The Prodigal Son:
Does the younger brother always care for his parents in old age?

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Abstract

Previous studies have shown that the older sibling often chooses to live away from his elderly parents with the aim of free-riding on the care provided by the younger sibling. In the presented model, we incorporate income effects to depict the alternative pattern frequently observed in Eastern countries, namely that the older sibling lives near his or her parents and cares for them in old age. By generalizing the existing model, we show the three cases of the elderly parents being looked after by (1) the older sibling, (2) the younger sibling, and (3) both siblings, in accordance with the relative magnitude of the income effect and the strategic incentive for one sibling to free-ride on the other. Our study also investigates the effect of changes in relative incomes on the level of total care received by their parents. We find that the overall care provided by both children increases as their aggregate income rises, but that at some point, this may be reduced because of the incentive to free-ride.

Keywords: Income effect, Voluntary provision of public good, Location choice, Sibling, Elderly care arrangement

JEL classification: H41, J17

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1. Introduction

A certain man had two sons. The younger of them said to his father, “Father, give me my share of your property.” He divided his livelihood between them. Not many days after, the younger son gathered all of this together and traveled into a far country (Luke, 15:11-13).

Caring for elderly parents has long been an important role of the family institution. Indeed, informal care by adult children even remains prevalent in many developed countries where social security and a residential care market are well established (OECD, 2005). Deciding who cares for elderly parents is a major issue in practice, especially when there is a smaller number of siblings because in this scenario each child shares a larger part of the financial burden. Moreover, the identity of the primary caregivers of elderly parents is of interest from an economics standpoint because caring for parents is a public good as long as they are all altruistic toward their parents. Hence, the voluntary provision of caregiving by children will undersupply care for those parents that have a significant free-rider problem.

The primary caregiver among family members differs between Western and Eastern countries. While studies of Western countries show that it is typically the younger son (Konrad et al., 2002; Fontaine et al., 2009), the oldest son more frequently takes on this responsibility in Eastern countries (McLaughlin and Braun, 1998; Liu and Kendig, 2000). On this difference, the pioneering work of Konrad et al. (2002) considers the private provision of parents’ care by two children in a game-theoretic model where their location choices affect the cost of visiting their parents. These authors show that the first-born child uses his first-mover advantage, so that he firstly chooses the location sufficiently away from his parents and free-rides on his altruistic younger brother. However, they suggest that this finding can change depending on parents’ bequest decisions, as originally proposed by Bernheim et al. (1985). In this vein, recent studies have theoretically considered that siblings may compete for the bequest they receive (Chang and Weisman, 2005; Faith et al. 2008); however, the causality of the strategic bequest motive remains inconclusive (Sloan et al. 1997; Perozek, 1998; Pezzin and Schone, 1999; Sloan et al., 2002; Kureishi and Wakabayashi, 2010; Wakabayashi and Horioka, 2009; Johar et al., 2014).

Different from the strategic motive mentioned above, the findings of the present study offer new insights into siblings’ caregiving behaviors, focusing on the effect of the income gap between two siblings on their location choices and caregiving decisions, which the

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2 At that time, he knew that he was supposed to receive only half of what the older sibling would do (Deuteronomy 21:17).
3 According to OECD (2005), 90% of informal care is provided by family and friends in OECD countries, with the care provided by children differing by nation: 24% in Australia, 28% in Germany, 48% in Ireland, 60% in Japan, 55% in Korea, 38% in Spain, 46% in Sweden, 43% in the United Kingdom, and 41% in the United States.
4 For instance, Agingcare.com estimates that 34 million Americans are personally providing care for older family members. Of these caregivers, 34% are spending $300 or more a month of their own money and 54% have sacrificed spending money on themselves to care for their parents.
5 Our study excludes the gender issue to clarify our contribution by only considering the problem of male siblings as in the theoretical mode of Konrad et al. (2002). If we allow the both male and female siblings, our results can be altered by the additional effects coming from the different productivity in domestic work including caregiving or the different opportunity cost due to gender wage gap. Some empirical studies explore the effects of children’s gender difference on care arrangement.
6 Other possible explanations for caregiving by the first-born child include Cox (1987)’s exchange model and Chu (1991)’s dynasty model.
7 For an excellent survey on intergenerational transfer from children to parents, see Maruyama and Nakamura (2012).
8 Pezzin et al. (2014) also give an interesting example that the distance from parents affects the care
quasi-linear utility function of Konrad et al. (2002) overlooks. Although location has been shown to affect decisions on caring for parents, the economic circumstances of siblings might also influence their location decisions and thereby their capacity to care for their elderly parents. In the model of Konrad et al. (2002), the older son always uses his position as a first-mover. However, the availability of first-mover advantage depends on the income differences between siblings since the older son cannot free-ride to younger son who has no income to spend for caregiving his parents. Income gap between siblings operates on the availability of first-mover advantage, so that the older son may serve as the primary caregiver. We therefore extend and generalize the work of Konrad et al. (2002) by additionally incorporating the role of the income differential between two siblings.  

Specifically, we consider the income effect so that the level of public good of caregiving depends not only on the marginal cost of caregiving provision (i.e., distance from parents) but also on the relative incomes of the siblings involved. In particular, income in our model is defined in a broad sense, including fixed wealth such as land. Until relatively recently, the eldest son took priority in inheriting the family estate, even in developed countries. Therefore, if siblings recognize that income gap is significant, this may affect the equilibrium characteristics in their strategic interactions.

By incorporating income effects, our generalized model classifies three cases of caregiving, namely by (i) only the older brother, (ii) only the younger brother, and (iii) both siblings. In particular, the presented findings show that the older brother cares for his parents when his income is sufficiently large compared with his younger brother, concurring with existing evidence of the positive relationship between the elder brother’s caregiving and the bequest he is expected to receive. By using our model, this relationship can then be interpreted as simply an income effect because the bequest decision can influence relative sibling income to a large degree. Our study also demonstrates that a higher aggregate income of all siblings may reduce overall care because the change in relative income between them accompanied by the change in aggregate income can induce the strategic incentive for one sibling to free-ride on the other.

The remainder of this paper is organized as follows. In section 2, we present the model, while sections 3 and 4 discuss the results of siblings’ location choices and provision of caregiving as a public good. Section 5 concludes.

2. Basic Model
Following Konrad et al. (2002), we consider the choices of location and care provision by adult children who are altruistic toward their elderly parent(s). Consider a family that consists of the parent(s), a first-born child \((i = 1)\), and a second-born child \((i = 2)\). We define the arrangement. They present a model in which every child avoid to live with their parents since they know that once one child live with his parents, he need to take the entire responsibility for caregiving.  

Reiner and Sielder (2009) also present an insightful model where the siblings negotiate at the third stage of care provision, and their choices of employment and location affect their bargaining power.

In studies such as Byrne et al. (2009) and Antman (2012), the monetary cost and the opportunity cost due to caring time are distinguished by considering both of formal and informal cares.

For instance, until 1947, the eldest son had the right to all the family assets in Japan, and the eldest son had been given a special status in Korea under the householder system until the legal reforms implemented in 2005.
utility function of child $i$ as

$$U_i = x_i^\alpha G^{1-\alpha},$$

(1)

where $x_i$ is the private consumption of child $i$ and $G$ is the total amount of care their parents receive from both children. Here, $1-\alpha$ represents the magnitude of altruism: $\alpha=1$ if the child displays no altruistic behavior and $\alpha=0$ if the child has extremely strong concern about his parents and no interest in private consumption. We thus simply assume that the care received by the parents is the sum of the care provided by both children (overall care hereafter):

$$G = g_1 + g_2$$

(2)

where $g_i$ is the care provided by child $i$. Following Konrad et al. (2002), we assume that $g_i$ denotes the number of visits by child $i$, and thus $G$ is the total number of visits that parents receive. The budget constraint of child $i$ is given by

$$y_i = x_i + (1+t_i)g_i$$

(3)

where $y_i$ is income and $t_i \in [0,1]$ denotes the spatial distance from the parents. While $y_i$ is an exogenous variable, $t_i$ is chosen by each child: $t_i = 0$ if he decides to live with his parents and $t_i = 1$ if he chooses his place of residence as remotely from his parents as possible. Here, $y_i$ does not depend on the location, which would be justified by the assumption that labor market is fully integrated and thus wage income does not depend on the location. To focus on the outcome when two children differ in timing of decision-makings and their income, we count out any gender-related differences among children.

Following the standard sequential-move decision-makings, the timings of the game are as follows:

1. Child 1 chooses the location $t_1$.
2. Child 2 chooses the location $t_2$.

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12 Eq. 3 can be interpreted as the constraint for individual who allocates his time for parental care and work. Suppose that $y_i$ is the exogenous (non-labor) income, $w$ the market wage, $g_i$ the hours for parental care, and $t_i$ the location factor that represents the additional travel time to visit his parents. Then, the budget constraint will be given by $X_i = y_i + w\left[1 - (1+t_i)g_i\right]$. Setting $w=1$ and rearranging, we have Eq. 3.
3. Both children decide on their levels of care \((g_1, g_2)\), simultaneously.

The outcome of this game can be obtained as the sub-game perfect Nash equilibrium. Hence, we apply the concept of backward induction and solve the problem from the final stage.

From (1)-(3), based on the premise of an interior solution, the reaction function in the third stage can be obtained as follows:

\[ g_1 = \frac{(1 - \alpha)y_1}{(1 + t_1)} - \alpha g_2, \]

\[ g_2 = \frac{(1 - \alpha)y_2}{(1 + t_2)} - \alpha g_1 \]

(4) (5)

To convey our main message in the clearest way, we simply assume \(\alpha = 1/2\) in the following analysis.

3. Equilibrium

Because the model contains corner solutions, we derive the equilibrium by classifying the outcomes into three cases: (i) both children care for their parents, \(g_1 > 0, g_2 > 0\); (ii) child 1 cares for his parents and child 2 free-rides, \(g_1 > 0, g_2 = 0\); and (iii) child 2 cares for his parents and child 1 free-rides, \(g_1 = 0, g_2 > 0\).

3.1. Both children care for their parents

We first analyze case (i). Based on (4) and (5), the conditions that result in case (i)'s equilibrium is given as follows:

\[ \frac{1 + t_2}{2(1 + t_1)} < \frac{y_2}{y_1} < \frac{2(1 + t_2)}{1 + t_1}. \]

(6)

If (6) holds, child \(i\) chooses the level of \(g_i\) as follows:

\[ g_1 = \frac{2y_1(1 + t_2) - y_2(1 + t_1)}{3(1 + t_1)(1 + t_2)} \text{ and } g_2 = \frac{2y_2(1 + t_1) - y_1(1 + t_2)}{3(1 + t_1)(1 + t_2)} \]

(7)

From (7), we have

\[ g_1 > g_2 \iff \frac{1 + t_2}{1 + t_1} > \frac{y_2}{y_1}, \]

implying that the smaller the distance from the parents and the higher his income, the more likely it is that child \(i\) provides a higher level of care.

By using (1)-(3) with (6), the utility of child 2 in the second stage is given by
\[ U_2 = \frac{(1 + t_2)y_1 + (1 + t_1)y_2}{3(1 + t_1)\sqrt{1 + t_2}}. \]

In stage 2, child 2 chooses \( t_2 \) in order to maximize \( U_2 \). The first- and second-order conditions are given as follows:

\[
\frac{\partial U_2}{\partial t_2} = \frac{(1 + t_2)y_1 - (1 + t_1)y_2}{6(1 + t_1)\sqrt{1 + t_2}},
\]

\[
\frac{\partial^2 U_2}{\partial t_2^2} = \frac{y_1}{4(\sqrt{1 + t_2})^3} \left( \frac{y_2}{y_1} - \frac{1 + t_2}{3(1 + t_1)} \right) > 0.
\]

The sign of (8) is ambiguous because the marginal benefit and marginal cost of living away from the parents work oppositely. If child 2 lives away from his parents, he can leave his older brother to provide more caregiving, which also increases the caregiving cost. The last inequality in (9) comes from (6), showing that the location choice of child 2 becomes the corner solution at either \( t_2 = 0 \) or \( t_2 = 1 \). Specifically, his choice is determined by the following equation:

\[
U_2|_{t_2=1} - U_2|_{t_2=0} = \frac{y_1(2 - \sqrt{2})}{6} \left( \frac{\sqrt{2}}{1 + t_1} - \frac{y_2}{y_1} \right).
\]

From (10), we have the reaction function of child 2 as follows:

\[
t_2 = 0 \quad \text{if} \quad \frac{y_2}{y_1} < \frac{\sqrt{2}}{1 + t_1}. \quad (11)
\]

\[
t_2 = 1 \quad \text{if} \quad \frac{y_2}{y_1} > \frac{\sqrt{2}}{1 + t_1}. \quad (12)
\]

(11) and (12) show that the location choice of child 1 in the first stage affects the location choice of child 2 in the second stage. Child 1 recognizes its influence on child 2’s location choice and thus strategically chooses where to live before his younger brother does so.

Suppose that child 1 chooses \( t_1 \) in order to satisfy (11). This choice makes child 2 live with his parents, i.e. \( t_2 = 0 \). In case (i), (6), which ensures \( g_1 > 0 \) and \( g_2 > 0 \), is limited to

\[
\frac{\sqrt{2}}{1 + t_1} < \frac{y_2}{y_1} < \frac{2}{1 + t_1}. \quad (13)
\]

When (13) is satisfied, the care provided by each child is given by

\[
g_1 = \frac{2y_1 - y_2(1 + t_1)}{3(1 + t_1)} \quad \text{and} \quad g_2 = \frac{2y_2(1 + t_1) - y_1}{3(1 + t_1)}.
\]
To derive the location of child 1 in the first stage, we insert (14) into the utility function of child 1. The objective function of child 1 in the first stage is now given as

$$U_1 = \frac{y_1 + (1 + t_1) y_2}{3\sqrt{1 + t_1}}.$$

The first- and second-order conditions for the maximization problem are thus obtained by

$$\frac{\partial U_1}{\partial t_1} = \frac{(1 + t_1) y_2 - y_1}{6\sqrt{1 + t_1}},$$

$$\frac{\partial^2 U_1}{\partial t_1^2} = \frac{3y_1 - (1 + t_1) y_2}{12\left(\sqrt{1 + t_1}\right)^3} > 0.$$

The last inequality in (16) comes from (13), showing that the location choice of child 1 becomes the corner solution at either $t_1 = 0$ or $t_1 = 1$. To determine child 1’s choice, we check the sign of

$$U_1|_{t_1=1} - U_1|_{t_1=0} = \frac{y_1(2 - \sqrt{2})}{3\sqrt{2}} \left( \frac{y_2}{y_1} - \frac{\sqrt{2}}{2} \right).$$

Hence, we find that

$$t_1 = 1 \quad \text{if} \quad \frac{y_2}{y_1} > \frac{\sqrt{2}}{2},$$

$$t_1 = 0 \quad \text{if} \quad \frac{y_2}{y_1} < \frac{\sqrt{2}}{2}.$$

(11) and (17) can hold at the same time, and thus $t_1 = 1$ and $t_1 = 0$ can be an equilibrium. In this case, based on (13), $\sqrt{2}/2 < y_2/y_1 < 1$ ensures that both children care for their parents. By contrast, (11) and (17) do not hold at the same time, and thus $t_1 = 0$ and $t_2 = 0$ do not hold at the equilibrium.

**Proposition 1.** If $\sqrt{2}/2 < y_2/y_1 < 1$, then $t_1 = 1$ and $t_2 = 0$. In this case, the care provided by each child is given by $g_1 = (y_1 - y_2)/3$ and $g_2 = (4y_2 - y_1)/6$.

Proposition 1 shows that overall care is $G = (y_1 + 2y_2)/6$ and that child 1 provides
less care than child 2 does ($g_1 < g_2$) even though child 1 has a higher income than child 2 has ($y_2/y_1 < 1$). This finding is explained as follows. Child 1 has an incentive to induce child 2 to provide more care and, therefore, he lives far away from his parents in the first stage, which forces child 2 to live at home. While the caregiving cost of child 1 is high, that of child 2 is low, and thus the care provided by child 2 is greater than that by child 1. Owing to the strategic location choice, child 1 places the burden of care on child 2 and enjoys higher private consumption.

Now suppose that child 1 chooses his location $t_1$ in order to satisfy (12). In this case, child 2 chooses $t_2 = 1$, and therefore the range given by (6), which ensures $g_1 > 0$ and $g_2 > 0$, is limited to

$$\frac{1}{1+t_1} < \frac{y_2}{y_1} < \frac{\sqrt{2}}{1+t_1}, \quad (19)$$

When $t_2 = 1$, the children choose

$$g_1 = \frac{4y_1 - y_2 (1+t_1)}{6(1+t_1)} \quad \text{and} \quad g_2 = \frac{y_2 (1+t_1) - y_1}{3(1+t_1)}.$$  

By inserting (20) into the utility function of child 1, we have

$$U_1 = \frac{2y_1 + (1+t_1)y_2}{6\sqrt{1+t_1}}.$$  

Thus, child 1 chooses $t_1$ to maximize his utility. The first- and second-order conditions are given as follows:

$$\frac{\partial U_1}{\partial t_1} = \frac{(1+t_1)y_2 - 2y_1}{12(1+t_1)^2}, \quad (21)$$  

$$\frac{\partial^2 U_1}{\partial t_1^2} = \frac{y_1}{24(1+t_1)^{\frac{7}{2}}} \left( \frac{6}{1+t_1} - \frac{y_2}{y_1} \right) > 0. \quad (22)$$  

The last inequality comes from (19). The first- and second-order conditions show that the location choice of child 1 becomes the corner solution at either of $t_1 = 0$ or $t_1 = 1$, which is determined by checking the sign of

$$U_1|_{t_1=1} - U_1|_{t_1=0} = \frac{y_1 (\sqrt{2} - 1)}{36} \left( \frac{y_2}{y_1} - \sqrt{2} \right) < 0.$$  

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The inequality comes from (19), and thus $t_1 = 0$. In this case, (19) is rewritten as

$$1 < \frac{y_2}{y_1} < \sqrt{2}.$$  

By summarizing the above discussion, we obtain the following result.

**Proposition 2.** If $1 < \frac{y_2}{y_1} < \sqrt{2}$, $t_1 = 0$ and $t_2 = 1$. In this case, the care provided by each child is given by $g_1 = \frac{4y_1 - y_2}{6}$ and $g_2 = \frac{y_2 - y_1}{3}$.

Proposition 2 shows that the overall care provided by both children is given by $G = \frac{2y_1 + y_2}{6}$. Furthermore, we find that child 1 provides greater care compared with child 2 even though the income of child 1 is smaller than that of the income of child 2 ($y_1 < y_2$); $g_1 > g_2$. This result is explained as follows. Since $y_1 < y_2$, child 2 has sufficient disposable income to care for his parents even if he lives a slightly remote distance from his parents. By contrast, the income of child 1 is small compared with child 2, and thus living far from the parents is costly for child 1. Since child 1 recognizes that even if child 2 chooses $t_2 = 1$, child 1 still provides care to their parents and thus he chooses $t_1 = 0$ to reduce the cost of caregiving. This choice forces child 2 to live far from the parents, reducing further the care he provides compared with child 1.

### 3.2. Corner solutions

In the previous subsection, we restricted our analysis to the case of the interior solution in which both children care for their parents. We now study the equilibrium pattern when $\frac{\sqrt{2}}{2} < \frac{y_2}{y_1} < \sqrt{2}$ does not hold. In this case, only one child cares for his parents and the other free-rides.

#### 3.2.1 Child 1 cares for his parents and child 2 free-rides

When $\frac{y_2}{y_1} < \sqrt{2}/2$, child 2 does not have sufficient income to care for his parents. In this case, the care provided by each child is given by

$$g_1 = \frac{y_1}{2(1 + t_1)} \quad \text{and} \quad g_2 = 0.$$  

By substituting (23) into the utility function of child 2, we have

$$U_2 = \frac{-y_1y_2}{2(1 + t_1)}.$$  

Since the utility of child 2 does not depend on his location, he is indifferent when choosing his location, showing that child 1 cannot select his location in the first stage to control the
location of child 2 determined in the second stage. We denote the location of child 2 as \( \tilde{t}_2 \in [0,1] \). Then, the utility of child 1 in the first stage is given by

\[
U_1 = \frac{y_1^2}{4(1 + t_1)}.
\]

Hence, child 1 chooses \( t_1 = 0 \) to maximize his utility. By using \( t_1 = 0 \) and \( t_2 = \tilde{t}_2 \) with (23), we have the following result.

**Proposition 3.** If \( y_2/y_1 < \sqrt{2}/2 \), then \( t_1 = 0 \), \( t_2 = \tilde{t}_2 \), \( g_1 = y_1/2 \), and \( g_2 = 0 \).

Since child 2 does not provide care to his parents, he has no preference on his location. Child 1 cannot use his location to induce child 2’s location to be the place child 1 prefers. In addition, child 1 cares for his parents and thus he chooses to live with them to minimize the caregiving cost.

### 3.2.2. Child 2 cares for his parents and child 1 free-rides.

When \( \sqrt{2} < y_2/y_1 \), the income of child 1 is so small that he cannot care for his parents. In this case, the care provided by each child is given by

\[
g_1 = 0 \quad \text{and} \quad g_2 = \frac{y_2}{2(1 + t_2)}.
\]  

(24)

Substituting (24) into the utility function of child 2, we have

\[
U_2 = \frac{y_2^2}{4(1 + t_2)}.
\]

From the utility maximization of child 2 with respect to \( t_2 \), he chooses \( t_2 = 0 \). Using (24) and \( t_2 = 0 \), the utility of child 1 in the first stage is given by

\[
U_1 = \frac{y_1y_2}{2}.
\]

The utility of child 1 does not depend on \( t_1 \), and thus he is indifferent about his location in \([0,1]\). Denoting the location of child 1 as \( \tilde{t}_1 \in [0,1] \), we have the following result.

**Proposition 4.** If \( \sqrt{2} < y_2/y_1 \), then \( t_1 = \tilde{t}_1 \), \( t_2 = 0 \), \( g_1 = 0 \), and \( g_2 = y_2/2 \).

The equilibrium pattern is depicted in Figure 1. When the income differential between the
two siblings is so large that it satisfies $y_2/y_1 \leq \sqrt{2}/2$ or $y_2/y_1 \geq \sqrt{2}$, and one of the two cares for their parents and the other free-rides, then child 1 cannot use his location as a strategic variable with which to control the location of his younger brother. In this case, the child who has the larger income lives with his parents and cares for them and the other free-rides. When the income differential between the two siblings is sufficiently small that it leads both children to care for their parents, child 1 uses his choice of location to induce his younger brother to choose the location he desires. If child 1 has the larger income compared with child 2, namely $y_2/y_1 < 1$, he recognizes that his younger brother provides less care. To make him provide more care, child 1 allows himself to live far from the parents, which makes him care for his parents less. This action leads the younger brother to care for their parents more, since they are in a situation of strategic substitution in terms of caregiving. Since the younger brother cares for their parents more, he lives with them. A similar argument explains the location pattern when $y_2/y_1 > 1$, in which child 1 lives with his parents.

Recalling our original question on the different care arrangements in Western and Eastern countries, one may raise the degree of remaining primogeniture in these cultures as a possible explanation. Compared with East Asian and some developing countries, Western nations have long abolished primogeniture. However, in East Asian countries, for example, such a custom guarantees the eldest child a sufficiently high income, leading the economy toward the left range in Figure 1. On the contrary, the economies in Western countries shift toward the right range of Figure 1. Although this interpretation seems to be similar to the strategic bequest motive, implying a positive relationship between the care provided by child 1 and the bequest he receives, the mechanism behind this result is different. In the context of the strategic bequest motive, the relation is obtained as a result of caregiving competition between the two children. However, in our model, the interpretation of this positive relationship is that the older sibling cares for his parents owing to his altruistic behavior, knowing the certain rule for his privileges as a first-born child.

4. Comparative statics
In this section, we consider how the change in the relative incomes of child 1 and child 2 influences overall care, $G$. For the analysis presented here, we assume that $y_1 = y$ and $y_2 = \beta y$, meaning that a change in $\beta$ is regarded as a change in the aggregate income of both children keeping child 1’s income constant.

We now redefine four regimes according to the level of $\beta$, namely Regime 1 ($\beta < \sqrt{2}/2$), Regime 2 ($\sqrt{2}/2 < \beta < 1$), Regime 3 ($1 < \beta < \sqrt{2}$), and Regime 4 ($\sqrt{2} < \beta$).

The care provided $G^j$ in each regime ($j = 1, 2, 3, 4$) is given by
\[ G^1 = \frac{y_1}{2} = \frac{y}{2} \]
\[ G^2 = \frac{y_1 + 2y_2}{6} = \frac{(1 + 2\beta)y}{6} \]
\[ G^3 = \frac{2y_1 + y_2}{6} = \frac{(2 + \beta)y}{6} \]
\[ G^4 = \frac{y_2}{2} = \frac{\beta y}{2} \]

From the equations above, it is clear that an increase in \( \beta \) leads to a rise in \( G^j \) except in Regime 1. Hence, to check the degree of continuity, we compare the level of \( G^j \) at the threshold of \( \beta \). The threshold of \( \beta \) between Regime 1 and Regime 2 is \( \sqrt{2}/2 \). By substituting \( \sqrt{2}/2 \) into \( \beta \), we have
\[ \frac{y}{2} = G^1 > G^2 = \frac{(1 + \sqrt{2})y}{6}. \]
The overall care is thus discontinuous between Regime 1 and Regime 2 (i.e., at the threshold of \( \beta = \sqrt{2}/2 \)). Note that although the aggregate income of the two siblings is higher in Regime 2 than it is in Regime 1, the level of \( G^2 \) drops around the threshold \( \beta = \sqrt{2}/2 \).

Next, the threshold of \( \beta \) between Regime 2 and Regime 3 is 1. Again, by substituting 1 into \( \beta \), we obtain
\[ G^2 = G^3 = \frac{y}{2}. \]
Now, overall care is continuous between Regime 2 and Regime 3 (i.e., at the threshold of \( \beta = 1 \)).

Finally, the threshold of \( \beta \) between Regime 3 and Regime 4 is \( \sqrt{2} \), meaning that substituting \( \sqrt{2} \) into \( \beta \) leads to
Again, overall care is discontinuous between Regime 3 and Regime 4 (i.e., at the threshold of $\beta = \sqrt{2}$).

The argument above is summarized in Figure 2. To interpret the effects of an increase in child 2’s income on the total amount of contribution, consider first the regime 1 in which the income of child 2 is sufficiently small. When $\beta < \sqrt{2}/2$, the income of child 2 is so small that he does not take care of his parents, $g_2 = 0$, and takes free ride on the care provided by child 1. In this case, child 1 lives with his parents, $t_1 = 0$, and chooses $g_1 = y_1/2$. In this context, the increase in child 2’s income, represented by an increase in $\beta$, change neither locational pattern nor contribution level. Once $\beta$ exceeds $\sqrt{2}/2$, however, child 2 now takes care of his parents. Knowing child 2’s incentives to take care of his parents, child 1 chooses $t_1 = 1$ in the first stage to make child 2 to involve more to take care of their parents. Although an increase in child 2’s income makes him choose positive amount of care, it reduces the contribution of child 1 through two channels. First, an increase in $g_2$ gives incentives for child 1 to take free-ride on child 2’s contribution, and that he reduces his contribution. Second, since child 1 chooses his location at $t_1 = 1$ to make his younger brother to contribute more, the cost of care for child 1 increases, and thereby he reduces his contribution further. At the immediate vicinity of $\beta = \sqrt{2}/2$ in regime 2, the positive effects of an increase in child 2’s income on child 2’s contribution is outweighed by the negative effects on child 1’s contribution, and thus the total amount of private contribution decreases. As $\beta$ further increases, the positive effects of an increase in child 2’s income tend to be strongly effective, and the total amount of contribution increases.

Once $\beta$ exceeds 1, child 2 now chooses $t_2 = 1$. This location choice partly offsets the positive effects of an increase in child 2’s income since it increases the cost of care, and hence the effects on child 2’s contribution of a switch from regime 2 to regime 3 are ambiguous. However, the regime switch impacts positively on child 1’s contribution because child 1 now chooses to live with his parents, $t_1 = 0$. This leads him to increase his contribution, and thus the total amount of contribution increases and that it exceeds the size of contribution under regime 1. Finally, when $\beta = \sqrt{2}$, child 1 takes free-ride on child 2’s
contribution by choosing $g_1 = 0$. This leads child 2 to choose $t_2 = 0$, which reduces the cost of care. The reduction in cost leads child 2 to contribute more to take care of his parents, and that the total amount of care increases.

To summary, the discontinuities between Regimes 1 and 2 and between Regimes 3 and 4 are explained by two elements: the strategic incentives between the two siblings and the changes in location choices of them. When the income gap between them is sufficiently large, the problem of providing elderly care substantially becomes that held by only one child (Konrad et al., 2002). In this case, overall care is solely determined by the income of the rich sibling living near from his parents, without being affected by strategic interactions between the siblings. On the contrary, when the income gap is sufficiently small, there exists an incentive for one child to free-ride on the other in terms of caregiving by living away from the parents. In this case, in addition to the different location choices (i.e., marginal cost of caregiving) in each regime, the level of care for both children tends to be below that held by either of them. Therefore, because of changes in location choices and of this externality, although aggregate income is increased from Regime 1 to Regime 2 (when fixing child 1’s income), we find that overall care temporarily decreases.

5. Discussion

We here present the analysis to study how the location patterns and the provision of care are changed when children cooperate in the second stage. Since the provision of care for parents has the property of public goods, the total amount of care for parents tends to be inefficiently low. This creates the scope for cooperation between siblings to provide care to parents.

Assume that, given their location determined in the first stage, the sibling cooperate to maximize their joint utilities when they provide care to their parents. The maximization problem is given by

$$\max_{g_1, g_2} U_1 + U_2$$

Solving the problem, we have

$$g_1 = \frac{2 + t_1 + t_2}{2(1 + t_1)} g_2 + \frac{y_1 + y_2}{2(1 + t_1)}$$

$$g_2 = \frac{2 + t_1 + t_2}{2(1 + t_2)} g_1 + \frac{y_1 + y_2}{2(1 + t_2)}$$

From these equations, it is easy to find that the joint utility maximization gives corner solutions:

$$t_1 > t_2 \rightarrow g_1 = 0, g_2 = \frac{y_1 + y_2}{2(1 + t_2)}$$
\[ t_1 < t_2 \rightarrow g_1 = \frac{y_1 + y_2}{2(1 + t_1)}, g_2 = 0 \]

Anticipating these outcomes in the second stage, the children determine their location non-cooperatively. As is the case with the previous section, the elder brother moves first and the younger brother follows. The equilibrium location patterns are obtained as follows:

\[ t_1 = \tilde{t}_1 > 0, t_2 = 0 \quad \text{when} \quad y_1 < y_2 \]

\[ t_1 = 0, t_2 = \tilde{t}_2 > 0 \quad \text{when} \quad y_1 > y_2 \]

The total amount of care provided to their parent is given by \( g_1 + g_2 = (y_1 + y_2)/2 \), implying that the cooperation among siblings increase the amount of care to the parents. The analysis reveals that the patterns to provide care and location patterns are slightly changed. When the siblings do not cooperate in providing care, there are four patterns for equilibrium; regime 1-4. However, when they do cooperate, only one child with higher income provides the care, and the regime 2 and 3 disappear.

6. Conclusion

In this study, we investigated the location choices and parents’ care arrangements of two siblings, taking account of their income differential. Specifically, we formulated a model in which their caregiving decisions are influenced by their relative incomes as well as the distance they live away from their parents (i.e., the marginal caregiving cost), in line with the approach taken by Konrad et al. (2002). By using this generalized model containing income effects, we then examined three cases of care arrangements given these income differences. First, when the income gap is sufficiently small, both children participate in caregiving. In this case, there exists a strategic incentive to live far away from the parents. This decision is made because relative distance is a determinant of the care each child needs to provide, and thus the older child can utilize his first-mover advantage (this case essentially corresponds to the result presented by Konrad et al. (2002)). When the income differential is sufficiently large, the highest-earning child (either the oldest or youngest) takes responsibility for caring for his parents irrespective of the other’s location choice. This novel result makes a unique contribution to the body of knowledge on this topic, and it partially explains the different care arrangements seen in Western and Eastern countries. Finally, we also investigated how changes in the relative incomes of the two siblings affect the overall care received. We show that overall care increases as aggregate income rises, but at some point this may reduce because of the strategic incentive for one child to free-ride on the other.

Before closing this study, some limitations should be mentioned. First, we analyzed siblings’ behaviors by treating income as exogenous. However, future works should aim to endogenize income by including former decisions such as educational and location choices. Second, distinguishing labor into labor and non-labor income may enable a rich description of adult children’s decisions taking account of the price effect coming from the opportunity cost of caregiving as in Byrne et al. (2009) and Antman (2012). Third, we specify the utility function to obtain analytical results. It is no wonder that our qualitative results still hold in
appropriate range of preference parameters, but the results might be affected quantitatively if child has extreme preferences. Finally, considering the vast literature on care arrangements, future studies could introduce a new cross-effect of incomes into the empirical analysis. Although some studies have investigated how one's education level influences the other's caregiving decisions for the elderly parents (Fontaine et al., 2009), few authors have considered income itself as the element of cross-effect. By taking account of income effects as one possible scenario, the empirical test of our model may be interesting for comparing the customs in Western and Eastern countries.
References
Rainer, H. & Siedler, T (2009), O brother, where art thou? The effects of having a sibling on geographic mobility and labour market outcomes, Economica, 76(303), 528-556.
Figure 1. Equilibrium pattern

Note $\bar{t}_i \in [0,1]$.

Figure 2. Overall care provided by both children ($y_1 = y$ and $y_2 = \beta y$)