firm 2. In other words, the best response of firm 2 is 0 when firm 1 chooses x_r^*. Consequently, we have the unique, partially dispersed configuration when $(t^2 - 6t + 12)/432 < k < (t^2 - 2t + 4)/144$, where two firms are located with the distance between them being x_r^* such that $f(x_r^*) = 0$ in each city.

The other candidate is a corner solution ($x_r^* = 0, 1/2$). Again, we analyze the behavior of firm 1. Because $f(x_r)$ is a decreasing function from (21), we find that $f(x_r)|_{x_r=0} \leq 0 \iff x_r^* = 0$ and $f(x_r)|_{x_r=1/2} \geq 0 \iff x_r^* = 1/2$. In addition, owing to symmetry, we can exclude an asymmetric equilibrium, $(x_A^*, x_B^*) = (0, 1/2)$ or $(1/2, 0)$. Reducing $f(\cdot) \leq 0$ at $x_A = x_B = 0$, we have $k \leq (t^2 - 6t + 12)/432$. Then, the best response of firm 2 is also 0; that is, the agglomerated configuration is a unique equilibrium in this case. Furthermore, reducing $f(\cdot) \geq 0$ at $x_A = x_B = 1/2$, we obtain $k \geq (t^2 - 2t + 4)/144$. In this case, the best response of firm 2 is also 1/2. This implies that the fully dispersed configuration is a unique equilibrium. In summary, three types of equilibria uniquely exist in each of the three parameter domains, which proves the proposition.

B. Proof of Proposition 2

Without loss of generality, we assume that firm 2 is located at 0 in each city. We use the same notation as in Section 3. From the results in Section 3, we have

$$\begin{aligned} SS &= 7t^4 \left[x_A^2(3-4x_A) - x_B^2(3-4x_B)\right]^2 /5832k - [48-24t \\ &\quad -t^2\left(7x_A^2(4x_A-3)+7x_B^2(4x_B-3)-4\right)]/108. \end{aligned} \tag{22}$$

First, we consider an interior solution ($0 < x_A, x_B < 1/2$). Differentiating (22) with respect to x_r, we have

$$\frac{\partial SS}{\partial x_r} = \frac{7t^2}{486k} x_r(1-2x_r)f(x_r),$$

where

$$f(x_r) = t^2\left[x_r^2(4x_r-3) - x_\zeta^2(4x_\zeta-3)\right] + 27k \quad \text{for } r \neq \zeta,\ r, \zeta \in \{A, B\}.$$

Therefore, we require $f(x_A) = f(x_B) = 0$ for an interior solution. Next, we have

$$f(x_A) - f(x_B) = 2t^2(x_A - x_B)g(x_A, x_B),$$

where $g(x_A, x_B) = 4x_Ax_B + 4x_A^2 + 4x_B^2 - 3x_A^2 - 3x_B^2$. We find $g(x_A, x_B) = 0 \iff (x_A, x_B) = (0, 0), (1/2, 1/2)$. Hence, an interior solution, if any, must be symmetric ($x_A = x_B$). Evaluating $f(x_A)$ at $x_A = x_B$, we have

$$f(x_A)|_{x_A=x_B} = 27k > 0,$$

which implies that no interior solution exists.

Next, we consider a corner solution ($x_r = 0, 1/2$). We find

$$\left.\frac{\partial SS}{\partial x_A}\right|_{x_A=0} = 0, \quad \left.\frac{\partial^2 SS}{\partial x_A^2}\right|_{x_A=0} > 0$$

for $\forall x_B \in [0, 1/2]$, which implies that the agglomerated configuration, $x_r = 0$, cannot be a solution. The only candidate for the second-best is $x_r = 1/2$. We find that

$$\begin{aligned} & SS|_{(x_A,x_B)=(1/2,1/2)} - SS|_{x_B=1/2} \\ = \ & \frac{7cst^2}{93312k}(1+4x_A)(1-2x_A)^2\left[216k + t^2(1+4x_A)(1-2x_A)^2\right] > 0 \end{aligned}$$

for $\forall x_A \in [0, 1/2)$. Hence, we conclude that a fully dispersed configuration, $(x_A, x_B) = (1/2, 1/2)$, maximizes SS if the second-best is a corner solution. Thus, we prove the proposition.

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